Internal multiple prediction with higher order terms and a new subtraction domain

Andrew Iverson, Scott Keating, Kris Innanen and Daniel Trad

ABSTRACT

Inverse scattering series internal multiple attenuation is a promising method for predicting and attenuating internal multiples. Though the method has displayed great potential, it is not a routine step in the seismic processing workflow. Some of the issues with the method include small errors with the predicted amplitudes, which are corrected with an adaptive subtraction. After subtraction, the question often arises whether primary energy is being damaged in the removal of internal multiples. A new domain to carry out the adaptive subtraction is introduced here, which may mitigate this risk. This domain is a more natural space to create the filter as systematic amplitude errors due to the prediction algorithm can be corrected in this space. To further assist the amplitude mismatches, additional terms from the scattering series are incorporated, and the importance of how these terms are implemented is discussed. Combining this higher dimensional adaptive subtraction space with the higher order terms significantly improves the accuracy of the prediction.

INTRODUCTION

Inverse scattering series (ISS) internal multiple attenuation is a powerful tool to attenuate internal multiples developed by Weglein et al. (1997). The method is data driven as the internal multiples are predicted using only the input data with no subsurface information requirements. The kinematics of predicted internal multiples are correct, but errors in the event amplitudes remain. These amplitude issues are often rectified through the use of adaptive subtraction after the prediction has been made. It has been shown that the error adjustments required can vary rapidly for separate internal multiples adjacent in time (Iverson and Innanen, 2017). When this occurs, it is difficult to build a subtraction filter that can account for these rapid variations in prediction errors without harming primaries. Another proposed method uses a variant of the algorithm, referred to as internal multiple elimination, which has improved on the issue (Ma and Weglein, 2015; Zou et al., 2016). It has also been shown that using higher order terms from the inverse scattering series can assist with certain amplitude errors and erroneous predictions (Liang et al., 2013). Here, we show that combining higher order prediction terms with in a new higher dimensional adaptive subtraction domain improves the amplitude predictions of internal multiples. The higher dimensional prediction domain was introduced previously (Iverson and Innanen, 2017), it is here outlined how completing the adaptive subtraction in this domain is beneficial.

ADAPTIVE SUBTRACTION IN DOWNWARD GENERATOR SPACE

The inverse scattering series internal multiple attenuation algorithm was first developed in the late 1990’s (Weglein et al., 1997). Though the series is infinite, in practice only the first term (b_3) in the series is generally utilized. It has been shown that the 3D algorithm can be simplified to a 1D version requiring only two integrals instead of three. This formulation is given in Equation 1 (e.g. Eaid et al., 2016).

$$b_3(\omega) = \int_{-\infty}^{\infty} dz_1 e^{-i2\frac{\omega}{c_0}z_1} b_1(z_1) \left[\int_{z_1+\varepsilon}^{\infty} dz_2 e^{i2\frac{\omega}{c_0}z_2} b_1(z_2) \right]^2 \tag{1}$$

Where b_3 is the internal multiple prediction, b_1 is the prepared input data z_1 and z_2 are the pseudo-depths chosen to satisfy lower-higher-lower relationship and ε is the search limiting parameter. Equation 1 is operating by solving for all possible combinations of events which obey the lower-higher-lower criteria generated by the higher term from the criteria or downward generating horizon z1. By storing every downward generator solution and solving for b3(z1, ω) this builds a 2D matrix of the internal multiple predictions which can then be inverse Fourier transformed over the frequency direction (Iverson and Innanen, 2017). It has been shown how this 2D space was used to visualize the various multiples and better understand the prediction solution, and to apply a precalculated scalar to adjust amplitudes (Iverson and Innanen, 2017). To further utilize this downward generator space 2D adaptive subtraction will be used.

Downward generator space adaptive subtraction

An adaptive subtraction algorithm has been developed which has been previously tested on internal multiple attenuation (Keating et al. 2015). This displayed results for cases with either the L1, L2 or hybrid L1/L2 norms were minimized. It was displayed how using the hybrid norm assisted the internal multiple prediction and allowed for the subtraction of unwanted multiples from a trace (Keating et al. 2015). The downward generator space was initially developed to assist in the issue of requiring different amplitude corrections due to transmission losses which was a function of the downward generator (Iverson and Innanen, 2017). This then gives a natural space to perform a 2D adaptive subtraction.

In the approach adopted here, the adaptive subtraction is performed by applying a filter designed in the downward generator - time space to the predictions and subtracting the result from the data in time. The filters used were allowed to vary over a small range of times, but all downward generators. This means that there were a very large number of filter coefficients used, due to the size of the downward generator space. In a conventional adaptive subtraction approach, where the filter used is given by minimizing

$$\phi(f) = ||\, d \,-\, Mf||^p{}_p, \tag{2}$$

where d is the time series for the data, M is a matrix representing convolution with the predicted multiples, f is the filter and p is a chosen parameter, having too many filter coefficients will result in overfitting of the data, potentially removing multiples (e.g. Guitton and Verschuur, 2004). To prevent this from happening, we instead minimize

$$\phi(f) = ||\, d - Mf||^p{}_p + \phi_R, \tag{3}$$

where φ_R is a term penalizing variation of the filter in downward generator space. If this penalty term is chosen to be large, the large number of filter coefficients will change only gradually in the downward generator space direction, preventing overfitting.

1D example

An example is used to display the benefit of 2D adaptive subtraction in the downward generator space. This example has three layers with several first order and higher order multiples (Figure 1) which has been used previously (Iverson and Innanen, 2017).

