JOINT SIMULTANEOUS INVERSION OF *PP* AND *PS* SEISMIC DATA

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Agenda

- Introduction
- Theory
- Synthetic example
- Case history
- Conclusions

Introduction

Current inversion practice involves independent estimation of reflectivity followed by inversion to impedance. For example:

Post-stack P-impedance Inversion

Invert stacked P-wave data to produce P-impedance.

Post-stack S-impedance Inversion

Invert stacked S-wave or C-wave data to produce S-impedance.

Elastic Impedance

Invert independent angle stacks to derive “elastic” impedance.

AVO Inversion

Analyze pre-stack data to derive R_P, R_S, and R_D, and invert independently to P-impedance, S-impedance, and density.

Introduction

- The objective of “simultaneous” inversion is to invert directly from multiple seismic traces to multiple inverted results.

- We will also include some form of coupling between the variables.

- This should add stability to a problem that is “ill-conditioned”:
  - - very sensitive to noise
  - - very non-unique.

- A second objective is to create an inversion process which is consistent with model-based inversion for the case of zero-offset *P*-impedance inversion.

- We call the process “joint” if different datasets (e.g. *PP* and *PS*) are input.

The Linearized *PP* equation

We start with the modification of Aki-Richards' equation as per Fatti et al. (Geophysics, 1994):

$$R_{PP}(\theta) = c_1 R_P + c_2 R_S + c_3 R_D$$

where:

$$c_1 = 1 + \tan^2\theta$$

$$c_2 = -8\gamma^2 \sin^2\theta$$

$$c_3 = -\frac{1}{2}\tan^2\theta + 2\gamma^2 \sin^2\theta$$

$$\gamma = \frac{V_S}{V_P}$$

$$R_P = \frac{1}{2}\left[\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right]$$

$$R_S = \frac{1}{2}\left[\frac{\Delta V_S}{V_S} + \frac{\Delta\rho}{\rho}\right]$$

$$R_D = \frac{\Delta\rho}{\rho}.$$

Extension to *PS* data

Similarly to the Fatti equation, we can write down a linearized expression for the PS reflectivity (Stewart, 1990; Larsen, 1999, Margrave et al., 2001):

$$R_{PS}(\theta,\phi) = c_4 R_S + c_5 R_D$$

$$\text{where:} \quad c_4 = \frac{\tan\phi}{\gamma}\left[4\sin^2\phi - 4\gamma\cos\theta\cos\phi\right],$$

$$c_5 = \frac{-\tan\phi}{2\gamma}\left[1 + 2\sin^2\phi - 2\gamma\cos\theta\cos\phi\right],$$

$$\text{and:} \quad \phi = \sin^{-1}(\gamma\sin\theta).$$

The "small reflectivity" approximation

We also use the "small reflectivity" approximation to relate the impedances (*Z*) to the reflectivities (*R*). In general:

$$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} \approx \frac{\Delta Z_i}{2Z_i} = \frac{\Delta \ln Z_i}{2} = \frac{1}{2}\left[L_{i+1} - L_i\right]$$

$$\text{where: } L_i = \ln Z_i$$

This applies to all three reflectivities:

$$R_{Pi} = \frac{1}{2}\left[L_{Pi+1} - L_{Pi}\right], R_{Si} = \frac{1}{2}\left[L_{Si+1} - L_{Si}\right], R_{Di} = \frac{1}{2}\left[L_{Di+1} - L_{Di}\right]$$

From seismic trace to impedance

If we add the effect of the wavelet, we can express the seismic trace in matrix notation as:

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{n-1} \end{bmatrix} = 0.5 \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ \vdots & w_1 & \ddots & \vdots \\ w_k & \vdots & \ddots & 0 \\ 0 & w_k & \vdots & w_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & w_k \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{bmatrix}$$

$$T = WR = WDL$$

Note that the trace and wavelet can be angle dependent.

The “uncoupled” equations

Combining all the previous ideas, the *PP* and *PS* equations now relate the angle dependent seismic traces to impedance:

$$T_{PP}(\theta) = c_1 W(\theta) DL_P + c_2 W(\theta) DL_S + c_3 W(\theta) DL_D$$

and:

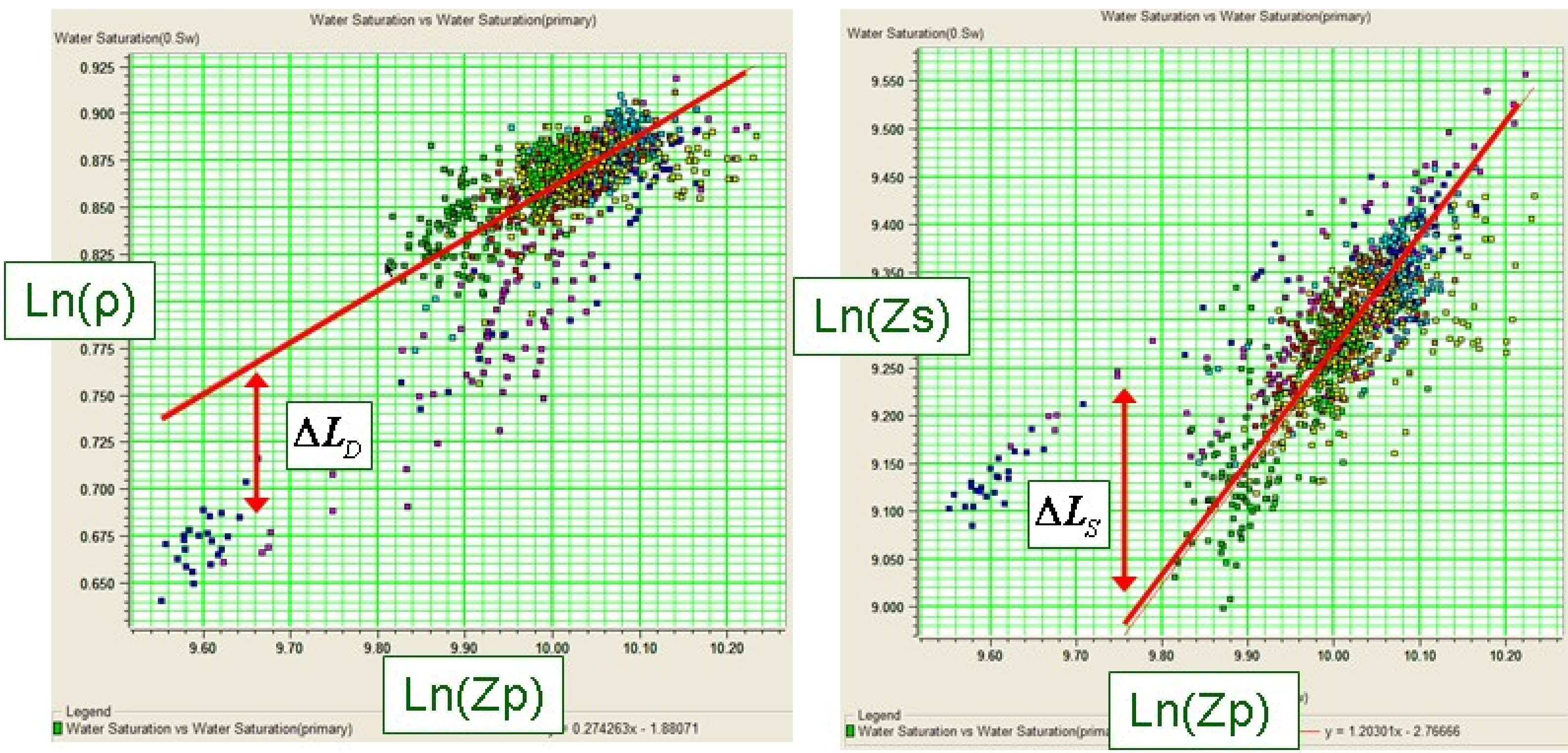
$$T_{PS}(\phi) = c_4 W(\phi) DL_S + c_5 W(\phi) DL_D$$

However, the above equations still have not been coupled in any way.

Coupling between variables

To do this we can make use of the fact that the resulting Z_S and ρ should be related to Z_P:

$$\ln(Z_S) = k\ln(Z_P) + k_c + \Delta L_S$$
$$\ln(\rho) = m\ln(Z_P) + m_c + \Delta L_D$$



The "coupled" equations

Substituting into the previous *PP* and *PS* equations we get:

$$T_{PP}(\theta) = \tilde{c}_1 W(\theta) DL_P + c_2 W(\theta) D\Delta L_S + 2c_3 W(\theta) D\Delta L_D,$$

$$\text{where}: \tilde{c}_1 = c_1 + kc_2 + mc_3$$

and:

$$T_{PS}(\phi) = \tilde{c}_4 W(\phi) DL_P + (c_4/2) W(\phi) DL_S + c_5 W(\phi) DL_D,$$

$$\text{where}: \tilde{c}_4 = kc_4/2 + mc_5$$

The above equations can now be written as a set of linearized and coupled equations, as shown on the next slide.

The final linearized form

Assuming that we have *N PP* angle stacks and *M PS* angle stacks, we can extend the earlier joint *PP* inversion matrix to the joint *PS* inversion matrix in the following way:

$$
\begin{bmatrix} T_{PP}(\theta_1) \\ \vdots \\ T_{PP}(\theta_N) \\ T_{PS}(\phi_1) \\ \vdots \\ T_{PS}(\phi_M) \end{bmatrix} = \begin{bmatrix} \tilde{c}_1(\theta_1)W(\theta_1)D & c_2(\theta_1)W(\theta_1)D & c_3(\theta_1)W(\theta_1)D \\ \vdots & \vdots & \vdots \\ \tilde{c}_1(\theta_N)W(\theta_N)D & c_2(\theta_N)W(\theta_N)D & c_3(\theta_N)W(\theta_N)D \\ \tilde{c}_4(\phi_1)W(\phi_1)D & (c_4(\phi_1)/2)W(\phi_1)D & c_5(\phi_1)W(\phi_1)D \\ \vdots & \vdots & \vdots \\ \tilde{c}_4(\phi_M)W(\phi_M)D & (c_4(\phi_M)/2)W(\phi_M)D & c_5(\phi_M)W(\phi_M)D \end{bmatrix} \begin{bmatrix} L_P \\ \Delta L_S \\ \Delta L_D \end{bmatrix}
$$

The solution is then found using an iterative solution.

The inversion algorithm

The algorithm looks like this:

(1) Given the following information:
- A set of *N* *PP* angle traces and *M* *PS* angle traces.
- A set of *N*+*M* wavelets, one for each angle.
- An initial model for Z_P.

(2) Calculate optimal values for *k* and *m* using the input logs.

(3) Set up the initial guess:

$$\begin{bmatrix} L_P & \Delta L_S & \Delta L_D \end{bmatrix}^T = \begin{bmatrix} \log(Z_P) & 0 & 0 \end{bmatrix}^T$$

(4) Solve the system of equations by conjugate gradients.

