Direct nonlinear inversion of viscoacoustic media using the inverse scattering series

Ali Fathalian and Kris Innanen

ABSTRACT

The objective of seismic exploration is obtaining structural subsurface information from seismic data by recording seismic waves motion of the ground. The recorded data have a non-linear relationship with the property changes across a reflector. In this work, the multi-parameter multi-dimensional direct non- linear inversion is investigated based on the inverse scattering task-specific sub-series. The result is direct and non-linear and has the potential to provide more accurate and reliable earth property predictions for larger contrast and more complex.The inverse scattering method has a direct response for imaging and inversion problems for a large contrast and a multi-dimensional corrugated target. We are derived the direct non-linear inversion equation for three parameter viscoacoustic cases. Numerical tests show that non-linear inversion results provide improved estimates in comparison with the standard linear inversion. When the non-linear term add to linear term the recovered value of parameters are much closer to the exact value..

INTRODUCTION

The objective of seismic exploration is obtaining structural subsurface information from seismic data by recording seismic waves motion of the ground. In fact, the main reason is to find hydrocarbon reservoirs in the earth. The wave motion is excitated by the seismic source that is located in the land (onshore) or in the marine (offshore) environments. The seismic source generates seismic waves, and the reflected wavefiled is measured by receivers that are located along lines (2D seismic) or on a grid (3D seismic). The seismic measurement is dependent to the seismic source environment. In the land (onshore) survey, there are strong noisy events, which are referred to surface waves that propagate along the surface. The reflection events are hidden by this strong noises. In the marine (offshore) survey shot record is more clean because of shear waves cannot propagate through water, and are therefore not measured for marine data. However, the structural map of the earth (imaging) and the mechanical properties of the target(inversion) are estimated by analysis of recorded data.There are many methods for considering subsurface information from seismic data that the data consist exclusively of primaries, means all other seismic events are considered as noise and removed(Carvalho, 1992; Verschuur and Wapenaar, 1992; Weglein et al., 1997; Matson, 1997; Weglein, 1999; Weglein and Dragoset, 2005). These methods for processing primaries can give useful results while ,under some circumstances (location structure beneath rapidly multi-D heterogeneity within layers(imaging) and the mechanical property of large contrast changes at a 1D or multi-D(inversion)), may become ineffective. The inverse scattering series is a direct multi-D inversion method that can perform the tasks associated with multiple removal, imaging and inversion(Weglein and Dragoset, 2005; Weglein, 2006). The inverse scattering method has a direct response for imaging and inversion problems for a large contrast and a multi-D corrugated target.The advantages of this method are involves explicit algorithms which directly provide improved estimates for medium properties without recourse to highly non-linear optimization procedures and determines data requirements for non-linear direct parameter estimation. The inversion method is direct and non-linear and has the potential to provide more accurate and reliable earth property predictions for larger contrast and more complex(Weglein et al., 2003).

The inverse scattering series

This section follow the derivation of Weglain et al.(2003)(Jost and Kohn, 1952). The basic wave equatins govering the wave propagation in the reference and actual medium are(Matson, 1997)

$$Lu = f \tag{1}$$

$$L_0 u = f \tag{2}$$

$$LG = \delta \tag{3}$$

$$L_0 G_0 = \delta \tag{4}$$

where L and L_0 are the actual and reference wave propagators, u is the displacement, f is the source term and, G and G_0 corresponding Green's operators for actual and reference mediums.

The perturbation operator, V(the difference between the reference and actual medium wave operators) and the scattered filed operator, ψ_s(the difference between the reference and actual medium Green'soperators) are defined as

$$V = L_0 - L \tag{5}$$

$$\psi_s = G - G_0 \tag{6}$$

The Lippmann-Schwinger equation is an operator identity:

$$\psi_s = G - G_0 = G_0 V G \tag{7}$$

By using the Born series, the scattered field can be expanded in an infinite series through self-substitution.

$$\begin{aligned} \psi_s = G - G_0 = G_0 V G \\ = G_0 V (G_0 + G_0 V G) \\ = G_0 V G_0 + G_0 V G_0 V G \\ = G_0 V G_0 + G_0 V G_0 V (G_0 + G_0 V G) \\ = G_0 V G_0 + G_0 V G_0 V G_0 + G_0 V G_0 V G_0 V G + ... \\ = (\psi_s)_1 + (\psi_s)_2 + (\psi_s)_3 + ... \end{aligned} \tag{8}$$

The measured values of the scattered wave filed is the data:

$$D = (\psi_s)_{measurement} \tag{9}$$

In the inverse scattering series, expanding V as an infinite series of data(Razavy, 1975; Stolt and Jacobs, 1981; Weglein et al., 1981; Innanen, 2004):

$$V = V_1 + V_2 + V_3 ...... \tag{10}$$

This form is substituted into the terms of the Born series, and terms of like order in the data are equated. The inverse scattering series form is :

$$\begin{aligned} D = (\psi_s)_m &= (G_0 V_1 G_0)_m \\ 0 &= (G_0 V_2 G_0)_m + (G_0 V_1 G_0 V_1 G_0)_m \\ &\ldots \end{aligned} \tag{11}$$

This series is a multi-D inversion procedure that directly determines physical properties using only reference medium information and reflection data. The perturbation$V$ can be calculated order by order in the data when the Green's operator for reference medium G_0 and the measured scattered wavefield D are known.This process is called inverse problem. Meanwhile, in the forward problem, G_0 and V are known and the forward series determin the total G. targets.