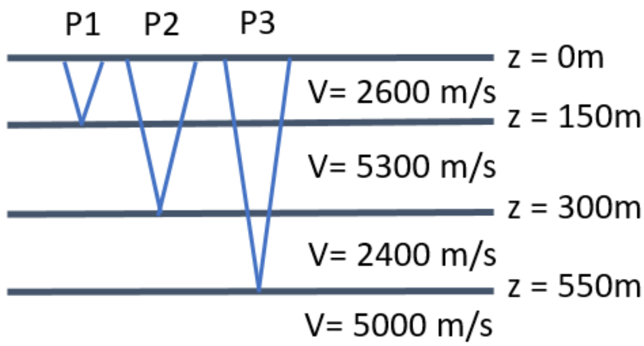

FIG. 1. Velocity and depth model used for the 1D prediction.

The prediction using the internal multiple attenuation algorithm is displayed below with no adaptive subtraction (Figure 2). It has predicted the kinematics of the event but with amplitude mismatches compared to the input trace.

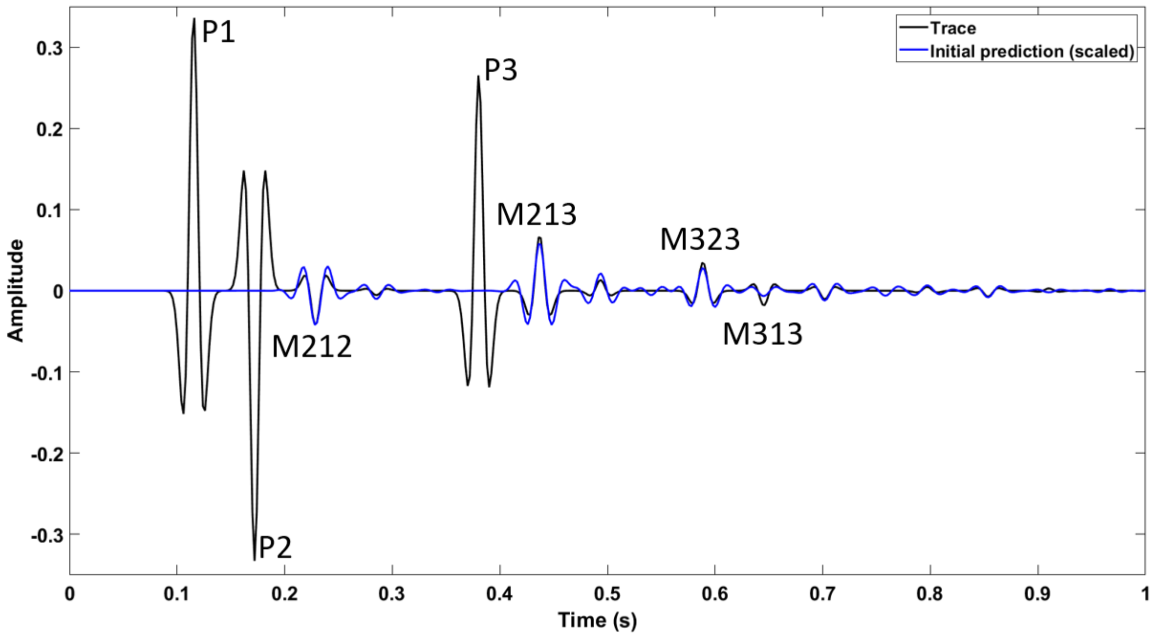

FIG. 2. Input trace and 1D internal multiple prediction.

While calculating the prediction the downward generator space was also created for the 2D adaptive subtraction (Figure 3). Note that each of the 2D wavelet like events in the space represent an internal multiple. If this was stacked over all off the rows (downward generator dimension) this would give the standard 1D result.

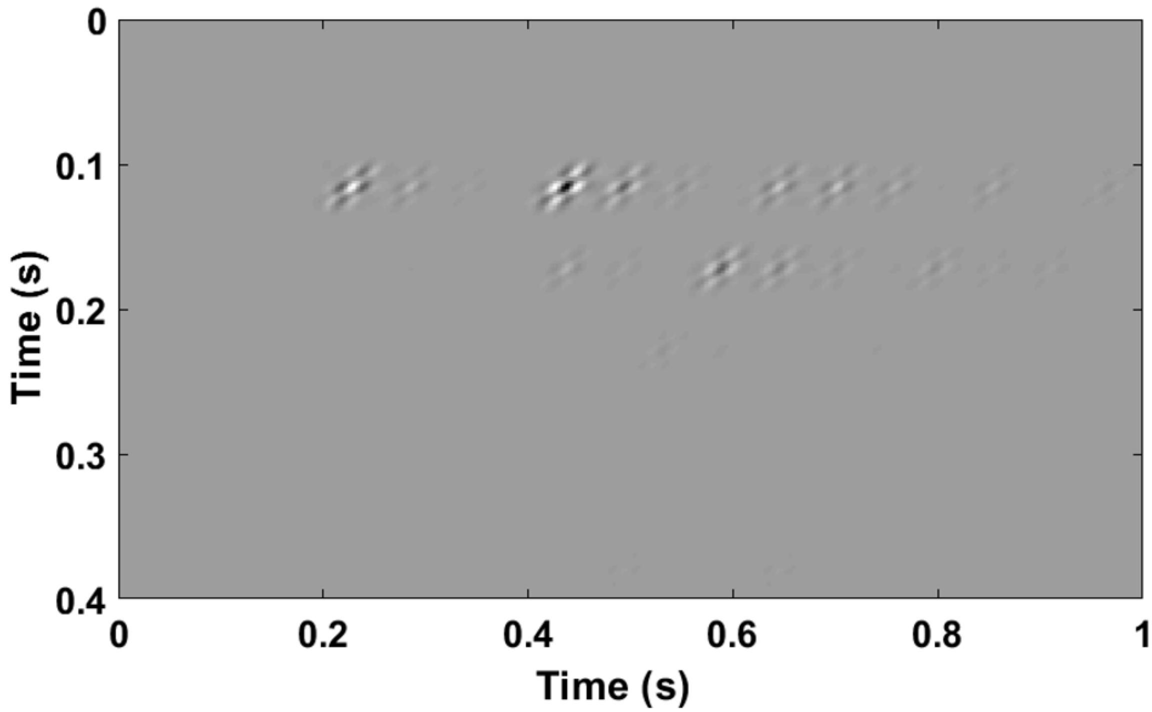

FIG. 3. 2D downward generator space displaying individual internal multiples. Prediction time is on the x axis and generator time on the y axis.

