Well-log parameterized full waveform inversion 1: analysis of the single parameter case

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ABSTRACT

Applications of multiparameter elastic full waveform inversion (EFWI) deal with several challenges posed by the intrinsic nonlinearity of technique. Parameter cross-talk is one of most critical examples of the latter. Though, existing resources like scattering radiation patterns and the addition of prior information into FWI formulations might provide means to prevent some level of cross-talk. Ultimately, they cannot guarantee the removal of these unwanted effects. As an alternative mitigation strategy, we are studying reducing the numbers of modeled parameters in an isotropic medium, from three to one in this study. We argue that the single parameter inversion can be constructed in any application where elastic parameters demonstrate a strong correlation in readily available data, like well-logs, by fitting a trendline. An example of strong parameter relationship was observed and formulated for the Carbon Management Canada Newell County Facility. Though, the single parameter formulation was previously analyzed. We are expanding its use in a synthetic time-lapse application for carbon dioxide (CO_2) sequestration at the same site. Current results demonstrate that inverted models converged towards the true solution with various levels of success. While inverted P-wave velocity (V_P) and S-wave velocity (V_S) values fluctuate between a small parameter overestimation and underestimation. Density (ρ) values are mostly overestimated.

INTRODUCTION

Multiparameter elastic full waveform inversion (EFWI) consist in the simultaneous determination of two or more elastic properties to describe a medium. Although, this technique can be applied under the assumption of anisotropy or isotropy, the latter is usually preferred in applications due to further simplification of the relationships that governs this theory over the former. This is essential to reduce some of the challenges that FWI faces due to its intrinsic nonlinearity, which includes parameter cross-talk. This occurs when a parameter (e.g., density or ρ) is erroneously mapped into the updates of another parameter (e.g., P-wave velocity or V_P).

To assess plausible parameter cross-talk before applications, FWI practitioners rely on several resources. One of them are qualitative studies like scattering radiation patterns, contamination kernels and others. These help to select a suitable parameterization given a seismic acquisition that might help to guide decisions during the inversion (e.g., holding fixed one parameter until convergence of others). Another example of resources is prior information (e.g., known geology, known correlation of elastic parameters). This can be employed as a weighting factor in FWI formulations or as a guiding parameterization. An example of the latter was demonstrated by Eaid et al. (2021) when inverting real data using a single parameter at the Carbon Management Canada Newell County Facility (CMC or CMC Newel County Facility). For this site, it was demonstrated through available well-log data, the existence of a strong relationship between density and P-wave velocity, and

S-wave velocity (V_S) and P-wave velocity. These observations were employed to define a single parameter (from heron trendline), which encapsulated prior information and prevented the effects of cross-talk during the inversion process. Since the trendline will bound V_P, V_S, and ρ predicted behaviour, only a particular combination of these elastic properties will be encouraged according to the trend.

Based on previous success of this log-guided parameterization at CaMI.FRS, we continue to analyze its use during this investigation, but now set in a time-lapse application. We expect that this will allows us to further elaborate on the parameterization applicability with respect to other established elastic FWI parameterizations. Likewise, we expect to use this study as a suitable benchmark to the addition of a second inversion parameter determined from the same log trendline, which is also reported in this volume by Amundaray et al. (2022).

METHOD

Reviewing the single parameter log-guided parameterization

For an isotropic-elastic medium, the wave equation in frequency domain can be expressed as:

$$\rho\omega^2\mathrm{u}+c_{11}\nabla(\nabla\cdot\mathrm{u})-c_{44}\nabla\times(\nabla\times\mathrm{u})+\nabla(c_{11}-2c_{44})(\nabla\cdot\mathrm{u})+\nabla c_{44}(\nabla\mathrm{u}+\nabla\mathrm{u}^T)+\mathrm{f}=0 \quad (1)$$

Where ρ, c_{11}, c_{44} represents the isotropic-elastic stiffness moduli, u describes the wave displacement and f is the source term.

Any model reparameterization of the Equation 1 is accomplished by modifying the parameters used to express the elastic moduli. These become relevant in FWI because the gradient, the Hessian, and the subsequent model updates are estimated using derivatives of the form $\partial\mathbf{S}/\partial m$, where **S** refers to the wavefield operator and m refers to the chosen elastic constant used in c_{11} and c_{44} in Equation 1.