THREE PARAMETERS VISCOACOUSTIC INVERSION

Viscoacoustic Scattering Potentials

In this section, we will consider a 1D viscoacoustic three parameter earth model . We start with the 3D viscoacoustic wave equations in the actual and reference medium (Jost and Kohn, 1952; Razavy, 1975; Stolt and Jacobs, 1981)

$$[K_0^2 + \nabla.\frac{1}{\rho_0(r)}\nabla]G_0(r, r_s; w) = \delta(r - r_s) \tag{12}$$

$$[K^2 + \nabla.\frac{1}{\rho(r)}\nabla]G(r, r_s; w) = \delta(r - r_s) \tag{13}$$

where $G(r, r_s; \omega)$ and $G_0(r, r_s; \omega)$ are respectively the free-space causal Green's functions that describe wave propagation in the actual and reference medium. We consider three variants on the viscoacoustic case, each utilizing wavenumbers which permit attenuation to be modelled in addition to acoustic behaviour. This requires moving away from the acoustic $K_0^2 = \omega/\rho c_0^2$, and adopting for the true medium(Jost and Kohn, 1952; Razavy, 1975; Stolt and Jacobs, 1981; Innanen and Lira, 2010):

$$K(z)^2 = \frac{w^2}{\rho c_0^2}[1 + \frac{i}{2Q(z)} - \frac{1}{\pi Q(z)}\ln(\frac{k}{k_r})]^2. \tag{14}$$

where $k_r = \omega_r/c_0$ is a reference wavenumber, and $k = \omega/c_0$. This form is re-writeable using an attenuation parameter $\zeta(z) = 1/Q(z)$ multiplied by a function F(k), of known form:

$$F(k) = \frac{i}{2} - \frac{1}{\pi}\ln(\frac{k}{k_r}) \tag{15}$$

which utilizes $\zeta(z)$ to correctly instill both the attenuation $(i/2)$ and dispersion $(-\frac{1}{\pi}\ln(\frac{k}{k_r})$. Notice that F(k) is frequency-dependent because of the dispersion term. Then

$$K(z)^2 = \frac{w^2}{\rho c_0^2}[1 + \zeta(z)F(k)]^2. \tag{16}$$

The linearized Born inversion is based on a choice for the form of the scattering potential V (the difference between the reference and actual medium wave operators)), which is given by

$$V = L_0 - L \tag{17}$$

For a homogeneous reference medium and 1D (Zhang and Weglein, 2009) case this amounts to

$$\begin{aligned} V(z,\nabla) &= K_0^2 - K(z)^2 + \nabla.(\frac{1}{\rho_0(r)} - \frac{1}{\rho(r)})\nabla \\ &\cong \frac{\omega^2}{\rho_0 c_0^2}[\alpha(z) - 2\zeta(z)F(k)] \\ &+\frac{1}{\rho_0}\beta(z)\frac{\partial^2}{\partial x^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\beta(z)\frac{\partial}{\partial z} \end{aligned} \tag{18}$$

$V(z,\nabla)$,$\alpha(z)$, $\beta(z)$ and $\zeta(z)$ can be expanded respectively as

$$V(z,\nabla) = V(z,\nabla)_1 + V(z,\nabla)_2 + V(z,\nabla)_3 + ... \tag{19}$$

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3 + ... \tag{20}$$

$$\beta = \beta_1 + \beta_2 + \beta_3 + ... \tag{21}$$

$$\zeta = \zeta_1 + \eta_2 + \zeta_3 + ... \tag{22}$$

Then we have

$$\begin{aligned} V_1 &\cong \frac{\omega^2}{\rho_0 c_0^2}[\alpha_1(z) - 2\zeta_1(z)F(k)] \\ &+\frac{1}{\rho_0}\beta_1(z)\frac{\partial^2}{\partial x^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\beta_1(z)\frac{\partial}{\partial z} \end{aligned} \tag{23}$$

and

$$\begin{aligned} V_2 &\cong \frac{\omega^2}{\rho_0 c_0^2}[\alpha_2(z) - 2\zeta_2(z)F(k)] \\ &+\frac{1}{\rho_0}\beta_2(z)\frac{\partial^2}{\partial x^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\beta_2(z)\frac{\partial}{\partial z} \\ &.... \end{aligned} \tag{24}$$

THE LINEAR TERM OF THE INVERSE SCATTERING SERIES

The estimation of the 1D contrast (i.e. in depth) of multiple parameters from seismic reflection data is considered.From the first equation of the inverse scattering series, Eq. (1.11), we have

$$D = G_0 V_1 G_0 \tag{25}$$

where V_1 is defined as

$$
\begin{aligned}
V_1 &\cong \frac{1}{\rho_0}[\frac{\omega^2}{c_0^2}[\alpha_1(z) - 2\zeta_1(z)F(k)] \\
&\quad +\beta_1(z)\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial z}\beta_1(z)\frac{\partial}{\partial z}] \\
&= \frac{1}{\rho_0}[k^2\alpha_1(z) - 2k^2F(k)\zeta_1(z) \\
&\quad +\beta_1(z)\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial z}\beta_1(z)\frac{\partial}{\partial z}]
\end{aligned} \tag{26}
$$

in space domain, for 1D media and 2D experiment, eq.25 can be written as

$$
\begin{aligned}
D(x_g, z_g; x_s, z_s; \omega) = \frac{1}{\rho_0}\int_{-\infty}^{\infty} dx^{'} \int_{-\infty}^{\infty} dz^{'} G_0(x_g, z_g; x^{'}, z^{'}; \omega) \\
\times[k^2\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'}) + \beta_1(z^{'})\frac{\partial^2}{\partial x'^2} + \frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}] \\
\times G_0(x^{'}, z^{'}; x_s, z_s; \omega)
\end{aligned} \tag{27}
$$

where x_g, z_g and x_s, z_s are respectively the positions of the receiver and source.The function G_0 describes propagation in the acoustic reference medium, and can be written as a 2D Green's function in bilinear form:

$$
G_0(x_g, z_g; x^{'}, z^{'}; \omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_x^{'} \int_{-\infty}^{\infty} dk_z^{'} \frac{e^{ik_x^{'}(x_g - x^{'})}e^{ik_z^{'}(z_g - z^{'})}}{k^2 - k_x'^2 - k_z'^2} \tag{28}
$$

$$
G_0(x\prime, z\prime; x^s, z^s; \omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_x^{''} \int_{-\infty}^{\infty} dk_z^{''} \frac{e^{ik_x^{''}(x\prime - x^s)}e^{ik_z^{''}(z\prime - z^s)}}{k^2 - k_x''^2 - k_z''^2} \tag{29}
$$

