Sequential Gaussian simulation using multi-variable cokriging

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ABSTRACT

Uncertainty analysis is a key element in reservoir properties prediction, and many techniques have been developed, such as simulation, which is usually performed by least square method. Least square estimation is a classic and well known approach as a best fit solver, which is equivalent to simple kriging or cokriging system in case of geostatistics filed. Inspired by the distinction of simple and ordinary system in geostatistics and benefited from the extended cokriging system, for reservoir properties prediction, we propose an approach to implement sequential simulation with multiple priori information using the extended cokriging system. As it implies, the conditional mean and variance in the posterior distribution are obtained by performing the extended cokriging system. Comparison between this approach and traditional sequential simulation using least square method are discussed in sense of semivariogram. The advantage are further analyzed through the estimated error map, which indicates that simulation using the extended geostatistics method can produce an more an accurate map, especially dealing with the data has the dramatical changes.

INTRODUCTION

Geostatistical inversion methods can reduce uncertainty in the reservoir properties prediction away from the control points, by integrating well log data (sampled sparsely but accurate) as primary data and seismic data (sampled well but band limited) as secondary data. Number of discussions have been presented. One of the classic methods is deterministic inversion, such as kriging, cokriging. To use variate secondary data, Russell et al. (2002) combined cokriging and multiattribute transforms. As Russell et al. (2002) illustrated, the improved secondary input of cokriging can be generated by multi-attribute analysis. Babak and Deutsch (2009) improved the cokriging model by merging all secondary data into a single super secondary data and then implementing the cokriging system with this single merged secondary data. Two or more than two secondary variables were employed in the estimation system without knowing stationary mean was presented by (Xu et al., 2015, 2016). However, a limitation is that deterministic methods provide the result that has trouble capturing the natural variability and heterogeneity of reservoirs due to the smoothing effect.

The sequential simulation technique provides a series of equal valid and possible realizations that indirectly reflect the distribution of reservoir properties. Deutsch and Journel (1992) calculated the kriging mean and variance at unmeasured locations and simulated the value at those points by using Sequential Gaussian Simulation (SGS). A posterior Gaussian distribution can be also described by a simple kriging system, therefore it is possible to draw samples of a posteriori probability density function by using sequential simulation(Hansen et al., 2006). Ligas and Kulczycki (2010) observed least squares prediction

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and geostatistical method of simple kriging are equivalent.

In this paper, we re-investigate the relation between least square method and geostatistics methods which shows that both least square and simple geostatistics methods are equivalent to conditional expectation in the posterior distribution. It indicates that the advantage of ordinary approach, compared to the simple system, are also superior to least square method. Furthermore, to take the benefit from the extended corkiging system, we implement the sequential simulation with the posterior mean and covariance calculated from the extended cokriging system. Finally, this approach is applied to Black foot data for porosity simulation.

METHOD

Any estimation, interpolation, projection or transformation problem, based on a given data $\mathbf{d}$, can be treated as the inversion of model parameters that generates the data under action of the operator $\mathbf{G}$ (Claerbout, 1992):

$$\mathbf{d} = \mathbf{G}\mathbf{m} \tag{1}$$

Therefore, the problem becomes to find the best fit model $\mathbf{m}$ with the projection operator $\mathbf{G}$ using the observed data $\mathbf{d}$. Here, the projection operator is relatively unconventional which leads to $\mathbf{G}$ does not needs to be dependent on any physical law. The classical principle to solve Eq. 1 is the estimated model has to satisfy the unbiased minimum error variance criteria, which turns the above problem into the well-known least-square prediction.

To further understand least-square prediction, the simplest case with one single estimate, instead of a set, is discussed in the following context. It's known that the least-square method gives an unique, best fit estimate if three criteria are met: *linearity*, *unbiasedness*, and *minimum error variance*. Based on these assumptions, an estimate ϕ can be described as the weighted ($\boldsymbol{\lambda}$) combination of a realization $\boldsymbol{\Psi}$, which can be considered as a second order stationary random function with zero mean (Heiskanen and Moritz, 1967; Dermanis, 1984). In matrix notation, the predictor can be written as

$$\phi = \boldsymbol{\lambda}^T \boldsymbol{\Psi} \tag{2}$$

If the mean of the second order stationary random realization $\boldsymbol{\Psi}$ is not zero, which means the second criteria, unbiasedness, does not hold, one has to transfer the observed data into a new zero-mean random function by subtracting its true (or estimated) mean (Dermanis, 1984; Ligas and Kulczycki, 2010). The optimal set for coefficient vector $\boldsymbol{\lambda}$ can be calculated by minimizing the mean square error (prediction error variance), and the solution can be expressed as

$$\boldsymbol{\lambda} = \mathbf{C}^{-1}\mathbf{c} \tag{3}$$

where, $\mathbf{C}$ is the covariance matrix of an observed dataset (i.e., $\boldsymbol{\Psi}$), $\mathbf{c}$ is the vector of covariances between the estimate point (ϕ) and the observed data ($\boldsymbol{\Psi}$).

The least square estimation can be obtained by substituting the coefficient vector $\boldsymbol{\lambda}$ into the predictor Eq. (2), written as

$$\phi = \mathbf{c}^T \mathbf{C}^{-1} \boldsymbol{\Psi} \tag{4}$$

and, in case of non-zero mean realization, it can be expressed as

$$\phi = \mu_\phi + \mathbf{c}^T\mathbf{C}^{-1}(\mathbf{\Psi} - \boldsymbol{\mu}_\mathbf{\Psi}) \tag{5}$$

with the prediction error variance calculated by

$$Var(R) = \mathbf{C}_{\phi 0} - \mathbf{c}^T\mathbf{C}^{-1}\mathbf{c} \tag{6}$$

where, $\mathbf{C}_{\phi 0}$ is the variance of ϕ, or the sill of the ϕ semivariogram.

The above derivation indicates that the least square prediction is equivalent to a simple kriging (SK) system when the prediction becomes to the interpolation problem ($\mathbf{\Psi} \rightarrow \mathbf{\Phi}$). The unbiased condition is satisfied in an automatic manner regardless of choice of weights. Beyond that, (Tarantola, 2005) demonstrated that least square problem (Eq.1) can also be described as the posterior Gaussian probability density in the model space

$$P(\mathbf{m}|\mathbf{d}) = const.exp\left[-\frac{1}{2}(\mathbf{m} - \boldsymbol{\mu}_{m|d})^T\mathbf{\Sigma}^{-1}_{m|d}(\mathbf{m} - \boldsymbol{\mu}_{m|d})\right] \tag{7}$$

with the conditional mean as

$$\boldsymbol{\mu}_{m|d} = \boldsymbol{\mu}_m + (\mathbf{GC}_m)^T(\mathbf{GC}_m\mathbf{G}^T + \mathbf{C}_d)^{-1}(\mathbf{d} - \boldsymbol{\mu}_d) \tag{8}$$

and the conditional covariance matrix

$$\mathbf{\Sigma}_{m|d} = \mathbf{C}_m - \mathbf{C}_m\mathbf{G}^T(\mathbf{GC}_m\mathbf{G}^T + \mathbf{C}_d)^{-1}(\mathbf{d} - \boldsymbol{\mu}_d) \tag{9}$$

where, $\boldsymbol{\mu}_m$ and $\boldsymbol{\mu}_d$ are the mean vector for model and given data respectively, $\mathbf{C}_m$ and $\mathbf{C}_d$ are the model and data covariance matrices. Here, as mentioned before, the conditional mean and covariance are identical to the simple kriging (SK) mean and covariance.