To incorporate prior information in a model reparameterization for FWI in cases where there is a robust relationship between the elastic parameters of a medium, Eaid et al. (2021) suggested preconditioning the terms in Equation 1 using a single parameter. As demonstrated by the authors, a method to estimate this parameter from available data can be achieve by the following steps:

1. Cross plotting well-log information: which helps to visualize whether there is a correlation between the elastic parameters of a medium.
2. Fitting a model through linear or nonlinear regression: this step facilitates the definition of a model that quantifies the predictability from one elastic parameter to another, aiding the construction of a well-behaved trendline. Because the latter should be constructed based on available data, this trend is case dependant, and it will encapsulate prior information.
3. Parameterizing the estimated trendline: a suitable parameterization for any well-behaved and differentiable trend is the arc length (η). The expression of the latter

in a $V_P - V_S - \rho$ space is given by

$$\eta_n = \eta_{n-1} + \sqrt{(\rho_n - \rho_{n-1})^2 + (V_{P_n} - V_{P_{n-1}})^2 + (V_{S_n} - V_{S_{n-1}})^2} \tag{2}$$

Where ρ is the density, V_P is the P-wave velocity, and V_S is the S-wave velocity. Addition of Equation 2 in FWI formulations results in waveform sensitivities in terms of the arc length, which are expressed as:

$$(\frac{\partial \rho}{\partial \eta}, \frac{\partial c_{11}}{\partial \eta}, \frac{\partial c_{44}}{\partial \eta}) \tag{3}$$

Once the trendline and its arc length parameterization is determined, the well-log values can be mapped from a space like $V_P - V_S - \rho$ to the actual single parameter to invert. This is done in four steps that are further explained in Eaid et al. (2021), by:

1. Identifying the values on the trendline at points n and $n + 1$, which bracket each well-log value.
2. Computing the value of η corresponding to the trendline at points n and $n + 1$.
3. Determining η_{V_P}, η_{V_S} and η_ρ by linearly interpolating the values at points n and $n+1$ weighted by the difference between the log and trend values.
4. Averaging the value of η given by η_{V_P}, η_{V_S} and η_ρ as a single parameter.

Modeled experiments

Three projected phases of carbon dioxide (CO_2) injection for the CMC Newell County Facility were modeled in this investigation as shown in Figure 1. These correspond to a stage prior to any gas injection, which we will refer as baseline, and two stages after 266 tons and 1664 tons of CO_2 are injected in the reservoir, which we will refer as medium and final stages. For the last two stages, it is assumed a horizontal migration of the CO_2 effects centered in an injection well at the reservoir level. This is identified between 295.65 m to 301.65 m in the Basal Belly River Formation (BBR) (Macquet et al., 2019).

To generate the true and initial model for the single parameter inversion, the well-log values were mapped from $V_P - V_S - \rho$ to η space following the steps discussed in the previous section. Figure 2 shows the fitted trendline for the well-log information from CMC Newell County Facility. As shown in Figure 3, we utilized a smoothed version of the true P-wave velocity, S-wave velocity, and density models to estimate the true models used in the inversion. These maintained the general trend observed in the logs, but removing fine details. We utilized as starting model for every stage, a model prior to any injection of CO_2, assuming unknown information about the migration of the gas.

