A simple way to improve PP and PS AVO approximations

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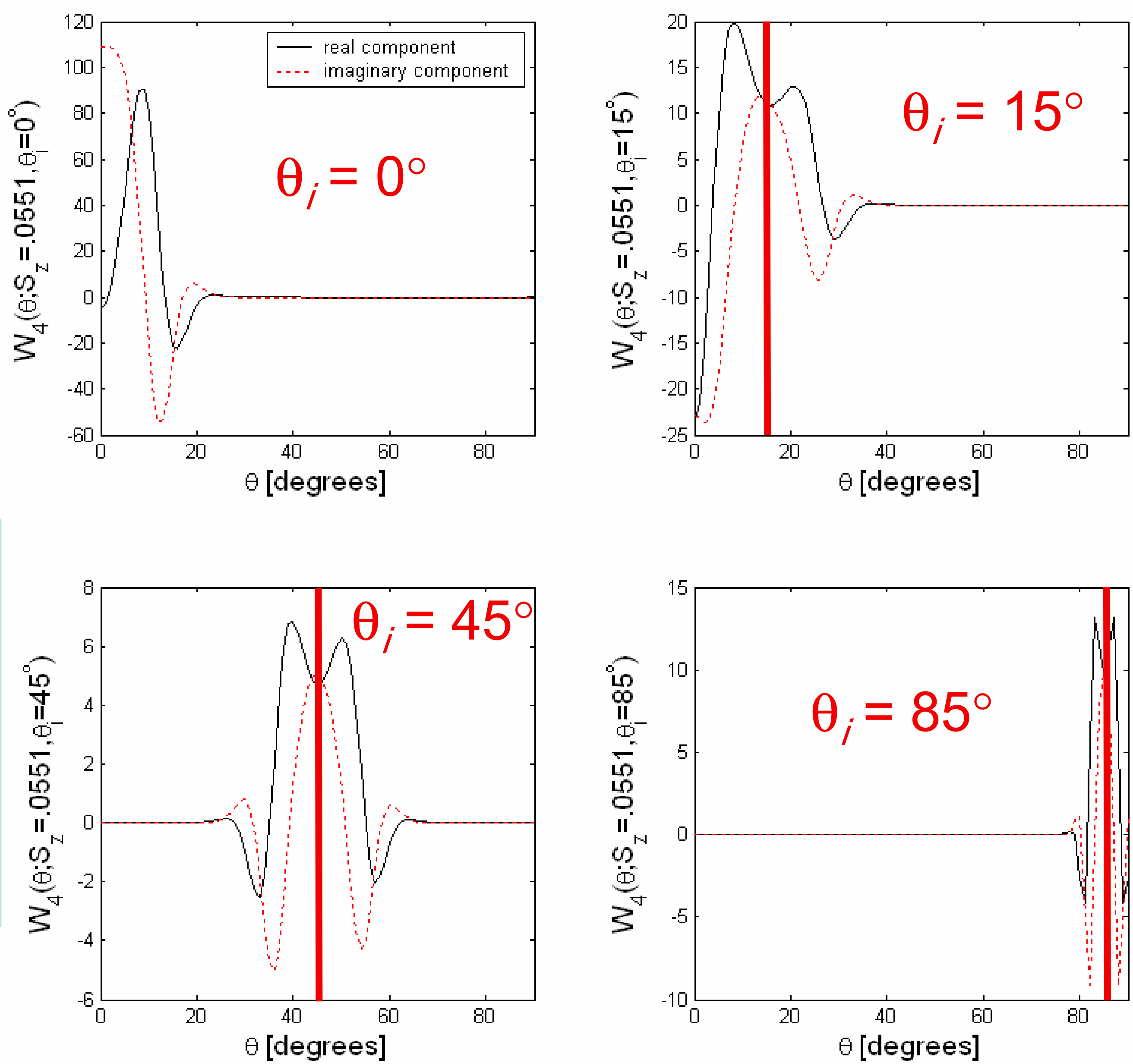
CREWES Sponsors Meeting

Thursday, December 1, 2005

Overview

- Notes on spherical-wave modeling ⟵
- Reflectivity Explorer observations
- Theoretical Explanations
- Improving AVO theories

Spherical-wave Reflection Coefficients



$$R_{\mathrm{PP}}^{\mathrm{spherical}}(\theta_i) \approx \int_0^\infty R_{\mathrm{PP}}(p) W_n(p;\theta_i) \frac{p}{\xi} dp$$