(5) Calculate the final values of Z_P, Z_S, and ρ:

$$Z_P = \exp(L_P)$$

$$Z_S = \exp(kL_P + k_c + \Delta L_S)$$

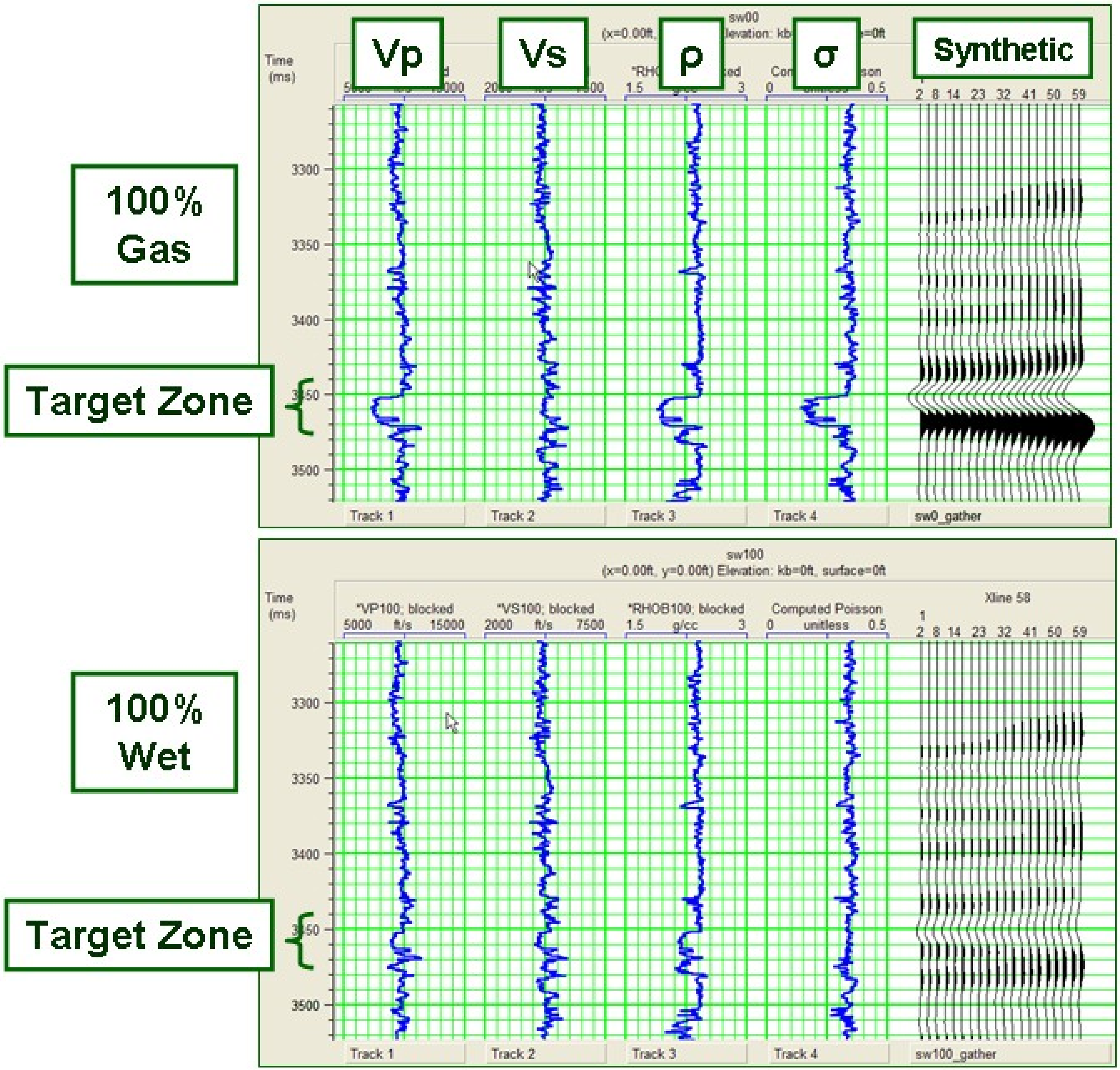
$$\rho = \exp(mL_P + m_c + \Delta L_D)$$

PP Synthetic test

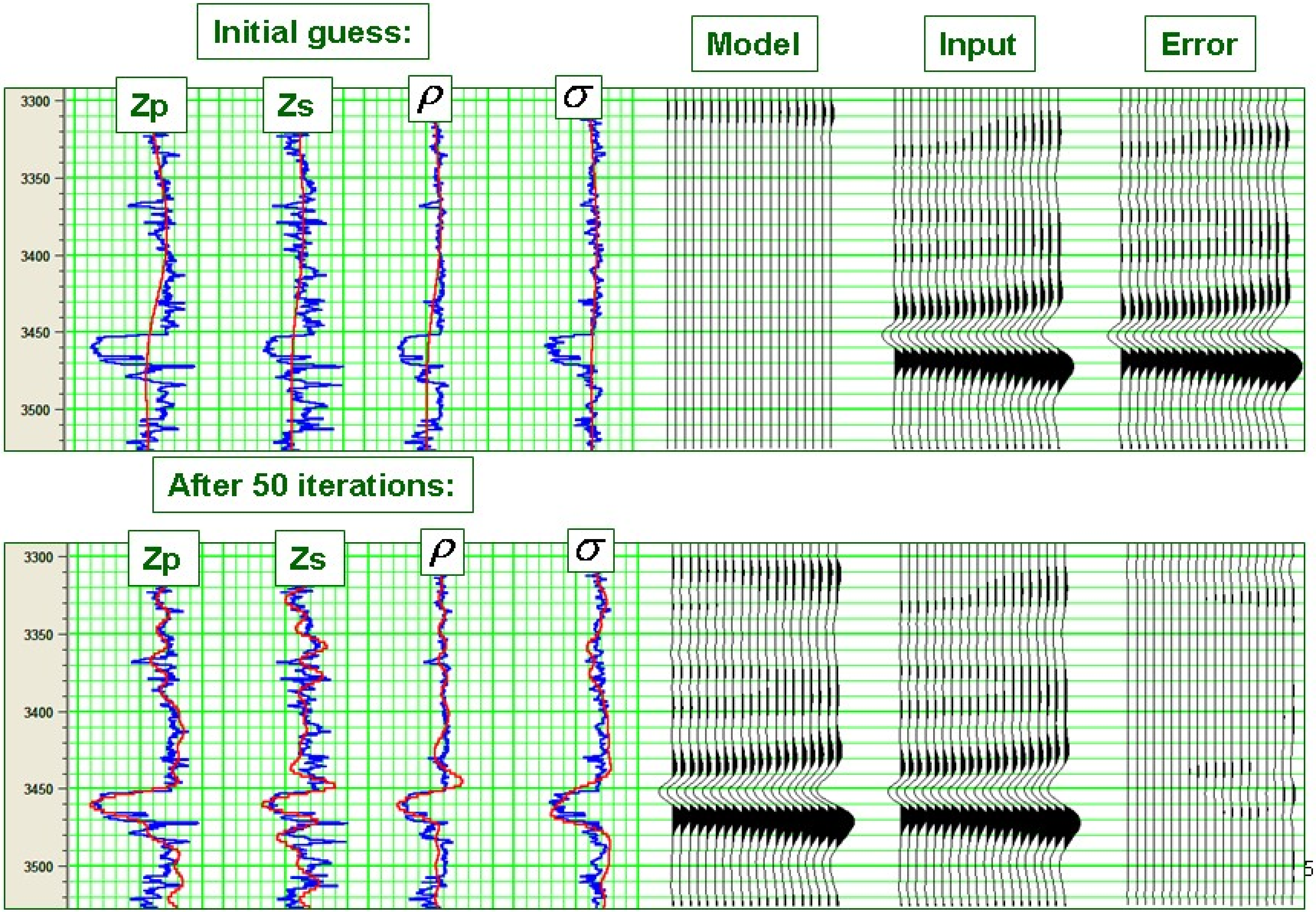
As a test, we produced a series of synthetic gathers corresponding to varying fluid effects.

The synthetics were created using Biot-Gassmann substitution and elastic wave modeling.

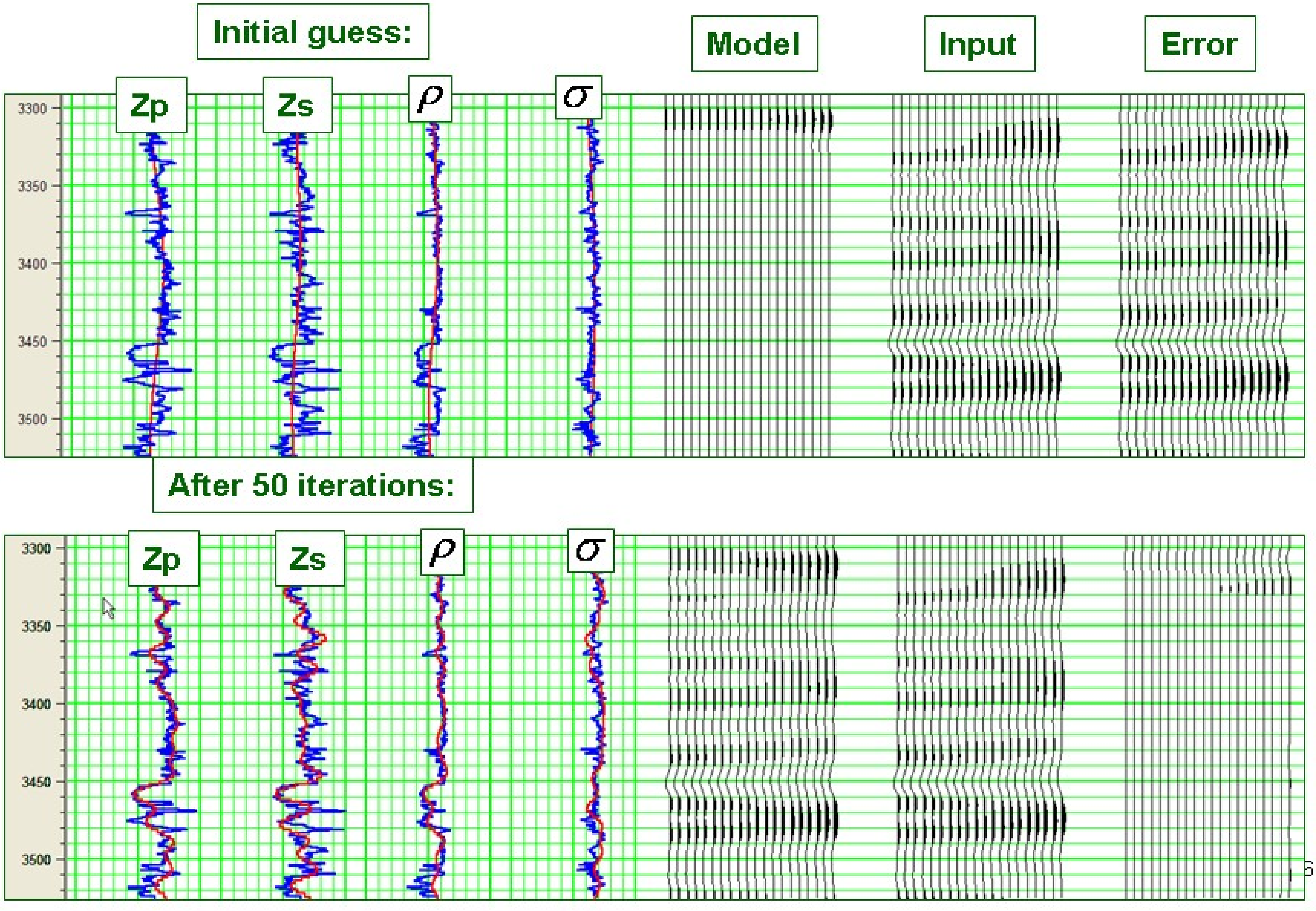
Two of these synthetics are shown here.