After the Fourier transformation over x_g and x_s on both sides, the left side of equation 26 can be written as

$$
D(k_g, z_g; -k^s, z^s; \omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dx^g \int_{-\infty}^{\infty} dx^s e^{-ik_g x_g} D(x_g, z_g; x_s, z_s; \omega) \times e^{ik_s x_s}
$$

and the right side become

$$
\begin{aligned}
\frac{1}{\rho_0}\int_{-\infty}^{\infty} dx^{'} \int_{-\infty}^{\infty} dz^{'} G_0(k_g, z_g; x^{'}, z^{'}; \omega)[k^2\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'}) \\
-k_s^2\beta_1(z^{'}) + \frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z}^{'}]G_0(x^{'}, z^{'}; -k_s, z_s; \omega)
\end{aligned} \tag{30}
$$

Measurements over a range of x_g will permit a Fourier transform to the coordinate k_g

in the scattered wavefield. So eq.1 can be written as

$$G_0(k_g, z_g; x^{'}, z^{'}; \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_x^{'} \int_{-\infty}^{\infty} dk_z^{'} \int_{-\infty}^{\infty} dx_g \frac{e^{i(k_g - k_m^{'})x_g} e^{-ik_x^{'} x^{'}} e^{ik_z^{'}(z_g - z^{'})}}{k^2 - k_x^{'2} - k_z^{'2}} \tag{31}$$
$$= \rho_0 \exp(-ik_g x^{'}) \frac{\exp(iq_g(z^{'} - z_g))}{4\pi i q_g}$$

and measurements over a range of x_s will permit a Fourier transform to the coordinate k_s in the scattered wavefield. After the Fourier transformation eq.2 become

$$G_0(x^{'}, z^{'}; -k_s, z_s; \omega) = \rho_0 \exp(ik_s x^{'}) \frac{\exp(iq_s(z^{'} - z_s))}{4\pi i q_s} \tag{32}$$

where $q_g^2 = k^2 - k_g^2$ and $q_s^2 = k^2 - k_s^2$.

Substitute eq. 31 and eq.32 into eq.30. We have

$$D(k_g, z_g; -ks_s, z_s; \omega) = -\frac{\rho_0}{4q_g q_s} (\frac{1}{2\pi})^2 e^{-i(q_g z_g + q_s z_s)} \int_{-\infty}^{\infty} dx^{'} \int_{-\infty}^{\infty} dz^{'} \tag{33}$$
$$\times e^{-i(k_g - k_s)x^{'}} [k^2 \alpha_1(z^{'}) - 2k^2 F(k) \zeta_1(z^{'}) - k_s^2 \beta_1(z^{'})] e^{i(q_g + q_s)z^{'}}$$
$$-\frac{\rho_0}{4q_g q_s} (\frac{1}{2\pi})^2 e^{-i(q_g z_g + q_s z_s)} \int_{-\infty}^{\infty} dx^{'} \int_{-\infty}^{\infty} dz^{'} e^{-i(k_g - k_s)x^{'}}$$
$$e^{iq_g z^{'}} \frac{\partial}{\partial z^{'}} \beta_1(z^{'}) \frac{\partial}{\partial z^{'}} e^{iq_s z^{'}}$$

After partial integration, where

$$\int_{-\infty}^{\infty} dz^{'} e^{iq_g z^{'}} \frac{\partial}{\partial z^{'}} \beta_1(z^{'}) \frac{\partial}{\partial z^{'}} e^{iq_s z^{'}} = q_g q_s \int_{-\infty}^{\infty} dz^{'} \beta_1(z^{'}) e^{i(q_g + q_s)z^{'}} \tag{34}$$

then eq.33 can be written as

$$D(k_g, z_g; -ks_s, z_s; \omega) = -\frac{\rho_0}{4q_g q_s} (\frac{1}{2\pi})^2 e^{-i(q_g z_g + q_s z_s)} \int_{-\infty}^{\infty} dx^{'} \int_{-\infty}^{\infty} dz^{'} \tag{35}$$
$$\times e^{-i(k_g - k_s)x^{'}} [k^2 \alpha_1(z^{'}) - 2k^2 F(k) \zeta_1(z^{'}) - k_g^2 \beta_1(z^{'}) + q_g q_s \beta_1(z^{'})] e^{i(q_g + q_s)z^{'}}$$

Then, we have

$$D(k_g, z_g; -ks_s, z_s; \omega) = -\frac{\rho_0}{4} e^{-iq_g(z_g + z_s)} [\frac{k^2}{q_g^2} \alpha_1(-2q_g) \tag{36}$$
$$-2\frac{F(k)k^2}{q_g^2} \zeta_1(-2q_g) - \frac{k_g^2}{q_g^2} \beta_1(-2q_g) + \beta_1(-2q_g)]$$

Using the relation $q_g = q_s = k\cos\theta$ and $k_g = k_s = k\sin\theta$, Eq. (2.12) becomes

$$D(k_g, z_g; -ks_s, z_s; \omega) = -\frac{\rho_0}{4}e^{-iq_g(z_g+z_s)}[\frac{1}{\cos^2\theta}\alpha_1(-2q_g) \\ -2\frac{F(k)}{\cos^2\theta}\zeta_1(-2q_g) + (1-\tan^2\theta)\beta_1(-2q_g)] \tag{37}$$