$$p = \frac{\sin\theta}{\alpha}$$

Efficient Explorer calculations

Assume that wavelet is of form

$$f_n(\omega) = \omega^n \exp(-s\,|\,\omega\,|)$$

Then ω-integration can be analytic

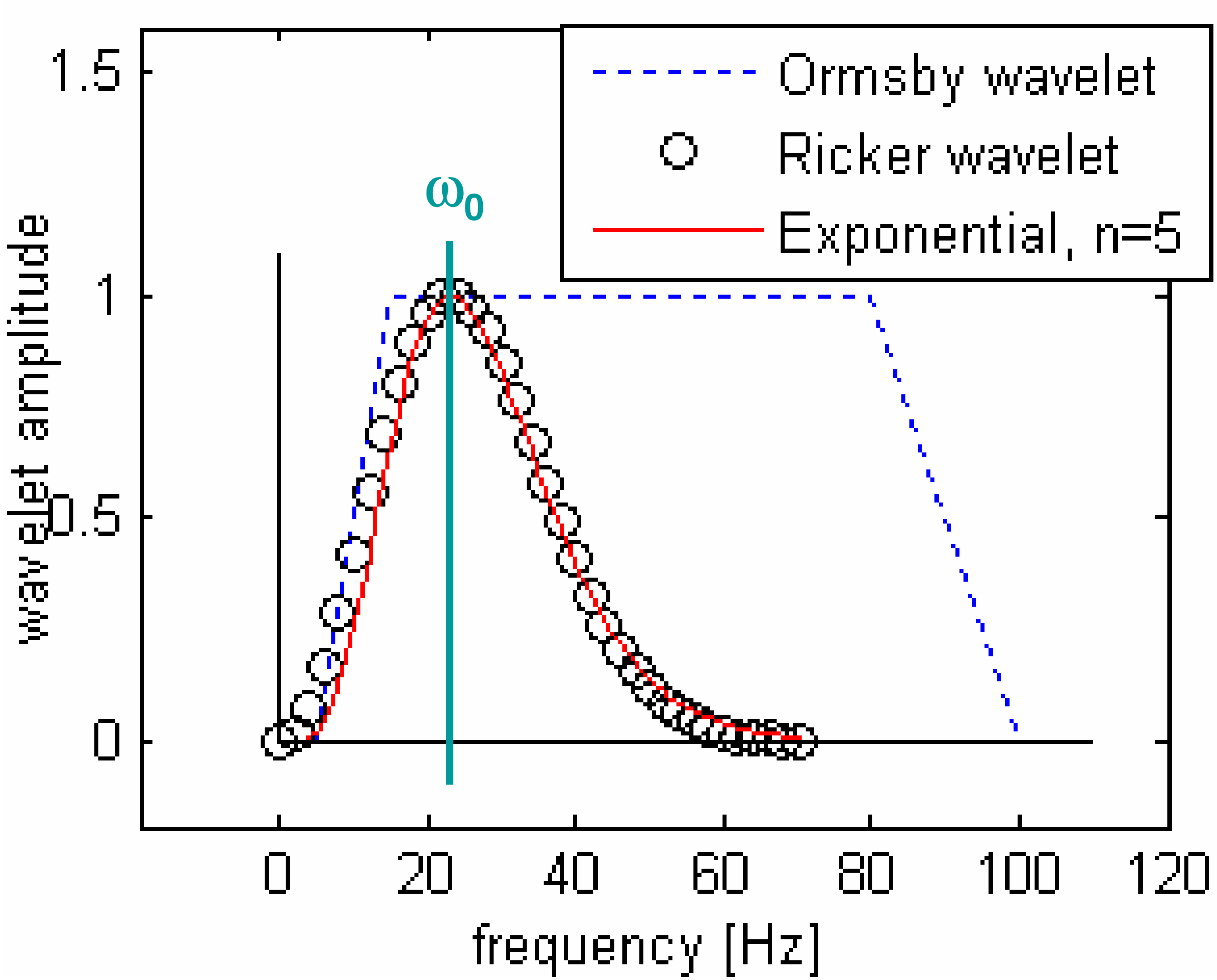
This form is similar to Ricker wavelet

$$f_{\text{Ricker}}(\omega) = \omega^2 \exp\left[-\left(\omega / \omega_0\right)^2\right]$$

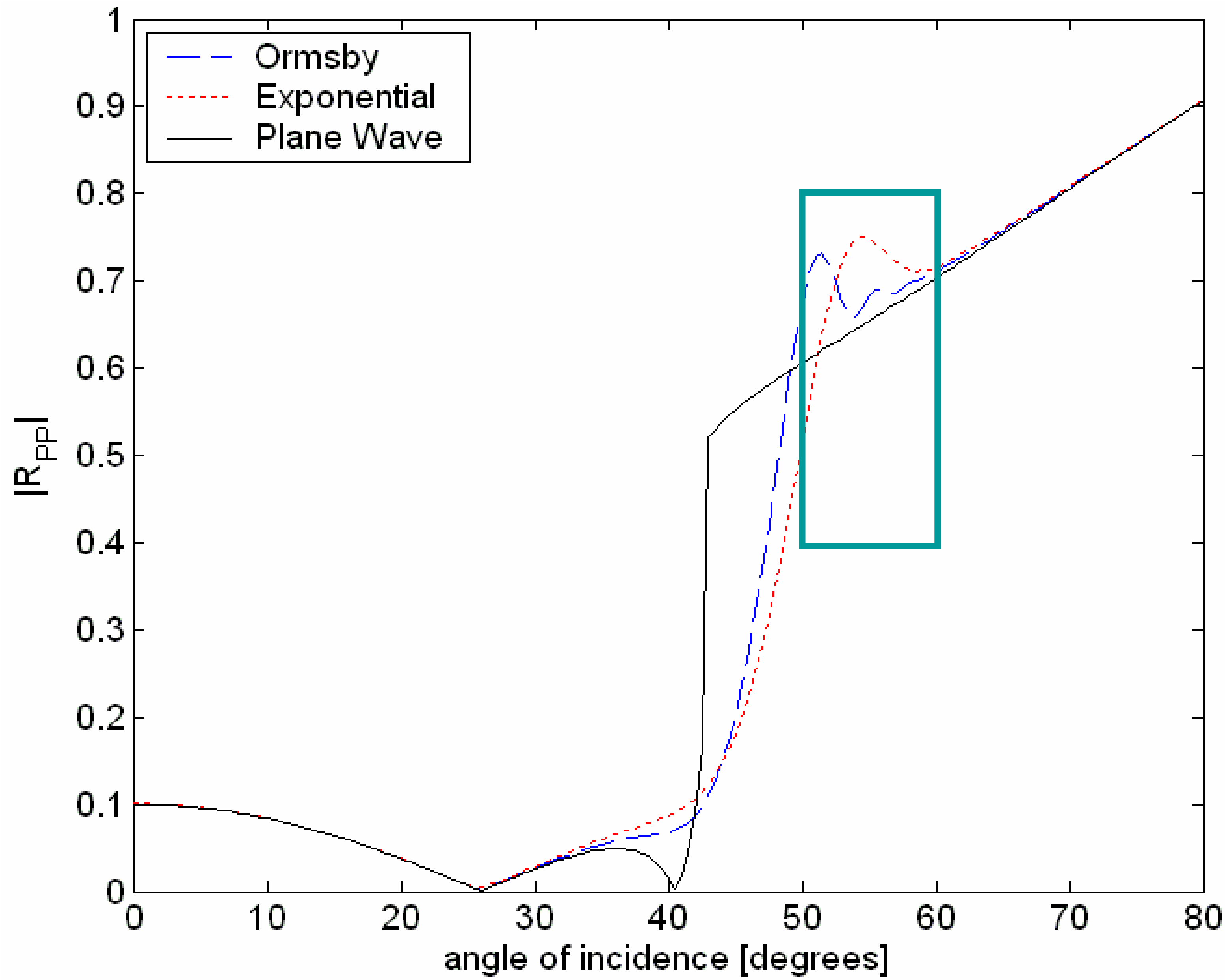
$$f_n(\omega) = \omega^n \exp(-n\,|\,\omega / \omega_0\,|)$$

