Sensitivity analysis of surface wave dispersion curves for various subsurface parameters

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ABSTRACT

The propagator matrix method is commonly used for inverting near-surface shear-wave velocity. An important precondition for its feasibility is the different sensitivity levels of dispersion curves with respect to shear-wave velocity, primary-wave velocity, layer thicknesses, and density. Here, we analyze the sensitivity of surface-wave dispersion-curves at different frequencies for various subsurface parameters. Not only the fundamental mode, higher-order modes extending to the fourth higher-order mode are all included in this study. From the analysis, higher modes are more sensitive to parameters in the deeper area. Besides, the sensitivity analysis is conducted on the phase-velocity, the group velocity, and their derivatives, respectively. Through comparing various types of observing data, the phase velocity derivative is more sensitive to parameter perturbations in the same wavelength range. However, its sensitivity shows simultaneously positivity and negativity at different depths. While the phase velocity shows positivity exclusively. At last, inversion stability and accuracy of different types of observing data are discussed.

INTRODUCTION

The accuracy of near-surface velocity structure determination greatly influences static correction, source parameter determination, and velocity modeling-building for deep area velocity inversion. Since surface wave dominates the shallow-seismic wavefield with high amplitude, surface-wave analysis has been used for near-surface geophysical study. The inversion of shear-wave velocity at shallow sites depends on a physical-meaningful principle and an appropriate measuring, analysis, and inversion approach, which are the propagator matrix method and the multichannel analysis of surface waves (MASW) (Thomson, 1950; Socco and Strobbia, 2004; Foti et al., 2014, 2018). The first part determines the dispersion relationship of the research region, which means under the layered earth-model assumption, with elasticity and thickness parameters specified for each layer and free surface boundary condition, the dispersion curves of the surface wave can be obtained (Haskell, 1953). The second part provides an effective workflow to measure the data, conduct the analysis, and inversion. Various developments have been made in both parts. As for the first part, calculation accuracy and computation efficiency have been improved through variable transformation, mathematical derivation, and formula simplification (Park et al., 1998). As for the second part, optimization on steps such as acquisition parameters setting and dispersion curves picking have been further studied (Xia et al., 1999). Besides these two parts, the inversion methods are important as well. It belongs to the MASW workflow but involves extensive content, which can be treated as an independent part. The theory of obtaining dispersive characteristics of the surface wave based on given models is relatively robust, stable results (dispersion curves) can be guaranteed. However, in real data processing, the dispersion curves picking and quality of the observing data seem to be more uncontrollable. Thus, details on obtaining clear dispersion spectra are more focused.

One hidden and determinant premise for the surface wave inversion to obtain shear-wave velocity is the sensitivity of dispersion curves on shear-wave velocity (Bhattacharya, 2015). To ensure the accuracy of shear-wave velocity, we need to make sure other elastic parameters and layer thickness do not influence the inversion for shear-wave velocity. In addition, for more robust inversion results, observing data which are more sensitive to subsurface shear-wave velocity should be explored.

In the previous study, the sensitivity analysis of phase-velocity and group-velocity on targeted subsurface parameters has been studied. Wang et al. (2020) extended the sensitivity analysis to a new type of observing data, the derivative of the Rayleigh-wave phase velocity, and found it shows higher sensitivity to shear-wave. In this study, we analyzed the sensitivity of four types of observing data: the group velocity, the phase velocity, the group velocity derivative, and the phase velocity derivative to various subsurface parameters, including primary-wave velocity, shear-wave velocity, and density. Besides, we extended all these analyses to higher modes. Analysis of using these data in inversion will also be conducted.

THEORY

Definition and formulation of the phase velocity derivative

In a multi-layered medium, phase and group velocities are dispersive. Through derivation (Wang et al., 2020), the expression of phase velocity derivative can be written as

$$\frac{dV_g}{df} = -\frac{V_{ph}\left(V_g - V_{ph}\right)}{fV_g} \tag{1}$$

where V_ph is Rayleigh-wave phase velocity, V_g is Rayleigh wave group velocity, f is the frequency.

