# Bayesian approaches to estimating rock physics properties from seismic attributes

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## ABSTRACT

Bayesian rock physics inversion refers to a set of probabilistic methods for the prediction of reservoir properties from elastic attributes, based on different statistical assumptions for the distribution of the model variables and different linear or nonlinear rock physics models. We have examined three Bayesian approaches using the well-log data at the Carbon Management Canada Newell County Facility, assuming Gaussian, Gaussian mixture, and non-parametric distributions of the rock physics variables. The solution is represented by the posterior distribution of the porosity and lithology parameters conditioned on the elastic data. In this application, because the nonlinearity of the rock physics model is not strong and the data are approximately Gaussian distributed, the three results are similar, all capturing the trend of the actual logs. However, the Gaussian mixture model might be a more appropriate solution, owing to its efficiency and its ability to recognize the multimodality of the data. The proposed methods can be combined with elastic inversion to implement a two-step workflow of seismic reservoir characterization at the field, if we assume that the rock physics model calibrated at the well location is also valid far away from the well.

## INTRODUCTION

In seismic reservoir characterization, the estimation of rock and fluid properties is generally achieved in two steps: seismic inversion and rock physics inversion. In seismic inversion, we invert the seismic data (e.g., amplitude, time, waveforms) for models of elastic attributes, such as velocity, density, and modulus. The inversion can be performed using complex forward models, for instance wave equations (Tarantola, 1986; Brossier et al., 2009; Alkhalifah and Plessix, 2014), or using forward models that are less computationally intense, for instance the linearized AVO (amplitude versus offsets) equations and the convolutional model (Aki and Richards, 2002; Buland and Omre, 2003; Russell et al., 2011). In rock physics inversion, we use the realizations of elastic attributes to estimate the reservoir properties of interest, such as porosity, lithology, and fluid saturation. The inversion requires a rock physics model to link elastic and reservoir properties. This model is generally nonlinear and the choice of the model depends on the geological environment. A review of the available methods combining rock physics and seismic inversion has been proposed by Doyen (2007), Bosch et al. (2010), and Grana et al. (2021).

A very common approach in inverse theory is Bayesian inversion, in which the prior distribution of the model is combined with the likelihood to observe the data given the model. In rock physics domain, the Bayesian framework allows integrating the prior knowledge of reservoir properties with a rock physics model that links the model parameters to seismic attributes through the probability density functions (PDFs). The solution of the inverse problem is then expressed as a PDF, from which multiple model realizations can be sampled. Under some restrictive assumptions, we can derive a closed-form solution of the Bayesian inverse problem. Generally, these assumptions include the linearity of the forward model, the Gaussian prior distribution of the model parameters, and the Gaussian distribution of the data errors (Buland and Omre, 2003; Tarantola, 2005; Grana, 2016). The closed-form solution of the posterior distribution has the advantage that the computational cost of the inversion is very limited. However, the reservoir properties are multimodal in many applications, especially when the lithofacies are complex, and the rock physics linearization might fail for highly nonlinear models.

To describe the multimodal behavior of rock and fluid properties, Grana and Rossa (2010) extend the Bayesian approach to Gaussian mixture models (GMM), which allow modeling each litho-fluid class as a single Gaussian component of the mixture. Moreover, the analytical results valid for Gaussian distributions can be extended to Gaussian mixtures, provided the rock physics model is not too far from linearity. If these assumptions with respect to the forward model and prior PDF are not in agreement with well log data, the posterior distribution must be evaluated numerically. In this case, Bayesian approaches based on kernel density estimation have been proposed in Mukerji et al. (2001), Doyen (2007), and Grana and Rossa (2010), where the joint distribution of model and data is described by a non-parametric PDF. With respect to the GMM approach, the non-parametric approach is more computationally demanding and it requires tuning of the kernel widths.

The main goal of this study is to examine the applicability of different Bayesian approaches for the estimation of rock properties from seismic attributes at the CMC Newell County Facility (Lawton et al., 2019; Macquet et al., 2019). Based on the well-log data of this area and the nonlinear rock physics model previously calibrated at the well (Hu et al., 2022), we analyze three different cases: 1) linearized rock physics model and Gaussian assumption of reservoir properties; 2) nonlinear rock physics model and Gaussian mixture assumption of reservoir properties; 3) nonlinear rock physics model and non-parametric distribution of reservoir properties.