ω_0 is the maximum frequency for both

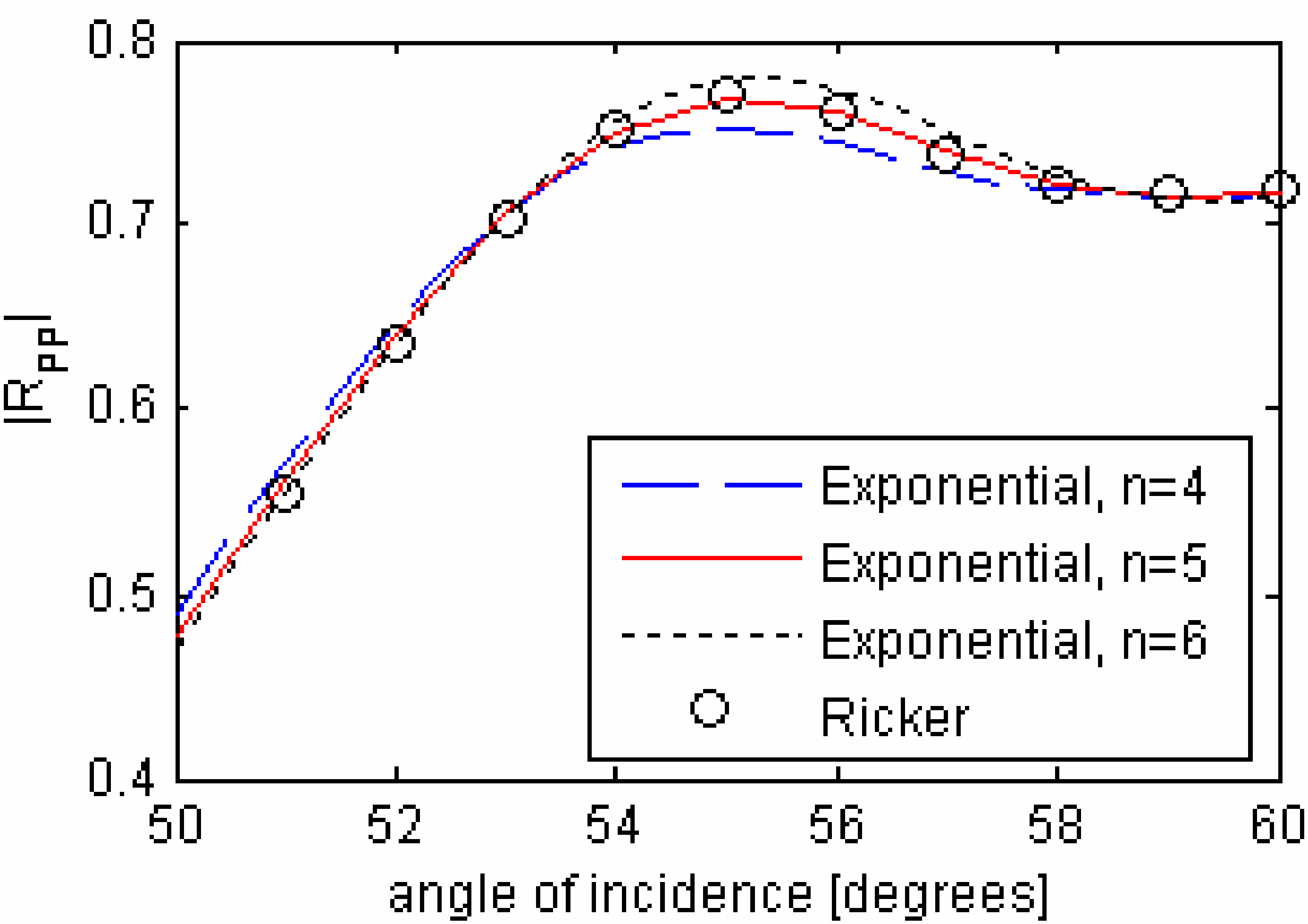
Wavelet comparison



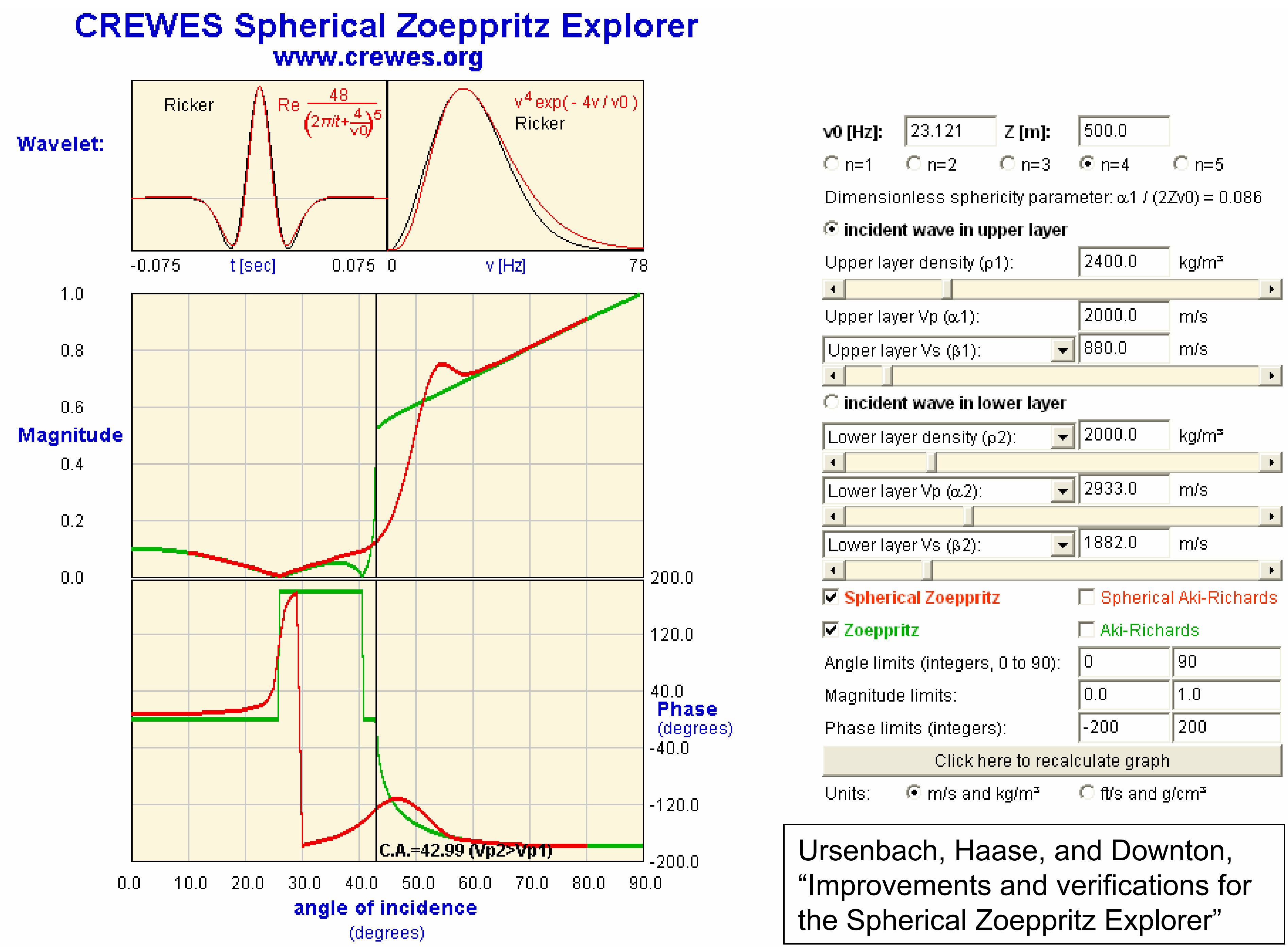
Spherical R_{PP} for Ormsby and n=4 wavelets



Representation of Ricker wavelet



(note range of axes)



Ursenbach, Haase, and Downton, "Improvements and verifications for the Spherical Zoeppritz Explorer"

Reflection of spherical waves in VTI media

– Posters –

- Ursenbach & Haase
  - Generalized reflections from point sources in a two-layer VTI medium: theory
- Haase & Ursenbach
  - Spherical-wave AVO-modelling in elastic VTI-media
  - Anelasticity and spherical-wave AVO-modelling in VTI-media

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Aki-Richards Approximation

$$R_{PP}^{\text{A-R}} = R_\rho + \frac{R_\alpha}{\cos^2\theta} - 4\gamma^2 \sin^2\theta (2R_\beta + R_\rho),$$