THE NONLINEAR TERM OF THE INVERSE SCATTERING SERIES

The second term of inversion is

$$G_0V_2G_0 = -G_0V_1G_0V_1G_0 \tag{38}$$

this equation in space domain can be written as

$$\int_{-\infty}^{\infty}dx^{'}\int_{-\infty}^{\infty}dz^{'}G_0(x_g, z_g; x^{'}, z^{'}; \omega)[k^2\alpha_2(z^{'}) - 2k^2F(k)\zeta_2(z^{'}) \\ +\beta_2(z^{'})\frac{\partial^2}{\partial x'^2} + \frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}]G_0(x^{'}, z^{'}; x_s, z_s; \omega) = \\ -\int_{-\infty}^{\infty}dx^{'}\int_{-\infty}^{\infty}dz^{'}\int_{-\infty}^{\infty}dx^{''}\int_{-\infty}^{\infty}dz^{''}G_0(x_g, z_g; x^{'}, z^{'}; \omega) \\ \times[[k^2\alpha_2(z^{'}) - 2k^2F(k)\zeta_2(z^{'}) \\ +\beta_2(z^{'})\frac{\partial^2}{\partial x'^2} + \frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}]G_0(x^{'}, z^{'}; x^{''}, z^{''}; \omega) \\ \times[[k^2\alpha_2(z^{''}) - 2k^2F(k)\zeta_2(z^{''}) \\ +\beta_2(z^{''})\frac{\partial^2}{\partial x''^2} + \frac{\partial}{\partial z^{''}}\beta_1(z^{''})\frac{\partial}{\partial z^{''}}]G_0(x^{''}, z^{''}; x_s, z_s; \omega) \tag{39}$$

After the Fourier transform over x_g and x_s, this equation become

$$\int_{-\infty}^{\infty} dz^{'}[\frac{k^2}{q_g^2}\alpha_2(z^{'}) - 2\frac{F(k)k^2}{q_g^2}\zeta_2(z^{'})$$
$$+(1-\frac{k_g^2}{q_g^2})\beta_2(z^{'})]e^{2iq_g z^{'}} =$$
$$\frac{i}{2}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''}\frac{e^{iq_g z^{'}}}{q_g}$$
$$\times[k^2(\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'})$$
$$-k_g^2\beta_1(z^{'}) + \frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}]\frac{e^{iq_g|z^{'}-z^{''}|}}{q_g}$$
$$\times[k^2(\alpha_1(z^{''}) - 2k^2F(k)\zeta_1(z^{''})$$
$$-k_g^2\beta_1(z^{''}) + \frac{\partial}{\partial z^{''}}\beta_1(z^{''})\frac{\partial}{\partial z^{''}}]\frac{e^{iq_g z^{''}}}{q_g} \tag{40}$$

The right hand of this equation can be written as

$$\frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''}[k^2\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'}) - k_g^2\beta_1(z^{'})]$$
$$[k^2\alpha_1(z^{''}) - 2k^2F(k)\zeta_1(z^{''}) - k_g^2\beta_1(z^{''})]e^{iq_g(z^{'}+z^{''})}e^{iq_g|z^{'}-z^{''}|}$$
$$+\frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''}e^{iq_g z^{'}}[k^2\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'})$$
$$-k_g^2\beta_1(z^{'})]e^{iq_g|z^{'}-z^{''}|}\frac{\partial}{\partial z^{''}}\beta_1(z^{''})\frac{\partial}{\partial z^{''}}e^{iq_g z^{''}}$$
$$+\frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''}e^{iq_g z^{'}}\frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}e^{iq_g|z^{'}-z^{''}|}[k^2\alpha_1(z^{''}) - 2k^2F(k)\zeta_1(z^{''})$$
$$-k_g^2\beta_1(z^{''})]e^{iq_g z^{''}}$$
$$+\frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''}e^{iq_g z^{'}}\frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}e^{iq_g|z^{'}-z^{''}|}\frac{\partial}{\partial z^{''}}\beta_1(z^{''})\frac{\partial}{\partial z^{''}}e^{iq_g z^{''}}$$
$$= a_1 + a_2 + a_3 + a_4 \tag{41}$$

and the left side become

$$\frac{1}{\cos^2\theta}\alpha_2(z) - \frac{2F(k)}{\cos^2\theta}\zeta_2(z) + (1-\tan^2\theta)\beta_2(z) \tag{42}$$

then, after Fourier transformation over $2q_g$ and divided by π, a_1, a_2, a_3, and a_4 become(see appendix A)

$$\begin{aligned}
a_1 = -\frac{1}{2\cos^4\theta}\alpha_1^2(z) - \frac{2F(k)^2}{\cos^4\theta}\zeta_1^2(z) \quad (43)\\
-\frac{\tan^4\theta}{2}\beta_1^2(z) + \frac{2F(k)}{\cos^4\theta}\alpha_1(z)\zeta_1(z) + \frac{\tan^2\theta}{\cos^2\theta}\alpha_1(z)\beta_1(z) - \frac{2F(k)\tan^2\theta}{\cos^2\theta}\zeta_1(z)\beta_1(z)\\
+(\frac{-1}{2\cos^4\theta}\alpha_1^{'}(z) + \frac{F(k)}{\cos^4\theta}\zeta_1^{'}(z) + \frac{\tan^2\theta}{2\cos^2\theta}\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\alpha_1(z^{'})\\
+(\frac{F(k)}{\cos^4\theta}\alpha_1^{'}(z) - \frac{2F(k)^2}{\cos^4\theta}\zeta_1^{'}(z) - \frac{F(k)\tan^2\theta}{\cos^2\theta}\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\zeta_1(z^{'})\\
+(\frac{\tan^2\theta)}{2\cos^2\theta}\alpha_1^{'}(z) - \frac{F(k)\tan^2\theta)}{\cos^2\theta}\zeta_1^{'}(z) - \frac{\tan^4\theta)}{2}\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\beta_1(z^{'})
\end{aligned}$$

$$\begin{aligned}
a_2 = \frac{1}{4\cos^2\theta}\alpha_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) - \frac{1}{4\cos^2\theta}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\alpha_1(z^{'})\\
-\frac{F(k)}{2\cos^2\theta}\zeta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) + \frac{F(k)}{2\cos^2\theta}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\zeta_1(z^{'})
\end{aligned} \quad (44)$$

$$\begin{aligned}
a_3 = \frac{1}{4\cos^2\theta}\alpha_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) - \frac{1}{4\cos^2\theta}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\alpha_1(z^{'})\\
-\frac{F(k)}{2\cos^2\theta}\zeta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) + \frac{F(k)}{2\cos^2\theta}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\zeta_1(z^{'})
\end{aligned} \quad (45)$$

and

$$\begin{aligned}
a_4 = -\frac{1}{2}\beta_1^2(z) - \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) + \frac{1}{2}\beta_1^2(z) + \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'})\\
+\beta_1^2(z) - \beta_1^2(z) + \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) - \frac{1}{2}\beta_1^2(z)\\
= -\frac{1}{2}\beta_1^2(z) + \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'})
\end{aligned} \quad (46)$$

