



Footprint

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CREWES 2005

**A look at seismic acquisition geometries using
3D prestack depth migration.**

Outline

- Model geometry and acquisition lattices
- Sampling Issues
- Modelling and migration methods
- Individual source records
- Complete geometry benchmark
- Source/receiver reduced geometry
- Random and surface noise

Motivation

- CREWES will soon be implementing 3D prestack depth migrations.
- Wavefield sampling is inextricably linked with imaging.
- What algorithms are best suited to land acquisition geometries?

Seismic Data is 5D

$u(x_r, y_r, x_s, y_s, t)$ Time domain wavefield.

(x_r, y_r) Receiver coordinates.

(x_s, y_s) Source coordinates.

$U(x_r, y_r, x_s, y_s, \omega)$ Frequency domain wavefield.

Each of these “components” has its own sampling lattice.

A Seismic Image is 3D

$$r(x, y, z) \quad \text{Reflectivity volume}$$

Migration, or imaging, is the name applied to the mapping

$$U(x_r, y_r, x_s, y_s, \omega) \rightarrow r(x, y, z) : (\mathbb{R}^5 \rightarrow \mathbb{R}^3)$$

Since the wavefield must satisfy a wave equation for each fixed source location, the spatial sampling lattice becomes critical to imaging performance.

Research Goal

- **Build a 3D prestack depth migration simulation that allows assessment of sampling and algorithmic details.**
- **Include source bandwidth and noise models.**
- **Use wave (not ray) theory.**
- **Strike a balance between simplicity and realism that captures something useful about “footprint”.**

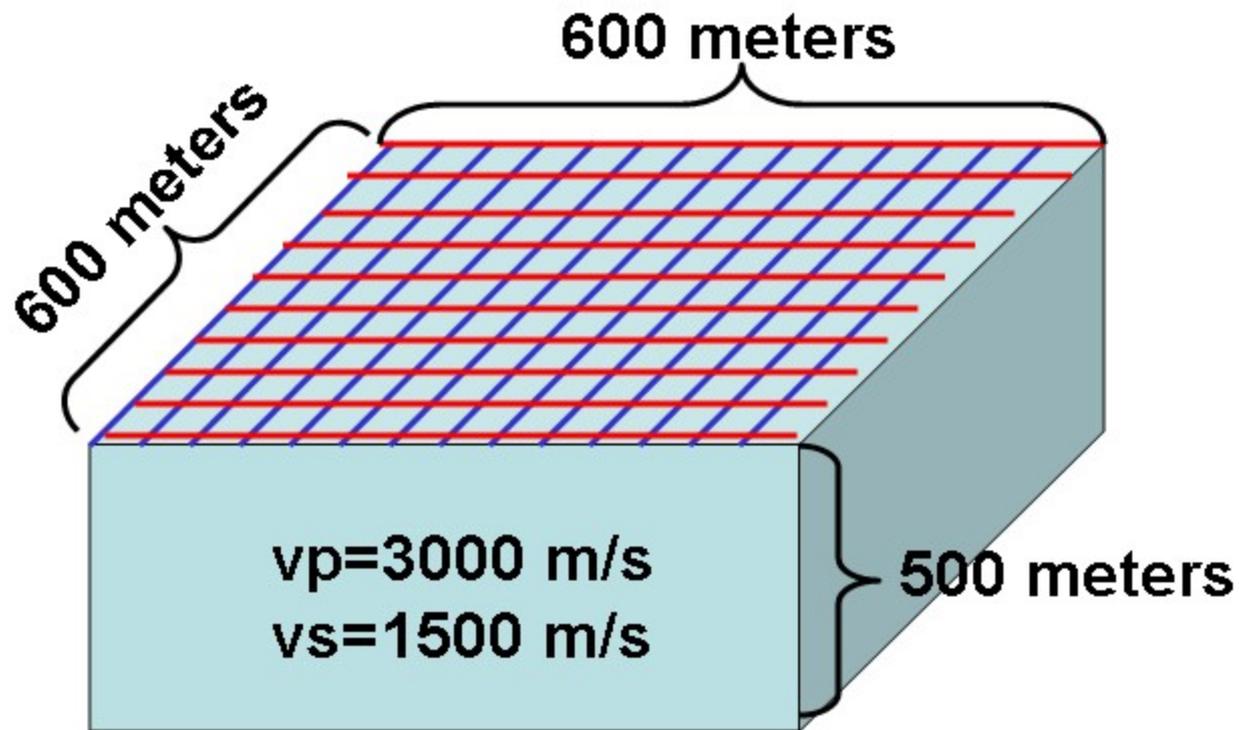
Definition

Acquisition Footprint : The seismic image of a horizontal, featureless reflector produced by a particular sampling lattice (5D) and a particular imaging algorithm.

It follows that the ideal footprint is a horizontal, featureless image.

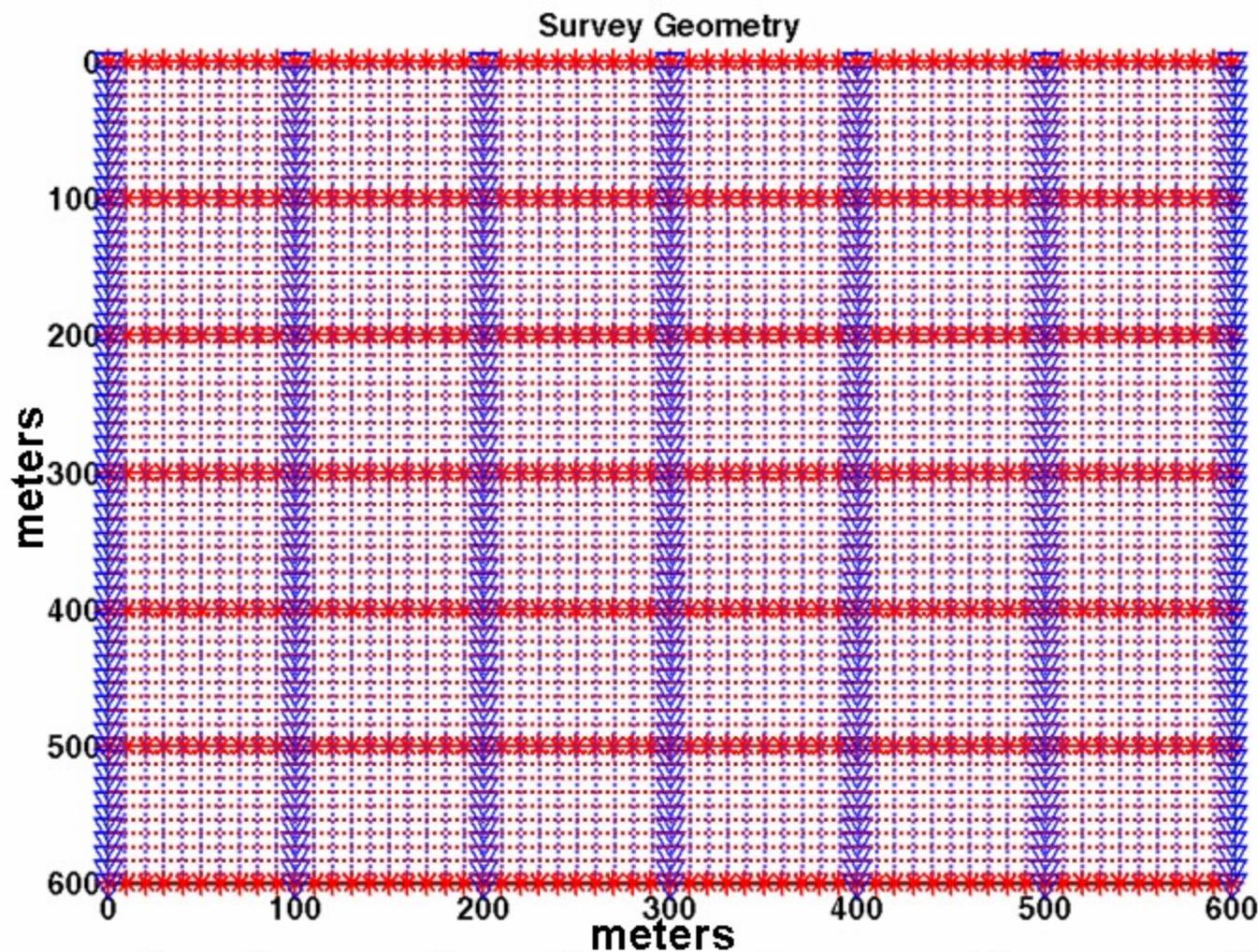
Model and Lattices

Model



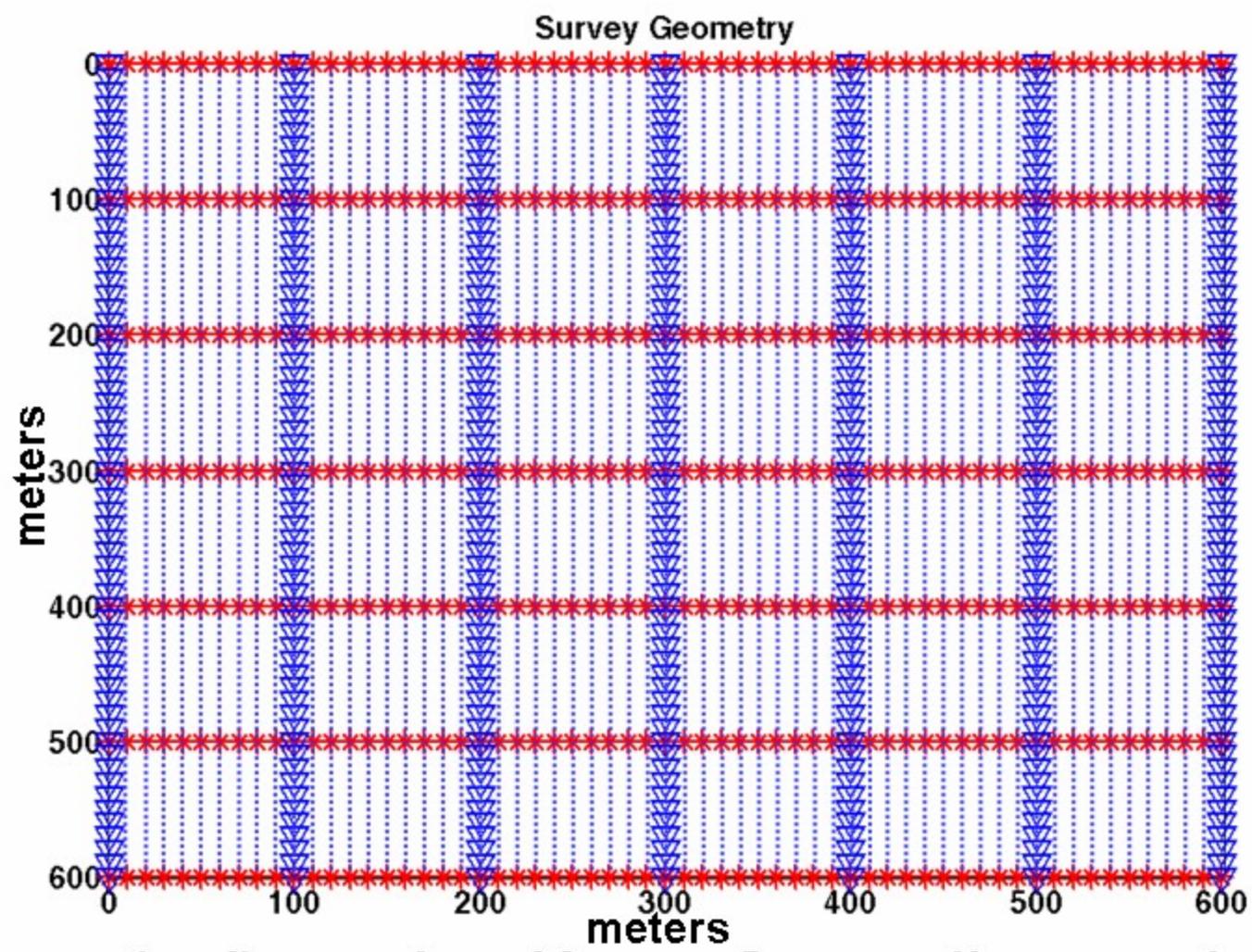
Flat, featureless reflector at 500 meters.
Image-shot modelled data 10-60 Hz.
Random noise and coherent noise options.

Complete Geometry



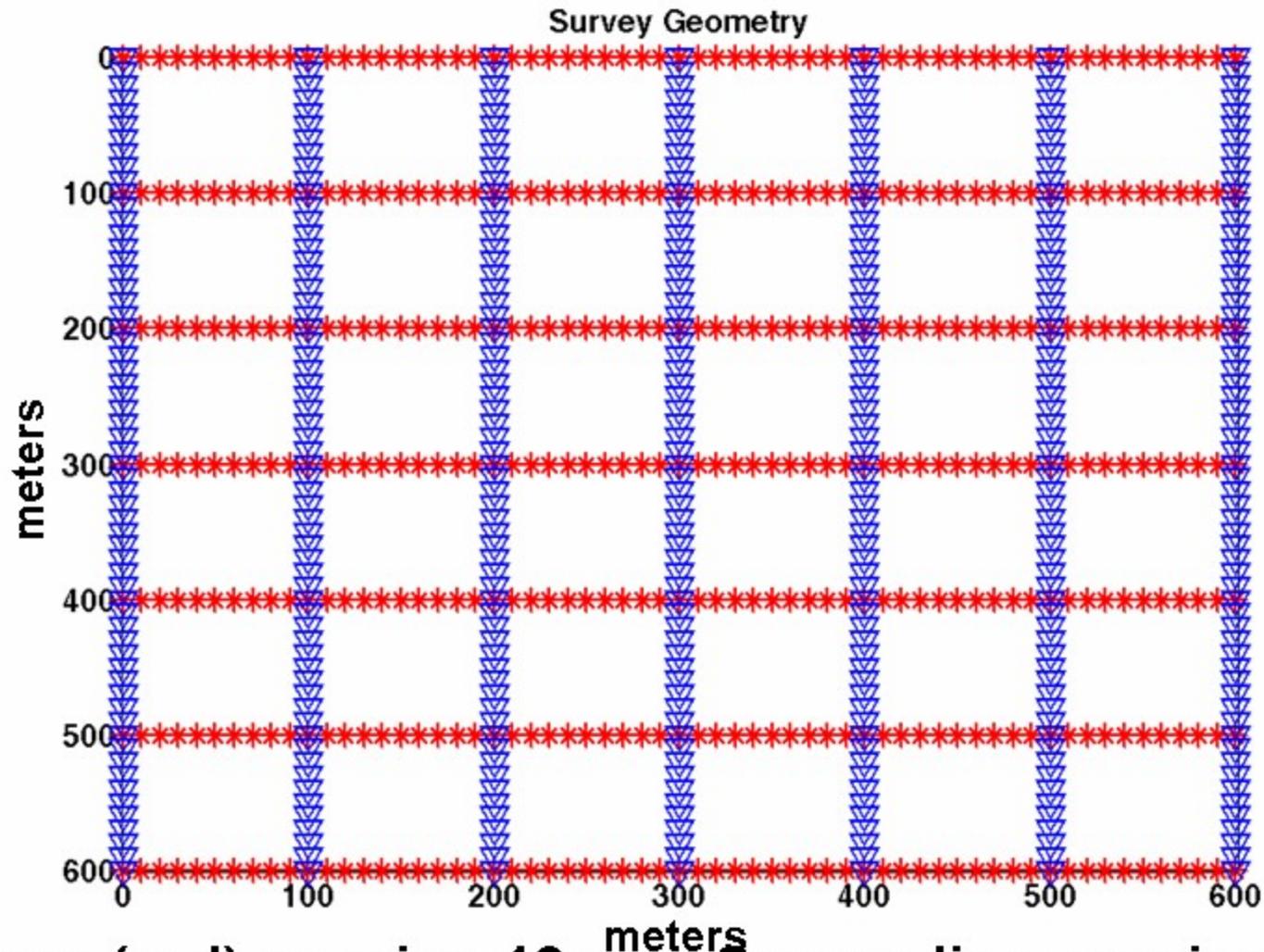
Source (red) spacing 10m --- Source line spacing 10m
Receiver (blue) spacing 10m --- Receiver line spacing 10m

Source Reduced Geometry



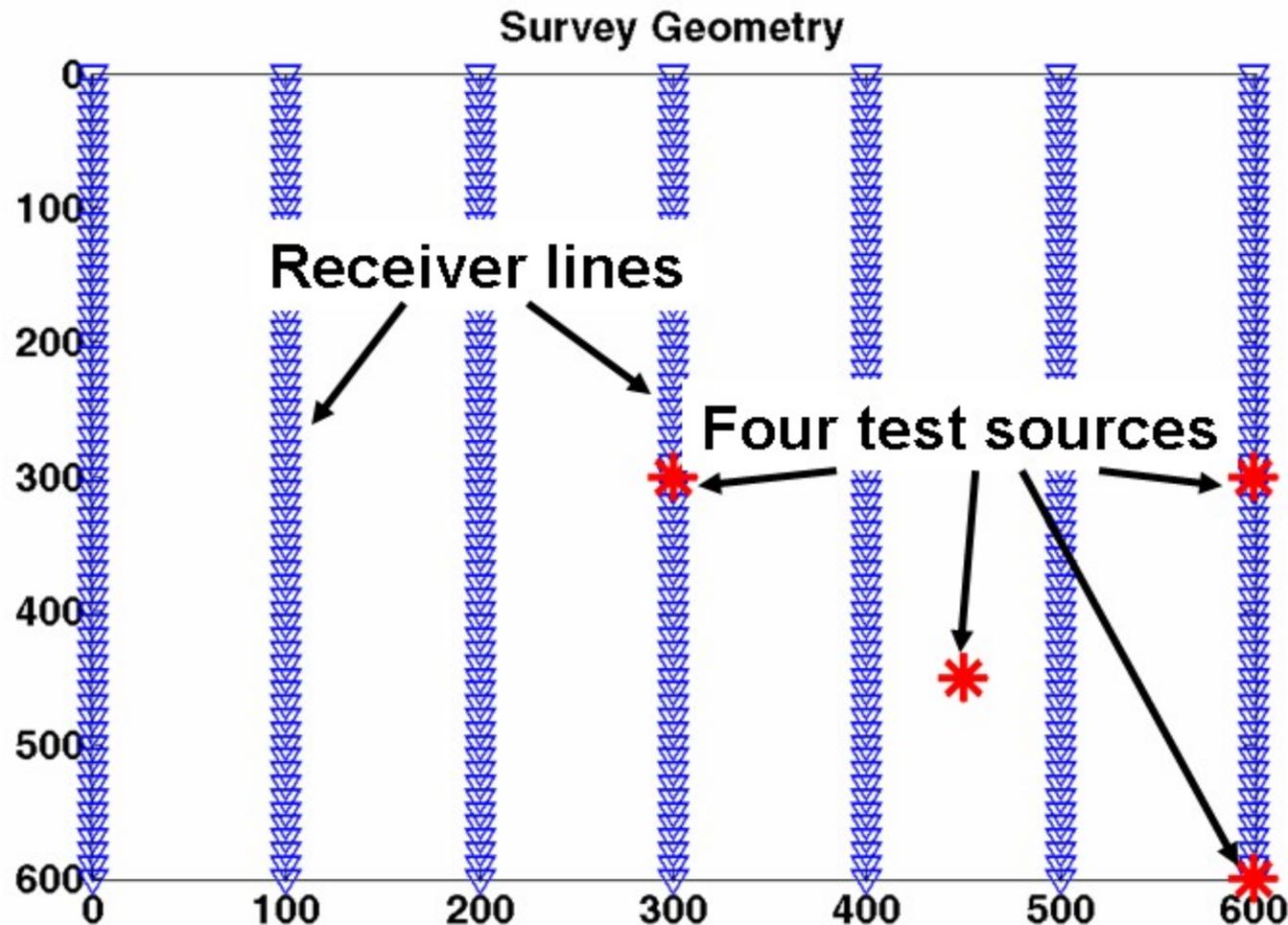
Source (red) spacing 10m --- Source line spacing 100m
Receiver (blue) spacing 10m --- Receiver line spacing 10m

Source /Rec Reduced Geometry



Source (red) spacing 10m --- Source line spacing 100m
Receiver (blue) spacing 10m --- Receiver line spacing 100m

Source Studies



Receiver interval 10 meters, receiver line spacing 10 meters, (only every 10th receiver line is shown).

Migration and Sampling Issues

Source-Record Migration

- Migrate each source record independently.
- Each source record gives a reflectivity estimate over a certain spatial area.
- Stack the source records to form a final reflectivity volume.

Imaging Conditions

a single source record

Deconvolution imaging condition

Downward continued data

$$r_j(x, y, z) = \frac{1}{n_\omega} \sum_{k=1}^{n_\omega} \frac{U_j(x, y, z, \omega_k)}{D_j(x, y, z, \omega_k)}$$

Modeled source

Imaging Conditions

a single source record

Deconvolution imaging condition

$$r_j(x, y, z) = \frac{1}{n_\omega} \sum_{k=1}^{n_\omega} \frac{U_j(x, y, z, \omega_k)}{D_j(x, y, z, \omega_k)}$$

Stabilized deconvolution imaging condition

$$r_j(x, y, z) = \frac{1}{n_\omega} \sum_k \frac{U_j(x, y, z, \omega_k) \bar{D}_j(x, y, z, \omega_k)}{D_j(x, y, z, \omega_k) \bar{D}_j(x, y, z, \omega_k) + \varepsilon}$$

Cross correlation imaging condition

$$r_j(x, y, z) = \frac{1}{n_\omega} \sum_k U_j(x, y, z, \omega_k) \bar{D}_j(x, y, z, \omega_k)$$

Migration Equations

Downward extrapolation of the reflected field

$$U_j(x, y, z + \Delta z, \omega) = \int_{\mathbb{R}^2} \hat{U}_j(k_x, k_y, \omega, z) \dots \\ \hat{W}(k_x, k_y, \omega, \Delta z) e^{i(k_x x + k_y y)} dk_x dk_y$$

$$\hat{W}(k_x, k_y, \omega, \Delta z) = \begin{cases} \exp\left(i\Delta z \sqrt{\omega^2 v^{-2} - k_x^2 - k_y^2}\right), & |\omega| > v \sqrt{k_x^2 + k_y^2} \\ \exp\left(-\Delta z \sqrt{k_x^2 + k_y^2 - \omega^2 v^{-2}}\right), & |\omega| \leq v \sqrt{k_x^2 + k_y^2} \end{cases}$$

Sampling Conditions

To avoid aliasing the propagating wavefield

$$\frac{\pi}{\delta x} = \frac{\pi}{\delta y} > \frac{\omega_{\max}}{v}$$

Equivalently

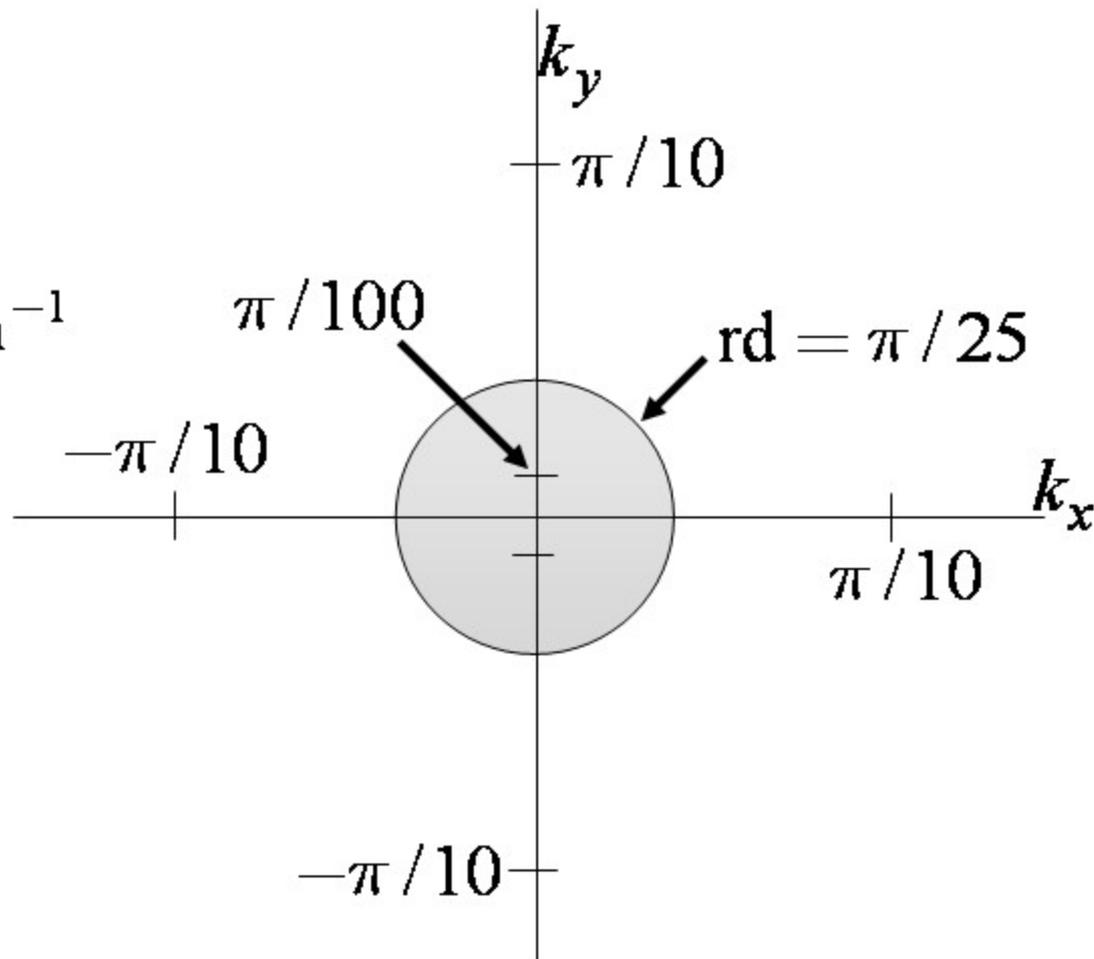
$$\delta x, \delta y < \frac{v}{2f_{\max}}$$

Analog Spectral Relationships

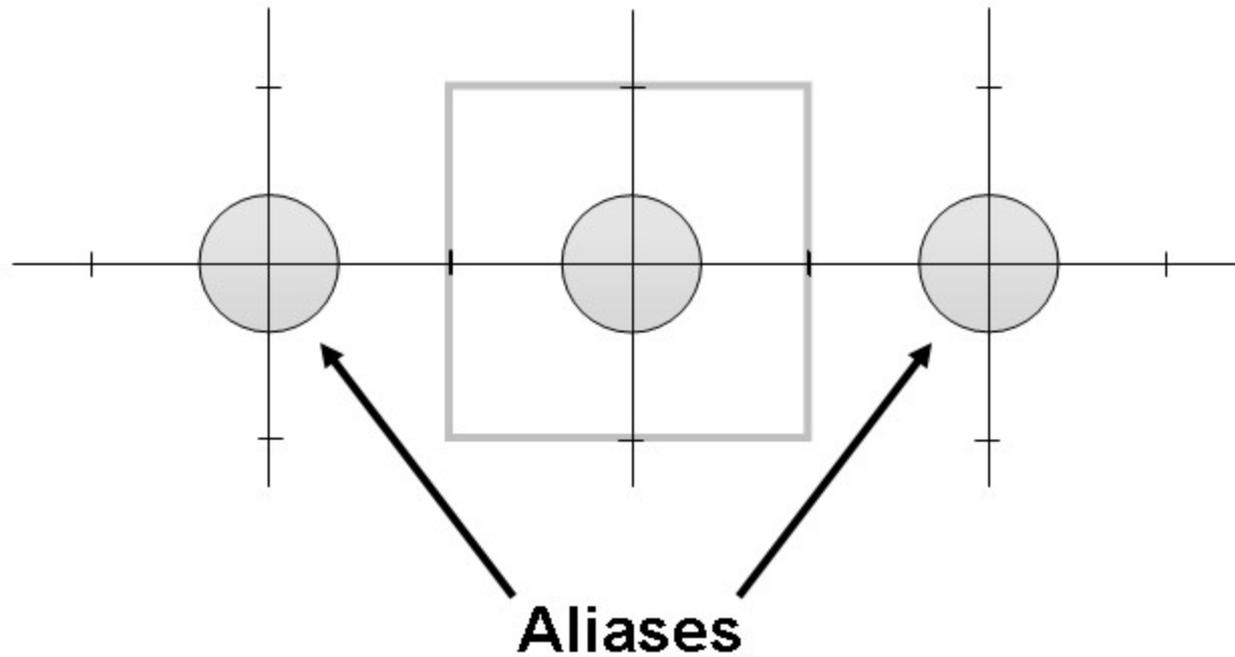
$$f_{\max} = 60 \text{ Hz}$$

$$v = 3000 \text{ m/s}$$

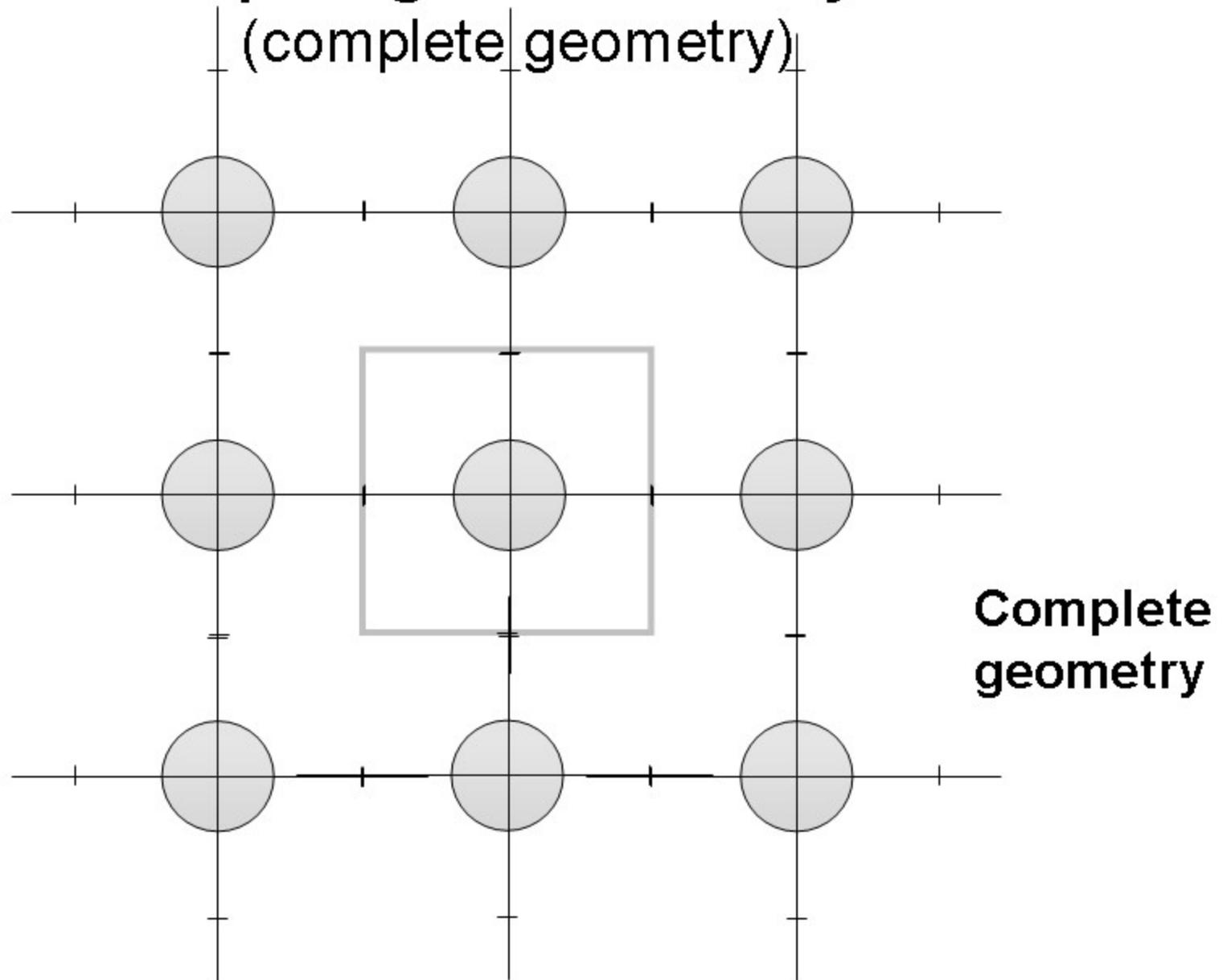
$$\omega_{\max} v^{-1} = \pi / 25 \approx .126 \text{ m}^{-1}$$



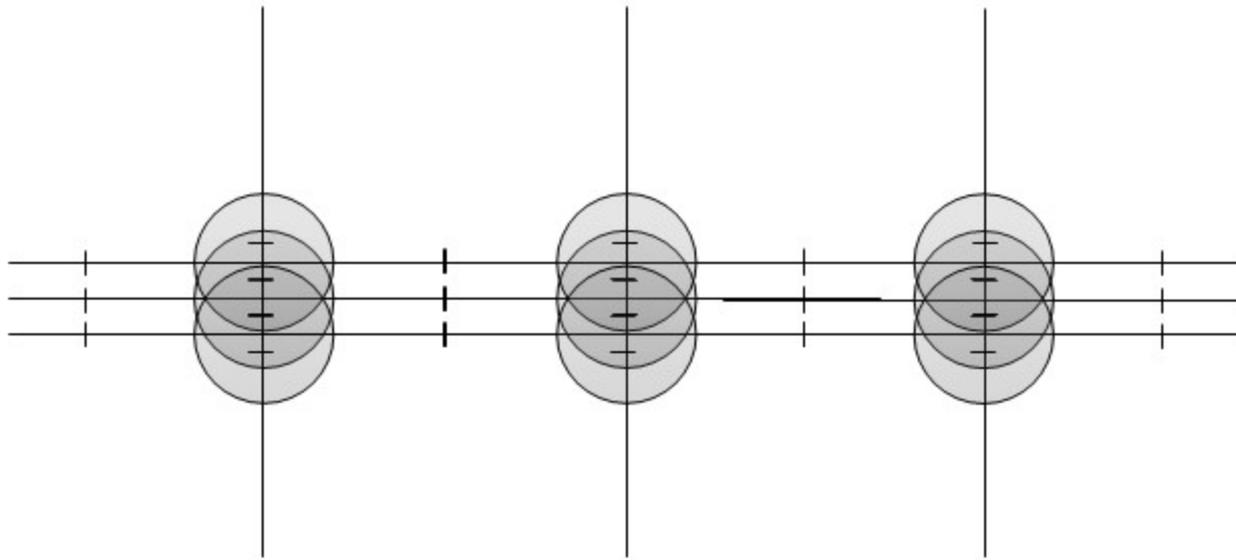
After sampling in x at 10 m



After sampling in x and y at 10 m



After sampling in x at 10 m
and y at 100 m
(receiver-reduced geometry)



Really, really, really bad aliasing

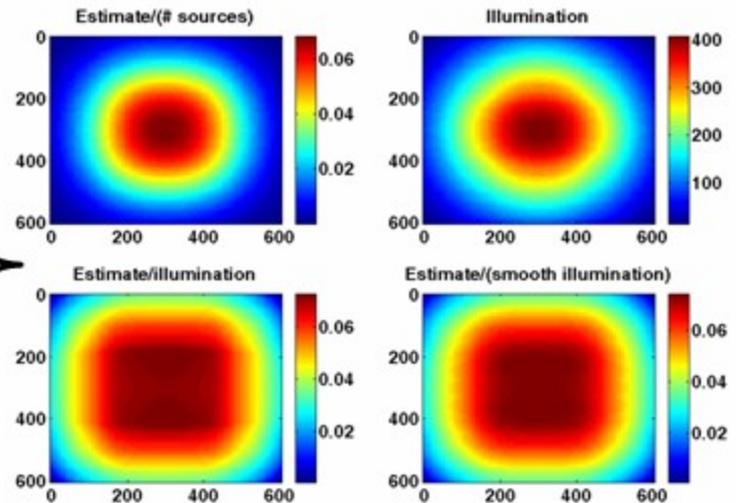
Simulation Code

Matlab Scripts

footprintPP.m

footprintPS.m

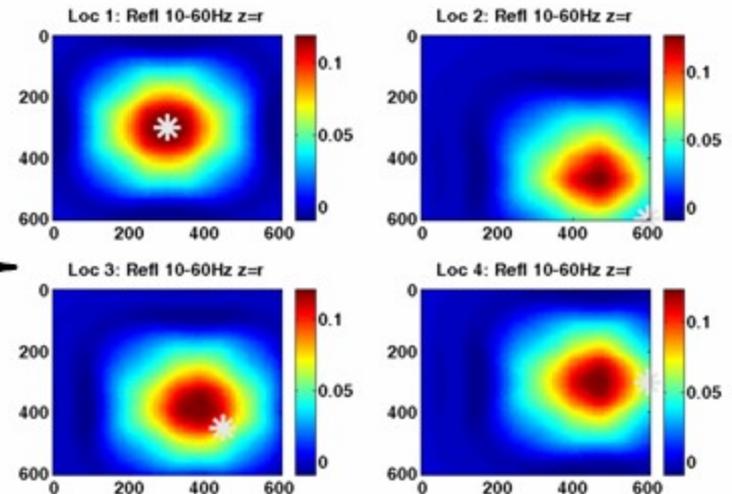
simulate
entire
surveys



illumination_functionsPP.m

illumination_functionsPs.m

Analyze
specific
source
points



Overall Scheme

Recall deconvolution imaging condition

$$r_j(x, y, z) = \frac{1}{n_\omega} \sum_{k=1}^{n_\omega} \frac{U_j(x, y, z, \omega_k)}{D_j(x, y, z, \omega_k)}$$

- **U(z=0)** Model data at surface with Green's Function.
- For receiver-reduced geometries, interpolation by Fourier zero padding.
- **U(z)**: Downward continuation of data by 3D phase shift.
- **D(Z)**: Direct modelling of source at reflector by Green's function.

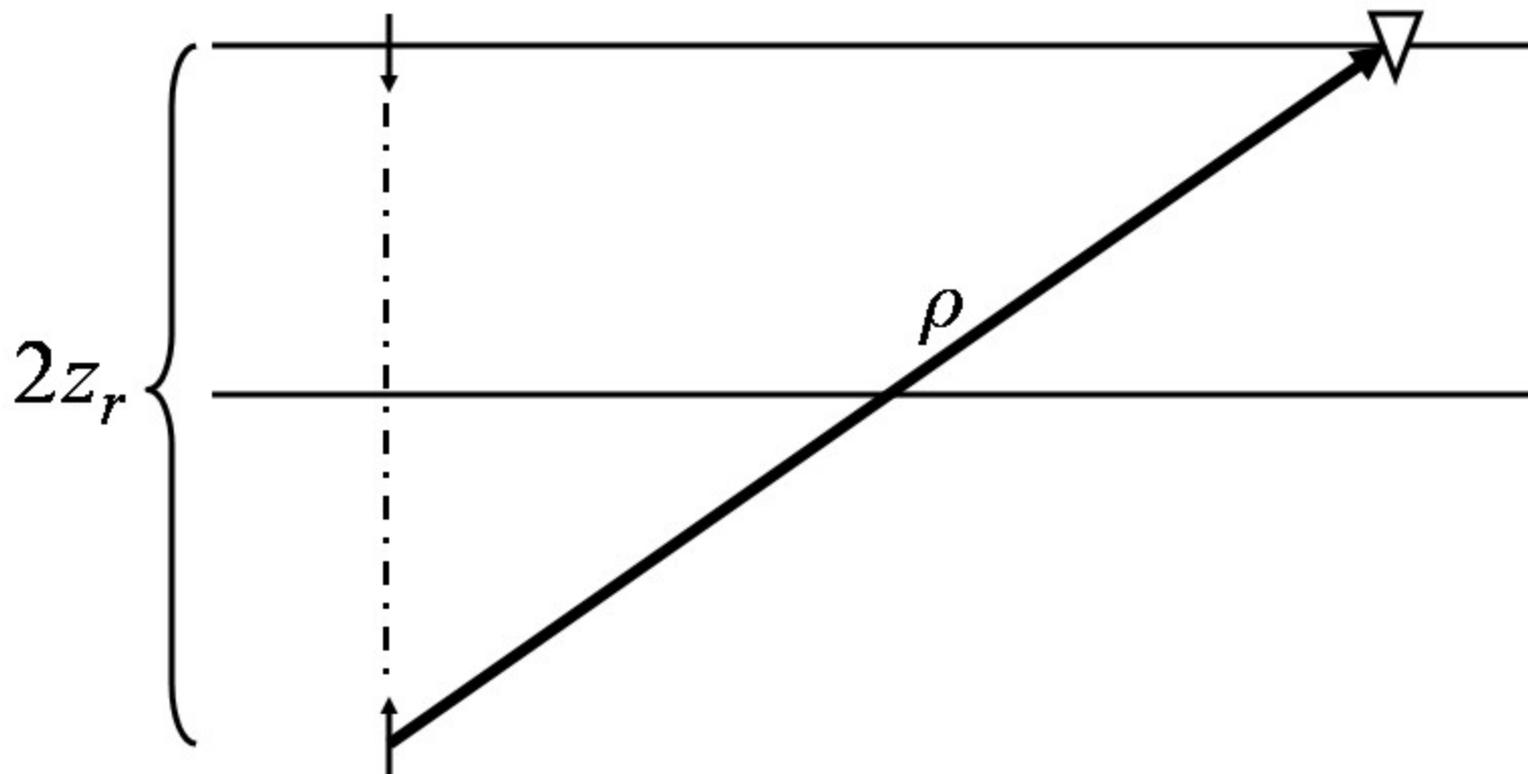
PP Data Model

Simple Image Source Model

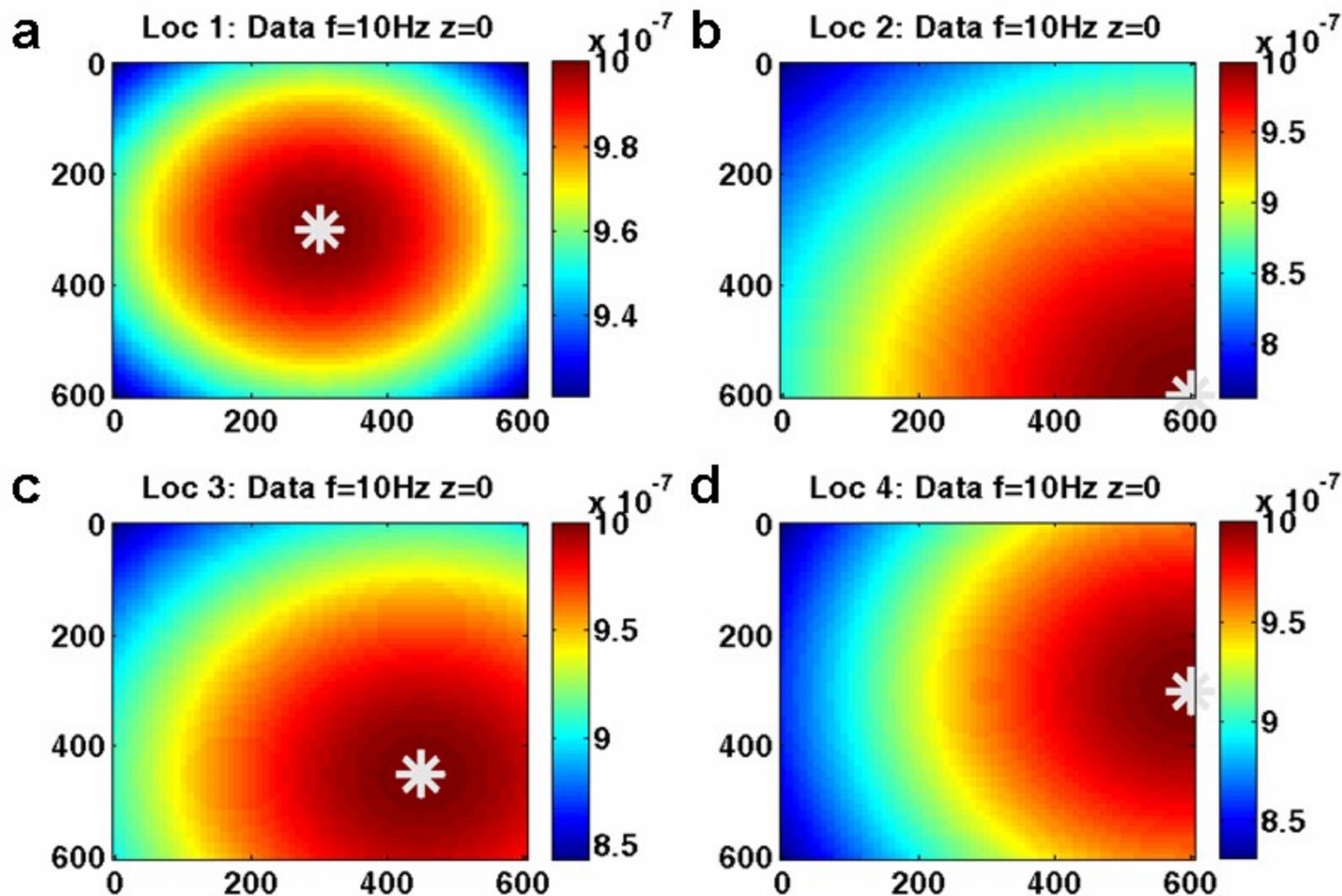
$$U_{j,PP}(x, y, z = 0, \omega) = c s(\omega) \frac{e^{-i\omega\rho/v_p}}{\rho}$$

c = reflection coefficient

$s(\omega)$ = source shape function



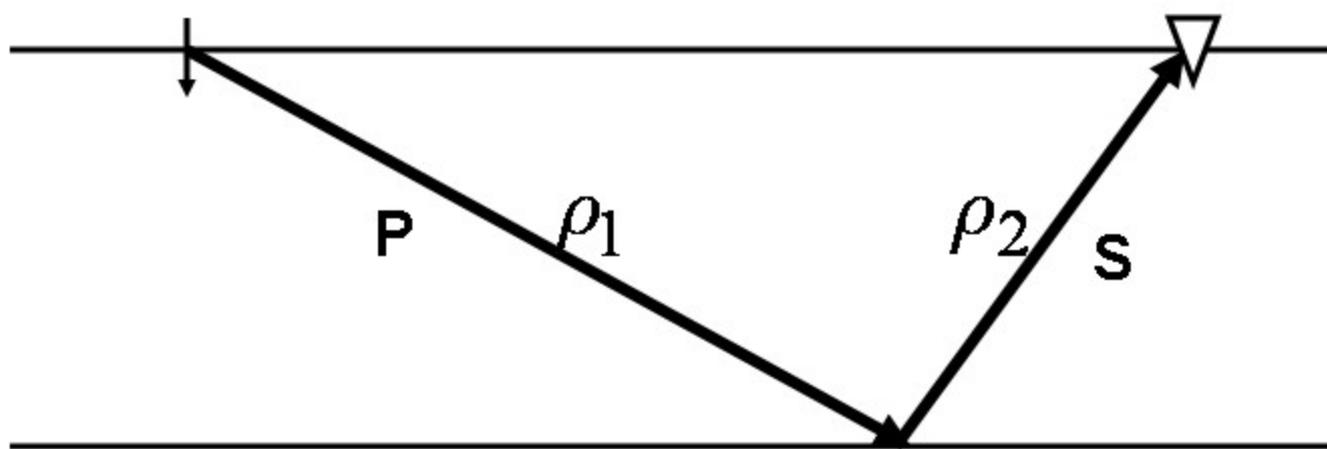
PP Data at $z=0$, 10 Hz



PS Data Model

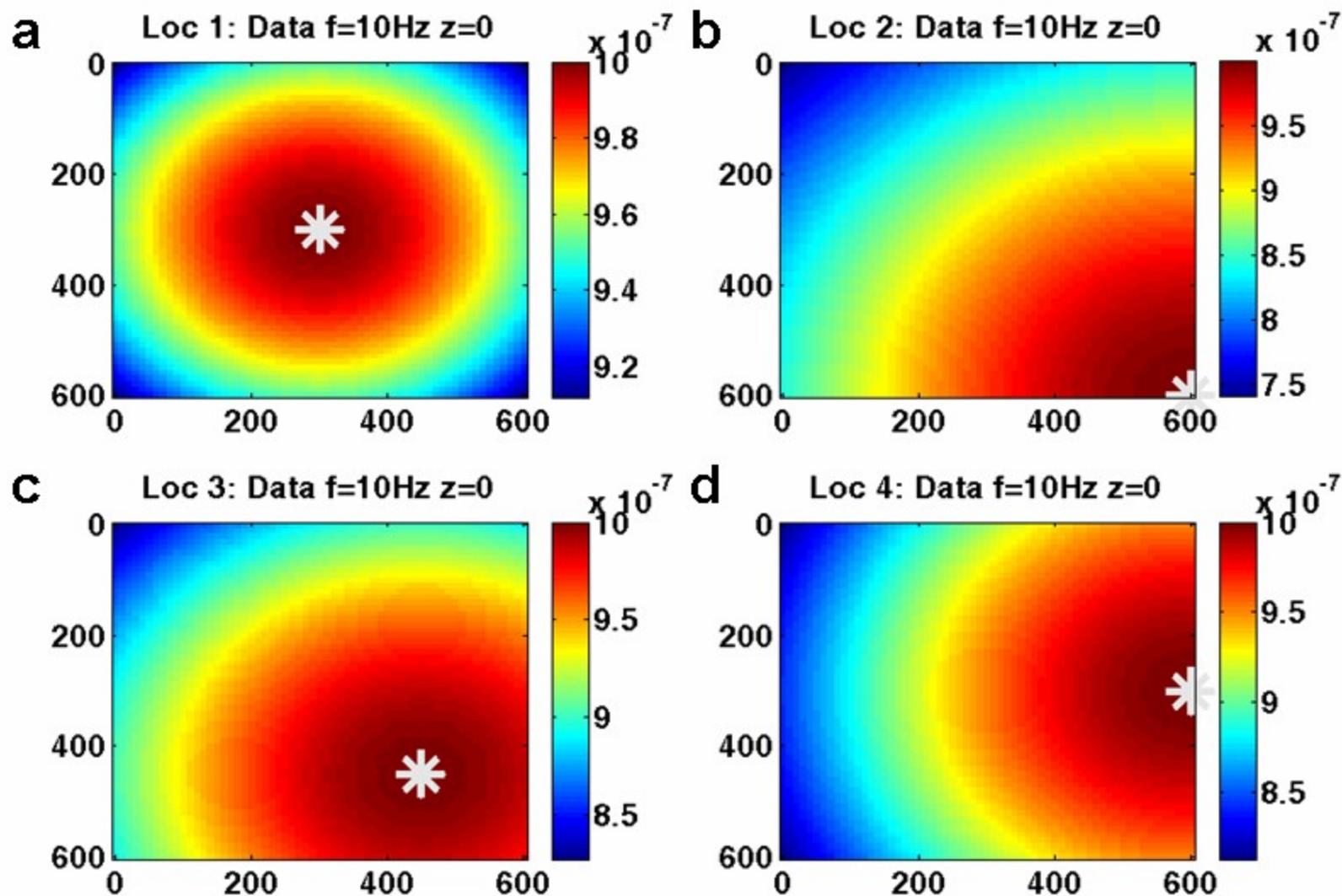
Modified Image Source Model

$$U_{j;PS}(x, y, z=0, \omega) = c s(\omega) \frac{e^{-i\omega(\rho_1/v_p + \rho_2/v_s)}}{\rho_1 + \rho_2}$$



PS conversion point determined by raytracing

PP Data at $z=0$, 10 Hz



Surface Noise Model

$$U_{j;surfnoise}(x, y, z = 0, \omega) = a_{surf} s_{surf}(\omega) \frac{e^{-i2\omega h/v_0}}{\sqrt{2h}}$$

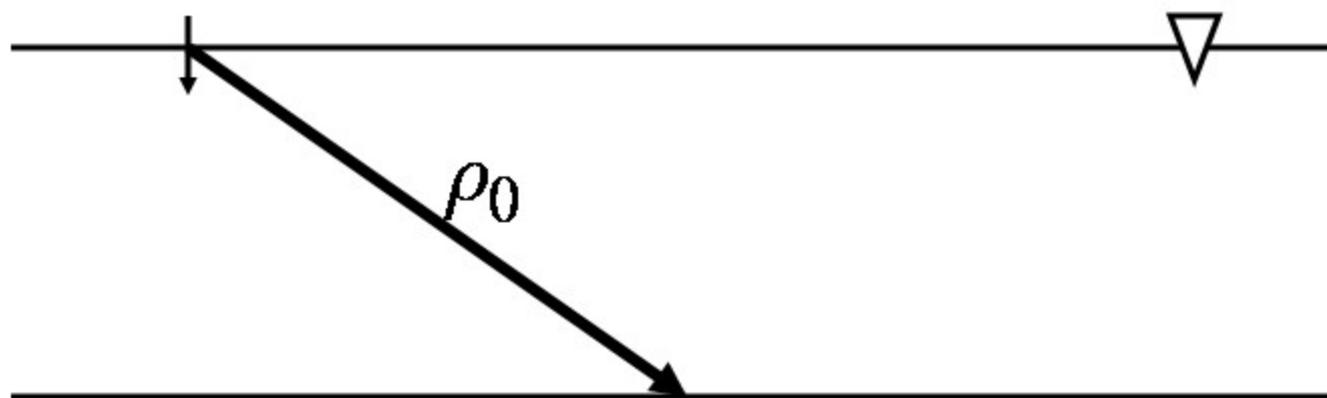
a_{surf} = scalar amplitude

$s_{surf}(\omega)$ = spectral shape function

h = half offset

Source Model at Reflector

$$D_{j,PP}(\mathbf{x}, \mathbf{y}, z = z_0, \omega) = s(\omega) \frac{e^{-i\omega\rho_0/v_p}}{\rho_0}$$



Stack Normalization

$$r_j(x, y, z) = \frac{1}{n_\omega} \sum_{k=1}^{n_\omega} \frac{U_j(x, y, z, \omega_k)}{D_j(x, y, z, \omega_k)}$$

Reflection coefficient estimation from one source.

$$r(x, y, z) = \frac{\sum_j r_j}{\text{something}}$$

Sum over shots to expand areal coverage and beat down noise.