Result at the gas location



Result at the wet location

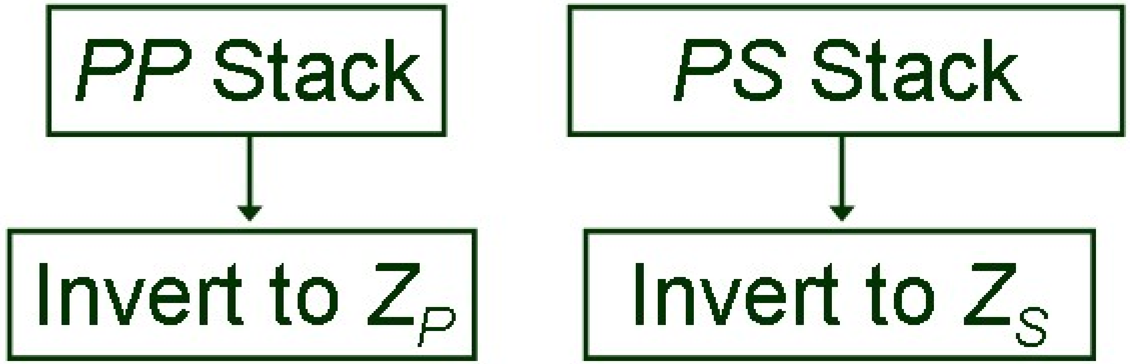


Case histories

- In previous presentations we have shown several case histories using PP angle gathers, including:
  - The Marlin field in GoM.
  - The Colony sand of Alberta
  - A heavy oil example from Alberta
- We have also used this method successfully using pre-stack PP and PS data on several international datasets for which we do not have permission to publish.
- In the following case study, we will use the simpler example of stacked PP and PS data as input.

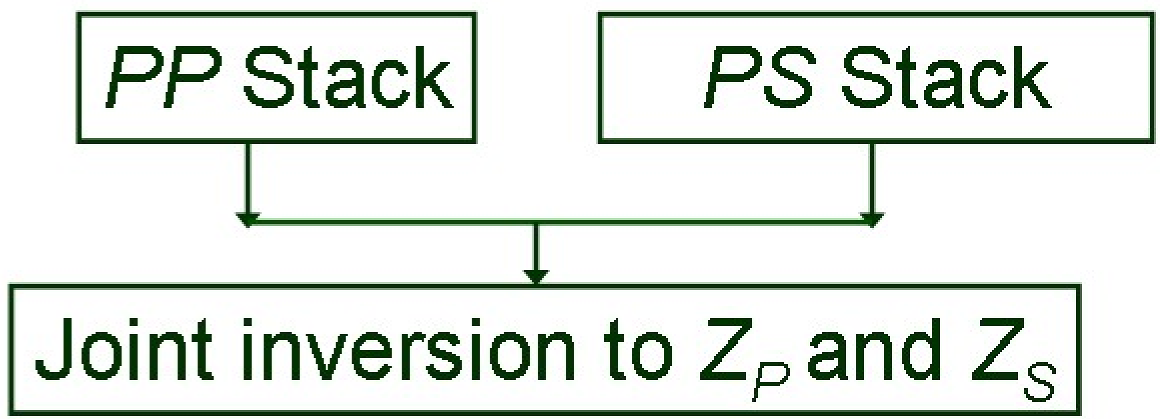
Case History

(A)



Current practice (A) is to invert separately for Z_P and Z_S from the *P* wave and converted wave datasets independently, which involves an assumption which estimates R_S reflectivity from the *PS* data.

(B)



Method (B) will involve joint inversion for Z_P and Z_S

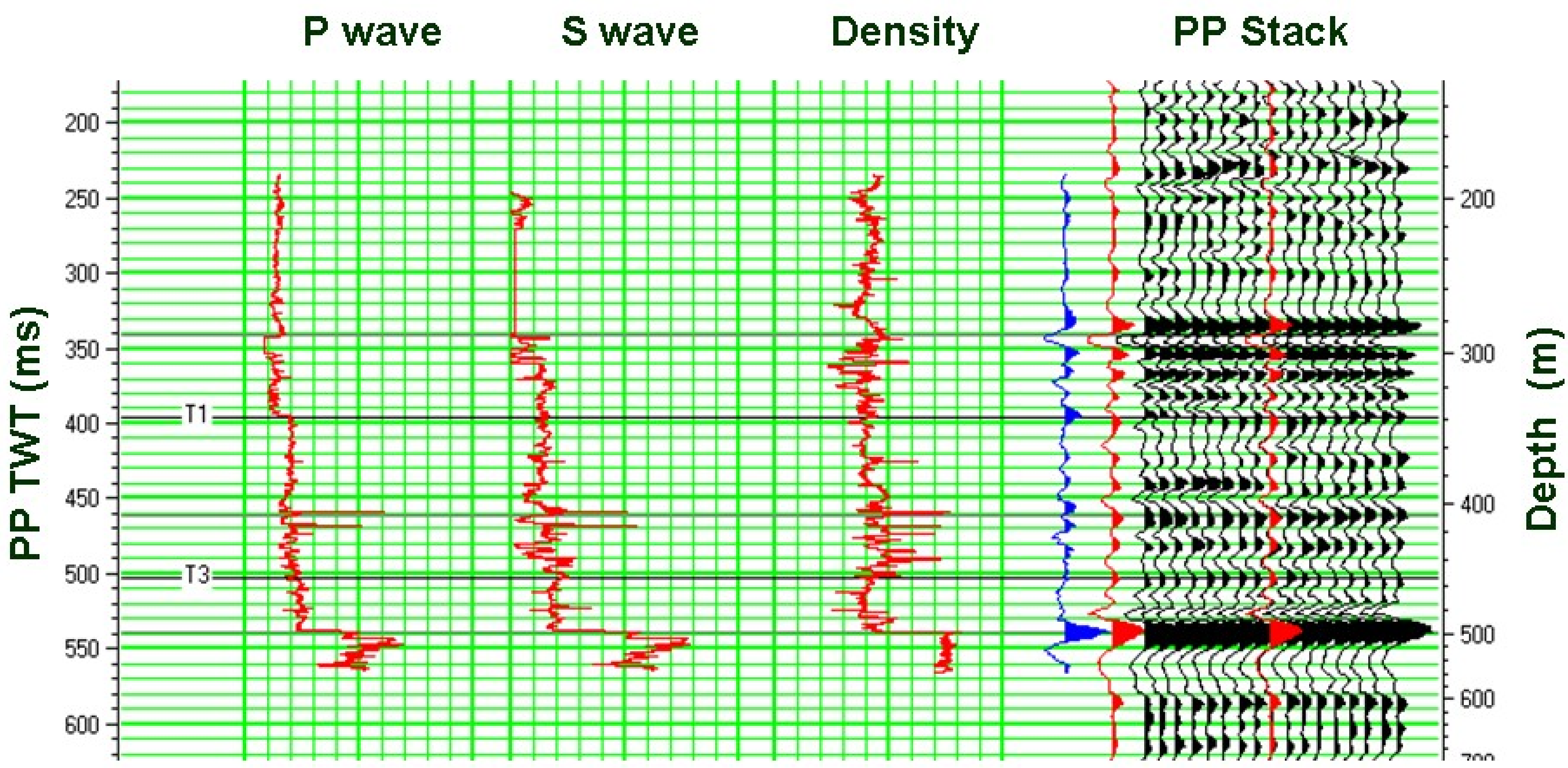
Case history

Study Location:
- Northeast Alberta, Canada

Study objective:
- Delineate areas of best sand development using P and PS data

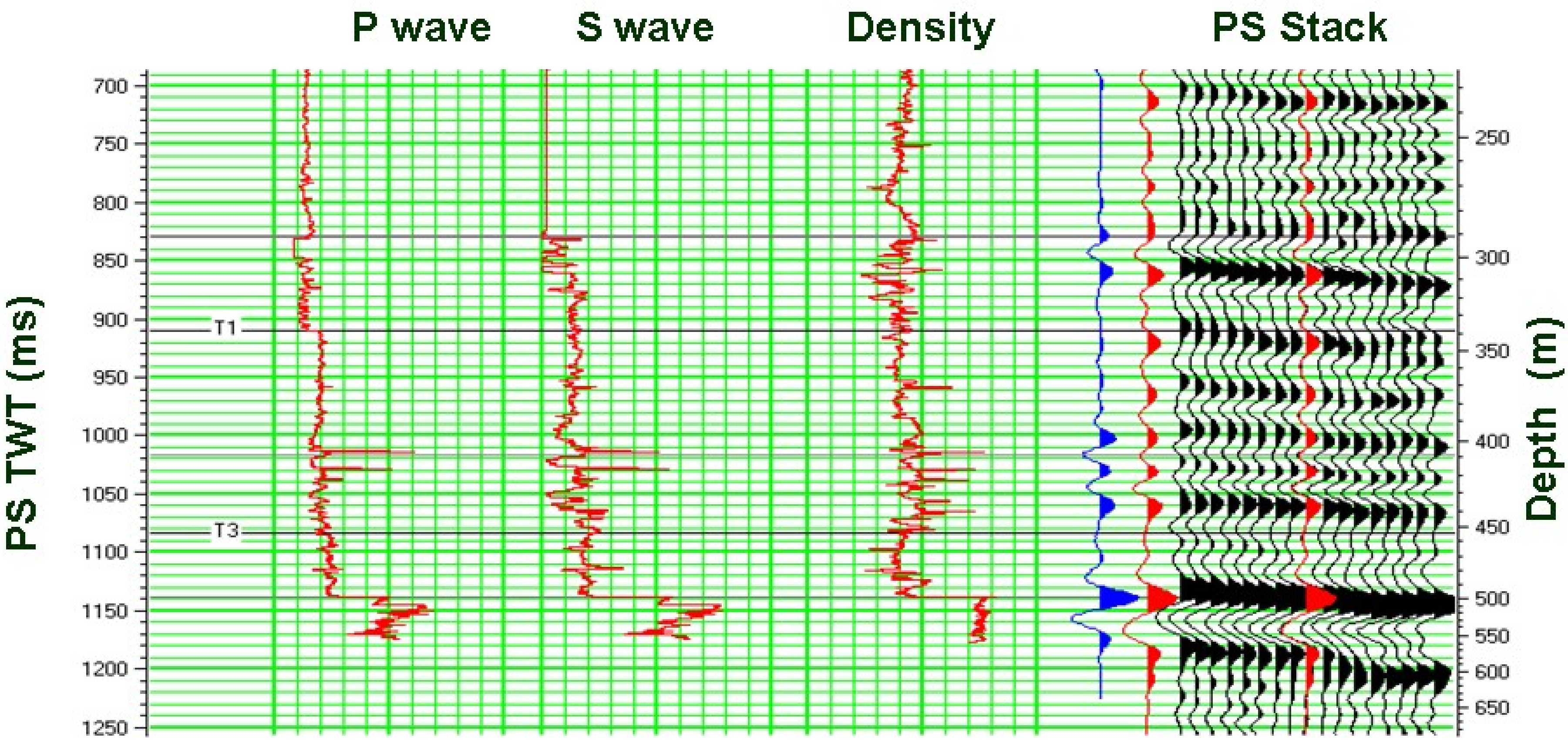
Workflow:
1. Correlate P and PS data to wells
2. Pick corresponding horizons on both datasets.
3. Use horizon based event matching to convert PS data to PP time.
4. Invert PP and PS data using independent inversion.
5. Invert PP and PS data using simultaneous inversion.