This is then put into the 2D adaptive subtraction, giving the result shown in Figure 4. In practice, the 2D trace is partially stacked along the row direction and this is done for two primary reasons. The first to reduce the computational expense of the subtraction by decreasing the dimensionality of the filter design problem, the second is to at a minimum stack over the width of the wavelet to reduce the potential for overfitting.

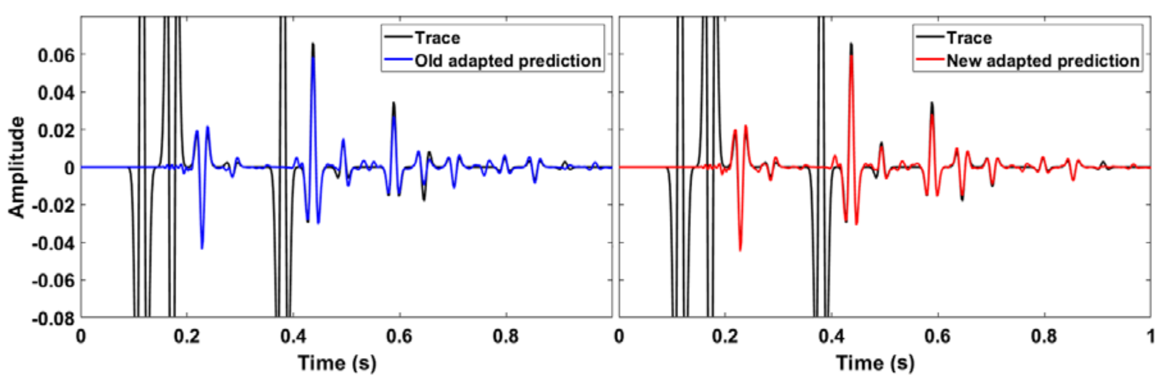

FIG. 4. (Left) Internal multiple prediction with 1D adaptive subtraction (Right) Internal multiple prediction with 2D adaptive subtraction

This figure demonstrates that utilizing this 2D space to carry out the adaptive subtraction can improve the prediction. Improvements in the subtracted result are especially evident at about 0.65s. This can attempt to account for the losses due to the downward generator which the original algorithm does not account for. The method also remains data driven as there are still no subsurface information requirements. The 2D adaptive subtraction improved the prediction but amplitude mismatches in the prediction remain. To further assist the prediction, higher order terms from the inverse scattering series will be explored.

HIGHER ORDER TERMS IN SERIES

Higher order multiples

The ISS multiple prediction algorithm uses all events in the data to predict internal multiples. When internal multiples are present in the data these events are also used in the algorithm to predict higher order multiples. This causes second order multiples to be generally overpredicted (Zhang & Shaw, 2010). The reason for the overprediction is that each second and higher order multiple can be predicted from several different combinations of events (Figure 5).

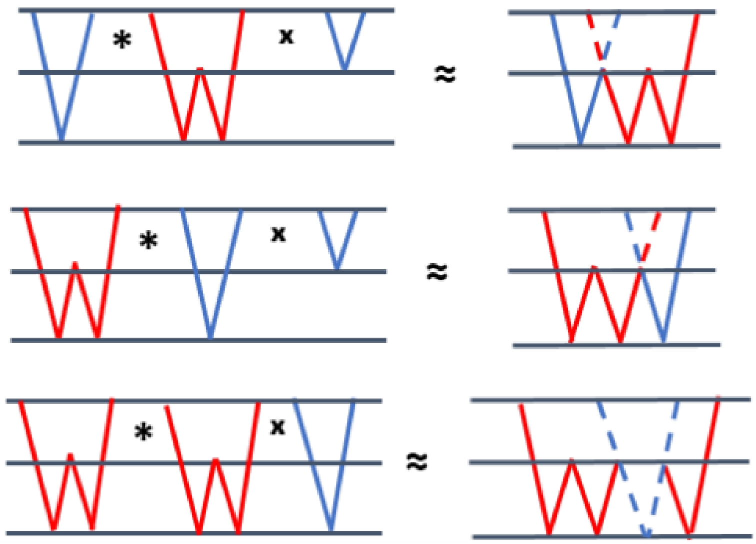

FIG. 5. Combinations of primaries (blue) and multiples (red) which will predict the same second order multiple M21212

These higher order multiples can also be seen in the downward generating space. This is shown in figure 6, where the second order multiple M21212 is outlined. Higher order multiples require different amplitude adjustments than first order multiples as they have been overpredicted. What is required is a means to isolate the higher order multiples from the first order multiples. To prevent this overprediction, we need an approach that can remove duplicate predictions of the same higher order event.