What “something” should be in the denominator?

Illumination Estimate #1

$$I_{r;j}(\mathbf{x}, \mathbf{y}, z) = \begin{cases} 1.0 \dots |r(\mathbf{x}, \mathbf{y}, z)| > \mu r_{\max} \\ \frac{|r(\mathbf{x}, \mathbf{y}, z)|}{\mu r_{\max}} \dots \text{otherwise} \end{cases}$$

$$\mu \sim 0.4 = \text{threshold}$$

This is just the absolute value of the reflectivity estimate, which is clipped at some threshold and smoothly scaled below the threshold.

Illumination Estimate #2

$$I_{cc;j}(x, y, z) = \begin{cases} 1.0 & \dots |cc(x, y, z)| > \nu_{cc_{\max}} \\ \frac{|cc(x, y, z)|}{\nu_{cc_{\max}}} & \dots \text{otherwise} \end{cases}$$

$$cc_j(x, y, z) = \sum_k \frac{U_j(x, y, z, \omega_k) \bar{D}_j(x, y, z, \omega_k)}{\sqrt{U_j^2(x, y, z, \omega_k) D_j^2(x, y, z, \omega_k) + \varepsilon}}$$

$$\nu \sim 0.9 = \text{threshold}$$

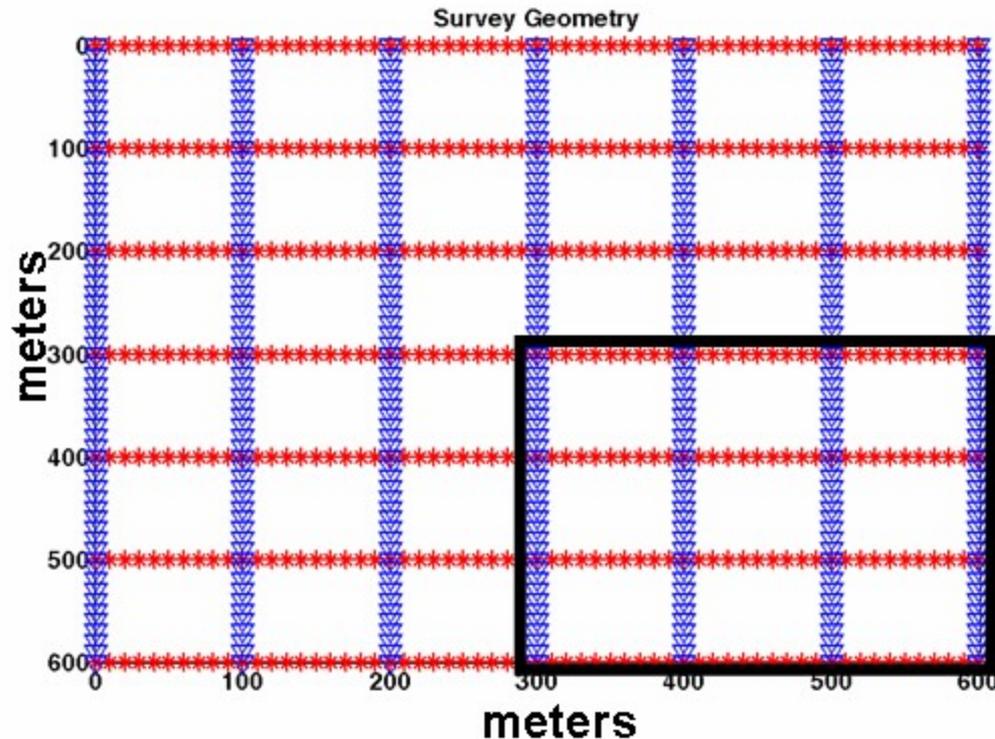
This is normalized crosscorrelation of the U and D wavefields, which is clipped at some threshold and smoothly scaled below the threshold.

Illumination Compensation

$$r_I(x, y, z) = \frac{\sum_j r_j}{\sum_j I_{r;j}} \quad \text{or} \quad r_{Icc}(x, y, z) = \frac{\sum_j r_j}{\sum_j I_{cc;j}}$$

These expressions are offered in analogy to the fold-normalization of a CMP stack.

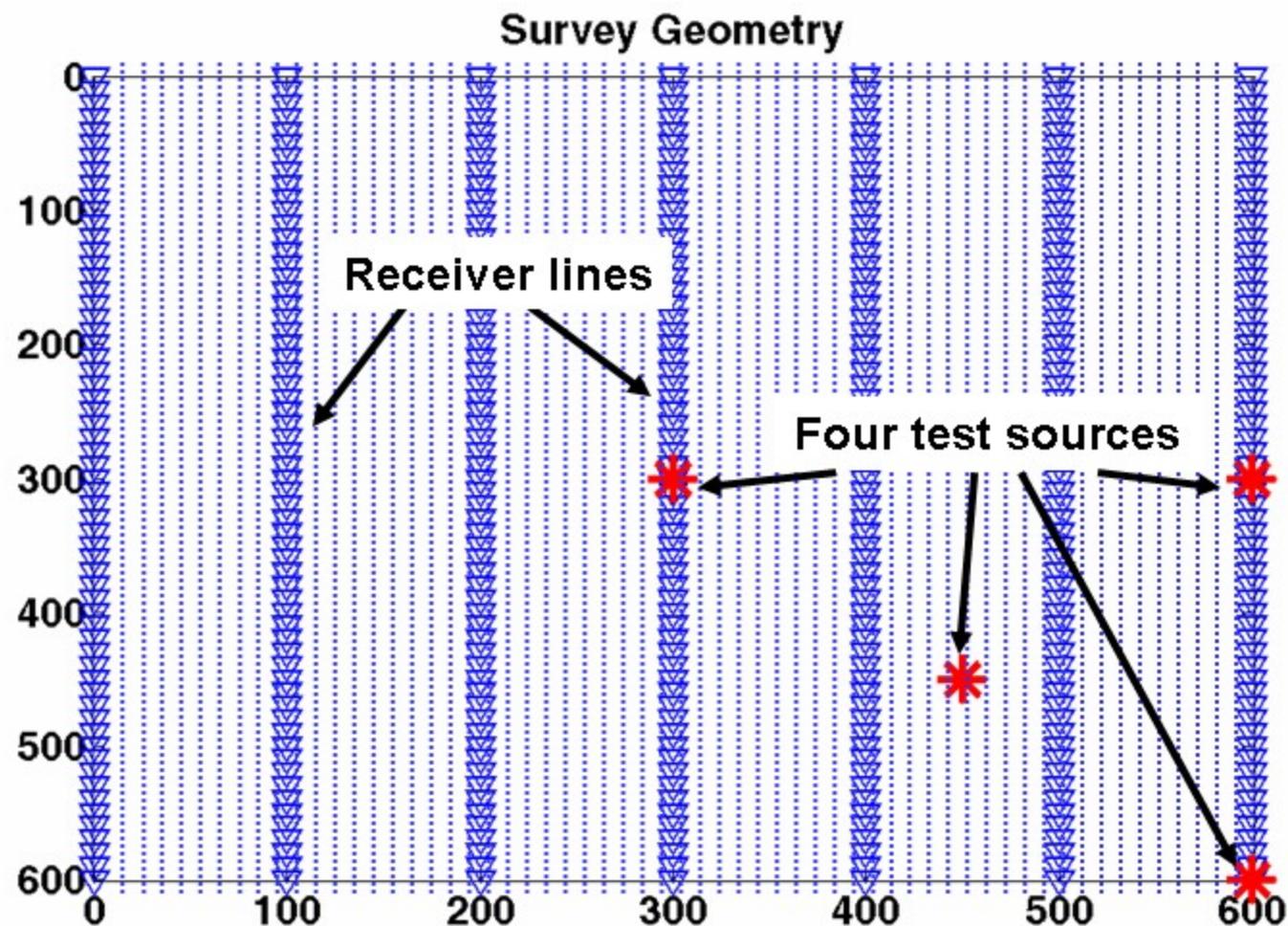
Computation Strategy



Only sources in the lower right quadrant are explicitly modelled. The others are deduced by symmetry. This has the unrealistic effect of symmetrising random noise as well.

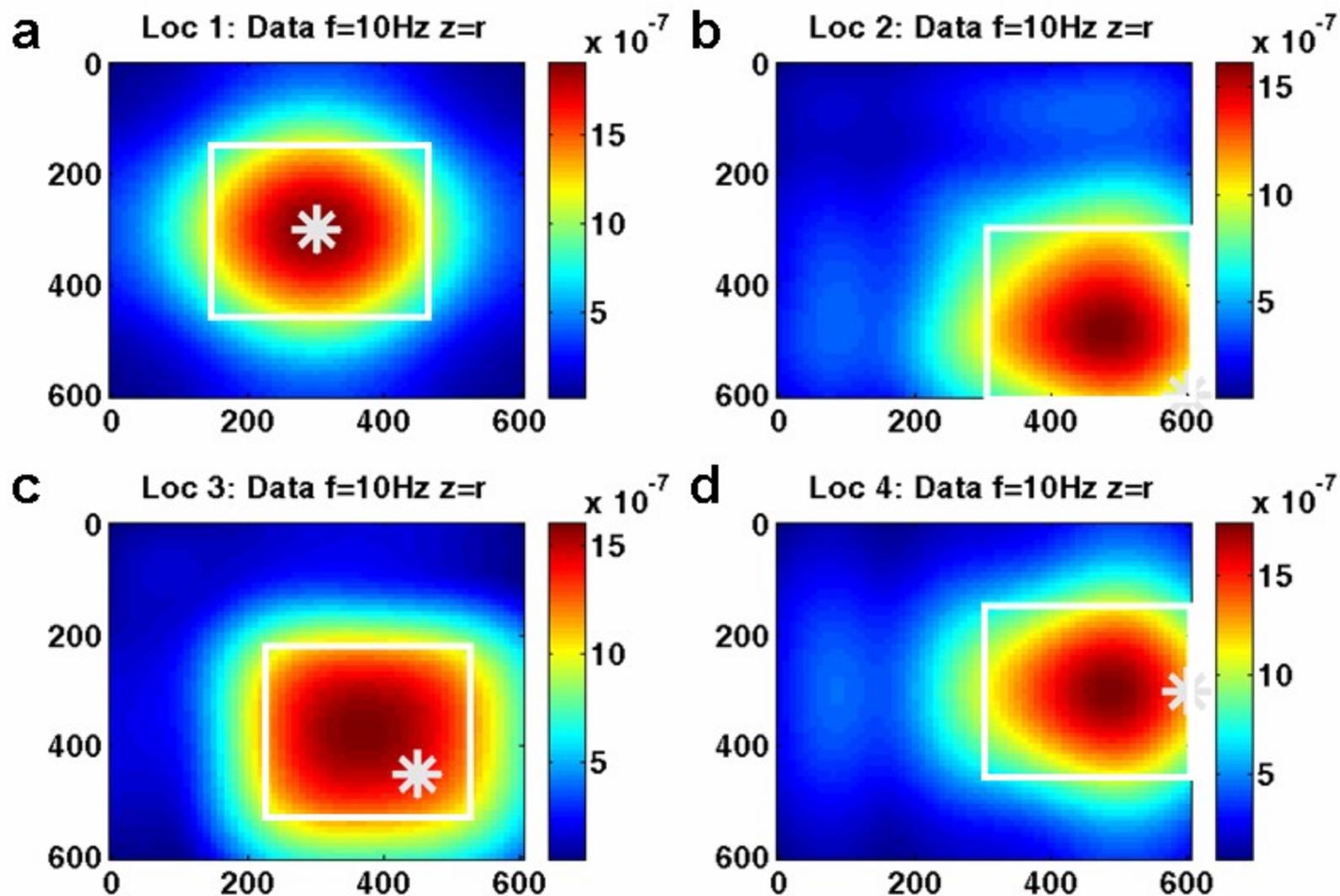
Individual Source Performance

Source Studies

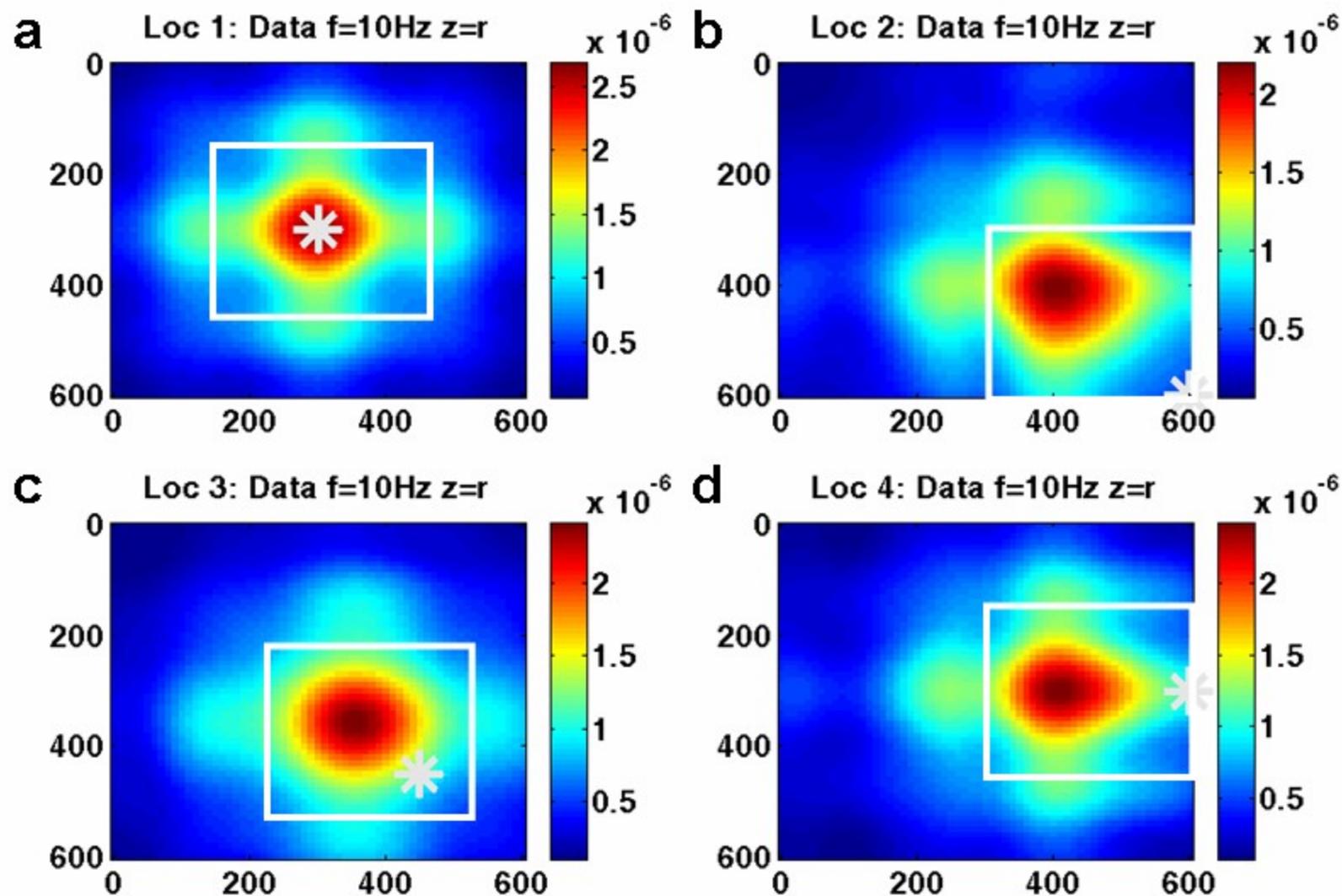


Receiver interval 10 meters and receiver line spacing 10 meters.

