3D viscoacoustic reverse time migration with attenuation compensation

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ABSTRACT

Processes of attenuation and dispersion always degrading the resolution of migrated images. To improve the image resolution, we have developed a new approach for the numerical solution of the 3D viscoacoustic wave equation in the time domain and we developed an associated reverse time migration (Q-RTM) method. The main feature of the Q-RTM approach is compensation of attenuation effects in seismic images during migration by separation of amplitude attenuation and phase dispersion terms. In the Q-RTM implementation, attenuation and dispersion compensated operators are constructed by reversing the sign of the amplitude attenuation and keeping the sign of dispersion operator unchanged. The Q-RTM approach is tested on a 3D model. We validate and examine the response of this approach by using it within an RTM scheme adjusted to compensate for attenuation. Our 3D numerical test focus on the amplitude recovery and resolution of the Q-RTM images as well as the interface locations. Numerical results show that the Q-RTM approach produced higher resolution images with recovered amplitude compared to the non-compensated RTM.

INTRODUCTION

Attenuation always affects the resolution of migrated images. To improve the resolution, one must correct for these attenuation effects. The earliest researches to mitigate the attenuation losses in seismic data involved using an inverse Q-filtering method based on 1D wave back-propagation (Bickel and Natarajan, 1985; Hargreaves and Calvert, 1991; Wang, 2002). These methods partially correct for the attenuation losses, but they are limited to 1D Q models, and the inverse filter is unstable (Wang, 2002, 2006; Zhang and Ulrych, 2007). In the prestack depth migration, many studies of attenuation compensation have examined using ray-tracing (Xin et al., 2008; Xie et al., 2009) and one-way wave-equation migration methods in the frequency domain (Dai and West, 1994; Mittet et al., 1995; Yu et al., 2002; Valenciano et al., 2011; Zhang et al., 2012).

Attenuation and dispersion can be compensated in a largely full-waveform consistent manner within the back-propagation and modeling components of RTM. Zhang et al. (2010) uses the dispersion relation of Kjartansson (1979) to formulate a pseudo-differential wave equation with separable the effects of amplitude loss and dispersion. They employ a regularization process to suppress high-frequency noises and stabilize the back-propagating wavefield. Bai et al. (2013) derive a new approach for attenuation compensation in RTM by introducing a new viscoacoustic wave equation defined without any memory variable. Fletcher et al. (2012) proposed applying separate amplitude and phase filters to twice-extrapolated source and receiver wavefields to compensate for attenuation effects. Dutta and Schuster (2014) used a least-squares RTM (LSRTM) method for attenuation compensation based on an standard linear solid (SLS) model and its adjoint operator (Blanch and Symes, 1995) with a simplified stress-strain relation.

Zhu and Harris (2014) proposed a constant-Q viscoacoustic wave equation with separated fractional Laplacians for compensating the attenuation effects. The average scheme is unsuitable for relatively sharp Q contrasts and just works for smoothly heterogeneous Q models. The amplification of high-frequency components during compensation for attenuation effects usually generates instability. To eliminate instability, Bai et al. (2013) and Zhu and Harris (2014) used a low-pass filter before or during wavefield extrapolate in RTM. Following Bai et al. (2013) approach, here we derive a new approach to the solution of the 3D viscoacoustic wave equation in the time domain based on an SLS model. We present the formulation for attenuating media that describes constant-Q wave propagation and includes independent terms for phase dispersion and amplitude attenuation. In this approach, there is no need for any extra memory variables, unlike the traditional viscoacoustic wave equations. We employ a 3D synthetic examples to test the proposed method in application to imaging and demonstrate that Q-RTM compensates in the receiver wavefields when using the source-normalized cross-correlation imaging condition.