## METHOD

The inverse problem can be written in the form

$$\mathbf{d} = g(\mathbf{m}) + \mathbf{e}, \tag{1}$$

where the model vector $\mathbf{m}$ comprises the rock physics properties to be estimated, the data vector comprises three elastic attributes: P-wave velocity, S-wave velocity and density $(V_\mathrm{P}, V_\mathrm{S}, \rho)$, $\mathbf{e}$ is the errors in the data, and $g$ is the physical relation that links the model to the data (a nonlinear rock physics model in our case).

We operate in a Bayesian setting to assess the posterior distribution $P(\mathbf{m}|\mathbf{d})$ of the model variables $\mathbf{m}$ conditioned on the data $\mathbf{d}$:

$$P(\mathbf{m}|\mathbf{d}) = \frac{P(\mathbf{m},\mathbf{d})}{\displaystyle\int P(\mathbf{m},\mathbf{d})d\mathbf{m}} = \frac{P(\mathbf{d}|\mathbf{m})P(\mathbf{m})}{\displaystyle\int P(\mathbf{d}|\mathbf{m})P(\mathbf{m})d\mathbf{m}}, \tag{2}$$

where $P(\mathbf{m},\mathbf{d})$ is the joint distribution of the model and data, $P(\mathbf{m})$ is the prior distribution of the model, and $P(\mathbf{d}|\mathbf{m})$ is the likelihood function of the data.

### Linear problem with Gaussian prior

Grana (2016) proposes a mathematical approach for the linearization of slightly non-linear rock physics models, using first-order Taylor series approximations:

$$\mathbf{d} \approx g(\mathbf{m_0}) + \mathbf{J_{m_0}}(\mathbf{m} - \mathbf{m_0}) + \mathbf{e} \tag{3}$$

where $\mathbf{J_{m_0}}$ is the Jacobian of the function $g$ evaluated at the value $\mathbf{m_0}$, which can be the mean of the prior distribution. Equation 3 can be rewritten as

$$\mathbf{d} \approx \mathbf{J_{m_0}}\mathbf{m} + g(\mathbf{m_0}) - \mathbf{J_{m_0}}\mathbf{m_0} + \mathbf{e} = \mathbf{G}\mathbf{m} + \mathbf{c} + \mathbf{e}, \tag{4}$$

where $\mathbf{G} = \mathbf{J_{m_0}}$ is the linearized rock physics model and $\mathbf{c} = g(\mathbf{m_0}) - \mathbf{J_{m_0}}\mathbf{m_0}$ is a constant.

If we assume that the model $\mathbf{m}$ is Gaussian distributed $\mathcal{N}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$ and the data error $\mathbf{e}$ is Gaussian $\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_e)$, then the posterior distribution $P(\mathbf{m}|\mathbf{d})$ is also Gaussian $\mathcal{N}(\boldsymbol{\mu}_{m|d}, \boldsymbol{\Sigma}_{m|d})$, and it can be analytically estimated through the following expressions for the conditional mean and covariance:

$$\begin{aligned} \boldsymbol{\mu}_{m|d} &= \boldsymbol{\mu}_m + \boldsymbol{\Sigma}_m \mathbf{G}^T (\mathbf{G}\boldsymbol{\Sigma}_m \mathbf{G}^T + \boldsymbol{\Sigma}_e)^{-1} (\mathbf{d} - \mathbf{G}\boldsymbol{\mu}_m) \\ \boldsymbol{\Sigma}_{m|d} &= \boldsymbol{\Sigma}_m - \boldsymbol{\Sigma}_m \mathbf{G} (\mathbf{G}^T \boldsymbol{\Sigma}_m \mathbf{G}^T + \boldsymbol{\Sigma}_e)^{-1} - \mathbf{G}\boldsymbol{\Sigma}_m. \end{aligned} \tag{5}$$

The mathematical derivation of these expressions can be found in Tarantola (2005). The main limitation of this approach is the Gaussian (unimodal) assumption of the model properties. Many rock properties in the subsurface, for example porosity and fluid saturation, are multimodal. In addition, owing to the linearization of the rock physics operator, this approach may fail for highly nonlinear models.

### Nonlinear problem with Gaussian mixture prior

In many applications, the multimodal behavior of rock properties can be approximated by Gaussian mixture distributions, i.e. linear combinations of Gaussian distributions:

$$P(\mathbf{m}) = \sum_{k=1}^{N_f} \lambda_k \mathcal{N}(\mathbf{m}; \boldsymbol{\mu}_m^k, \boldsymbol{\Sigma}_m^k), \tag{6}$$

where the distributions $\mathcal{N}(\mathbf{m}; \boldsymbol{\mu}_m^k, \boldsymbol{\Sigma}_m^k)$ represent the $k^{\text{th}}$ Gaussian component and the coefficients $\lambda_k$ represent the weights of the linear combination. One of the advantages of Gaussian mixture models in geophysical applications is the possibility to identify the components of the mixtures with geological classifications, such as facies and geobodies.