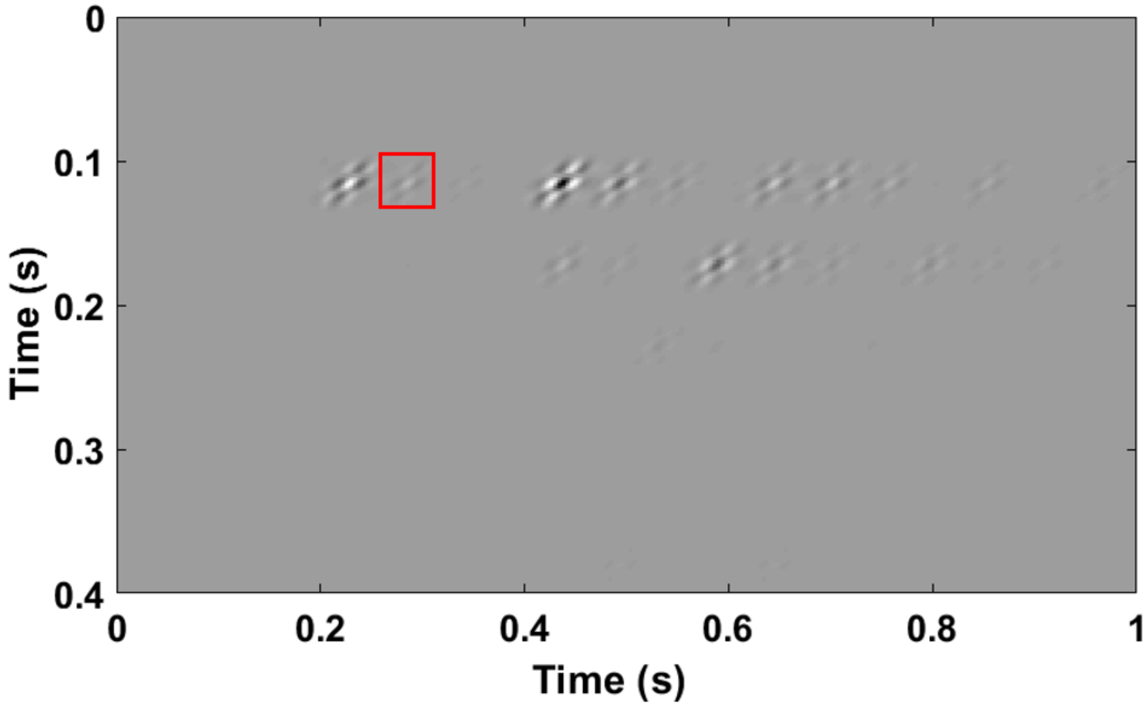

FIG. 6. 2D downward generator space displaying individual internal multiples, red box displaying multiple M21212

Inverse scattering series

The ISS multiple prediction algorithm was derived by selecting a subset of the inverse scattering series which obeys the lower-higher-lower criterion. The series is infinite but often truncated to only use the first term for prediction. Because the full series should correctly predict all multiples, the problems caused by overpredicted higher order events must be addressed by higher order terms in the series. The original algorithm calculates b3 from the input data b1 through the prediction

$$b_3(\omega) = \int_{-\infty}^{\infty} dz_1 e^{i2\frac{\omega}{c_0}z_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-i2\frac{\omega}{c_0}z_2} b_1(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{i2\frac{\omega}{c_0}z_3} b_1(z_3). \quad (4)$$

Some higher order terms can be calculated by taking the multiple prediction output (b3) and using this in place of b1 in one of the terms of the ISS multiple prediction algorithm (Equation 4) (Liang et al., 2013). This yield

$$b_5^{PPI}(\omega) = \int_{-\infty}^{\infty} dz_1 e^{i2\frac{\omega}{c_0}z_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-i2\frac{\omega}{c_0}z_2} b_1(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{i2\frac{\omega}{c_0}z_3} b_3(z_3), \quad (5)$$

The first higher order term to be used is referred to as b5PPI since it uses two data traces and then the internal multiple trace (Liang et al., 2013). It has been noted that this term should be scaled by a factor of 2 to account for both the b5PPI and b5IPP. It will be displayed for these tests that a scalar of 1 was used successfully. By having one of the input traces being the predicted internal multiples there are theoretically no primaries in this trace. This can then be used to predict higher order multiples which are causing the previously noted issue. There is also the potential for pure artifacts to arise in the prediction when only the first term is used. This term is referred to as b5PIP (Equation 6) (Liang et al., 2013). By using a similar approach but in a different portion of the algorithm we can also attempt to attenuate this issue. This higher order term is used to correct an event that is predicted that is purely an artifact. (Liang et al., 2013)

$$b_5^{PIP}(\omega) = \int_{-\infty}^{\infty} dz_1 e^{i2\frac{\omega}{c_0}z_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-i2\frac{\omega}{c_0}z_2} b_3(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{i2\frac{\omega}{c_0}z_3} b_1(z_3), \quad (6)$$

Implementing the higher order terms

The higher order terms require the output from the original algorithm (b3). A key step to have success with the higher order terms (b5) is to scale the data prior to putting the internal multiple trace back in the algorithm for the higher order terms calculation. At a minimum a scalar is generally required to fit the amplitudes back to the input data. Another option is to use adaptive subtraction and attempt to both scale the data and adjust the frequency content from the algorithm due to convolving the data with itself (Iverson and Innanen, 2017).

Higher order terms in downward generator space

Using the same synthetic trace, the downward generator space is used to display b3 and the two b5 higher order terms (Figure 7). Outlined in the b3 space is the four first order multiples. The second image displays the predictions of the b5PPI term. It shows that the higher order terms can be separated from the first order multiples. Finally, the last displays events that are purely artifacts in the prediction. Remembering that the final 1D trace is created by summing over all the rows, this highlights how different internal multiples can contribute at a single point in time. There may be an option to use all three spaces to see if a different scalar besides the value of one may benefit the final prediction. At this point this is not tested. If there were any problems with scaling of the initial prediction of b3 prior to being put into higher order algorithm, then this could still be adjusted in this space.