A 3D VISCOACOUSTIC WAVE EQUATION FOR EXTRAPOLATING WAVEFIELDS

The 3D velocity-stress formulation of the viscoacoustic wave equation is expressed as (Robertsson et al., 1994)

$$\partial_t u_x = -\rho^{-1}\partial_x P, \tag{1}$$

$$\partial_t u_y = -\rho^{-1}\partial_y P, \tag{2}$$

$$\partial_t u_z = -\rho^{-1}\partial_z P, \tag{3}$$

$$\partial_t P = -K\left(\partial_x u_x + \partial_y u_y + \partial_z u_z\right)\left(\tau_\varepsilon \tau_\sigma^{-1}\right) - r, \tag{4}$$

$$\partial_t r = -\tau_\sigma^{-1} r + K\left(\partial_x u_x + \partial_y u_y + \partial_z u_z\right)\tau_\sigma^{-1}\left(1 - \tau_\varepsilon \tau_\sigma^{-1}\right). \tag{5}$$

where $u_x(x,t)$, $u_y(x,t)$, and $u_z(x,t)$ are the particle velocity components in the x-, y- and z-directions respectively, $P(x,t)$ is pressure wavefield , $\rho(x)$ is density, r is a memory variable, K represents the bulk modulus of the medium, τ_ε and τ_σ are related to the quality factor Q and the reference angular frequency ω as (Robertsson et al., 1994)

$$\tau_\sigma = \frac{\sqrt{1 + 1/Q^2} - 1/Q}{\omega}, \tag{6}$$
$$\tau_\varepsilon = \frac{1}{\omega^2 \tau_\sigma}.$$

In attenuative media, the wave experiences two main effects, reduced amplitude, and phase distortion due to dispersion. By separation between the amplitude attenuation and phase dispersion effects, wave propagation can be simulated in three scenarios, i.e., only the amplitude loss effect, only the phase dispersion effect, or both effects concurrently. In this

paper, we present an approach for the solution of the viscoacoustic wave equation in the time domain to explicitly separate phase dispersion and amplitude attenuation. We first apply the Fourier transform to the first-order linear differential equations 1-5 in the time domain to obtain the frequency domain viscoacoustic wave equation:

$$i\omega\tilde{u}_x = -\rho^{-1}\partial_x\tilde{P}, \tag{7}$$

$$i\omega\tilde{u}_y = -\rho^{-1}\partial_y\tilde{P}, \tag{8}$$

$$i\omega\tilde{u}_z = -\rho^{-1}\partial_z\tilde{P}, \tag{9}$$

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_y\tilde{u}_y + \partial_z\tilde{u}_z\right)\left(\tau_\varepsilon\tau_\sigma^{-1}\right) - \tilde{r}, \tag{10}$$

$$i\omega\tilde{r} = -\tau_\sigma^{-1}\tilde{r} + K\left(\partial_x\tilde{u}_x + \partial_y\tilde{u}_y + \partial_z\tilde{u}_z\right)\tau_\sigma^{-1}\left(1 - \tau_\varepsilon\tau_\sigma^{-1}\right). \tag{11}$$

From equation 11, the memory variable in the frequency domain can be calculated as a function of the particle velocity and the relaxation time

$$\tilde{r} = K\left(\partial_x\tilde{u}_x + \partial_y\tilde{u}_y + \partial_z\tilde{u}_z\right)\frac{\tau_\sigma^{-1}\left(1 - \tau_\varepsilon\tau_\sigma^{-1}\right)}{\left(i\omega + \tau_\sigma^{-1}\right)}. \tag{12}$$

By substituting equation 12 into equation 10, the memory variable equation is removed. The new first-order viscoacoustic wave equation in the frequency domain without source components is

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_y\tilde{u}_y + \partial_z\tilde{u}_z\right)\left[\left(\tau_\varepsilon\tau_\sigma^{-1}\right) + \frac{\tau_\sigma^{-1}(1 - \tau_\varepsilon\tau_\sigma^{-1})}{\left(i\omega + \tau_\sigma^{-1}\right)}\right], \tag{13}$$

which can be re-written, after some algebraic manipulation, as

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_y\tilde{u}_y + \partial_z\tilde{u}_z\right)\left[\frac{(\omega^2\tau_\varepsilon\tau_\sigma + 1)}{\omega^2\tau_\sigma^2 + 1} + i\frac{(\omega\tau_\varepsilon - \omega\tau_\sigma)}{\omega^2\tau_\sigma^2 + 1}\right]. \tag{14}$$