Definition and formulation of the group velocity derivative

Since the phase velocity are dependent on frequency in a multi-layered medium, the phase velocity can be written as

$$V_{ph} = \frac{\omega}{k} \tag{2}$$

Thus,

$$\omega = kV_{ph} \tag{3}$$

The Rayleigh-wave velocity is also a function of frequency, wave number, and model parameters (V_p, V_s, ρ etc), take the derivative of ω with respect to wavenumber k,

$$\frac{d\omega}{dk} = V_{ph} + k\frac{dV_{ph}}{dk} = V_g \tag{4}$$

assuming the derivative of model parameters with respect wavenumber k is zero. Then, we take the derivative of group velocity V_g with respect to angle frequency ω,

$$\frac{dV_g}{d\omega} = \frac{dV_{ph}}{d\omega} + \frac{dk}{d\omega}\frac{dV_{ph}}{dk} + k\frac{d^2V_{ph}}{dkd\omega}. \tag{5}$$

The partial derivatives of phase velocity with respect to frequency $\frac{dV_{ph}}{d\omega} = \frac{1}{k}$ and to wavenumber $\frac{dV_{ph}}{dk} = -\frac{\omega}{k^2}$. Substituting them in the above equation,

$$\frac{dV_g}{d\omega} = \frac{1}{k} + \frac{1}{V_g}\frac{-\omega}{k^2} + k\frac{-1}{k^2}. \tag{6}$$

$$\frac{dV_g}{d\omega} = -\frac{1}{V_g}\frac{\omega}{k^2}. \tag{7}$$

$$\frac{dV_g}{d\omega} = -\frac{1}{V_g}\frac{V_{ph}^2}{\omega}. \tag{8}$$

$$\frac{dV_g}{df} = -\frac{V_{ph}^2}{fV_g}. \tag{9}$$

Thus, the analytical formulation of the group velocity derivative with respect to frequency is obtained.