Following the demonstration prensented by Tarantola (2005), Hansen et al. (2006) expand the Gaussian linear inverse problem involving two types of dataset. The projection between model parameter and two given datasets is given by

$$\mathbf{d}_0 = \mathbf{Gm} \tag{10}$$

where,

$$\mathbf{d}_0 = \begin{bmatrix}\mathbf{a}_0 \\ \mathbf{b}_0\end{bmatrix}, \mathbf{C}_d = \begin{bmatrix}\mathbf{C}_{aa} & \mathbf{C}_{ab} \\ \mathbf{C}^T_{ab} & \mathbf{C}_{bb}\end{bmatrix}, \boldsymbol{\mu}_0 = \begin{bmatrix}\boldsymbol{\mu}_{a_0} \\ \boldsymbol{\mu}_{b_0}\end{bmatrix} \tag{11}$$

Again, it can be described as the posterior Gaussian probability density in the model space based on two datasets

$$P(\mathbf{m}|\mathbf{a}_0, \mathbf{b}_0) = const.exp\left[-\frac{1}{2}(\mathbf{m} - \boldsymbol{\mu}_{m|a_0,b_0})^T\mathbf{\Sigma}^{-1}_{m|a_0,b_0}(\mathbf{m} - \boldsymbol{\mu}_{m|a_0,b_0})\right] \tag{12}$$

Here, the conditional mean $\boldsymbol{\mu}_{m|a_0,b_0}$ and covariance $\mathbf{\Sigma}_{m|a_0,b_0}$ have the same matrix notation as the previous illustrated in Eq. (8) and Eq. (9), except the data covariance matrix

is replaced by Eq. (11). Therefore, the conditional mean $\boldsymbol{\mu}_{m|a_0,b_0}$ and covariance $\boldsymbol{\Sigma}_{m|a_0,b_0}$ coincide with the mean and covariance solved by a traditional simple cokriging (SCK) system.

Analogously, the inverse problem can be expanded and be described as the posterior Gaussian probability distribution based on given N-datasets ($\mathbf{d} = \left[\mathbf{d}_1^T, \mathbf{d}_2^T, \ldots, \mathbf{d}_n^T\right]^T$),

$$P(\mathbf{m}|\mathbf{d}_1, \ldots, \mathbf{d}_n) = const.exp\left[-\frac{1}{2}(\mathbf{m} - \boldsymbol{\mu}_{m|(\mathbf{d}_1,\ldots,\mathbf{d}_n)})^T \boldsymbol{\Sigma}^{-1}_{m|(\mathbf{d}_1,\ldots,\mathbf{d}_n)}(\mathbf{m} - \boldsymbol{\mu}_{m|(\mathbf{d}_1,\ldots,\mathbf{d}_n)})\right] \tag{13}$$

where the conditional mean $\boldsymbol{\mu}_{m|(\mathbf{d}_1,\ldots,\mathbf{d}_n)}$ and covariance $\boldsymbol{\Sigma}_{m|(\mathbf{d}_1,\ldots,\mathbf{d}_n)}$ are equivalent to the mean and prediction error covariance calculated from an extended simple cokriging (SCK) system involving N-datasets demonstrated by Xu et al. (2015, 2016).

By implementing one of the above three methods in a random sequence, a sequential conditional (co)simulation can be achieved (Goovaerts, 1997; Gloaguen et al., 2005b,a; Gomez-Hernandez et al., 2005; Hansen and Mosegaard, 2008; Journel and Huijbregts, 1978). As we know, sequential simulation is a technique to generate a series of independent realizations based on the known model parameters and given data. The posterior probability density function based on known model parameters and given data, at any location $\mathbf{x}_i$, needs to be calculated and then be moved to the next location depending on the random sequence. Assume we have known model parameters $\boldsymbol{\Phi}_0(\mathbf{x}_i)$ at m locations ($\mathbf{x}_i \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m\}$) and two given data $\mathbf{A}$ and $\mathbf{I}$, one estimated realization at other locations ($\mathbf{x}_j \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$) using sequential conditional simulation based on known model parameters $\boldsymbol{\Phi}(\mathbf{x}_i)$ and given data $\mathbf{A}$ and $\mathbf{I}$ can be performed as following steps,

(1) Sorting the estimated locations into a random sequence $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$.

(2) Visit the grid point $\mathbf{x}_j$ according to the path generated in step (1).

(3) Calculating the conditional mean and covariance at visited location $\mathbf{x}_j$ based on known $\boldsymbol{\Phi}_0$ and pre-simulated model parameters $\boldsymbol{\Phi}(\mathbf{x}_1, ..., \mathbf{x}_{j-1})$ and two given datasets ($\mathbf{A}$ and $\mathbf{I}$), by performing one of three methods mentioned above, i.e., least-square prediction, simple geostatistical methods, or conditional expectation approach.

(4) Build the posterior probability distribution $P\{\boldsymbol{\Phi}(\mathbf{x}_j)|\boldsymbol{\Phi}_0, \mathbf{A}, \mathbf{I}, \boldsymbol{\Phi}(\mathbf{x}_1, ..., \mathbf{x}_{j-1})\}$, and draw a random value.

(5) Add the simulated value at location $\mathbf{x}_j$ into the known dataset.

(6) Move to the next location $\mathbf{x}_{j+1}$ in the sequence, and repeat step 2-5 until the last grid point in the sequence is encountered.

Note that the conditional mean and covariance are calculated by least square (LS) or simple geostatistical methods (SK, SCK, or extended SCK), and both of them are equivalent to conditional expectation in the sense of Gaussian random field (Ligas and Kulczycki, 2010). In other words, one shares the strengths as others do, as well as the drawbacks. One of the defects is that, either least-square and conditional expectation or the simple geosta-

tistical methods, requires a zero-mean second order stationary random realization as the observed data. Again, the known mean needs to be utilized to transfer the data into a zero-mean random function if the unbiasedness does not hold. However, in practice, the mean is usually unknown and is replaced by an estimated mean which can be calculated, for example, by the trend removal technique (Dermanis, 1984). Beyond that, the automatic-meet unbiased condition implies that no constraint is applied to the coefficient (weight) vector during the process. This may generate singular value occurring in the estimation when the significant changes of magnitude between the objective and observed data occur.