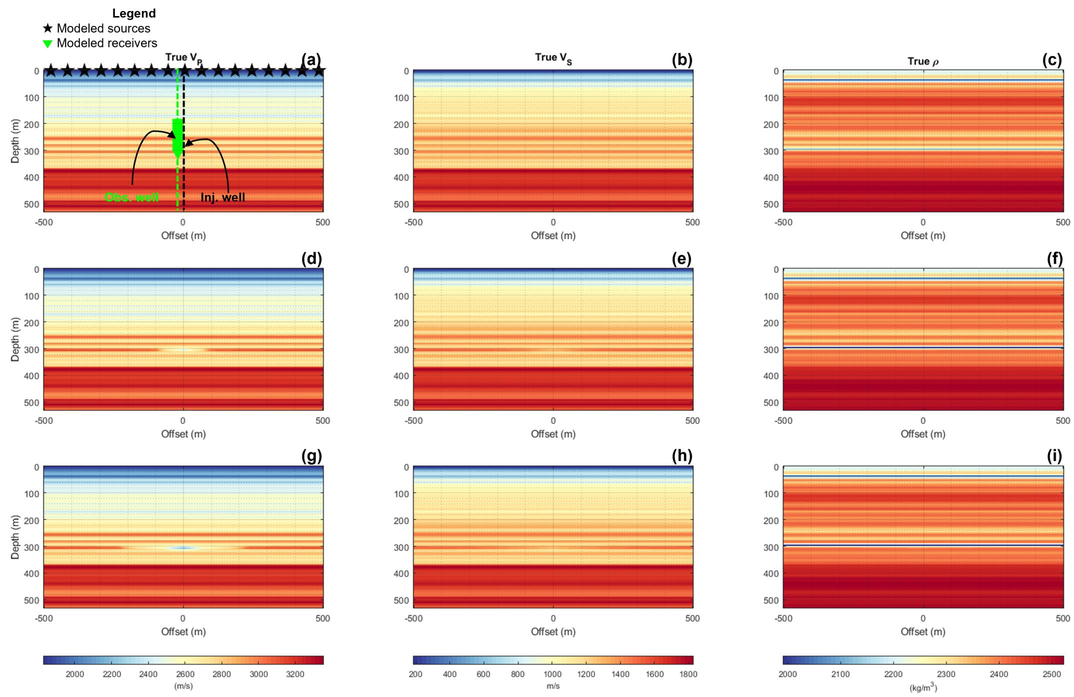

FIG. 1. True V_P, V_S, and ρ models corresponding to the three simulated stages. Models (a), (b), and (c) correspond to the baseline stage, models (d), (e), and (f) correspond to the medium injection stage, and models (g), (h), and (i) correspond to the final injection stage. Furthermore, an schematic of the seismic experiment and configuration of CMC Newell County Facility is overlaid on (a). The black stars indicate the source locations, the green triangles indicate the receivers, the green dashed line indicates the observation well, and the black dashed line indicates the injection well.

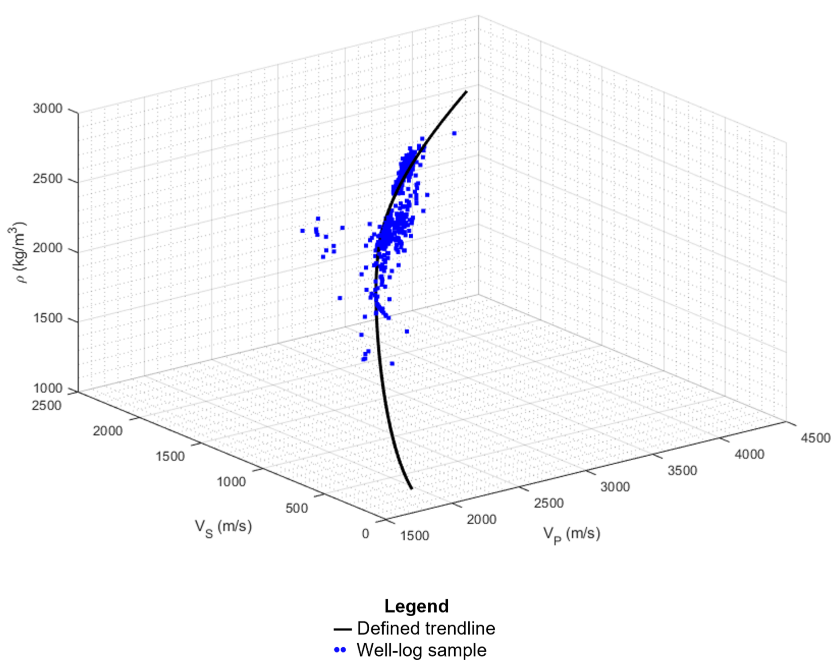

FIG. 2. 3D plot of the well-log data in $V_P - V_S - \rho$ space for the CMC Newell County Facility. The blue dots are log samples and the black line is the fitted trendline.