SENSITIVITY ANALYSIS

Rayleigh-wave phase velocity

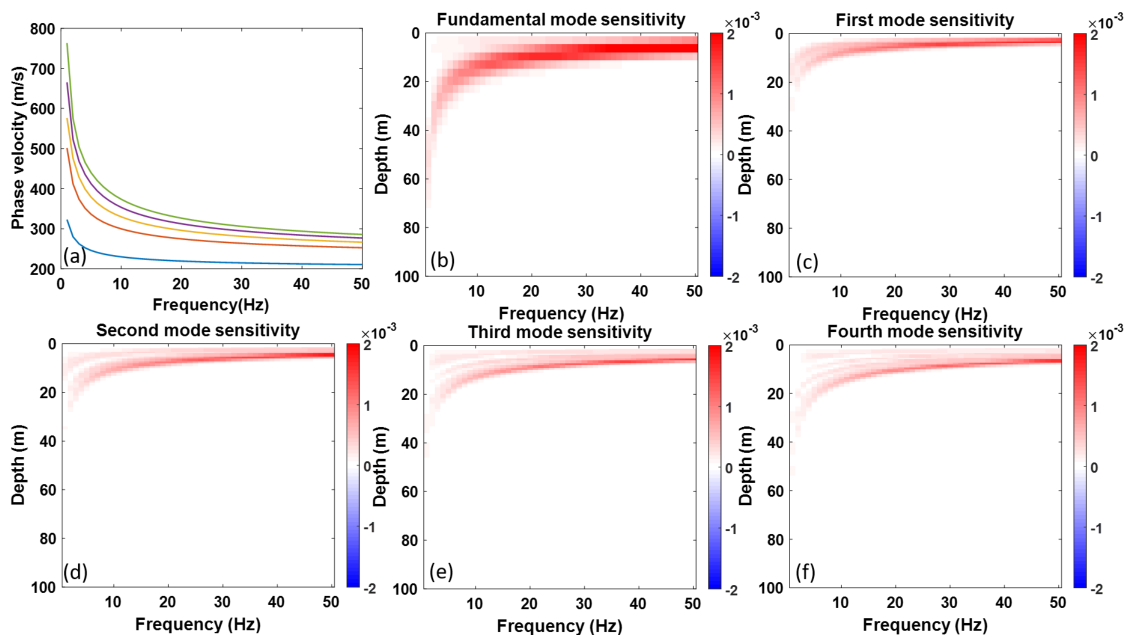

FIG. 1. Sensitivity of Rayleigh-wave phase velocity with respect to shear-wave velocity. (a) Multimode phase velocity. (b), (c), (d), (e), and (f) are the sensitivity of each mode.

A multi-layered model with 100 equal thin layers is created for the dispersion curves generation. The shear-wave velocity (Vs) is set from top 410 m/s to bottom 1390 m/s with a constant increase. The primary-wave velocity (Vp) is set to be 1.9 multiplied by the shear wave velocity. The density is set from the top 1002 g/m^3 to the bottom 1200 g/m^3. The calculation frequency range is set from 1 Hz to 50 Hz. Perturbations are added to the various parameters from the top to the bottom layers. The values of the perturbation are set to be relatively small and constant. Since the Vp and Vs are in ratio, their perturbations are in ratio as well. Here, the shear-wave velocity perturbation is set to be 6 m/s, primary-wave velocity perturbation is set to be 10 m/s, and the density perturbation is set to be 2 g/cm^3. For a better comparison, all the dispersion discrepancies are normalized by dividing the perturbation values.

The sensitivity of multimode Rayleigh-wave phase velocity with respect to shear-wave velocity, primary-wave velocity, and density are shown in FIG. 1, FIG. 2, and FIG. 3, respectively. From Figure1, In these figures, (a) shows the dispersion curves generated