When transformed back to the time domain, equation 14 produces a viscoacoustic wave equation that maintains the approximate constant-Q attenuation and dispersion behaviours during wave propagation. To apply this equation within RTM, we write viscoacoustic forward and backward extrapolation as:

$$\partial_t P = -K\left(\partial_x u_x + \partial_y u_y + \partial_z u_z\right)\left[a_1(2/A) + ia_2(2/AQ)\right], \tag{15}$$

where $A = 1 + (\sqrt{1 + 1/Q^2} - 1/Q)^2$. The quantities $2/A$ and $2/AQ$ are dispersion-dominated and amplitude-attenuation-dominated operators, respectively. The only difference between viscoacoustic and acoustic wave equations is the complex- valued term $2/A + i2/AQ$. The coefficients a_1 and a_2 are constants of unit magnitude, whose signs are important for the forward and backward extrapolation. Note that when $Q \to \infty$, the
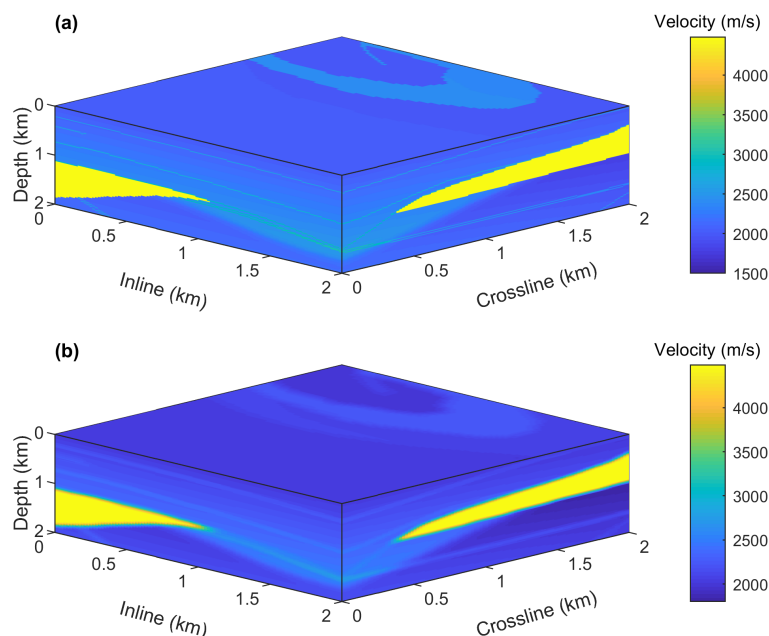
FIG. 1. Velocity cubes in the 3D SEG/EAGE salt models: (a) true velocity; (b) migration velocity.

dispersion-dominated operator goes to 1 and the amplitude-loss-dominated operator disappears, i.e., the viscoacoustic case approaches to the acoustic case. The positive sign of a_2 represents the amplitude loss in extrapolating source propagation. To amplifies the amplitude, the sign of the attenuation term ($a_2 = -1$) will be reversed. The viscoacoustic wave equation also contains a dispersion term that affects the phase during wave propagation. The sign of this term ($a_1 = 1$) does not need to be changed.

For backward modeling, the 3D viscoacoustic wave equation with compensation of attenuation effects ($a_1 = 1$ and $a_2 = -1$) can be written as

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left[a_1(2/A) - i a_2(2/AQ) \right]. \tag{16}$$

