# Wave mode separation for a surface with slope using the free-surface response

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## ABSTRACT

A method for wave mode separation for multicomponent receivers on a sloping surface is presented in this work. It takes into account the free surface effect and assumes that receivers are placed vertically on the earth's surface. The method was tested with synthetic data, showing promising results.

## INTRODUCTION

Multicomponent receivers have orthogonal components, which record P and S- wave modes. These wave-modes can be mixed on the field records, depending on the relationship between the wave polarization direction and the component direction. The separation of these two wave-modes makes it easier to handle and process each one of them, so it can be considered an important first step in analyzing these data.

If the data are recorded on the earth's surface, the free surface has an effect on both wavefield phase and amplitude. Methods for wave-mode separation taking into account the free surface effect have been proposed by some authors, such as Dankbaar (1985), assuming a horizontal free surface, with receivers normal to it. The free surface effect is defined as a function of the angle of incidence and the elastic properties at that location, and the incident wavefield is decomposed into plane waves corresponding to different angles of incidence to apply these properties.

However, if the receivers are placed vertically and there is a slope, as in the case of rough topography, this method requires additional considerations. As mentioned by some authors (e.g. Ronen et al., 2005), the multicomponent method could contribute to improvement of the seismic data in rough terrain, and multicomponent receivers can improve P- and S-wave data. Also, Guevara et al. (2006), from ray trace modeling over a rough topography location with a low velocity near surface layer, noticed that the waves arrive almost normal to the surface, that is to say, with in a different direction to the vertical and horizontal receivers; thus in this case, wave mode separation would be profitable even for the normal incidence case.

This work extends the method to the case of a free surface with slope. The relationship between the horizontal and the sloping free surface is derived, and the free surface effect and plane wave decomposition are applied to obtain separated wave fields.

## THEORY

Dankbaar (1985) proposed a method for wave mode separation taking into account the free surface effect. The response characteristics of the vertical and horizontal receivers are defined as a function of the velocities and horizontal slowness (ratio between angle of incidence and velocity), by the following equations:

$$R_z^P(p) = 2T^{-1} * E * (2V_S^2p^2 - 1)/R_0(p)$$

$$R_z^S(p) = 4V_Sp * E * N/R_0(p)$$

$$R_x^P(p) = 4V_Pp * E * N/R_0(p)$$

$$R_x^S(p) = 2 * N * (1 - 2V_S^2p^2)/R_0(p)$$

where the $R$ subscripts indicate the direction and the superscripts the wave mode, $V_P$ and $V_S$ refer to P- and S-wave velocities, $p$ to the horizontal slowness, and $T = V_S/V_P$, $E = (T^2 - V_S^2p^2)^{1/2}$, $N = (1 - V_S^2p^2)^{1/2}$, and $R_0(p) = (1 - 2V_S^2p^2)^2 + 4p^2V_S^2 * E * N$.

Thus, the response of the vertical and horizontal receivers can be defined as:

$$U_z = P_{in}R_z^P + S_{in}R_z^S$$

and

$$U_x = P_{in}R_x^P + S_{in}R_x^S$$

where $P_{in}$ is the incident P-wave and $S_{in}$ is the incident S-wave.

According to these last equations, it would be possible to obtain the incident P- and S–wave fields by an inversion method, if we have the vertical and horizontal components (recorded in the field), the P and S-wave velocities, and the horizontal slowness. To this purpose Dankbaar (1985) applies a Fourier approach, and Donati (1996) uses the Radon Transform as a function of the slowness (τ-p transform); both methods are angle decompositions of the wavefield.

The sloping surface case is illustrated in Fig. 1 for an incident P-wave. It is required to define two orthogonal coordinate axes for analysis purposes, one corresponding to the vertical and horizontal directions ($x_0$-$z_0$) and the other one to the directions parallel and normal to the surface ($x_\xi$-$z_\xi$) as illustrated in Figure 1, assuming that the slope has an angle $\boldsymbol{\xi}$. $P_I$ corresponds to the incident P –wave, $P_R$ to the reflected P-wave, and $S_R$ to the reflected S-wave.

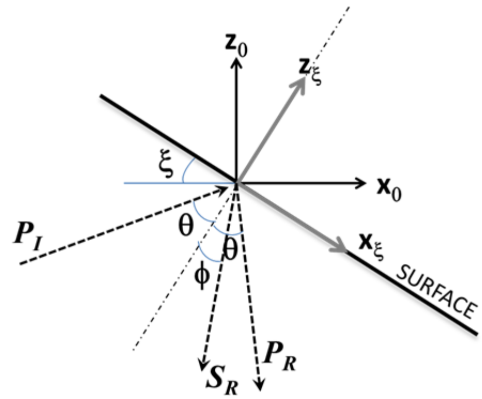


FIG.. 1. Analysis of P-wave incidence on the sloping surface. Two coordinate axes are defined, $x_0$-$z_0$ corresponding to the horizontal and vertical directions, and $x_\xi$-$z_\xi$ to the directions parallel and normal to the surface. $P_I$ corresponds to the incident P–wave, $P_R$ to the reflected P-wave, and $S_R$ to the reflected S-wave.