P wave log correlation



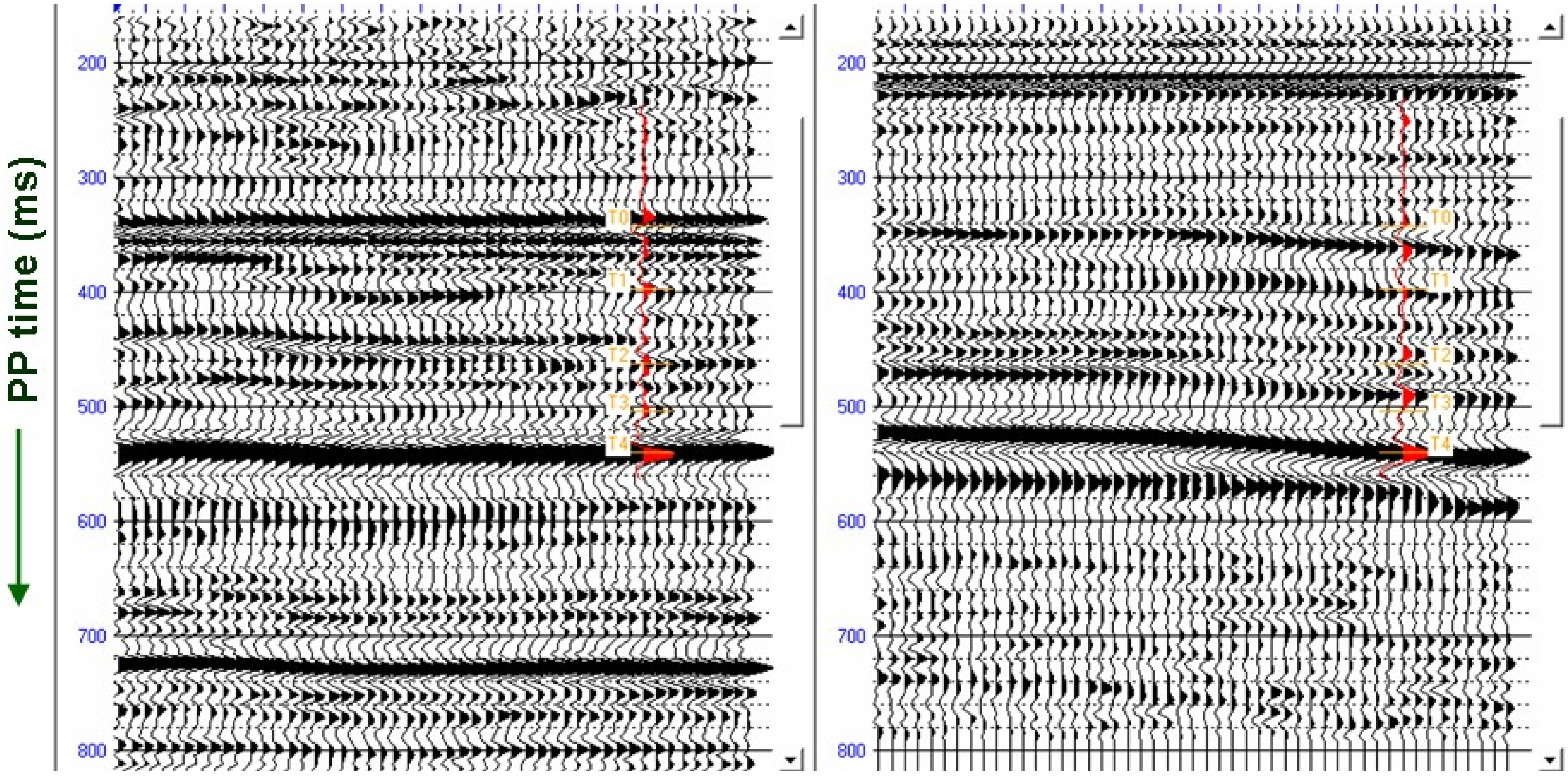
Data displayed in PP time with correlation between PP synthetic and PP stack.

PS log correlation



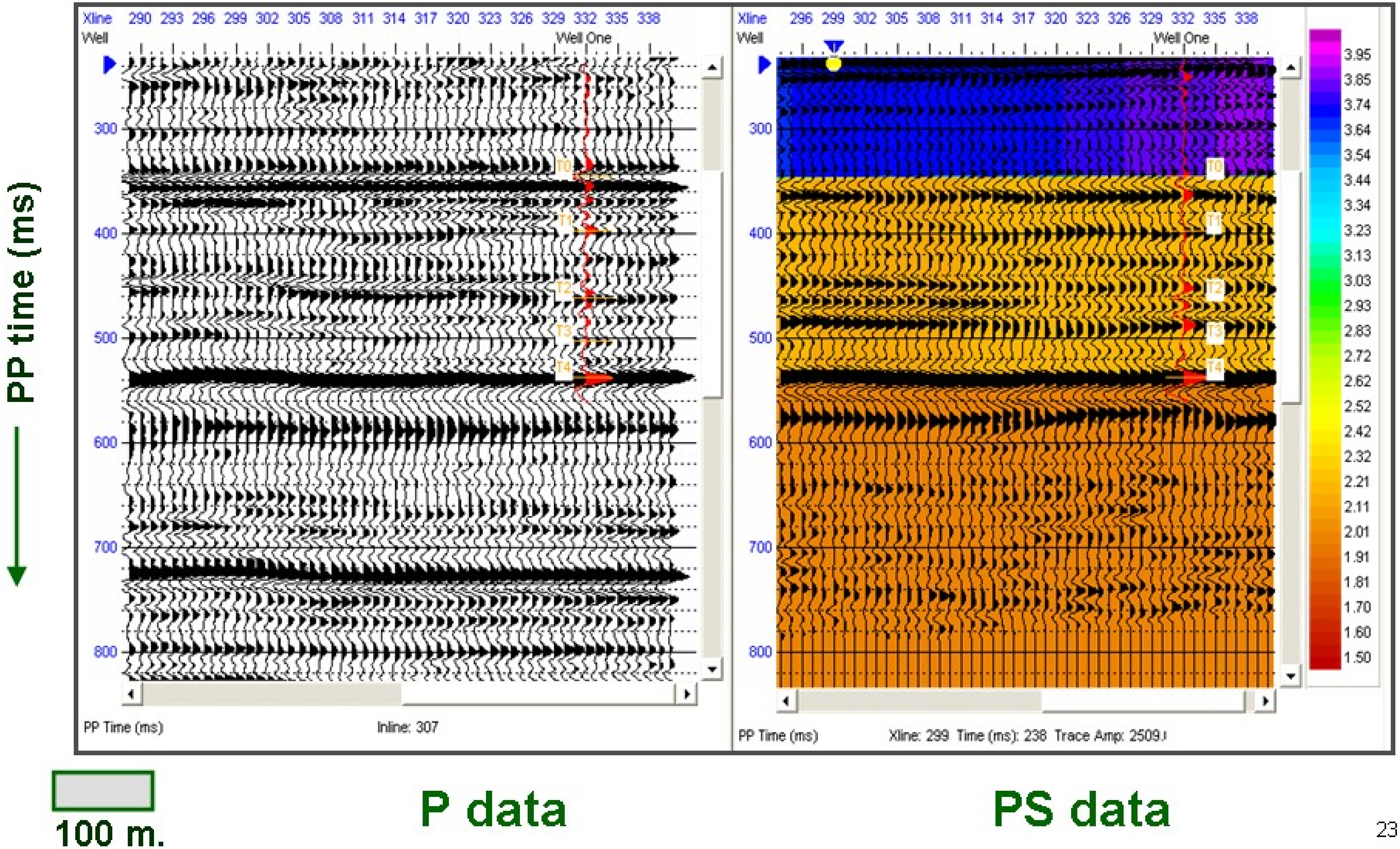
Data displayed in PS time with correlation between PS synthetic and PS stack.

P - PS seismic and synthetic ties (PP time)



Data displayed in PP time – structure differences due to variations in Vp/Vs

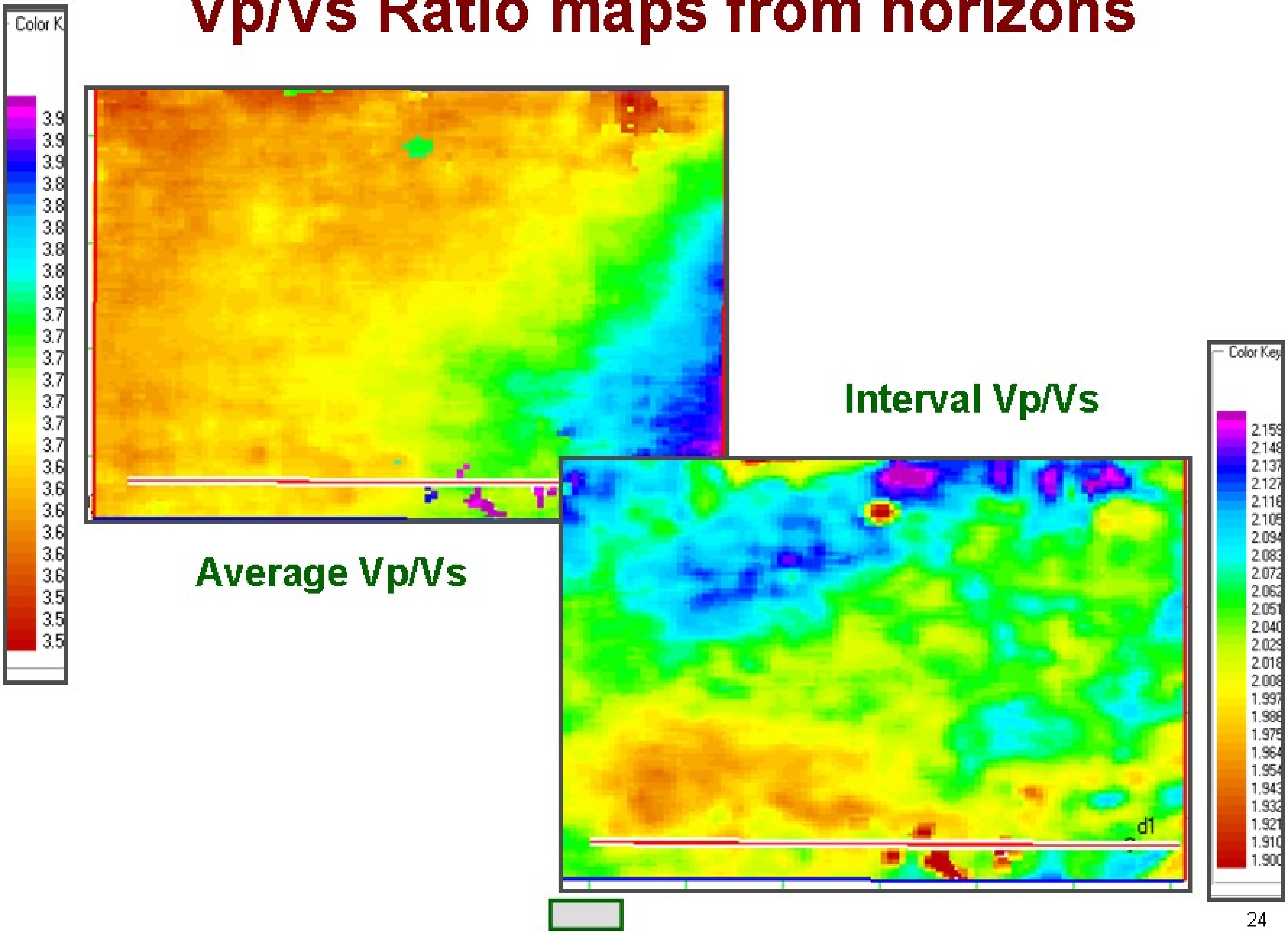
Horizon matching showing PS data with derived VpVs ratio (in PP time)