3D SYNTHETIC EXAMPLE IN COMPLEX MEDIA

The synthetic 3D SEG/EAGE salt model (Aminzadeh et al., 1997) offers an opportunity to test the Q-RTM equation in a 3D setting (see Appendix C). We chose a small portion of this model that contains thin sand layers and a salt body with a complex topology. The true and migration velocity models with a Q anomaly are plotted in Figures 1 and 2. The Q model contains a high attenuation anomaly with Q=20. The data set is generated by finite differences with a Ricker source with central frequency 15 Hz. The grid spacings for the three directions are 20 m, and the sampling interval is 1 ms. Figure 3 shows the horizontal component of acoustic and viscoacoustic data respectively. The data from the viscoacoustic simulation shows reduced amplitude and shifted phase compared to those of the acoustic case beneath the strongly attenuating zones. We apply acoustic and viscoacoustic RTM imaging to viscoacoustic seismic data. The RTM images include the RTM of viscoacoustic data without compensation (Figure 4), the compensated viscoacoustic RTM (Figure 5), and
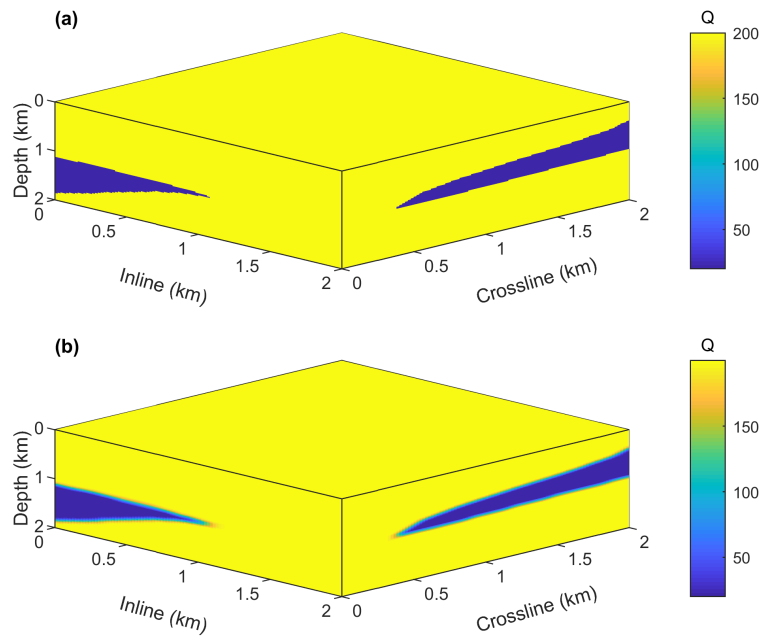
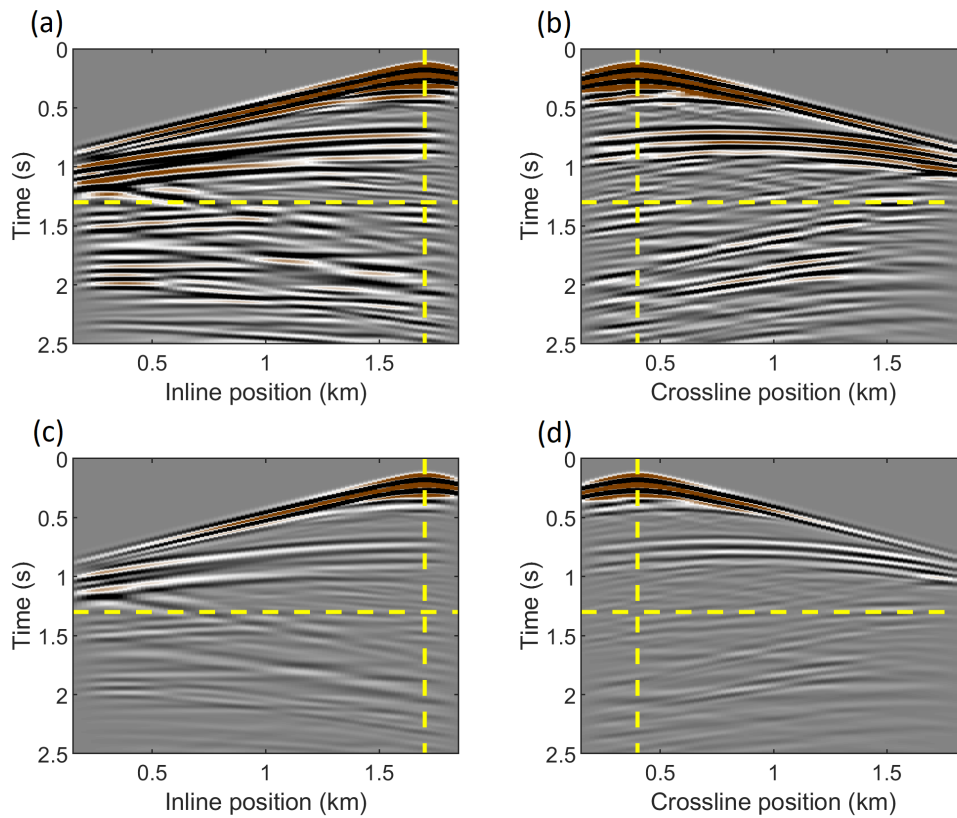FIG. 2. Q models: (a) true Q; (b) migration Q.

