Formulating full waveform inversion as a least squares migration-type problem

Scott Keating and Kris Innanen

ABSTRACT

The imaging of seismic reflectors can be iteratively improved by using a least squares migration (LSM) approach. Full waveform inversion (FWI) is usually posed as an inverse problem for the long to intermediate scale features of a seismic model, with little contribution from reflectors. Here, we show that full waveform inversion can be formulated to perform a similar role to LSM by choosing an appropriate choice of model parameterization. This approach specifically recovers the small wavelength features of the subsurface and does not require a complicated objective function or data filtering. Numerical tests show that this approach can be more effective than conventional FWI in recovering seismic reflector information.

INTRODUCTION

Full waveform inversion (FWI) is a strategy for estimating the seismic properties of the subsurface by finding the subsurface model which best reproduces the full information content of a measured seismic experiment (Tarantola, 1984; Lailly, 1983; Virieux and Operto, 2009). This approach has been very successful in creating high resolution velocity models, and is rapidly developing as an inversion approach for anisotropic, viscoelastic, and poroelastic subsurface models (e.g. Tarantola, 1986; Hicks and Pratt, 2001; Choi et al., 2008; Alkhalifah and Plessix, 2014).

In FWI, the inversion is usually formulated as an optimization problem, where the minimum of an objective function is sought (Virieux and Operto, 2009). This objective function is a measure of the discrepancy between the observed data and the data predicted from the current model of the subsurface. The minimization is performed with respect to a set of variables parameterizing the subsurface model. While the first objective function proposed was the L_2 norm of the difference between these data sets (Tarantola, 1984; Lailly, 1983), the inversion result can be improved in many problems by considering different choices for the objective function. For instance, when considering inversion with a poor initial model, objective functions measuring phase differences (Fichtner et al., 2008), mass transport distances (Engquist and Froese, 2014), or filter coefficients (Warner and Guasch, 2016) often improve the inversion result. Other authors have found that emphasizing the contribution of reflection data in the objective function can improve the long-wavelength inversion results in deep parts of the model (Yang and Sava, 2011; Xu et al., 2012).

Another key consideration in the FWI optimization problem is the set of variables used to parameterize the model. Finite difference or finite element techniques are usually used to simulate wavefield propagation in FWI, and typically the variables defined in the inversion are the variables used in the forward modeling, specifying elastic properties at points in a grid. Unlike the objective function, this part of the optimization problem is rarely changed, except when considering different physical properties in the inversion. Studies which have experimented with different parameterizations have often been motivated by efficiency concerns (Bunks et al., 1995; Debens et al., 2015; Datta and Sen, 2016; Keating and Innanen, 2017). However, parameterization choice may also have the capacity to change the behaviour of the optimization problem, and this is the key concept explored in this report. In particular, we investigate the possibility of using an FWI approach to recover an accurate reflectivity model.

Seismic migration is a procedure for estimating a seismic reflectivity model of the subsurface (Claerbout, 1971). Reflectivity is not an inherent property of materials, but instead depends on property contrasts and wave characteristics, such as frequency and angle of incidence. Migration typically involves the assumption that reflections are caused by abrupt spatial changes in the subsurface properties that occur on much smaller scales than all seismic wavelengths, while the velocity model used to simulate wave propagation varies on much longer scales than seismic wavelengths, and thus doesn't introduce reflections. This assumption allows for a frequency-independent reflectivity. An angle-independent reflectivity is also usually assumed. While migration is highly effective, it often struggles to recover an accurate model in regions poorly illuminated by the seismic experiment.

To help mitigate this problem, least-squares migration (LSM) has been proposed (Nemeth et al., 1999). In LSM, migration is reformulated as an inversion problem in which reflectivity is recovered. The problem becomes an attempt to find the reflectivity model which best reproduces the data. Often, this reflectivity is assumed to be angle and frequency independent. This limits the effectiveness of LSM, as the measured data can often have significant angle and frequency dependence in reflectivity. In this report, we investigate the treatment of least-squares migration as an FWI problem with a specific choice of model parameterization. This may allow us to avoid some of the assumptions in LSM and so improve the inversion results.

THEORY

Appropriate parameterization for a LSM-type problem

The goal of the FWI procedure is to find an accurate approximation of chosen elastic properties of the subsurface. The best subsurface model is assumed to be that which most accurately reproduces the measured data in numerical modeling of the corresponding seismic experiment. At its core, the FWI approach is an optimization problem. Finding the best model is equivalent to finding the minimum of an objective function quantifying the data mismatch with respect to a set of variables describing the subsurface. This set of variables will be referred to as the inversion model. Under the constant density acoustic approximation, a simple choice for the inversion model is the set of variables $a_{i,j}$, where for any point (x_n, z_m) defined in the subsurface

$$\frac{1}{v(x_n, z_m)^2} = a_{n,m}, \tag{1}$$

and v represents P-wave velocity. In effect, each variable a is a coefficient for a corresponding basis function, where the basis consists of delta functions, as in figure 1. The squared slowness parameterization is often chosen for its simplicity and good convergence properties (Anagaw, 2014).

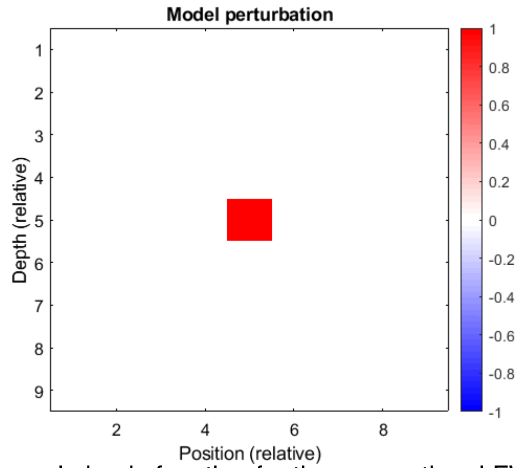


FIG. 1. Example basis function for the conventional FWI approach.