$$R_{PS}^{\text{A-R}} = -\gamma \tan\varphi \left[R_\rho + 2\gamma \cos(\theta - \varphi)(2R_\beta + R_\rho) \right],$$

$$\theta = (\theta_1 + \theta_2)/2, \quad \varphi = (\varphi_1 + \varphi_2)/2$$

$$R_\alpha = \frac{\Delta\alpha}{2\alpha}$$

$$R_\beta = \frac{\Delta\beta}{2\beta}$$

$$R_\rho = \frac{\Delta\rho}{2\rho}$$

$$\gamma = \frac{\beta_1 + \beta_2}{\alpha_1 + \alpha_2}$$

$$R_{PP}^{\text{Shuey}} = \left(R_\rho + R_\alpha\right) + \left[R_\alpha - 4\gamma^2(2R_\beta + R_\rho)\right]\sin^2\theta + \sin^2\theta \tan^2\theta R_\alpha$$

$$\equiv A + B\sin^2\theta + C\sin^2\theta\tan^2\theta,$$

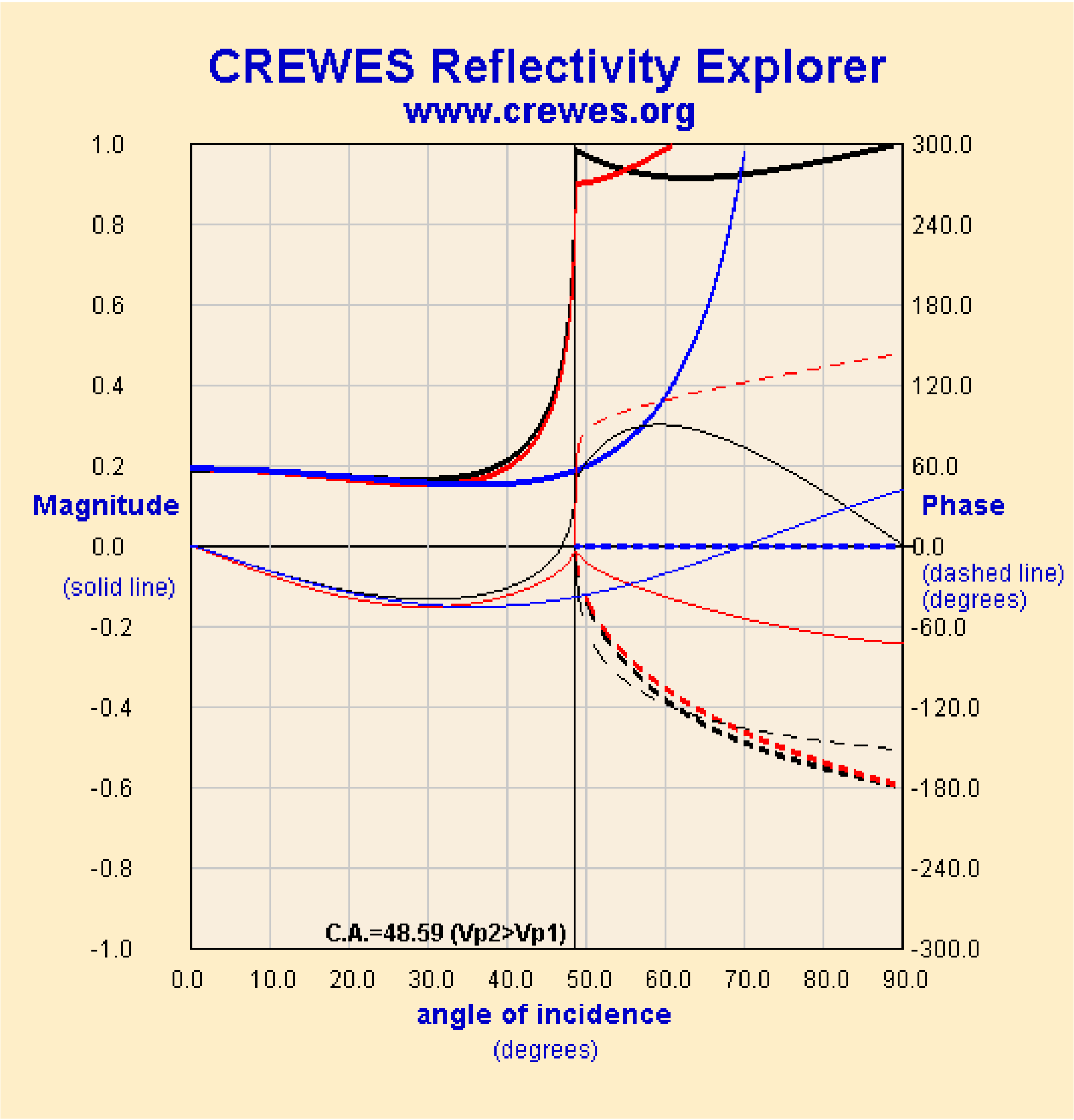
$$R_{PS}^{\text{Shuey-like}} = -\gamma\left[R_\rho + 2\gamma(2R_\beta + R_\rho)\right]\sin\theta + O(\sin^3\theta)$$