For nonlinear rock physics models, Grana and Rossa (2010) propose a semi-analytical approach based on Monte Carlo simulations. According to this approach, we randomly sample the prior distribution (Equation 6) and apply the rock physics model to obtain the corresponding set of elastic properties $\mathbf{d}$. We then use these samples as a training dataset to estimate the joint distribution of rock and elastic properties assuming a Gaussian mixture distribution:

$$P(\mathbf{m}, \mathbf{d}) = \sum_{k=1}^{N_f} \lambda_k' \mathcal{N}(\mathbf{y}; \boldsymbol{\mu}_y^k, \boldsymbol{\Sigma}_y^k), \tag{7}$$

where $\mathbf{y} = (\mathbf{m}, \mathbf{d})$. The joint mean and covariance of each component are given by

$$\boldsymbol{\mu}_y^k = \begin{bmatrix} \boldsymbol{\mu}_m^k \\ \boldsymbol{\mu}_d^k \end{bmatrix}, \quad \boldsymbol{\Sigma}_y^k = \begin{bmatrix} \boldsymbol{\Sigma}_{m,m}^k & \boldsymbol{\Sigma}_{m,d}^k \\ \boldsymbol{\Sigma}_{d,m}^k & \boldsymbol{\Sigma}_{d,d}^k \end{bmatrix}, \tag{8}$$

These quantities of the joint distribution can be inferred from the training dataset using maximum likelihood estimation methods such as the expectation–maximization algorithm. Then the conditional distribution $P(\mathbf{m}|\mathbf{d})$ is again a Gaussian mixture:

$$P(\mathbf{m}|\mathbf{d}) = \sum_{k=1}^{N_f} \lambda_k'' \mathcal{N}(\mathbf{m}; \boldsymbol{\mu}_{m|d}^k, \boldsymbol{\Sigma}_{m|d}^k), \tag{9}$$

with conditional mean and covariance given by

$$\begin{aligned} \boldsymbol{\mu}_{m|d}^k &= \boldsymbol{\mu}_m^k + \boldsymbol{\Sigma}_{m,d}^k (\boldsymbol{\Sigma}_{d,d}^k)^{-1} (\mathbf{d} - \boldsymbol{\mu}_d^k) \\ \boldsymbol{\Sigma}_{m|d}^k &= \boldsymbol{\Sigma}_{m,m}^k - \boldsymbol{\Sigma}_{m,d}^k (\boldsymbol{\Sigma}_{d,d}^k)^{-1} \boldsymbol{\Sigma}_{d,m}^k. \end{aligned} \tag{10}$$

This approach is robust when the rock physics model is not highly nonlinear.

## Nonlinear problem with non-parametric prior

If the assumptions for the aforementioned approaches are not in agreement with well-log data, a non-parametric approach for the conditional probability estimation $P(\mathbf{m}|\mathbf{d})$ should be adopted. Kernel density estimation is a non-parametric technique that allows us to estimate the probability distribution by fitting a base function at each data point including only those observations close to it.

Let $\mathbf{m} = (m_1, ..., m_M)$ represent a model vector with $M$ variables, for instance $M = 3$ when the model consists of porosity, clay content, and water saturation at a single point, $\mathbf{d} = (d_1, ..., d_D)$ represent a data vector with $D$ parameters, for instance $D = 3$ when using the P- and S-wave velocities plus density at the same point as input data, and $\{\mathbf{m}_i, \mathbf{d}_i\}_{i=1,...,N_s}$ represent the set of $N_s$ Monte Carlo samples, then the joint distribution $P(\mathbf{m}, \mathbf{d})$ can be estimated as

$$P(\mathbf{m}, \mathbf{d}) = \frac{1}{N_s \prod_{u=1}^{M} h_{m_u} \prod_{v=1}^{D} h_{d_v}} \sum_{i=1}^{N_s} \prod_{u=1}^{M} K(\frac{m_u - m_{u_i}}{h_{m_u}}) \prod_{v=1}^{D} K(\frac{d_v - d_{v_i}}{h_{d_v}}), \tag{11}$$

where $K$ is the kernel function, such as the Gaussian kernel and the Epanechnikov kernel, and $\mathbf{h}_m$ and $\mathbf{h}_d$ are the vectors of kernel bandwidths of each variable (Grana et al., 2021). Then, the conditional distribution $P(\mathbf{m}|\mathbf{d})$ can be numerically evaluated using Equation 2. Because the joint and conditional distributions are numerically evaluated in a discretized domain for all the possible combinations of $(\mathbf{m}, \mathbf{d})$, one of the limitations of the proposed approach is the memory requirement for fine discretization of the model and data domains.