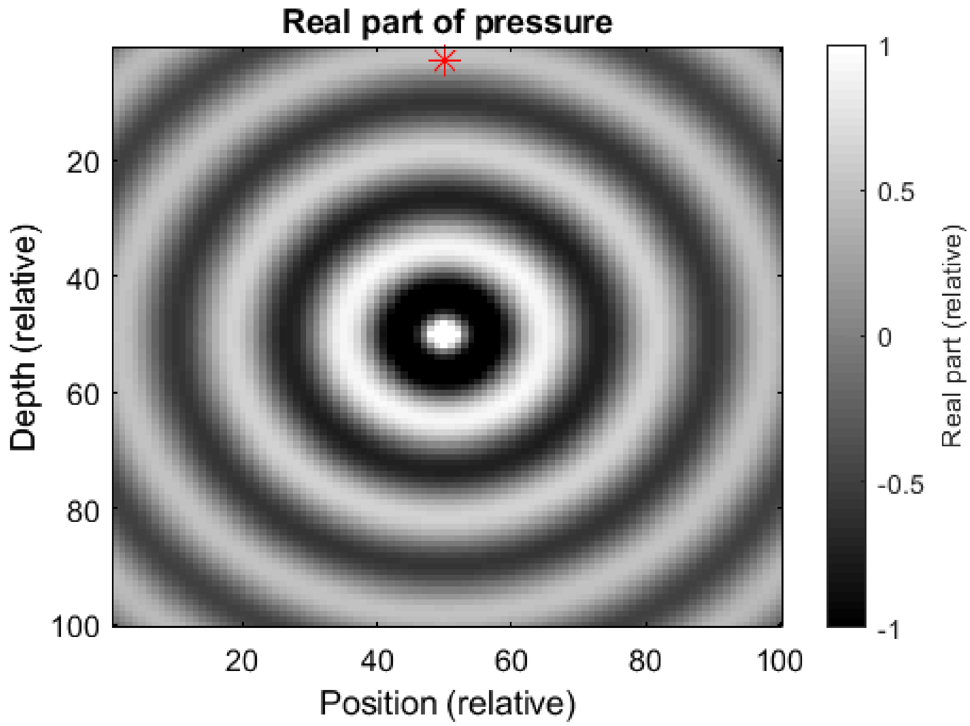
In this report we define the inversion model m such that it relates to a by the equation

$$a = Pm, \tag{2}$$

where P is a matrix defining a set of basis functions, and the vector a contains the elements $a_{i,j}$ for each subsurface location considered. In the conventional FWI approach, P is an identity matrix, and $m = a$. In this case the basis vectors in the columns of P, p_n, are delta-like functions (Figure 1) with strong capacity to change both transmissive and reflective wave behaviour. By altering the matrix P, we will try to formulate a LSM-type problem of reflectivity recovery, where long wavelength model features are neglected in the inversion.

Ideally, a LSM-type FWI will alter the data only by changing reflectors in the model, and not by affecting the traveltimes of seismic waves. The effect of altering a model variable on the wavefield can be investigated by studying the radiation pattern of the variable, the derivative of a modeled wavefield with respect to the variable. If a variable generates a radiation pattern with negligible amplitudes at scattering angles corresponding to transmission, it has little impact on transmitted waves. Numerical radiation patterns can be calculated by taking the difference between modeled wavefields in a reference and perturbed medium. The radiation pattern associated with a velocity perturbation like the basis function in figure 1 is shown in figure 2. The source location is indicated by a red star, and the result is shown with a frequency domain wavefield at 15Hz with constant-density acoustic finite difference wave propagation. For this type of model perturbation, the scattered wavefield is equal amplitude in all directions. This means that changing variables like these affects both waves transmitting through and reflecting from this part of the model. These variables are consequently ill-suited for recovering reflectivity if transmission related errors also exist in the data residuals.

Seismic reflectivity is caused by abrupt changes in the seismic properties of the subsurface. Traveltimes are instead determined by a sum of wave slownesses along a path from source to receiver. This suggests that primarily reflective variables will have sharp spatial changes, but no low frequency component. An appropriate basis function for introducing reflections with minimal impact on transmission, and its corresponding radiation pattern are shown in figures 3 and 4. This variable is a Laplacian operator instead of a delta