Finally, we have

$$\frac{1}{\cos^2\theta}\alpha_2(z) - \frac{2F(k)}{\cos^2\theta}\zeta_2(z) + (1-\tan^2\theta)\beta_2(z) = \tag{47}$$
$$-\frac{1}{2\cos^4\theta}\alpha_1^2(z) - \frac{2F(k)^2}{\cos^4\theta}\zeta_1^2(z) - \frac{\tan^4\theta}{2}\beta_1^2(z)$$
$$+\frac{2F(k)}{\cos^4\theta}\alpha_1(z)\zeta_1(z) + \frac{\tan^2\theta}{\cos^2\theta}\alpha_1(z)\beta_1(z) - \frac{2F(k)\tan^2\theta}{\cos^2\theta}\zeta_1(z)\beta_1(z)$$
$$+(\frac{-1}{2\cos^4\theta}\alpha_1^{'}(z) + \frac{F(k)}{\cos^4\theta}\zeta_1^{'}(z) + \frac{1}{2\cos^2\theta}(\tan^2\theta - 1)\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\alpha_1(z^{'})$$
$$+(\frac{F(k)}{\cos^4\theta}\alpha_1^{'}(z) - \frac{2F(k)^2}{\cos^4\theta}\zeta_1^{'}(z) - \frac{F(k)}{\cos^2\theta}(\tan^2\theta - 1)\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\zeta_1(z^{'})$$
$$+(\frac{1}{2\cos^2\theta}(\tan^2\theta - 1)\alpha_1^{'}(z) - \frac{F(k)}{\cos^2\theta}(\tan^2\theta - 1)\zeta_1^{'}(z) - \frac{1}{2}(\tan^4\theta - 1)\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\beta_1(z^{'})$$

NUMERIC EXAMPLES: SINGLE INTERFACE

Similar to the acoustic case , for a single interface 1D viscoacousic medium case,the analytic data is define(Stolt and Jacobs, 1981)

$$D(q_g, \theta) = \rho_0 R(\theta)\frac{e^{2iq_g a}}{4i\pi q_g} \tag{48}$$

where a is the depth of the interface. In this section, we numerically test the direct inversion approach for a specific model, $c_0 = 1500m/s$, $c_1 = 1700m/s$, $\rho_0 = 1.0g/cm^3$ and $\rho_1 = 1.2g/cm^3$. When assuming the data(D) is available, first, we can compute the linear solution for α_1, β_1, and ζ_1 from Eq. (3.27) by choosing three different frequencies k_1, k_2 and k_3. Then, substituting the solution into the Eq. (3.37)(nonlinear term) and computing the non-linear solution for α_2, β_2, and ζ_2. For this model the first order and the first order plus the second order are plotted. The actual values are defined by the green lines. Figure 3.1-3.2 show sets of recovered parameters (α, β and ζ). For the sake of illustration, frequency pairs $k_1 = k_2$, for which the inversion equations are singular, are smoothed using averages of adjacent ($k_1 \neq k_2$) results.The results show that all the second order solutions provide improvements over the linear solutions. When the non-linear term is added to linear order, the results become much closer to the corresponding exact values.

CONCLUSION

In this chapter, we consider three parameters direct non-linear inversion for viscoacoustic media. Both the linear and nonlinear processing and inversion are investigated. The numerical results indicate that all the second order solutions provide improvements over the linear solutions. When the second term is added to linear order, the results become much closer to the corresponding exact values. The inversion method is direct and nonlinear and has the potential to provide more accurate and reliable earth property predictions for larger contrast and more complex without knowing the specific properties of the target.

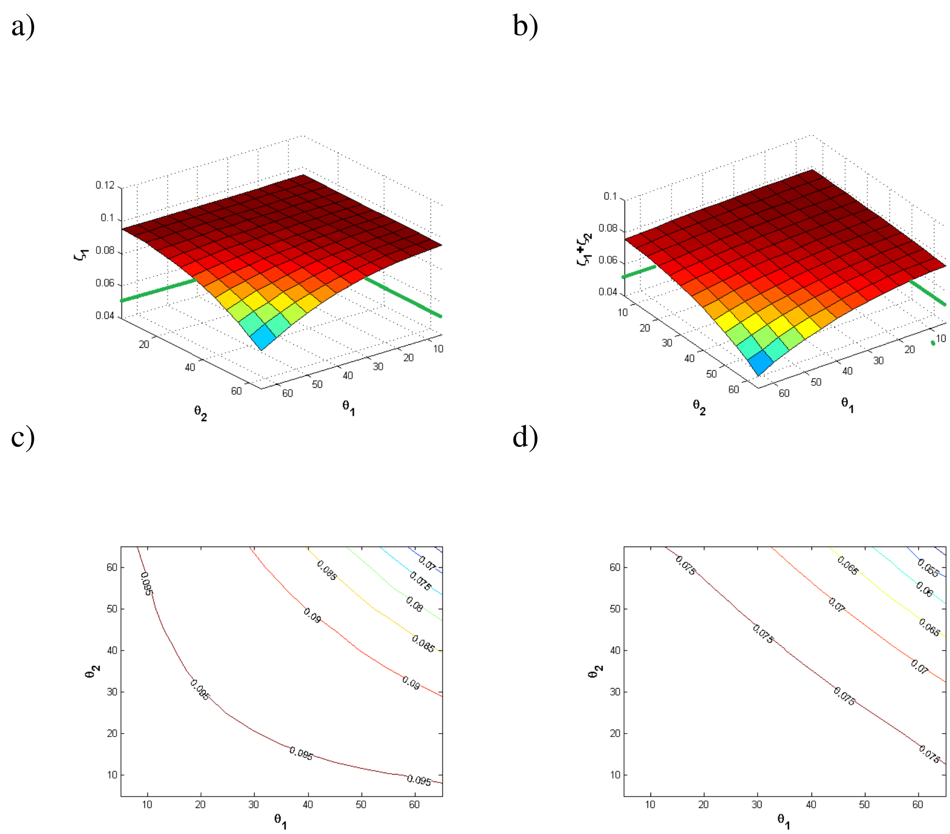

FIG. 1. Recovered parameter from the normal incidence for ζ. The exact value of ζ is 0.1. The linear approximation ζ_1 (a) and the sum of linear and first non-linear $\zeta_1 + \zeta_2$(b). The graphs on the bottom are the corresponding contour plots of the graphs on the top.