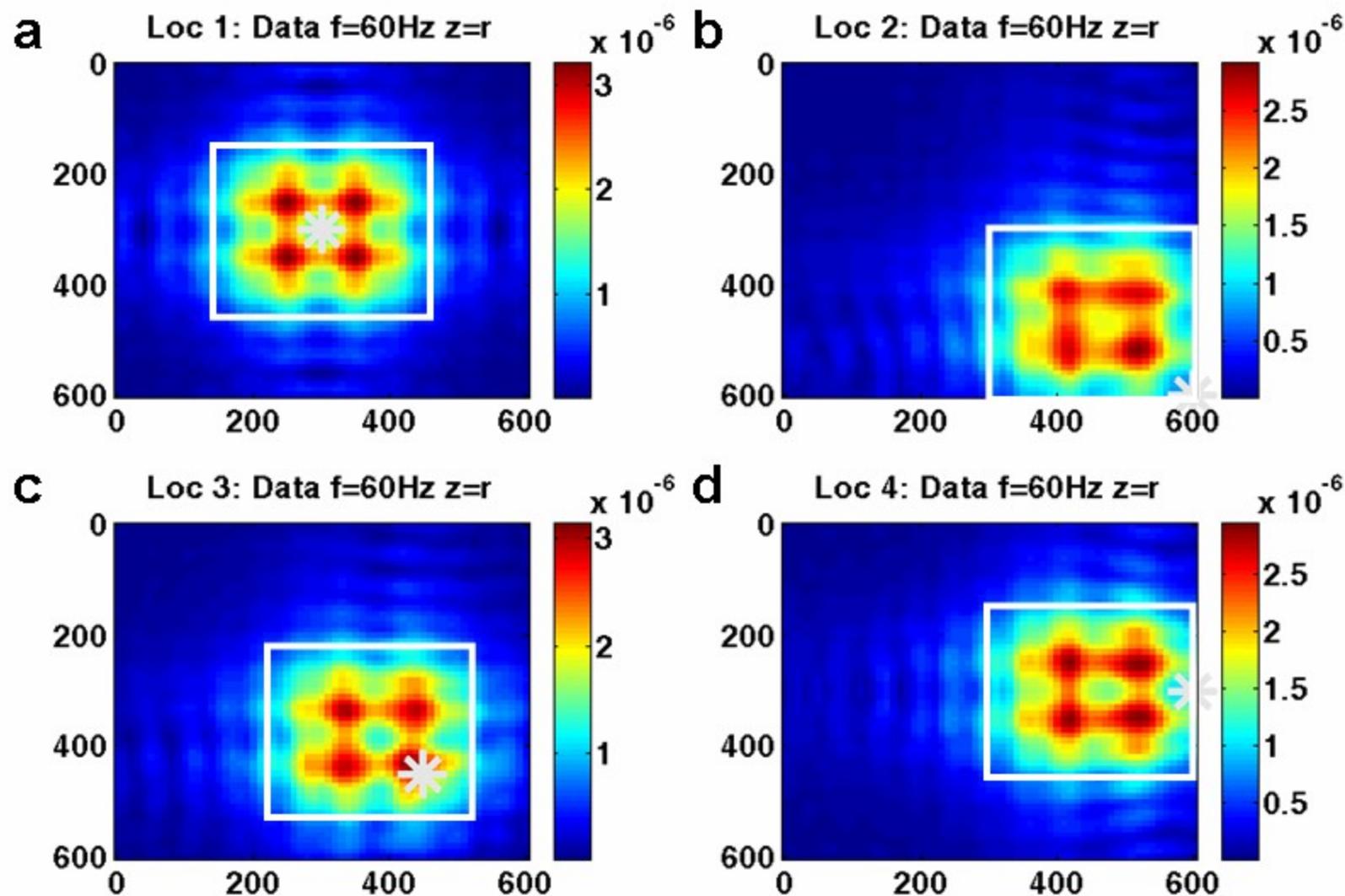
PP Data at $z=500$, 10 Hz



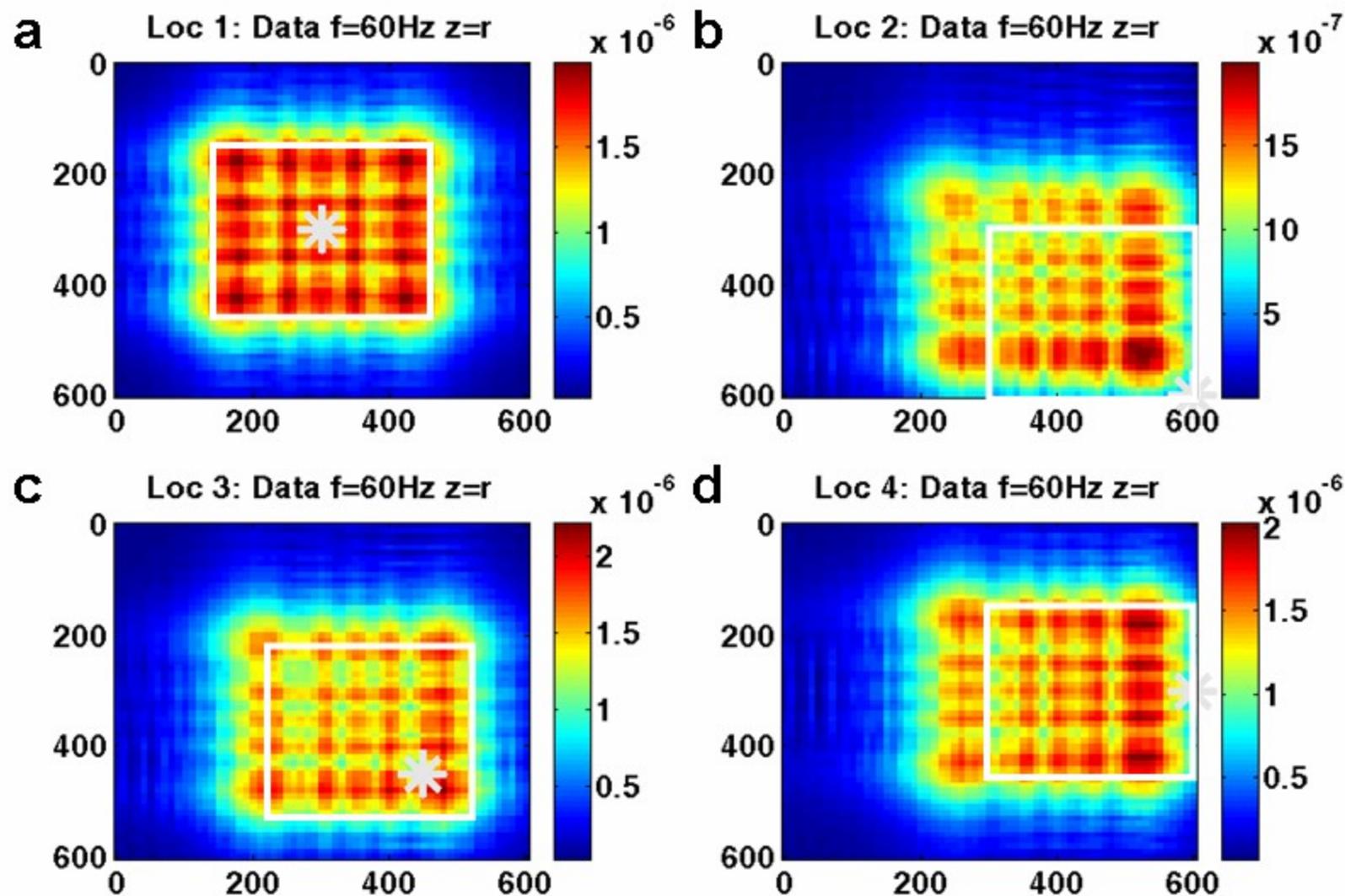
PS Data at $z=500$, 10 Hz



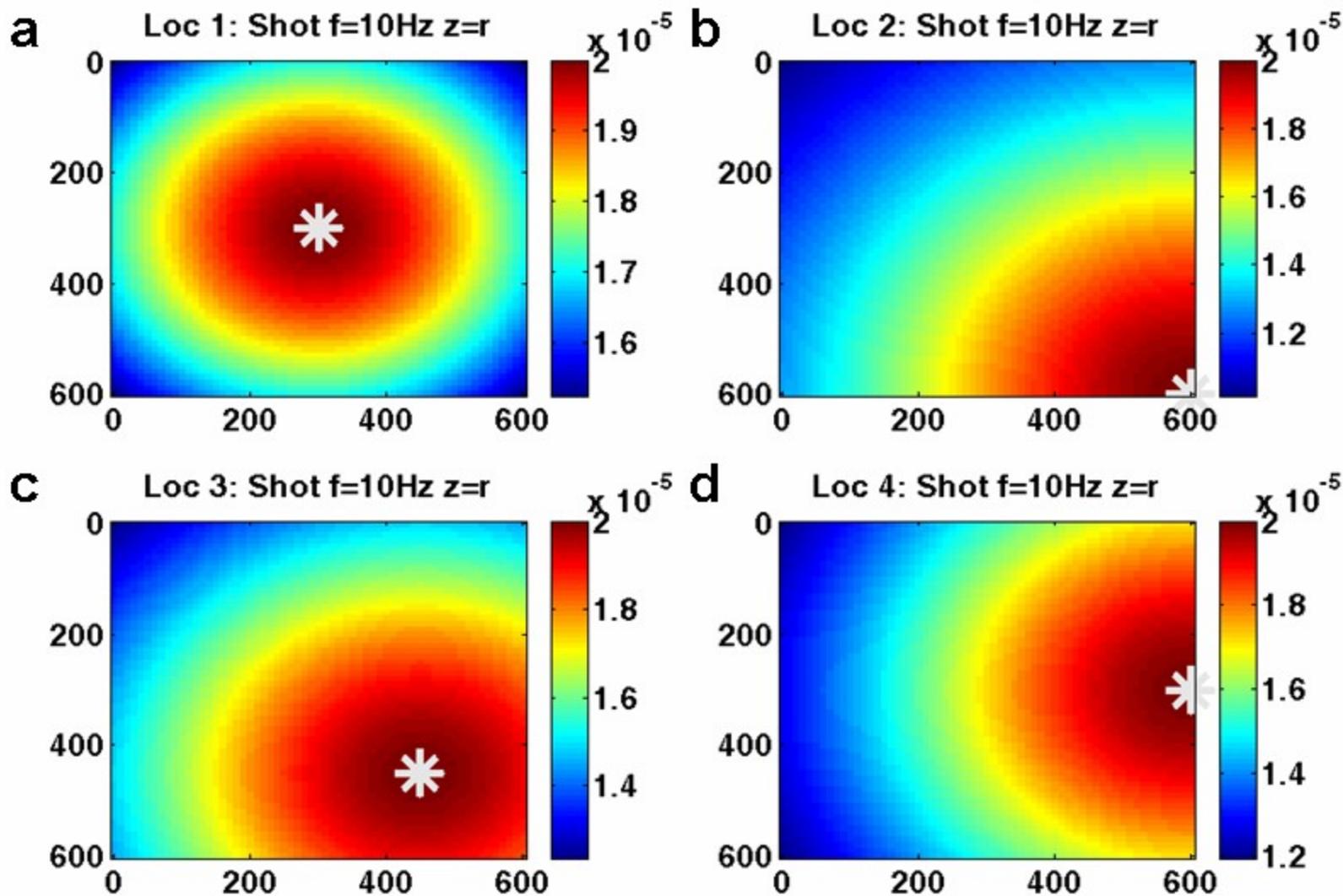
PP Data at $z=500$, 60 Hz



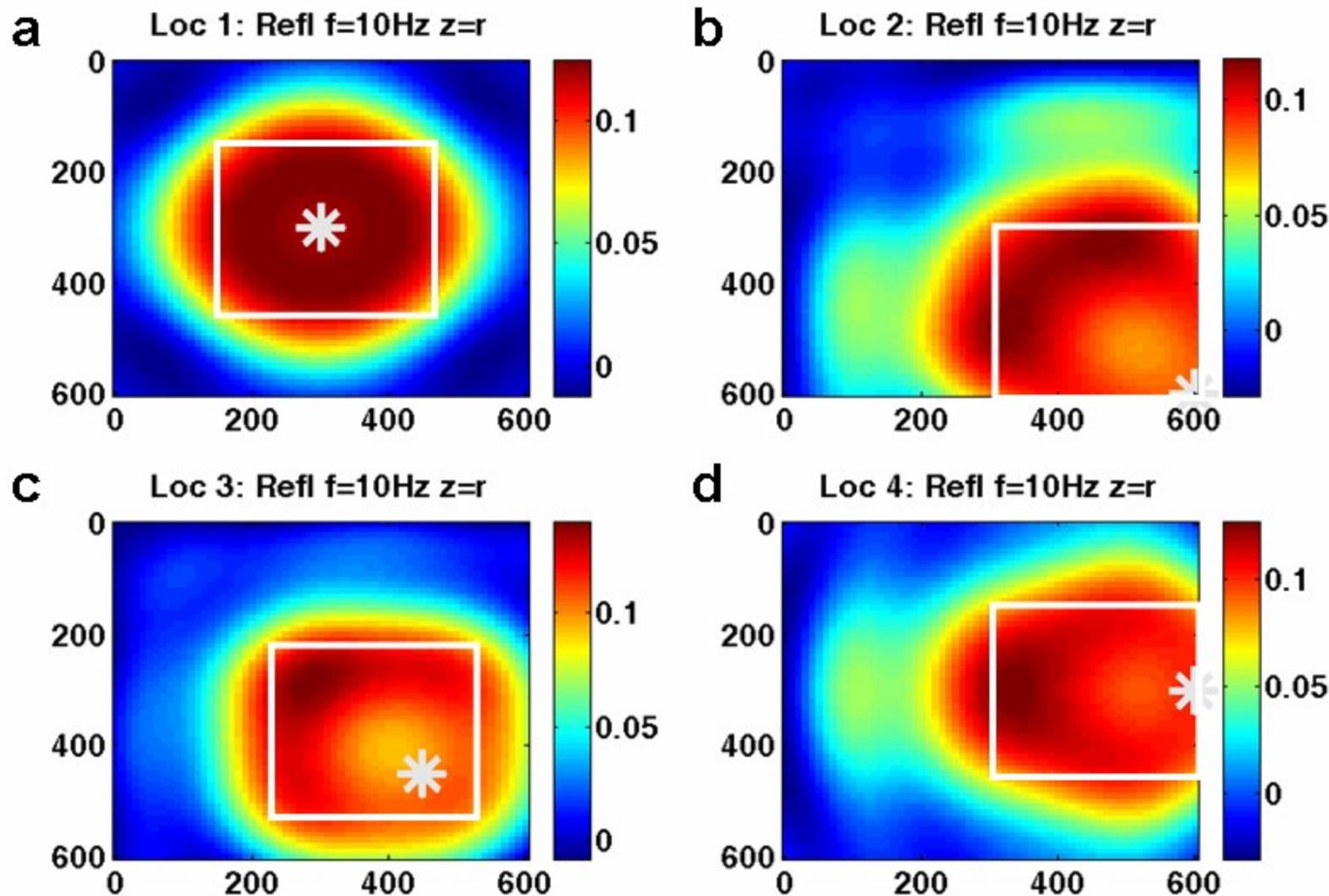
PS Data at $z=500$, 60 Hz



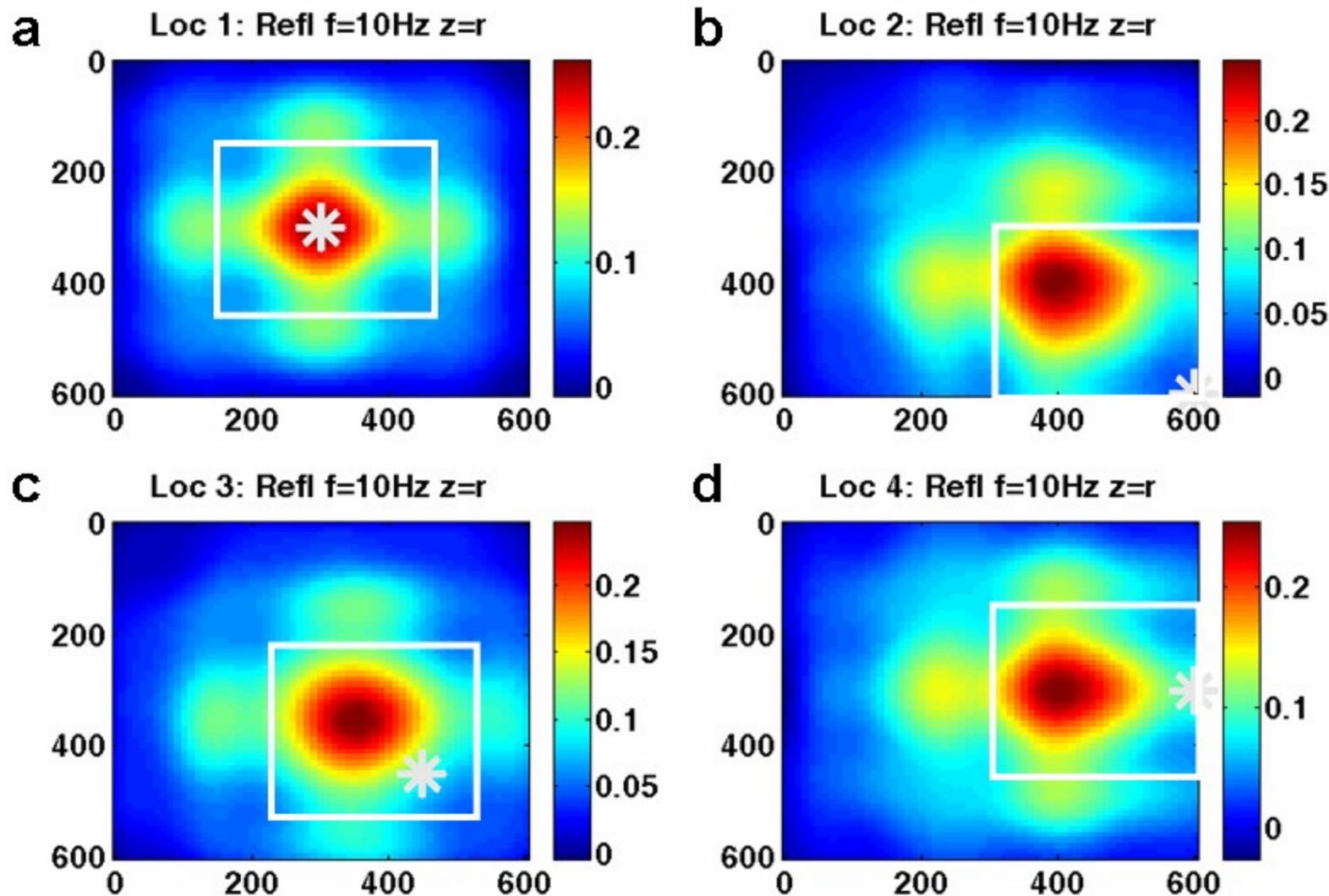
Source at $z=500$, 10 Hz



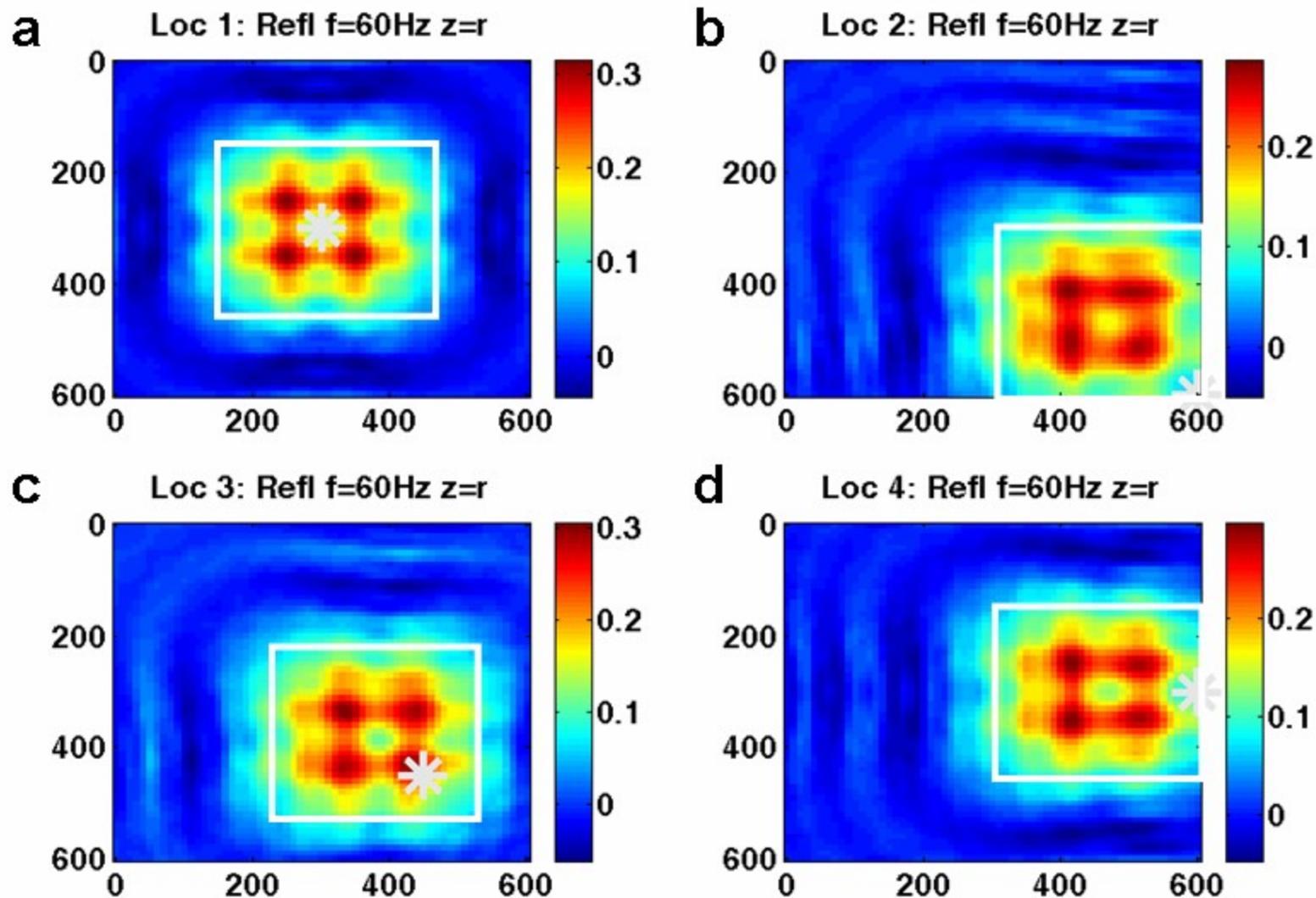
PP Reflectivity at 10 Hz



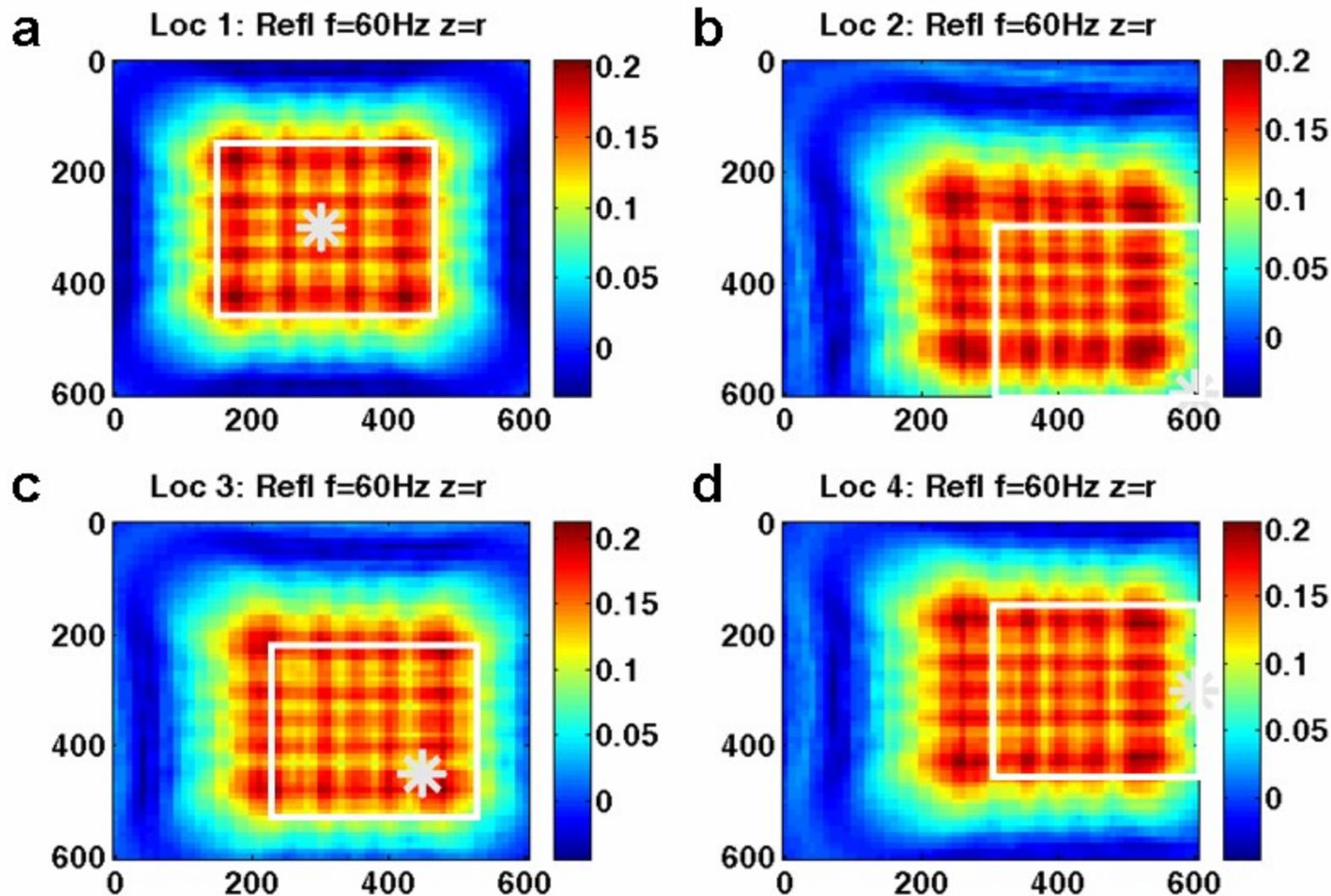
PS Reflectivity at 10 Hz



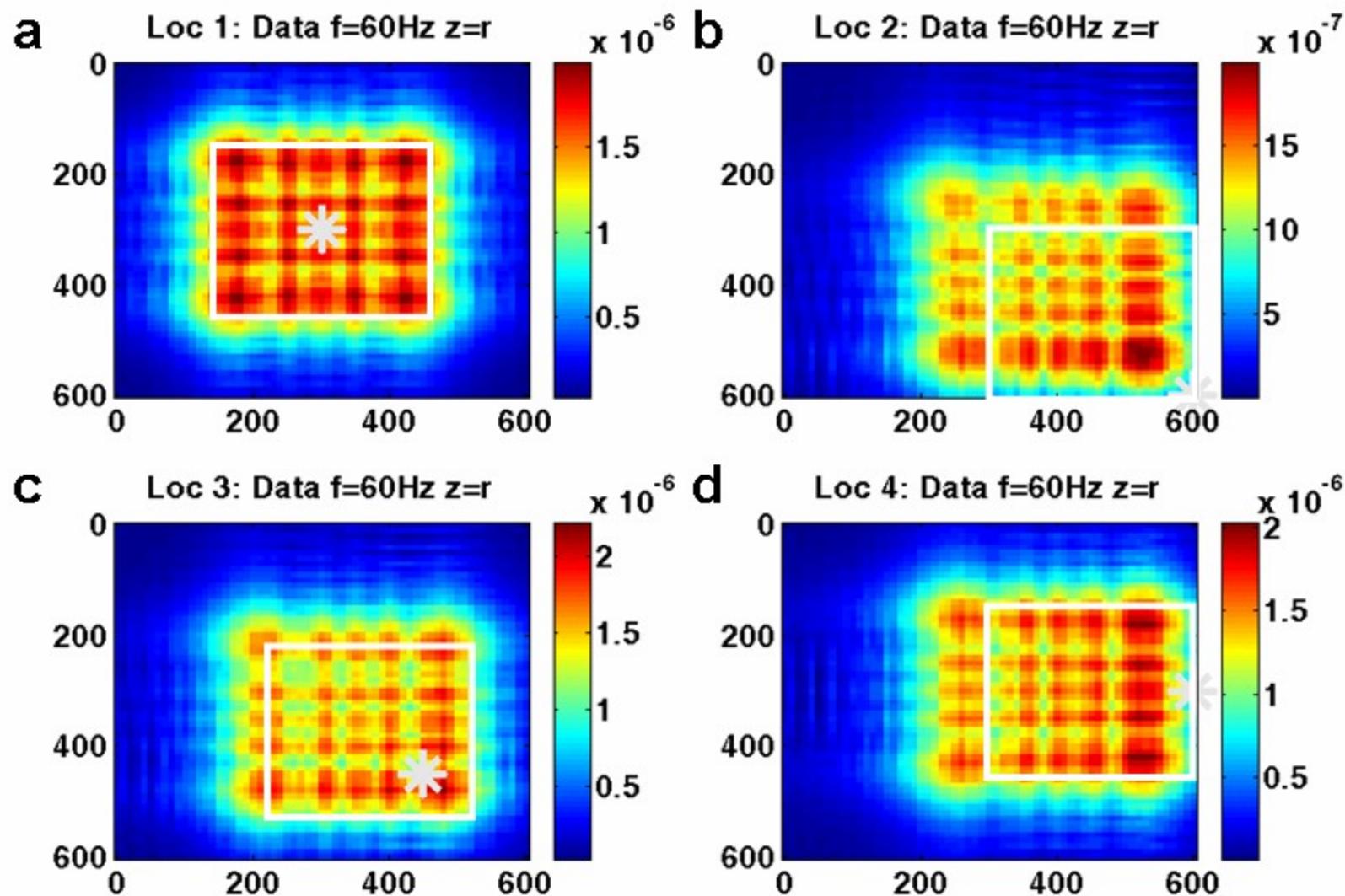
PP Reflectivity at 60 Hz



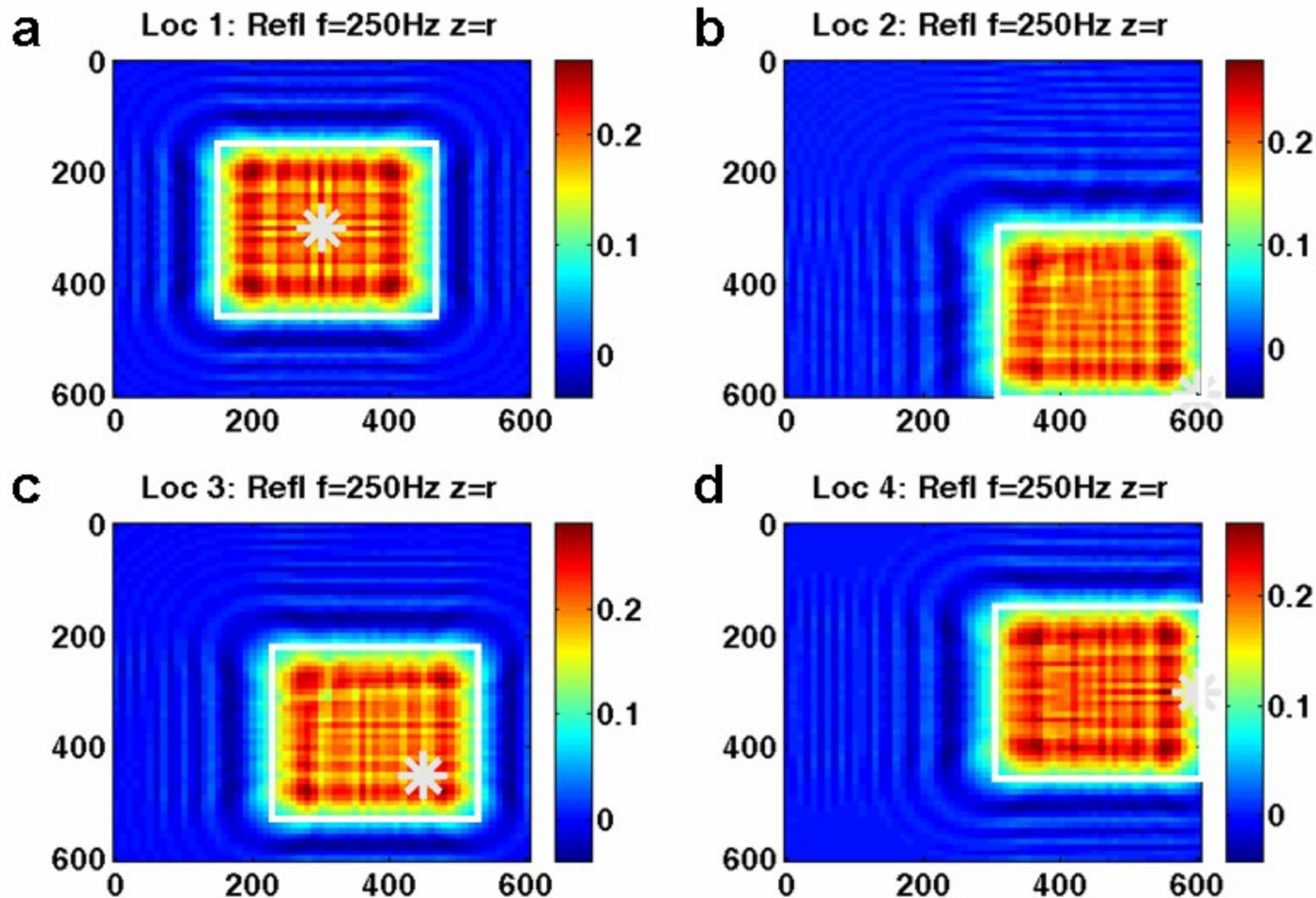
PS Reflectivity at 60 Hz



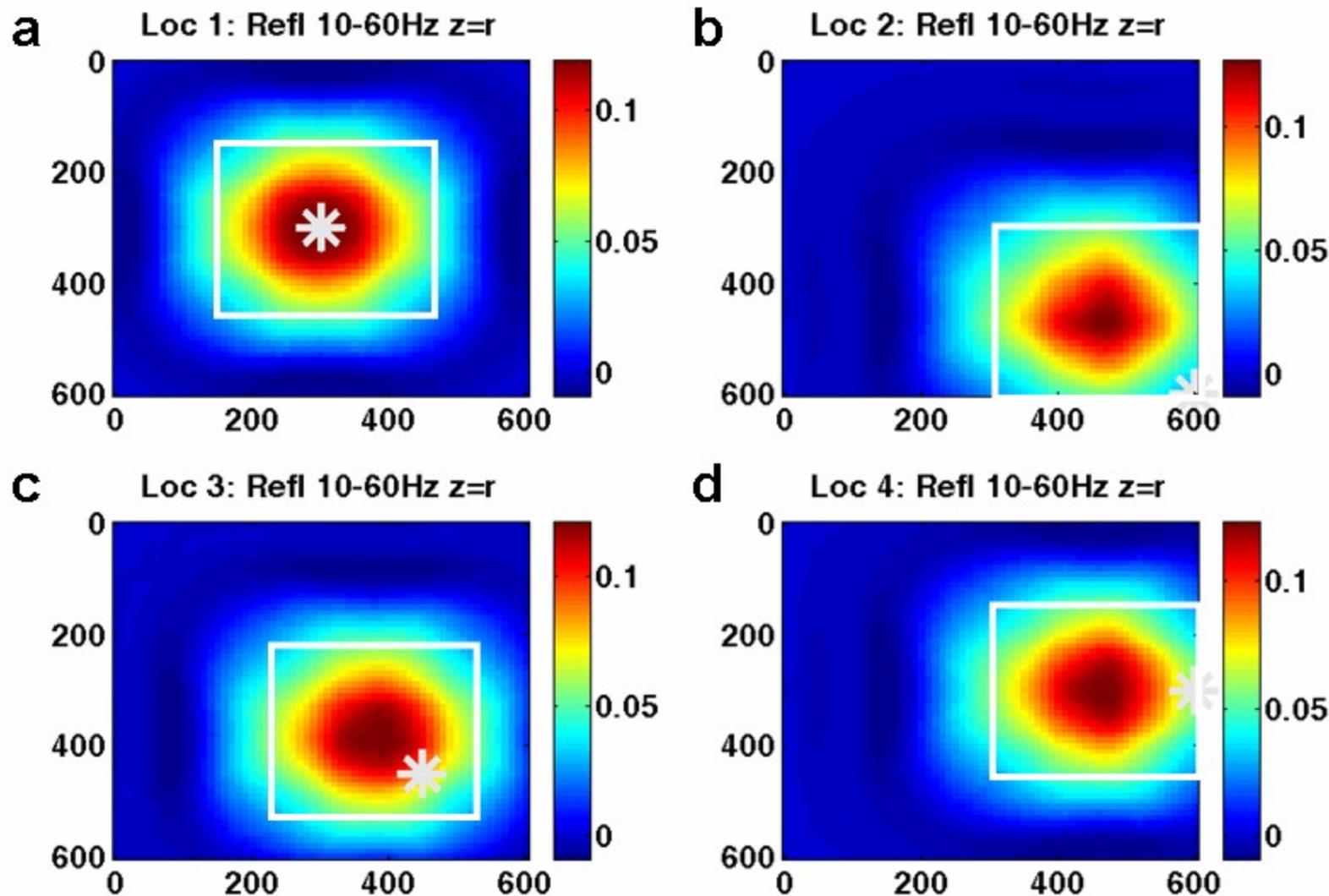
PS Data at $z=500$, 60 Hz



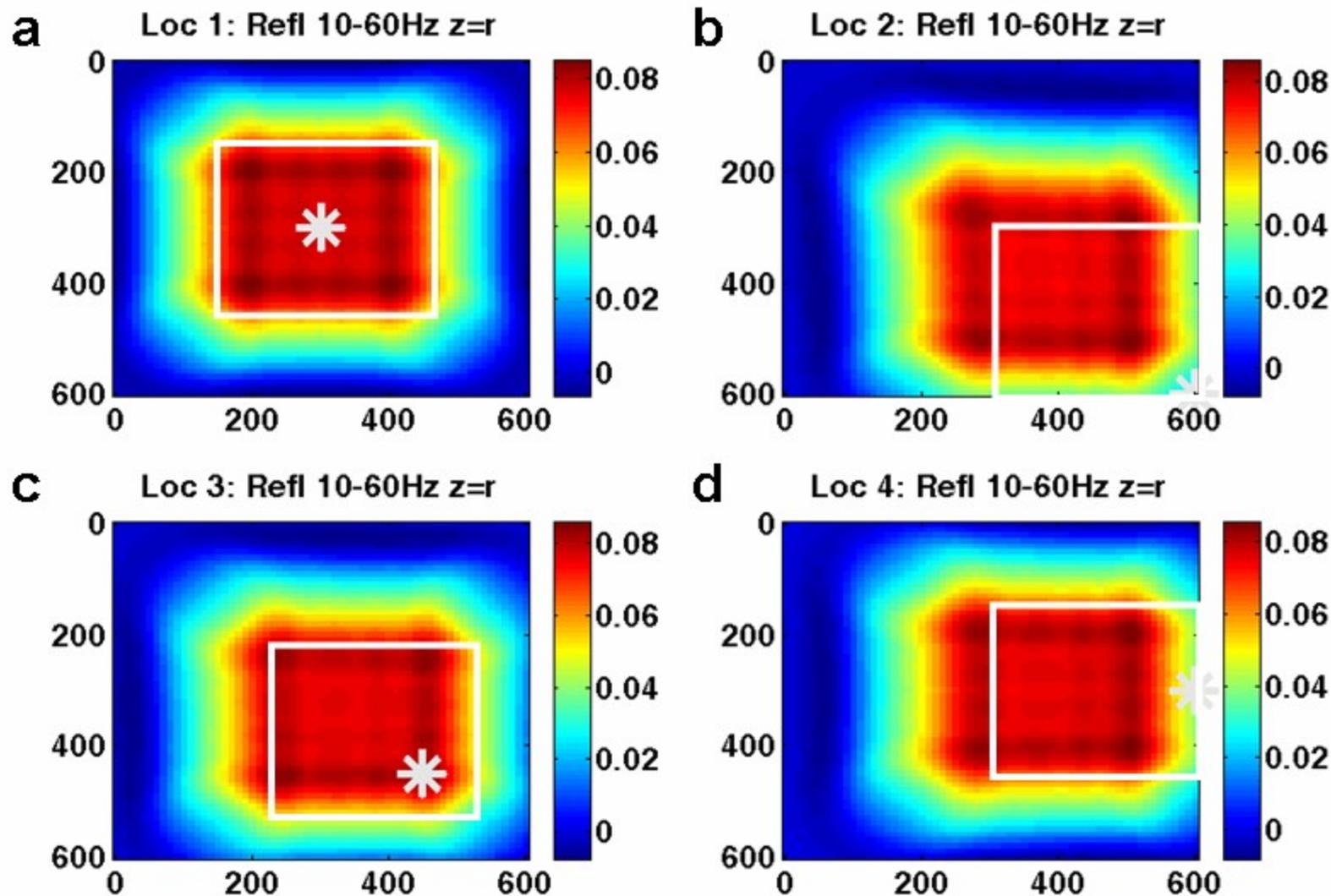
PP Reflectivity at 250 Hz



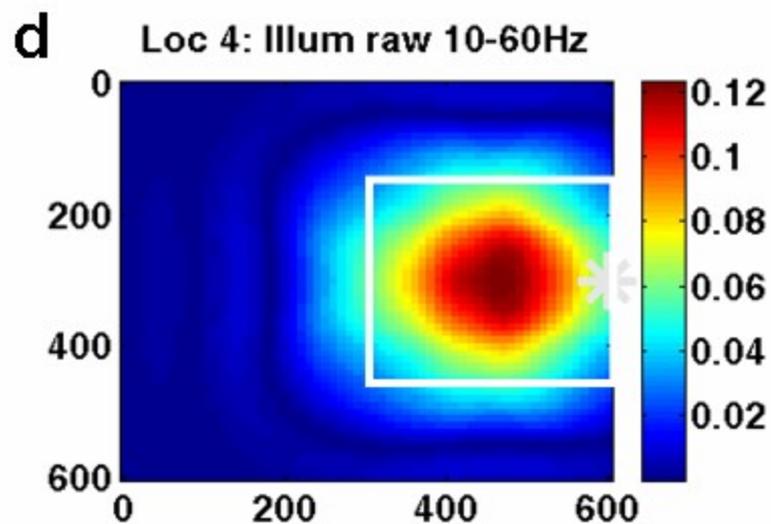
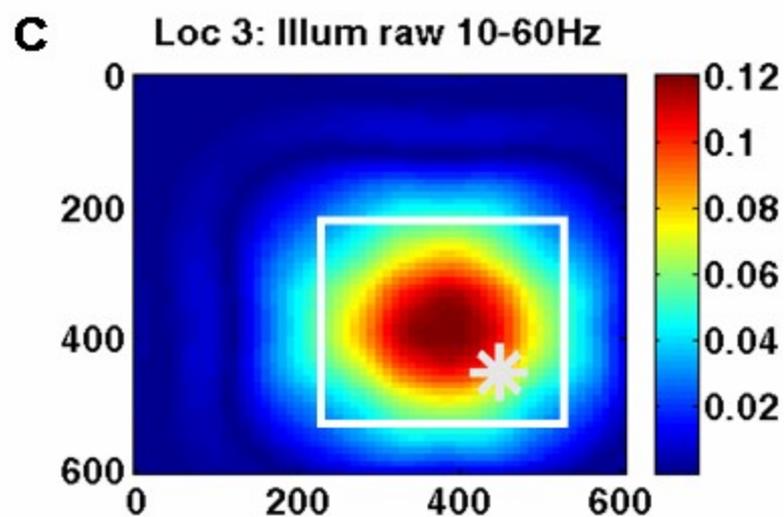
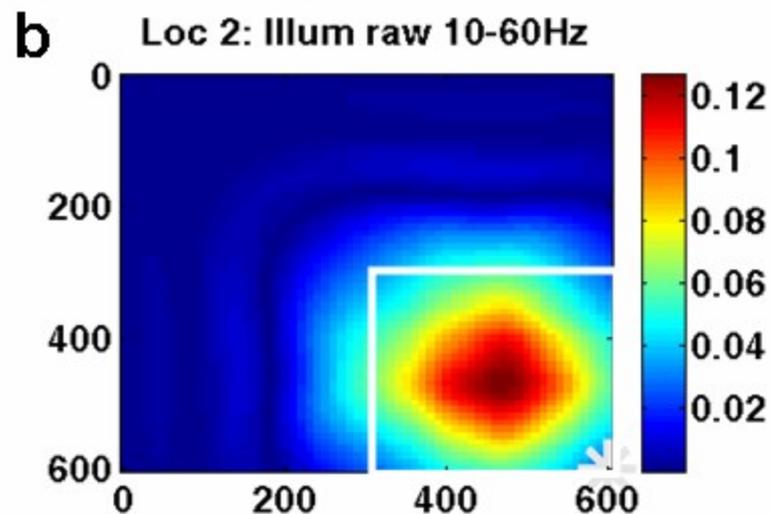
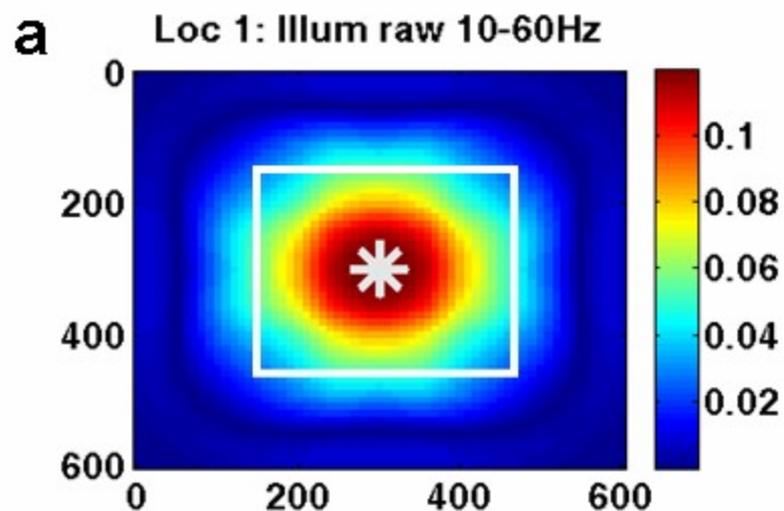
PP Reflectivity 10-60 Hz



PS Reflectivity 10-60 Hz

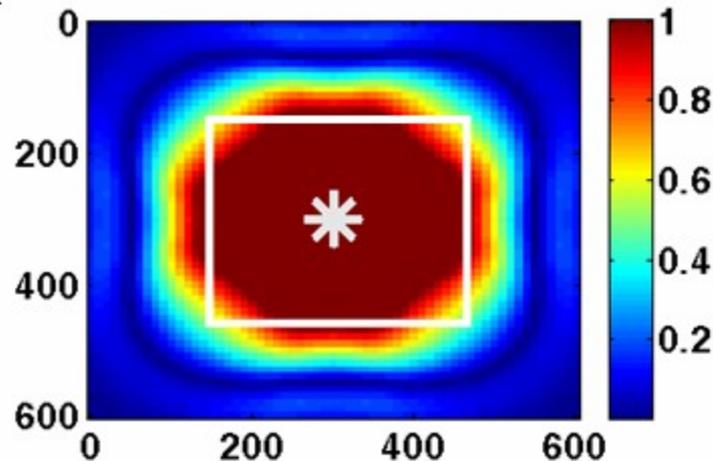


Illumination (data) Raw

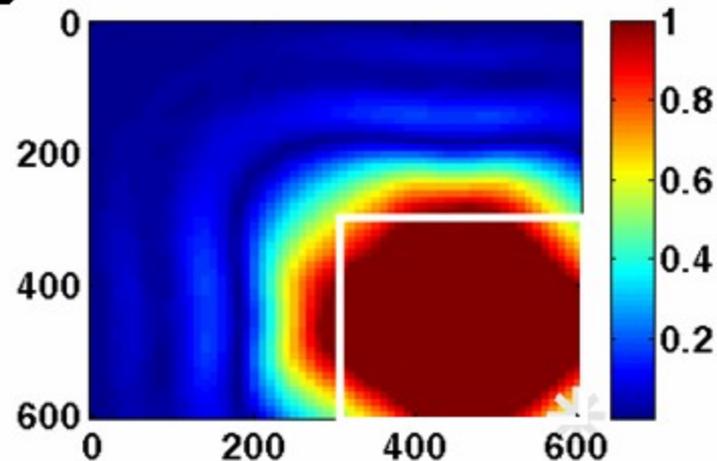


Illumination (data) Thresholded

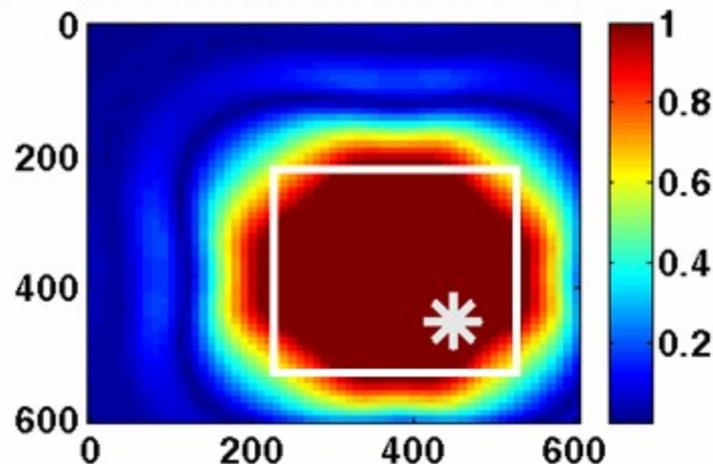
a Loc 1: Illum thresh 10-60Hz



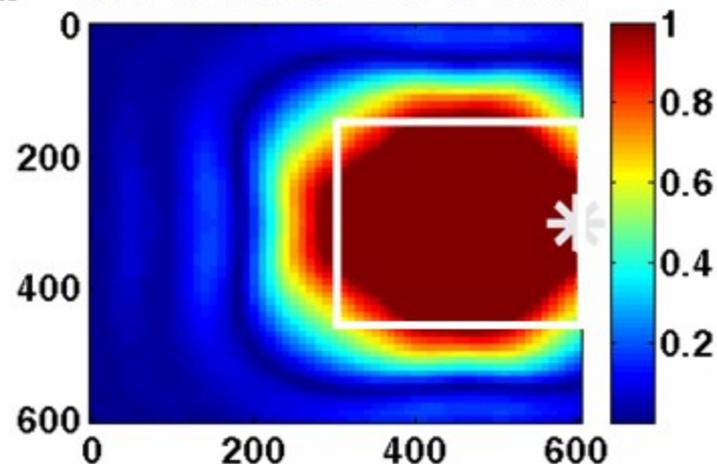
b Loc 2: Illum thresh 10-60Hz



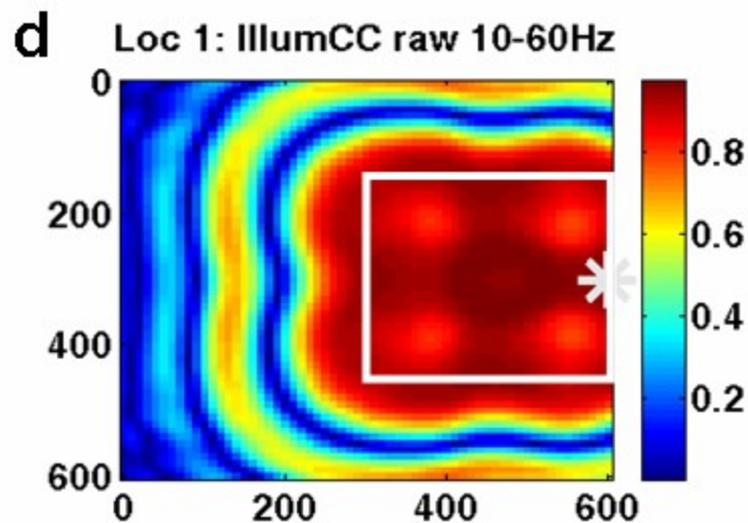
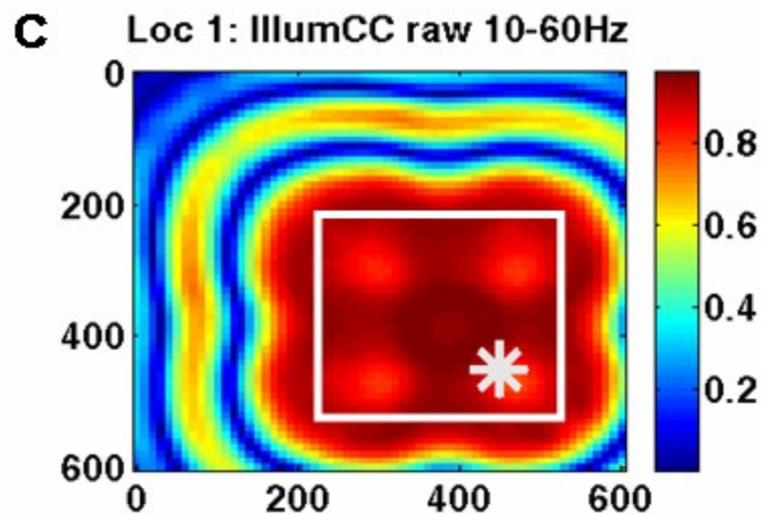
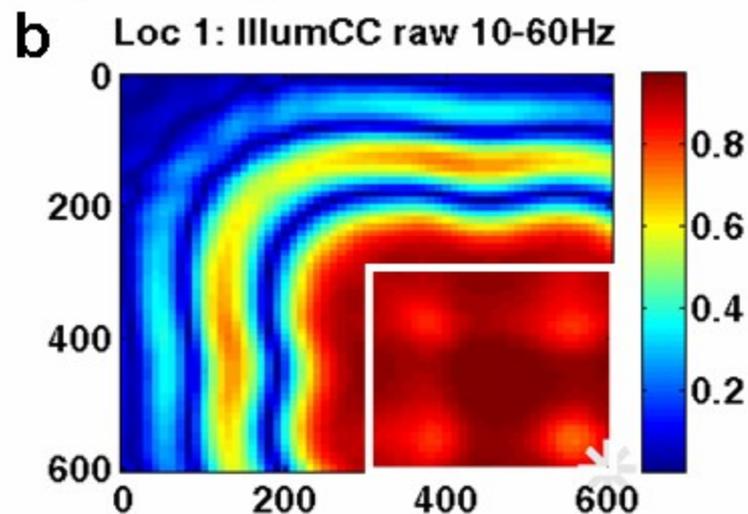
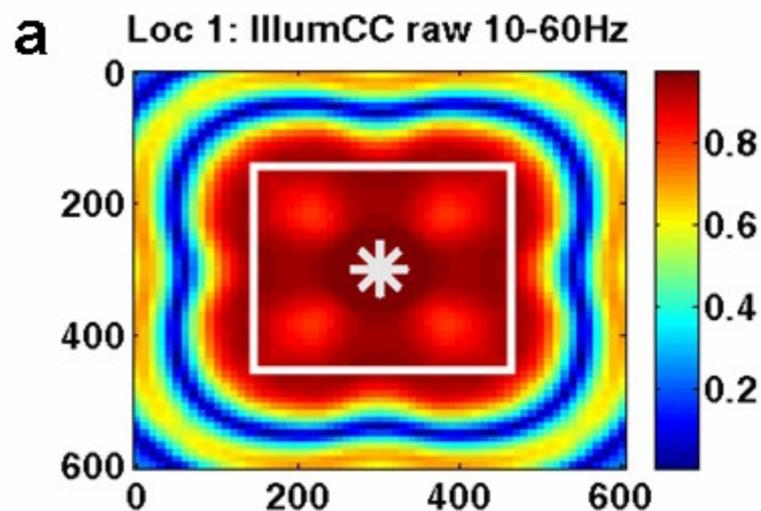
c Loc 3: Illum thresh 10-60Hz



d Loc 4: Illum thresh 10-60Hz

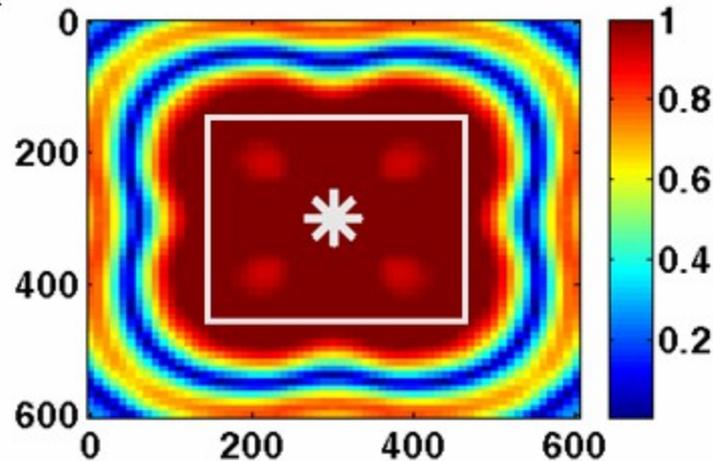


Illumination (CC) Raw

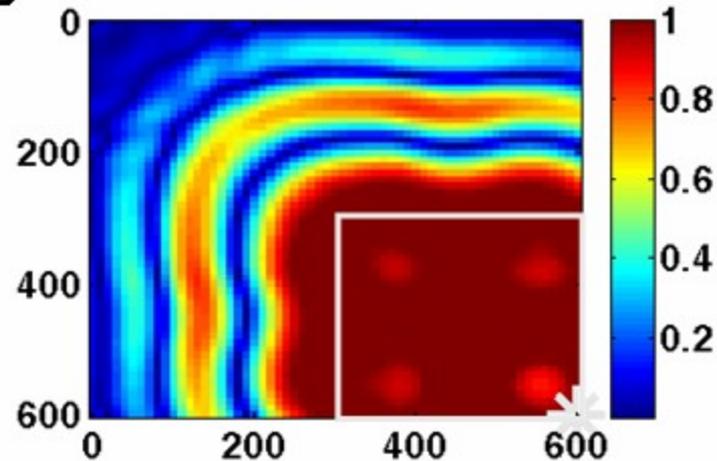


Illumination (CC) Thresholded

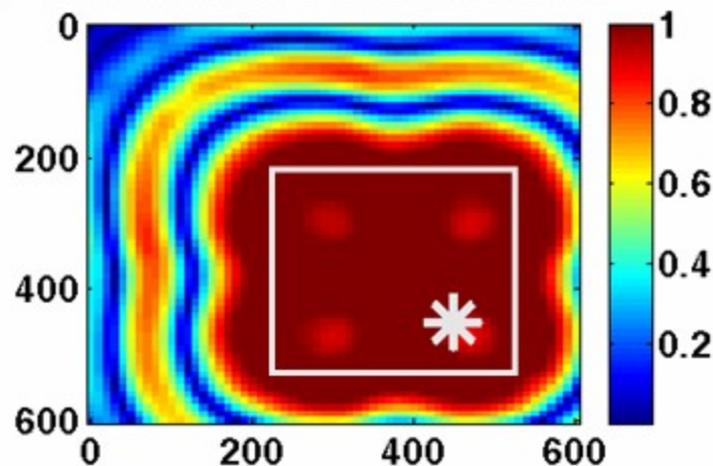
a Loc 1: IllumCC thresh 10-60Hz



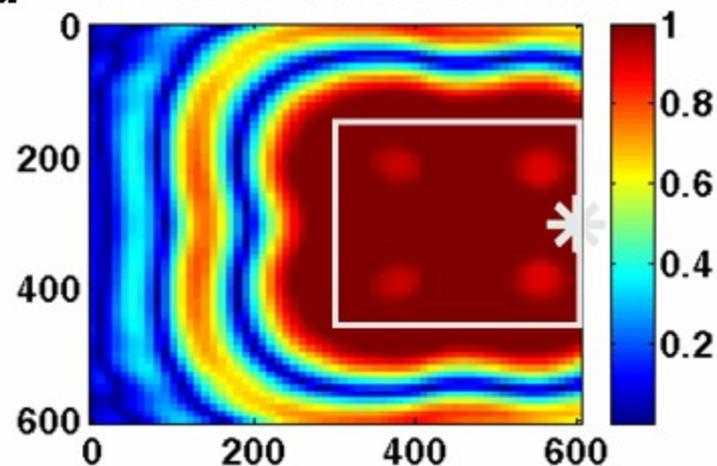
b Loc 2: IllumCC thresh 10-60Hz



c Loc 3: IllumCC thresh 10-60Hz

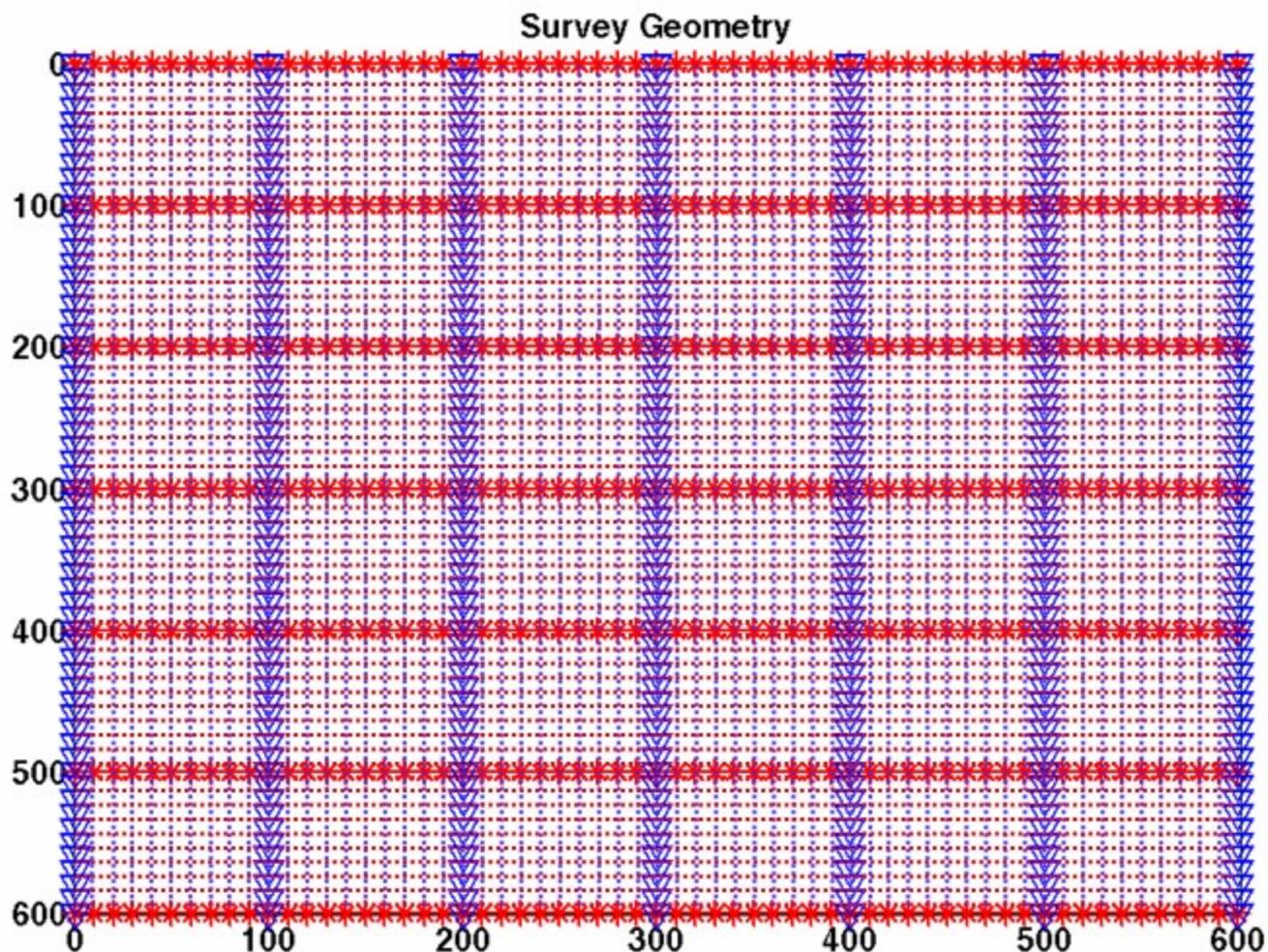


d Loc 4: IllumCC thresh 10-60Hz



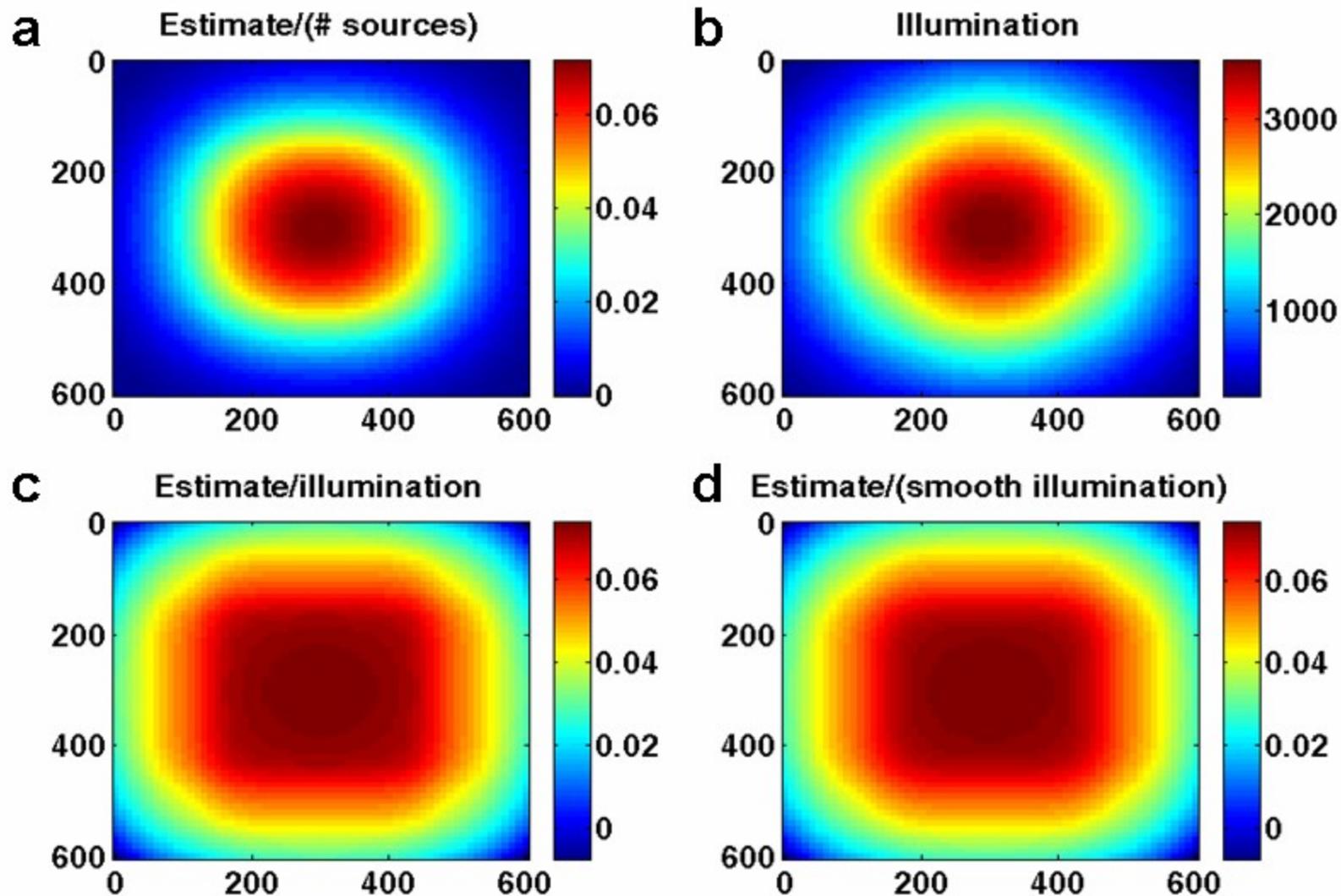
Complete Geometry
no noise

Complete Geometry

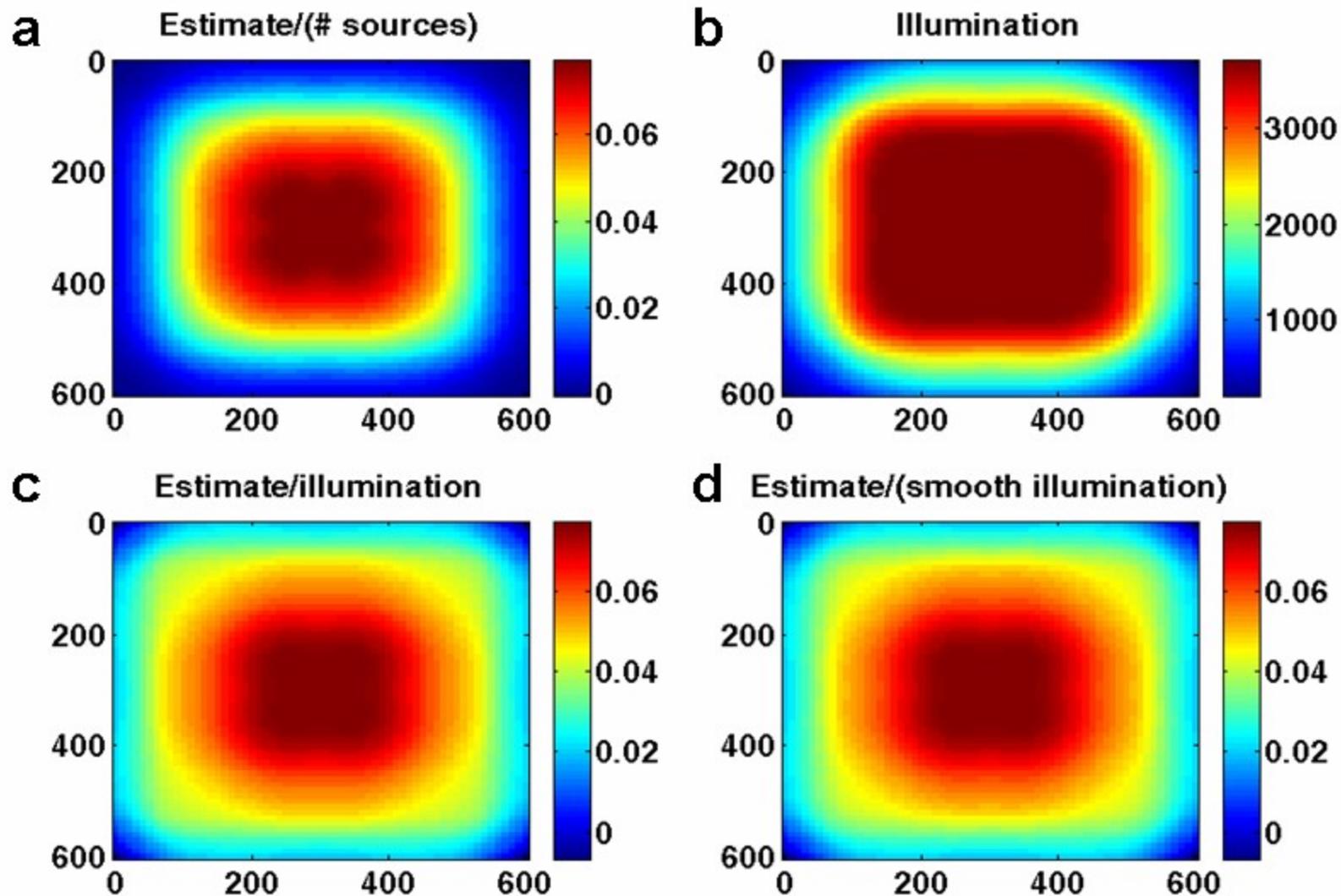


Shot (red) spacing 10m --- Shot line spacing 10m
Receiver (blue) spacing 10m --- Receiver line spacing 10m