P data

PS data

Vp/Vs Ratio maps from horizons



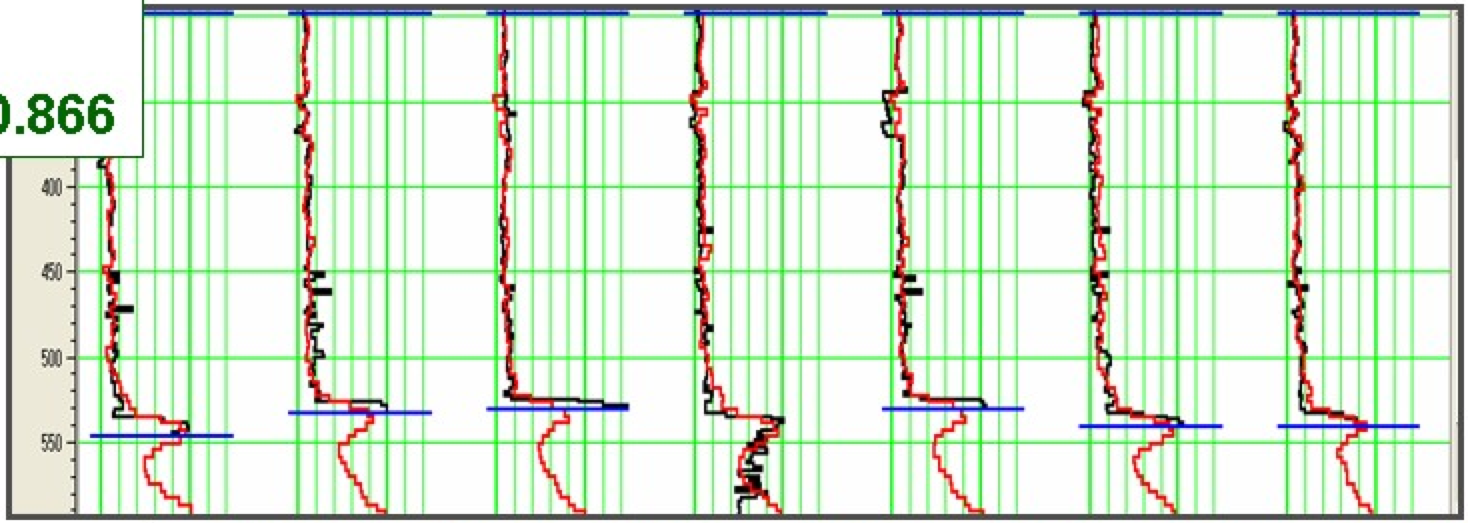
The inversion procedure

- In the following comparison, we will perform two different types of inversion: independent inversion, and joint inversion.
- For the independent inversion, we invert the PP and PS stacks independently, as the name suggests. The assumption was zero-offset for both stacks, where a scale factor was computed to transform the PS stack.
- For the joint inversion, we invert the two stacks jointly, and assume that the inputs are angle stacks. We therefore need an average angle for each stack.
- Empirically, we found that the PP angle stack was at an average angle of 20^0, and the PS angle stack was at an average angle of 10^0.