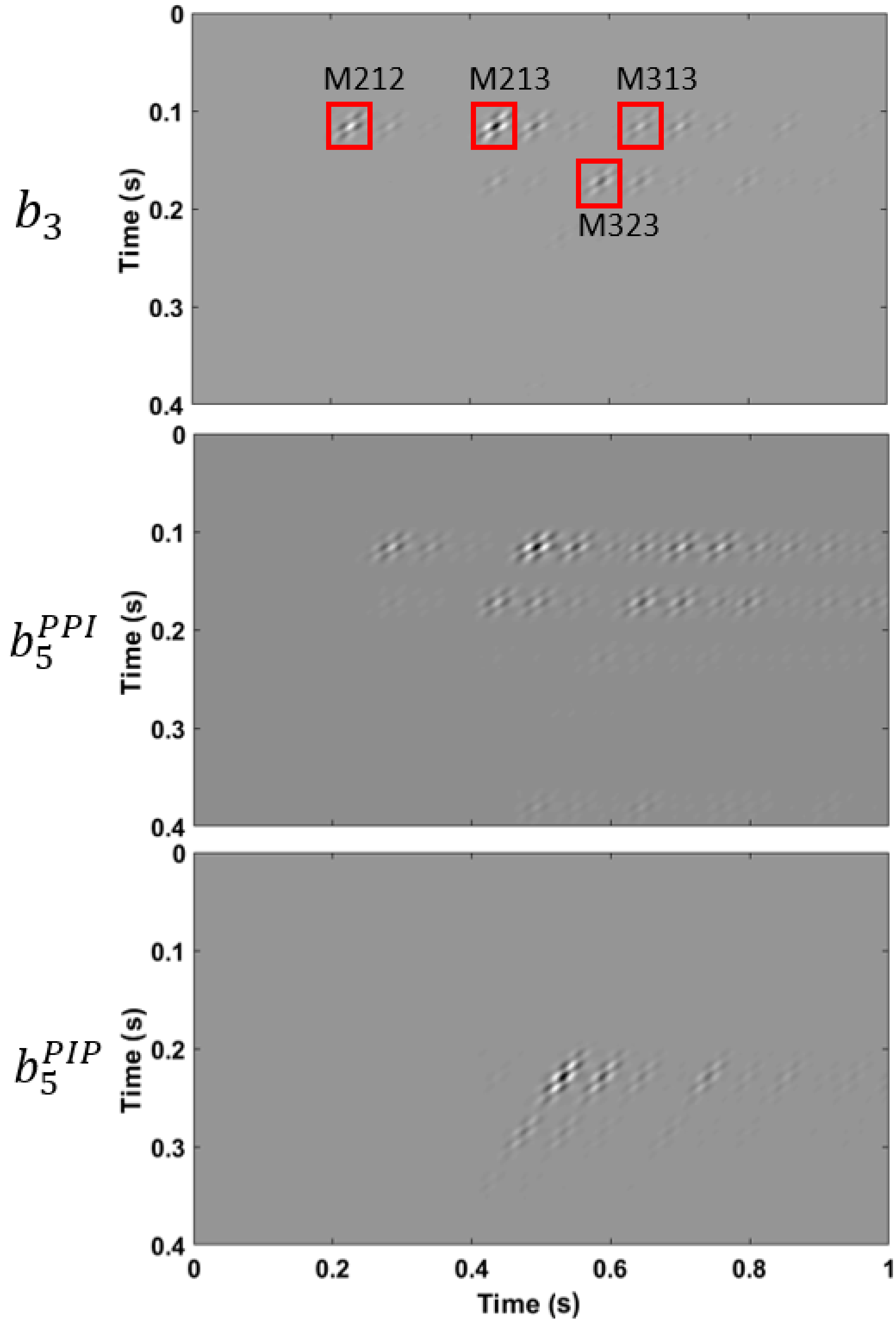

FIG. 7. 2D downward generator space displaying individual internal multiples (Top) b3 term with first order multiples displayed with red box (Middle) b5PPI higher order multiples (Bottom) b5PIP artifacts from the prediction

These spaces will be summed to create one 2D space to be used for the prediction (Figure 8). By summing all these terms in the 2D space the result is a downward generator space which has been corrected for the overpredictions. This 2D space can now be used as previously shown with either the scalar or the 2D adaptive subtraction to obtain a final prediction.