The procedure we propose for sequential conditional Gaussian simulation (SCGS) is inspired by the ongoing development into the extend corkging system (Xu et al., 2015). Detailed comparisons between simple, ordinary, and rescaled ordinary methods for kriging and cokriging were discussed by Goovaerts (1998). It concluded that, instead of a stationary mean in simple (co)kriging system, the (rescaled) ordinary cokriging (OCK and ROCK) utilizing the estimated local mean. Further more, compared to two constraints of the ordinary cokriging (OK), only one unbiasedness constraint applied for the weight vector in the rescaled cokriging (ROCK) system lessens the risk of producing unacceptable estimates, such as negative values (Goovaerts, 1998; Xu et al., 2016).

Also the extended (rescaled) ordinary cokriging equation allows for more than two secondary variables participating in the estimation system without requiring stationary mean of model parameters to be known (Xu et al., 2015, 2016). Therefore, in this paper, we perform the sequential conditional Gaussian simulation (SCGS) by replacing the 3rd step illustrated above with the extend (rescaled) ordinary cokriging (OCK and ROCK) system to obtain the conditional mean and covariance. Investigation of differences between simulation using least square method, simulation with extend ordinary cokriging (OCK), and simulation with extended rescaled ordinary cokriging (ROCK) system are analyzed.

IMPLEMENTATION

Neighborhood search and pre-calcuated covariance

Before performing the simulation process, the computation efficiency will be discussed by considering extend ordinary cokriging system. Based on the previous research (Xu et al., 2015), the extend ordinary cokriging (OCK) system involving two secondary variables can be written in matrix notation, as

$$\begin{bmatrix} \mathbf{C}_{\Phi\Phi} & \mathbf{C}_{\Phi A} & \mathbf{C}_{\Phi I} & \mathbf{1} & 0 & 0 \\ \mathbf{C}_{A\Phi} & \mathbf{C}_{AA} & \mathbf{C}_{AI} & 0 & \mathbf{1} & 0 \\ \mathbf{C}_{I\Phi} & \mathbf{C}_{IA} & \mathbf{C}_{II} & 0 & 0 & \mathbf{1} \\ \mathbf{1}^T & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1}^T & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \\ \boldsymbol{\lambda}_3 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{\phi_0\Phi} \\ \mathbf{C}_{\phi_0 A} \\ \mathbf{C}_{\phi_0 I} \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

In the implementation of sequential conditional Gaussian simulation (SCGS), the covariance matrix becomes larger and larger as the iteration number of simulation increases, which makes the approach considerably time consuming, or possibly even being incalculable. However, in the view of geostatistics (Deutsch et al., 1998), data far beyond the

range of semivariogram has little contribution in the conditioning mean and covariance of posterior probability distribution. Therefore, the covariance matrix in extended ordinary cokriging (OCK) system can be limited into an acceptable subset with the range in grid point.

Even though the covariance matrix is restricted in a small set, the calculation of covariance in each iteration is still a computational burden. To make the utmost of calculated covariance in previous iterations, and to avoid the repetitive computation of covariance at the same location, the pre-calculated covariance table is utilized. This ensures the covariance between two identical locations is only calculated once. In addition, for an isotropic model, the covariance function does not depend on absolute locations, but rather on the distance between two locations. Therefore, the pre-calculated covariance table is very computational efficient. Another advantage is, the covariance between two specified locations is identified with an unique label, which can be easily referenced during the iteration.

Initial input of LS, extend OCK and ROCK method

The survey data was recorded from the Blackfoot field located in southern Alberta in 1995 for Canadian Petroleum. There were twelve wells involved in this study case, all of which contained the calculated porosity logs. An average porosity value between the picked top and base of the zone of interest in each well, is considered as the sampled known "model parameters" ($\mathbf{\Psi}_0$). Figure 1 shows that the well distribution on the survey area and the porosity value at each well location.

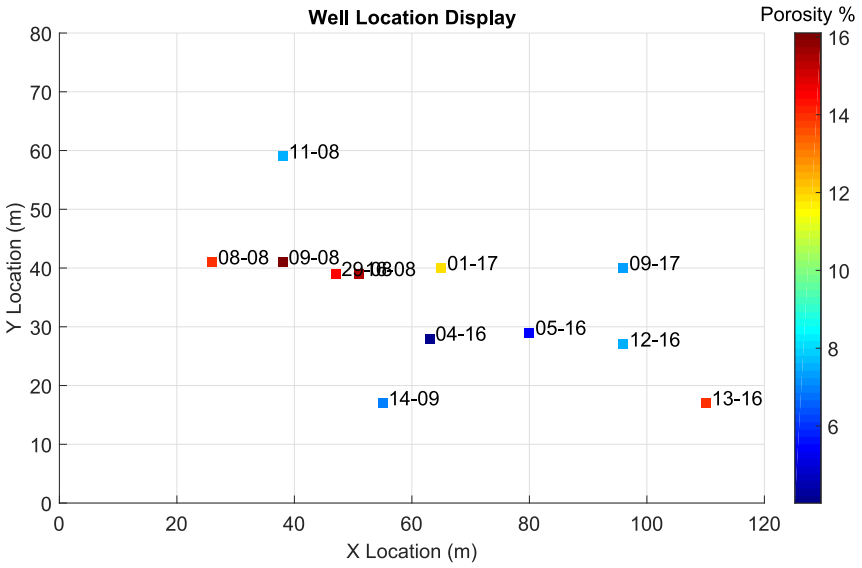
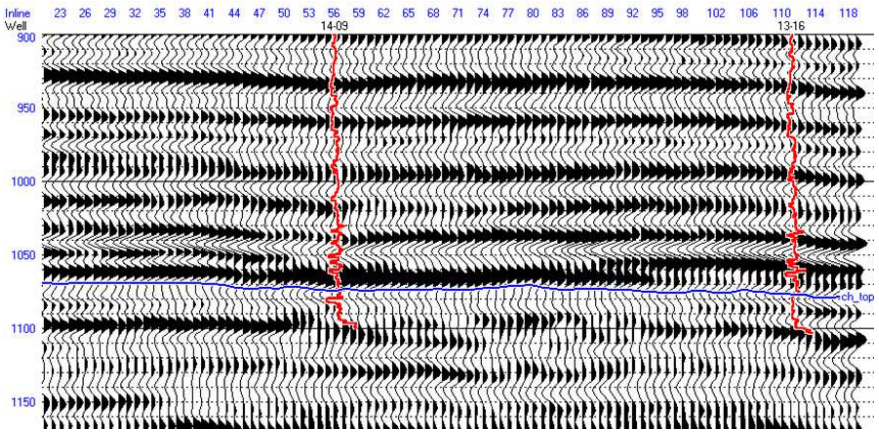
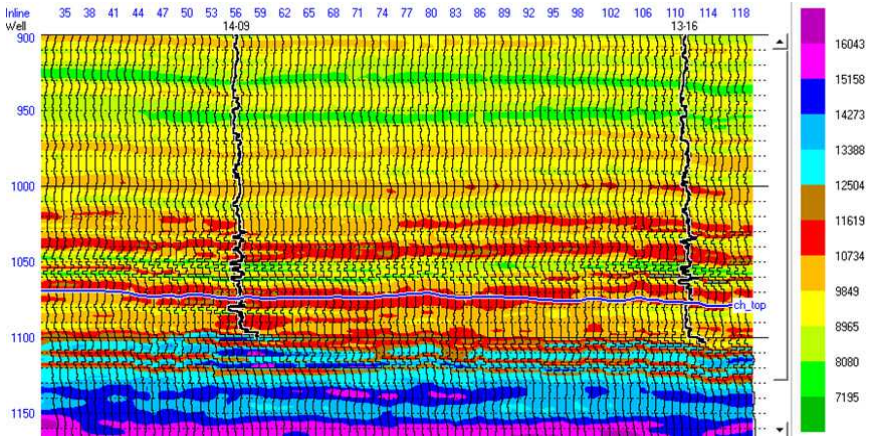


FIG. 1: Well distribution and average porosity value in the zone of interest.