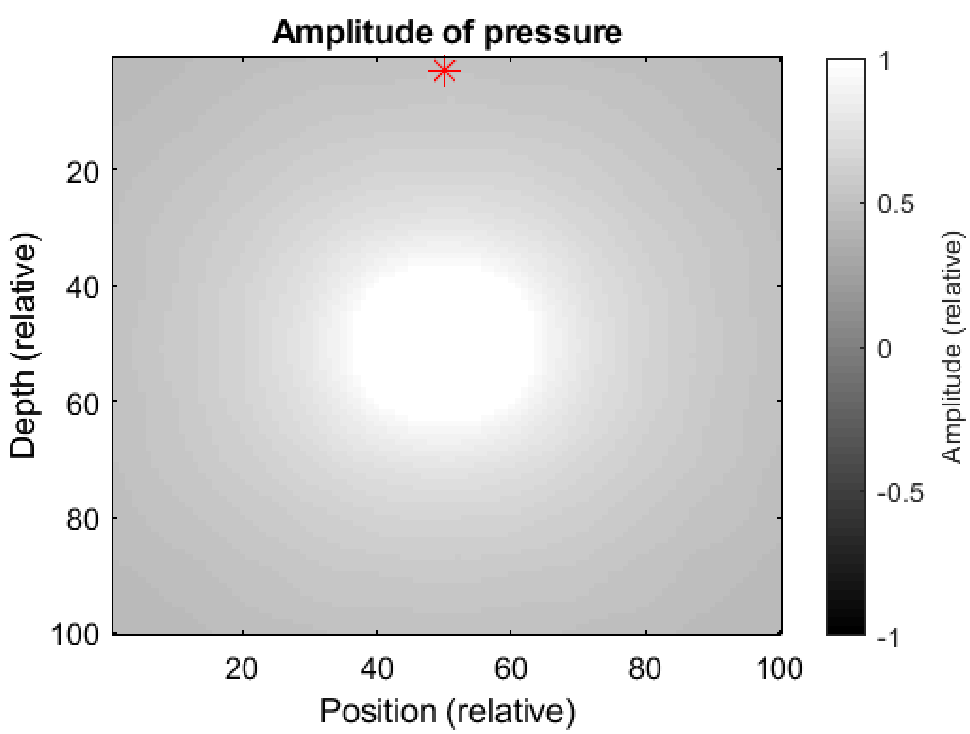


FIG. 2. Real part (top) and amplitude (bottom) of the frequency domain numerical radiation pattern of the variable in figure 1. The red star marks the source location used.

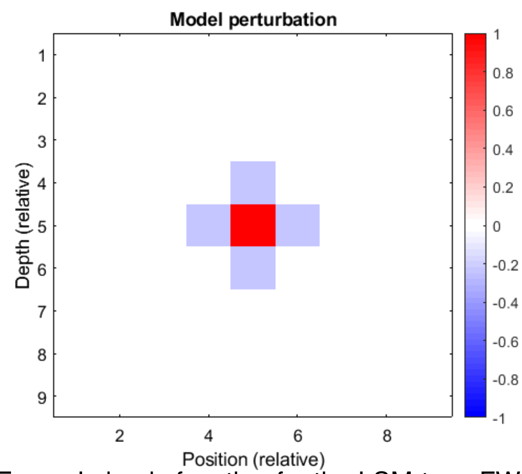


FIG. 3. Example basis function for the LSM-type FWI approach.

spike, it has sharp changes in velocity, but no zero frequency component. The amplitude of the wave back-scattered toward the source in figure 4 is still large, but there is almost no contribution to the radiation pattern from the forward-scattered wave. Perturbations in variables like this one primarily act to introduce reflections and have limited impact on travel times. This type of variable may be well suited for a LSM-type optimization. For the LSM-type FWI approach used here, P is defined such that each column p_n is a spatially shifted version of the basis shown in figure 3.

Gradient calculation

In the FWI problem it is usually necessary to take the gradient of the objective function with respect to the model m. The gradient with respect to m can be determined from the conventional FWI gradient with respect to a by using the relation

$$\frac{d\phi}{dm} = \frac{d\phi}{da}\frac{da}{dm}, \tag{3}$$

where ϕ is the objective function. Using the relation between m and a defined in equation 2, this simplifies to

$$\frac{d\phi}{dm} = g = P^T g_a, \tag{4}$$

where g_a is the gradient of the objective function with respect to the variables a, $\frac{d\phi}{da}$. Combining equations 2 and 4, the model changes in the model space of a described by the gradient g are $PP^T g_a$. As each column of P is only different from the others by a spatial shift, the matrix PP^T describes a convolution process. In effect, g can be obtained by applying a filter to g_a. If the chosen variables behave as intended, this filter will remove long wavelength contributions and preserve short wavelength, reflectivity type characteristics. An appropriate choice of variable can guarantee that the gradient does not include long wavelength features that are undesirable in a LSM-type problem. This also means that one iteration of this procedure is simply one iteration of steepest-descent FWI followed by a high-pass filtering, a process highly similar to reverse time migration.