$$\equiv A_S \sin\theta + O(\sin^3\theta).$$

A further approximation

- Shuey (1985) also suggested substituting θ_1 for θ as an approximation.
- What behavior does this give?

θ vs. θ_1 approximations



$$R_\alpha = 0.143$$

$$R_\beta = 0.143$$

$$R_\rho = 0.0476$$

$$\gamma = 0.5$$

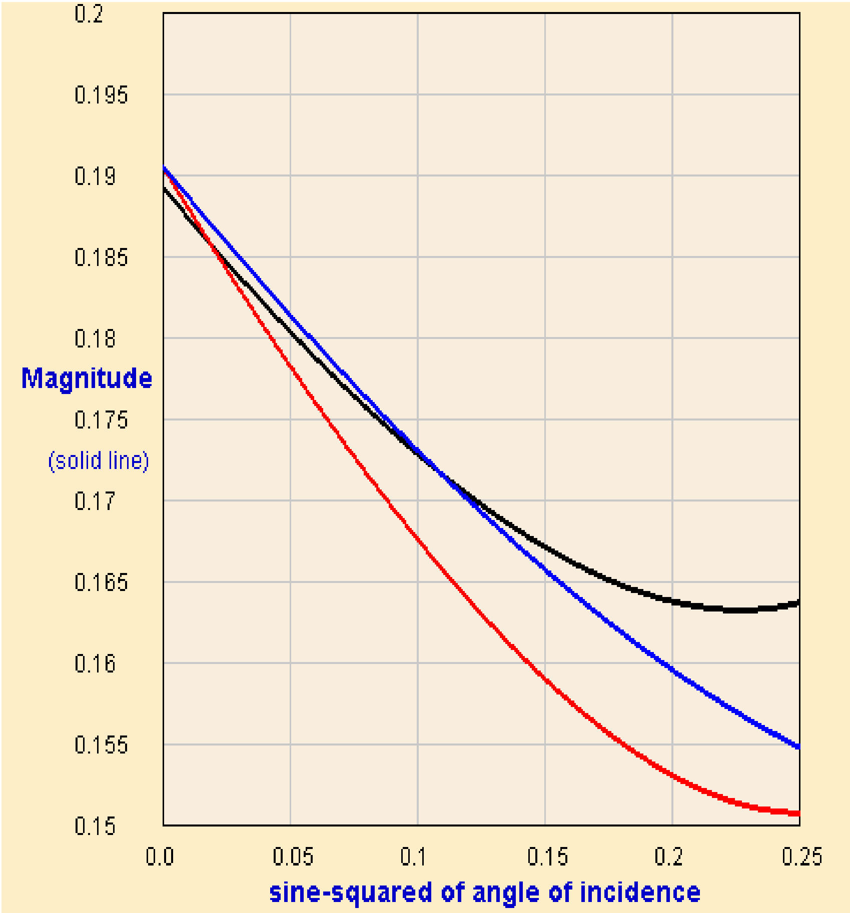
$$\frac{\Delta\alpha}{\alpha} = 0.286$$

$$\frac{\Delta\beta}{\beta} = 0.286$$

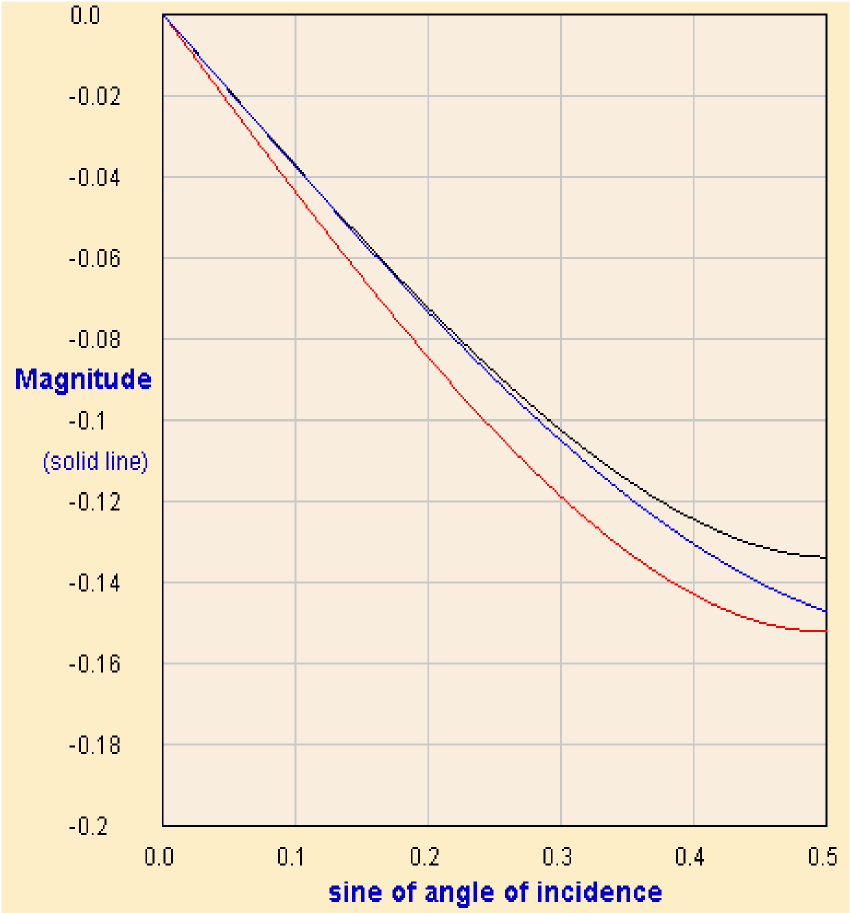
$$\frac{\Delta\rho}{\rho} = 0.0952$$

θ vs. θ_1 approximations

R_{PP}

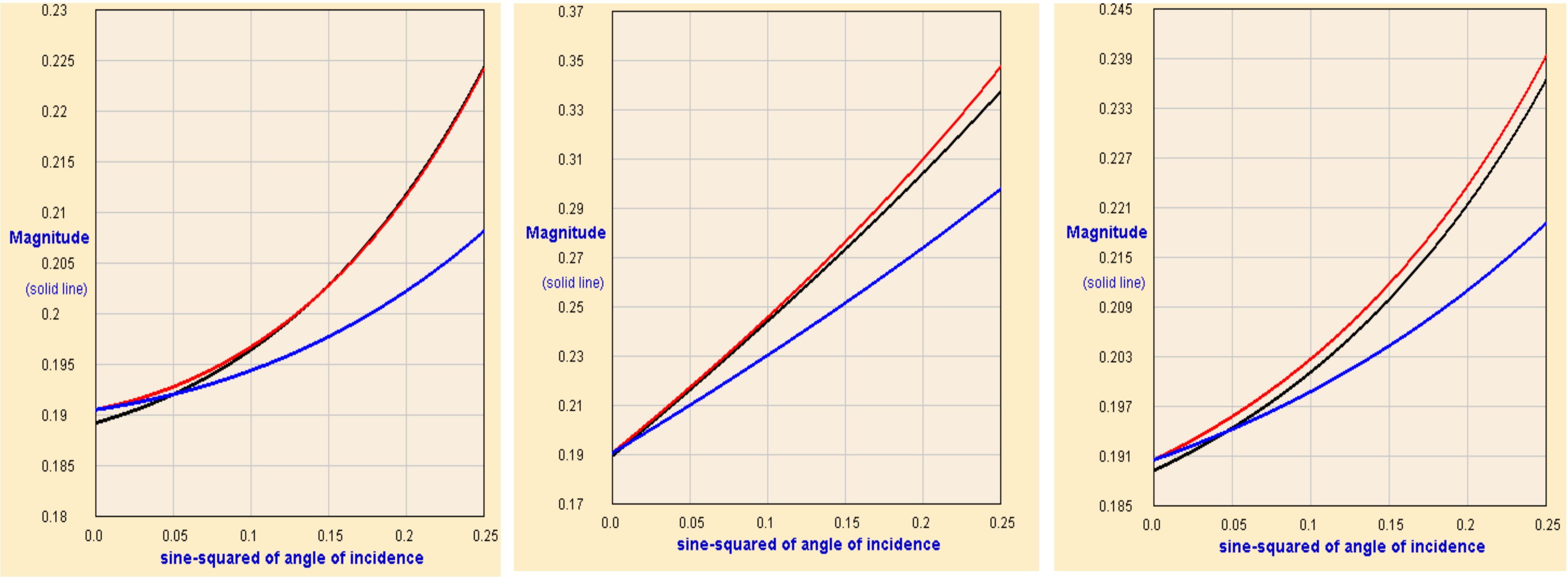


R_{PS}



θ vs. θ_1 approximations

a) $\gamma^{new} = 0.3$ b) $R_\beta^{new} = -R_\beta$ c) $R_\beta^{new} = 0.1R_\beta$



Effect of $\theta \rightarrow \theta_1$

- True linear behavior: no critical point
- R_{PS} more accurate in $0° < \theta_1 < 30°$ range
- R_{PP} more accurate if
  - $\gamma > .35$
  - $|R_\beta| > |R_\alpha|$, same sign
- R_{PP} less accurate if
  - $\gamma < .3$
  - $|R_\beta|$ small, or opposite sign to $|R_\alpha|$

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Why is θ_1 better at low angles?

- Differences disappear for $R_\alpha = 0$ ($\theta = \theta_1$)