Representative secondary inputs are attribute key elements for geostatistical methods. Instead of directly using acquired seismic data, two slices extracted from different 3-D volumes, the acoustic impedance inversion and the stacked P-wave seismic data, are used. The inversion volume was obtained using Hampson-Russell Software (Russell et al., 2002). First, build an initial model from the well logs and pick horizon on the seismic section. Second, stop perturbing this model when the synthetic seismogram has a best match with



(a) The final CDP stack and interested zone



(b) P-wave impedance section and interested zone

FIG. 2: Crossline 18 from the 3-D seismic volume.

the original data. Crossline 18 extracted from the seismic volume is illustrated in Figure 2 showing a seismic input line (Figure 2a) and an inverted impedance line (Figure 2b).

An inversion slice was trimmed by picking 10ms average window below the channel top from 3D inverted volume. Similarly, we extracted three data slices, seismic amplitude, amplitude envelope, and instantaneous phase, by applying a 10ms window of RMS average on the zone of interest. To choose appropriate inputs for simulation, correlation coefficients were calculated between the porosity and all four created data slices. The best two correlation coefficients were observed from inversion slice and seismic amplitude slice, which were -0.65 and 0.41, respectively. Thus, in this case, seismic amplitude slice (indicated as **A**) shown in Figure 3 and inversion slice (indicated as **I**) shown in Figure 4 are considered as the two conditioning datasets for the sequential simulation study.

As discussed previously, to make the approach computationally efficient, the distance and covariance table are calculated before the iteration, delineated in Figure 5 and Figure 6, respectively. Figure 6 shows the covariance tables among porosity, RMS amplitude, and acoustic P-impedance, are diagonal symmetric due to the isotropic nature of relations. The limited covariance matrices in Eq. 14 can be readily indexed from the pre-calculated

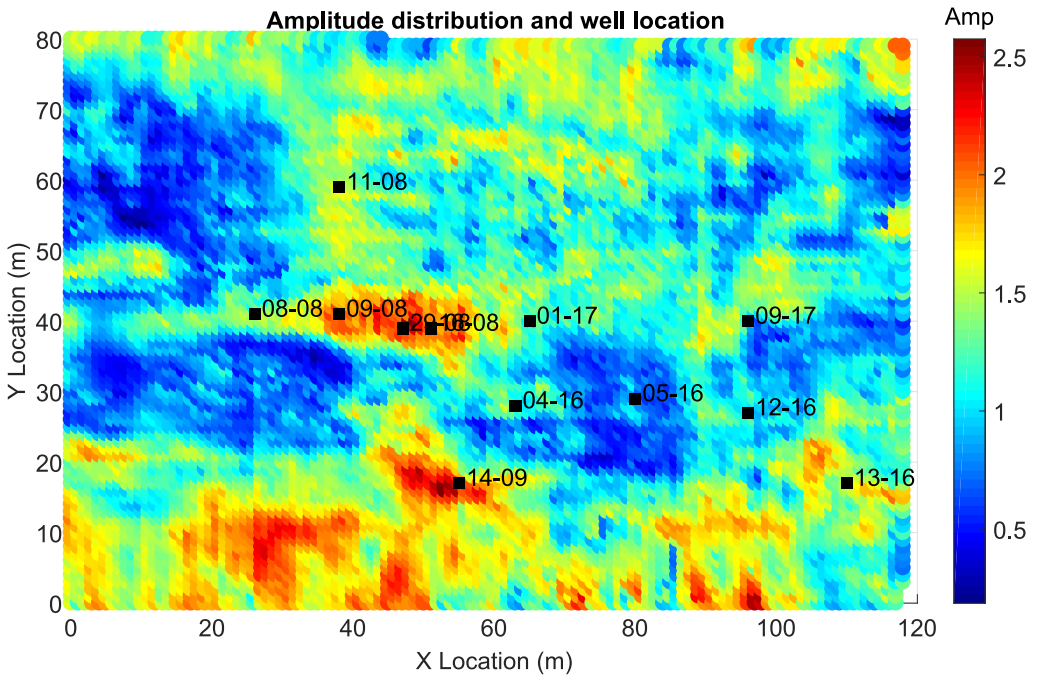


FIG. 3: The average RMS amplitude slice at the interested zone.

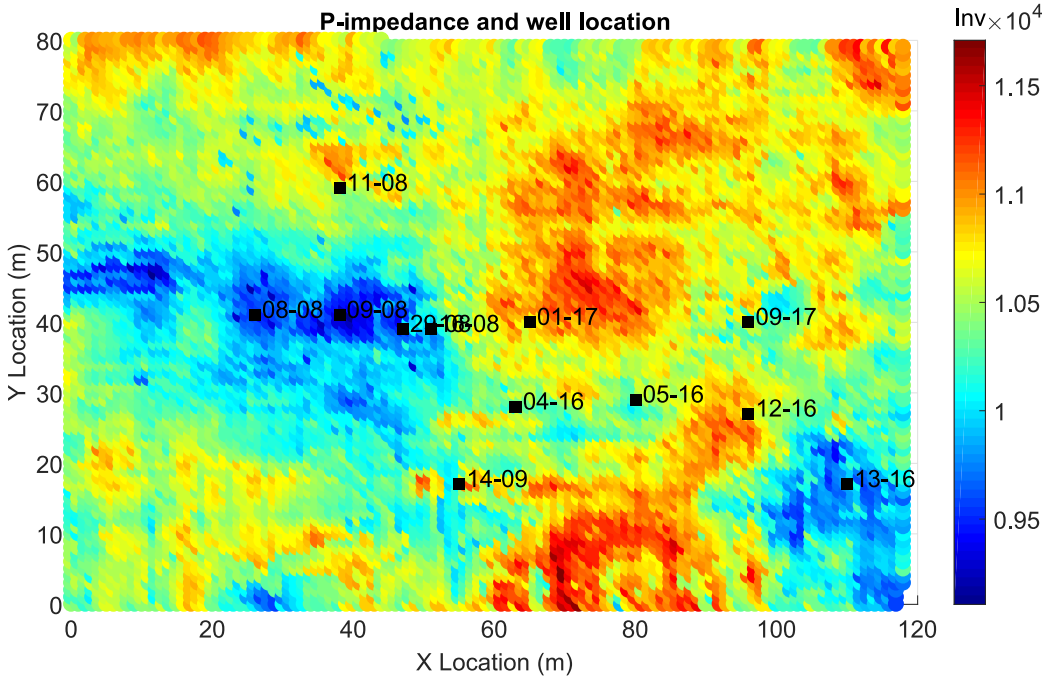


FIG. 4: The average acoustic P-impedance slice at the interested zone.

covariance tables, respectively.