As illustrated in Figure 1(a), we assumed a fixed VSP configuration for every modeled stage as seismic acquisition, which mimics the deployed permanent experiment at CMC Newell County Facility (Hall et al., 2018). Vertical receivers were positioned in an observation well every 5 m between 190-305 m, which is located 20 m from the injection well. Surface sources were simulated with a separation of 60 m under the hypothesis that repeatability will not be a pressing issue.

To draw comparisons between an established elastic parameterization and the proposed single parameter constructed from the arc length of a well-log trendline (Eaid et al., 2021). We reproduced the same stages of CO_2 injection (i.e., baseline, medium stage, and final stage) using $V_P - V_S - \rho$ as parametrization. According to Amundaray and Innanen (2021), the latter has reproduced encouraging results in previous feasibility studies for site.

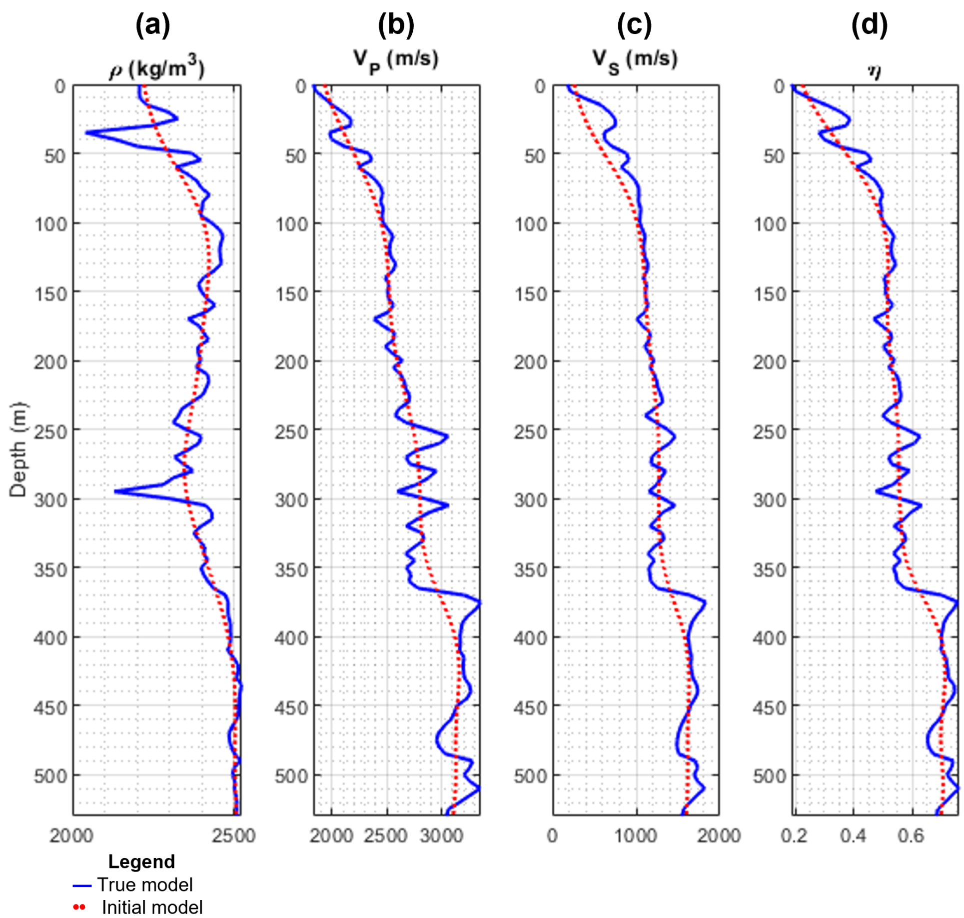

FIG. 3. Adapted well-logs and the mapped parameter used during the inversion. In (a) density, (b) P-wave velocity, (c) S-wave velocity, and (d) η parameter. For each case, the blue lines represent the true model and the red dotted lines represent the initial models.

A relevant difference between the two parameterizations in use for this investigation is related to the employed inversion scheme. While, in the log-guided parameterization V_P, V_S, and ρ are simultaneously inverted by updating η. In the established elastic parameterization, we had to use a scaled approach due to parameter cross-talk. To accomplish this, we simultaneously inverted V_P and V_S using only the vertical component of the VSP data, until reaching some level of model convergence. Following, we continue to invert for V_P and V_S simultaneously, but using both, the vertical and horizontal components of the VSP data, until reaching the last defined frequency band. Lastly, we repeated the inversion process but only for ρ, holding fixed as initial models both inverted V_P and V_S results.