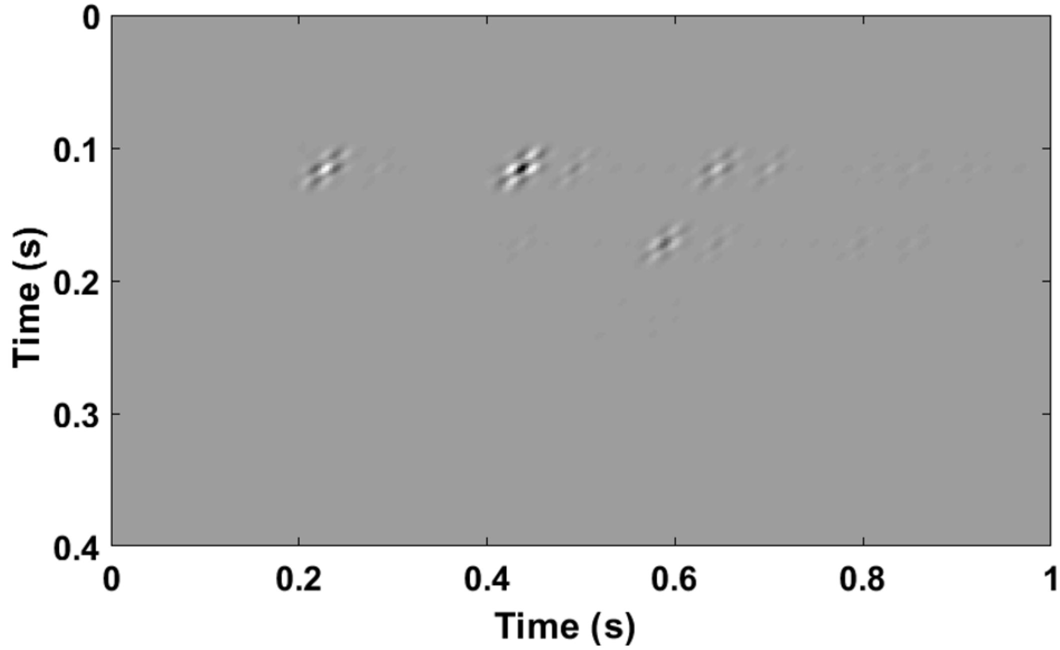

FIG. 8. 2D downward generator space displaying individual internal multiples after including higher order terms

By using the 2D adaptive subtraction the result is displayed (Figure 9). This has improved the relative amplitudes significantly.

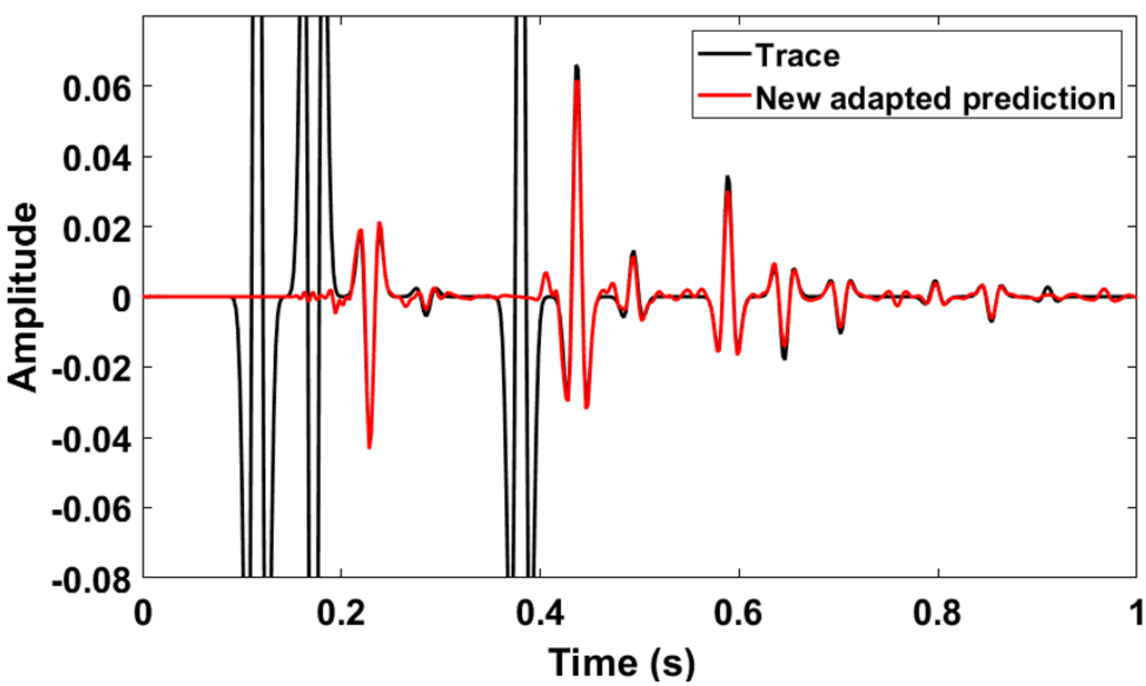

FIG. 9. Internal multiple prediction with higher order terms and 2D adaptive subtraction. Compare to figure 4.

By combining both higher order terms and the 2D space a significant improvement in the prediction is found. Previously to correct these amplitudes a significantly non-stationary or harsh adaptive subtraction would be required. Through the combination of both the 2D space and higher order terms these multiples can now be more accurately attenuated.

1D example with small primary

To display the potential uplift of these terms and why improved amplitude prediction is necessary a subtle addition to this example is displayed. An additional layer is added to the previous model (Figure 10).

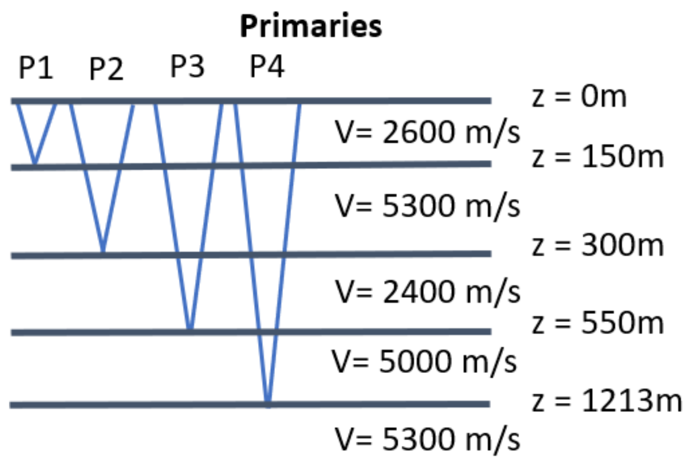

FIG. 10. Velocity and depth model with additional layer used for 1D prediction.

The addition of this layer was done to create a multiple which occurs at the same time with opposite polarity of an internal multiple (Figure 11). The result is an input trace which has no evidence of the primary as it has destructively interfered with the multiple.