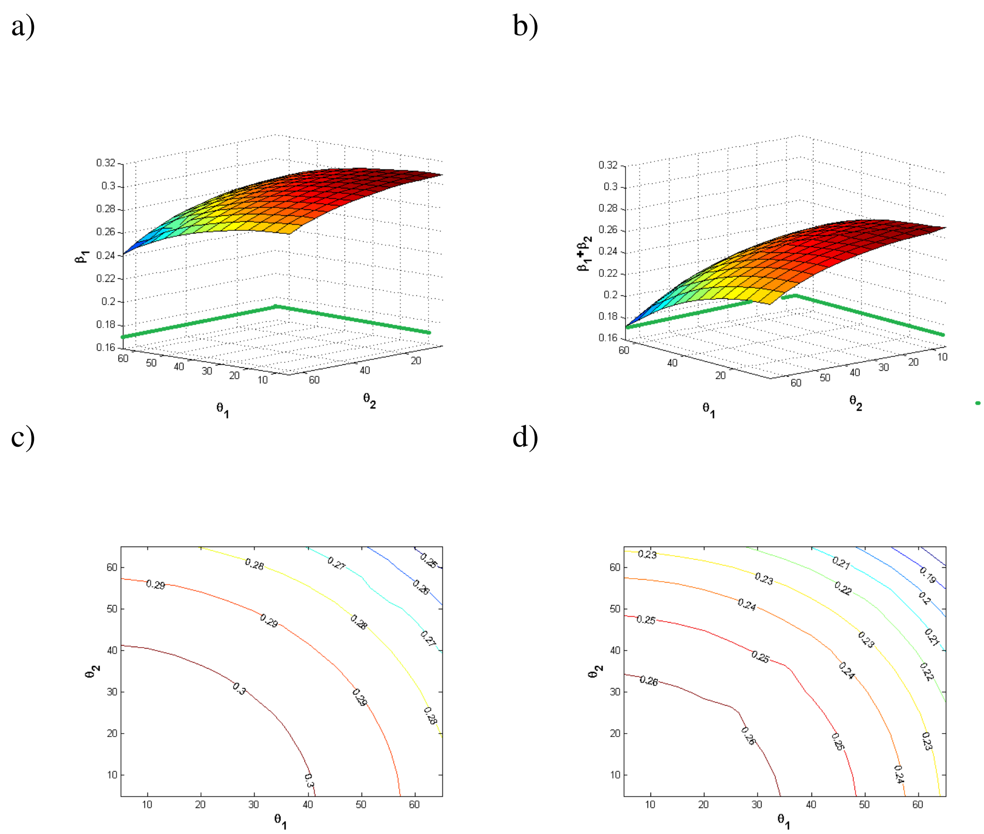

FIG. 2. Recovered parameter from the normal incidence for β. The exact value of β is 0.17. The linear approximation β_1 (a) and the sum of linear and first non-linear $\beta_1 + \beta_2$(b). The graphs on the bottom are the corresponding contour plots of the graphs on the top.

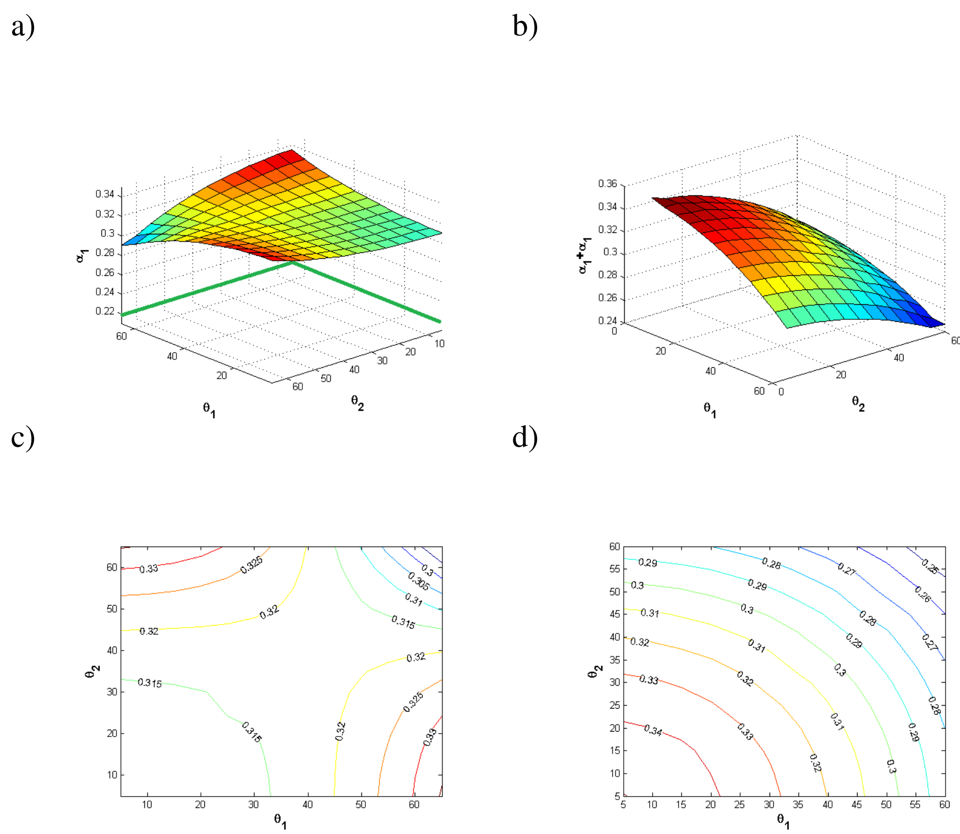

FIG. 3. Recovered parameter from the normal incidence for α. The exact value of α is 0.22. The linear approximation α_1 (a) and the sum of linear and first non-linear $\alpha_1 + \alpha_2$(b).