Simulated realizations using SCGS with extend LS, OCK, and ROCK

The sequential conditional Gaussian simulation is performed by implementing the procedure 1-6 already demonstrated, except that the least-square (or simple geostatistics) con-

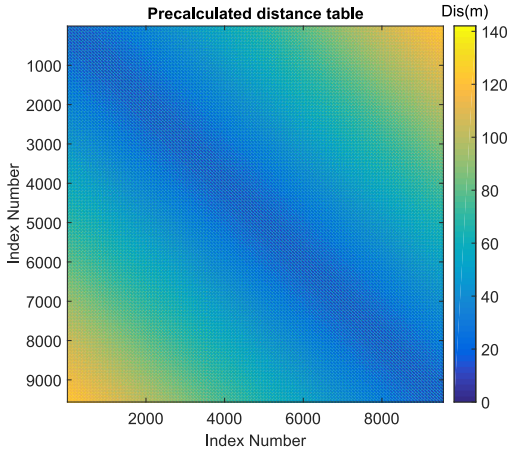


FIG. 5: Precalculated distance table on the entire area.

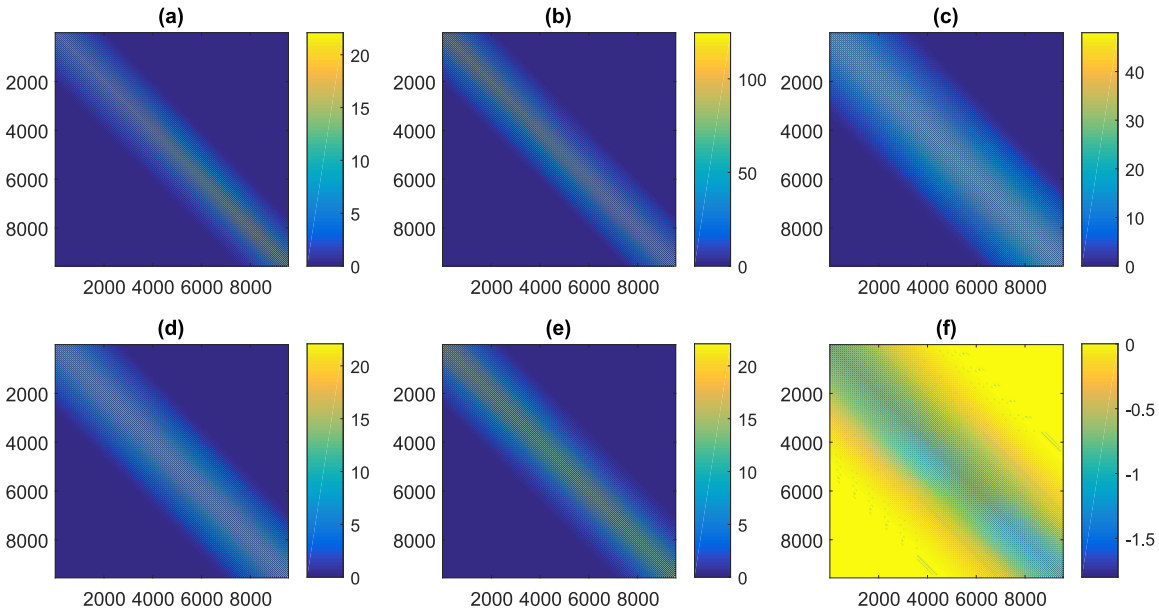


FIG. 6: Precalculated covariance table on the entire area among the porosity, P-impedance, and RMS amplitude. (a) $\mathbf{C}_{\Phi\Phi}$, (b) $\mathbf{C}_{AA}$, (c) $\mathbf{C}_{II}$, (d) $\mathbf{C}_{\Phi A}$, (e) $\mathbf{C}_{\Phi I}$, (f) $\mathbf{C}_{AI}$

ditional mean and variance calculation is replaced by an extend ordinary cokriging, or by an extended rescaled ordinary cokriging system, respectively. 1000 independent realizations are generated by performing the sequential simulation process with 1000 random paths, conditioned to known porosity (Figure 1), inverted P-impedance data (Figure 4), and RMS amplitude slice (Figure 3). Figure 7 and Figure 9 shows 9 of all 1000 realizations by performing sequential conditional simulation using extend OCK and ROCK systems, respectively.

Note that all realizations shown in Figure 7 and in Figure 9 delineate the correct trends of porosity variation, i.e., high porosity values around well 08-08, 09-08, 29-08, 16-08 and around the right bottom corner with well 13-16, which can be identified using the prior information of known "model parameters" shown in Figure 1. However, the SCGS with extend OCK method generated negative values in all realizations (see in Figure 7) which

didn't occurred in simulation with extend ROCK approach (shown in Figure 9) due to only one constraint applied for the weight vector. Therefore, the extend rescaled ordinary cokriging with one unbiasedness condition advocates a much more reasonable and stable version of sequential conditional Gaussian simulation.