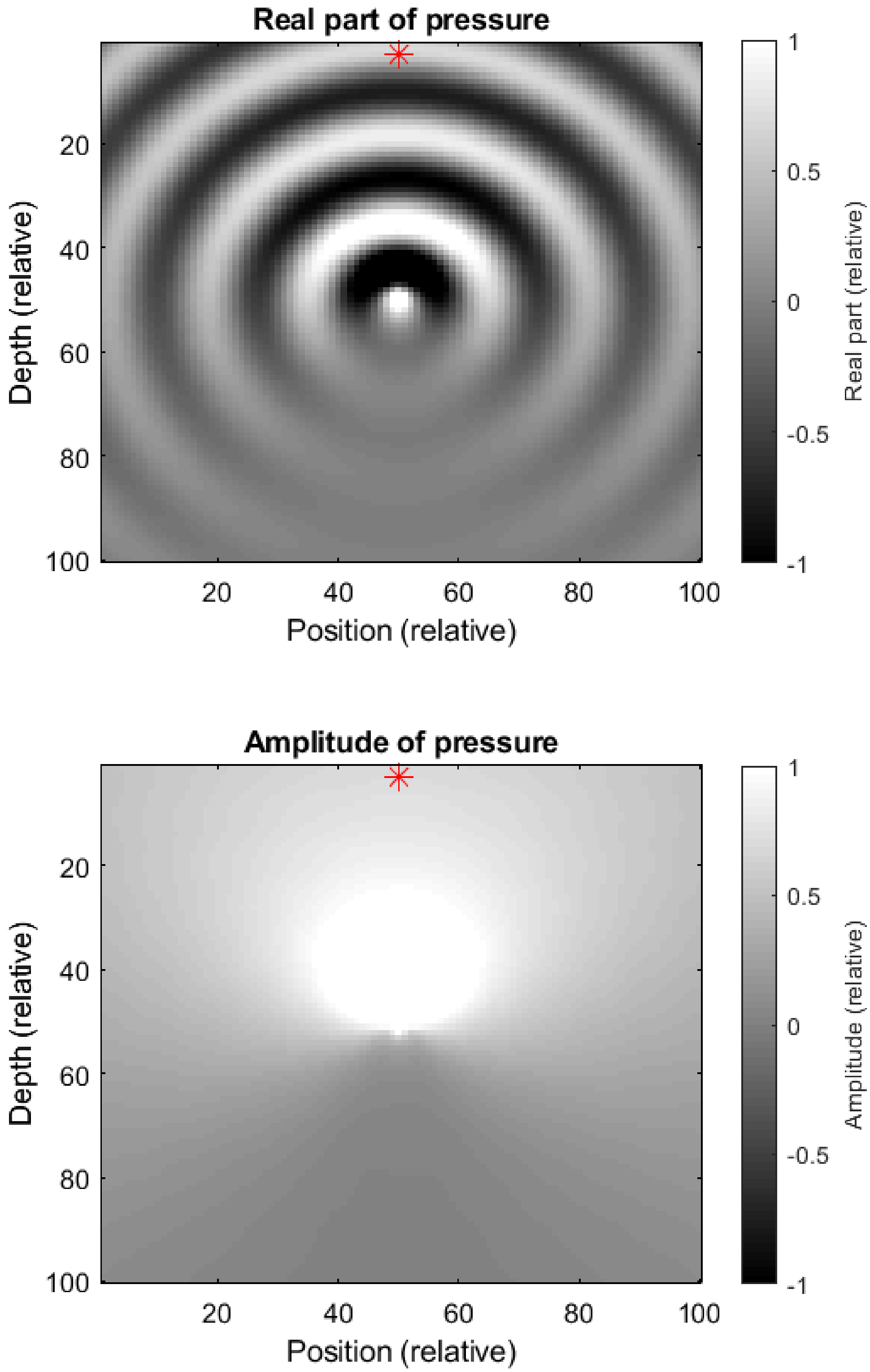


FIG. 4. Real part (top) and amplitude (bottom) of the frequency domain numerical radiation pattern of the variable in figure 3. The red star marks the source location used.

Regularization

While the gradient can be made safe from any long wavelength contributions, many optimization strategies make use of second derivative information to determine the update direction in an FWI procedure. These strategies may not be similarly immune. It is possible that basis functions which produce a good gradient may still result in a long-wavelength update when considering second-derivative information. Additionally, it may be possible that a combination of short wavelength steepest descent updates combine to create a long wavelength change in the model. Behaviour of this kind will be problematic in a LSM-type problem, and should be prevented. This could be achieved by choosing a set of basis functions that cannot be combined to create a low frequency change. Such an approach might restrict the inversion too severely, however. An alternate approach would be to penalize or constrain the recovered models in such a way that long wavelength changes are not produced. This is the approach explored here.

A regularization term penalizing long wavelength changes is given by

$$\phi_R = \frac{1}{2}||m_{smooth}||_2^2, \tag{5}$$

where m_{smooth} is calculated by applying a spatial smoothing to m. By introducing this term into the objctive function, reflectivity trends on scales larger than the smoothing can be penalized in the inversion. This should help to prevent length wavelength changes from being introduced by the LSM-type FWI optimization procedure.

Objective function

While the radiation pattern of the LSM-type variable (figure 4) has limited energy in the forward scattering part, it is not zero in this region. This means that changes in the LSM-type model variables have some capacity to change transmission behaviour. Transmission behaviour is important for relatively high energy events in measured data, such as the direct wave and diving waves. The large amplitudes of these events causes them to be heavily weighted in an objective function based on the L_2 measure of data misfit. Artifacts may then be introduced in an LSM-type approach corresponding to these events despite the relatively small capacity of the variables considered to influence them. In order to avoid such artifacts, an L_1 objective function (Brossier et al., 2009) is used here, given by

$$\phi = \phi_R + \sum_{n=1}^{N} \sqrt{(D_n - (Ru)_n)(D_n - (Ru)_n)^*}, \tag{6}$$

where D is the measured data, u is the frequency domain forward-modeled wavefield, and R is a matrix which applies the receiver sampling. This objective puts less emphasis on large amplitudes than the L_2 norm, helping to prevent artifacts caused by non-reflection events.

NUMERICAL EXAMPLES

To investigate the approach discussed above, a series of numerical tests were performed. A surface acquisition was simulated, with 99 sources and 49 receivers evenly spaced along

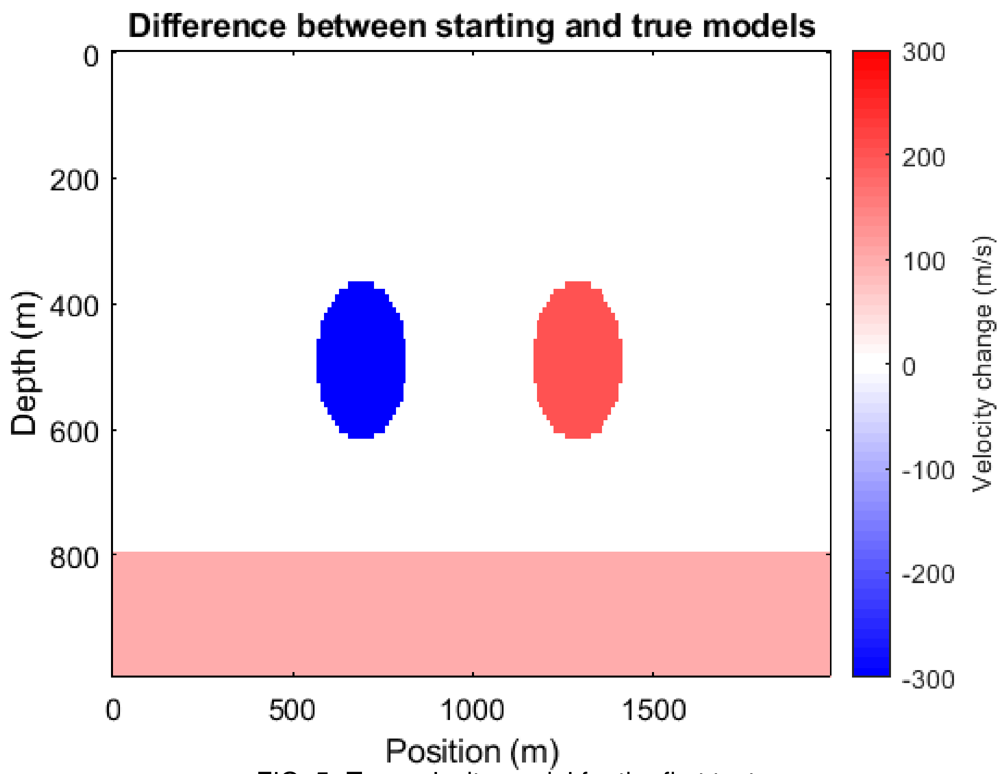


FIG. 5. True velocity model for the first test.