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APPENDIX A

Viscoacoustic case

$$
\begin{aligned}
a_1 = \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'} \int_{-\infty}^{\infty} dz^{''}[k^2\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'}) - k_g^2\beta_1(z^{'})] \\
[k^2\alpha_1(z^{''}) - 2k^2F(k)\zeta_1(z^{''}) - k_g^2\beta_1(z^{''})]e^{iq_g(z^{'}+z^{''})}e^{iq_g|z^{'}-z^{''}|} \\
= \frac{i}{q_g^3}\int_{-\infty}^{\infty} dz^{'} \int_{-\infty}^{\infty} dz^{''}[k^2\alpha_1(z^{'}) - 2k^2F(k)\zeta_1(z^{'}) - k_g^2\beta_1(z^{'})] \\
[k^2\alpha_1(z^{''}) - 2k^2F(k)\zeta_1(z^{''}) - k_g^2\beta_1(z^{''})]e^{2iq_g z^{'}} H(z^{'} - z^{''})
\end{aligned}
$$

After the Fourier transformation over $2q_g$ and divided by π, we have

$$
\begin{aligned}
a_1 = -\frac{1}{2}\frac{\partial}{\partial z}\int_{-\infty}^{\infty} dz^{'} \int_{-\infty}^{\infty} dz^{''}[\frac{k^2}{q_g^2}\alpha_1(z^{'}) - 2\frac{k^2}{q_g^2}F(k)\zeta_1(z^{'}) - \frac{k_g^2}{q_g^2}\beta_1(z^{'})] \\
[\frac{k^2}{q_g^2}\alpha_1(z^{''}) - 2\frac{k^2}{q_g^2}F(k)\zeta_1(z^{''}) - \frac{k_g^2}{q_g^2}\beta_1(z^{''})]e^{2iq_g z^{''}}\delta(z - z^{'})H(z^{'} - z^{''}) = \\
-\frac{1}{2}\frac{\partial}{\partial z}\{[\frac{\alpha_1(z)}{\cos^2\theta} - \frac{2F(k)}{\cos^2\theta}\zeta_1(z) - \tan^2\theta\beta_1(z)]\int_{-\infty}^{\infty} dz^{''}[\frac{\alpha_1(z^{''})}{\cos^2\theta} \\
-\frac{2F(k)}{\cos^2\theta}\zeta_1(z^{''}) - \tan^2\theta\beta_1(z^{''})]H(z - z^{''})\}
\end{aligned}
$$

This equation cab be written as

$$
\begin{aligned}
a_1 = -\frac{1}{2}[\frac{1}{\cos^2}\alpha_1(z) - 2\frac{F(k)}{\cos^2}\zeta_1(z) - \tan^2\theta\beta_1(z)]^2 \\
-\frac{1}{2}[\frac{1}{\cos^2}\alpha_1^{'}(z) - 2\frac{F(k)}{\cos^2}\zeta_1^{'}(z) - \tan^2\theta\beta_1^{'}(z)]\int_{-\infty}^{\infty} dz^{'} \\
[\frac{1}{\cos^2}\alpha_1(z^{'}) - 2\frac{F(k)}{\cos^2}\zeta_1(z^{'}) - \tan^2\theta\beta_1(z^{'})] = -\frac{1}{2\cos^4\theta}\alpha_1^2(z) - \frac{2F(k)^2}{\cos^4\theta}\zeta_1^2(z) \\
-\frac{\tan^4\theta}{2}\beta_1^2(z) + \frac{2F(k)}{\cos^4\theta}\alpha_1(z)\zeta_1(z) + \frac{\tan^2\theta}{\cos^2\theta}\alpha_1(z)\beta_1(z) - \frac{2F(k)\tan^2\theta}{\cos^2\theta}\zeta_1(z)\beta_1(z) \\
+(\frac{-1}{2\cos^4\theta}\alpha_1^{'}(z) + \frac{F(k)}{\cos^4\theta}\zeta_1^{'}(z) + \frac{\tan^2\theta}{2\cos^2\theta}\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\alpha_1(z^{'}) \\
+(\frac{F(k)}{\cos^4\theta}\alpha_1^{'}(z) - \frac{2F(k)^2}{\cos^4\theta}\zeta_1^{'}(z) - \frac{F(k)\tan^2\theta}{\cos^2\theta}\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\zeta_1(z^{'}) \\
+(\frac{\tan^2\theta)}{2\cos^2\theta}\alpha_1^{'}(z) - \frac{F(k)\tan^2\theta)}{\cos^2\theta}\zeta_1^{'}(z) - \frac{\tan^4\theta)}{2}\beta_1^{'}(z))\int_{-\infty}^{\infty} dz^{'}\beta_1(z^{'})
\end{aligned}
$$