## APPLICATION TO WELL LOGS

We present the application of the method to well-log data analysis. The well logs in this example were measured in the Countess 10-22 well at the Carbon Management Canada (CMC) Newell County Facility, located in Southwest of Brooks, Alberta. The well was drilled to a depth of 550 m, with the shallow stratigraphy composed of interbedded mudstone, fine-grained sandstone, and uncleated coals (Lawton et al., 2019). A comprehensive log suite including a dipole sonic log was acquired at the well. The wireline logs were further studied using Schlumberger's elemental log analysis (ELAN) that provided depth profiles of porosity and lithology parameters.

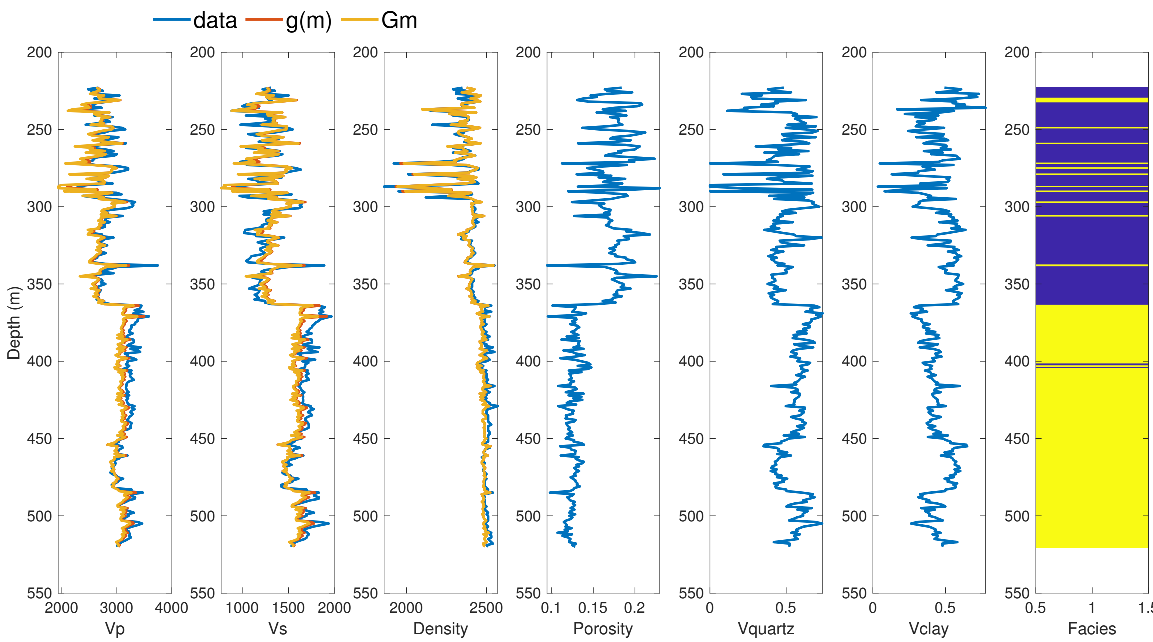


FIG. 1. Well-log data of the Countess 10-22 well at the CMC Newell County Facility. From left to right: P-wave velocity, S-wave velocity, density, porosity, and the volume fractions of quartz and clay. The blue, red, and yellow curves denote the real data, the data predicted by the nonlinear rock physics model, and the data predicted by the linearized rock physics model. The rightmost column shows the result of facies classification based on the measured data, assuming two facies.

Figure 1 plots the well logs of velocities, density, porosity ($\phi$) and the volume fractions of quartz ($V_{\mathrm{qu}}$) and clay ($V_{\mathrm{cl}}$) within the depth interval 220m-530m. Previously, we constructed a rock physics model combining the soft-sand model and Gassmann's equations to link the elastic and rock property logs (Hu et al., 2022). The rock physics model $g$ can be written as

$$(V_{\mathrm{P}}, V_{\mathrm{S}}, \rho) = g(\phi, V_{\mathrm{qu}}, V_{\mathrm{cl}}, V_{\mathrm{co}}, P_{\mathrm{eff}}), \tag{12}$$

where $V_{\mathrm{co}}$ is the coal volume and $V_{\mathrm{qu}} + V_{\mathrm{cl}} + V_{\mathrm{co}} = 1$. The brine saturation was assumed to be 100%, according to Macquet et al. (2019). The effective pressure initially used in the rock physics modeling increases linearly with depth. However, using an average pressure value of 5 MPa, we can still obtain a good match between the predicted and measured logs, as shown in Figure 1. Therefore, we define three model variables, $\phi$, $V_{\mathrm{qu}}$, and $V_{\mathrm{cl}}$. Equation 12 is then reduced to

$$(V_{\mathrm{P}}, V_{\mathrm{S}}, \rho) = g(\phi, V_{\mathrm{qu}}, V_{\mathrm{cl}}), \tag{13}$$

The implementation of the linear Bayesian approach requires the linearization of the rock physics model. In Figure 1, we notice that the linearized model approximates the exact rock physics model very well.