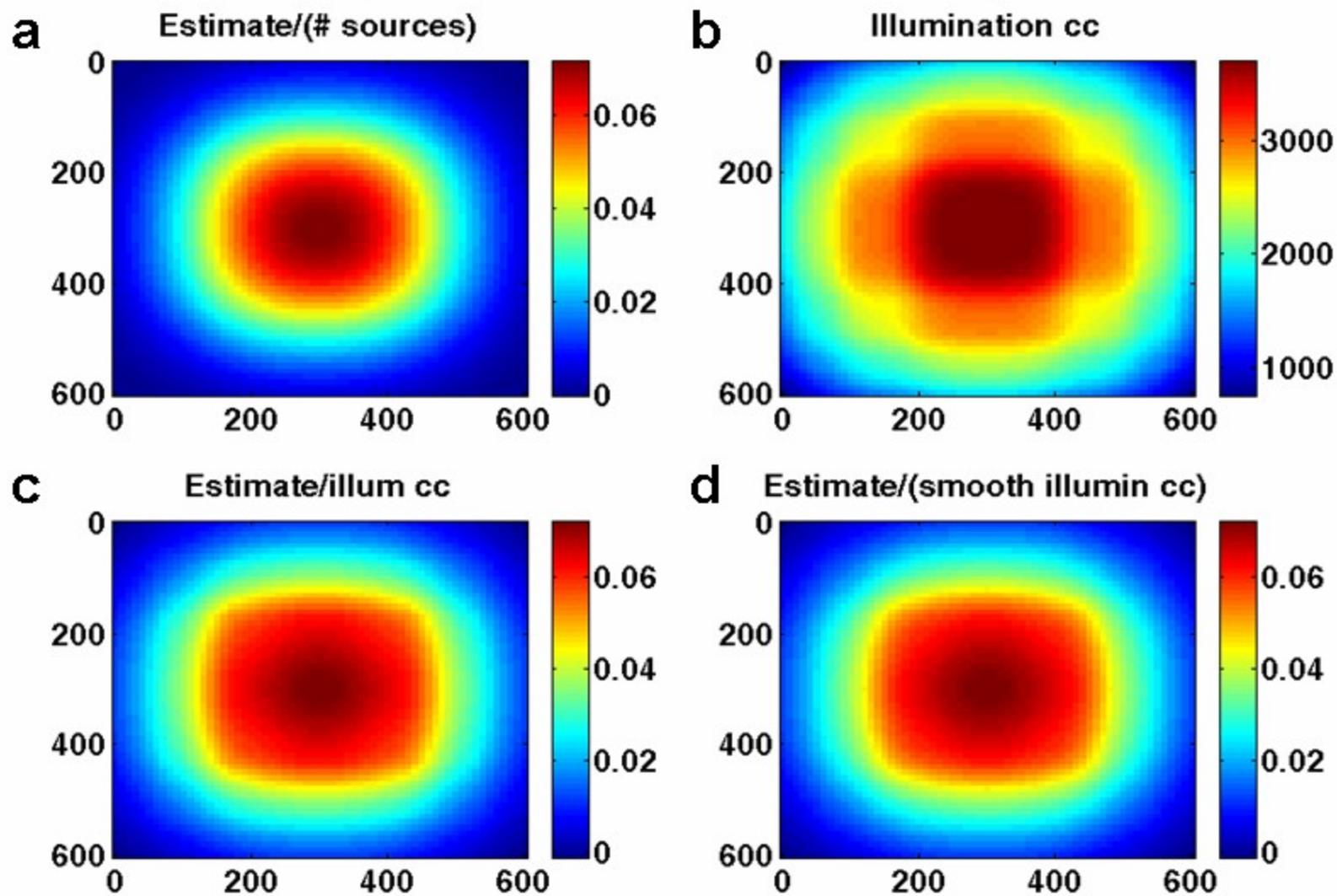
Complete Geometry PP



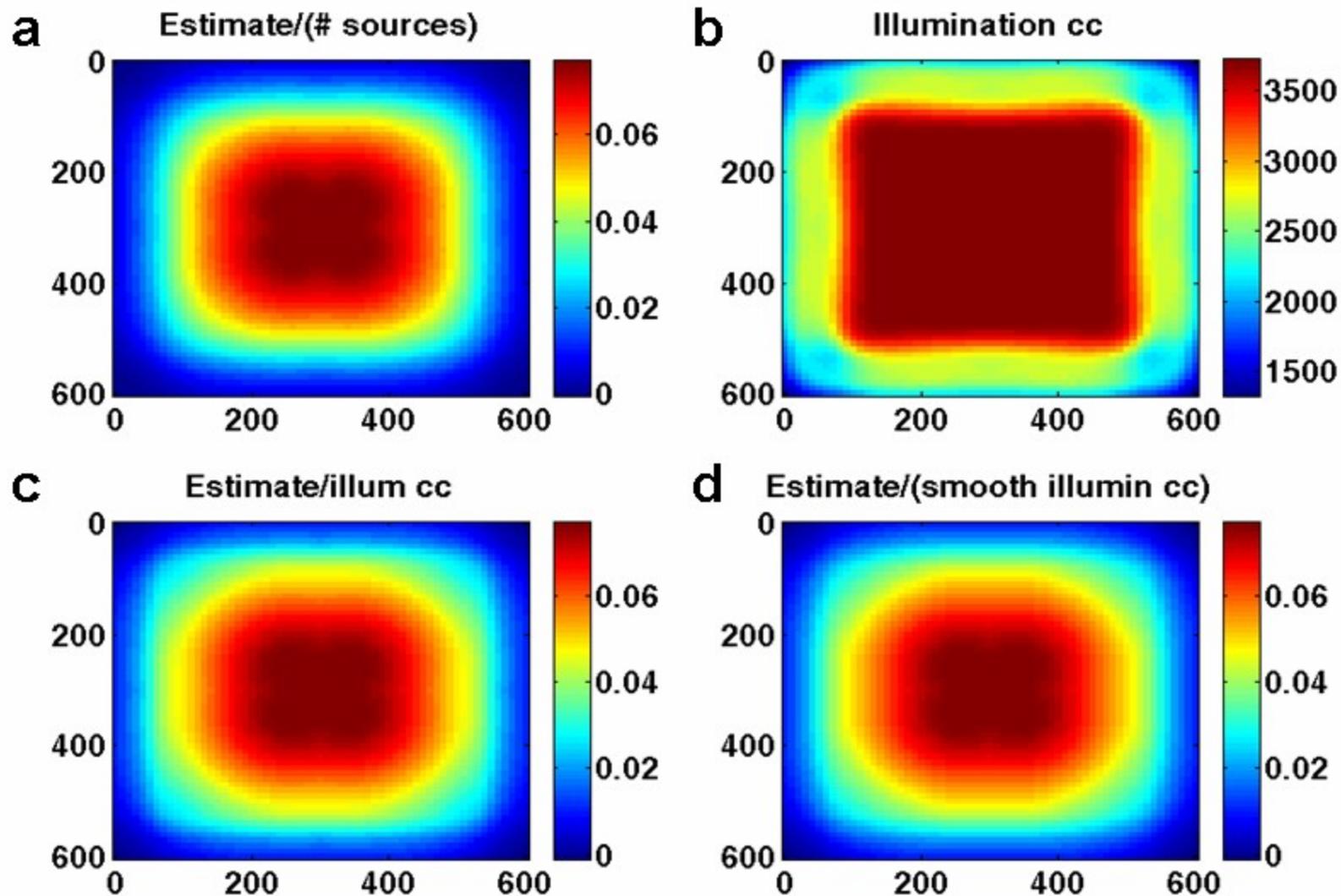
Complete Geometry PS



Complete Geometry PP



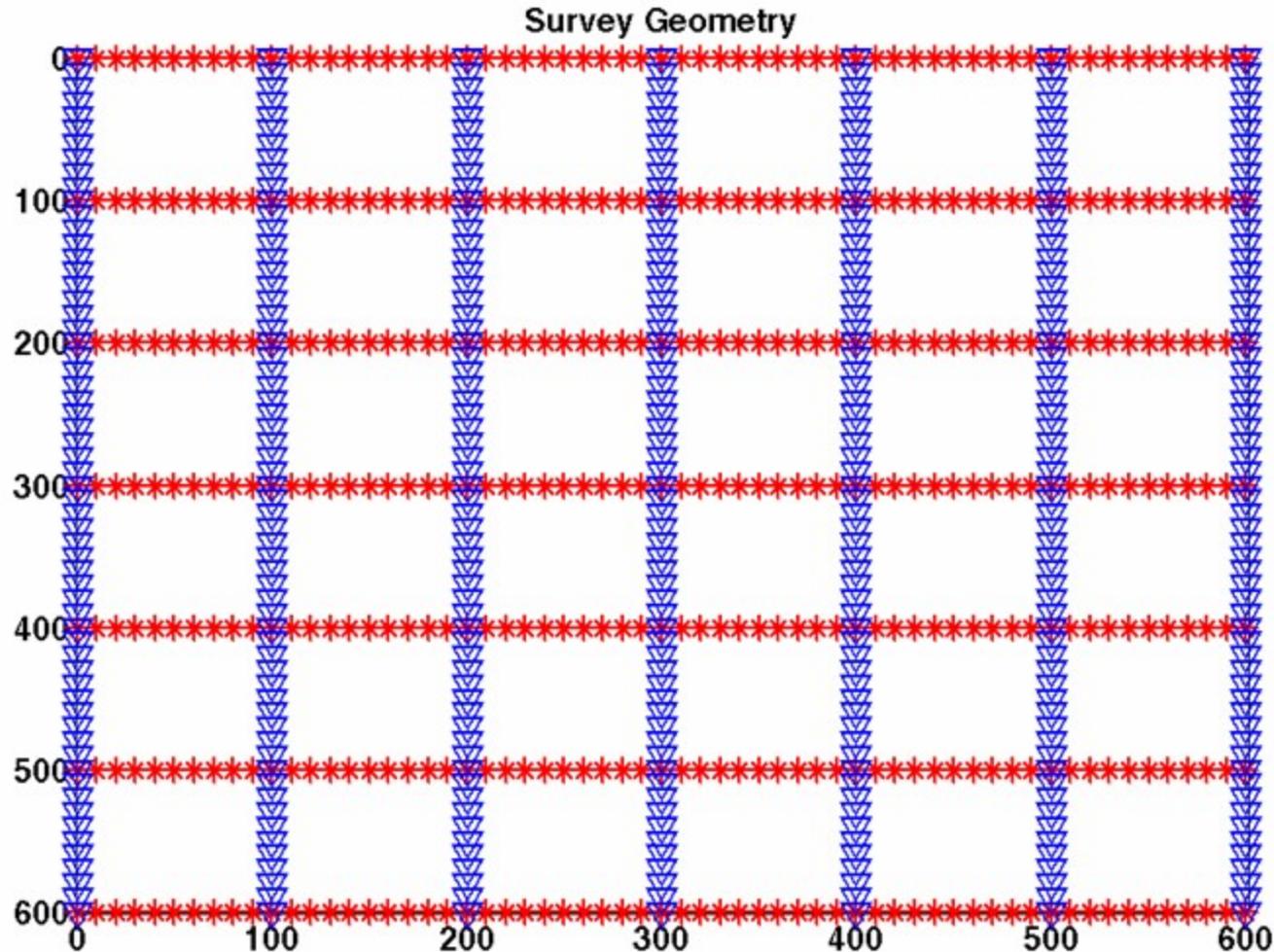
Complete Geometry PS



Source Reduced Geometry
no noise
See Paper

Source and Receiver Reduced
Geometry
no noise

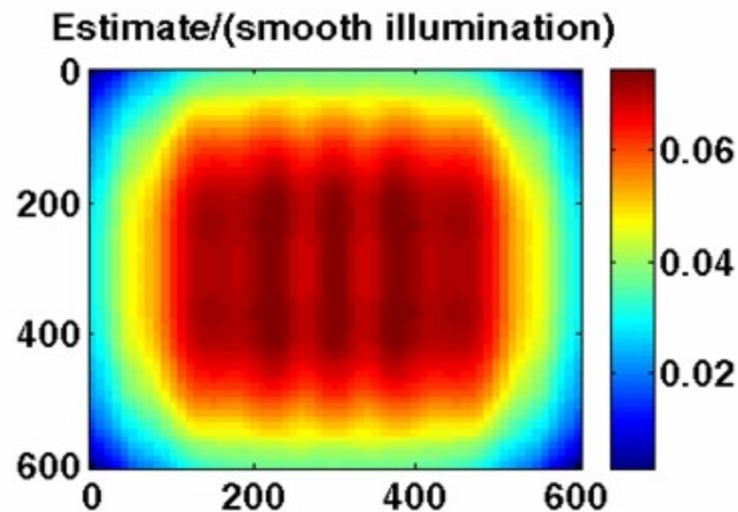
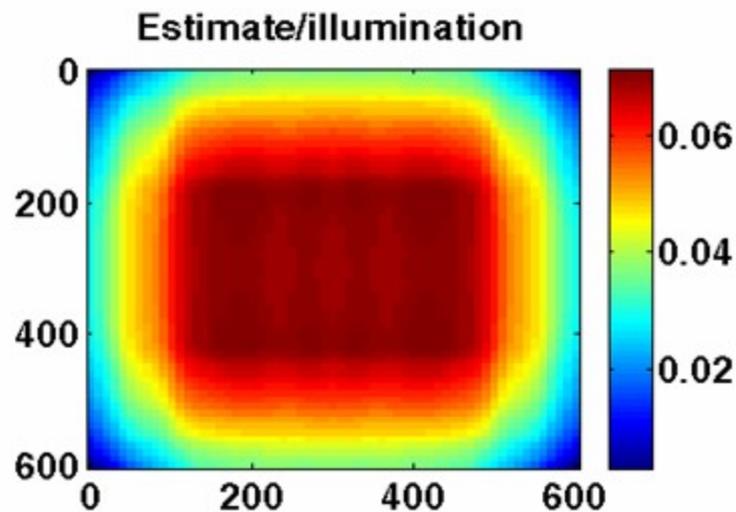
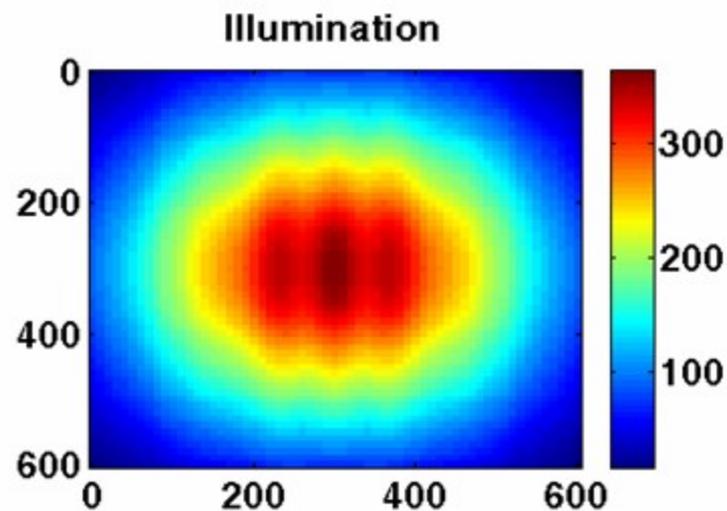
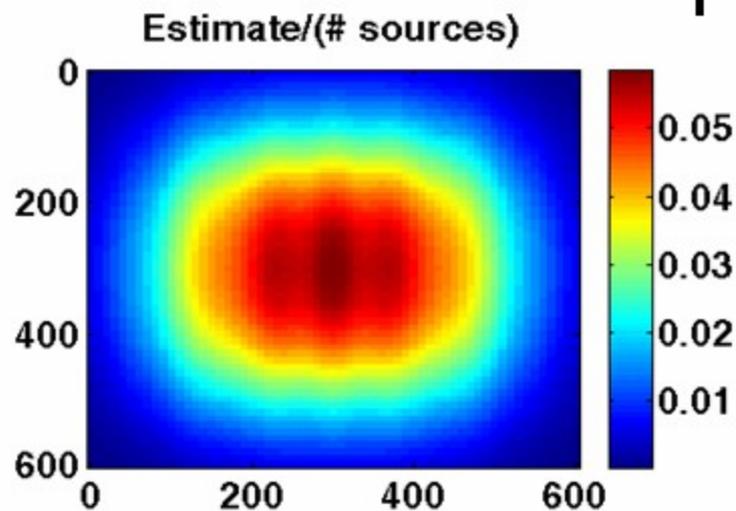
Source /Receiver Reduced Geometry



Shot (red) spacing 10m --- Shot line spacing 100m
Receiver (blue) spacing 10m --- Receiver line spacing 100m

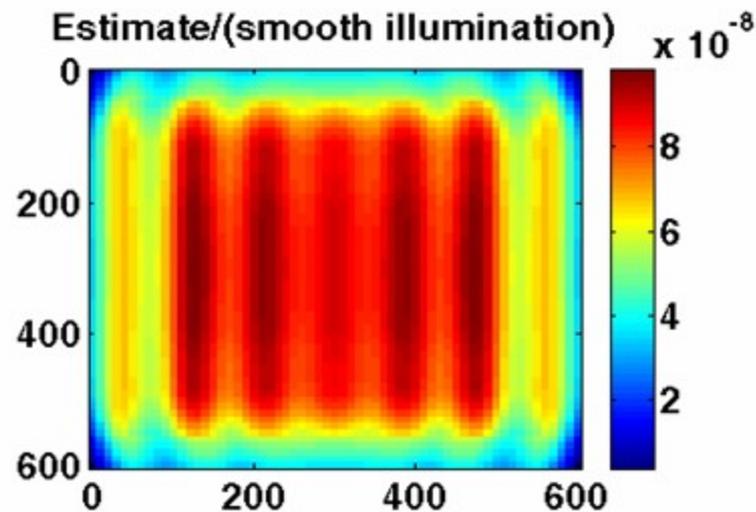
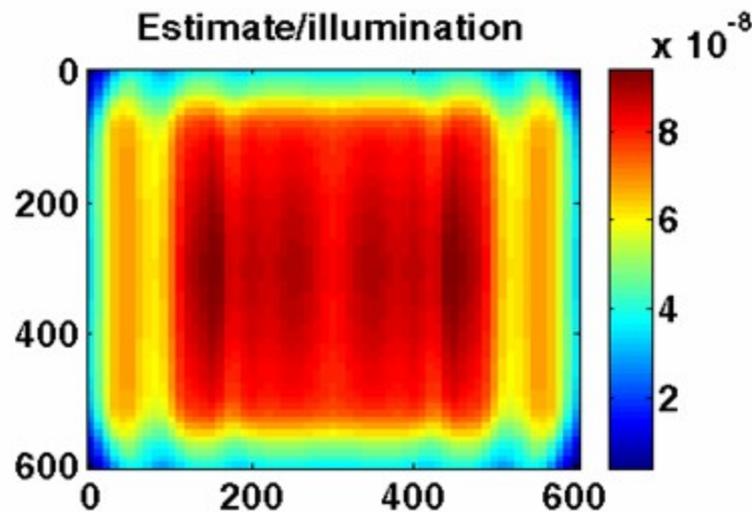
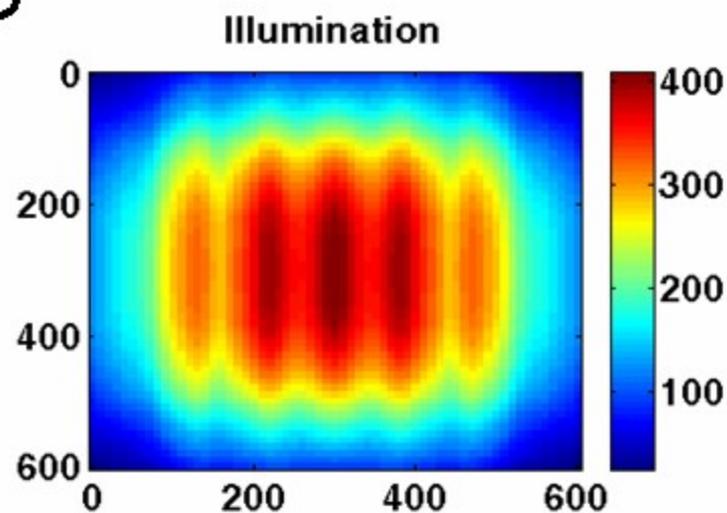
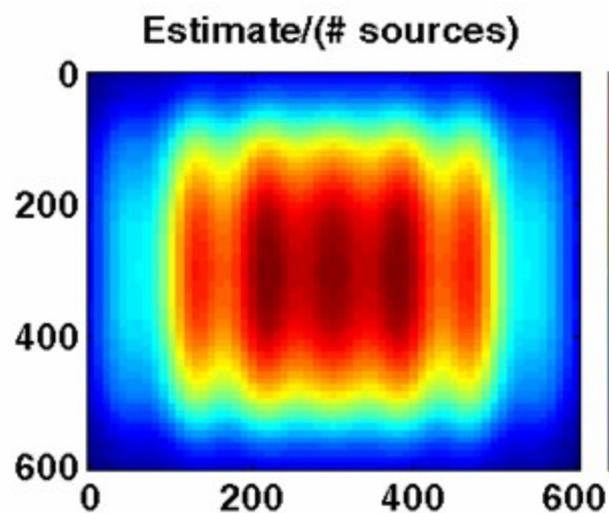
Source/Receiver Reduced Geometry

PP



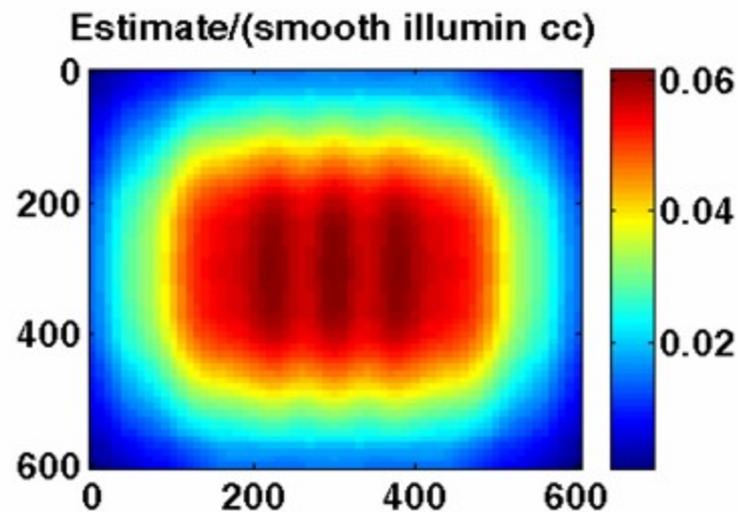
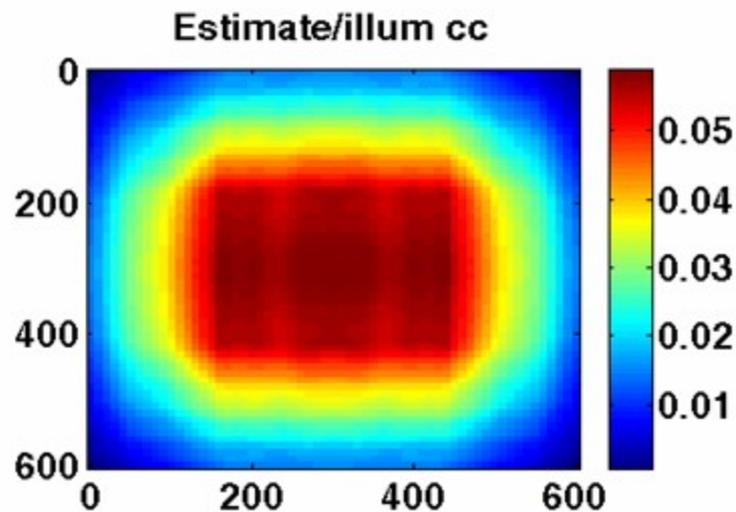
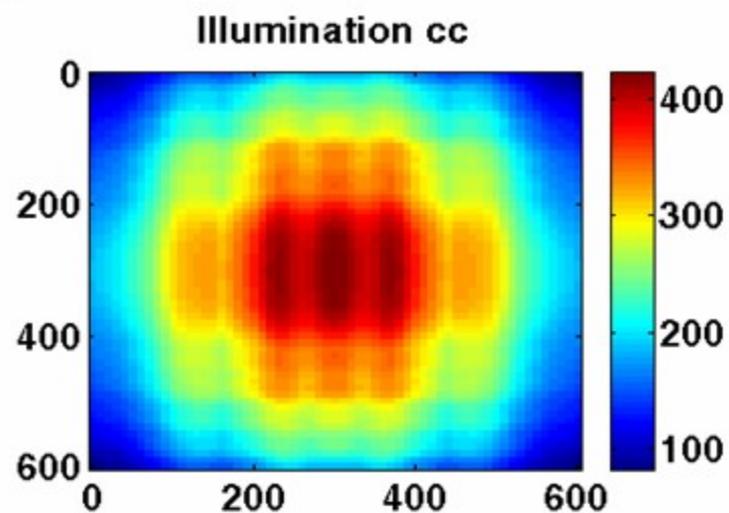
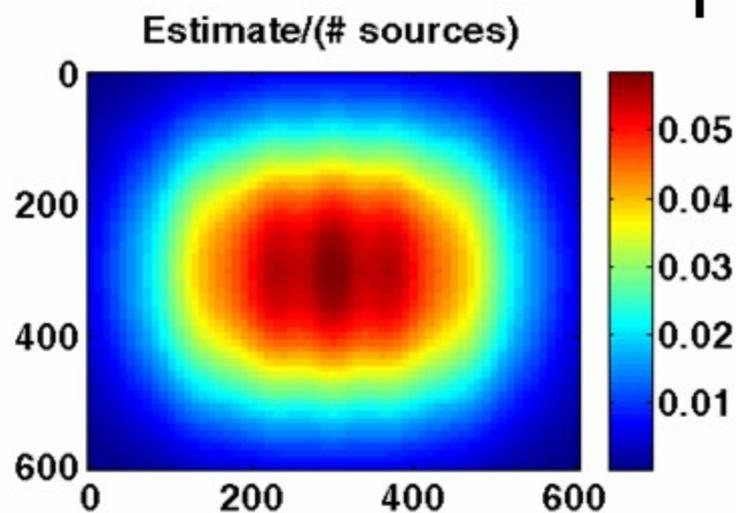
Source/Receiver Reduced Geometry

PS



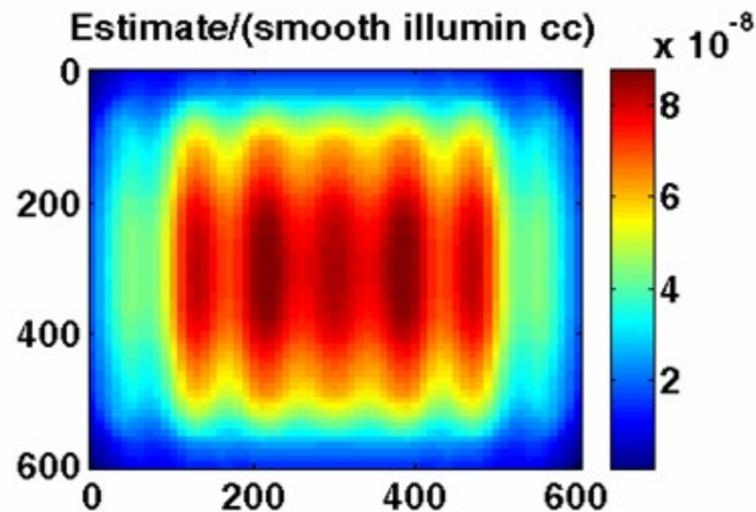
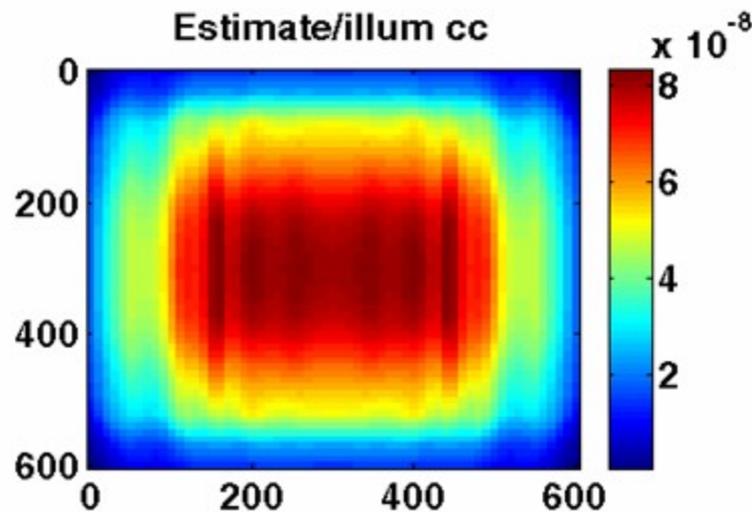
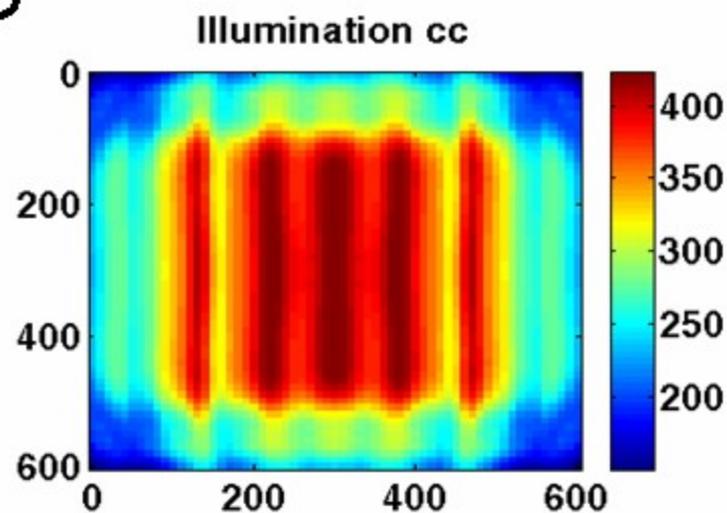
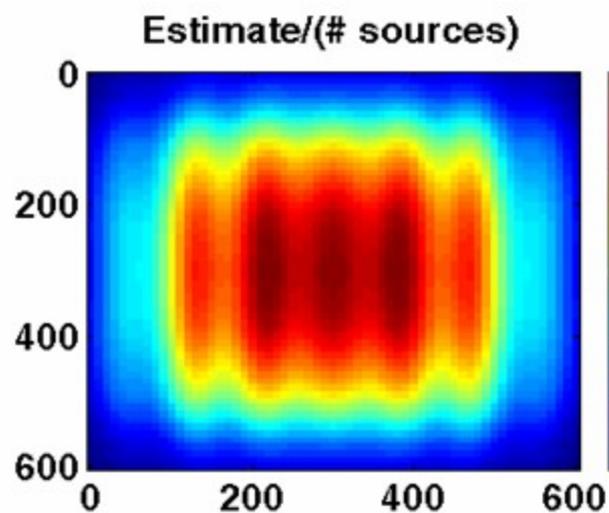
Source/Receiver Reduced Geometry

PP

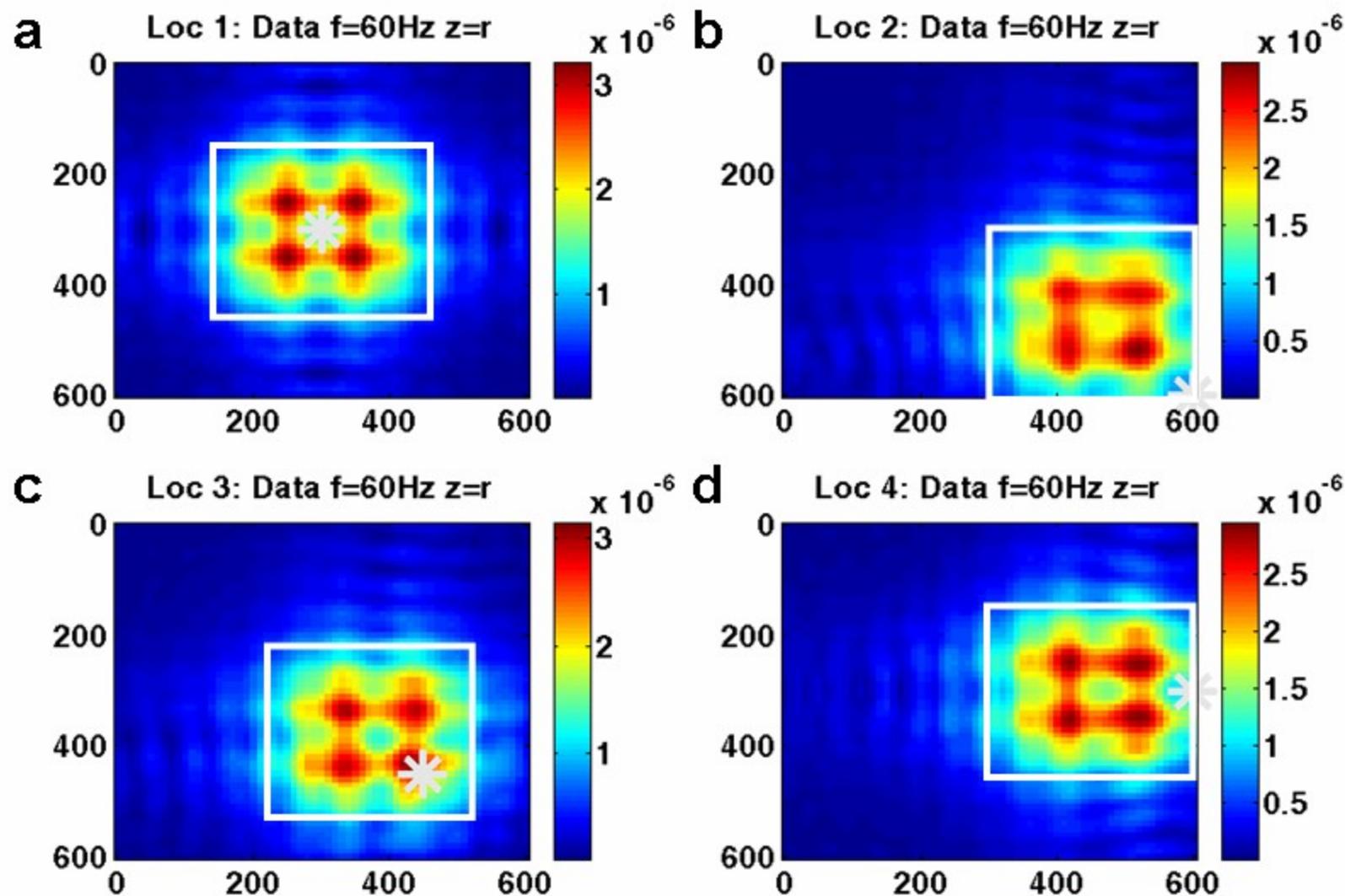


Source/Receiver Reduced Geometry

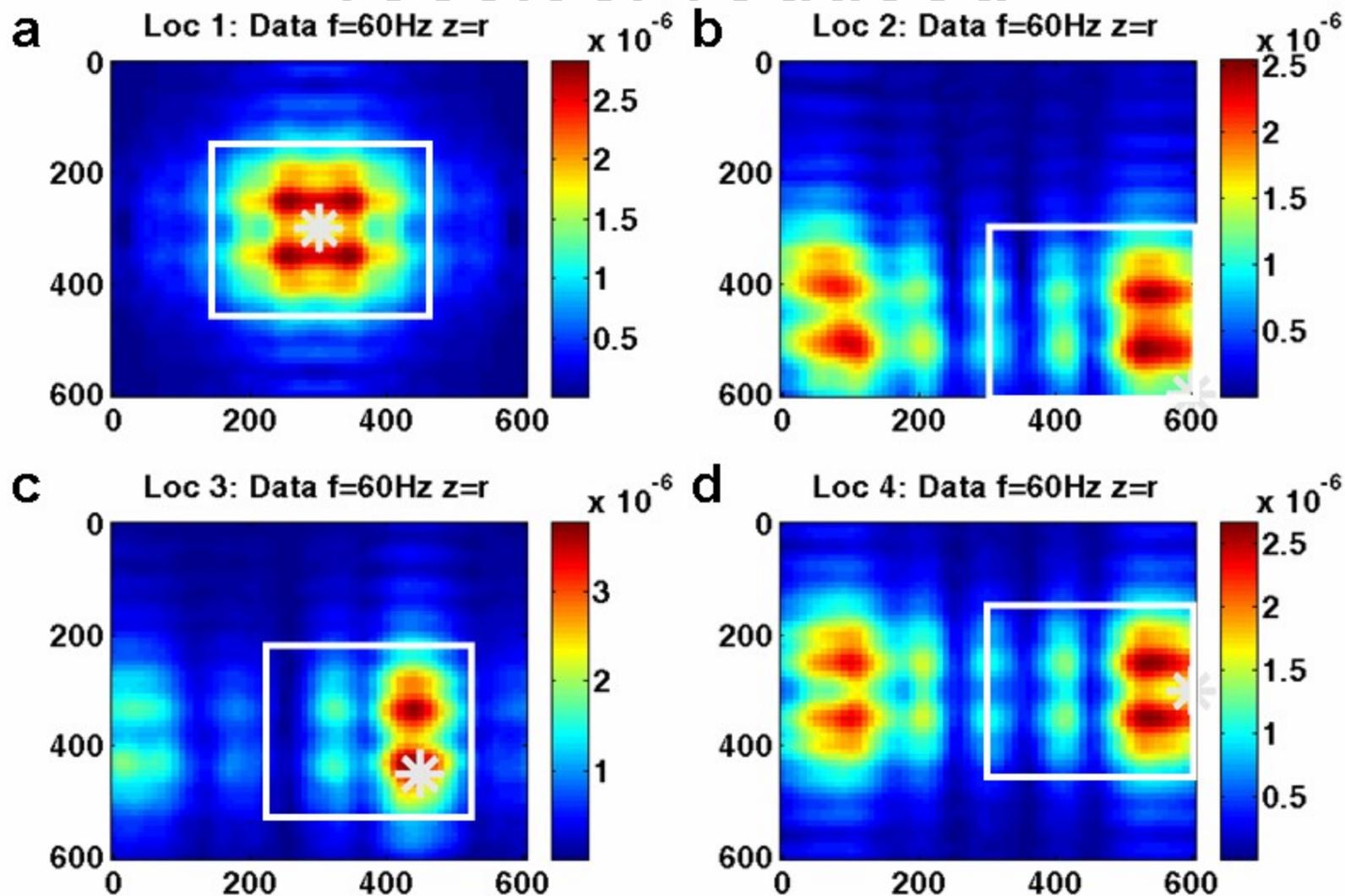
PS



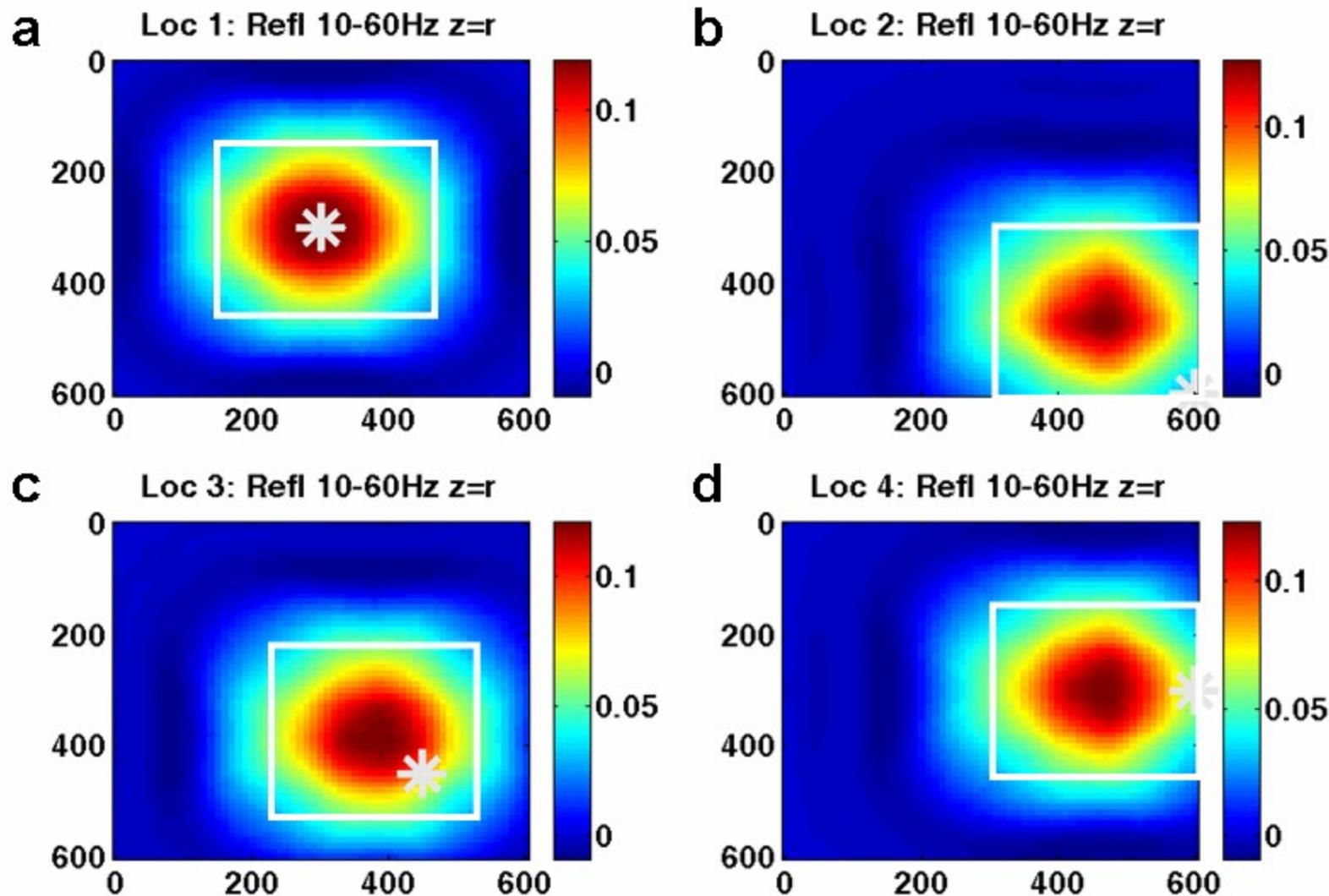
PP Data at $z=500$, 60 Hz



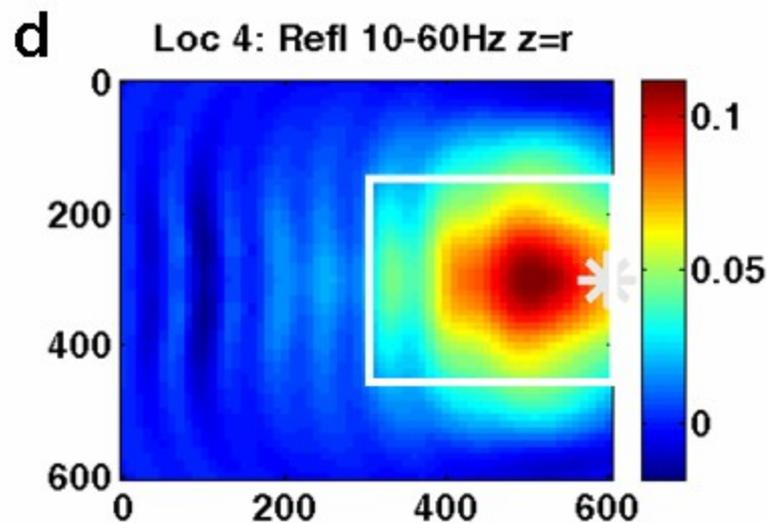
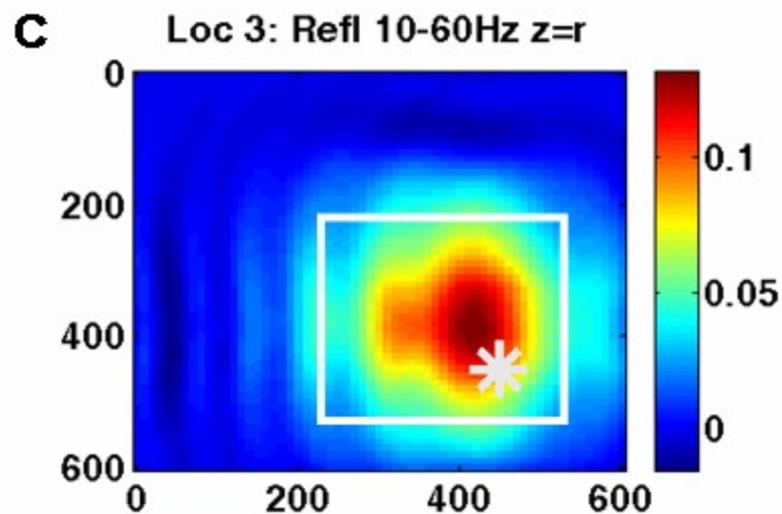
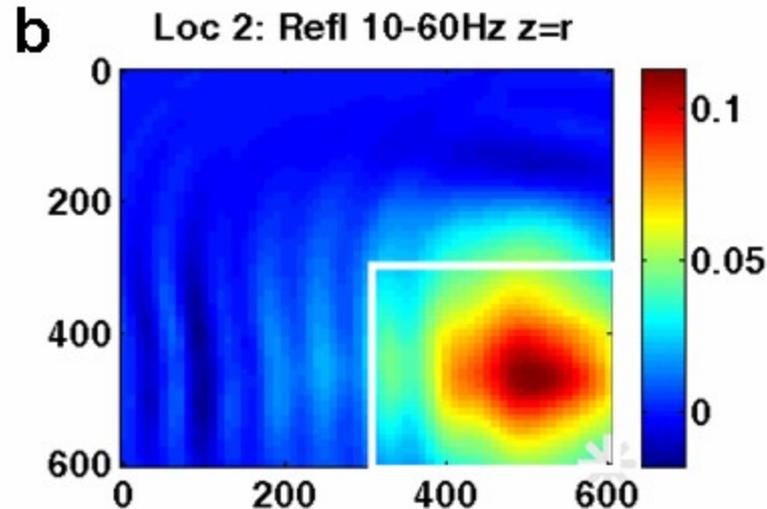
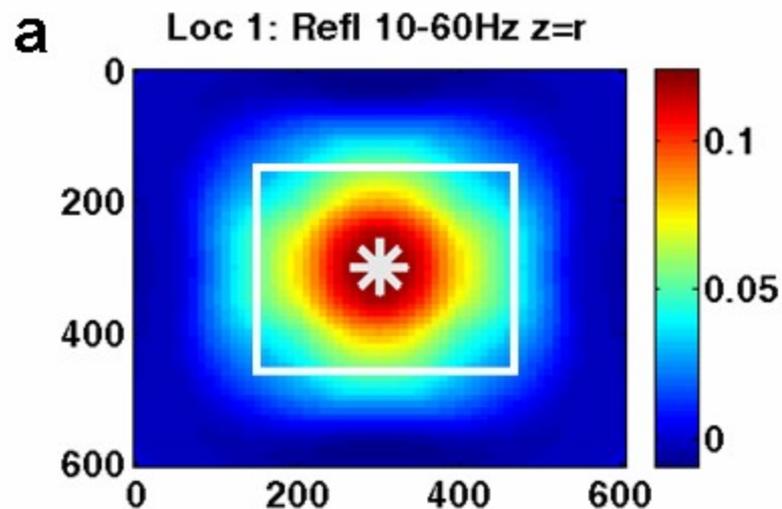
PP Data at $z=500$, 10 Hz receiver reduced



PP Reflectivity 10-60 Hz

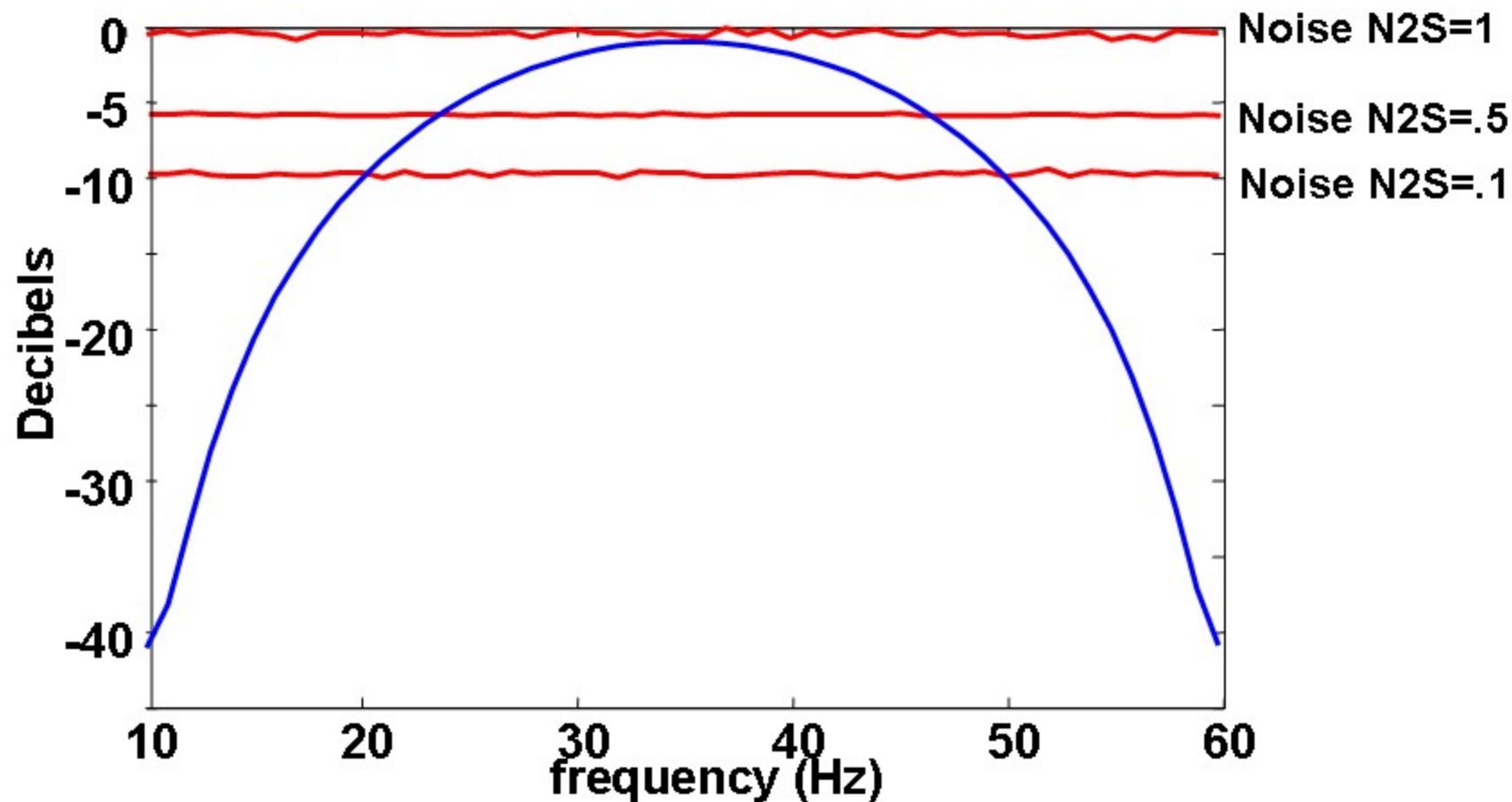


PP Reflectivity 10-60 Hz receiver reduced



Inclusion of Random Noise

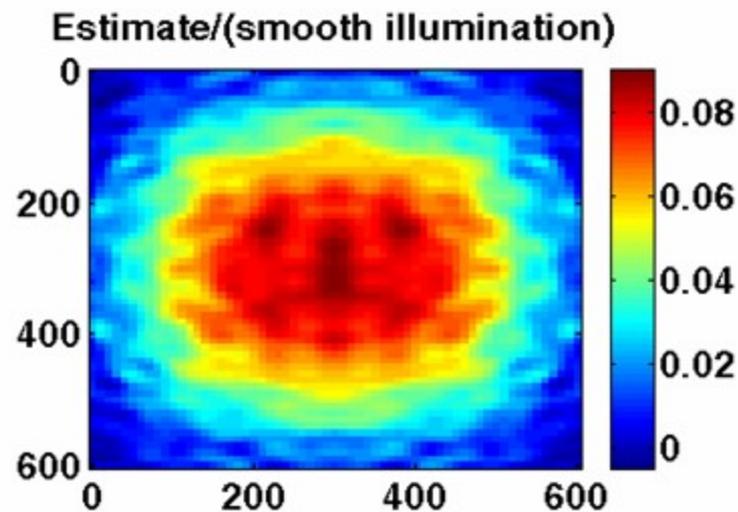
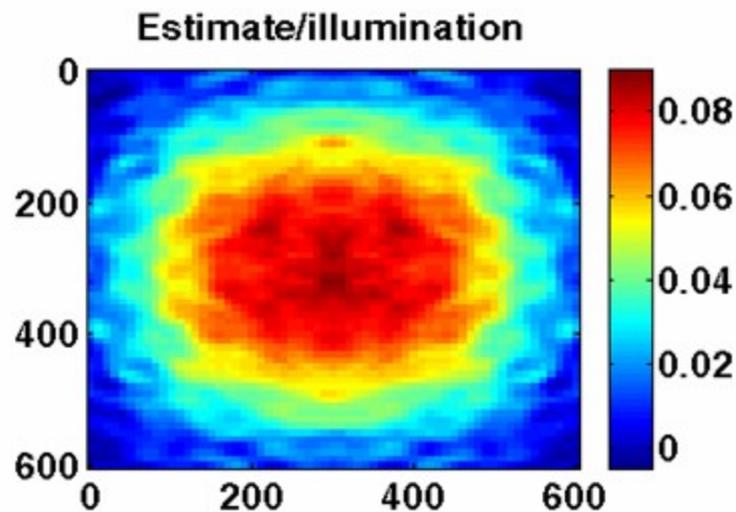
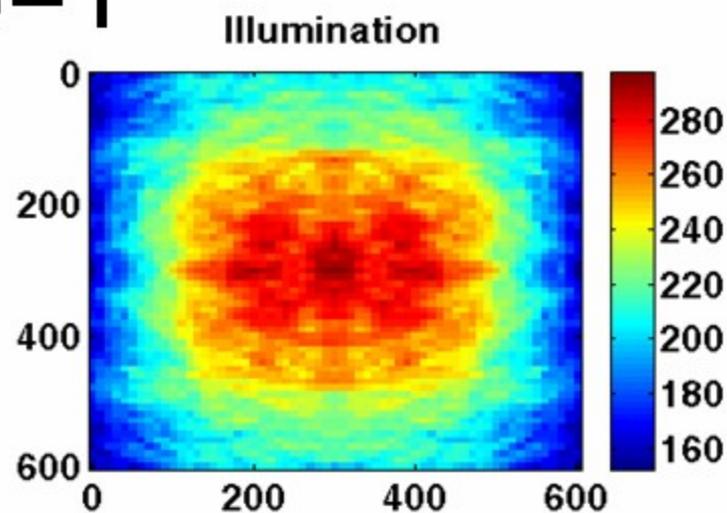
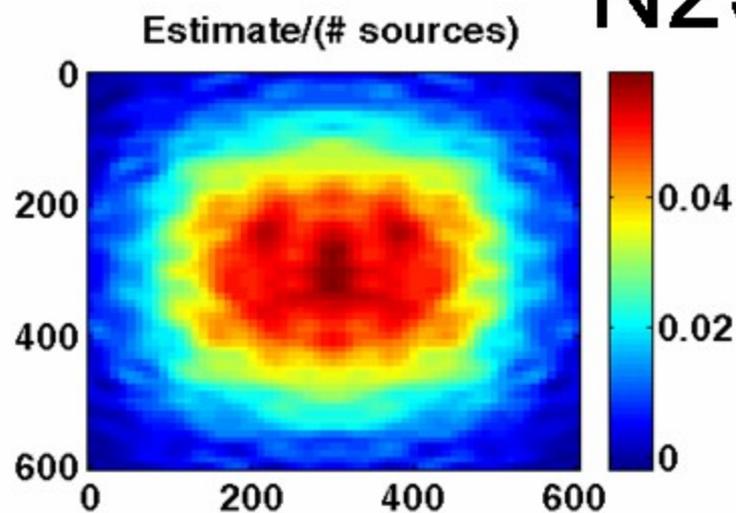
Random Noise Levels



Gaussian random noise, zero mean, standard deviation equal to the strength of the data at dominant frequency.