- Used MAPLE to linearize R_{PP}^{exact}, R_{PS}^{exact} in R_β, R_ρ
- Find coefficient of sine-powers:

$$R_{PP}(\theta_1) = A + B\sin^2\theta_1 + \cdots, \qquad B = [R_\alpha - 4\gamma^2(2R_\beta + R_\rho)]\frac{1+R_\alpha}{1-R_\alpha}$$

$\frac{1+R_\alpha}{1-R_\alpha} \approx 1 + 2R_\alpha$; $\to (1-R_\alpha)^2$

$$R_{PP}(\theta) = A + B\sin^2\theta + \cdots, \qquad B = [R_\alpha - 4\gamma^2(2R_\beta + R_\rho)](1-R_\alpha^2)$$

$$R_{PS}(\theta_1) = A_S\sin\theta_1 + \cdots, \qquad A_S = -[R_\rho + 2\gamma(2R_\beta + R_\rho)]$$

$\to 1 - R_\alpha$

$$R_{PS}(\theta) = A_S\sin\theta + \cdots, \qquad A_S = -[R_\rho + 2\gamma(2R_\beta + R_\rho)](1-R_\alpha)$$

Alternate expression for *B*

- Used MAPLE to linearize R_{PP}^{exact} in R_ρ only

$$B^{\theta_1}(\text{nonlinear in } R_\beta;\, R_\rho = 0) = (R_\alpha - 8\gamma^2 R_\beta)\frac{(1+R_\alpha)}{1-R_\alpha} + \frac{16\gamma^3 R_\beta^2}{1-R_\alpha}$$

Set $R_\beta = R_\alpha, \quad \gamma = 1/2$

Then $B^{\theta_1} = -R_\alpha$

$$B^{\theta} = -R_\alpha (1 - R_\alpha)^2$$

$$B^{\text{Shuey}} = -R_\alpha$$

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A better expression?