the top of the model. Wavefield modeling was done in the frequency domain, with 20 frequencies between 5 Hz and 20 Hz used. No noise was added to the synthetic data. Finite difference modeling was used to generate the data and to perform the inversion. A grid spacing of ten meters was used for the modeling. The optimization used in the inversions considered all frequencies simultaneously. To minimize the objective function the BFGS optimization approach was used (Nocedal and Wright, 1999).

In the first test a toy model was imaged, with a known, constant background velocity. The model used is shown in figure 5. The result after a single iteration of LSM-type FWI is shown in figure 6, and that after five iterations is shown in figure 7. These results match our expectations; the single iteration result, which should be similar to an RTM image, correctly identifies the strongest reflectors in the model, but fails to image well in areas of poor illumination, such as near the edges of the model. The reflectivity model is also ringy due to the imperfect acquisition geometry. After five iterations (figure 7), the recovered model is substantially less ringy, and the edges of the model are much better imaged. This result is encouraging, but is little different from what a conventional FWI approach might produce (figure 8). A more significant test of the approach would be a scenario in which FWI does not do a good job of recovering an accurate reflectivity model.

If the initial velocity model is incorrect on long wavelength scales, FWI will generally act to improve it. This behaviour is desirable when trying to obtain an improved velocity model, but may be prioritized over the accurate recovery of reflectors. In this environment LSM-type FWI may be able to achieve better imaging than conventional FWI. To investigate this possibility, we consider numerical simulations on the model shown in figure 9.

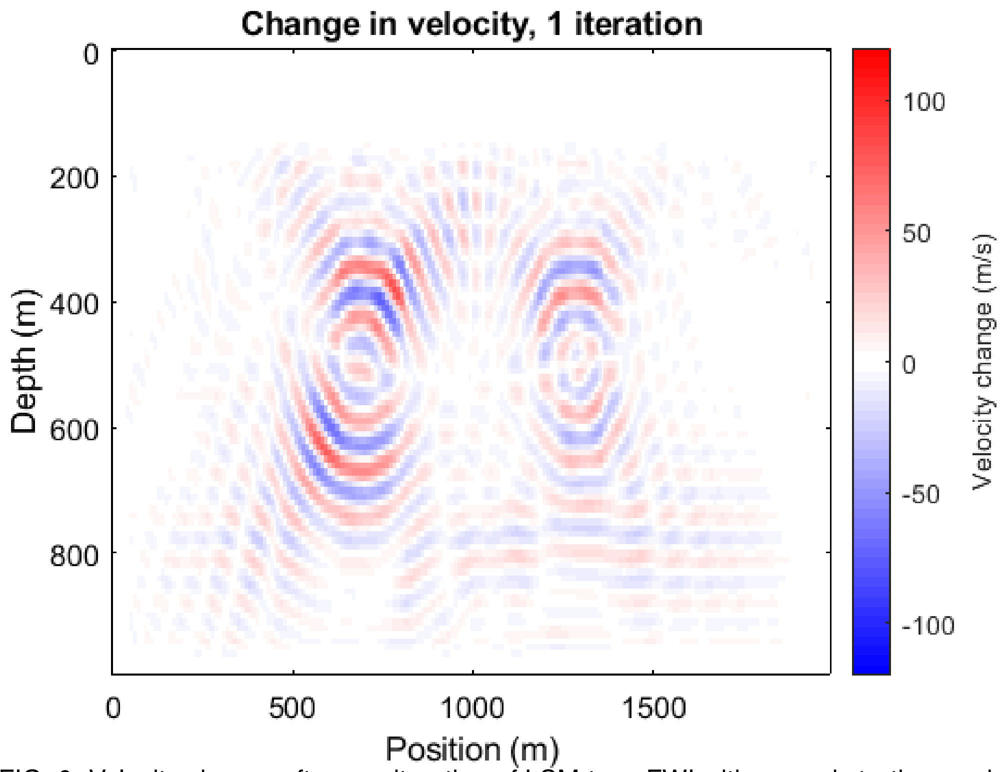


FIG. 6. Velocity change after one iteration of LSM-type FWI with a good starting model.