and a_2 is

$$
\begin{aligned}
a_2 = \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' e^{iq_g z'}[k^2\alpha_1(z') - 2k^2 F(k)\zeta_1(z') \\
k_g^2\beta_1(z')]e^{iq_g|z'-z''|}\frac{\partial}{\partial z''}\beta_1(z'')\frac{\partial}{\partial z''}e^{iq_g z''} = \\
\frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' e^{iq_g z'}[k^2\alpha_1(z') - 2k^2 F(k)\zeta_1(z') \\
-k_g^2\beta_1(z')]e^{iq_g|z'-z''|}[(iq_g)^2\beta_1(z'')e^{iq_g z''} + iq_g\beta_1'(z'')e^{iq_g z''}] = -\frac{iq_g}{2\cos^2\theta}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' \\
\alpha_1(z')\beta_1(z'')[e^{2iq_g z'}H(z'-z'') + e^{2iq_g z''}H(z''-z')] + \frac{iF(k)q_g}{\cos^2\theta}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' \\
\zeta_1(z')\beta_1(z'')[e^{2iq_g z'}H(z'-z'') + e^{2iq_g z''}H(z''-z')] \\
+\frac{iq_g\tan^2\theta)}{2}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz''\beta_1(z')\beta_1(z'')e^{2iq_g z'}H(z'-z'') - \frac{1}{2\cos^2\theta}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' \\
\alpha_1(z')\beta_1'(z'')[e^{2iq_g z'}H(z'-z'') + e^{2iq_g z''}H(z''-z')] + \frac{F(k)}{cos^2\theta}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' \\
\zeta_1(z')\beta_1'(z'')[e^{2iq_g z'}H(z'-z'') + e^{2iq_g z''}H(z''-z')] + \frac{\tan^2\theta}{2}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' \\
\beta_1(z')\beta_1'(z'')[e^{2iq_g z'}H(z'-z'') + e^{2iq_g z''}H(z''-z')]
\end{aligned}
$$

Then, after the Fourier transformation over $2q_g$ and divided by π, we have

$$
\begin{aligned}
a_2 = \frac{1}{4\cos^2\theta}\alpha_1'(z)\int_{-\infty}^{z} dz'\beta_1(z') - \frac{1}{4\cos^2\theta}\beta_1'(z)\int_{-\infty}^{z} dz'\alpha_1(z') \\
-\frac{F(k)}{2\cos^2\theta}\zeta_1'(z)\int_{-\infty}^{z} dz'\beta_1(z') + \frac{F(k)}{2\cos^2\theta}\beta_1'(z)\int_{-\infty}^{z} dz'\zeta_1(z')
\end{aligned}
$$

a_3 can be written as

$$
\begin{aligned}
a_3 = \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' e^{iq_g z'}\frac{\partial}{\partial z'}\beta_1(z')\frac{\partial}{\partial z'}e^{iq_g|z'-z''|}[k^2\alpha_1(z'') - 2k^2F(k)\zeta_1(z'') \\
-k_g^2\beta_1(z'')]e^{iq_g z''} = \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz'' e^{iq_g z'}\frac{\partial}{\partial z'}[\beta_1(z')(iq_g)sign(z'-z'')e^{iq_g|z'-z''|}] \\
[k^2\alpha_1(z') - 2k^2F(k)\zeta_1(z') \\
-k_g^2\beta_1(z')]e^{iq_g z''} = \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dz''[\beta_1'(z')(iq_g)sign(z'-z'')e^{iq_g|z'-z''|} \\
+\beta_1(z')(2iq_g)\delta(z'-z'') + \beta_1(z')(iq_g)^2e^{iq_g|z'-z''|}][k^2\alpha_1(z'') - 2k^2F(k)\zeta_1(z'') \\
-k_g^2\beta_1(z'')]e^{iq_g(z'+z'')}
\end{aligned}
$$

This equation can be written as

$$a_3 = -\frac{1}{2}\int_{-\infty}^{\infty} dz^{'} \int_{-\infty}^{\infty} dz^{''} \beta_1^{'}(z^{'})[\frac{1}{\cos^2\theta}\alpha_1(z^{''}) - \tan^2\theta\beta_1(z^{''}) - \frac{2F(k)}{\cos^2\theta}\zeta_1(z^{''})]$$
$$\times[e^{2iq_g z^{'}}H(z^{'} - z^{''}) - e^{2iq_g z^{''}}H(z^{''} - z^{'})] - \int_{-\infty}^{\infty} dz^{'}\beta_1(z^{'})[\frac{1}{\cos^2\theta}\alpha_1(z^{'}) - \tan^2\theta\beta_1(z^{'})$$
$$-\frac{2F(k)}{\cos^2\theta}\zeta_1(z^{'})]e^{2iq_g z^{'}} - \frac{iq_g}{2}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''}\beta_1(z^{'})[\frac{1}{\cos^2\theta}\alpha_1(z^{''}) - \tan^2\theta\beta_1(z^{''}) - \frac{2F(k)}{\cos^2\theta}\zeta_1(z^{''})]$$
$$\times[e^{2iq_g z^{'}}H(z^{'} - z^{''}) + e^{2iq_g z^{''}}H(z^{''} - z^{'})]$$

Afther the fourier transformation over $2q_g$ and divided by π, we have

$$a_3 = \frac{1}{4\cos^2\theta}\alpha_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) - \frac{1}{4\cos^2\theta}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\alpha_1(z^{'})$$
$$-\frac{F(k)}{2\cos^2\theta}\zeta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) + \frac{F(k)}{2\cos^2\theta}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\zeta_1(z^{'})$$

Finally, a_4 is

$$a_4 = \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''} e^{iq_g z^{'}}\frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}e^{iq_g|z^{'}-z^{''}|}\frac{\partial}{\partial z^{''}}\beta_1(z^{''})\frac{\partial}{\partial z^{''}}e^{iq_g z^{''}}$$
$$= \frac{i}{2q_g^3}\int_{-\infty}^{\infty} dz^{'}\int_{-\infty}^{\infty} dz^{''} e^{iq_g z^{'}}\frac{\partial}{\partial z^{'}}\beta_1(z^{'})\frac{\partial}{\partial z^{'}}e^{iq_g|z^{'}-z^{''}|}[(iq_g)^2\beta_1(z^{''})e^{iq_g}z^{''} + iq_g\beta_1^{'}(z^{''})e^{iq_g}z^{''}]$$

Then, afther the Fourier transformation over $2q_g$, we have

$$a_4 = -\frac{1}{2}\beta_1^2(z) - \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) + \frac{1}{2}\beta_1^2(z) + \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'})$$
$$+\beta_1^2(z) - \beta_1^2(z) + \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'}) - \frac{1}{2}\beta_1^2(z)$$
$$= -\frac{1}{2}\beta_1^2(z) + \frac{1}{2}\beta_1^{'}(z)\int_{-\infty}^{z} dz^{'}\beta_1(z^{'})$$