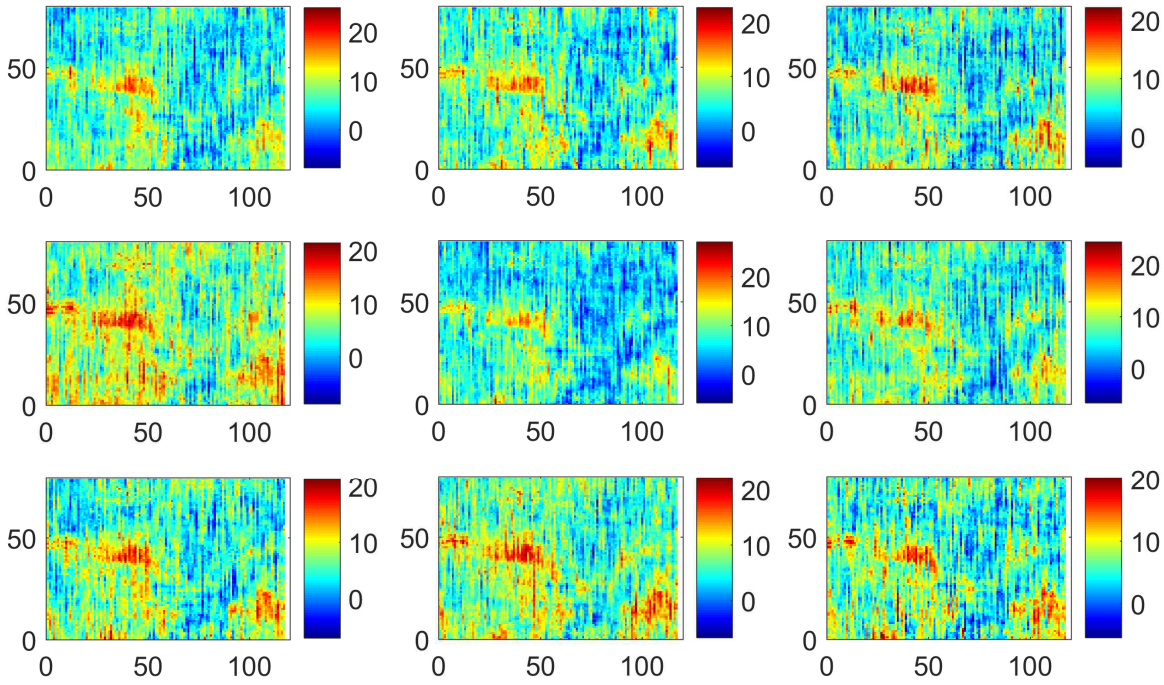


FIG. 7: All 9 of 1000 realizations using sequential conditional Gaussian simulation with extend ordinary cokriging (OCK) system.

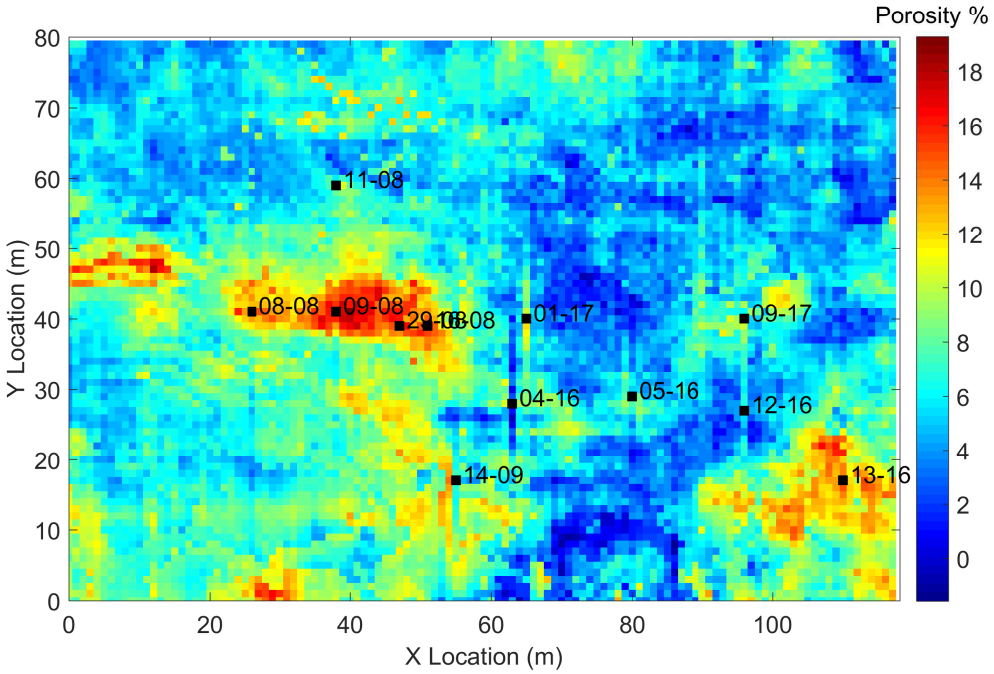


FIG. 8: Porosity mean value map of all 1000 realizations using SCGS with extend OCK.

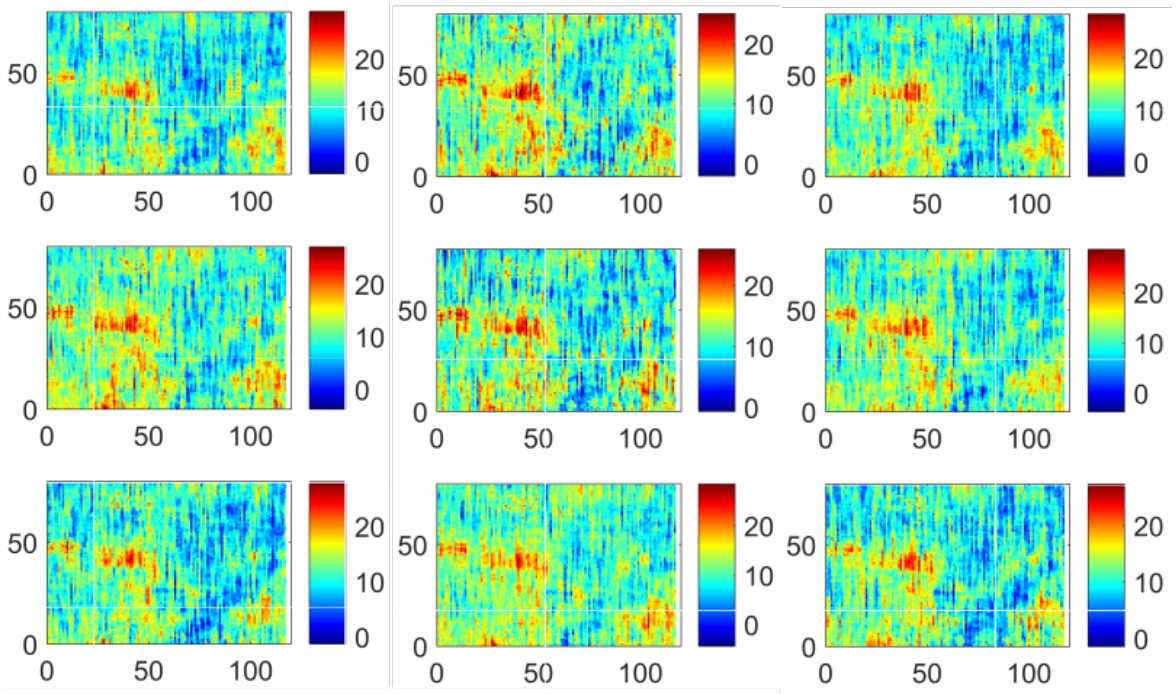


FIG. 9: All 9 of 1000 realizations using sequential conditional Gaussian simulation with extend rescaled ordinary cokriging (ROCK) system.