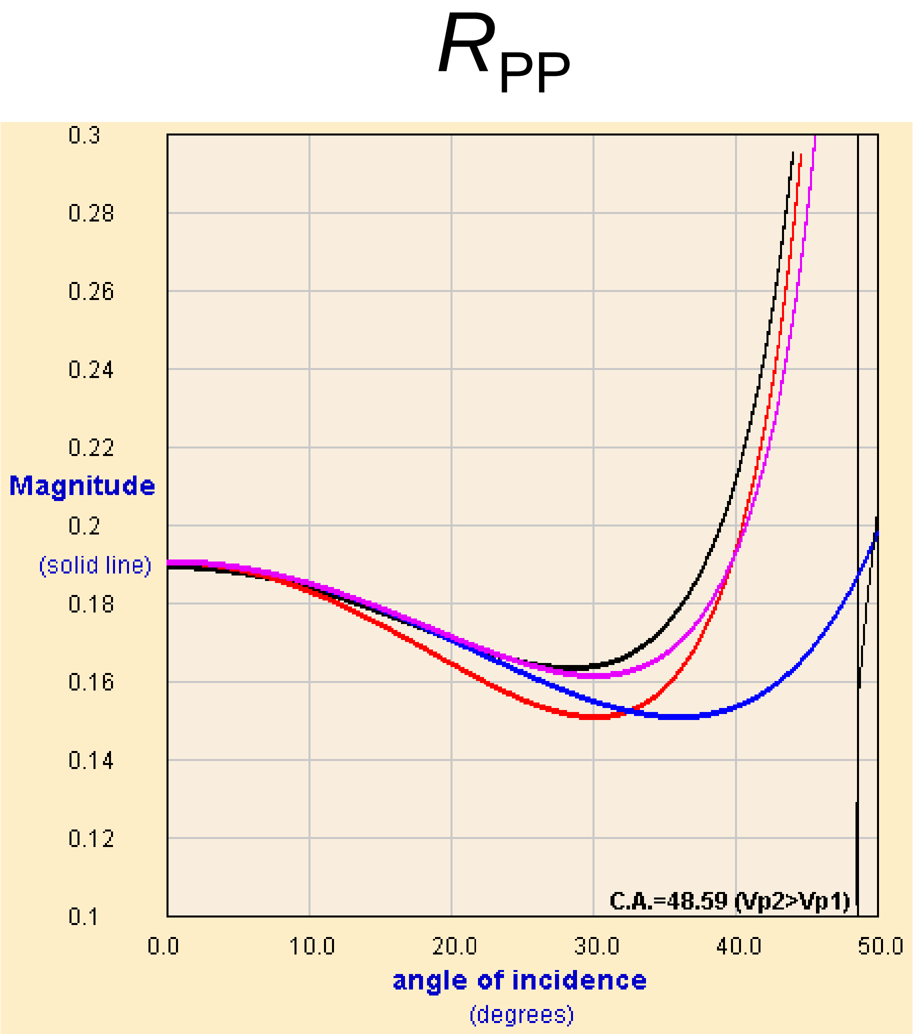
Note that $\sin\theta_1 = \sin\theta \dfrac{1 - R_\alpha}{\sqrt{1 + R_\alpha^2 \tan^2\theta}}$