Figures 2-4 show the parametric and non-parametric distributions fitted to the well-log data of the rock physics properties. For illustration purpose, we only display the bivariate projections in the domain of porosity and clay content. We assume a Gaussian mixture distribution of two components, which represent two different rock types, namely high-porosity sand and low-porosity shale, based on the result of facies classification at the well (Figure 1); Compared to the Gaussian assumption (Figure 2), the Gaussian mixture PDF might be more appropriate for this application, given the bimodality of the data (Figure 3). The non-parametric PDF is calculated using the Epanechnikov kernel, and it provides a more accurate description of the joint distribution (Figure 4).

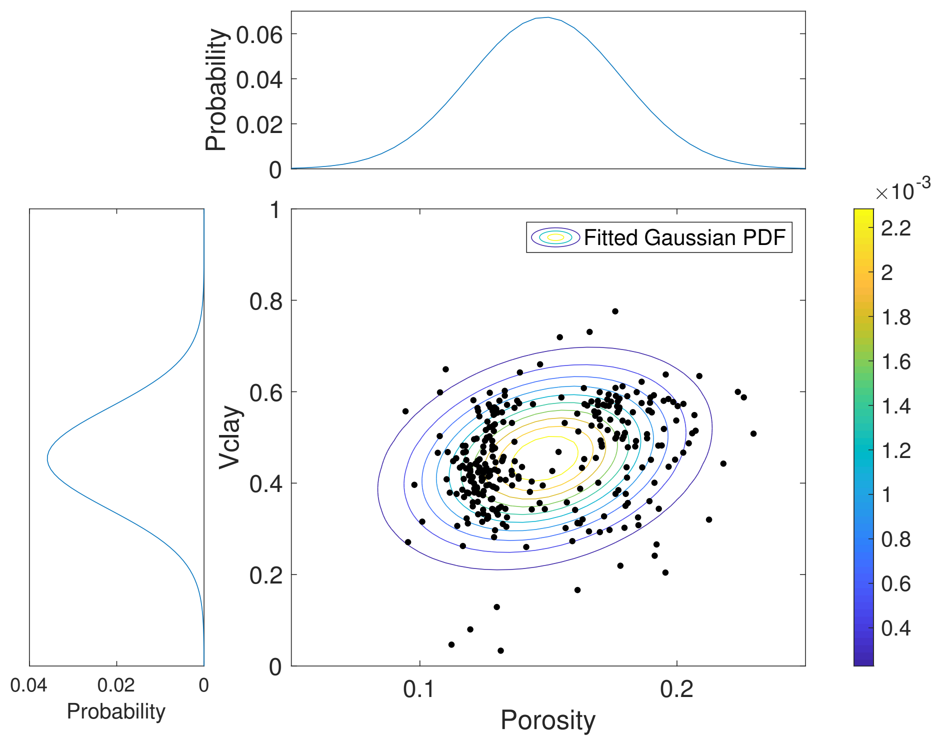


FIG. 2. Prior distribution of rock physics variables: Gaussian case. The curves represent the 2D joint distribution of porosity and clay content with associated 1D marginal distribution, obtained by fitting the well-log data (black dots).