Inverted P-Impedance

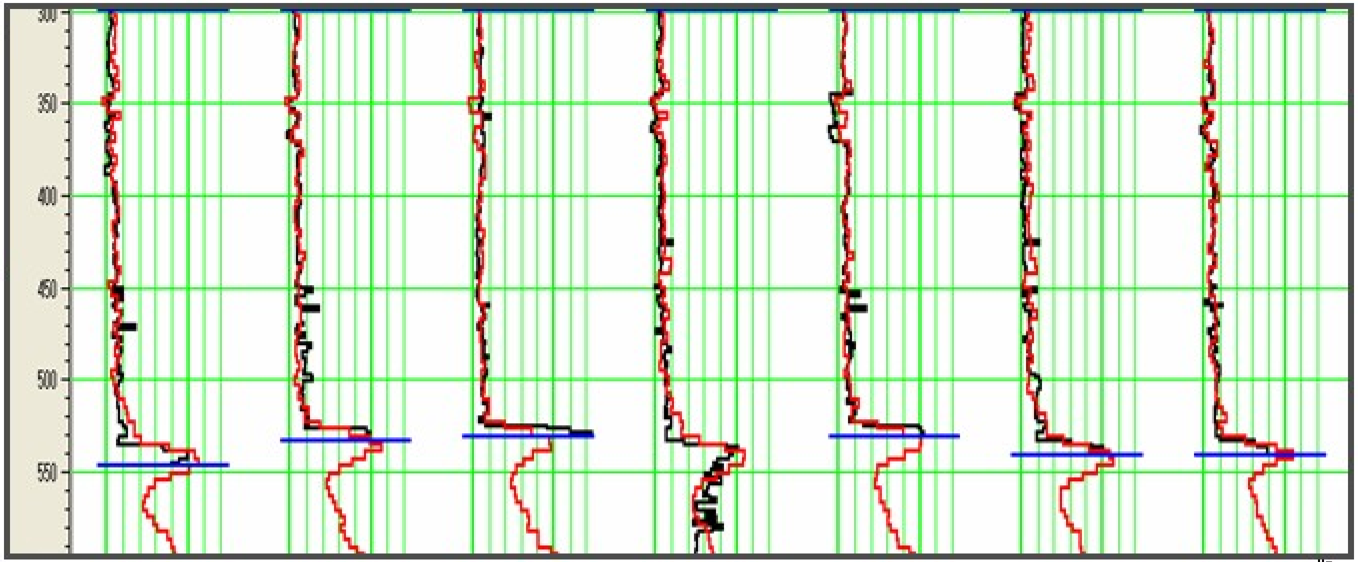
(A) Independent Inversion

Correlation 0.866



(B) Joint Inversion

Correlation: 0.87



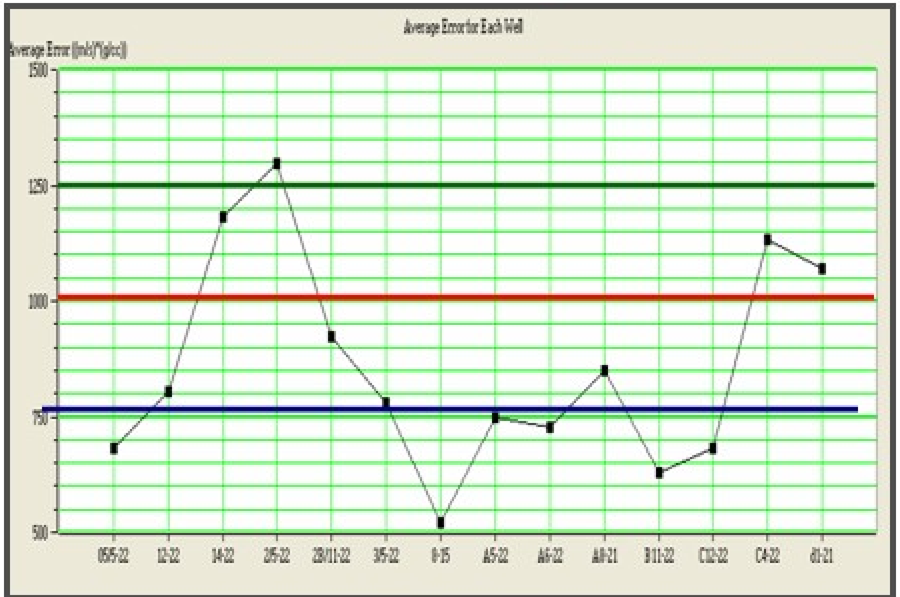
Z_P: Average Error Well by Well

(A) Independent Inversion

1250

1000

750

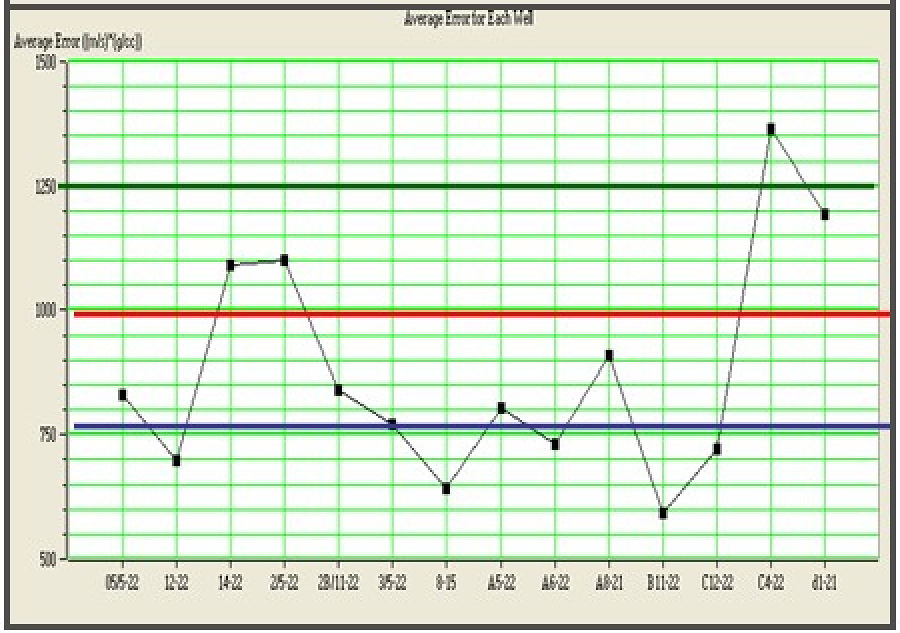


(B) Joint Inversion

1250

1000

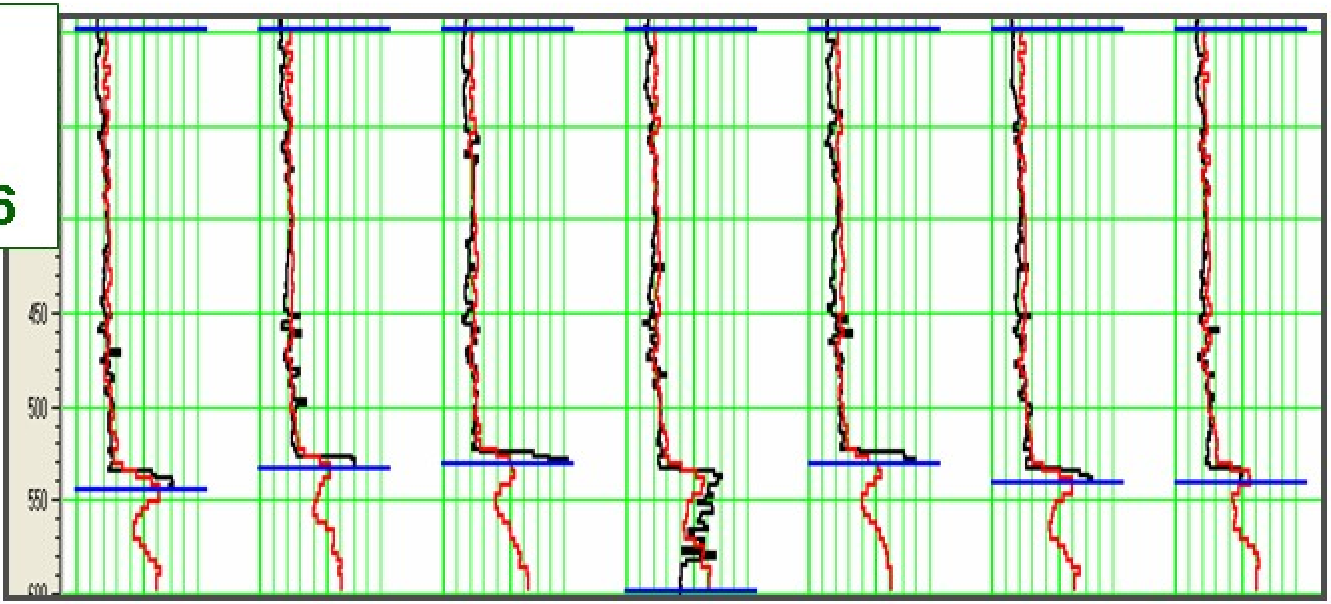
750



Inverted S-Impedance

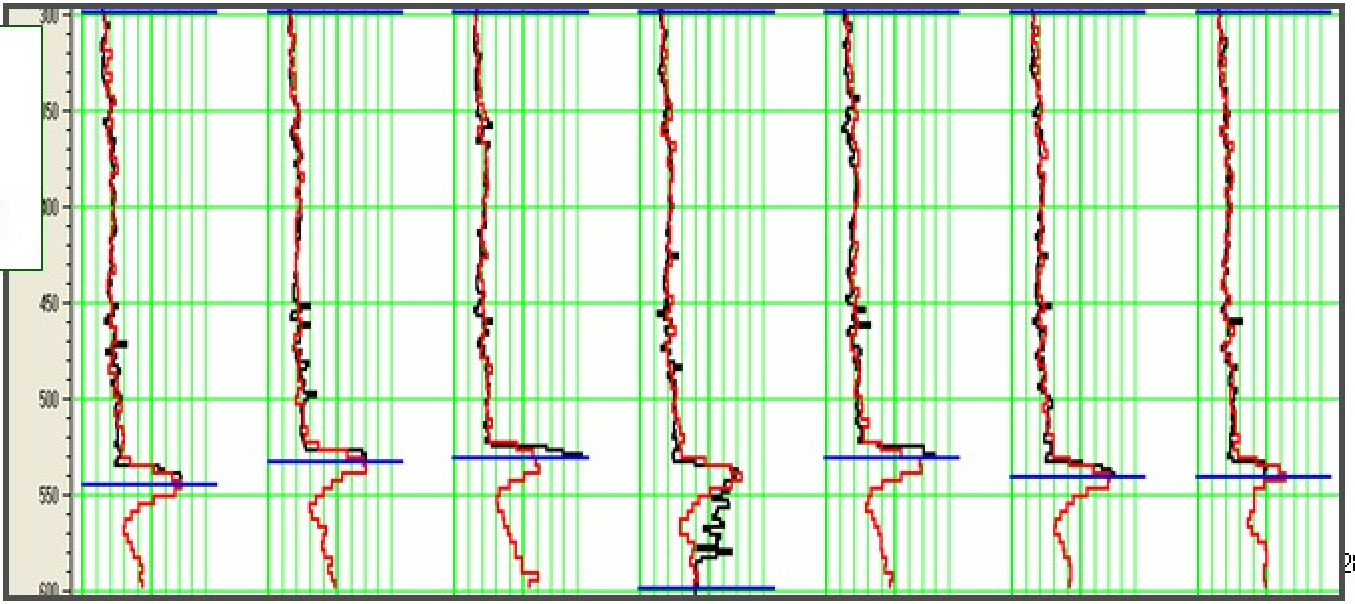
(A) Independent Inversion

Correlation: 0.856



(B) Joint Inversion