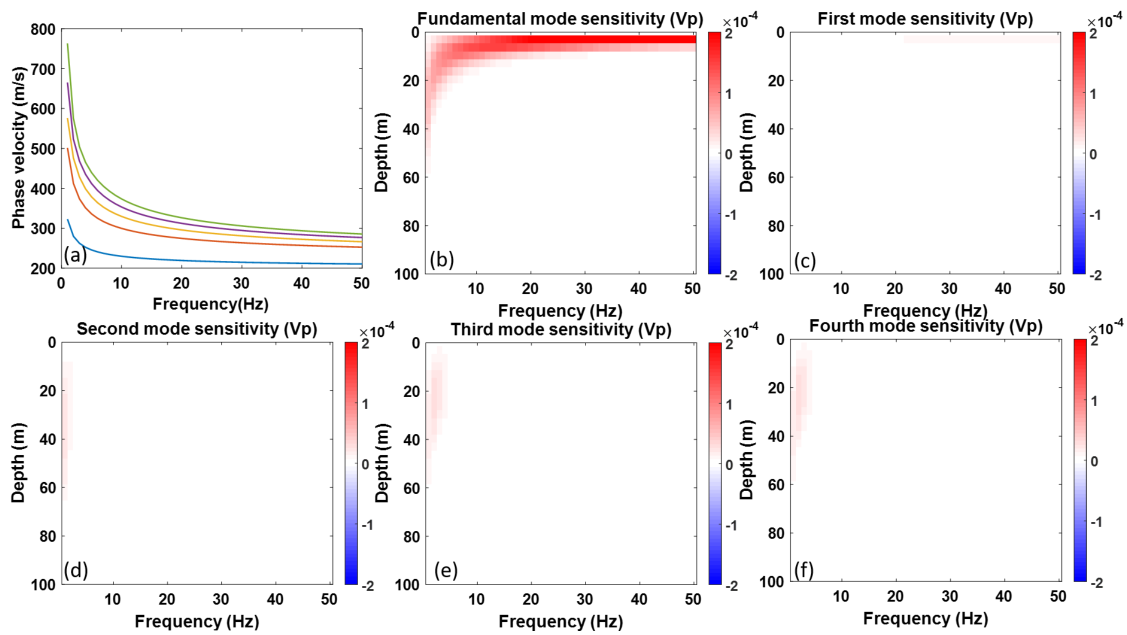

FIG. 2. Sensitivity of Rayleigh-wave phase velocity with respect to primary-wave velocity. (a) Multimode phase velocity. (b), (c), (d), (e), and (f) are the sensitivity of each mode.

using propagator matrix method. (b)-(f) shows the sensitivity of phase velocity from the fundamental mode to the fourth mode. In FIG. 1, we can find the Rayleigh-wave phase velocity shows a positive response to a positive Vs perturbation. Higher modes are more sensitive to the deep layers. Thus, using higher modes in inversion can improve the resolution of the deep area. Lower frequency dispersion curves are more sensitive to the modification of the deep layers.

In FIG. 2, it shows a similar pattern to the FIG. 1, that lower frequency data are more influenced by the deep layers, but exists several differences. For instance, the sensitivities between fundamental and higher modes are different. Higher modes seem to have much lower sensitivity compared with the fundamental mode. Besides, another significant difference is the sensitivity level. Since these results are all normalized, the sensitivity level comparison is meaningful. For shear-wave velocity analysis, the displaying scale is -2×10^{-3} to 2×10^{-3}, while for primary-wave velocity analysis, the displaying scale is -2×10^{-4} to 2×10^{-4}.

In FIG. 3, a staggered positive and negative sensitivity pattern is shown. The higher modes seem to extend to deeper layers. The sensitivity scale for density is from -5×10^{-5} to 5×10^{-5}. From the comparisons, we can easily find, the sensitivity level between shear-wave velocity, primary-wave velocity, and density is close to 40:4:1. There are significant

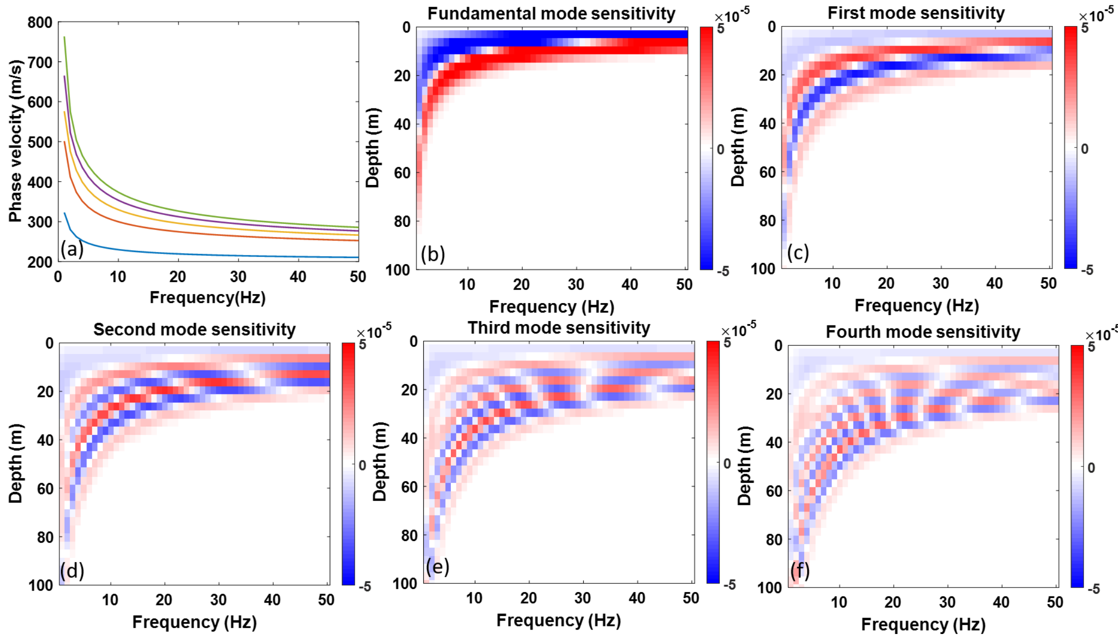

FIG. 3. Sensitivity of Rayleigh-wave phase velocity with respect to density. (a) Multimode phase velocity. (b), (c), (d), (e), and (f) are the sensitivity of each mode.