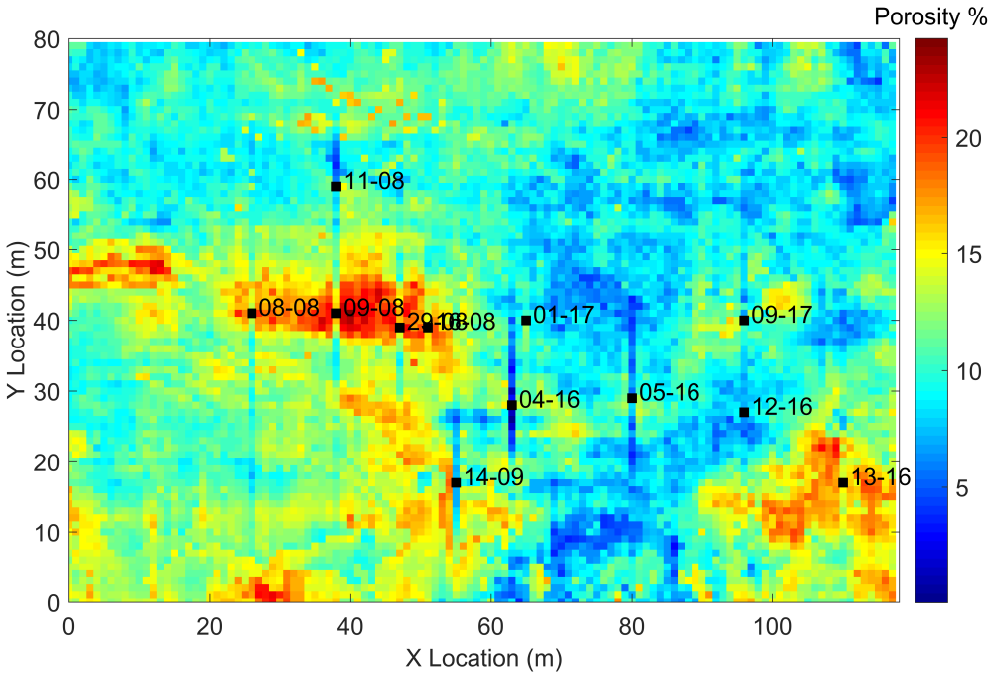


FIG. 10: Porosity mean value map of all 1000 realizations using SCGS with ROCK.

The mean porosity maps obtained by SCGS with extend OCK and ROCK based on 1000 realizations are shown in Figure 8 and Figure 10. A similar trend of porosity variation on mean maps are achieved by SCGS, either with extended OCK system or using extended ROCK approach. Both are analogous to the estimated map we illustrated previously (Xu et al., 2015), which reinforces the validity and feasibility of the sequential

conditional simulation using an extend cokriging system presented in this paper. Also note the difference between SCGS with extend OCK and with ROCK, porosity values estimated by ROCK are higher than those estimated using OCK, and negative values are avoided.

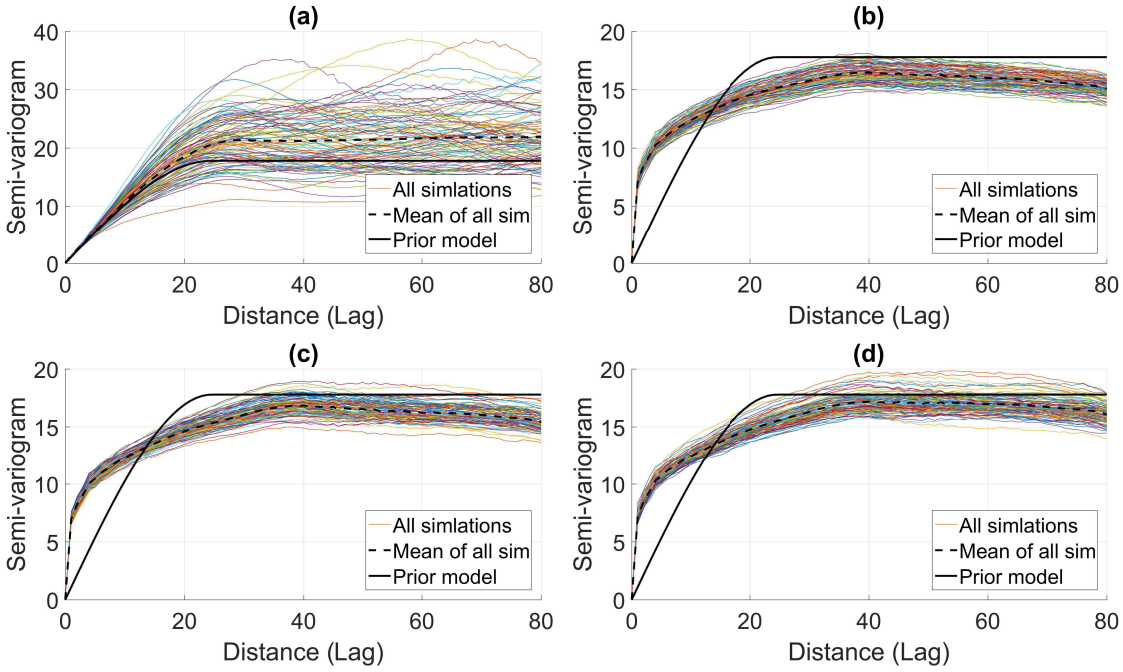


FIG. 11: Comparisons of semi-variograms. (a) semi-variograms of 100 realizations and its mean using unconditional simulation, (b) semi-variograms of 100 realizations and its mean using SCGS with least-square method, (c) semi-variograms of 100 realizations and its mean using SCGS with extend OCK system, (d) semi-variograms of 100 realizations and its mean using SCGS with extend ROCK system.

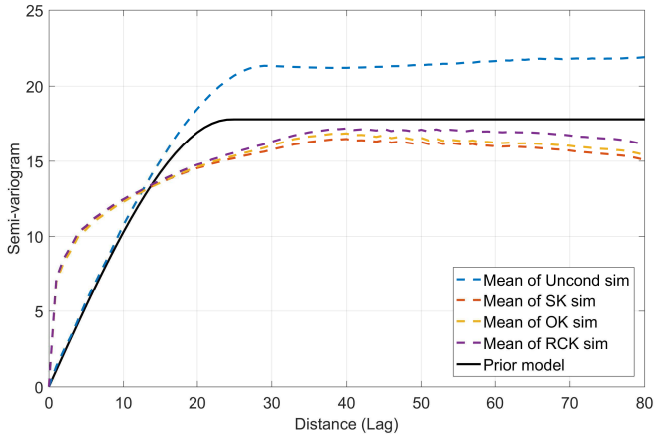


FIG. 12: Probability map, using SCGS with extend ROCK, where porosity value is higher than 12%.

To analyze the difference of SCGS with LS approach and with extend geostatistical methods, the unconditional simulation and SCGS with least square method are also performed. Figure 11a shows the semi-variogram calculated from each of 100 realizations using unconditional simulation. In Figure 11b, the semi-variogram are obtained from each

of 100 realizations using SCGS with least square method. Figure 11c illustrates the semi-variogram obtained from each of 100 realizations using SCGS with extend OCK system. And comparisons of semi-variograms of each of 100 realizations using SCGS with extend ROCK approach are delineated in Figure 11d. The ergodic fluctuations of conditional simulation (Figure 11b,c,d) are noticeable smaller as compared to those observed for unconditional simulation (Figure 11a). Further analysis are discussed by comparing the semi-variogram of their mean map, shown in Figure 12. The sill of semi-variogram obtained from unconditional simulation is much higher than conditional simulations with other three approaches.