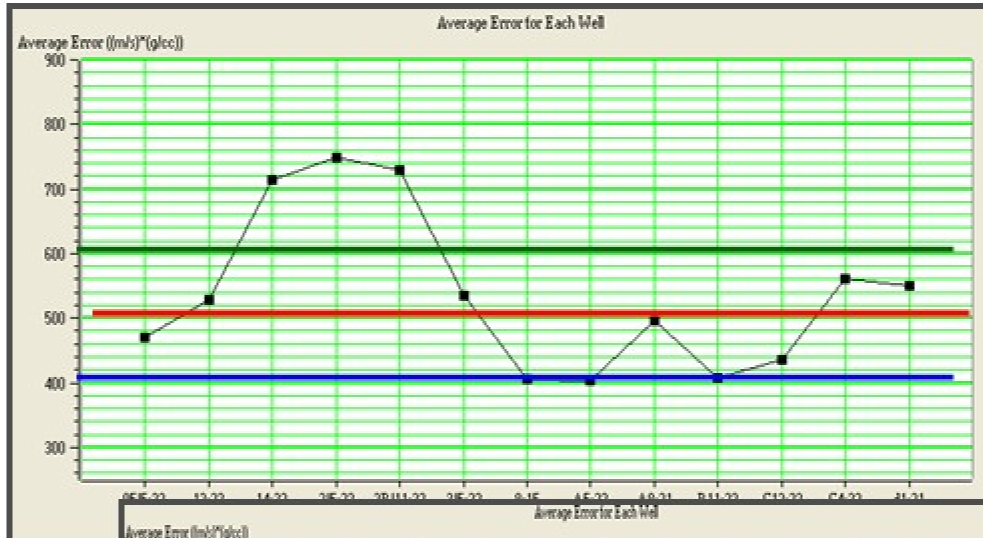
Correlation 0.872



Zs: Average Error Well by Well

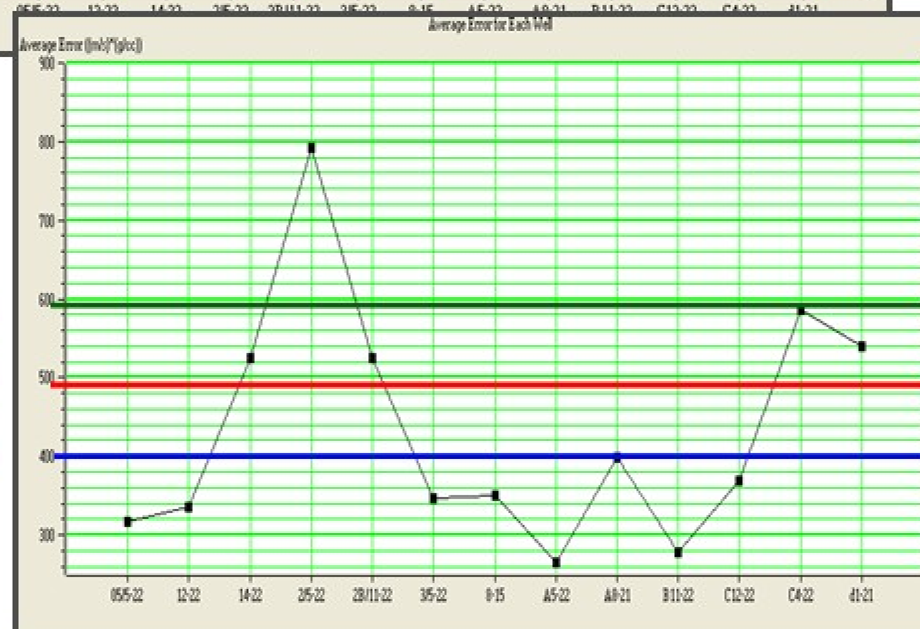
(A) Independent Inversion

600
500
400

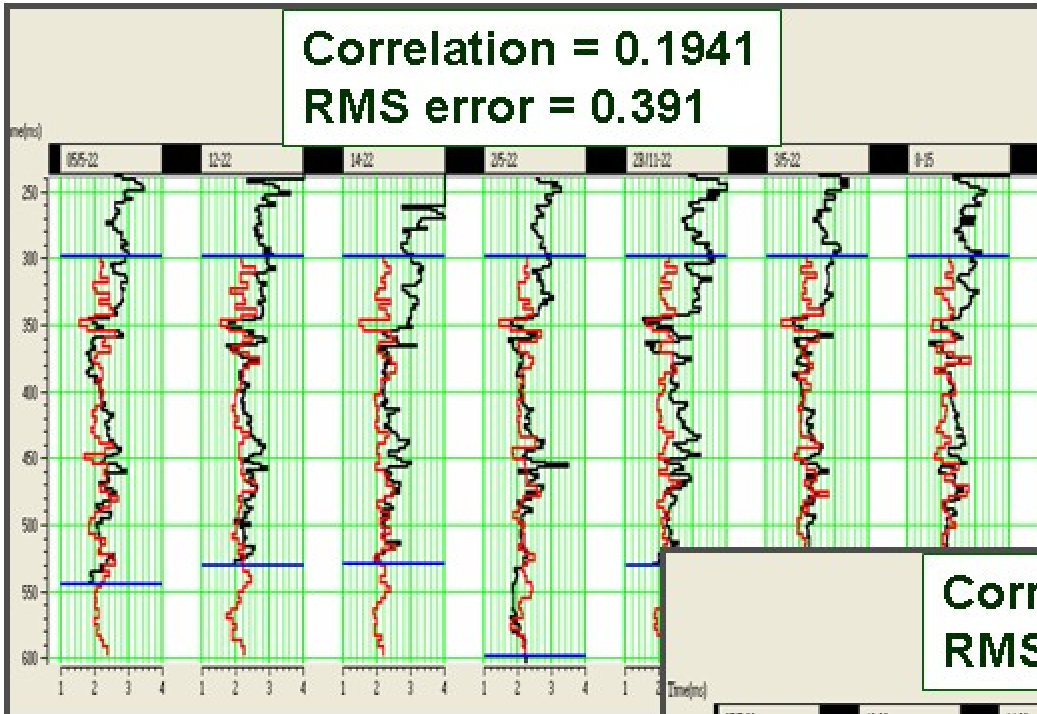


(B) Joint Inversion

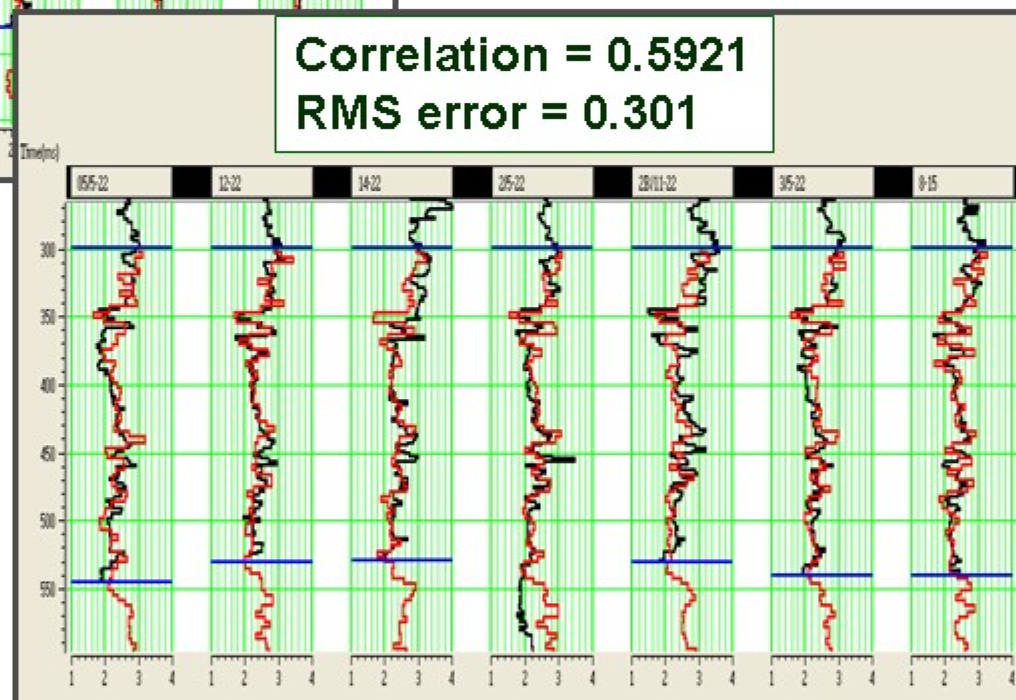
600
500
400



Vp/Vs Ratio



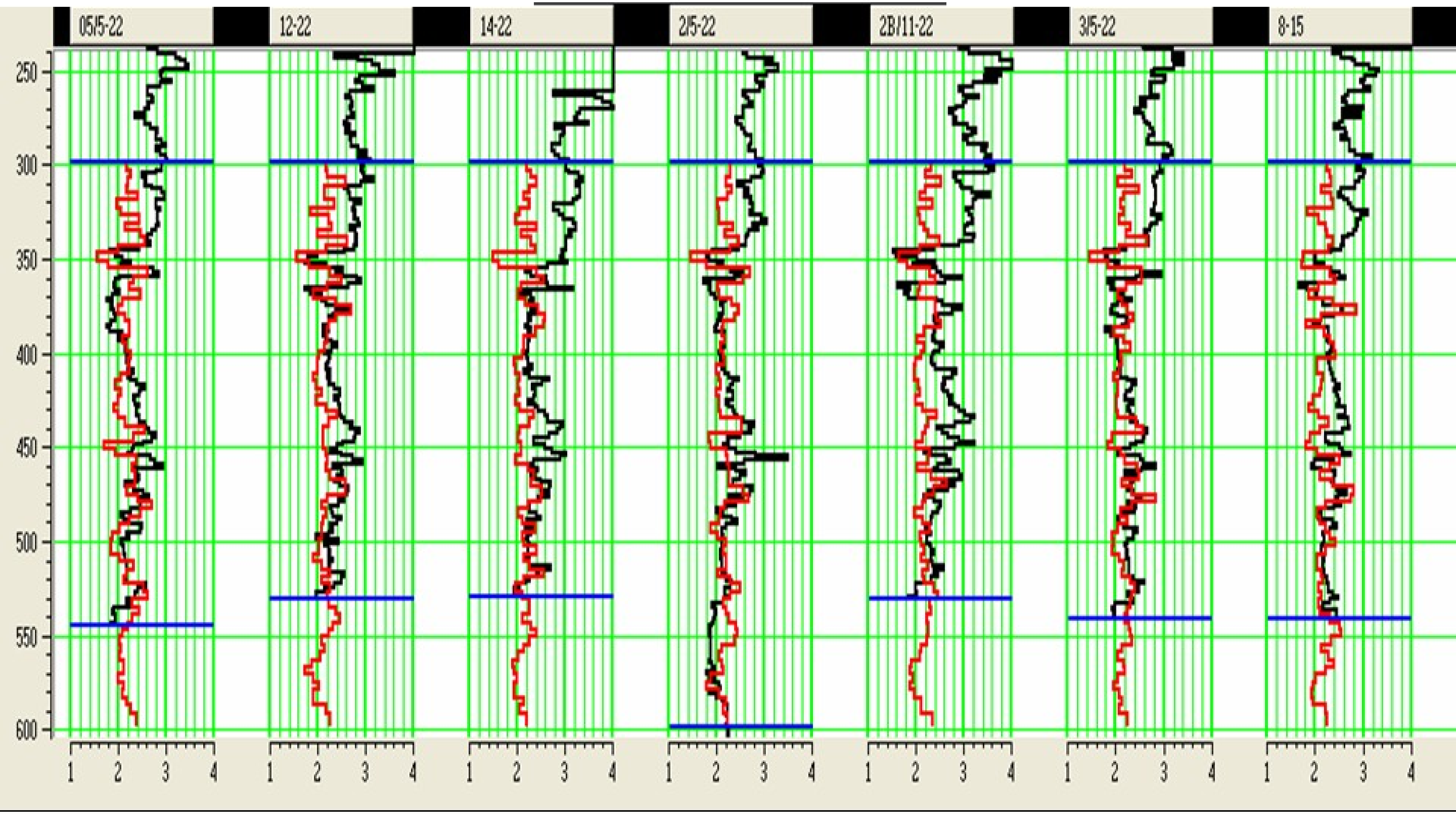
(A) Independent Inversion



(B) Joint Inversion

Vp/Vs Ratio

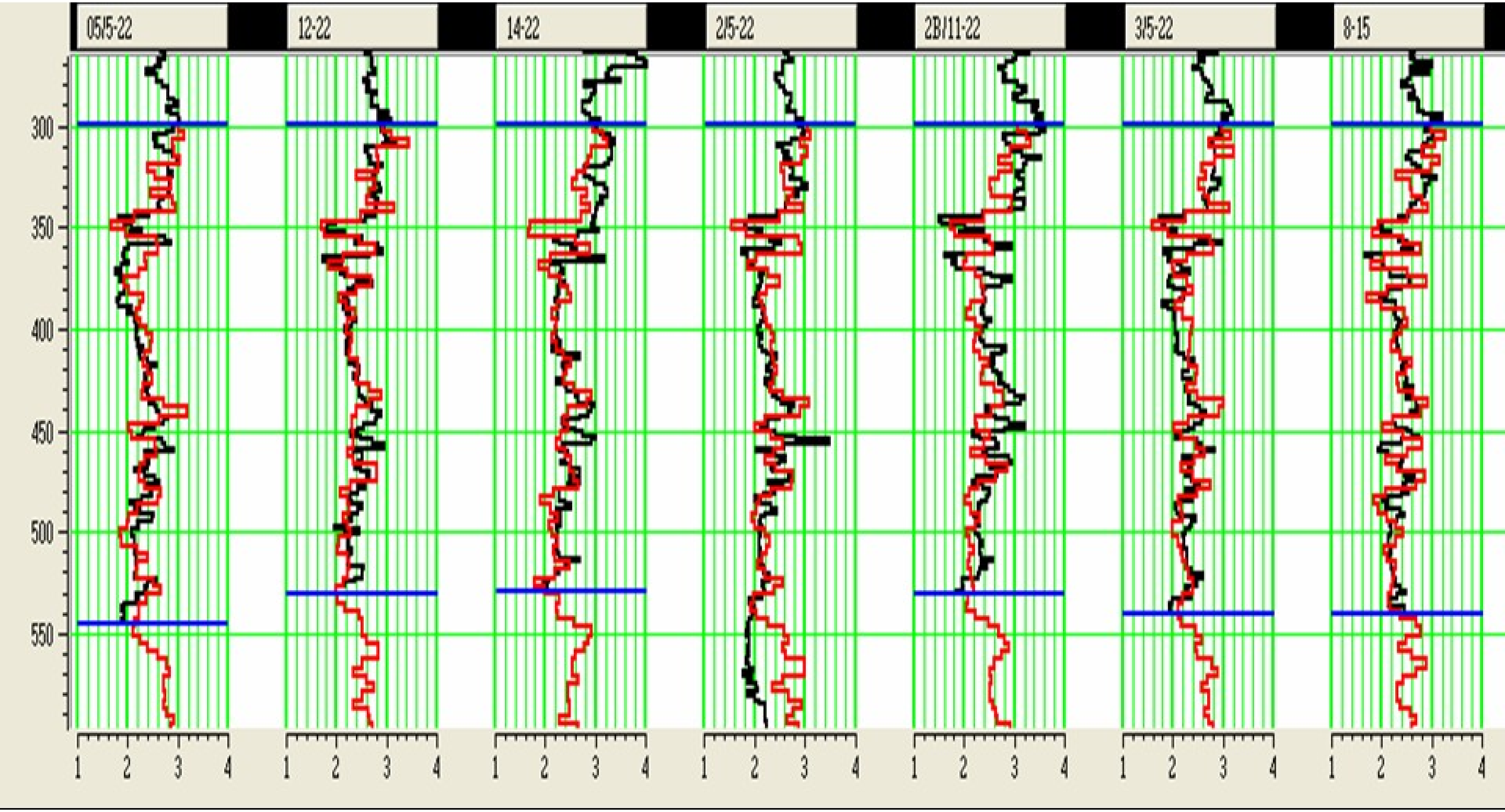
Correlation = 0.1941
RMS error = 0.391



(A) Independent Inversion

Vp/Vs Ratio

Correlation = 0.5921