RESULTS

Figure 4 shows the inverted V_P, V_S, and ρ models between 100 m to 530 m depth, after the inverted η model was mapped back to $V_P - V_S - \rho$ space. Though, inverted values in the shallow section demonstrated poor convergence, leading us to reduce the effective area of inversion. The deeper sections of most models reproduce interpretable results with respect to the true models. This is in terms of (1) the measured values for each elastic parameter, and (2) the imaged limits of expected CO_2 effects around the reservoir level.

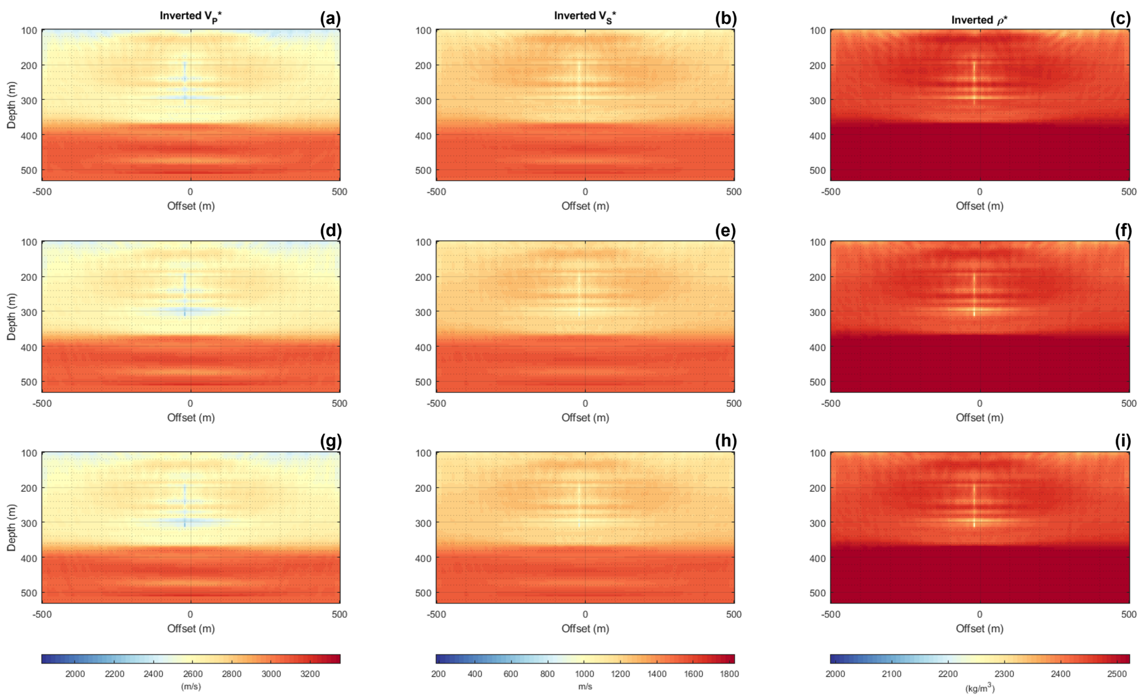

FIG. 4. Inverted models after η was mapped back to $V_P - V_S - \rho$ space. Models (a), (b) and (c) correspond to the baseline stage, models (d), (e) and (f) corresponds to the medium injection stage, and models (g), (h) and (i) corresponds to the final injection stage.

To further inspect the inverted models, an offset point distanced 40 m from the modeled observation well and 20 m from the injection well, respectively, was defined. Figure 5 shows the inverted values for each parameter. As seen Figure 5(a)-(c), inverted V_P roughly follows the true model at most depth levels. While most values between 100 m and 250 m show a slight overestimation, values between 250 m to 530 m tend to show signs of model underestimation. This is a consistent behaviour for each simulated stage.