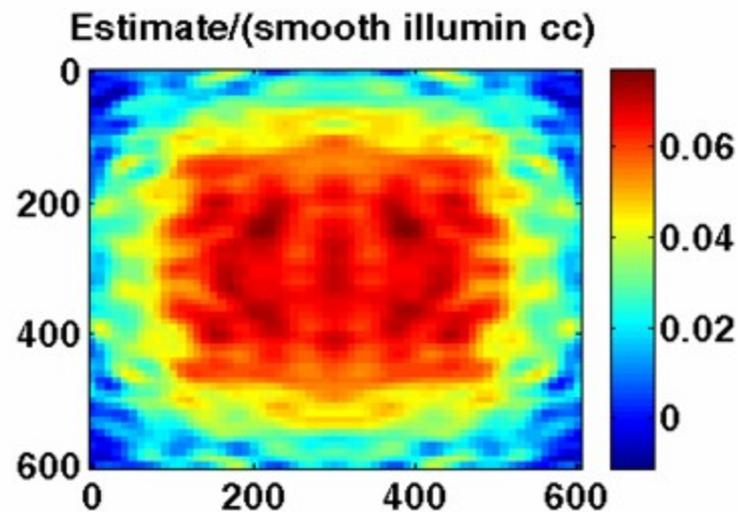
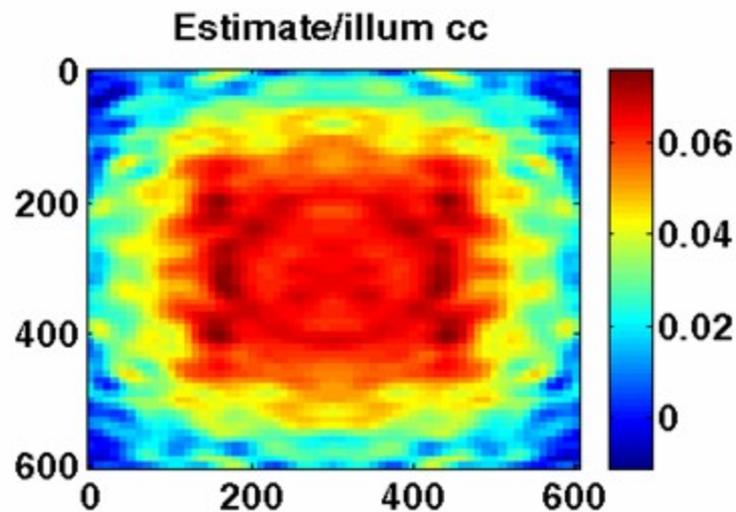
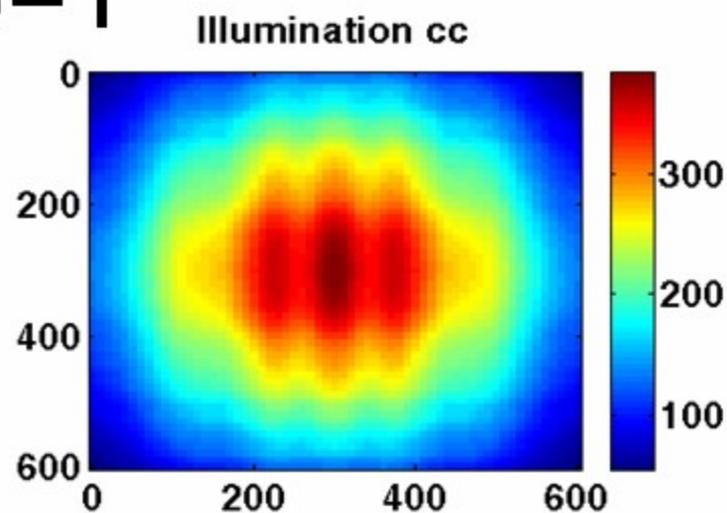
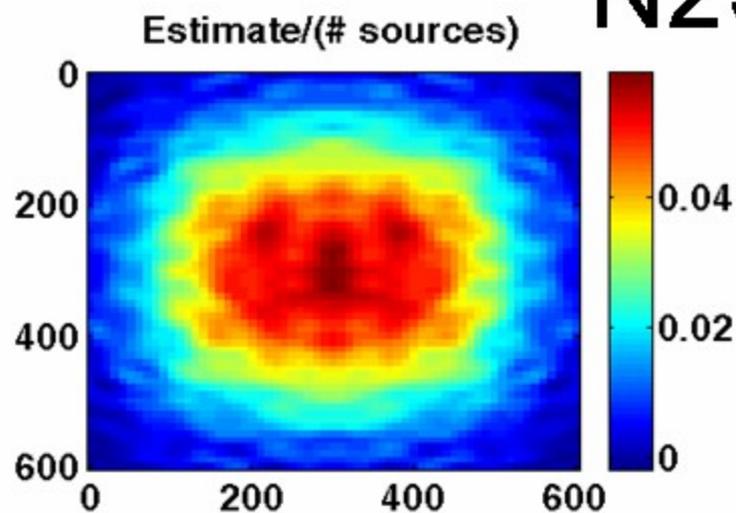
Source/Rec Reduced Geom PP -

N2S=1



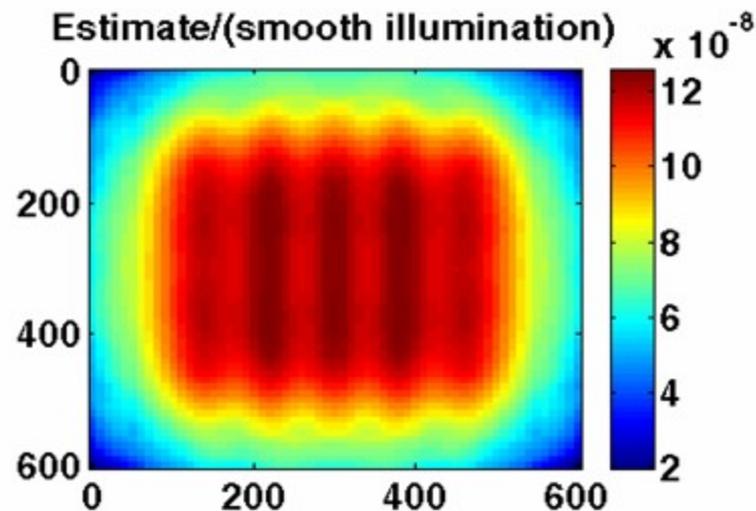
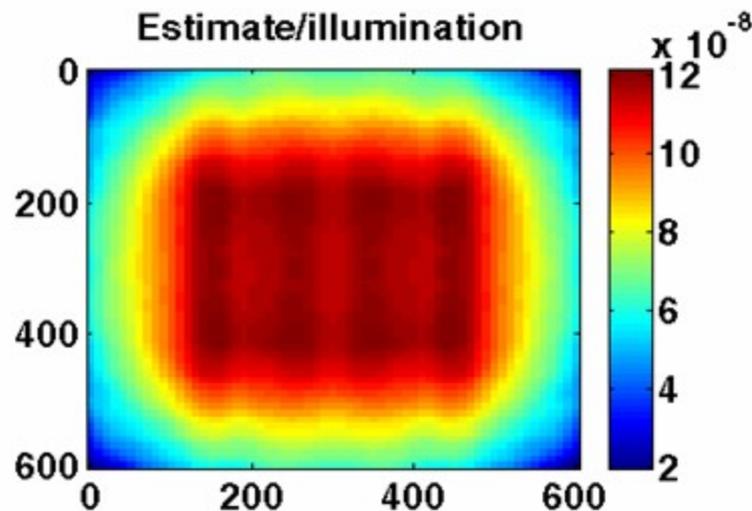
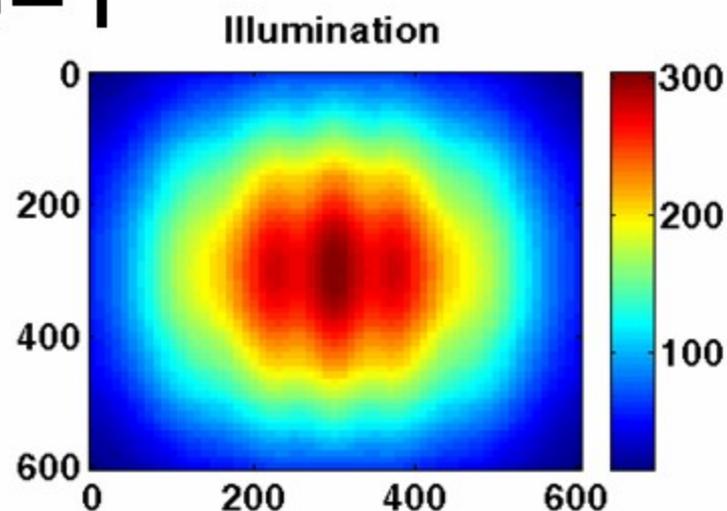
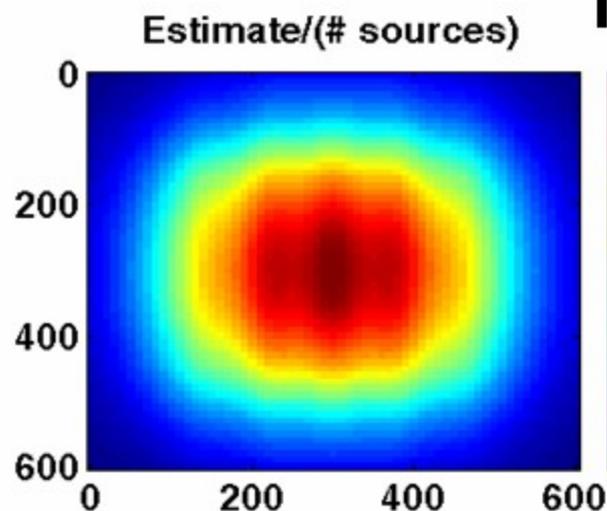
Source/Rec Reduced Geom PP -

N2S=1



Source/Rec Reduced Geom PP -

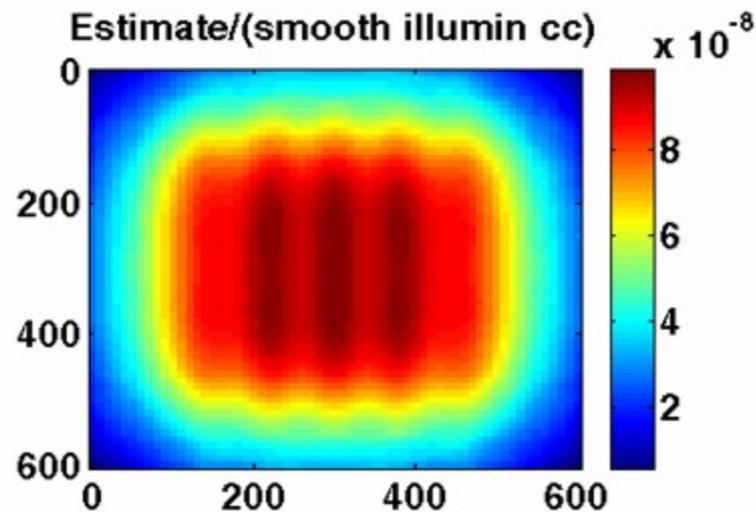
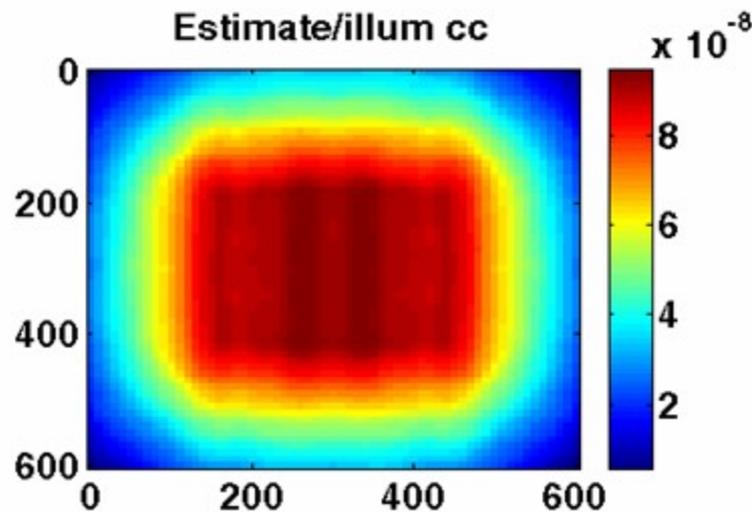
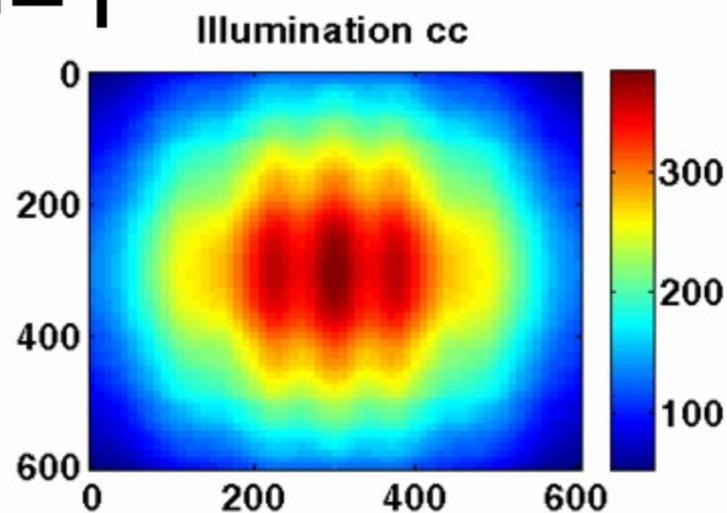
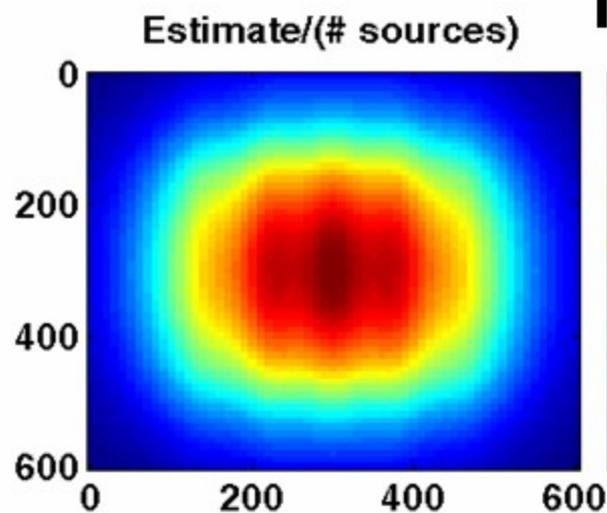
N2S=1



Crosscorrelation imaging condition

Source/Rec Reduced Geom PP -

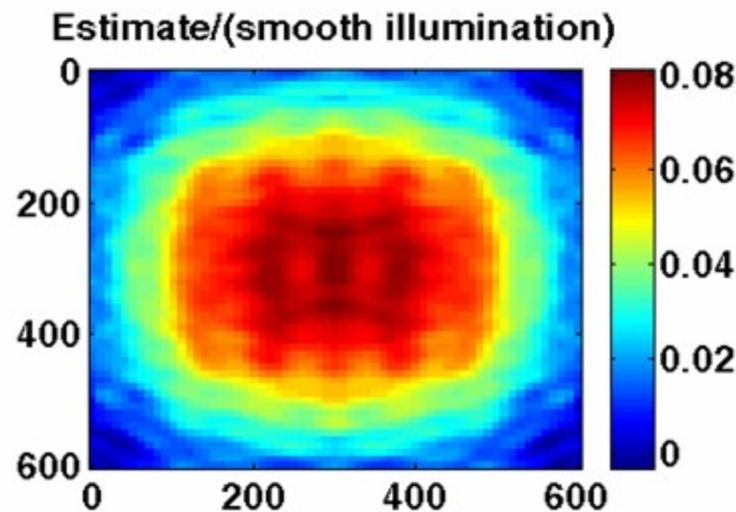
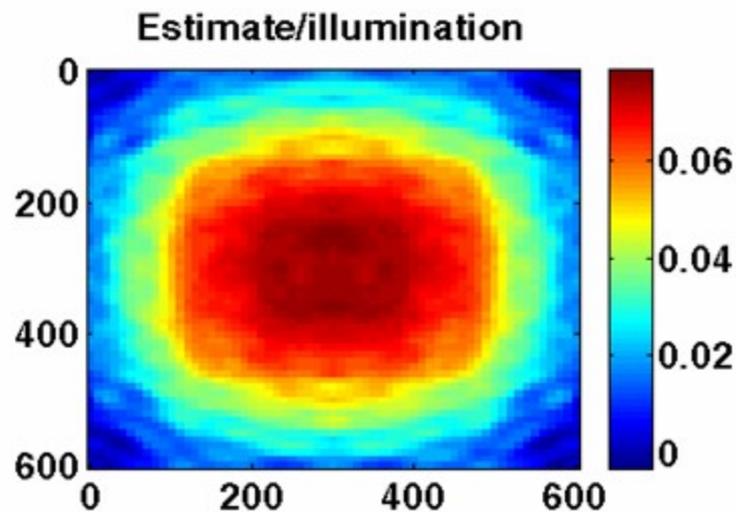
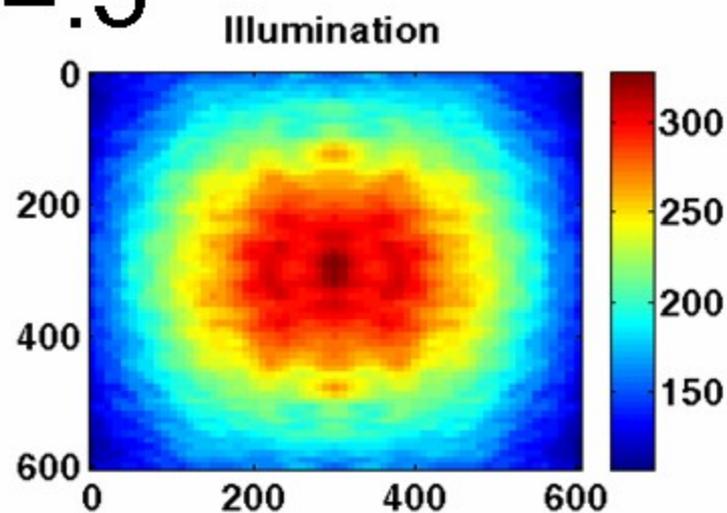
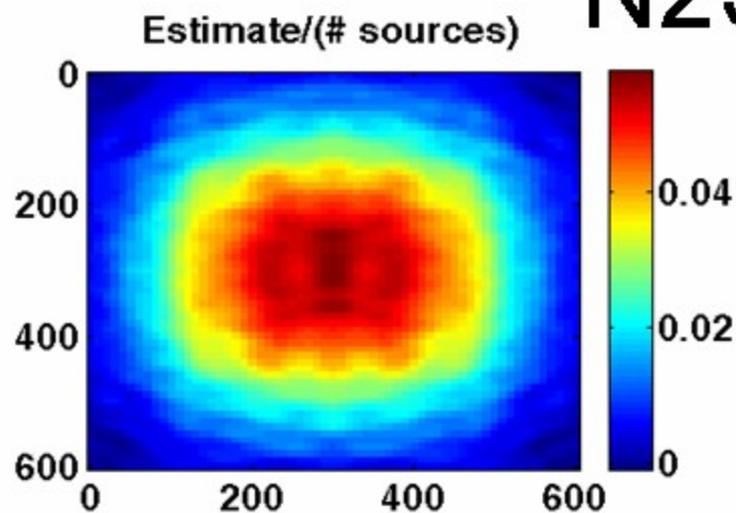
$N2S=1$



Crosscorrelation imaging condition

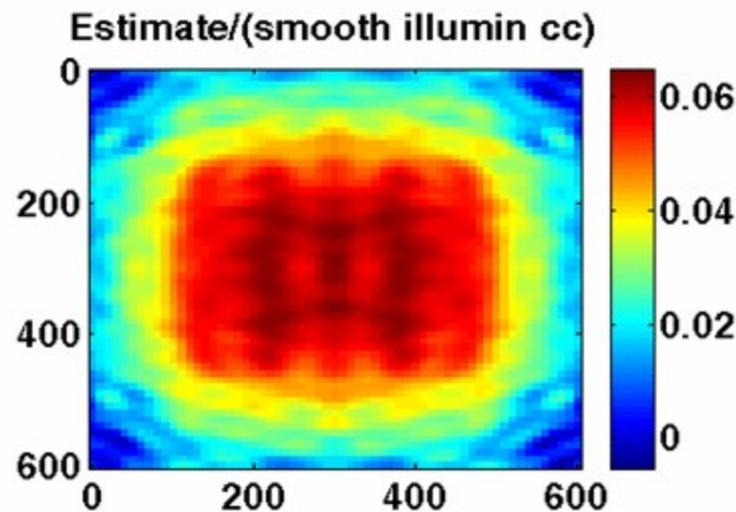
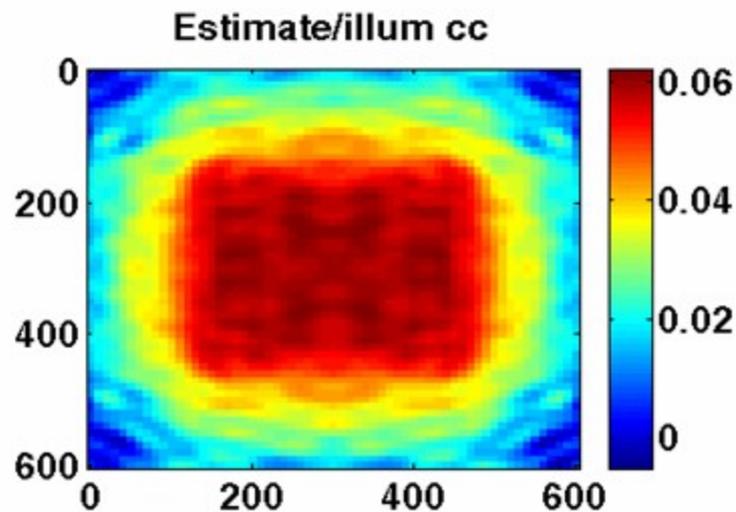
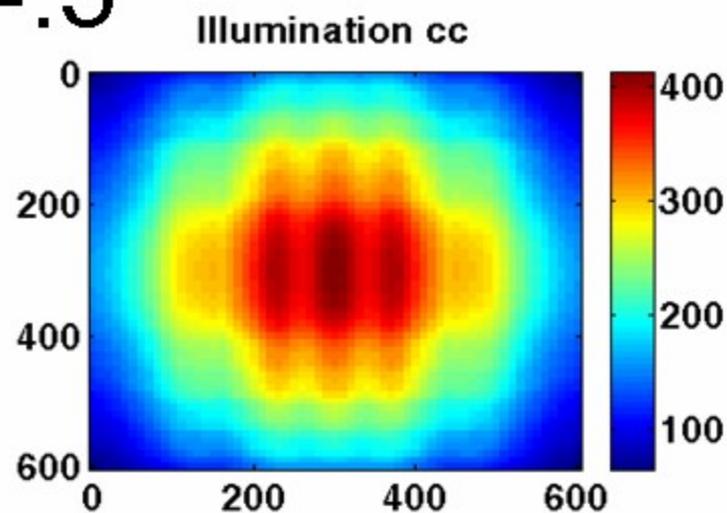
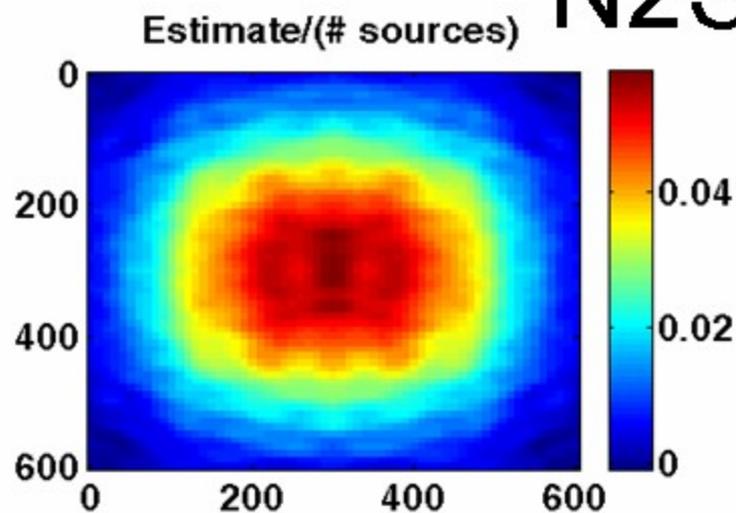
Source/Rec Reduced Geom PP -

N2S=.5

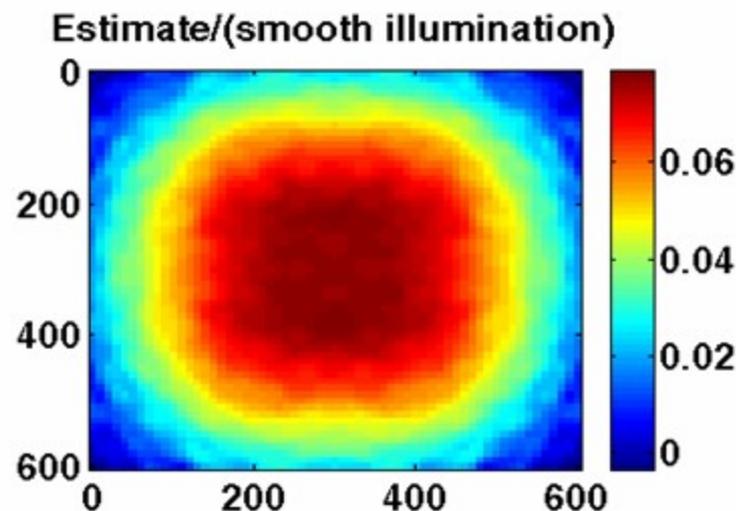
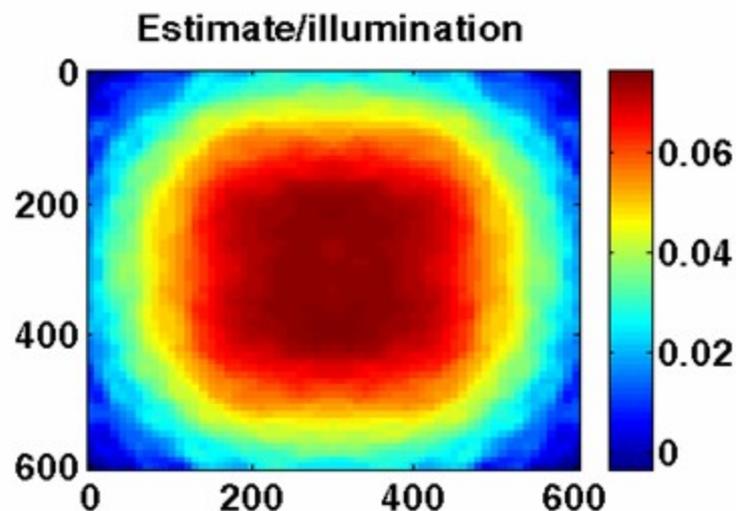
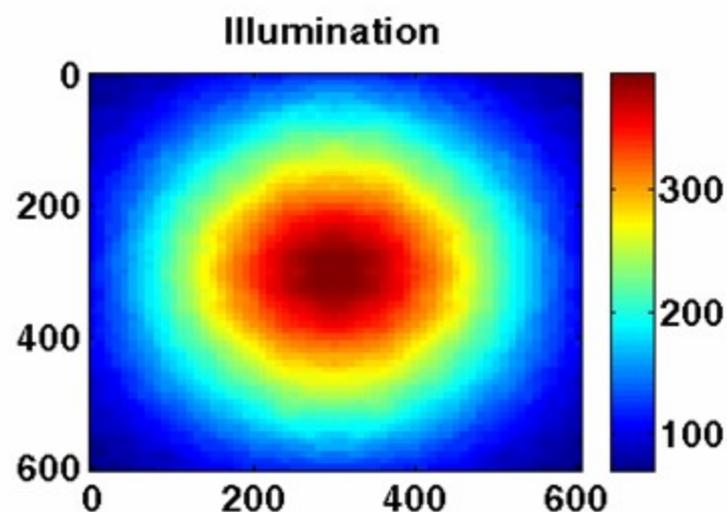
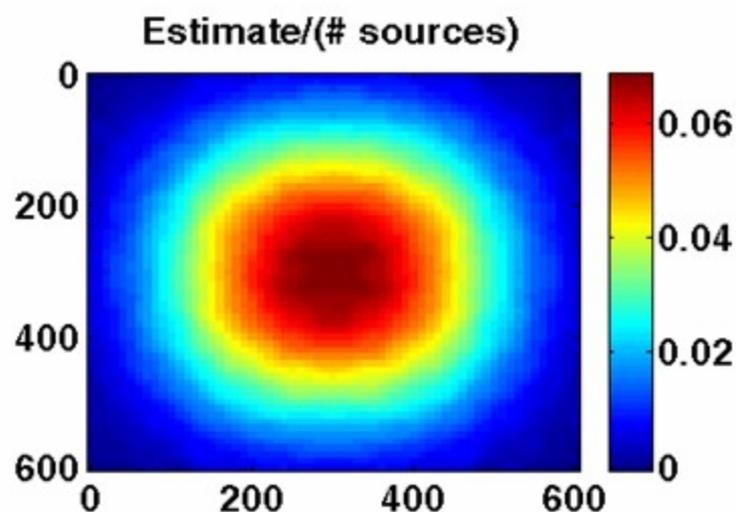


Source/Rec Reduced Geom PP -

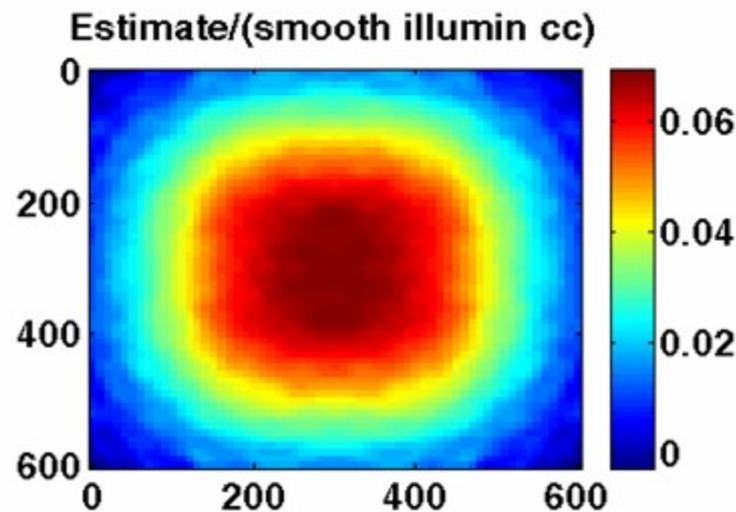
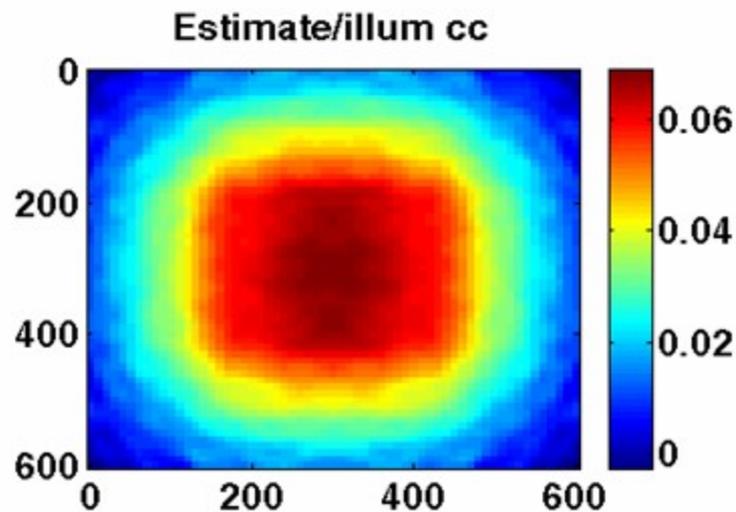
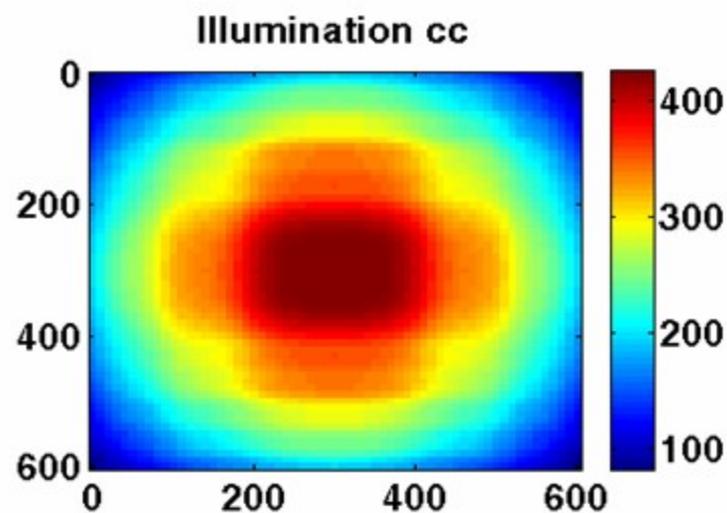
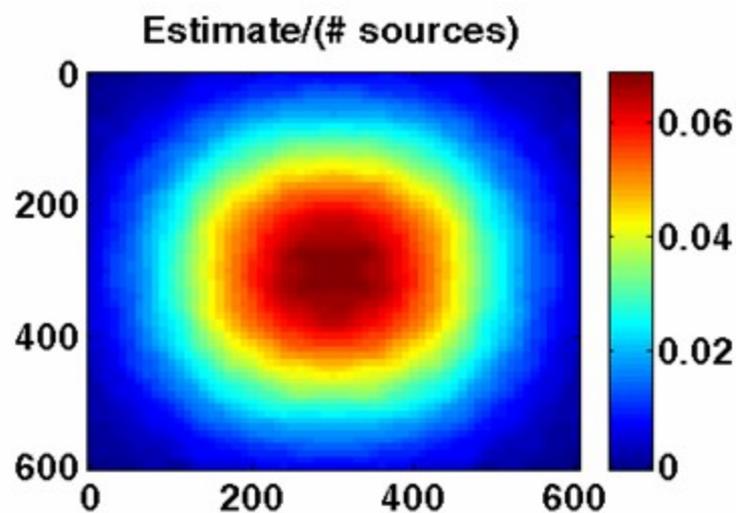
N2S=.5



Source Reduced Geom PP -N2S=.5

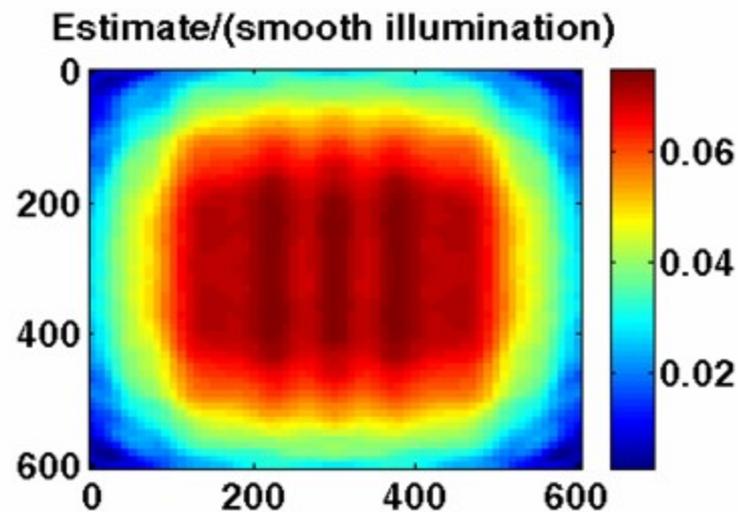
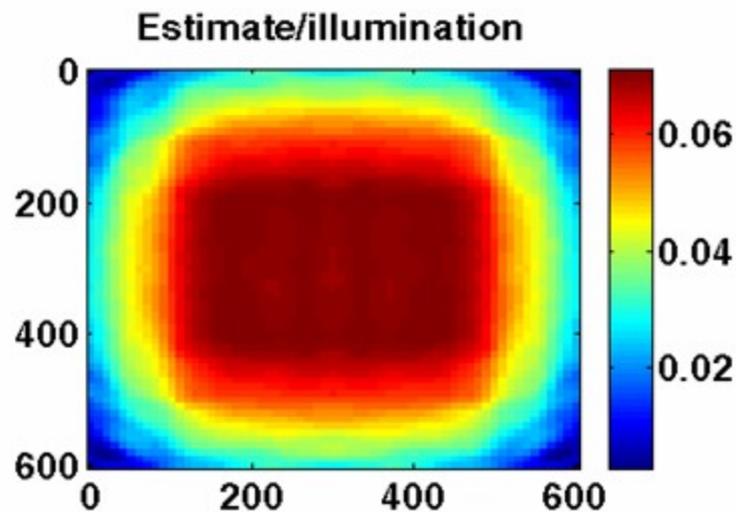
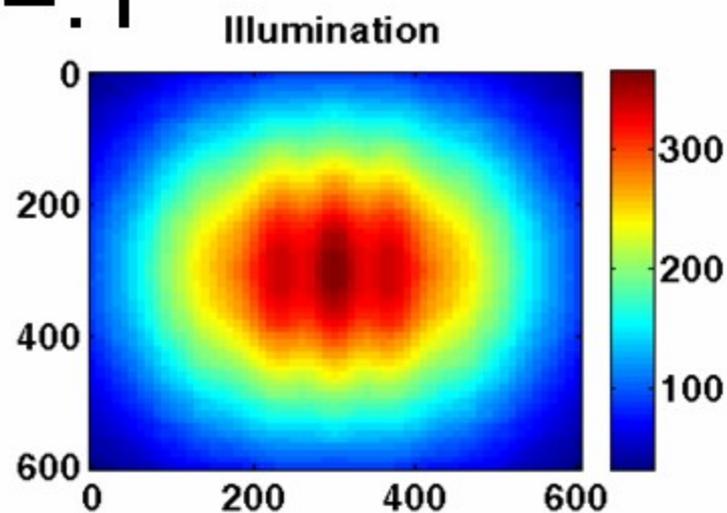
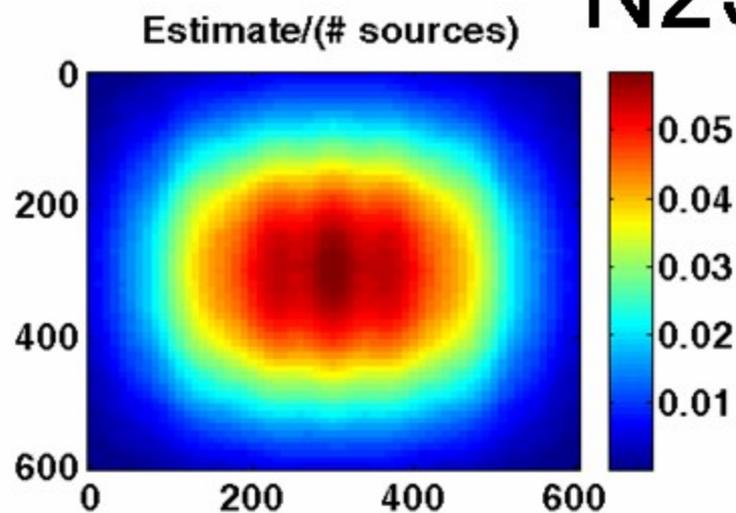


Source Reduced Geom PP -N2S=.5



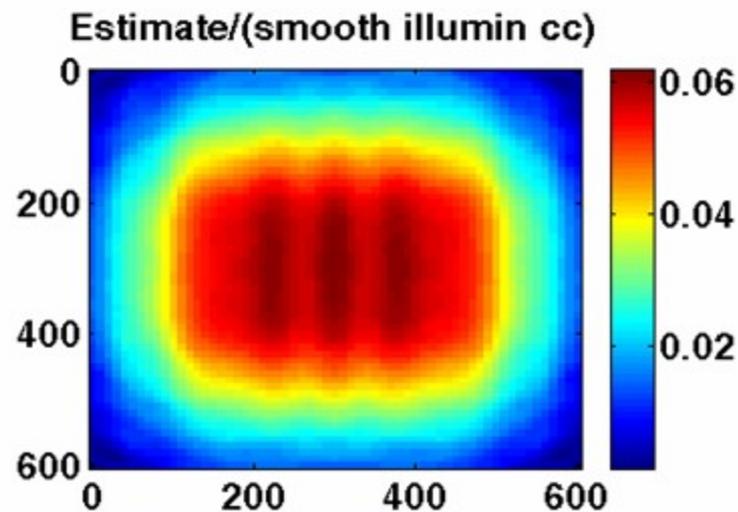
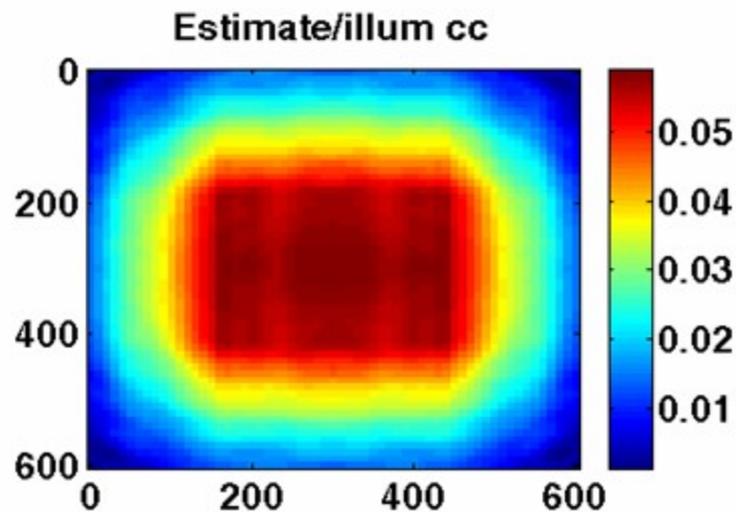
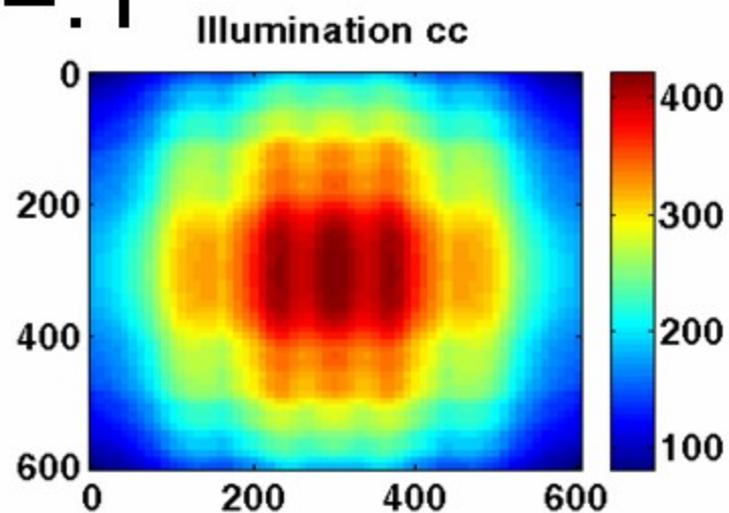
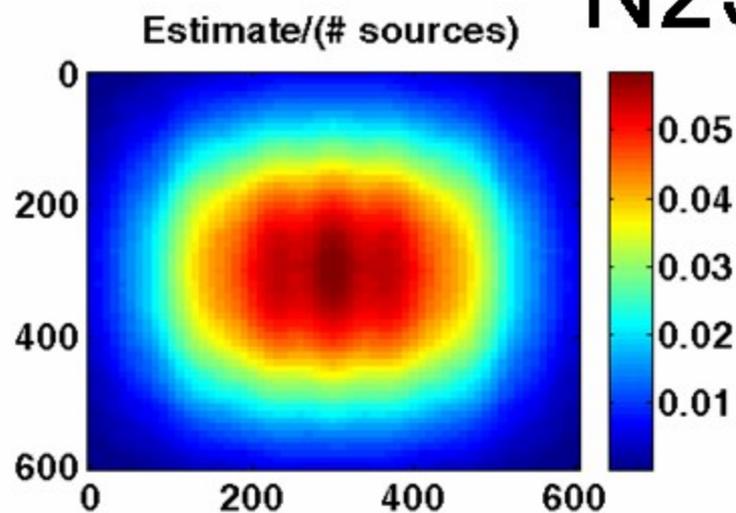
Source/Rec Reduced Geom PP -