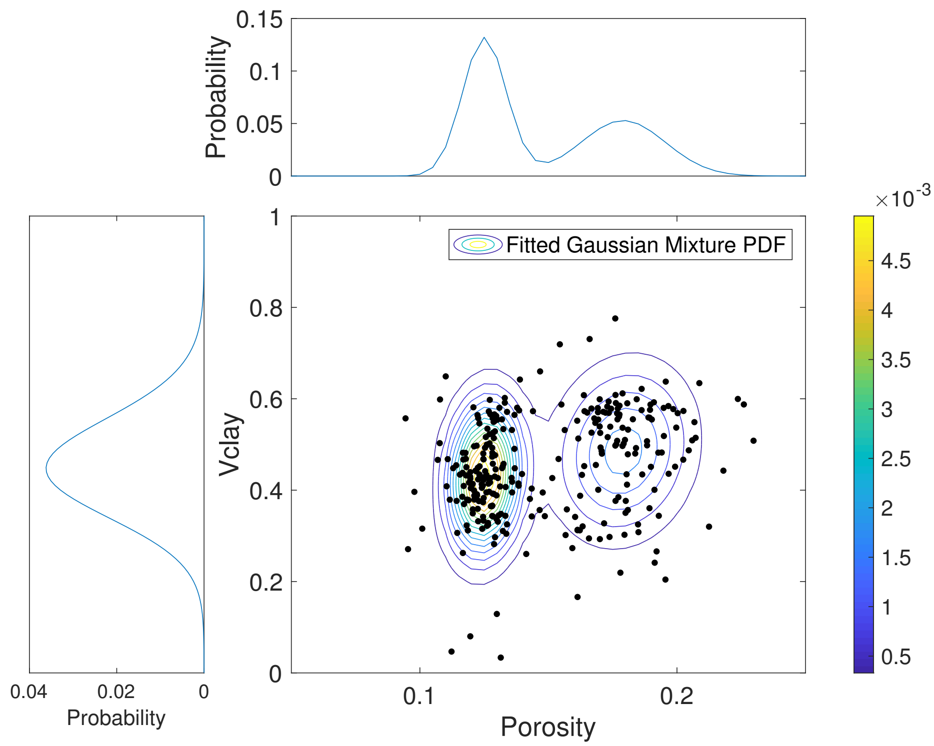


FIG. 3. Prior distribution of rock physics variables: Gaussian mixture case. The parameters of the Gaussian mixture model is obtained from the facies classification of the well-log data

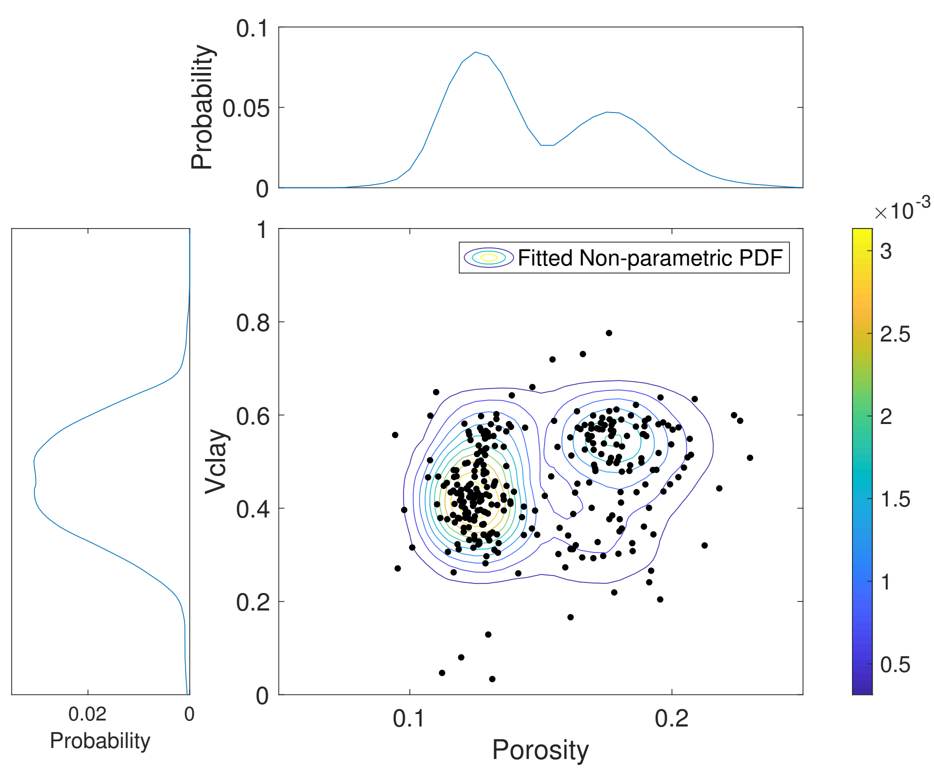


FIG. 4. Prior distribution of rock physics variables: non-parametric case. The Epanechnikov kernel is used.

To mimic seismic resolution, we filter the velocity and density logs using a step size of 5m (Figure 5), and then use the smooth logs as input data for the rock physics inversion. The Bayesian linearized inversion provides the posterior mean, the posterior covariance matrix, and the full posterior distribution evaluated on the multidimensional grid for each measured data. The marginal distributions are obtained by numerically integrating the posterior distribution. Overall, the probability distributions capture the trend of the actual logs (Figure 6). Given the good match between the actual rock-physics model and the linearized approximation, we point out that the prediction errors are mainly caused by the errors in the original rock-physics model and those in the input data (smooth elastic logs).