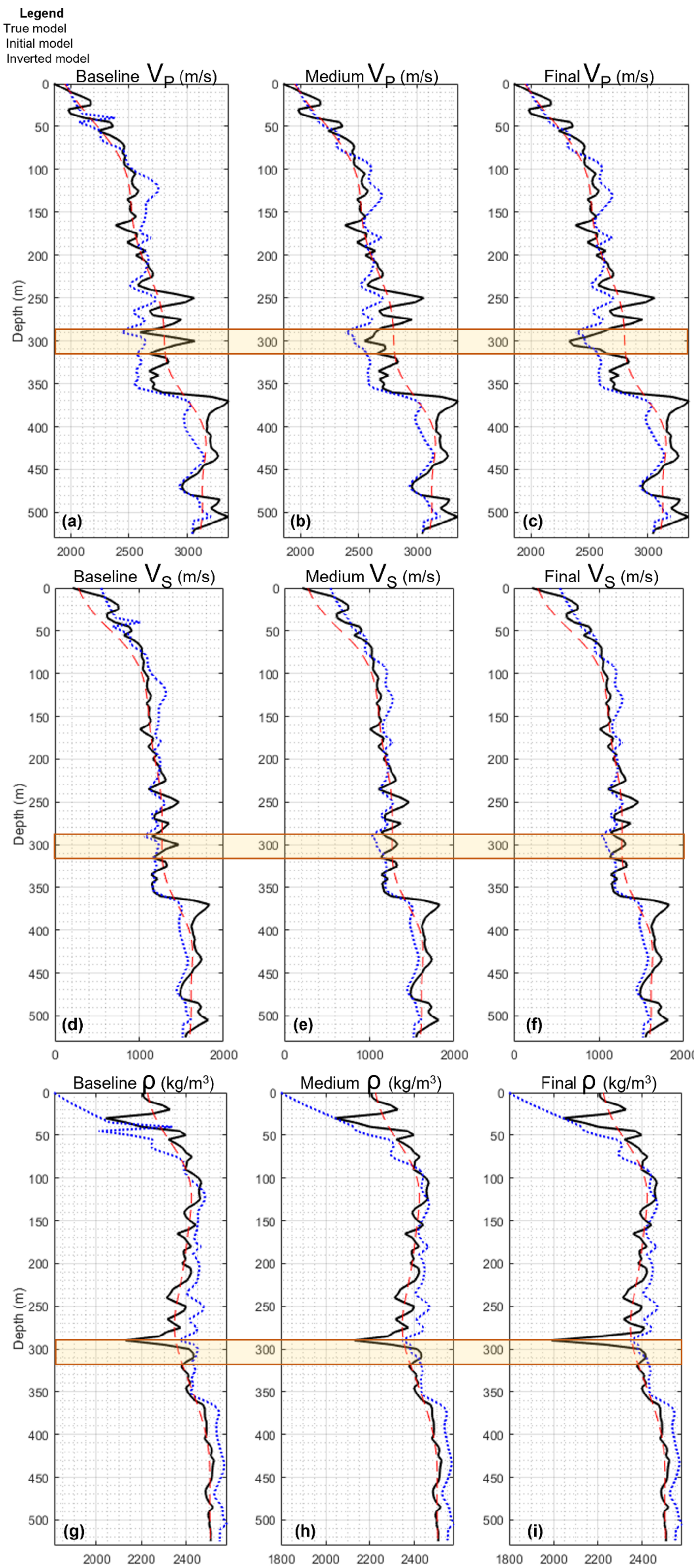

FIG. 5. Inverted V_P, V_S, and ρ using the single parameter log-guided paramaterization at an offset location set 20 m right from the zero offset. Profile views (a), (b), and (c) are associated to V_P, (d), (e), and (f) are associated to V_S, and (g), (h), and (i) are associated to ρ. For each, the black lines are the true models, the red dashed lines are the initial models, the blue dotted lines are the inverted models, and the yellow boxes indicate the reservoir level.

In the case of V_S, Figure 5(d)-(f) shows that the inverted values follows a similar trend, in terms of model overestimation and underestimation, with respect to V_P. Nonetheless, visual inspection suggest a slightly better model convergence for this parameter between 240 m to 500 m depth. This is further discussed quantitative by Amundaray et al. (2022) using the normalized root mean square error (NRMSE).

Figure 5(g)-(i) shows the inverted ρ at the same defined offset location. Though, inverted values smoothly follow the general trend of the true model, most are an overestimation that averages 100 kg/m^3 above the expected value. This a consistent behaviour for each simulated stage.