N2S=.1



Source/Rec Reduced Geom PP -

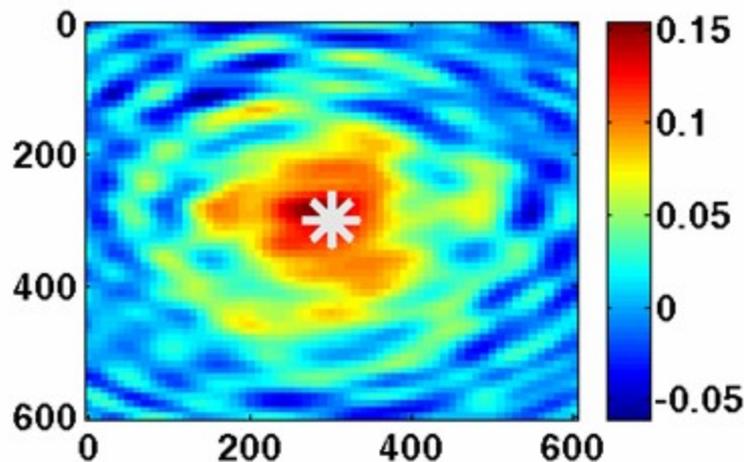
$N2S=.1$



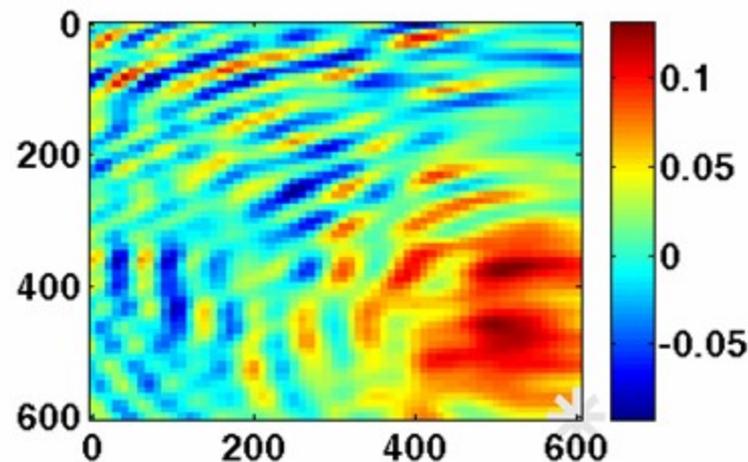
Source/Rec Reduced Geom PP -

N2S=.5

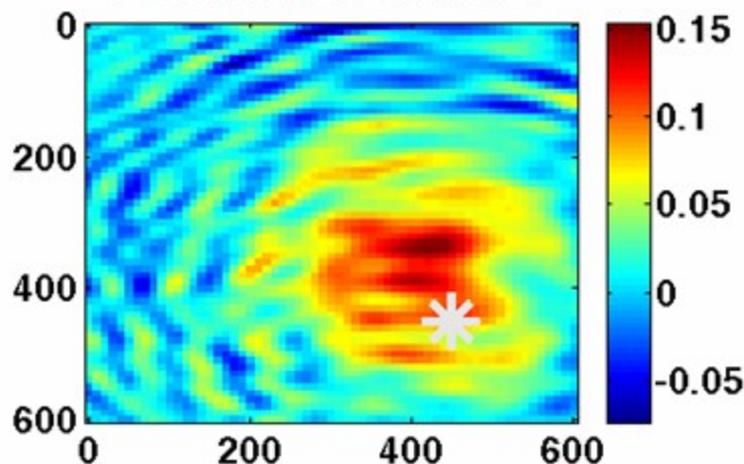
Loc 1: Refl 10-60Hz z=r



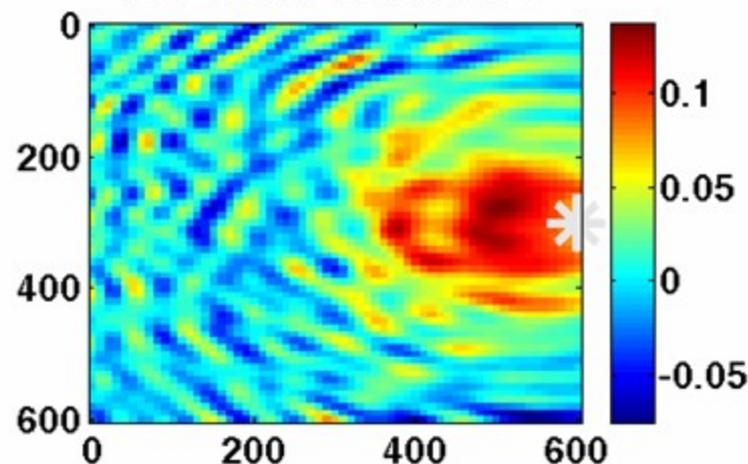
Loc 2: Refl 10-60Hz z=r



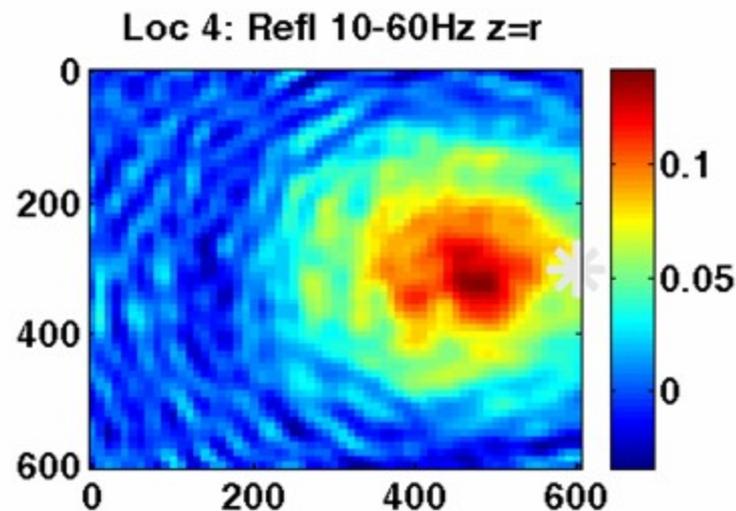
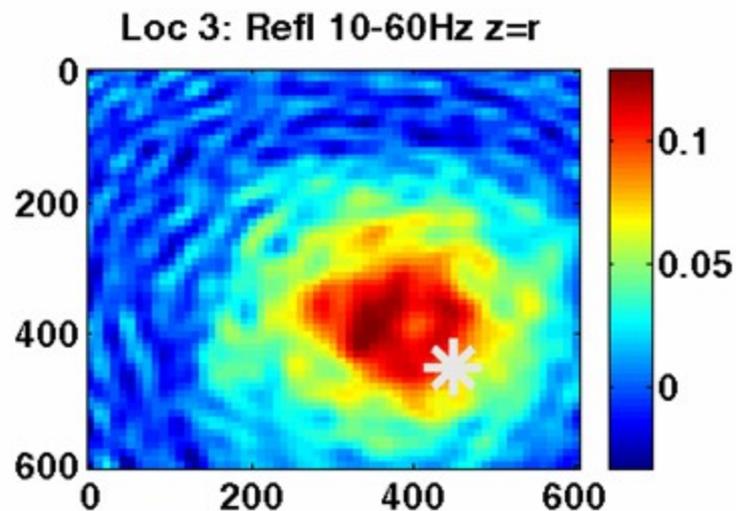
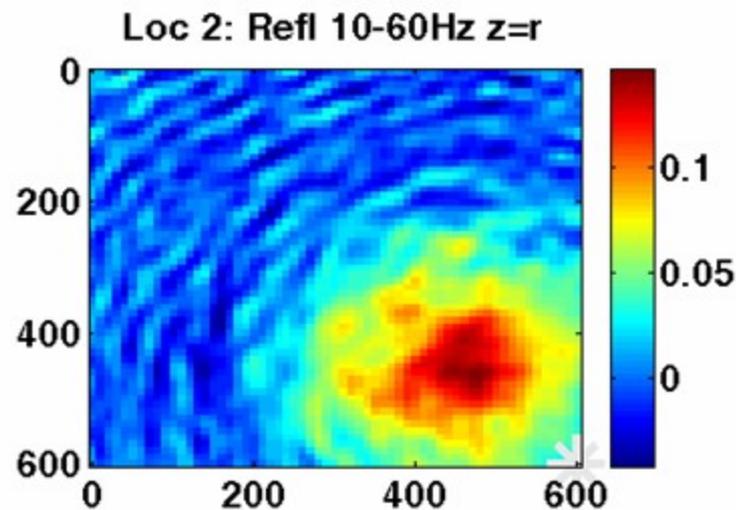
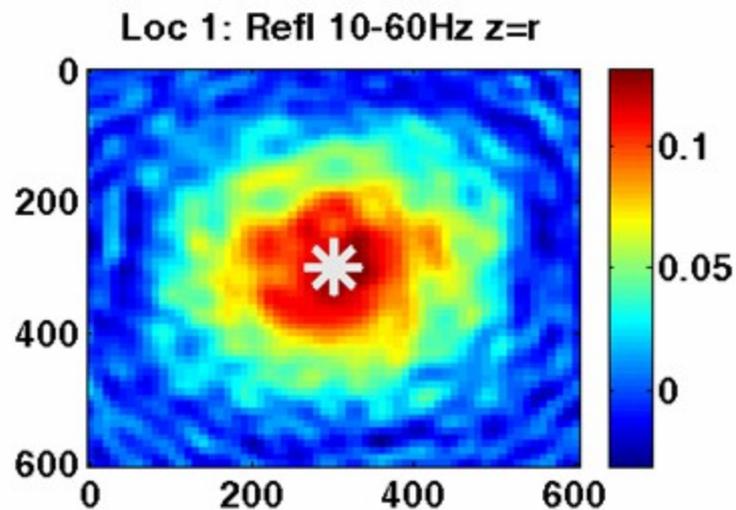
Loc 3: Refl 10-60Hz z=r



Loc 4: Refl 10-60Hz z=r



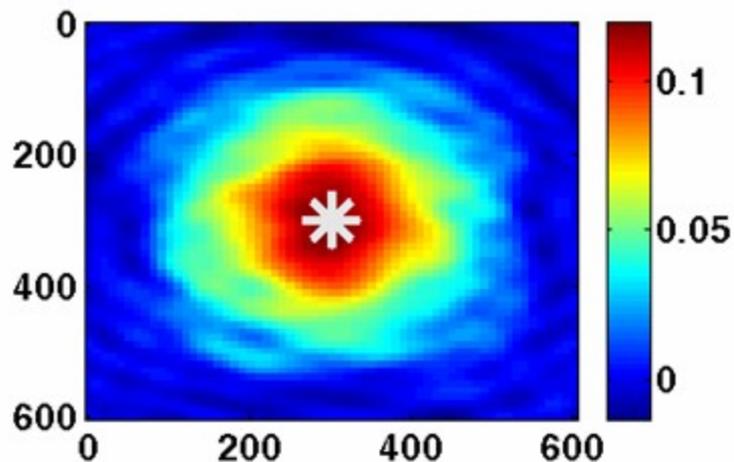
Source Reduced Geom PP -N2S=.5



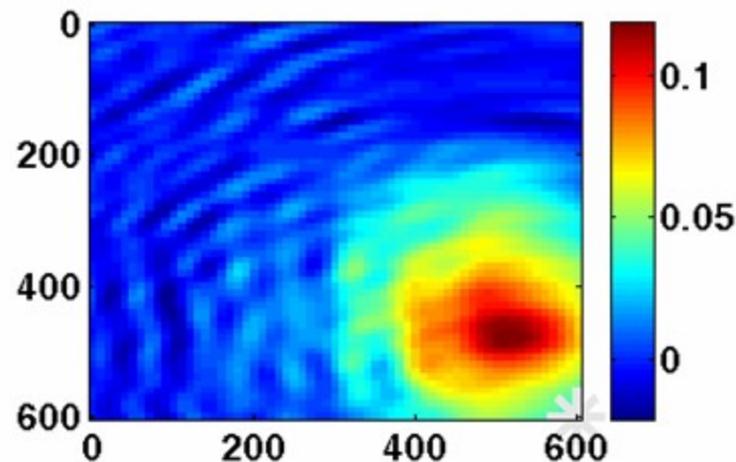
Source/Rec Reduced Geom PP -

$N2S = .1$

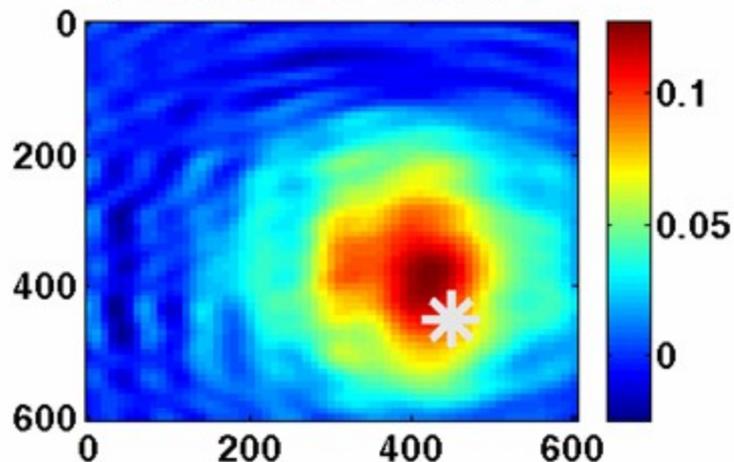
Loc 1: Refl 10-60Hz z=r



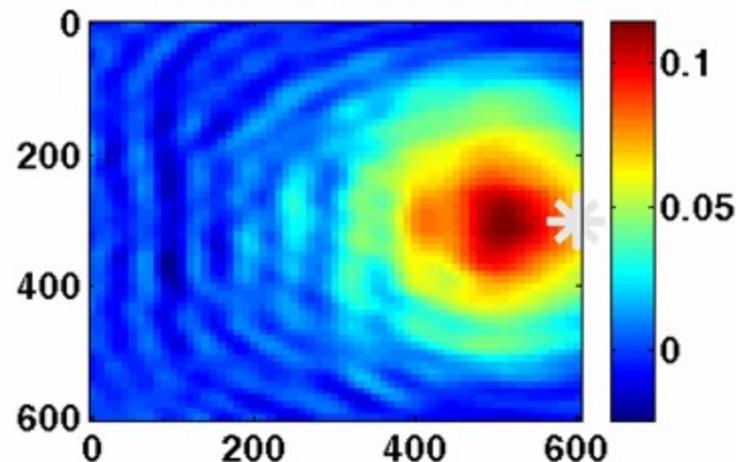
Loc 2: Refl 10-60Hz z=r



Loc 3: Refl 10-60Hz z=r



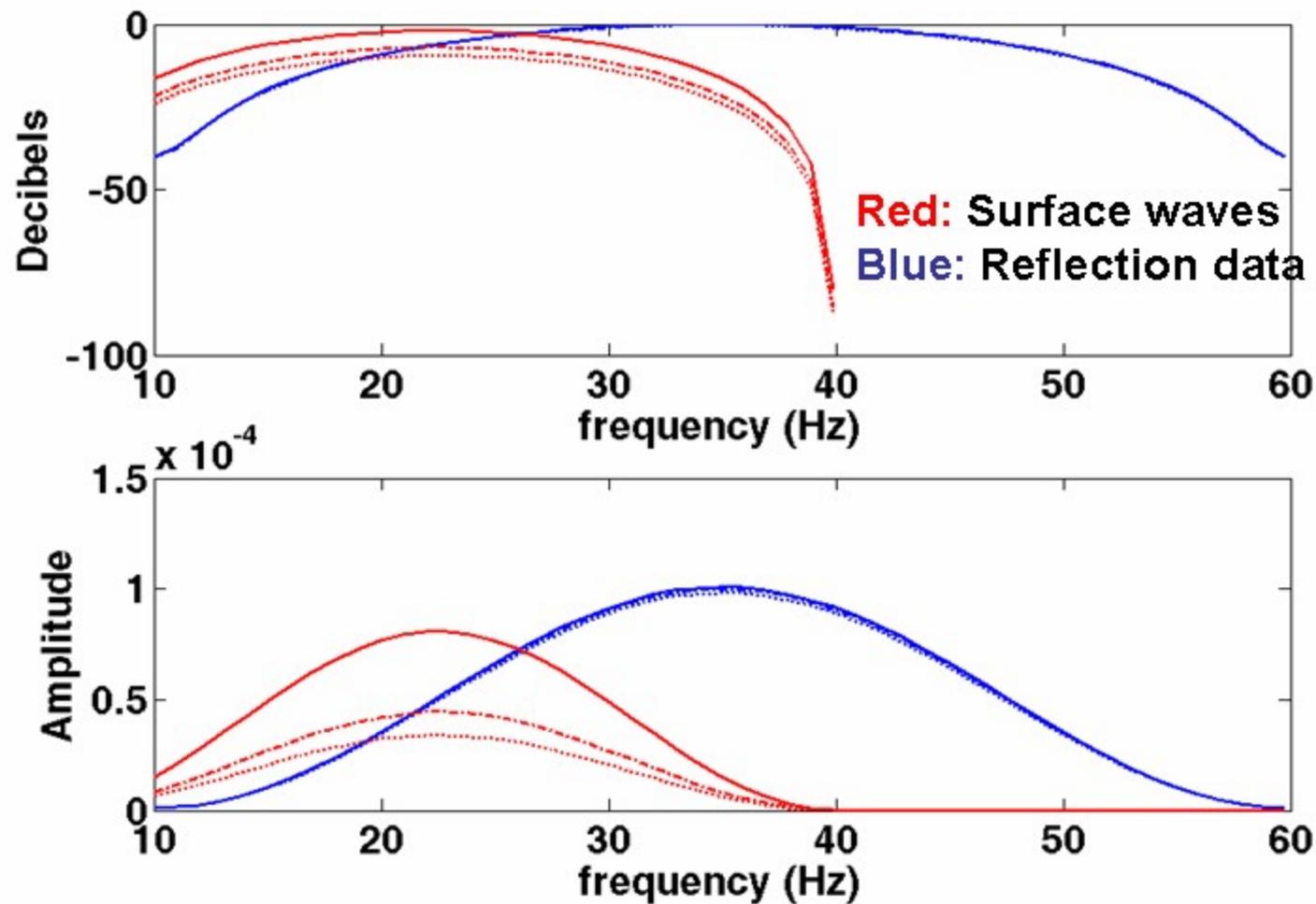
Loc 4: Refl 10-60Hz z=r



Surface Waves

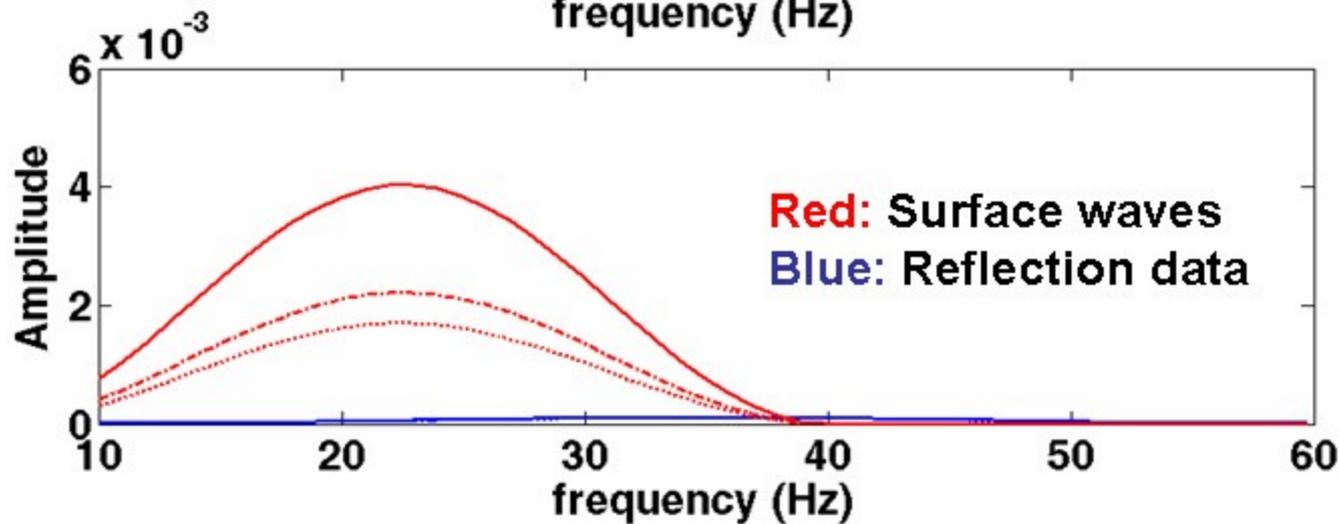
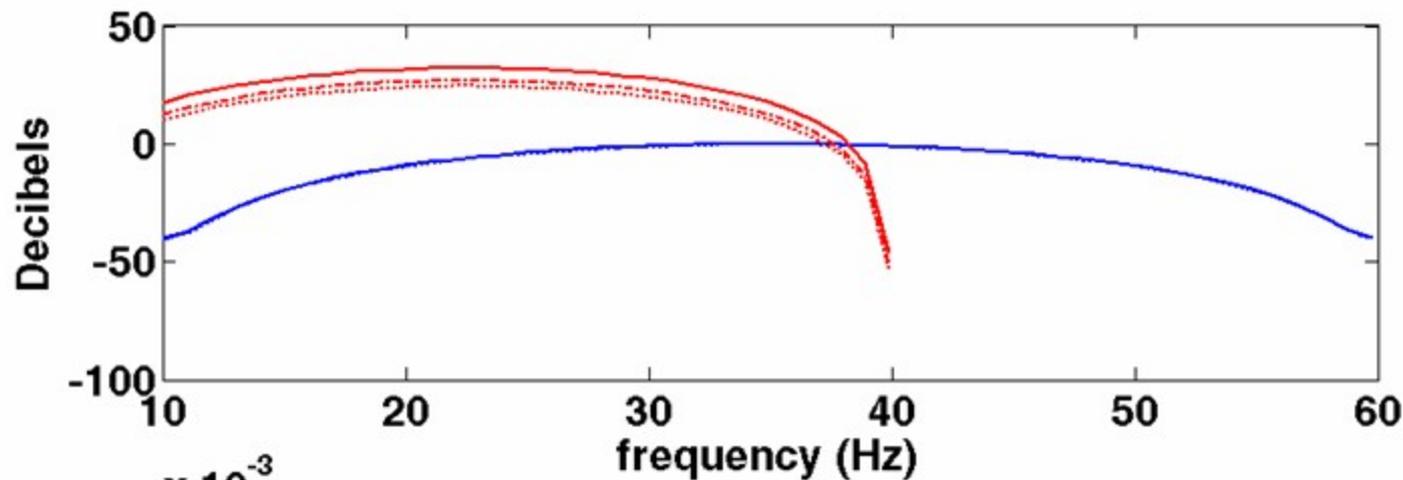
Surface Wave Power

$$a_{\text{relsurf}} = 1 \quad f_{\text{ref}} = 25, \quad h_{\text{ref}} = 50$$

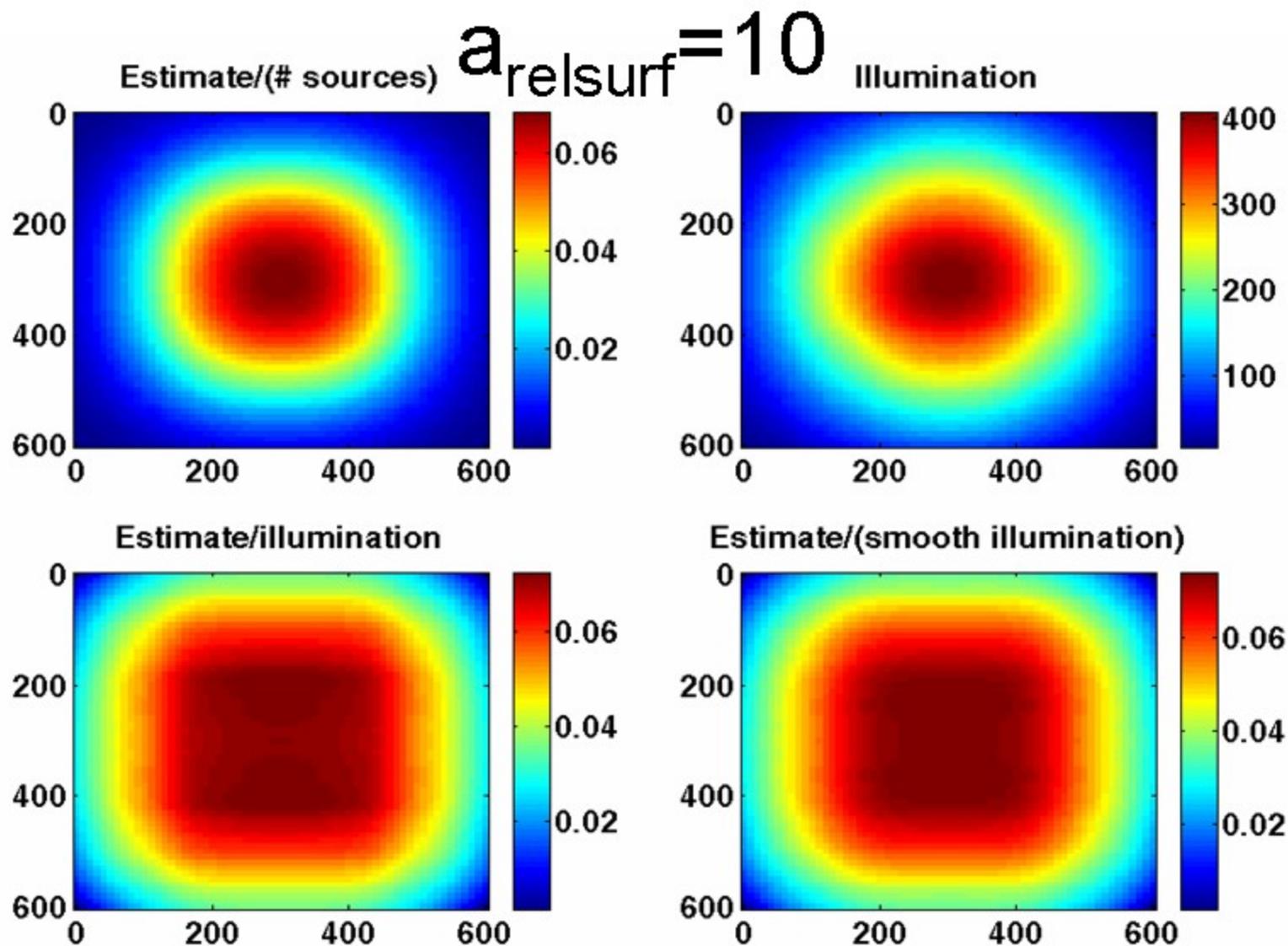


Surface Wave Power

$$a_{\text{relsurf}} = 50 \quad f_{\text{ref}} = 25, \quad h_{\text{ref}} = 50$$

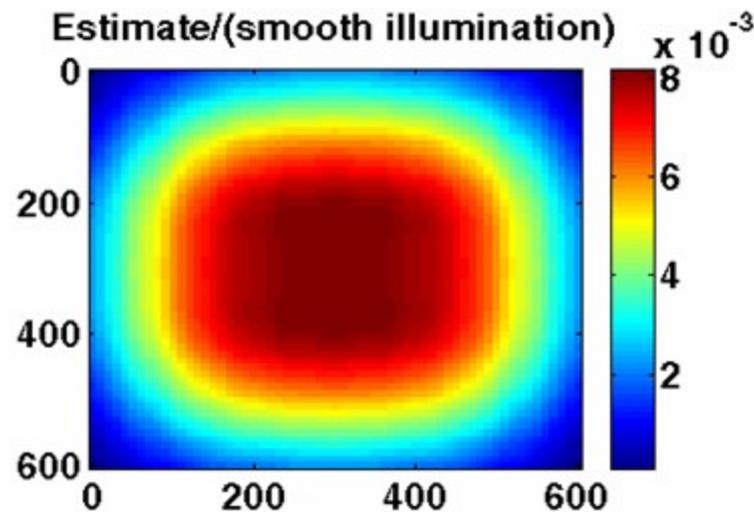
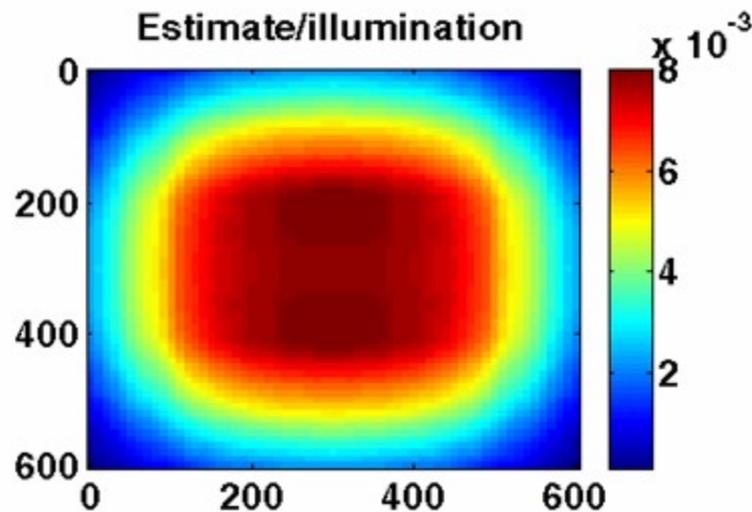
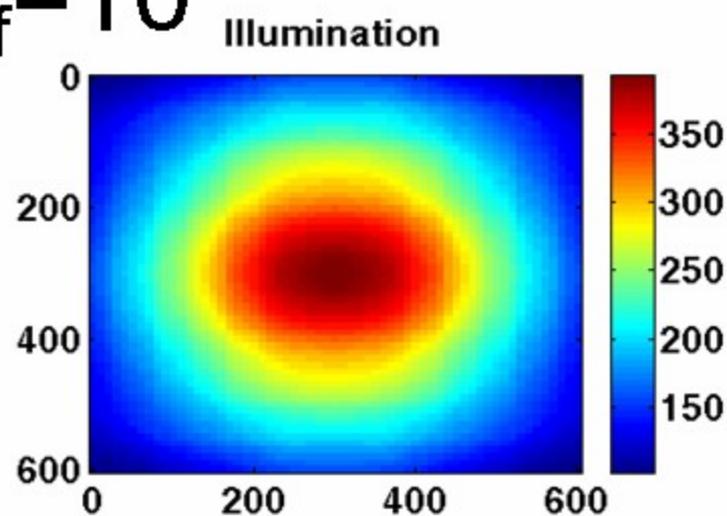
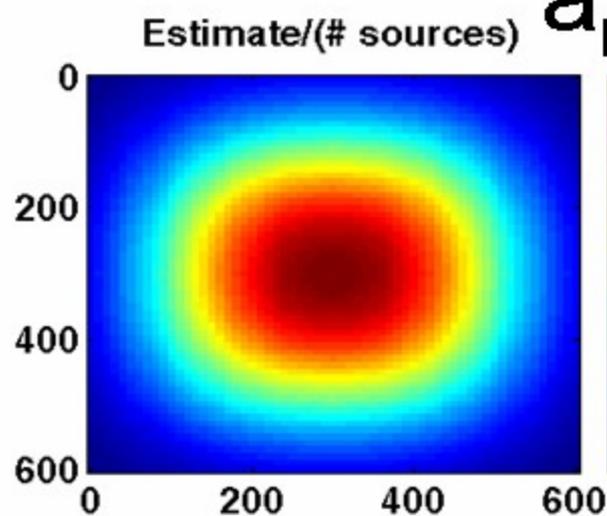


Source Reduced Geom PP

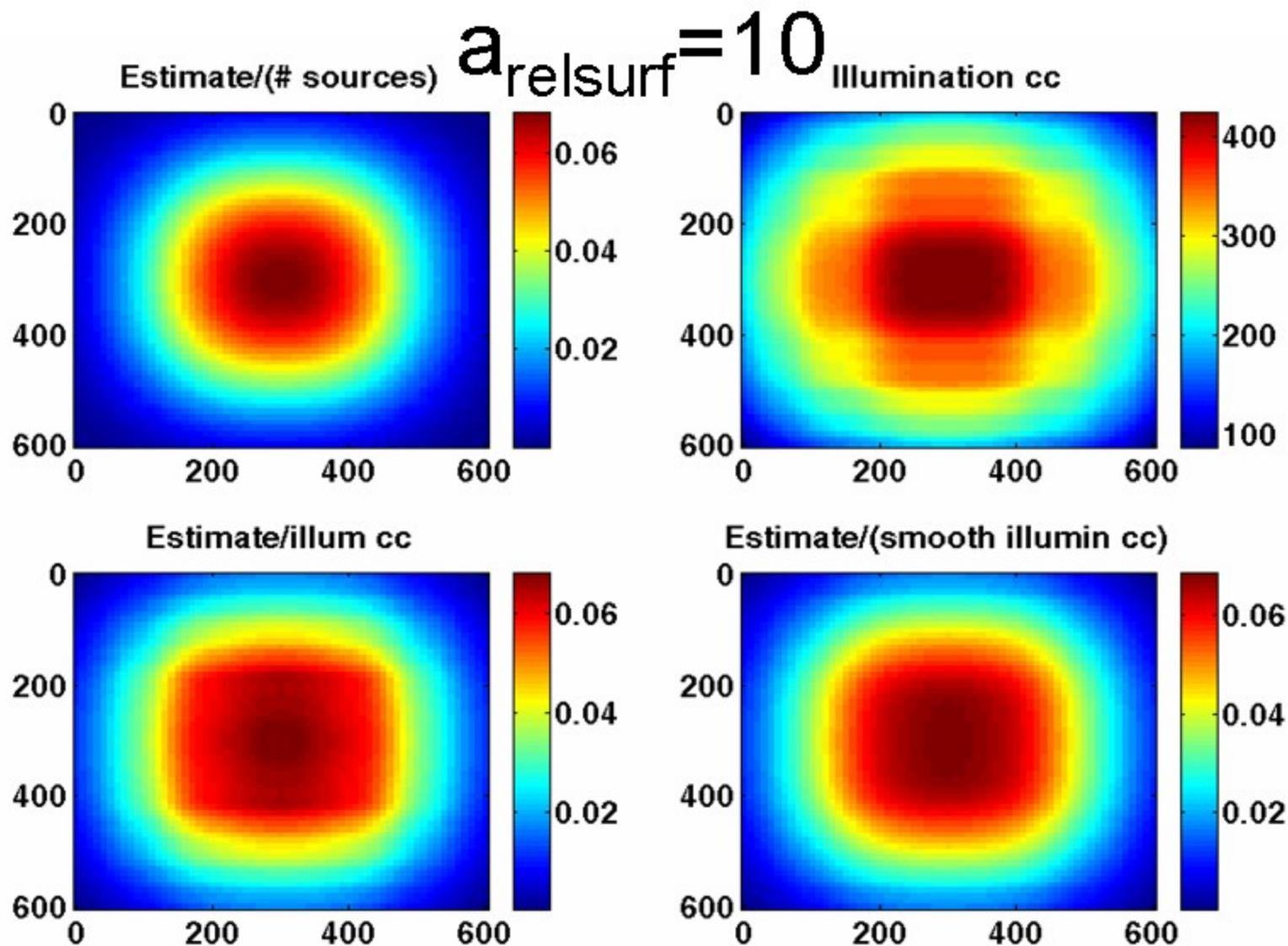


Source/Rec Reduced Geom PP

$a_{\text{rel surf}} = 10$

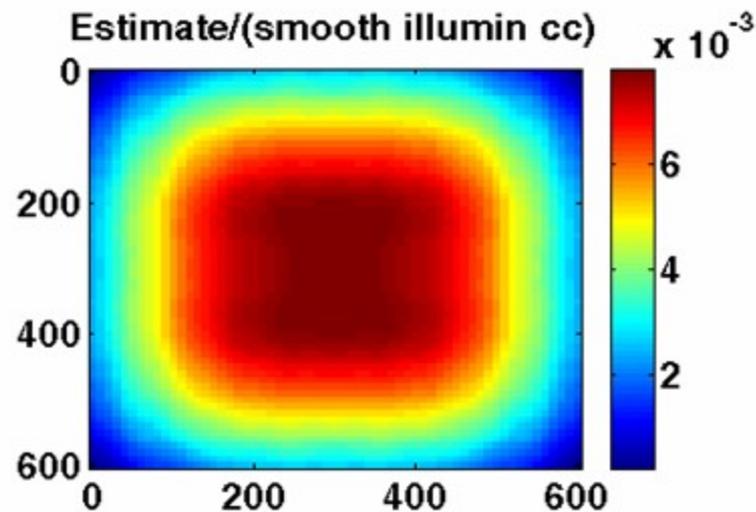
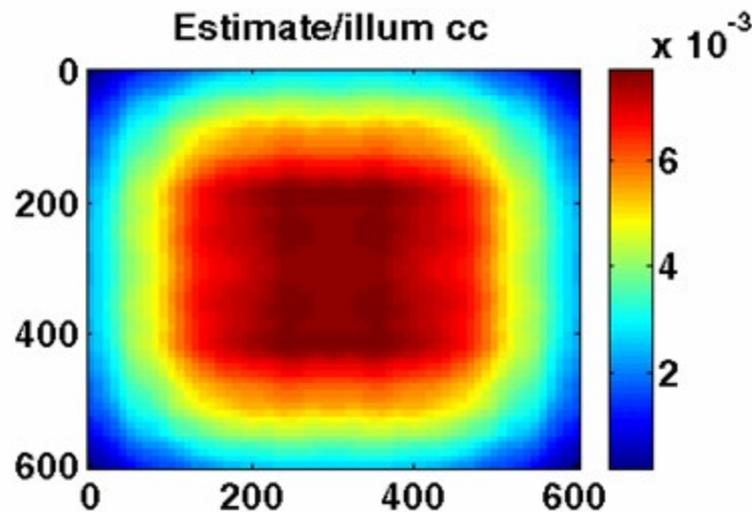
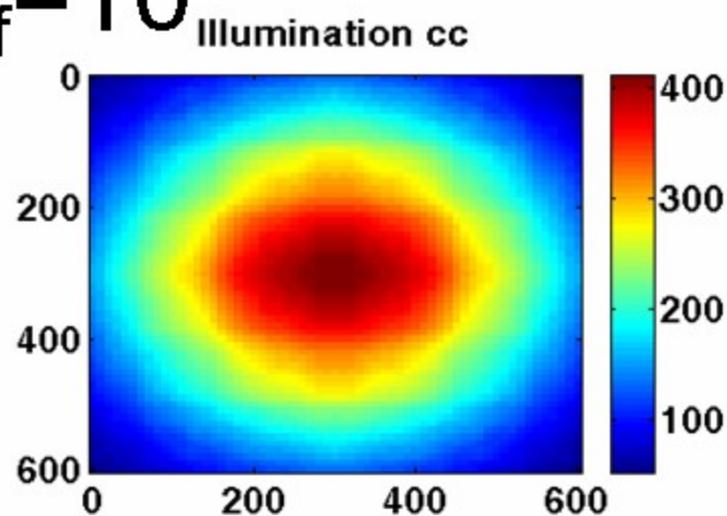
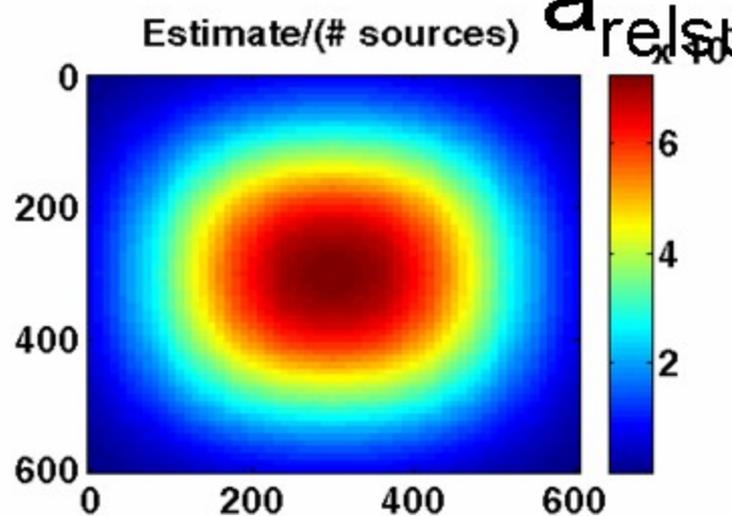