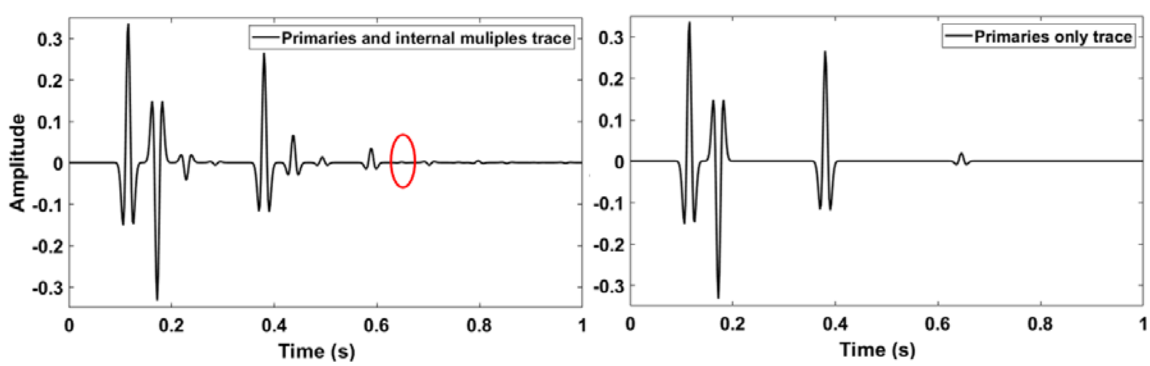

FIG. 11. (Left) Primaries and multiples trace highlighting location of missing primary due to internal multiples (Right) Primaries only trace

Following a similar methodology to that outlined above, the results are displayed to show the uplift of both the higher order terms and combing it with the 2D adaptive subtraction (Figure 12). This first plot displays both the b3 prediction with adaptive subtraction and the 2D adaptive subtraction with higher order terms.

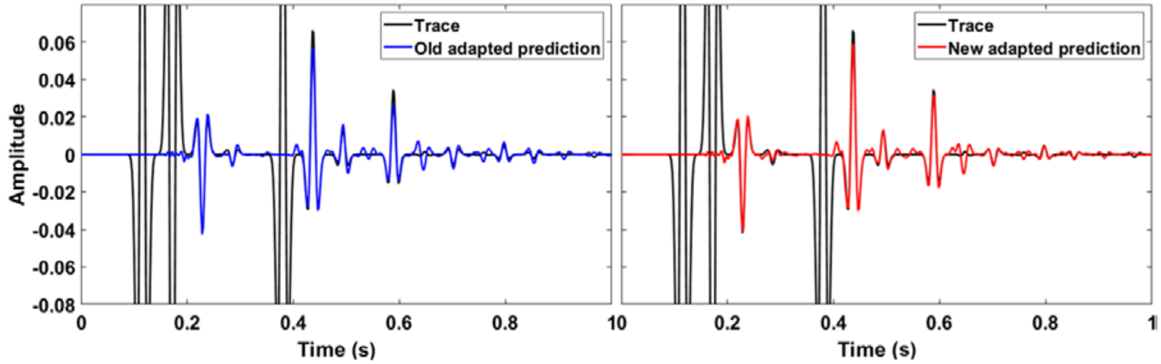

FIG. 12. (Left) Internal multiple prediction with 1D adaptive subtraction (Right) Internal multiple prediction with 2D adaptive subtraction and higher order terms

The multiple attenuated trace is then created by subtracting the trace and the 1D adaptive subtraction (Figure 13). The result shows a number of small amplitude events which make it difficult to distinguish if any of these are primaries.

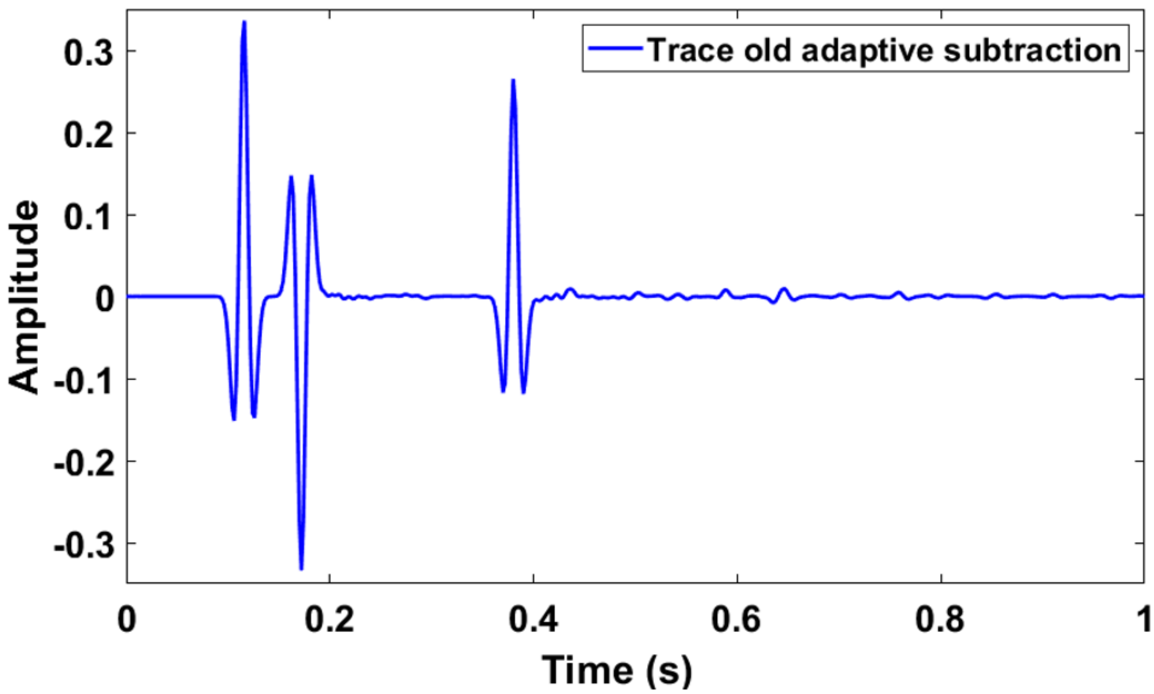

FIG. 13. Trace with internal multiple attenuation using the 1D adaptive subtraction

The second multiple attenuated trace is created by subtracting the trace with higher order terms and the 2D adaptive subtraction (Figure 14). The small fourth primary event is now visible, and this display the importance of correcting for these amplitude issues. This gives increased confidence in the prediction to reduce potential harm to primary events.