Figure 6 and Figure 7 show V_P, V_S, and ρ residuals for the medium and final stage of injection, taking as reference the inverted baseline obtained from each parameterization. Though, both parameterizations reproduce the expected decrease on the elastic moduli associated to the CO_2 effects. The measured magnitudes for each parameter are captured better when using $V_P - V_S - \rho$ parameterization instead of the log-guided parameterization. For the latter, inverted values are similar for all simulated stages, in terms of imaged limits gas effects and magnitude. This does differs from the predictions set for the site by the true models.

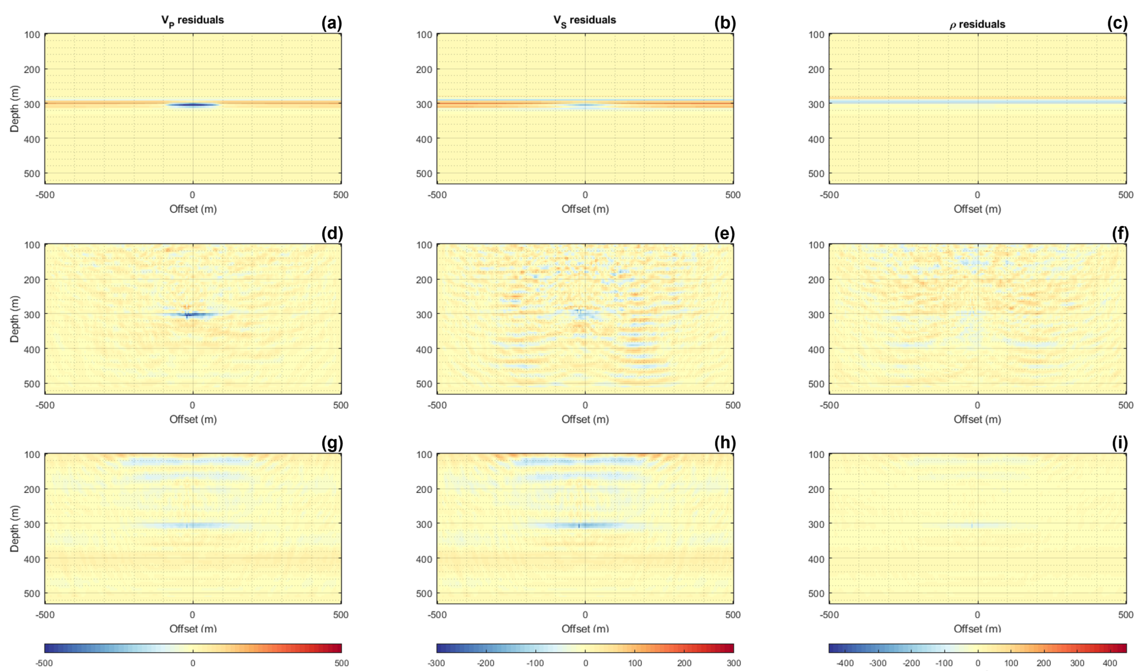

FIG. 6. V_P, V_S, and ρ model residuals for the medium stage of injection, using as reference the inverted baseline from each parameterization. Models (a), (b), and (c) are true residuals, models (d), (e), and (f) are residuals obtained using $V_P - V_S - \rho$ as parameterization, and (g), (h), and (i) are residuals obtained using the single term log-guided parameterization.