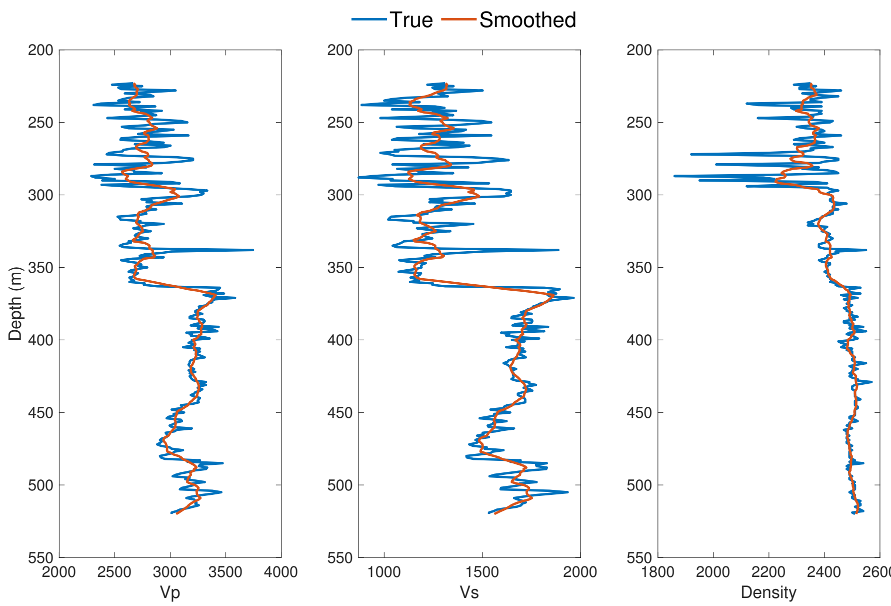


FIG. 5. Filtered velocity and density logs used as input data for the rock physics inversion.

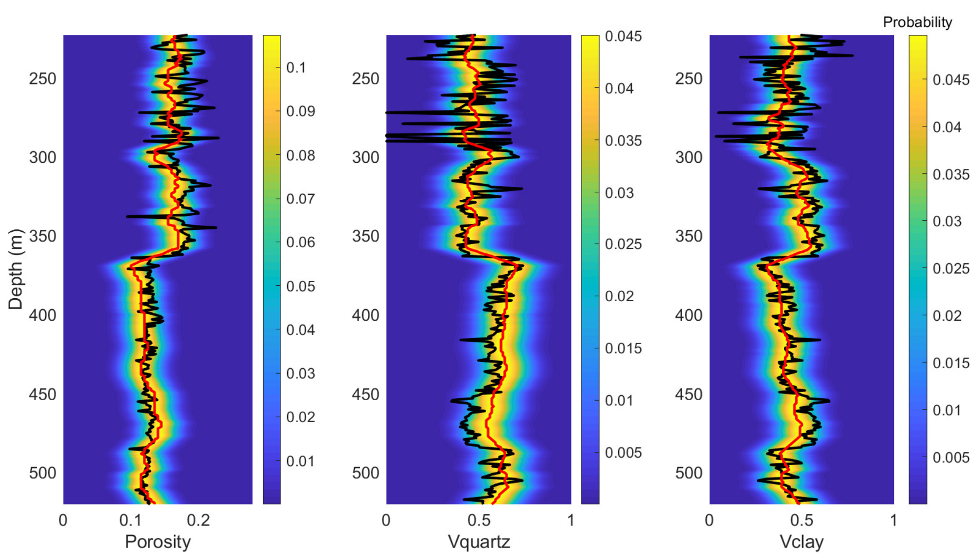


FIG. 6. Results of the Bayesian linearized rock physics inversion. The background color represents the posterior distribution and the solid red curves represent the maximum a posteriori predictions. The solid black curves represent the actual well logs.