Then the relations between the recorded components in the vertical and horizontal axes and the components in the axes normal and parallel to the surface are:

$$U_{z\xi} = U_{z0} * \cos \boldsymbol{\xi} - U_{x0} * \sin \boldsymbol{\xi}$$

and

$$U_{x\xi} = U_{x0} * \cos \boldsymbol{\xi} + U_{z0} * \sin \boldsymbol{\xi}$$

These equations allow us to rotate the data recorded in the vertical and horizontal directions, and then apply the equations corresponding to the directions parallel and normal to the surface, as proposed by Dankbaar (1985) and Donati (1996).

## EXAMPLES

A simple geological model with sloping surface allowed us to test this method. This model is illustrated in Figure 2. Ray tracing was used to generate the synthetic data, with the software Norsar-2D.

Two case examples are presented in the following. The first one corresponds to the receiver located at x=1000 m (see Fig. 2). Figure 3 illustrates the resulting vertical and horizontal component gathers for 100 shots. Figure 4 illustrates the resulting P- and S-wave fields after wave mode separation using the method described in the theory section.

The second example corresponds to the receiver located at x=500. Figure 5 illustrates the resulting vertical and horizontal component gathers also for 100 shots. Figure 6 illustrates the resulting P- and S- wave fields after wave mode separation. As expected in both cases the S-wave arrives later and shows expected features, such as low energy at short offsets and reversal of polarity in both directions.

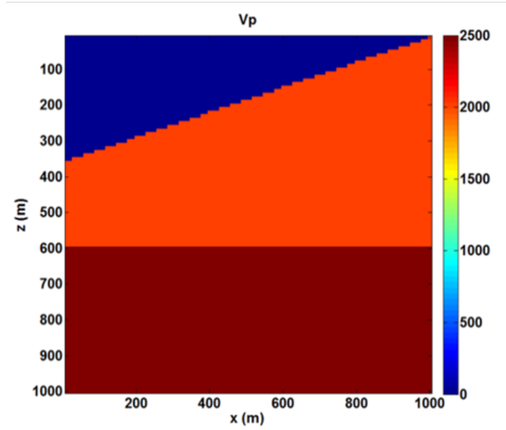


FIG. 2. Geological model to test the method, illustrated by the P-wave velocity. Sources and receivers are located on the sloping surface.

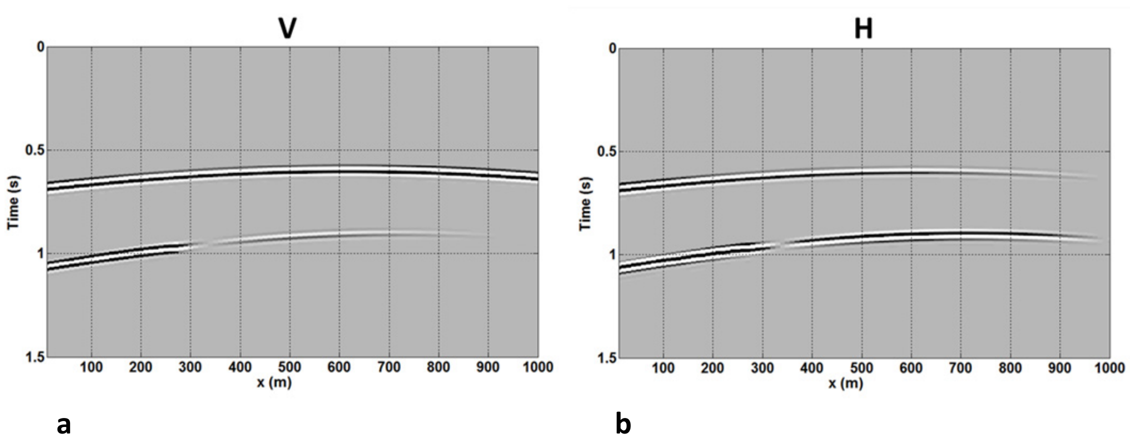


FIG. 3. Common receiver gathers at the receiver located in x=1000.