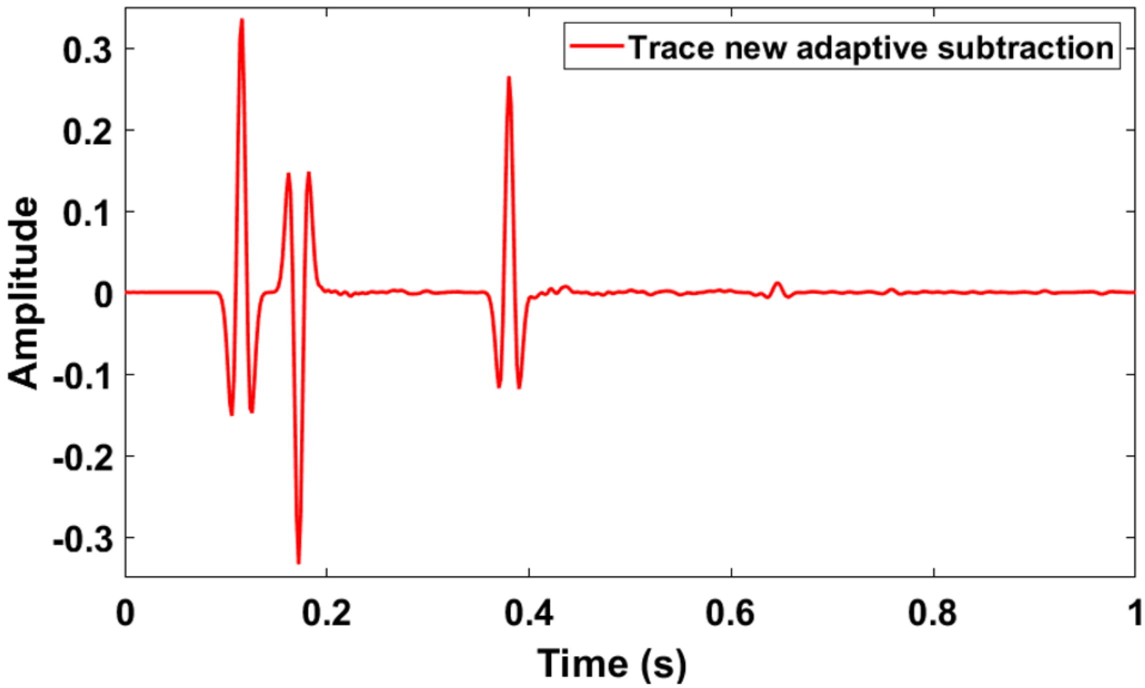

FIG. 14. Trace with internal multiple attenuation with 2D adaptive subtraction and higher order terms

CONCLUSIONS

Using the 2D downward generator space can assist in the adaptive subtraction of internal multiples. By also including higher order terms in the prediction series, the accuracy of the prediction can be further increased. A key question to be addressed is how practical this extra dimension will become with real data. Once various amplitude recovery techniques have been used, or if there are other factors such as attenuation or other losses across the downward generator will this extra dimension be able to assist.

ACKNOWLEDGMENTS

The authors would like to thank the sponsors of CREWES for the support of this work. This work was also funded NSERC through the grant CRDPJ 461179-13. Andrew Iverson would also like to thank NSERC and SEG for their support.

REFERENCES

Eaid, M., Sun, J., Keating, S., and Innanen, K. A., 2016, 1D and 1.5D internal multiple prediction in MatLab, CREWES Annual Report, 28.

Guitton, A., and Verschuur, D., 2004, Adaptive subtraction of multiples using the l1-norm, Geophysical Prospecting,52.

Iverson, A., and Innanen, K. A., 2017, Inverse scattering series internal multiple prediction with depth dependent scalars, CREWES Annual Report, 29.

Keating, S., Sun, J., Pan, P., and Innanen, K. A., 2015, Nonstationary l1 adaptive subtraction with application to internal multiple attenuation, CREWES Annual Report, 27.

Liang, H., Ma, C., and Weglein, A. B., 2013, General theory for accommodating primaries and multiples in internal multiple algorithm: Analysis and numerical tests, SEG Technical Program Expanded Abstracts 2013, pp 4178-4183.

Ma, C.*, and Weglein, A. B., 2015, A new Inverse Scattering Series (ISS) internal-multiple-attenuation algorithm that predicts the accurate time and approximate amplitude of the first-order internal multiples and addresses spurious events: Analysis and Tests in 2D, SEG Technical Program Expanded Abstracts 2015, pp 4402-4407.

Weglein, A. B., Gasparotto, F. A., Carvalho, P. M., and Stolt, R. H., 1997, An inverse-scattering series method for attenuating multiples in seismic reflection data, Geophysics, 62(6), 1975–1989.

Zhang, H., and Shaw, S., 2010, 1D analytical analysis of higher order internal multiples predicted via the inverse scattering series based algorithm, SEG Technical Program Expanded Abstracts 2010, pp 3493-3498.

Zou, Y., Ma, C., and Weglein, A. B., 2016, The first inverse-scattering-series internal multiple elimination method for a multidimensional subsurface, SEG Technical Program Expanded Abstracts 2016, pp 4550-4554.