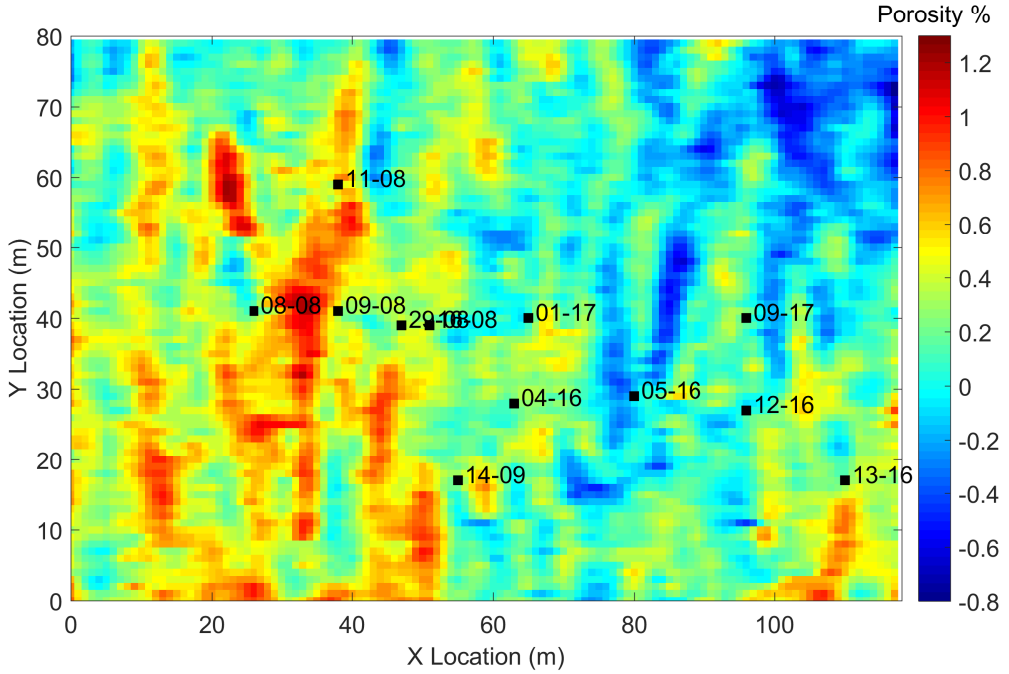


FIG. 13: The error of mean map from 100 realizations between SCGS with least square and with extend ROCK.

The difference of SCGS with least square compared to the extended ROCK system can also be ivestigated by calculating the error of mean map between two different approachs, shown in Figure 13. Figure 13 also verifies the demonstration made by Goovaerts (1998) that, the departures of estimated primary and all secondary local means from stationary means result in the difference between simple cokriging and (rescaled) ordinary cokriging. The related error difference between extend simple cokriging (SCK) and extend rescaled ordinary cokriging (ROCK) at a particular location $\mathbf{x}$, can be written as

$$Z_{ROCK}(\mathbf{x}) - Z_{SCK}(\mathbf{x}) = \left[\lambda_1^{SCK} + \lambda_2^{SCK} + \lambda_3^{SCK}\right] \left[m_{ROCK}(\mathbf{x}) - m_1\right] \qquad (15)$$

where, $Z_{ROCK}(\mathbf{x})$ and $Z_{SCK}(\mathbf{x})$ denote estimations calculated from extend ROCK and SCK systems, respectively. m_1 represent the stationary mean, and $m_{ROCK}(\mathbf{x})$ are the estimated local mean at each $\mathbf{x}$ location using both primary and all secondary data in the neighbourhoods. Therefore, Eq. (15) indicates that extended ROCK estimates are higher than extended SCK estimates where the local mean is higher than the global mean, and vice versa. To observe the porosity distribution when it is higher than a particular value over

the entire area institutive, we present the probability map,using SCGS with extend ROCK, in percentage with porosity value higher than 12% (Figure 14), which also conforms to the porosity variation on the mean map shown in Figure 10.

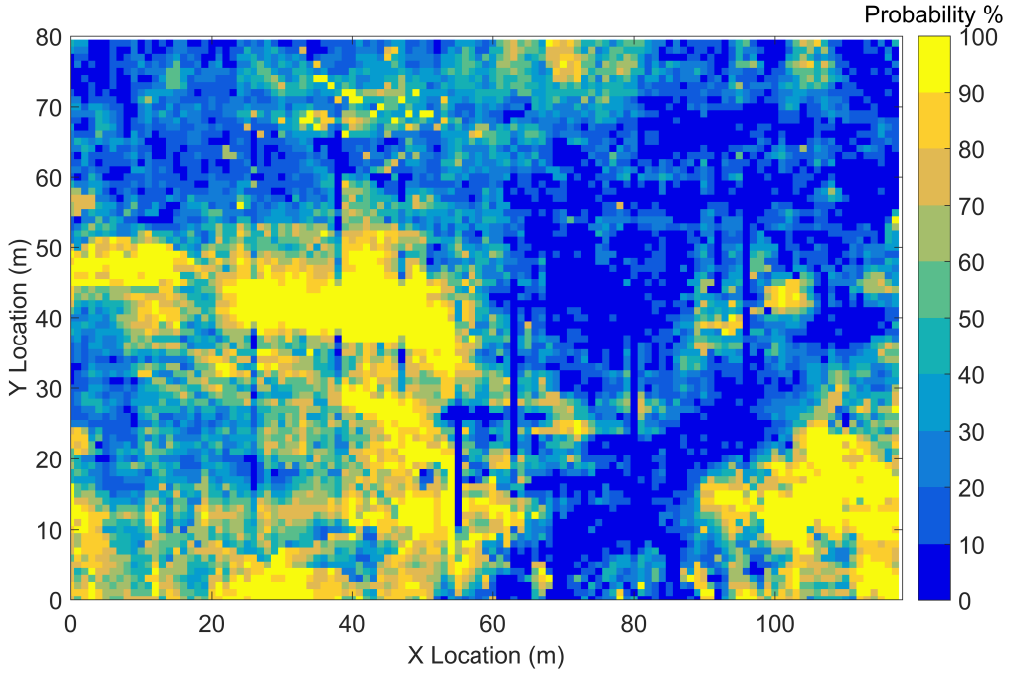


FIG. 14: Probability map, using SCGS with extend ROCK, where porosity value is higher than 12%.

CONCLUSIONS

Sequential conditional simulation is an effective tool to generate independent realizations for a second order stationary random field. In the case of geostatistics, the extend ordinary cokriging system allows more than one secondary variables to participate in the estimation process, and also prevents unacceptable (singular, or negative values) values to be produced in the final result by adding the unbiasedness constraint for the coefficient vector. In this paper, inspired by study of the extended cokriging system, we presented the sequential conditional Gaussian simulation with the extend ordinary cokriging method. By pre-calculating the covariance tables among known and conditioning datasets, and restricting the neighbourhood search strategy to be limited to the covariance matrix size, the sequential conditional simulation with an extend cokriging approach can be made computationally efficient. Furthermore, the constraint applied to the weights leads to an more adaptable and feasible version of sequential simulation.

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