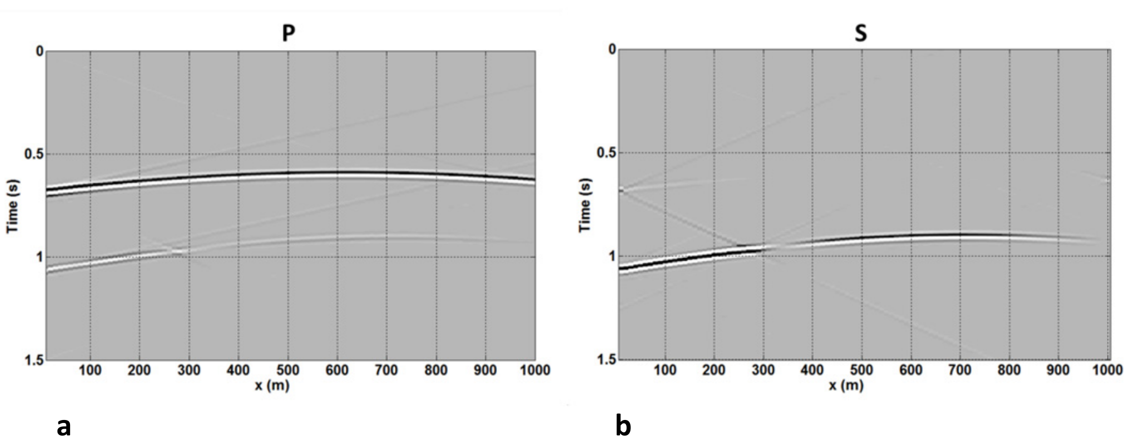


FIG. 4. P- and S- wave fields after the wave mode separation for the receiver at x=1000 m..

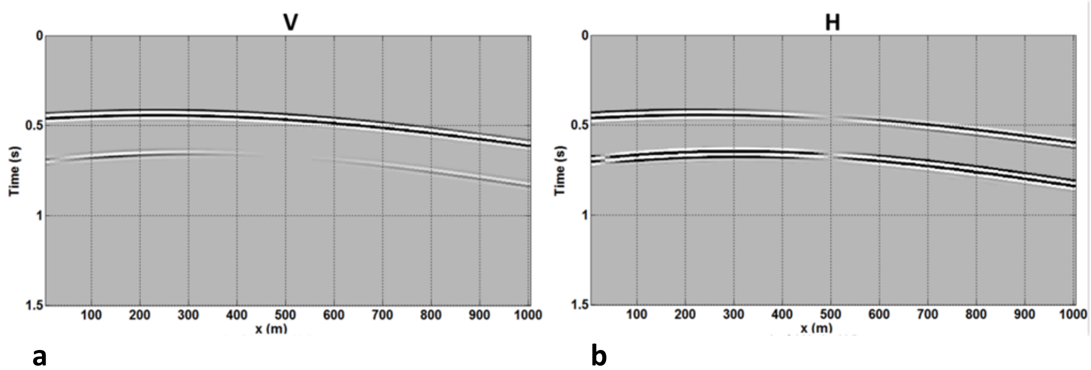


FIG. 5. Common receiver gathers at the receiver location x=500 m.

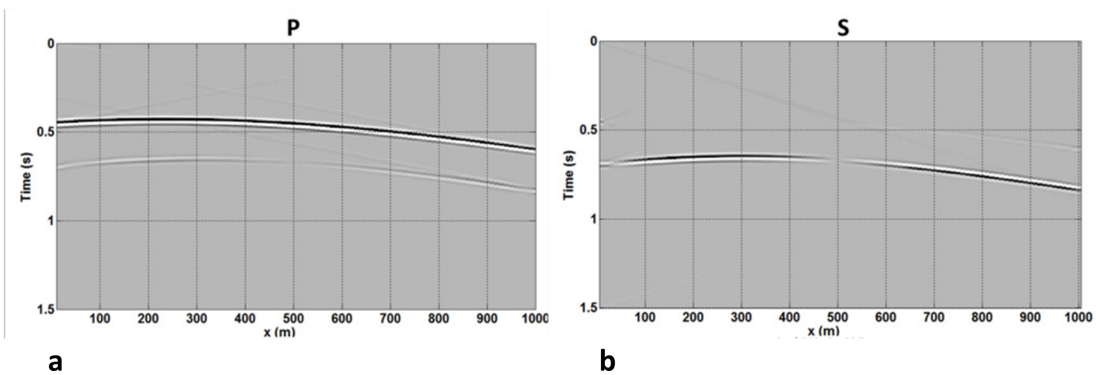


FIG. 6. P and S-wave after wave mode separation for the receiver at x=500 m.

## CONCLUSIONS

A method for wave mode separation for a sloping surface has been proposed, taking into account the free surface response, and based on the approach of Dankbaar (1985).

The method works in the common receiver gather domain, and the near surface velocities for P and S-waves at this location are required. Vertical deployment of receivers is assumed.

The examples, from a simple model and with data generated by using ray tracing, show expected characteristics for such a wave mode separation method.

A possible application of this method is the improvement of P and S-wave data in rough terrain.

## ACKNOWLEDGEMENTS

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