differences in the sensitivity levels with respect to various elastic parameters. Therefore, it is reasonable to exclusively invert the shear-wave velocity using the Rayleigh-wave phase velocity. Since meaningful conclusions have been obtained from the above test, thus, the sensitivity with respect to primary-wave velocity and density will not be discussed in the following sections.

Rayleigh-wave group velocity

The sensitivity of Rayleigh-wave group velocity with respect to shear-wave velocity is shown in FIG. 4. The Rayleigh-wave group velocity has two fewer points in frequency than the phase velocity, as it takes derivative. In FIG. 4, the pattern is similar to the phase-velocity case, except that both positivity and negativity are shown in the sensitivity analysis. To be more specific, the perturbations in the deep layers have a negative influence on the multimode group velocity dispersion curves, while the perturbations in the shallow layers have a positive influence. The sensitivity level is on the same scale as the phase velocity case.

Rayleigh-wave phase velocity derivative

The sensitivity of the Rayleigh-wave phase velocity derivative with respect to shear-wave velocity is shown in FIG. 5. FIG. 5 seems to show higher sensitivity of phase velocity derivative to deep layers, especially the high modes. However, staggered positive and negative responses of the dispersion curve with positive perturbations are displayed. One significant advantage of the Rayleigh-wave phase velocity derivative is the sensitivity scale is large, which is 25 times of the phase velocity.

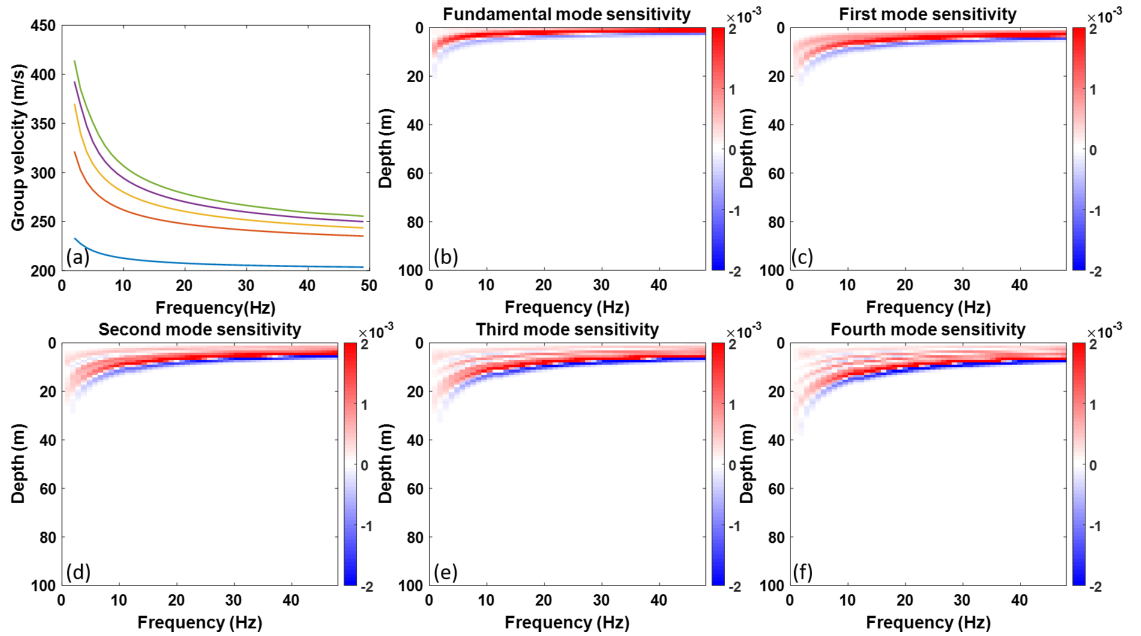

FIG. 4. Sensitivity of Rayleigh-wave group velocity with respect to shear-wave velocity.(a) Multimode group velocity. (b), (c), (d), (e), and (f) are the sensitivity of each mode.