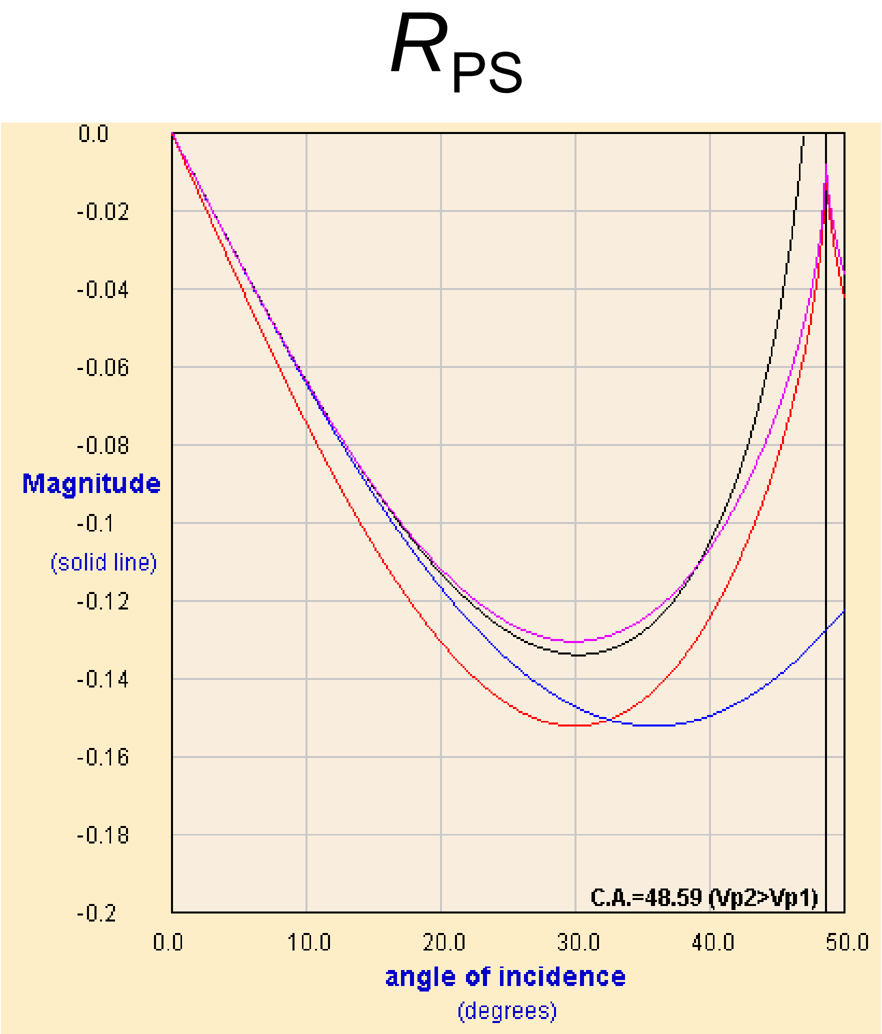
$$\approx \sin\theta(1 - R_\alpha)$$

Substitute $\sin\theta_1 = \sin\theta(1 - R_\alpha)$
in the initial gradients of the $\sin\theta_1$ expressions.

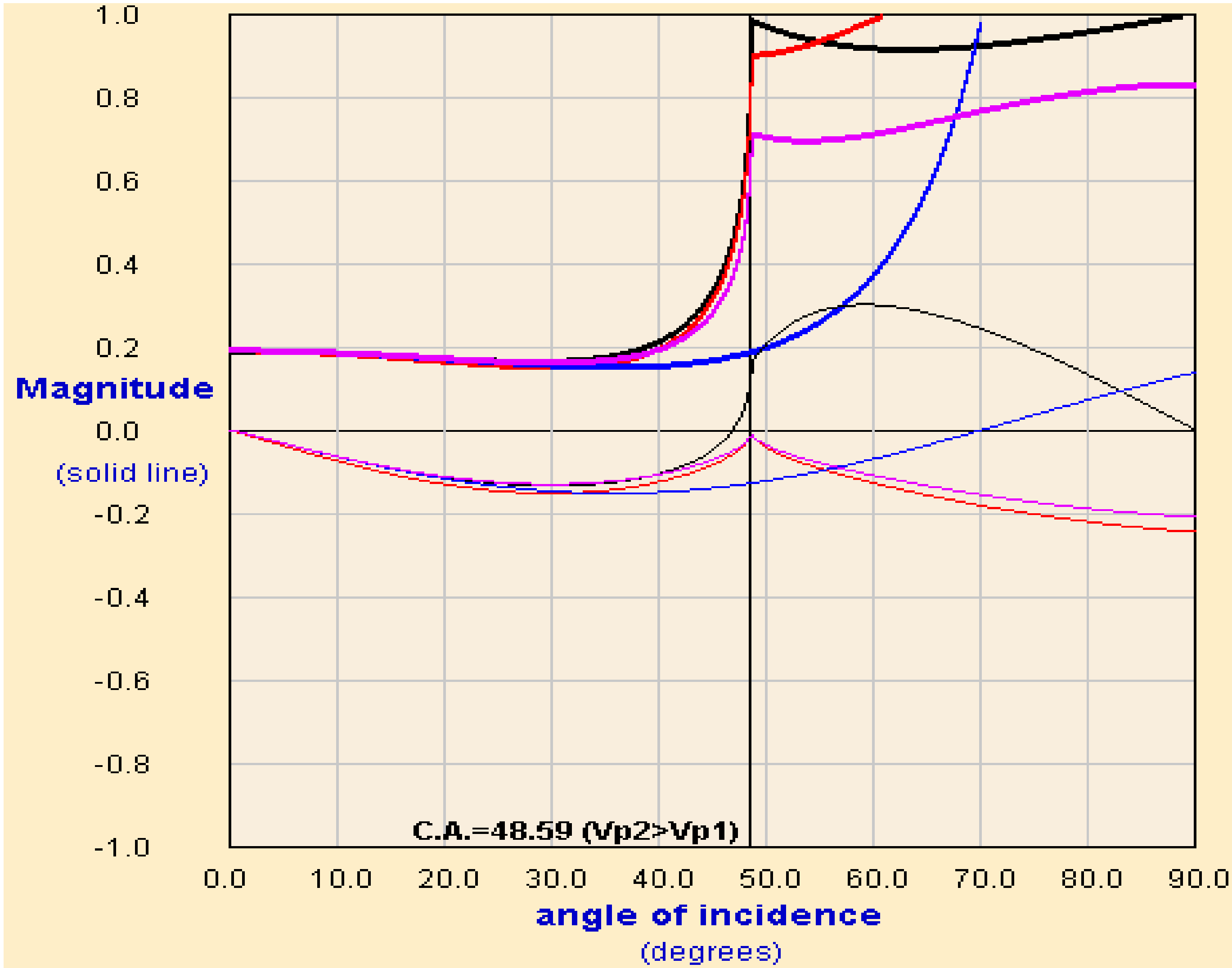
This should give better behavior at low angles *and* a critical point.

New approximation





New approximation



The Smith-Gidlow approximation

$$R_{\mathrm{PP}}^{\mathrm{S\text{-}G}} = \frac{1}{4}R_\alpha + \frac{R_\alpha}{\cos^2\theta} - 4\gamma^2 \sin^2\theta(2R_\beta + \frac{1}{4}R_\alpha)$$

The Fatti approximation

$$R_{\mathrm{PP}}^{\mathrm{Fatti}} = \frac{R_I}{\cos^2\theta} - 8\gamma^2 \sin^2\theta R_J + \left(4\gamma^2 \sin^2\theta - \tan^2\theta\right) R_\rho$$

Conclusions

- The Aki-Richards expression has been compared using both θ and θ_1 as the dependent variable
- The expression in terms of θ is best near the critical point
- The expression in terms of θ_1 is best at low angles for R_{PS} and certain regions of R_{PP}
- The quality of the θ_1 expression has been justified by theoretical analysis

Conclusions

- A new version of the Aki-Richards approximation is given in which $\sin\theta$ is multiplied by $(1 - R_\alpha)$
- An estimate of R_α is already required to obtain θ, so this requires no new information
- The new expression is more accurate for a wider range of low angles and has a correctly located critical point.