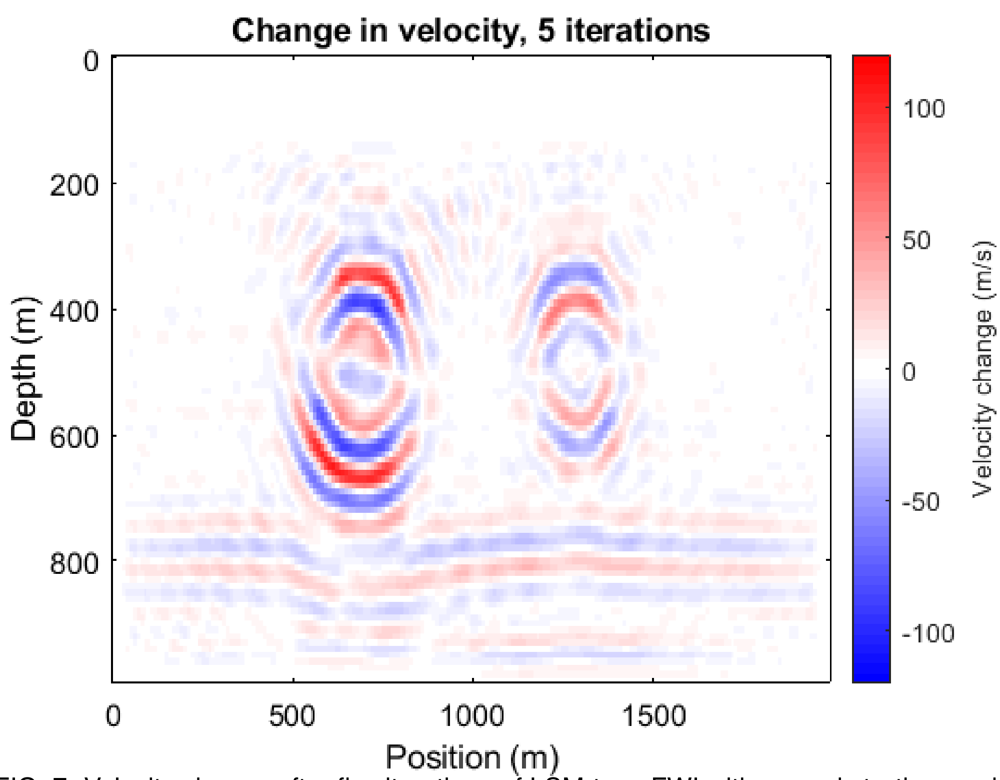


FIG. 7. Velocity change after five iterations of LSM-type FWI with a good starting model.

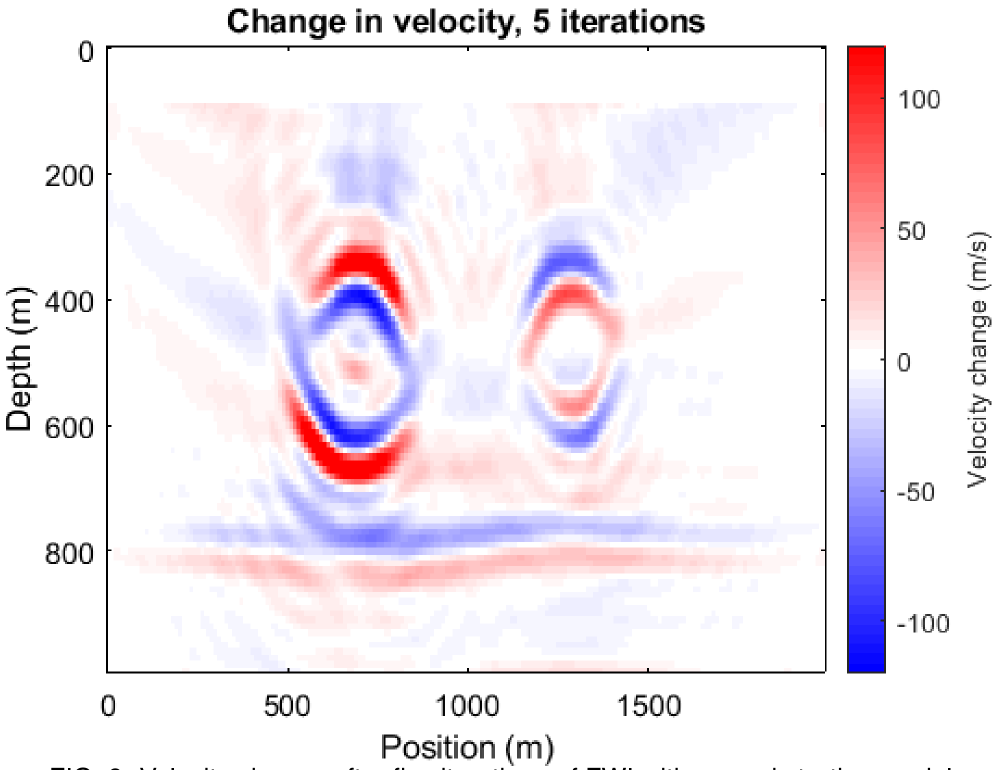


FIG. 8. Velocity change after five iterations of FWI with a good starting model.

While this model has a background velocity which changes with depth, the initial velocity used in the inversion is still a constant value. In consequence, FWI will work to improve the long wavelength velocity features of the model. The result after five iterations of FWI is shown in figure, (10). In this case, the shallow part of the velocity model has been significantly improved, but the underlying reflector has been poorly imaged in comparison to the result with the good starting velocity (figure 8). By contrast, the LSM-type approach continues to recover just the short scale behaviour of the model (figure 11). This difference has important implications. Note that the LSM-type approach was able to obtain a good estimate of the poorly illuminated areas, while the FWI result wasn't. This suggests that by removing the ability of the inversion to reproduce the long wavelength behaviour of the model, the short wavelength characteristics can be more accurately recovered at the same cost.

DISCUSSION

The numerical examples investigated here indicate that the LSM-type approach explored offers some potential to recover the short wavelength structure of the subsurface. This is achieved without restricting the data allowed into the model and without exotic modifications to the FWI objective function. By appropriate choice of the variables used in defining the subsurface model, the inverse problem was restricted to describing only reflection type data.