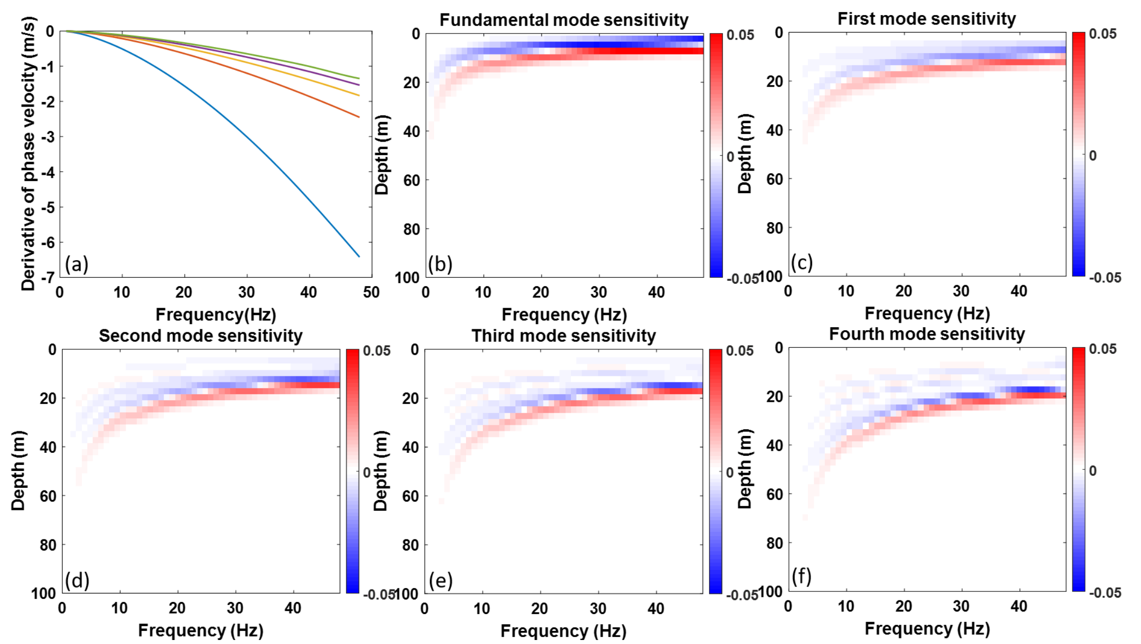

FIG. 5. Sensitivity of Rayleigh-wave phase velocity derivative with respect to shear-wave velocity.(a) Multimode phase velocity derivative. (b), (c), (d), (e), and (f) are the sensitivity of each mode.

Rayleigh-wave group velocity derivative

The sensitivity of Rayleigh-wave phase velocity derivative with respect to shear-wave velocity is shown in FIG. 6. The Rayleigh-wave group velocity derivative shows a similar pattern with group velocity in the same scale. We draw the sensitivity at 10 Hz for all these four types of observing data.