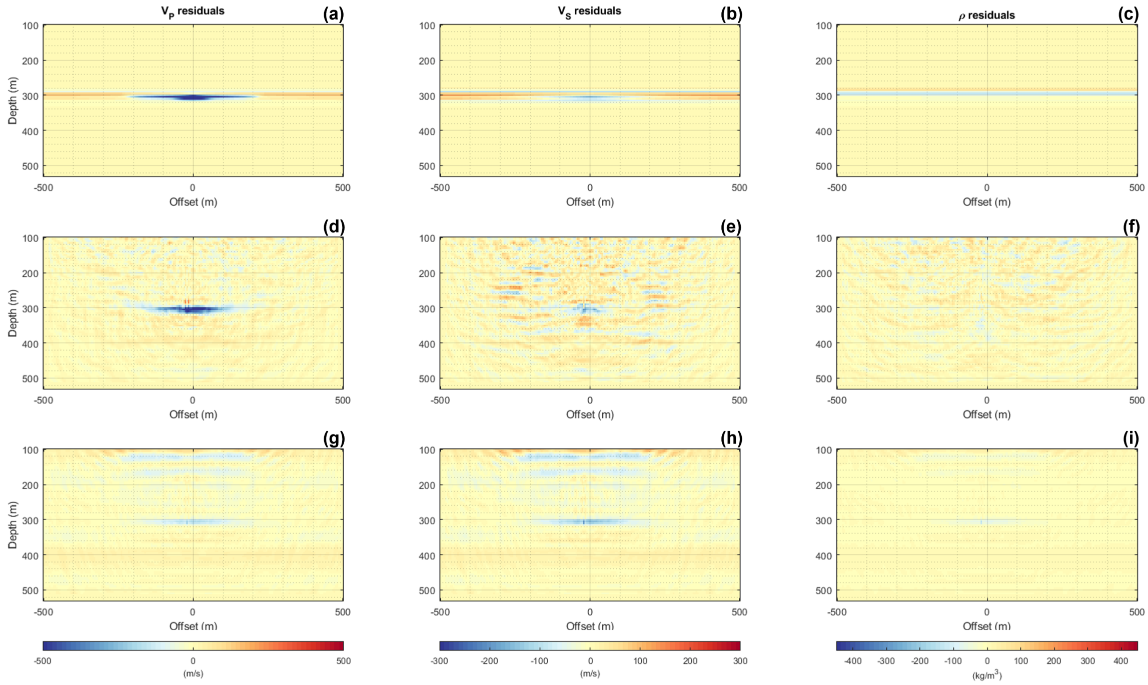

FIG. 7. V_P, V_S, and ρ model residuals for the final stage of injection, using as reference the inverted baseline from each parameterization. Models (a), (b), and (c) are true residuals, models (d), (e), and (f) are residuals obtained using $V_P - V_S - \rho$ as parameterization, and (g), (h), and (i) are residuals obtained using the single term log-guided parameterization.

CONCLUSIONS

Inverted models using the single parameter log-guided parameterization in a time-lapse application for the CMC Newell County Facility demonstrate a good level of model convergence. This is true for depth levels below 100 m. Above the latter, many artifacts were introduced, probably due to the proximity to the modeled sources. Once bypassed the shallow section, inverted ρ values are constantly overestimated with respect to the true model. Whereas, in the case of V_P and V_S, inverted values tend to fluctuate more similarly around the true model values, with small model underestimation and overestimation at similar depths. This can be regarded as a by-product of using the trendline parameterization during the inversion, where only a single combination of parameter is encouraged.

Although, the inverted parameter magnitudes using the single term parameterization fall behind results obtained using $V_P - V_S - \rho$ as parameterization by at least half. Its implementation reproduces models that image the lateral limits of the CO_2 effects. These can be distinguished without ambiguities in the model residuals generated for each parameter with respected to the inverted baseline.

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REFERENCES

Amundaray, N., and Innanen, K. A., 2021, Time-lapse elastic full waveform inversion of CO2 injection at CaMI FRS using VSP: a feasibility study: CREWES Research Report, **33**, No. 1.

Amundaray, N., Keating, S., Eaid, M., and Innanen, K. A., 2022, Well-log parameterized full waveform inversion 2: formulation of the two parameter case: CREWES Research Report, **this volume**, No. 2.

Eaid, M., Keating, S., and Innanen, K. A., 2021, Full waveform inversion of DAS field data from the 2018 CaMI VSP survey: CREWES Research Report, **33**, No. 7.

Hall, K. W., Bertram, K. L., Bertram, M. B., Innanen, K. A. H., and Lawton, D. C., 2018, CREWES 2018 multi-azimuth walk-away VSP field experiment: CREWES Research Report, **30**, No. 16.

Macquet, M., Lawton, D. C., Saeedfar, A., and Osadetz, K. G., 2019, A feasibility study for detection for detections thresholds of co2 at shallow depths at the cami field research station, newell county, alberta, canada: Petroleum Geoscience, **25**, No. 4, 509–518.