This approach has several significant challenges, however, which may limit its usefulness in its current form. One major obstacle is the imperfect choice of variables. The

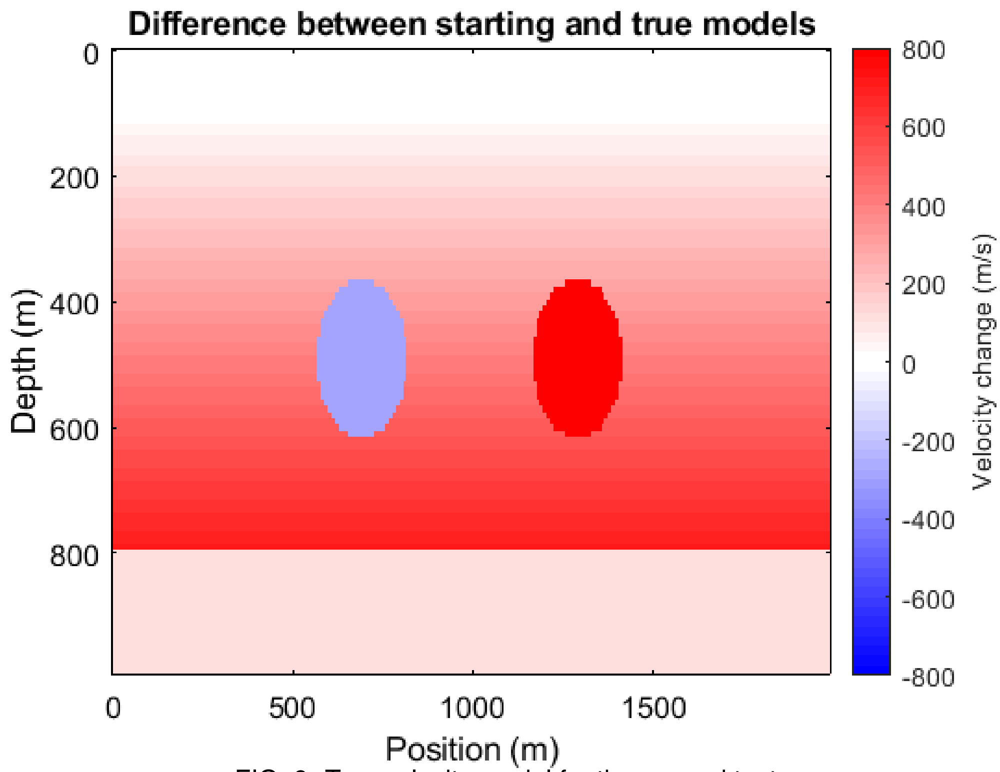


FIG. 9. True velocity model for the second test.

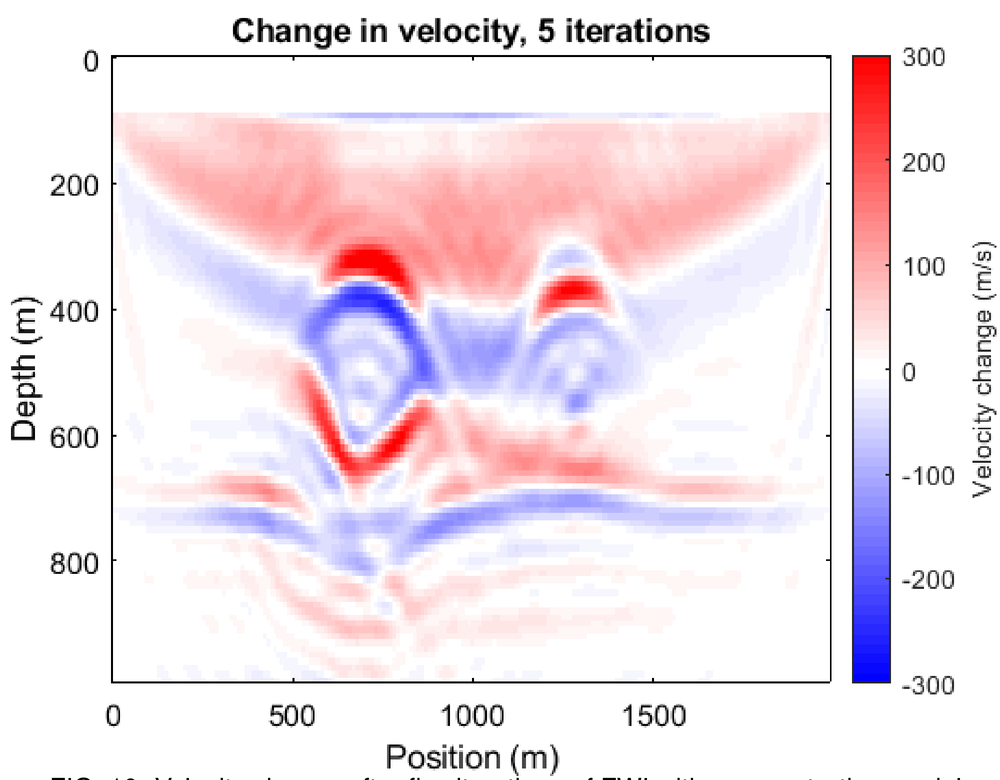


FIG. 10. Velocity change after five iterations of FWI with a poor starting model.