Source Reduced Geom PP

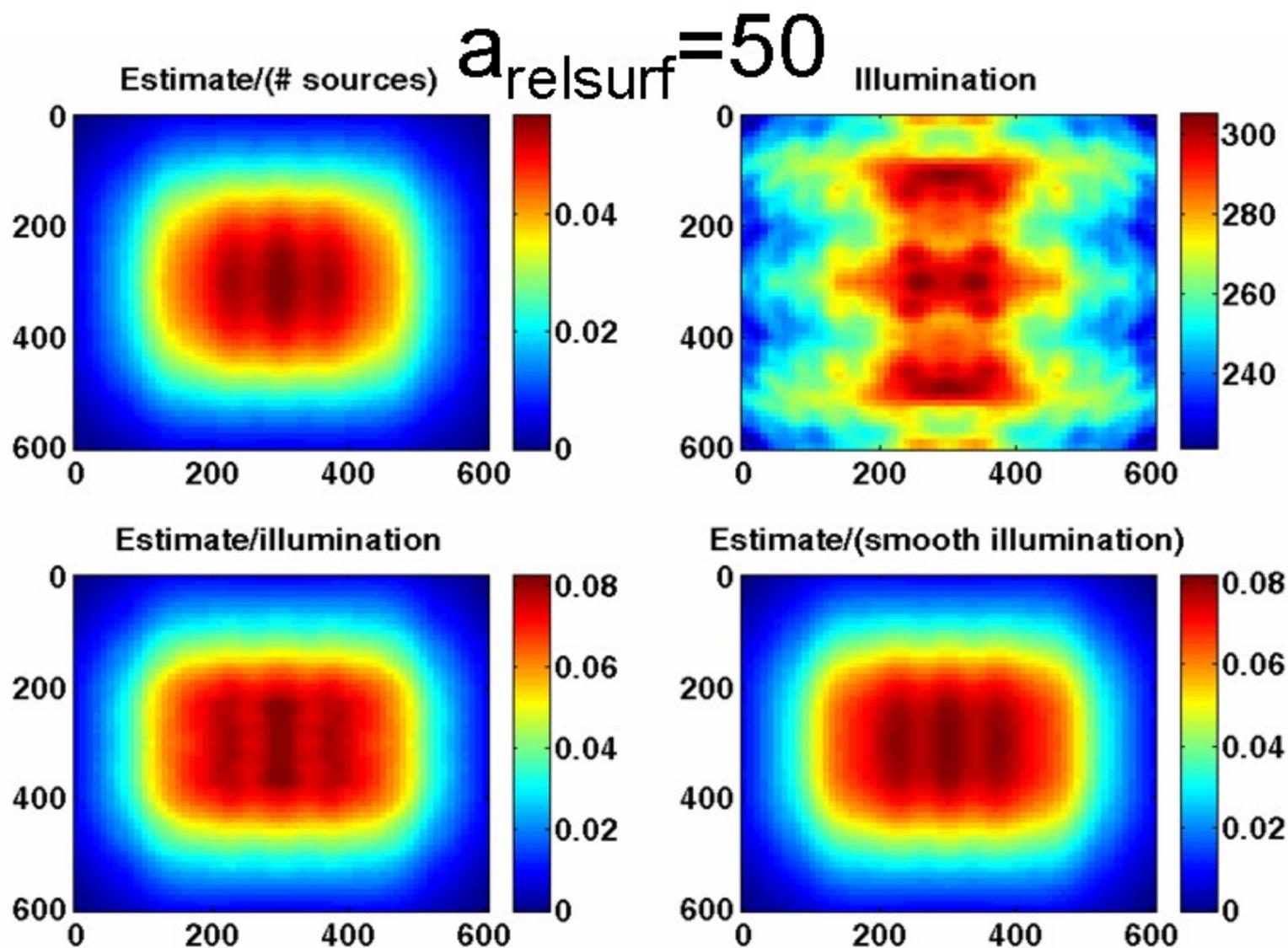


Source/Rec Reduced Geom PP

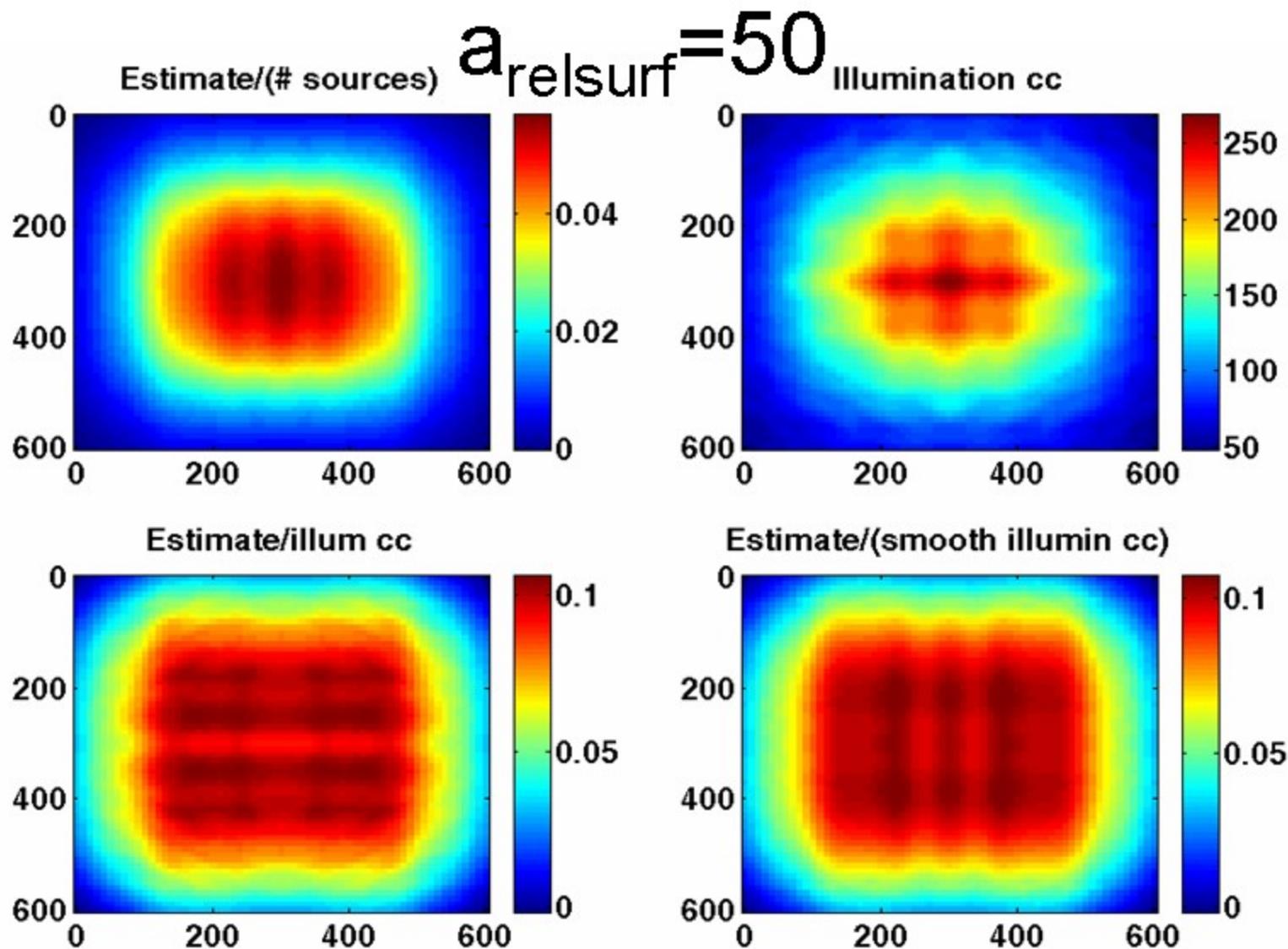
$a_{\text{rel surf}} = 10$



Source/Rec Reduced Geom PP

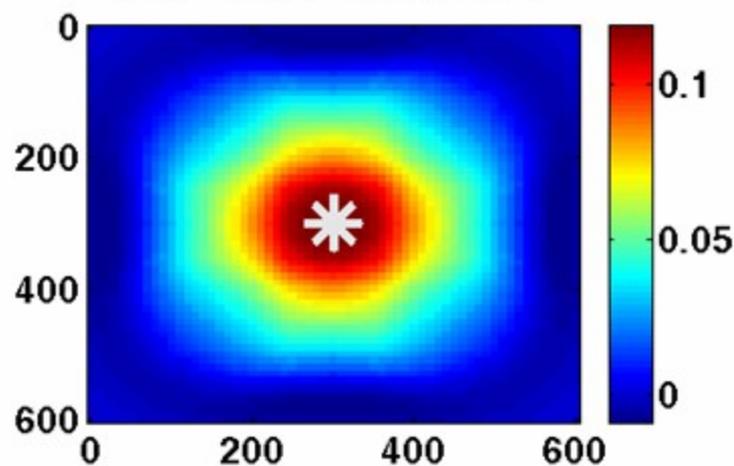


Source/Rec Reduced Geom PP

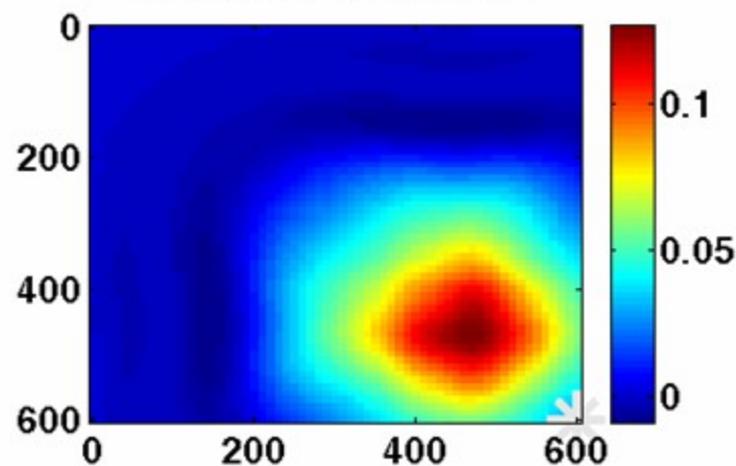


Full receiver geom, PP, $a_{\text{relsurf}}=0$

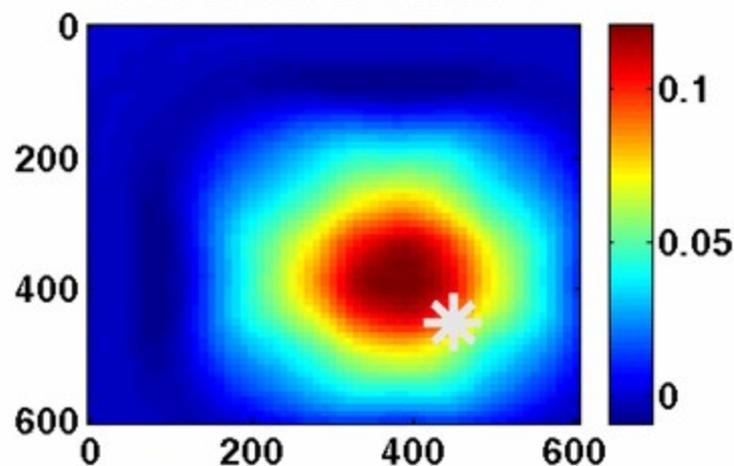
Loc 1: Refl 10-60Hz z=r



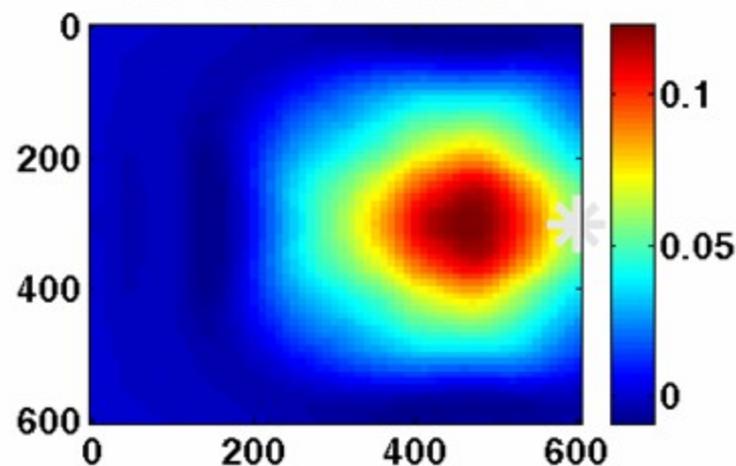
Loc 2: Refl 10-60Hz z=r



Loc 3: Refl 10-60Hz z=r

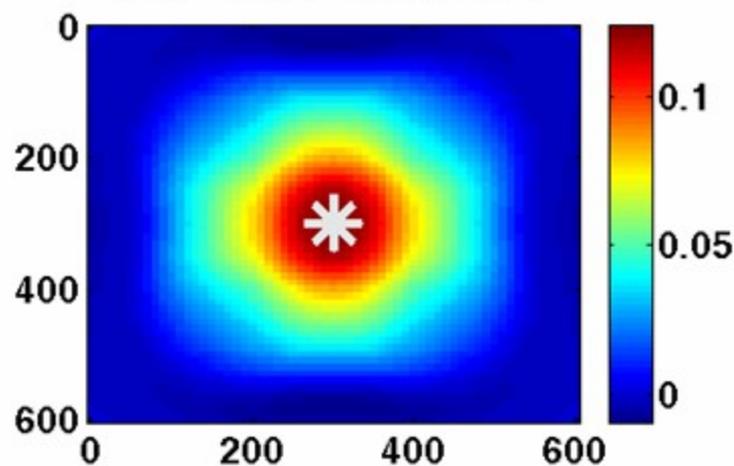


Loc 4: Refl 10-60Hz z=r

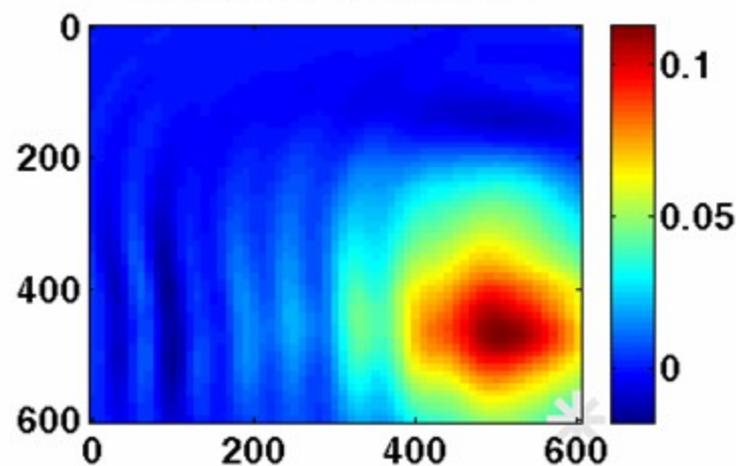


Receiver red. geom, PP, $a_{\text{relsurf}}=0$

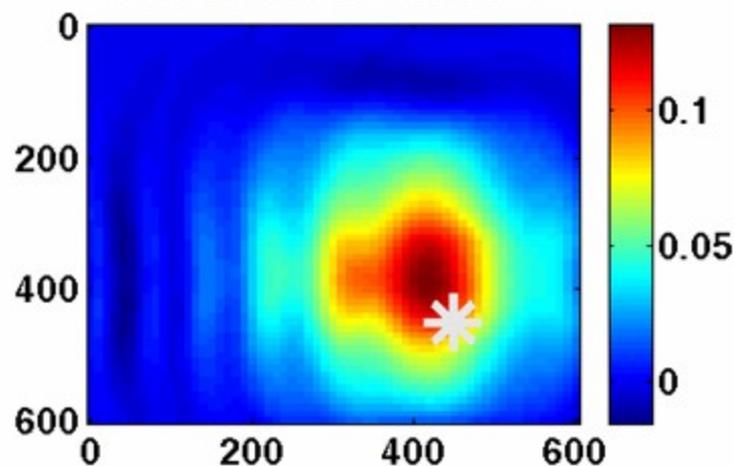
Loc 1: Refl 10-60Hz z=r



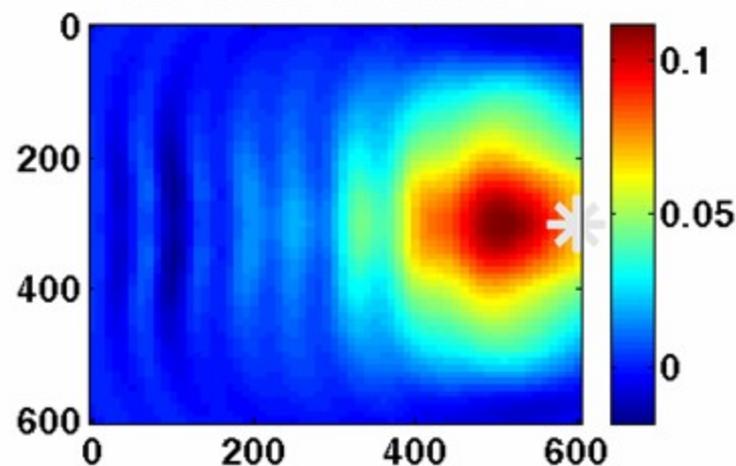
Loc 2: Refl 10-60Hz z=r



Loc 3: Refl 10-60Hz z=r

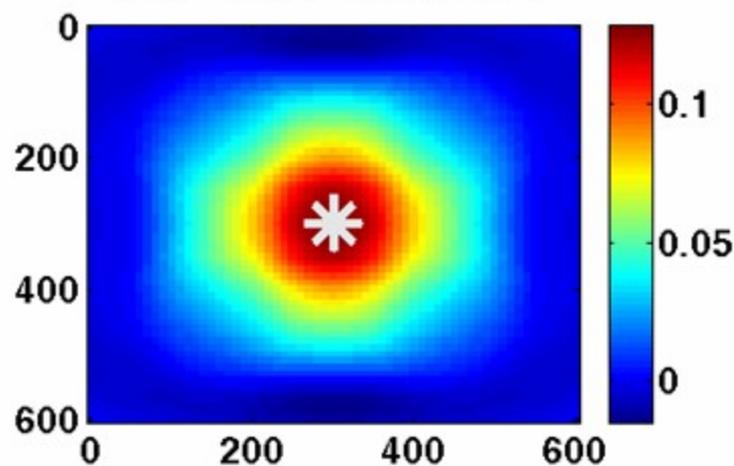


Loc 4: Refl 10-60Hz z=r

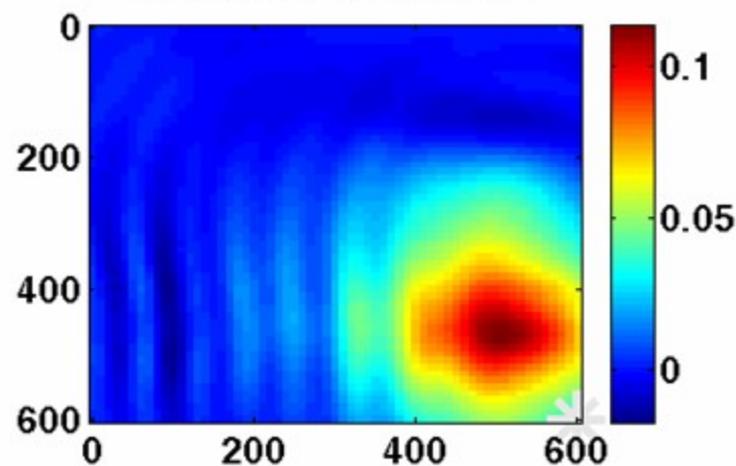


Receiver red. geom, PP, $a_{\text{relsurf}}=1$

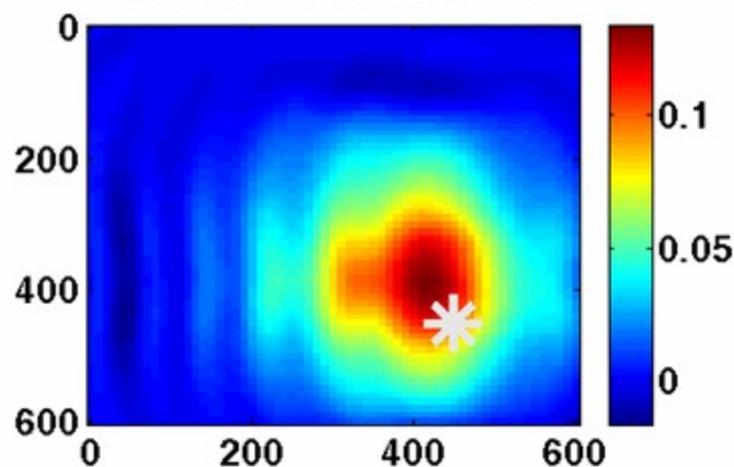
Loc 1: Refl 10-60Hz z=r



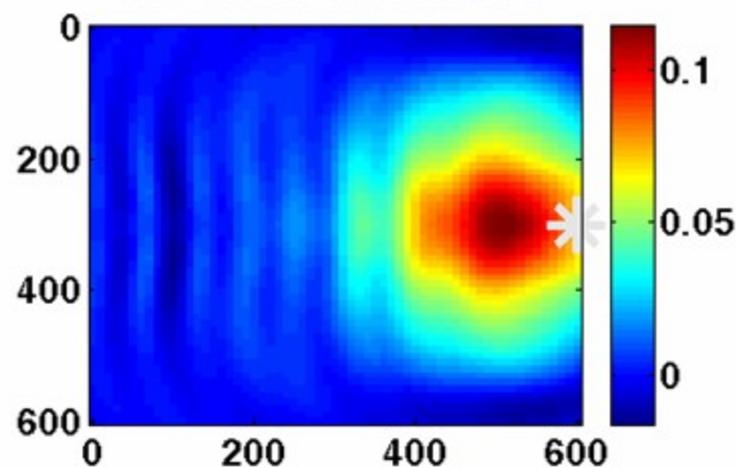
Loc 2: Refl 10-60Hz z=r



Loc 3: Refl 10-60Hz z=r

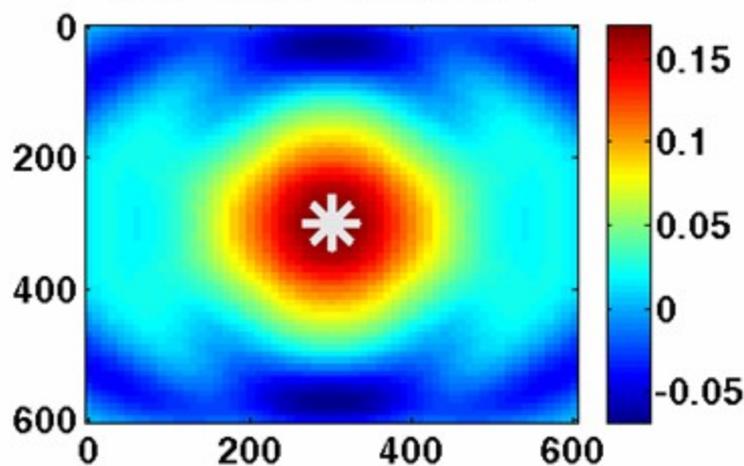


Loc 4: Refl 10-60Hz z=r

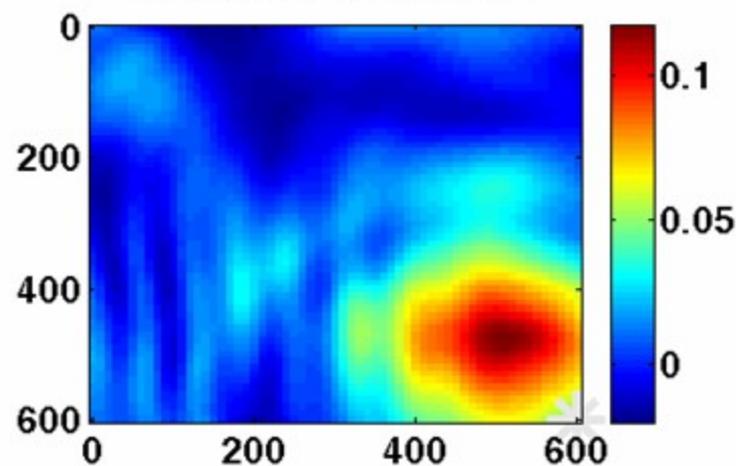


Receiver red. geom, PP, $a_{\text{relsurf}}=10$

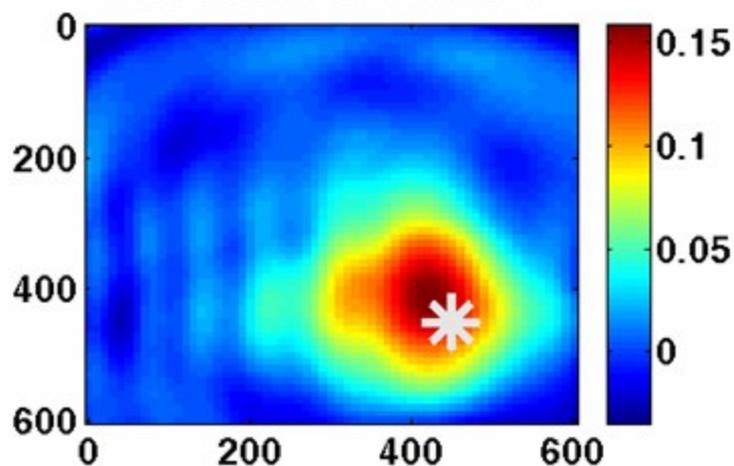
Loc 1: Refl 10-60Hz z=r



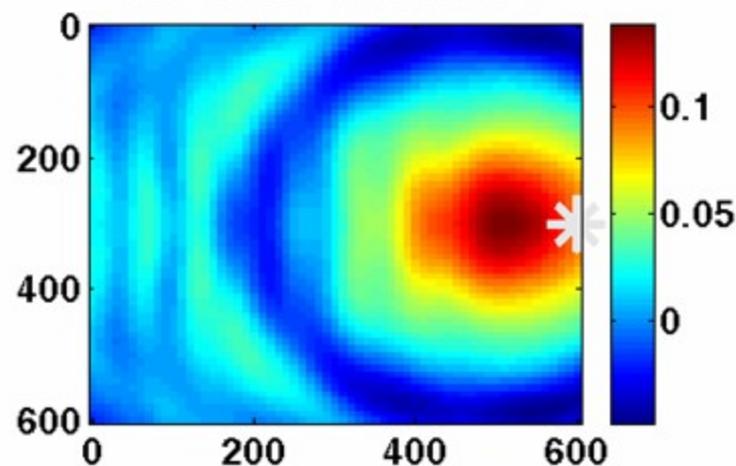
Loc 2: Refl 10-60Hz z=r



Loc 3: Refl 10-60Hz z=r

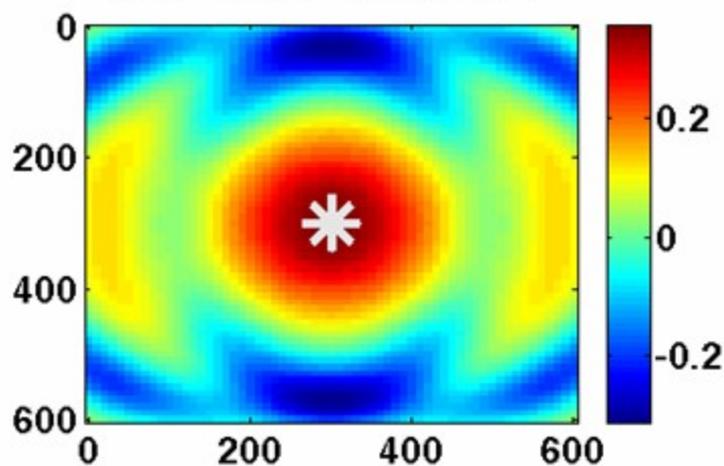


Loc 4: Refl 10-60Hz z=r

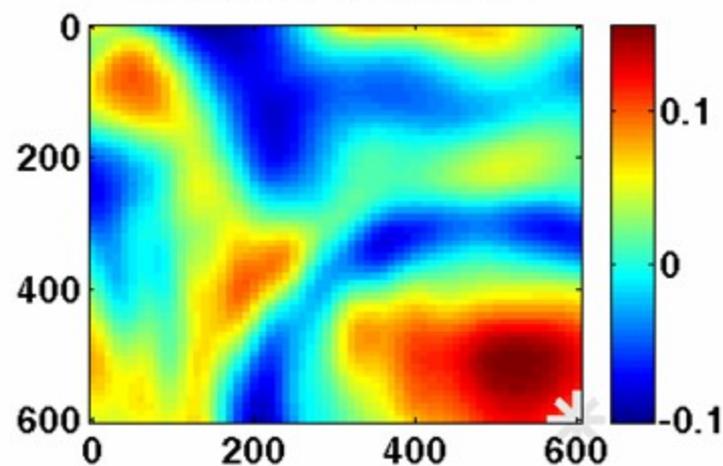


Receiver red. geom, PP, $a_{\text{relsurf}}=50$

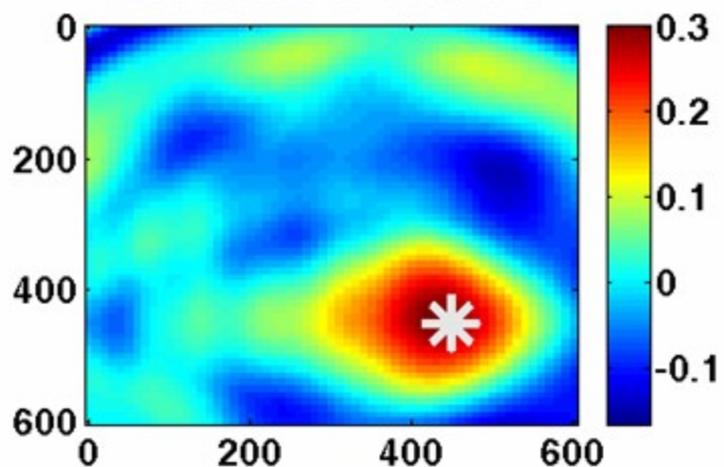
Loc 1: Refl 10-60Hz z=r



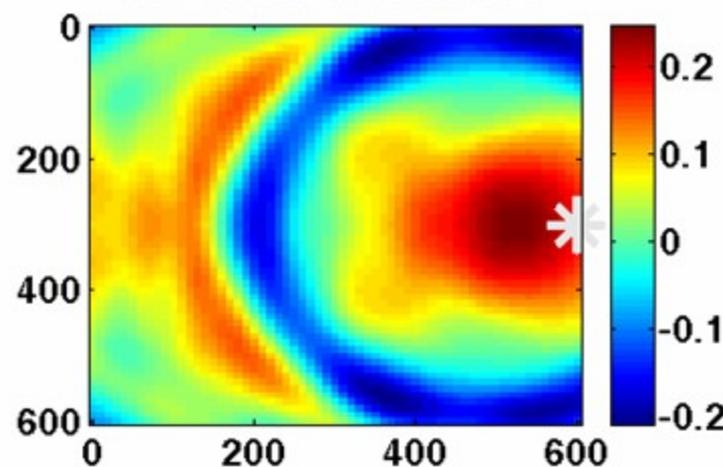
Loc 2: Refl 10-60Hz z=r



Loc 3: Refl 10-60Hz z=r

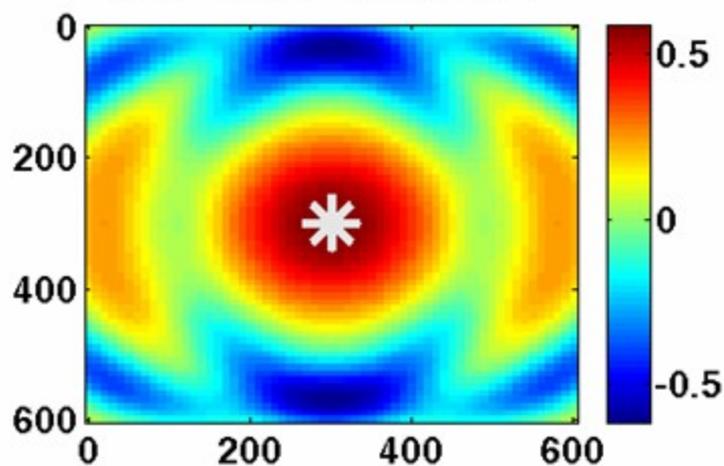


Loc 4: Refl 10-60Hz z=r

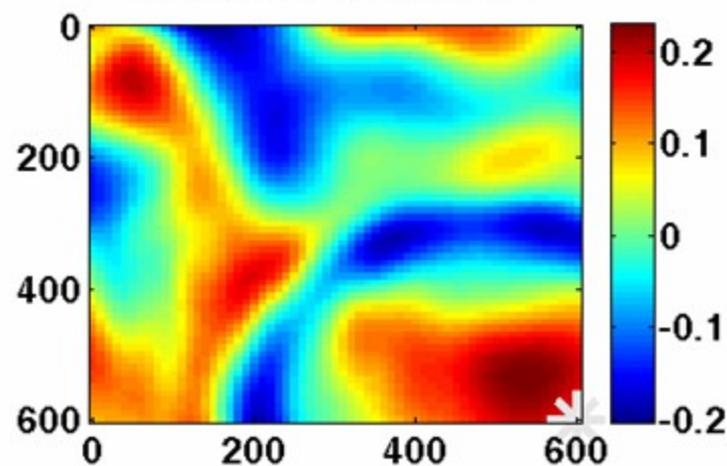


Receiver red. geom, PP, $a_{\text{relsurf}}=100$

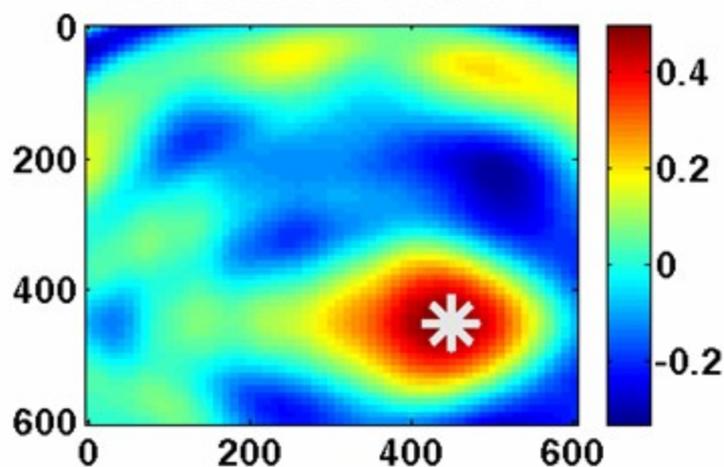
Loc 1: Refl 10-60Hz z=r



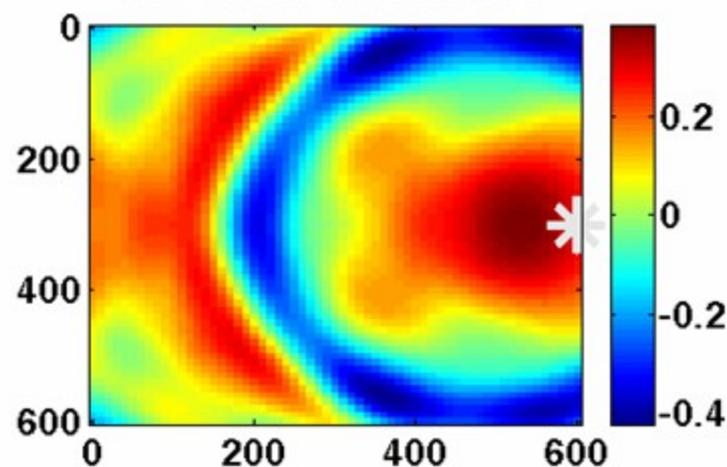
Loc 2: Refl 10-60Hz z=r



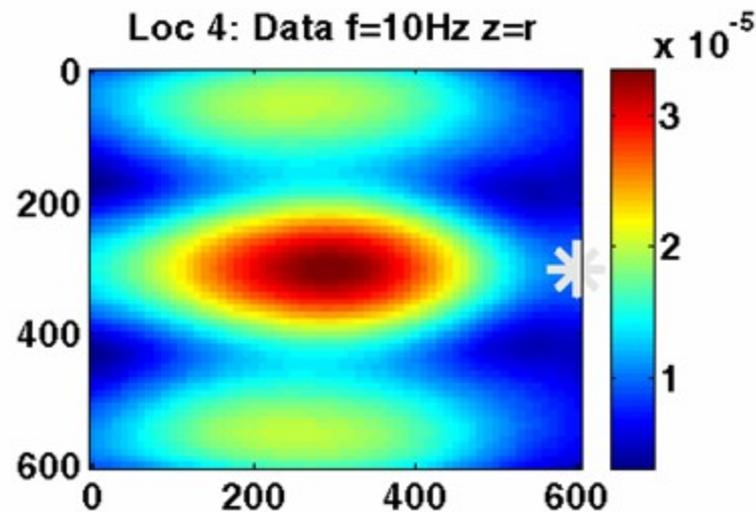
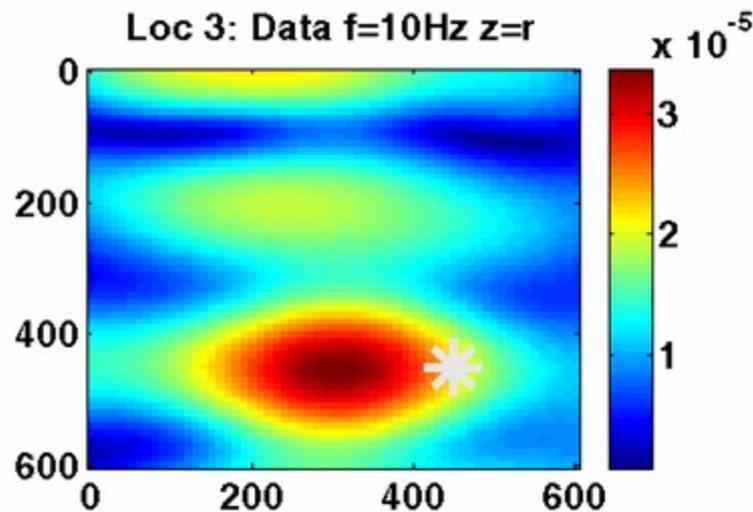
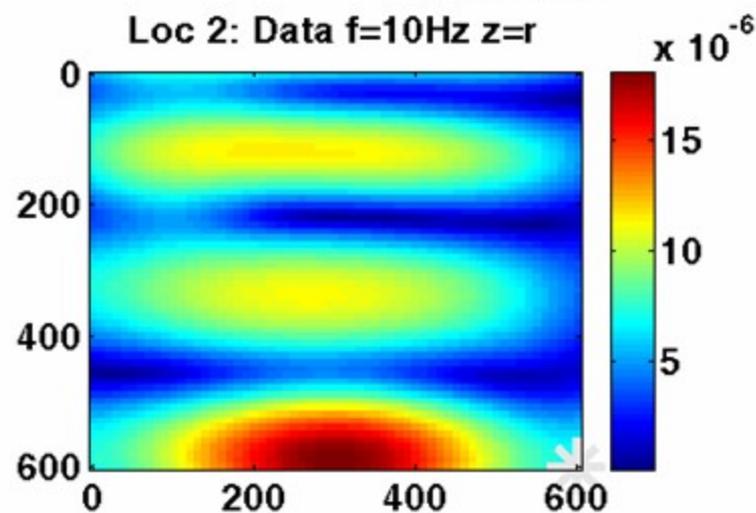
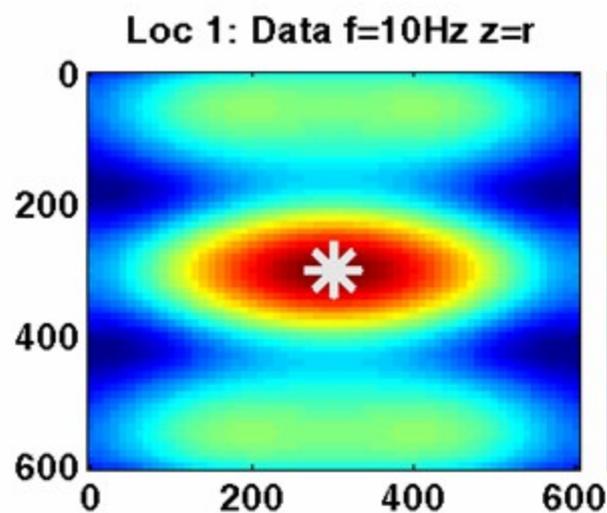
Loc 3: Refl 10-60Hz z=r



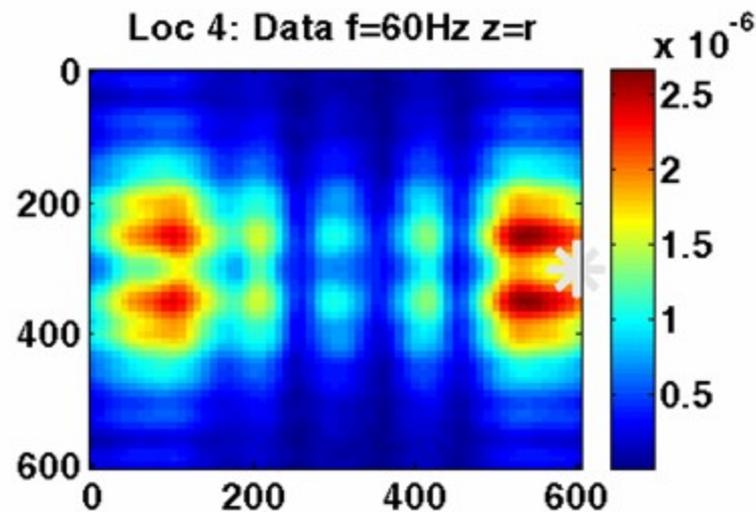
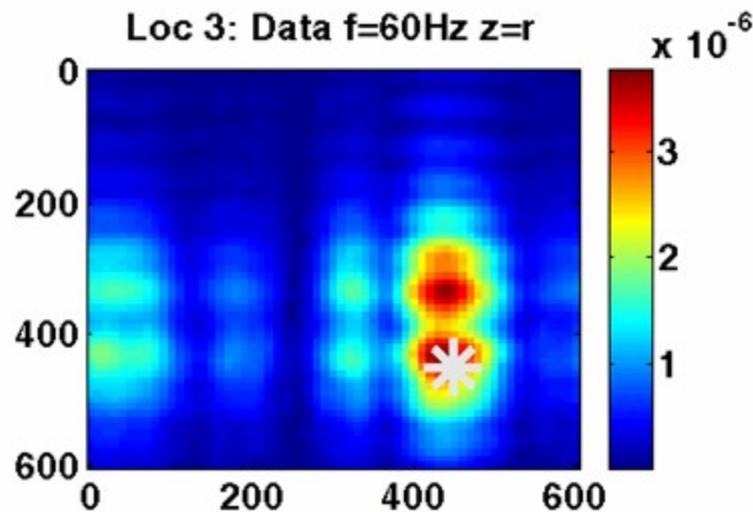
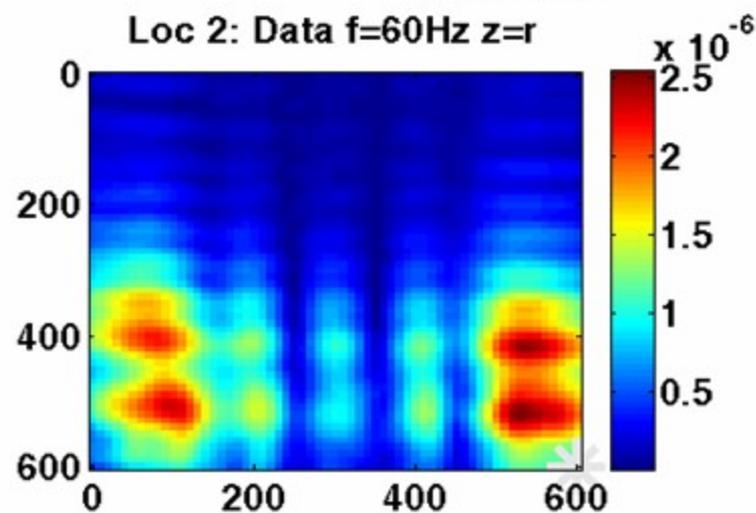
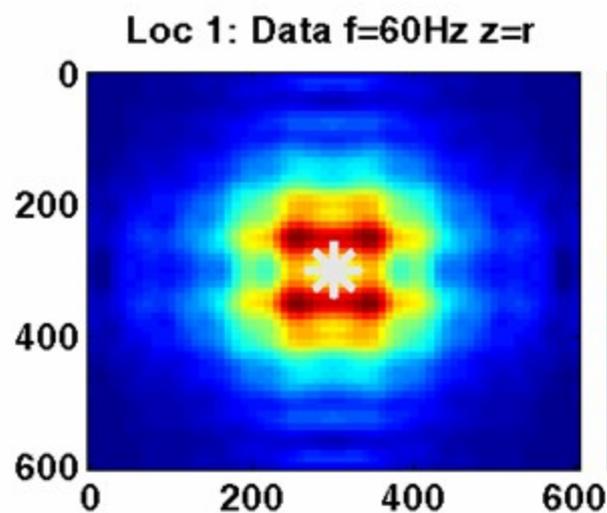
Loc 4: Refl 10-60Hz z=r