FIG. 3. Synthetic data example. (a) shot record in the acoustic approximation; (b) crossline shot record in the acoustic approximation; (c) inline and (d) shot records in the viscoacoustic case.
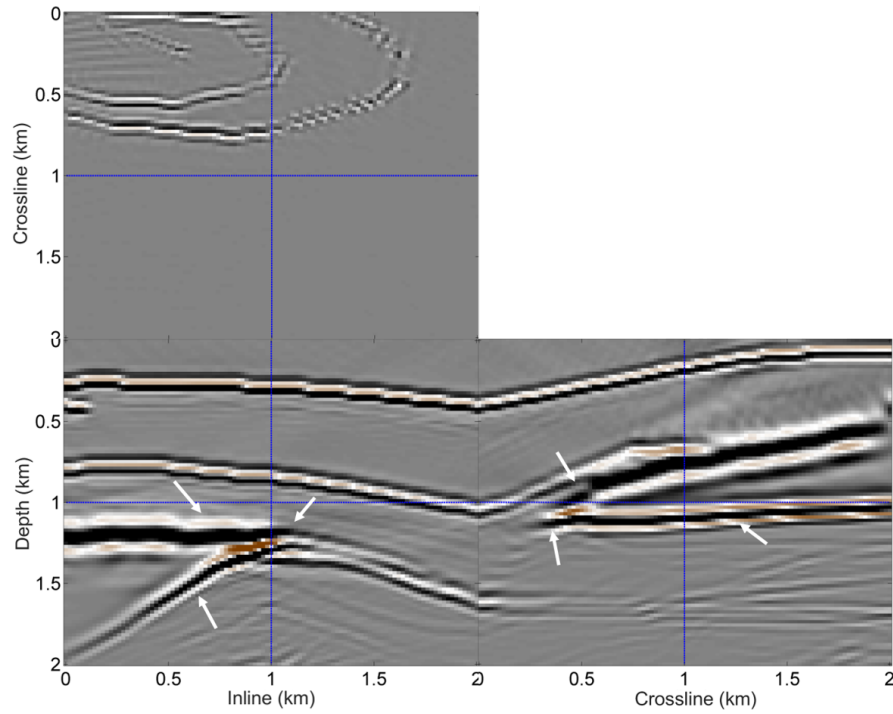
FIG. 4. Reference acoustic image by applying acoustic RTM on acoustic data.
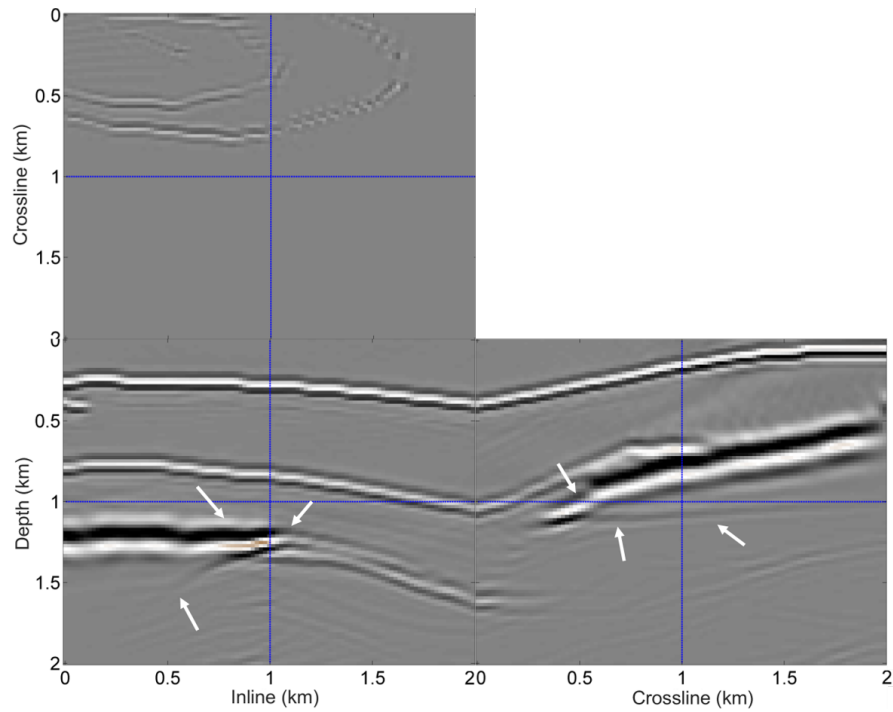