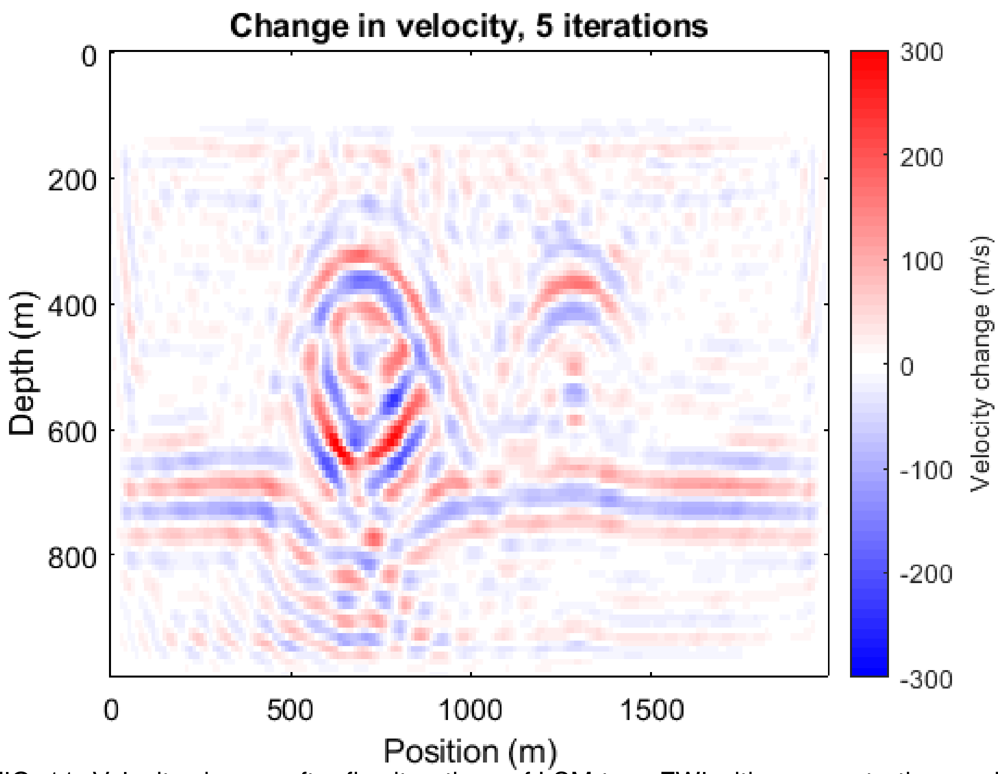


FIG. 11. Velocity change after five iterations of LSM-type FWI with a poor starting model.

inversion only neglects the transmissive effects of the model because the variables chosen have no capacity to change them. When the variables only imperfectly represent reflectors, however, the reflectivity estimate could be impacted by low frequency behaviours as well. If these artifacts are large enough they could prevent the model from effectively updating. This issue may be solvable with a more appropriate choice of variables, but is a significant concern.

In its current form, this strategy suffers from stability issues. When large numbers of iterations are performed, extreme velocity values are produced by the inversion. This instability does not occur when the background velocity is known, and could indicate that more work needs to be done to remove background related artifacts.

The wave propagation modeled here used the constant density acoustic approximation, but this was not necessary for the concepts explored. Extending these results to elastic or anelastic models is a topic of future research.

CONCLUSIONS

An inverse problem for the seismic reflectivity of the subsurface can be formulated as FWI with a specific choice of parameterization. This approach does not require exotic reformulations of the objective function, or significant data filtering. In this report, an L_1 objective function and smoothness-penalizing regularization were used together with the reflectivity oriented parameterization to reduce unwanted artifacts. Numerical tests showed that when long wavelength errors exist in the velocity model, the LSM-type FWI has the potential to outperform traditional FWI in imaging reflectors. While the constant-density acoustic approximation was employed here, LSM-type FWI could conceptually be applied to more complex models of wave propagation (e.g. elastic). Numerical artifacts and instability are concerns when large numbers of iterations are performed, this is a topic of ongoing research.

ACKNOWLEDGEMENTS

The authors thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13. Scott Keating was also supported by the Earl D. and Reba C. Griffin Memorial Scholarship.

REFERENCES

Alkhalifah, T., and Plessix, R., 2014, A recipe for practical full-waveform inversion in anisotropic media: An analytical parameter resolution study: Geophysics, **79**, R91–R101.

Anagaw, A. Y., 2014, Full waveform inversion using simultaneous encoded sources based on first- and second-order optimization methods: Ph.D. thesis, University of Alberta.

Brossier, R., Operto, S., and Virieux, J., 2009, Robust frequency-domain full-waveform inversion using the l1 norm: Geophysical Research Letters, **36**, L20,310.

Bunks, C., Salek, F., Zaleski, S., and Chavent, G., 1995, Multiscale seismic waveform inversion: Geophysics, **60**, 1457–1473.

Choi, Y., Min, D., and Shin, C., 2008, Two-dimensional waveform inversion ofmulti-component data in acoustic-elastic coupled media: Geophysical Prospecting, **56**, 863–881.

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, No. 3, 467–481.

Datta, D., and Sen, M. K., 2016, Estimating a starting model for full-waveform inversion using a global optimization method: Geophysics, **81**, R211–R223.

Debens, H., Warner, M., Umpleby, A., and da Silva, N., 2015, Global anisotropic 3d fwi: SEG Expanded Abstracts.

Engquist, B., and Froese, B., 2014, Application of the wasserstein metric to seismic signals: Commun. Math. Sci., 12 5 979 988.

Fichtner, A., Kennett, B., Igel, H., and Bunge, H., 2008, Theoretical background for continental- and global-scale full-waveform inversion in the time-frequency domain.

Hicks, G., and Pratt, R. G., 2001, Reflection waveform inversion using local descent methods: estimating attenuation and velocity over a gas-sand deposit: Geophysics, **66**, 598–612.

Keating, S., and Innanen, K., 2017, Multi-resolution newton optimization in full waveform inversion: CREWES Annual Report, **29**.

Lailly, P., 1983, The seismic inverse problem as a sequence of before stack migrations: Conference on Inverse Scattering, Theory and Application, Society for Industrial and Applied Mathematics, Expanded Abstracts, 206–220.

Nemeth, T., Wu, C., and Schuster, G., 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, 208–221.

Nocedal, J., and Wright, S. J., 1999, Numerical Optimization, Springer Series in Operations Research: Springer-Verlag.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266.

Tarantola, A., 1986, A strategy for nonlinear inversion of seismic reflection data: Geophysics, **51**, 1893–1903.

Virieux, J., and Operto, S., 2009, An overview of full-waveform inversion in exploration geophysics: Geophysics, **74**, No. 6, WCC1.

Warner, M., and Guasch, L., 2016, Adaptive waveform inversion: Theory: Geophysics, **81**, R429–R445.

Xu, S., Wang, D., Chen, F., Lambaré, G., and Zhang, Y., 2012, Inversion on reflected seismic waves: SEG Expanded Abstracts.

Yang, T., and Sava, P., 2011, Wave-equation migration velocity analysis with time-shift imaging: SEG Expanded Abstracts.