Receiver red. geom, PP, $a_{\text{relsurf}}=10$

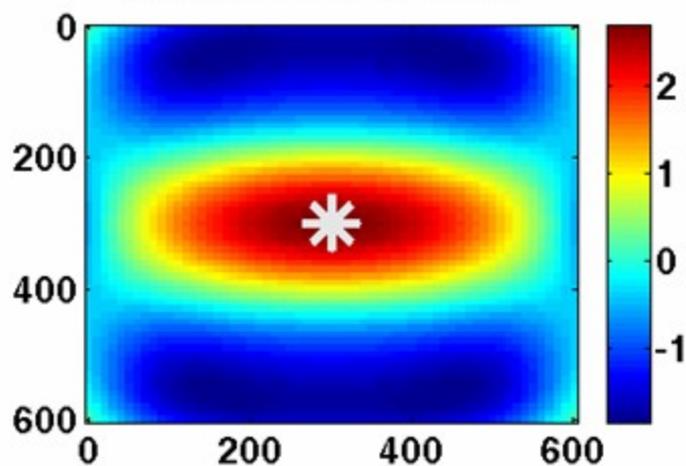


Receiver red. geom, PP, $a_{\text{relsurf}}=10$

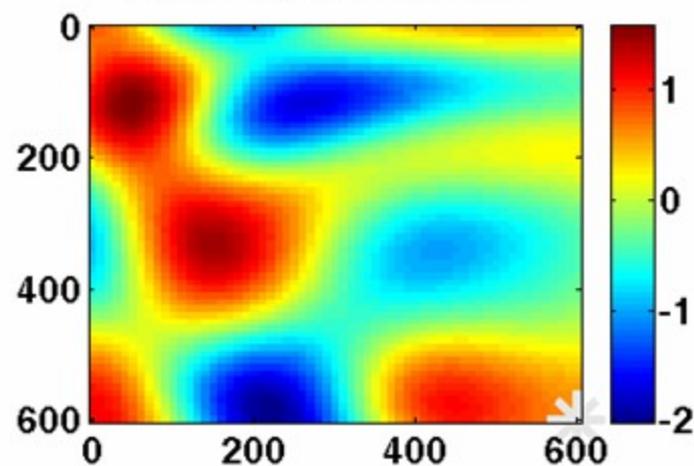


Receiver red. geom, PP, $a_{\text{relsurf}}=10$

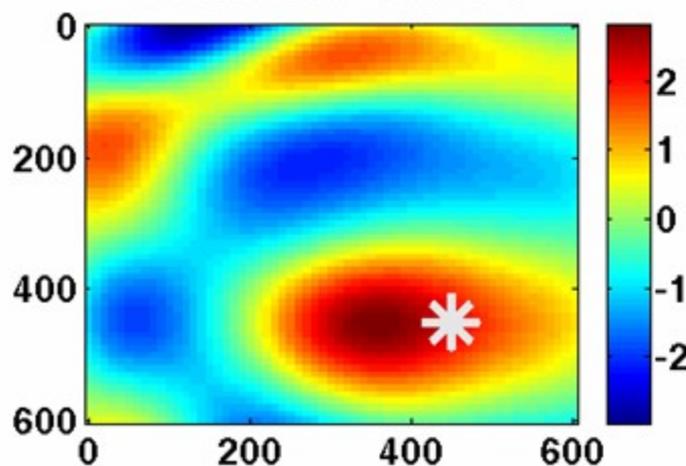
Loc 1: Refl f=10Hz z=r



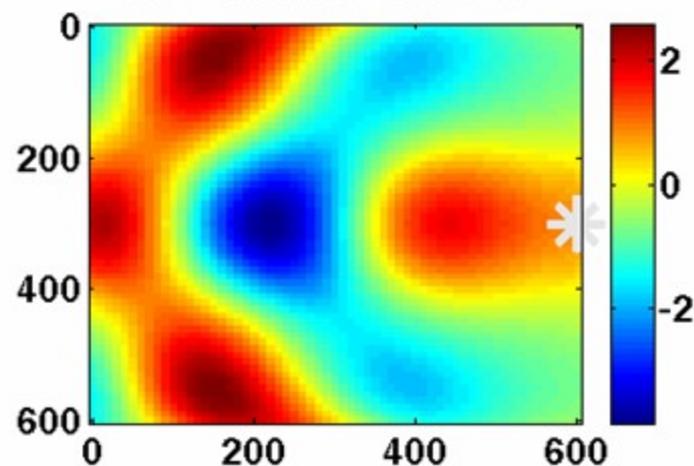
Loc 2: Refl f=10Hz z=r



Loc 3: Refl f=10Hz z=r

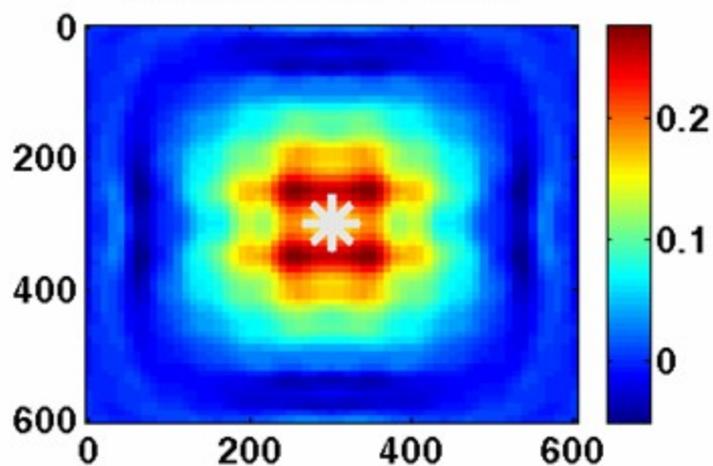


Loc 4: Refl f=10Hz z=r

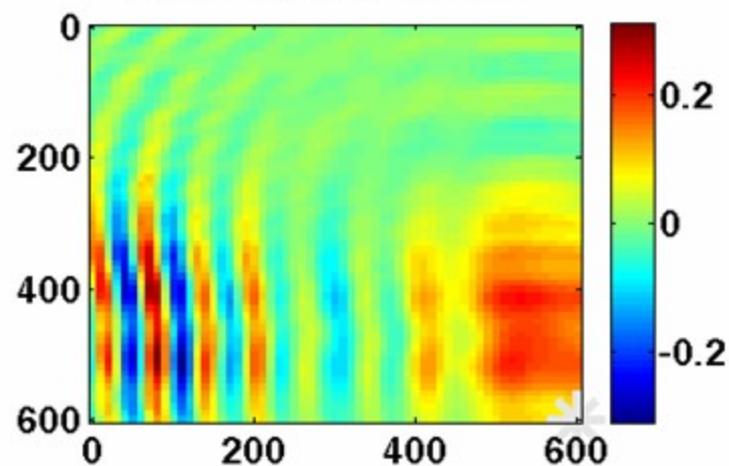


Receiver red. geom, PP, $a_{\text{relsurf}}=10$

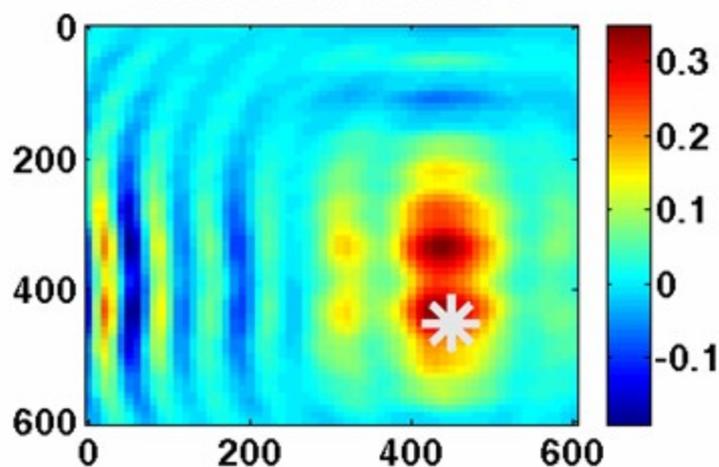
Loc 1: Refl f=60Hz z=r



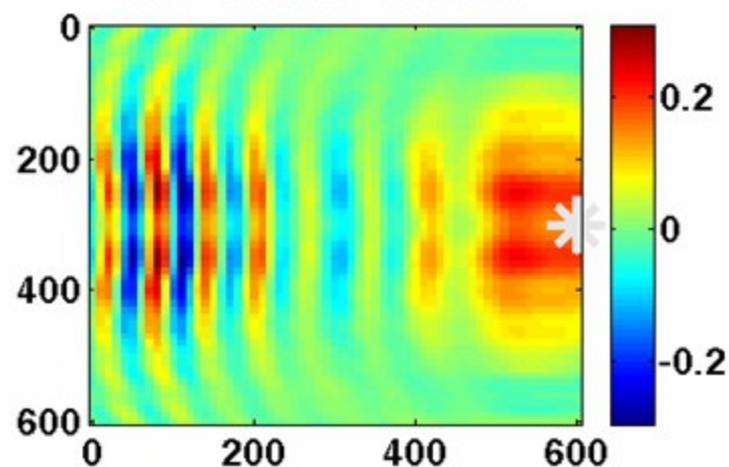
Loc 2: Refl f=60Hz z=r



Loc 3: Refl f=60Hz z=r

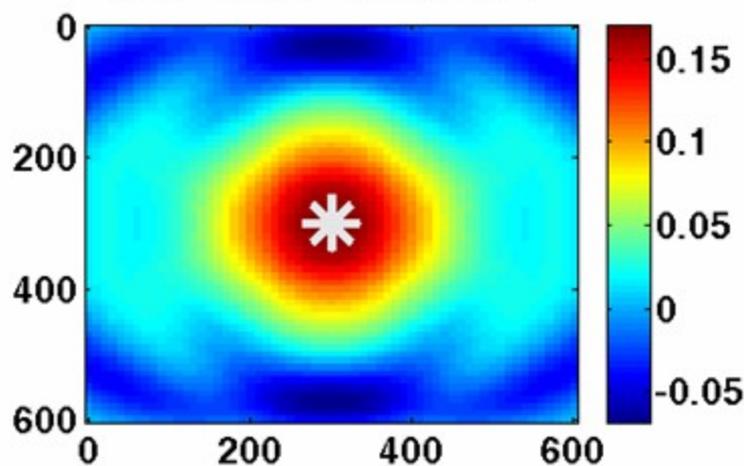


Loc 4: Refl f=60Hz z=r

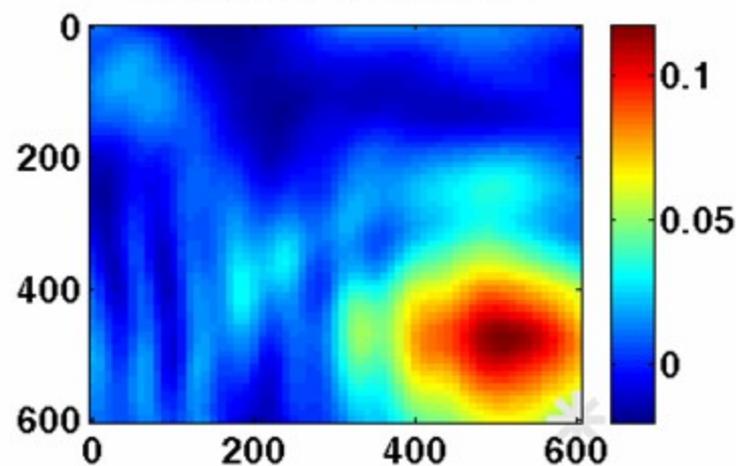


Receiver red. geom, PP, $a_{\text{relsurf}}=10$

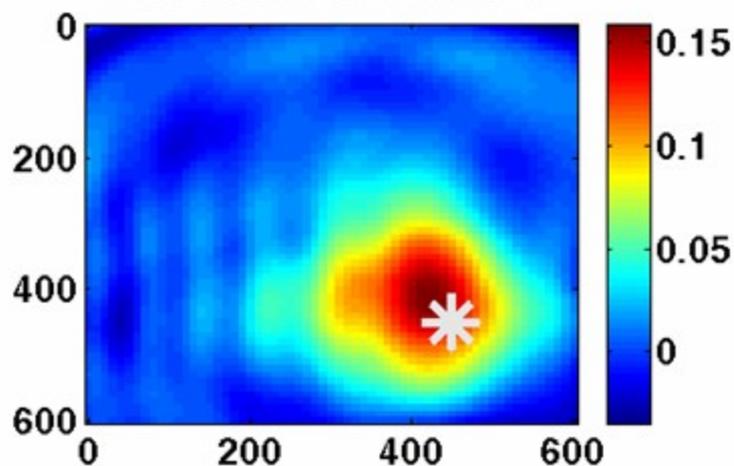
Loc 1: Refl 10-60Hz z=r



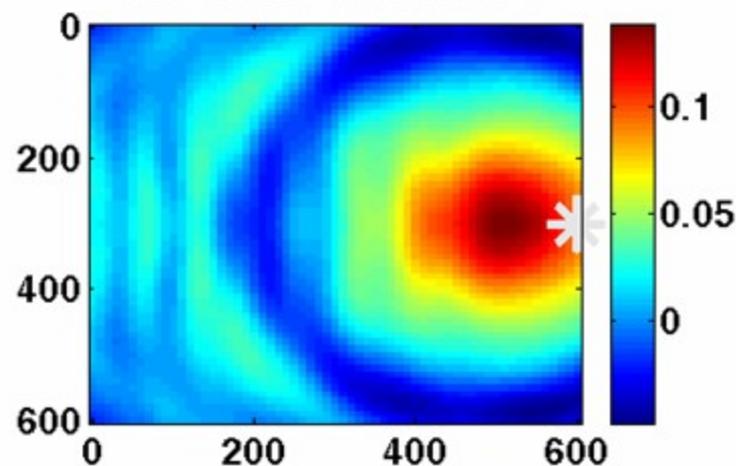
Loc 2: Refl 10-60Hz z=r



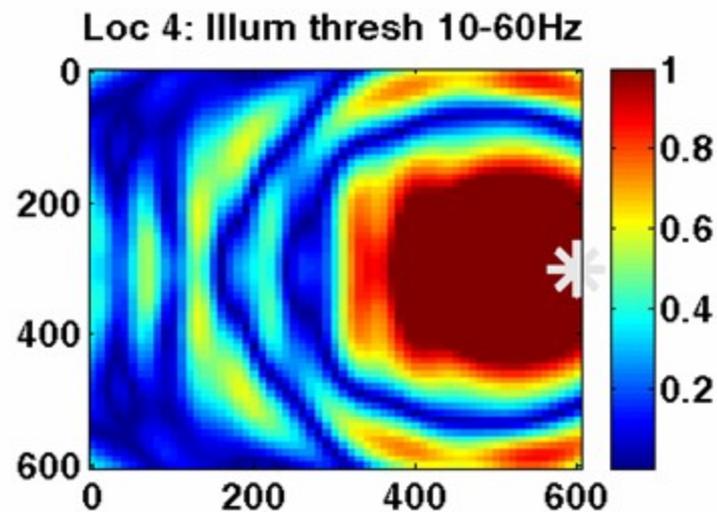
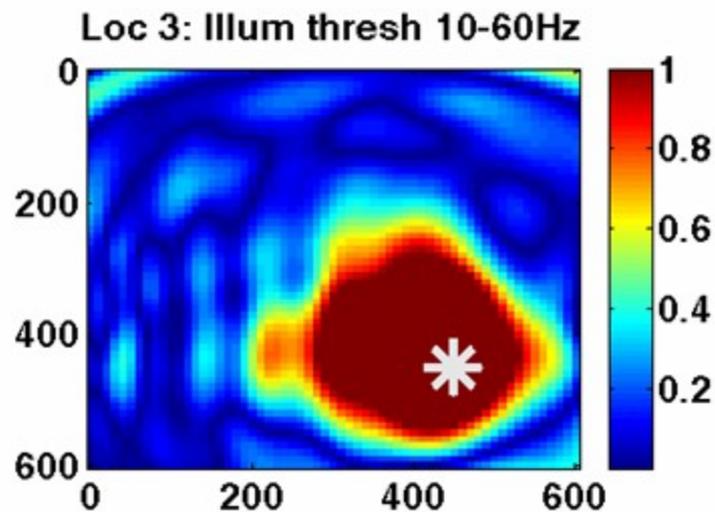
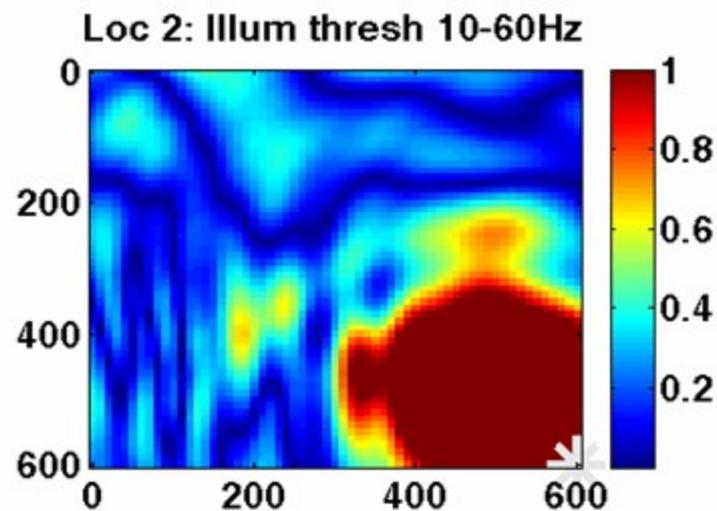
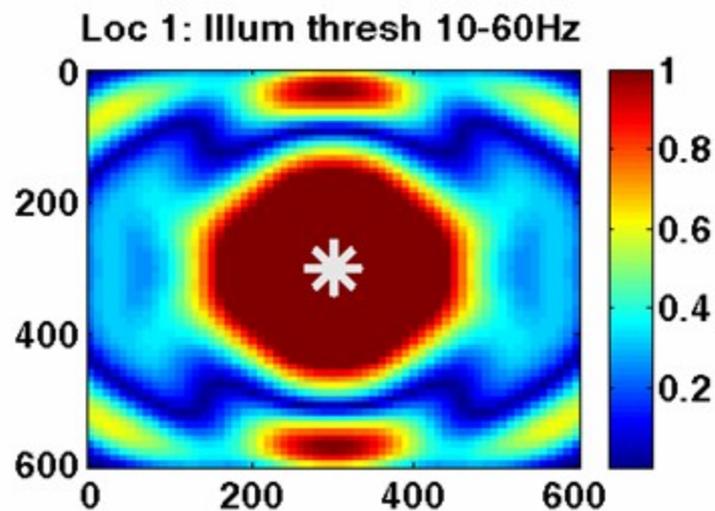
Loc 3: Refl 10-60Hz z=r



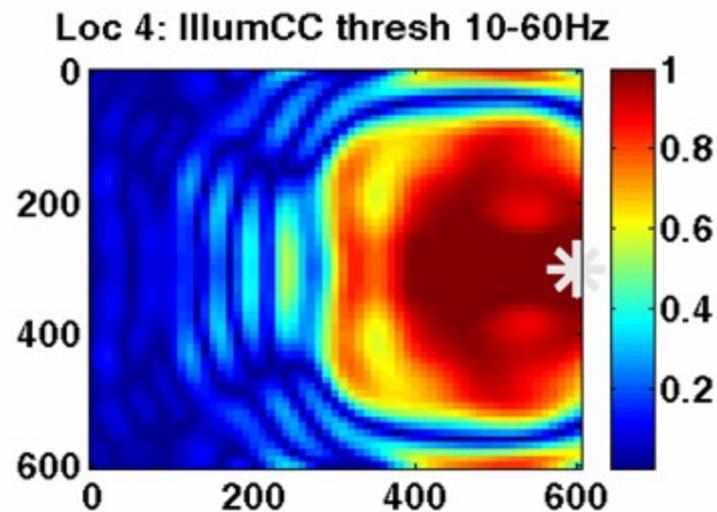
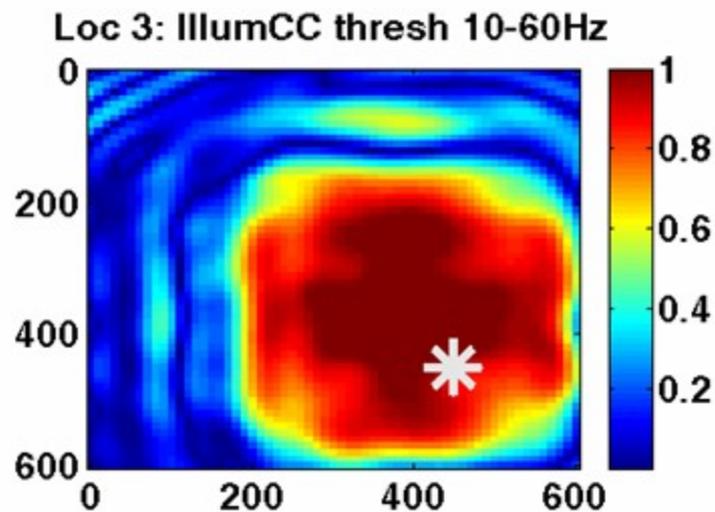
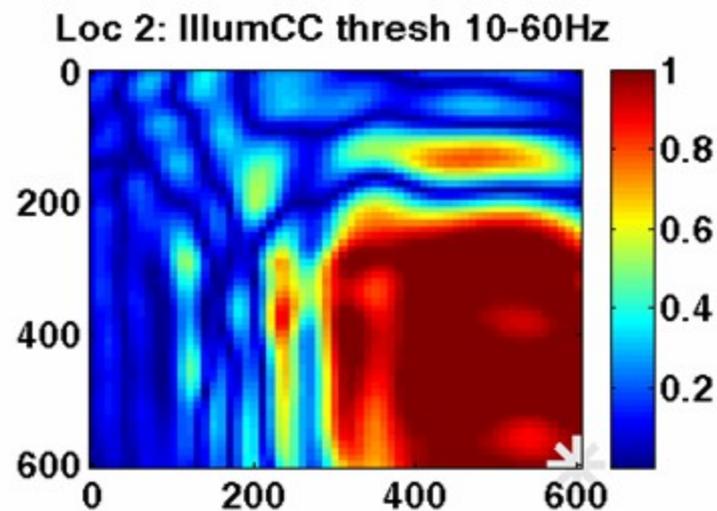
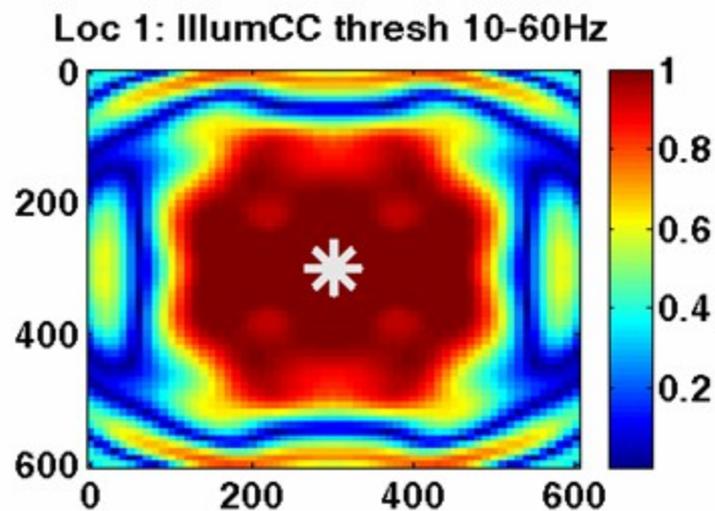
Loc 4: Refl 10-60Hz z=r



Receiver red. geom, PP, $a_{\text{relsurf}}=10$



Receiver red. geom, PP, $a_{\text{relsurf}}=10$



Conclusions

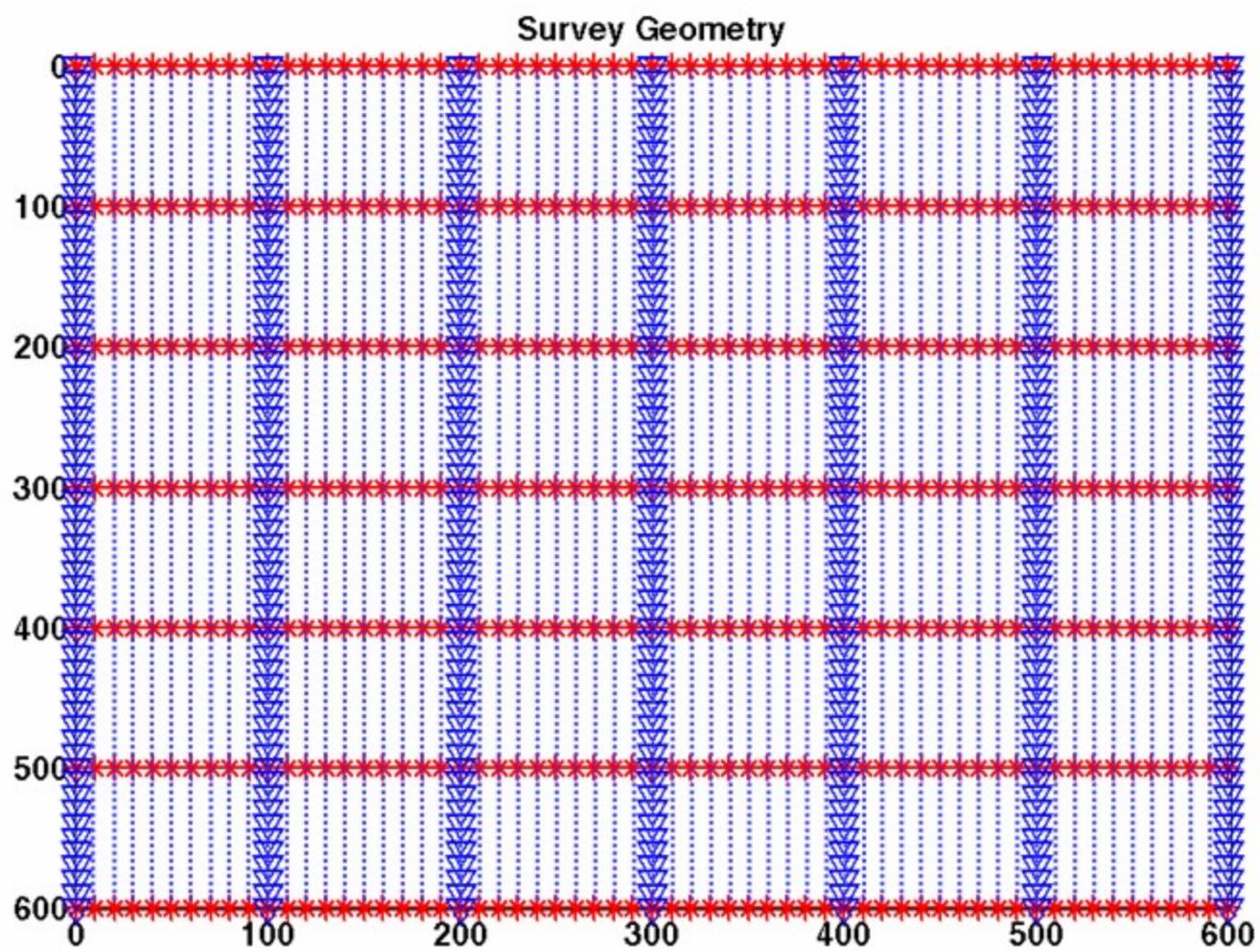
- You sure can make a lotta color pictures with a good Matlab script.
- The simulator gives plausible results.
- Illumination normalization seems like a good thing.
- Noise is a bad thing.
- The receiver-reduced geometry needs help.
- There's a lot more to learn here. Tune in next year!

Acknowledgements



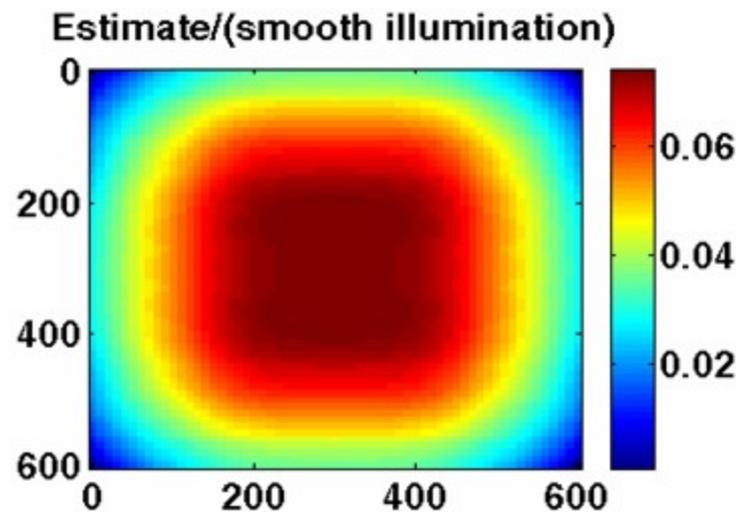
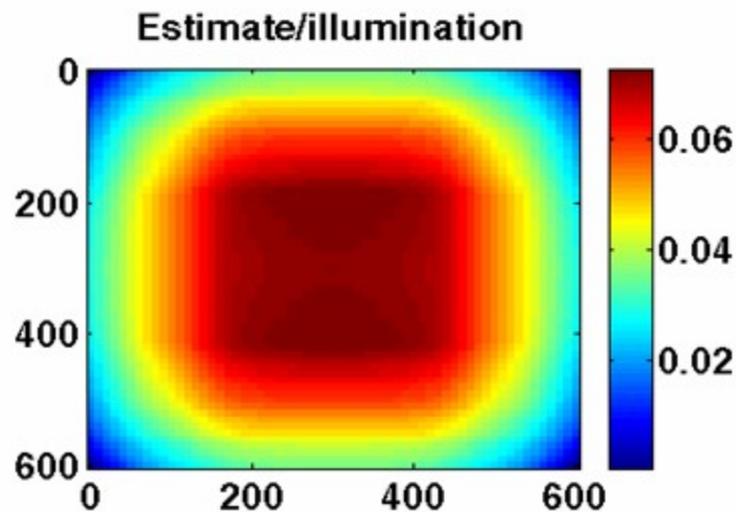
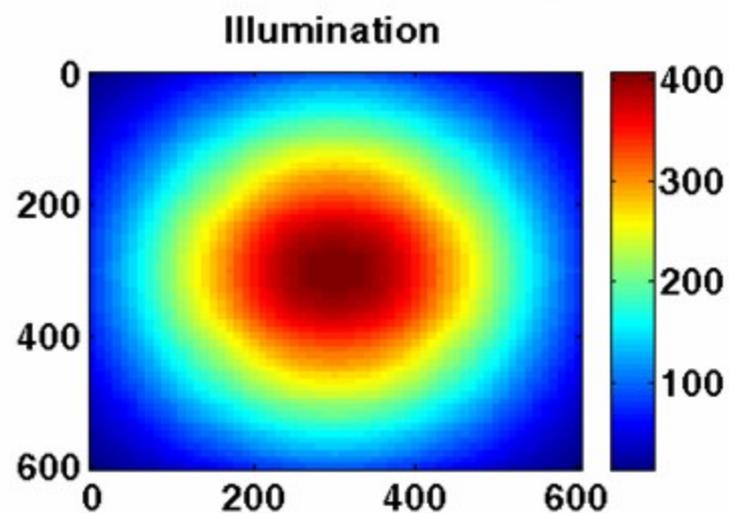
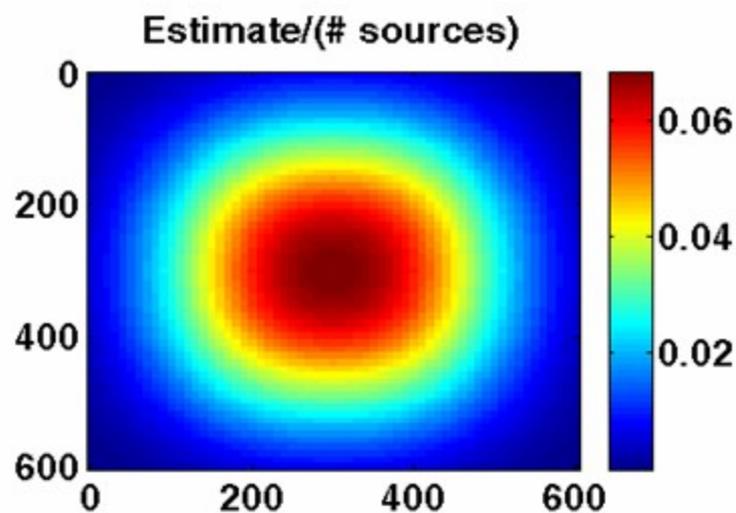
**Thanks to
Heidi
for her
unwavering
support**

Source Reduced Geometry



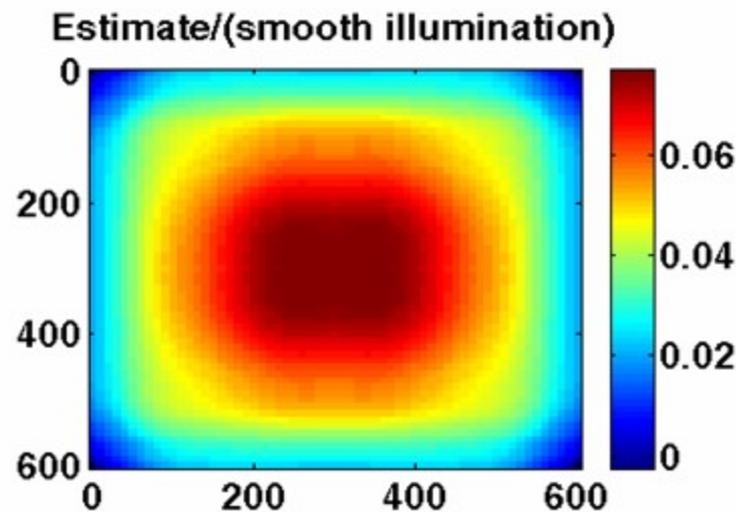
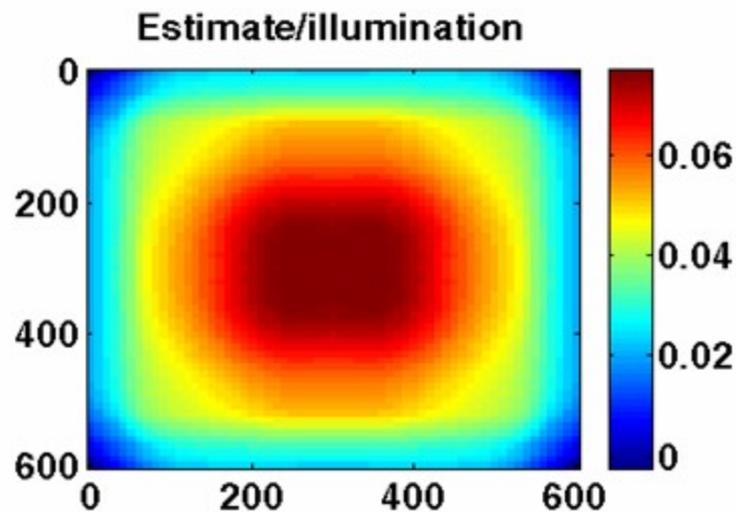
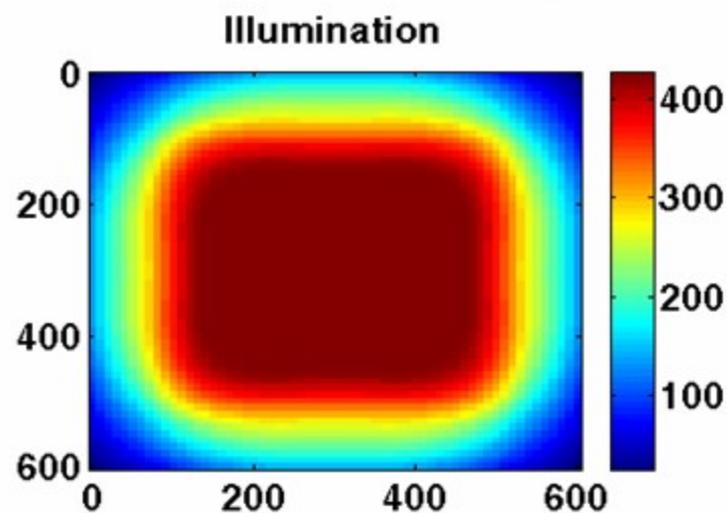
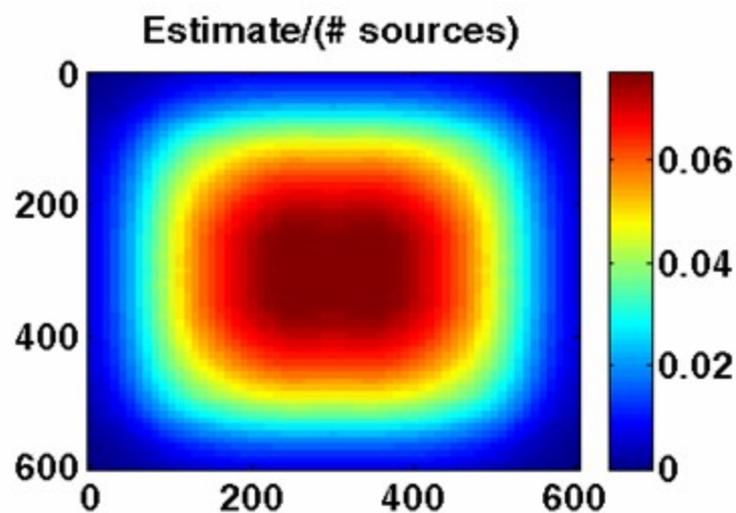
Shot (red) spacing 10m --- Shot line spacing 100m
Receiver (blue) spacing 10m --- Receiver line spacing 10m

Source Reduced Geometry PP



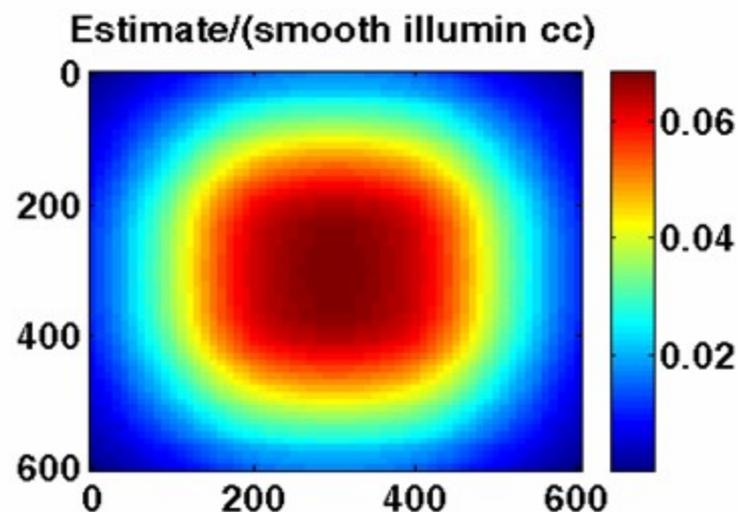
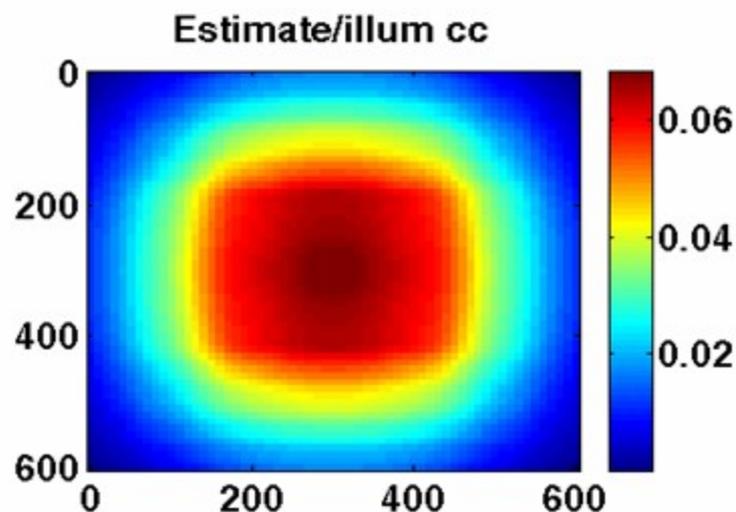
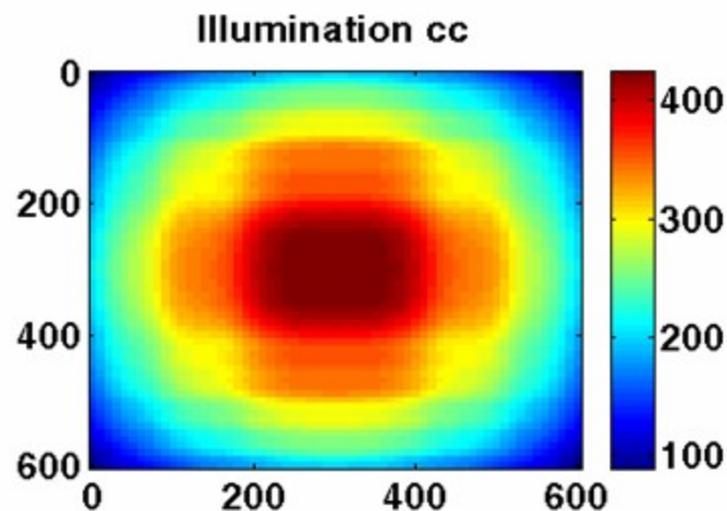
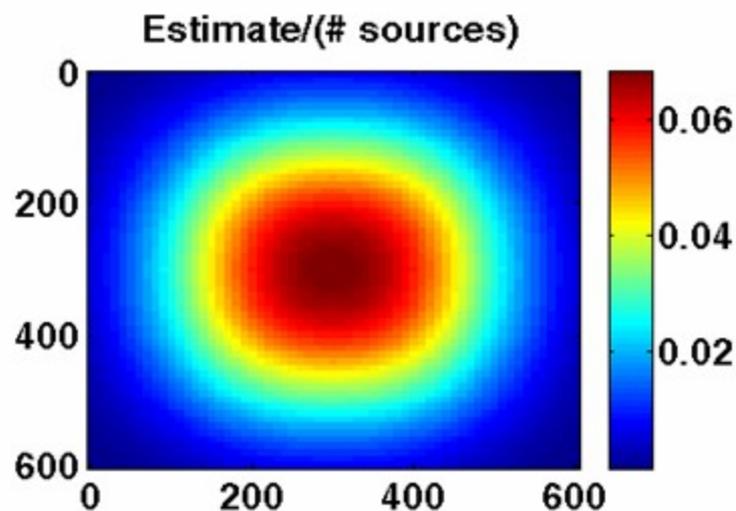
Threshold = 0.4

Source Reduced Geometry PS



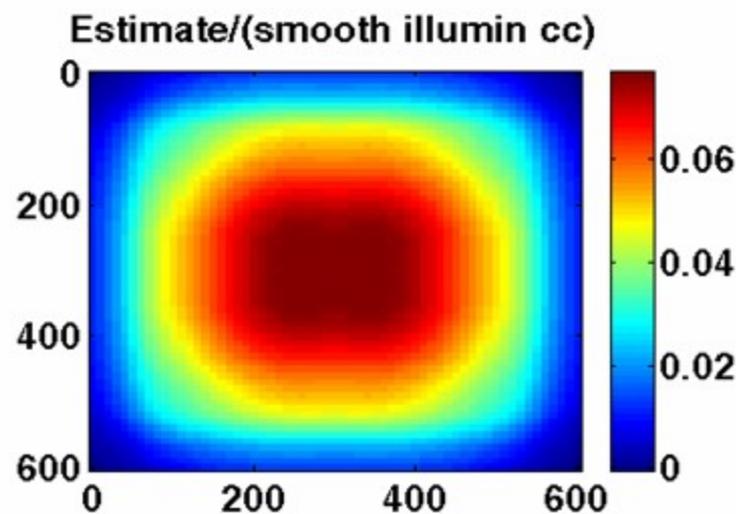
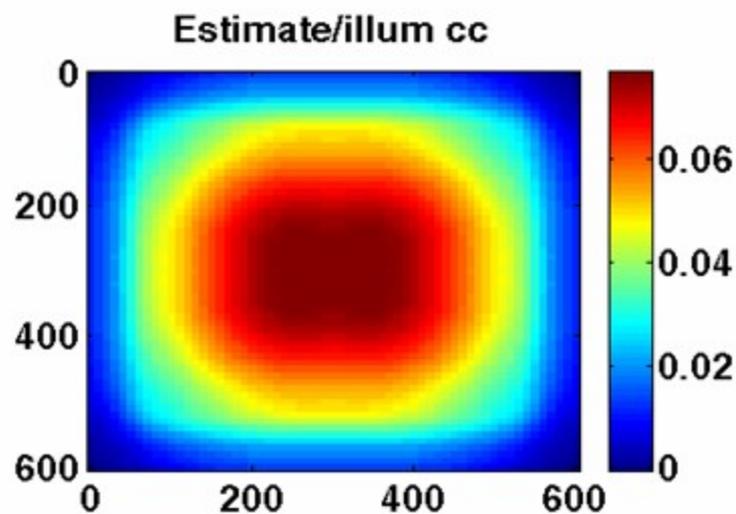
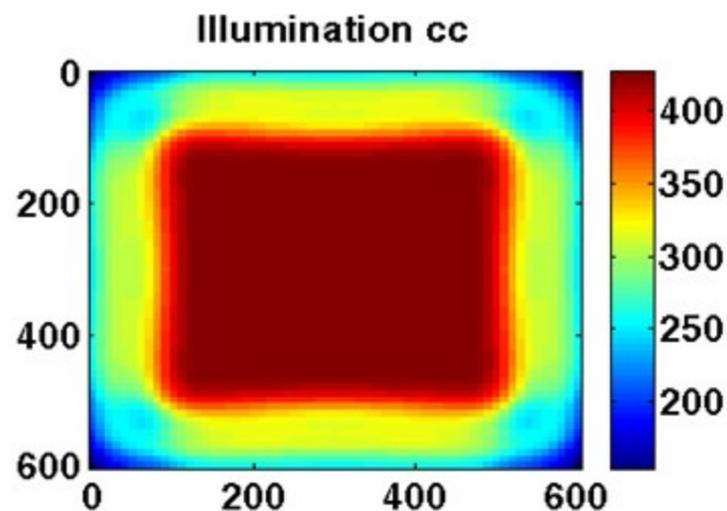
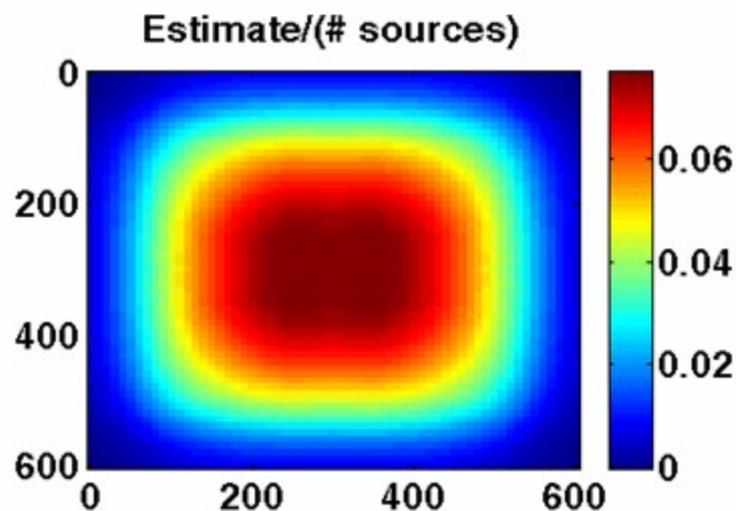
Threshold = 0.9

Source Reduced Geometry PP



Threshold = 0.4

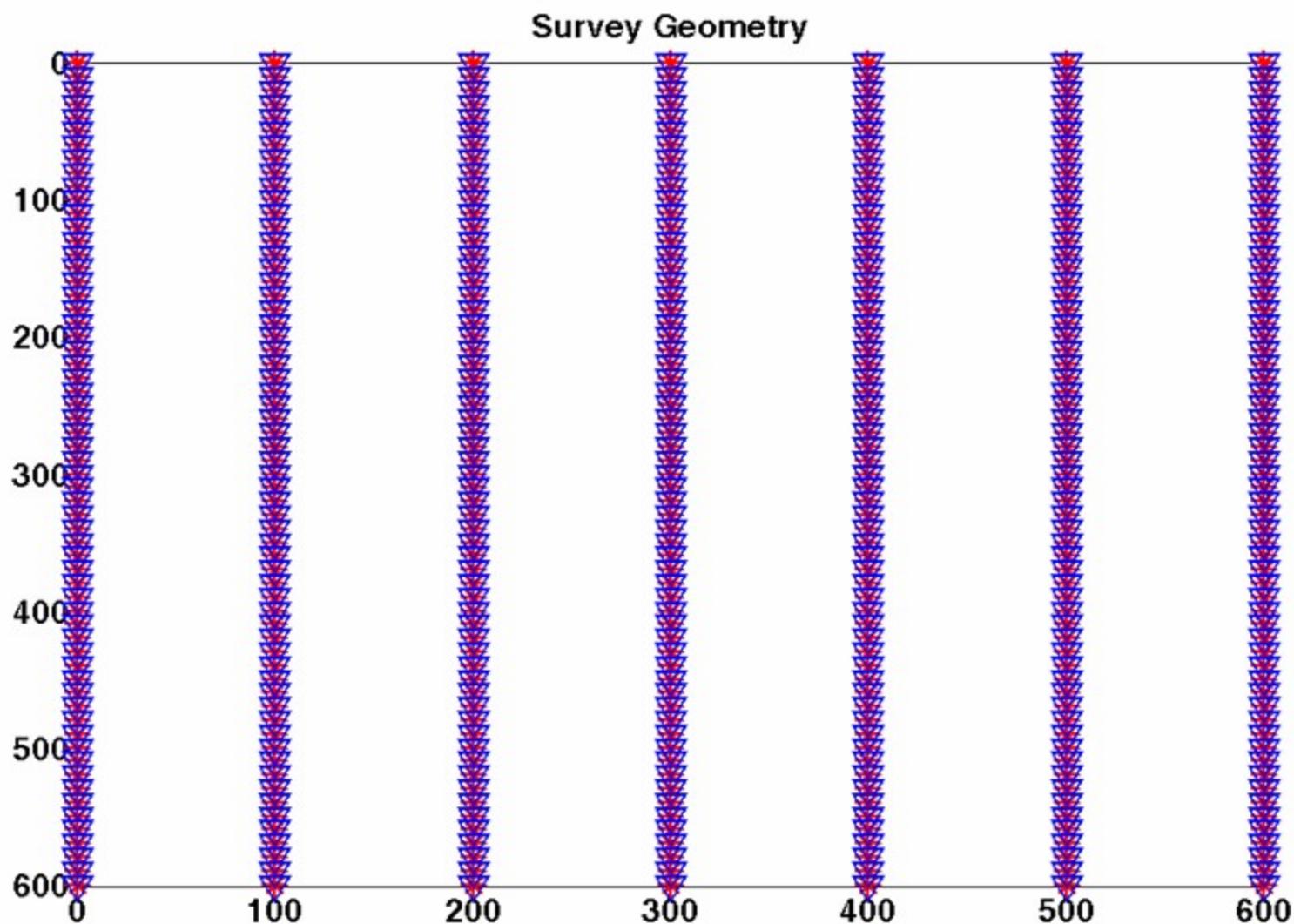
Source Reduced Geometry PS



Threshold = 0.9

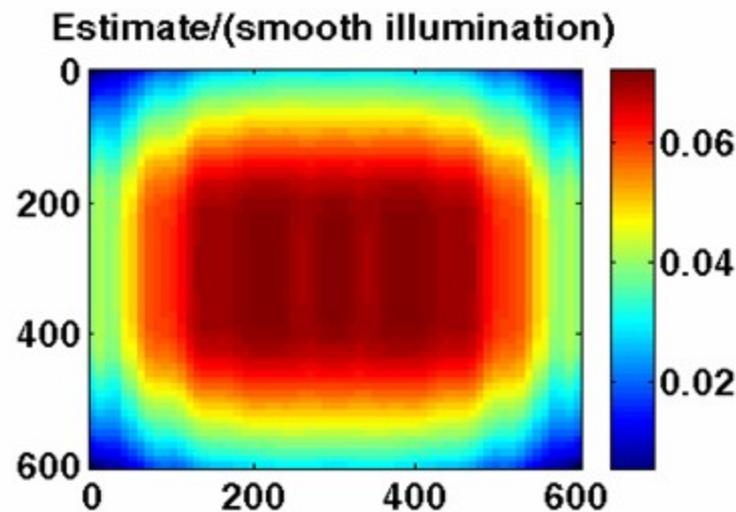
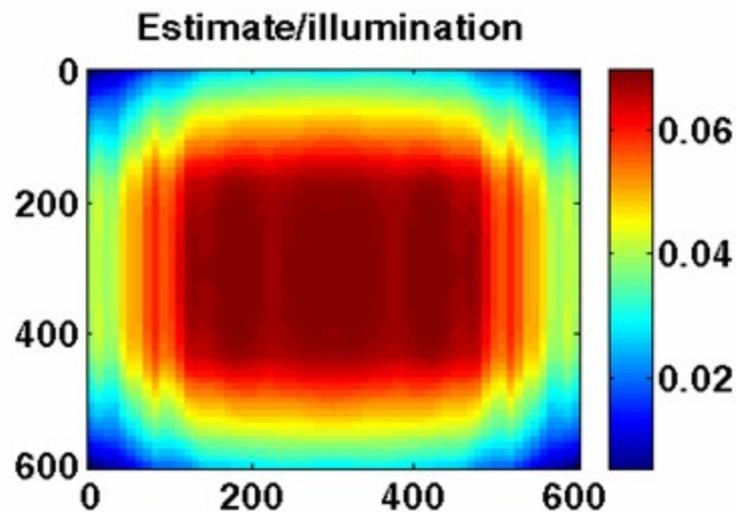
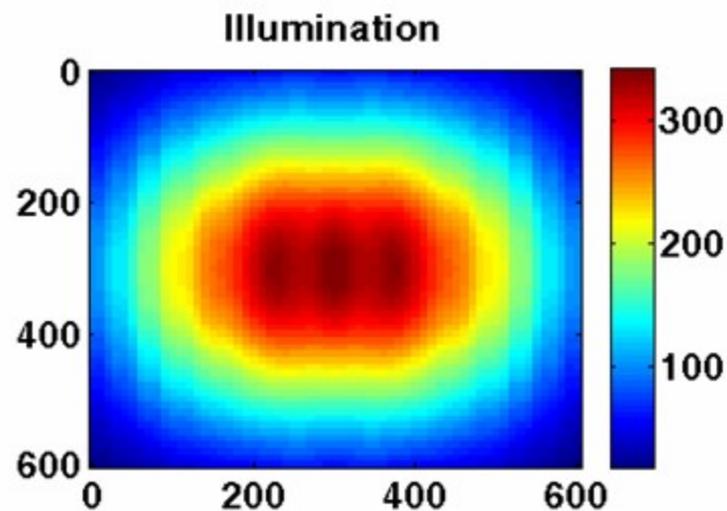
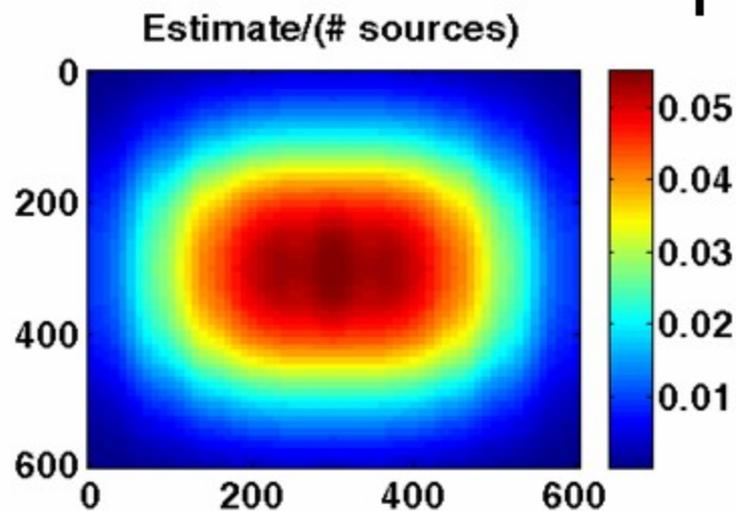
A Silly Thing To Do

* Silly Shot/Receiver Reduced Geometry



Silly Source/Receiver Reduced Geometry

PP



Silly Source/Receiver Reduced Geometry

PP