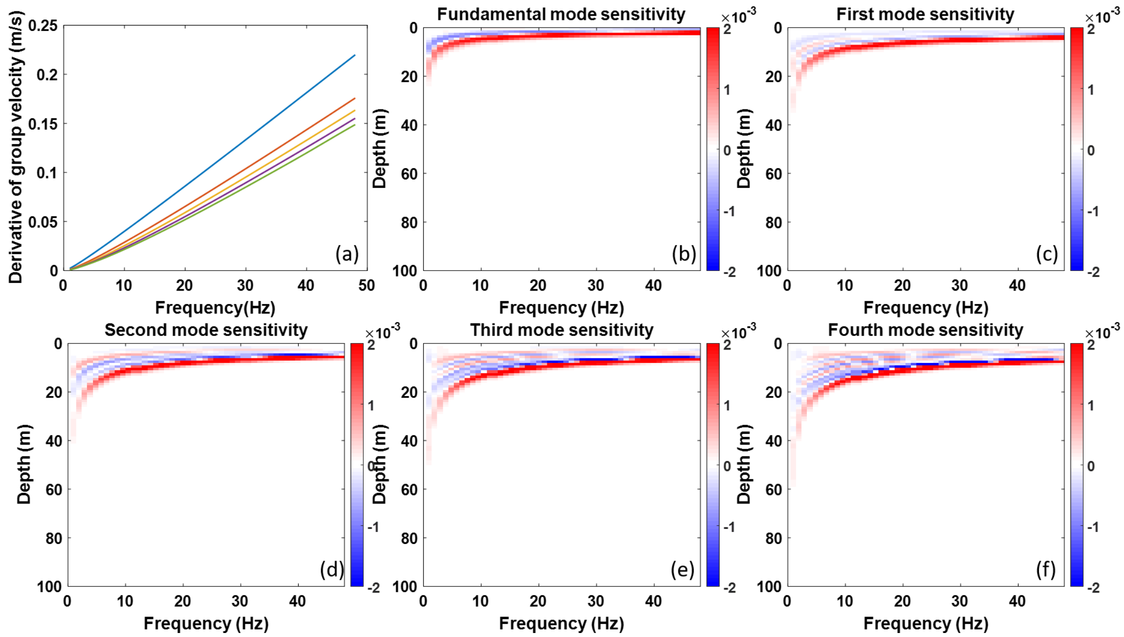

FIG. 6. Sensitivity of Rayleigh-wave group velocity derivative with respect to shear-wave velocity.(a) Multimode group velocity derivative. (b), (c), (d), (e), and (f) are the sensitivity of each mode.

It is clear from FIG. 7 that the sensitivity scale of the phase velocity derivative is much larger than the other three. Therefore, based on the tests above, we find it is an appropriate approach to adopt phase velocity derivative as the new observing data. However, the inversion using only fundamental mode may generate trade-off results since the sensitivity along depth is not all positive or negative. Higher modes are recommended to incorporate in the inversion for adding more constraints on the model parameters.

As for inversion, a preliminary comparison on trans-dimensional inversion using Rayleigh-wave phase velocity and phase velocity derivative is conducted (Dettmer et al., 2010). Our tests show that if the data are noise-free, with the same iteration times, the phase velocity derivative shows less uncertain result compared with phase velocity since it has higher sensitivity to shear-wave velocity. If we add Gaussian noise to the phase velocity and obtain the phase velocity derivative based on the noisy data. The robustness and stability of phase velocity inversion are better than its derivative. When noises are added, it seems the noises combined with the two-sided sensitivity influence the stability of the inversion result greatly. This can also because our data size is limited, so that the noise may not be Gaussian enough.

CONCLUSIONS

We have analyzed the sensitivity of Rayleigh-wave dispersion curves for various subsurface model parameters, including primary wave velocity, shear wave velocity, and density. We find the sensitivity levels with respect to these three parameters are in a ratio close to 4:40:1. Meanwhile, we explore several observing data types: Rayleigh wave phase velocity, group velocity, frequency derivative of phase velocity, frequency derivative of group velocity. We find the normalized sensitivity levels of these observing data to shear-wave velocity are in a ratio close to 1:2:12:2. Therefore, the phase velocity derivative shows the highest sensitivity to subsurface shear-wave velocity. However, we find only the phase velocity