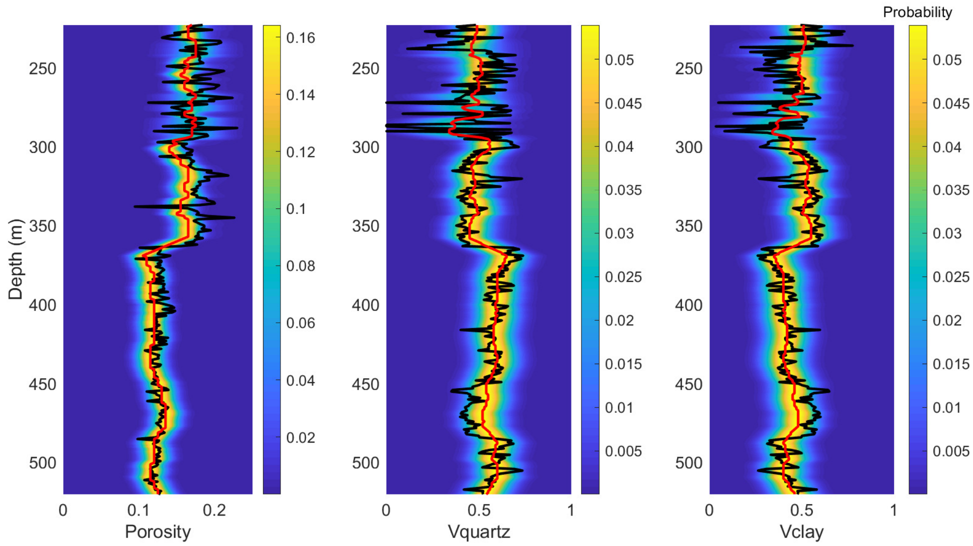


FIG. 7. Results of the Bayesian Gaussian mixture approach.

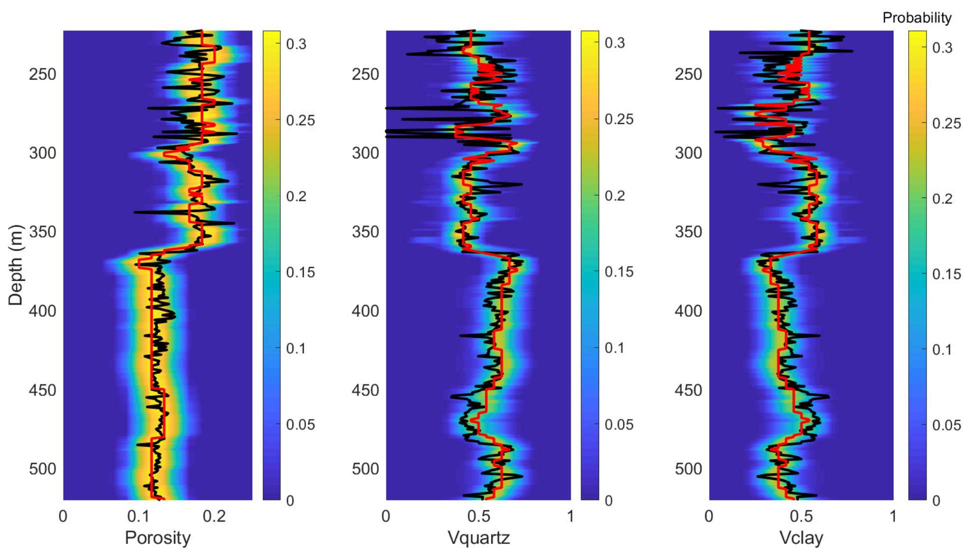


FIG. 8. Results of the Bayesian non-parametric approach.

We then repeat the inversion using the Gaussian mixture approaches. To compensate the parameter values not sampled at the well, we explore the prior model space using Monte Carlo simulations and then apply the rock physics model to obtain the corresponding set of elastic samples. A training dataset of 1000 samples is then used to evaluate the joint distribution of the model and data variables. The result of the Gaussian mixture approach (Figure 7) is slightly more accurate and precise than the linearized one, because the facies classification of well data allows a good discrimination of the rock physics and elastic properties. However, the two results are similar. This is because for the depth range of the data, the nonlinearity of the rock physics model is weak and the Gaussian assumption is acceptable.

For the non-parametric approach, we numerically estimate the joint PDF of the model and data variables from the training dataset using kernel density estimation. The computation is performed on a discretized six-dimensional grid. The kernel widths are assumed to be equal to 1/10 of the length of the domain of each property. Also, this approach provides a good estimation of the posterior probability. However, the uncertainty of the porosity estimate is larger than the Gaussian and Gaussian mixture cases, and we might need a finer discretization for this property, which is more computationally demanding.

## CONCLUSIONS

We have applied different parametric and non-parametric Bayesian rock physics inversion approaches to the well-log data at the CMC Newell County Facility. The solution is represented by the posterior distribution of the porosity and lithology parameters conditioned on the elastic data. All these approaches provide reasonably accurate results. In particular, the Gaussian mixture model is a suitable solution because of its analytical convenience and its ability to capture the features of different litho-fluid classes. The non-parametric approach based on kernel density estimation does not exhibit clear advantage in this application because the model variables are approximately Gaussian distributed and because it suffers from computational issues.

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