FIG. 5. Acoustic RTM image with viscoacoustic data (noncompensated). It is obvious the attenuation effect especially for Salt flanks under the high-attenuation zone.
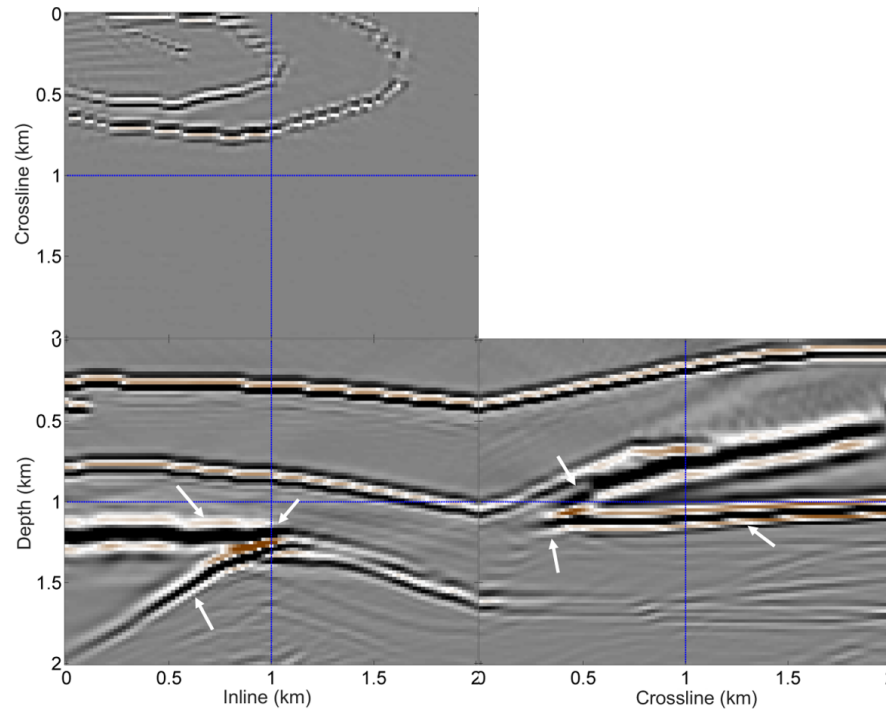
FIG. 6. Viscoacoustic RTM image (compensated). RTM image is clear, and the position is accurate from compensated viscoacoustic RTM, compared with the acoustic RTM image.

the reference RTM results (Figures 6). For viscoacoustic RTM, without compensating for amplitude loss, the image amplitudes and positions are inaccurate for the reflectors beneath the strongly attenuating layers, and salt flanks in the image are shifted down relative to the reference depth. After compensation, in the viscoacoustic RTM, the migrated amplitudes of layers and salt flanks are more accurate by resolved than in the non-compensated RTM images, and the reflectors are imaged at the correct locations compared with the reference images.

CONCLUSIONS

3D viscoacoustic RTM imaging algorithm based on the time-domain constant-Q wave propagation that can correct the attenuation and dispersion effects in migrated images are investigated. The attenuation effects in the source and receivers wavefields can be recovered by applying compensation operators on the measured receiver wavefield. The phase dispersion and amplitude loss operators in Q-RTM approach are separated, and the compensation operators are constructed by reversing the sign of the attenuation operator without changing the sign of the dispersion operator. Numerical test in 3D on synthetic data shows that this approach can improve the image resolution, especially beneath areas with strong attenuation.

ACKNOWLEDGMENTS

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