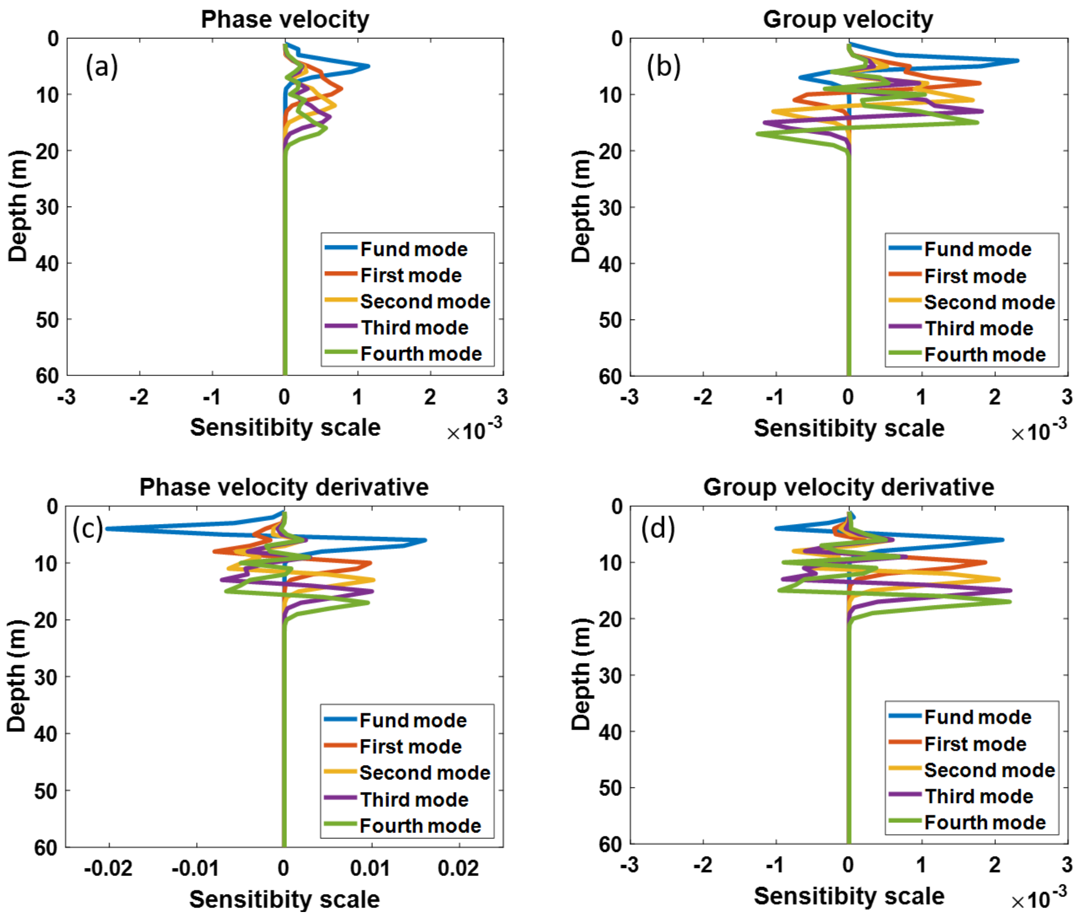

FIG. 7. Sensitivity of four types of observing data to shear-wave velocity. (a) Phase velocity. (b) Group velocity. (c) Phase velocity derivative. (d) Group velocity derivative.

shows one-sided sensitivity, while the other three show two-sided sensitivity. If higher modes are not incorporated in the inversion, the non-uniqueness of solutions will be more amplified. Therefore, these will all influence the inversion process. Both the phase velocity and phase velocity derivative have their advantages and limitations. Further study can be conducted on the combination of these two types of data or interleaved inversion.

ACKNOWLEDGEMENTS

We thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors, NSERC(Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13 and CRDPJ 543578-19. Luping was partially supported by a scholarship from SEG.

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