RMS error = 0.301



(B) Joint Inversion

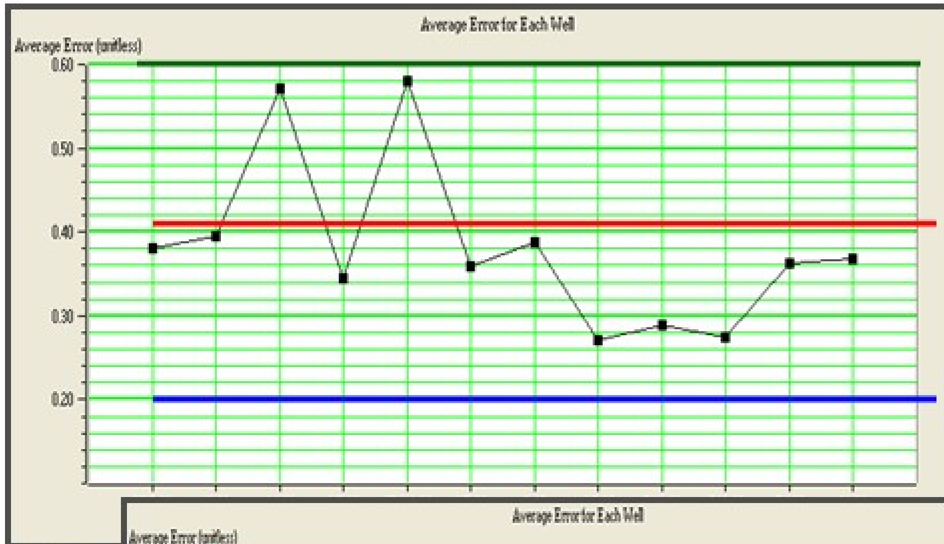
Vp/Vs Average Error Well by Well

(A) Independent Inversion

0.6

0.4

0.2

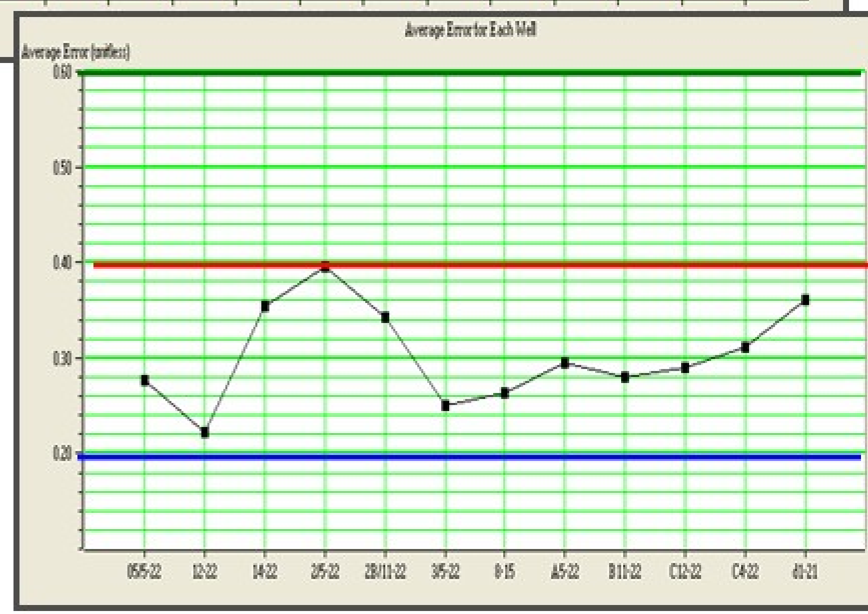


(B) Joint Inversion

0.6

0.4

0.2



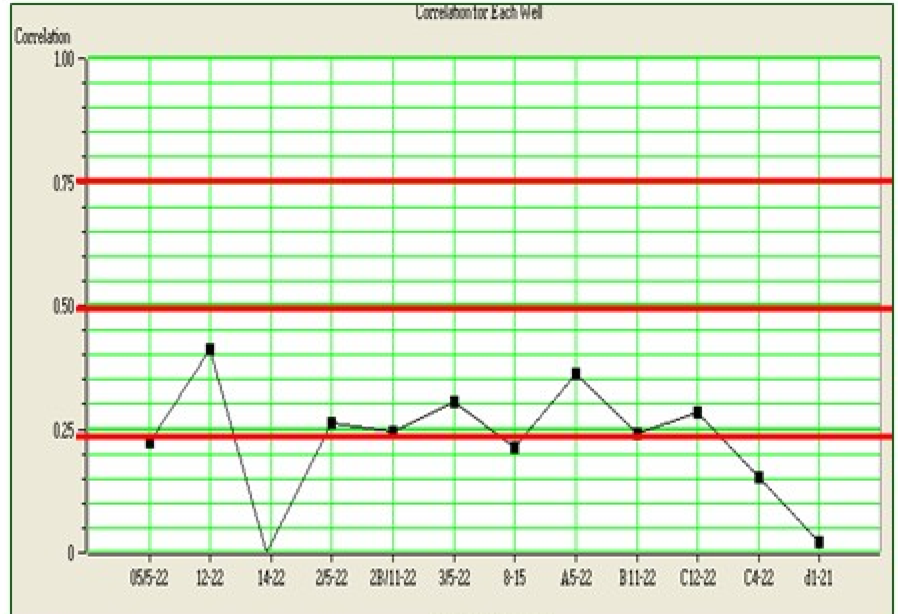
VpVs Correlation Well by Well

(A) Independent Inversion

0.75

0.50

0.25



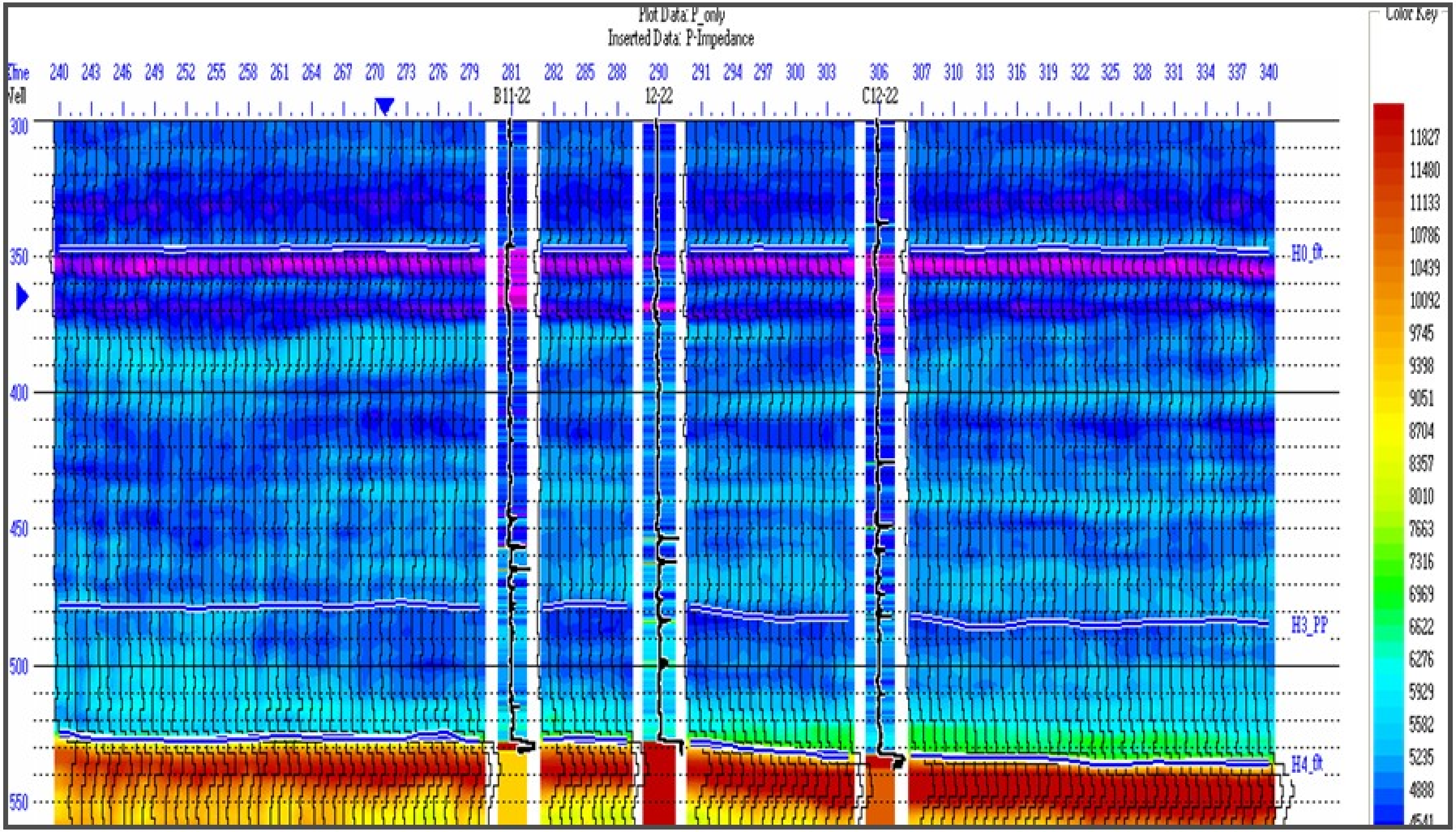
(B) Joint Inversion

0.75

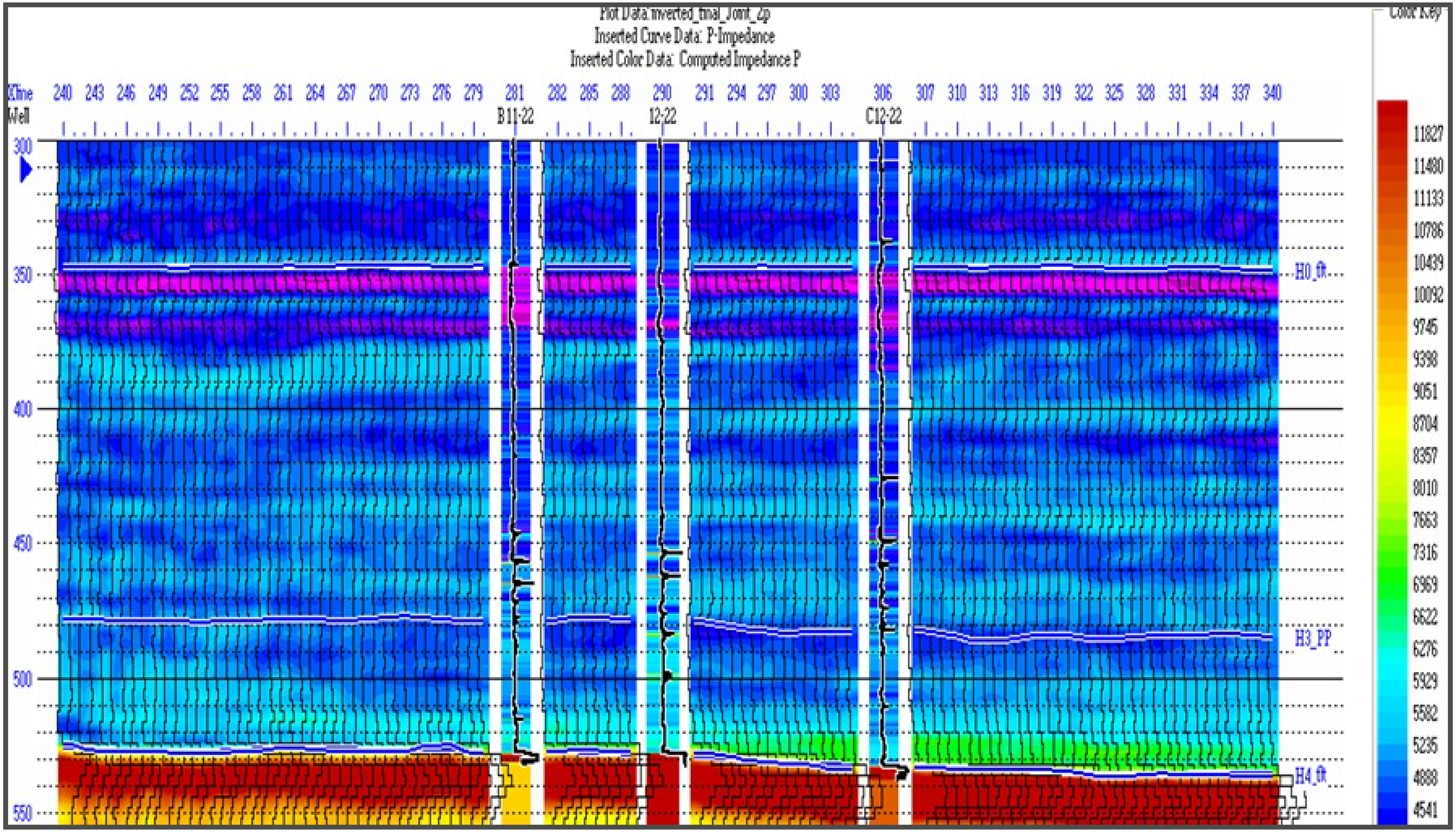
0.50

0.25

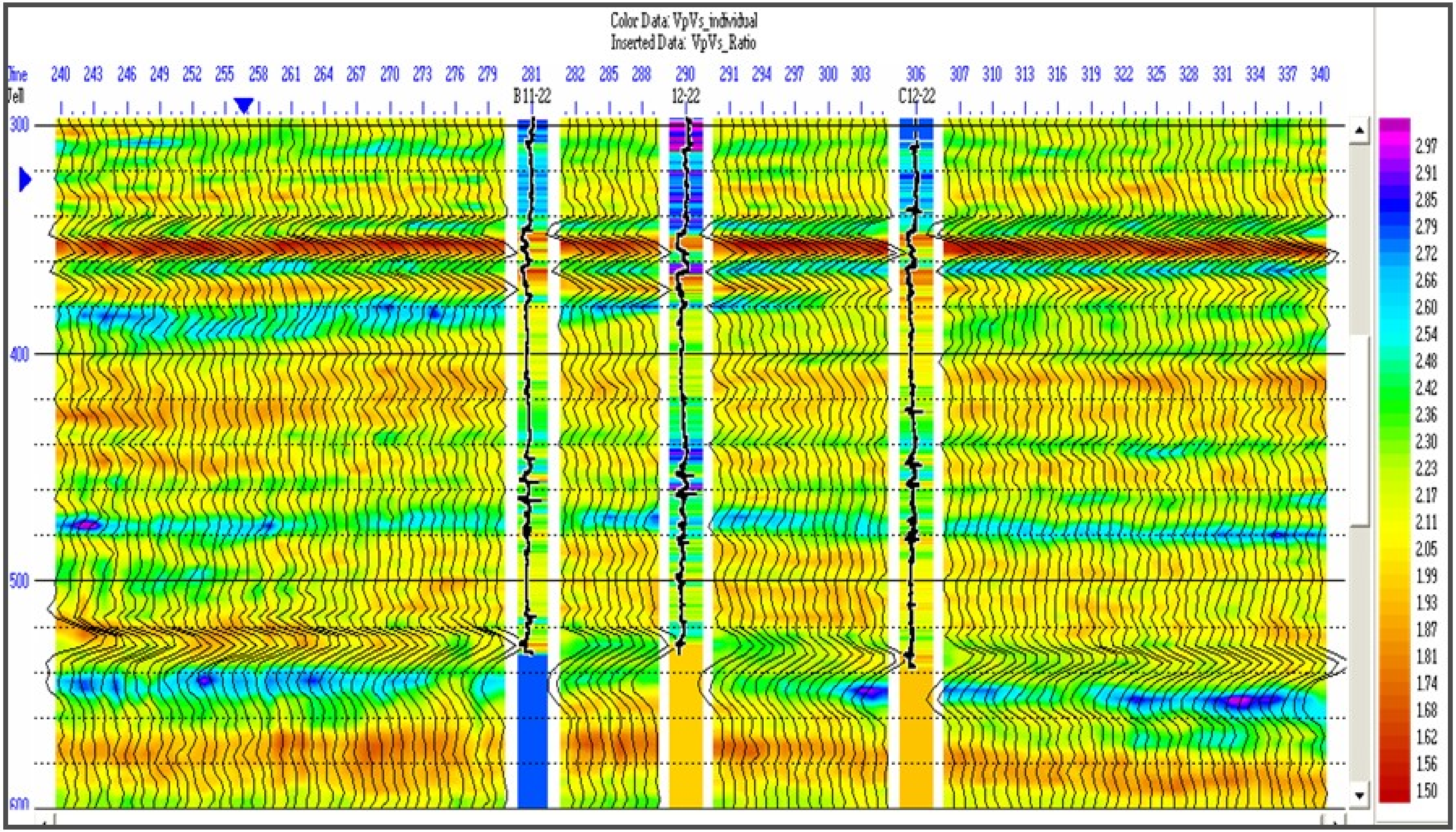
Independent Inverted P impedance



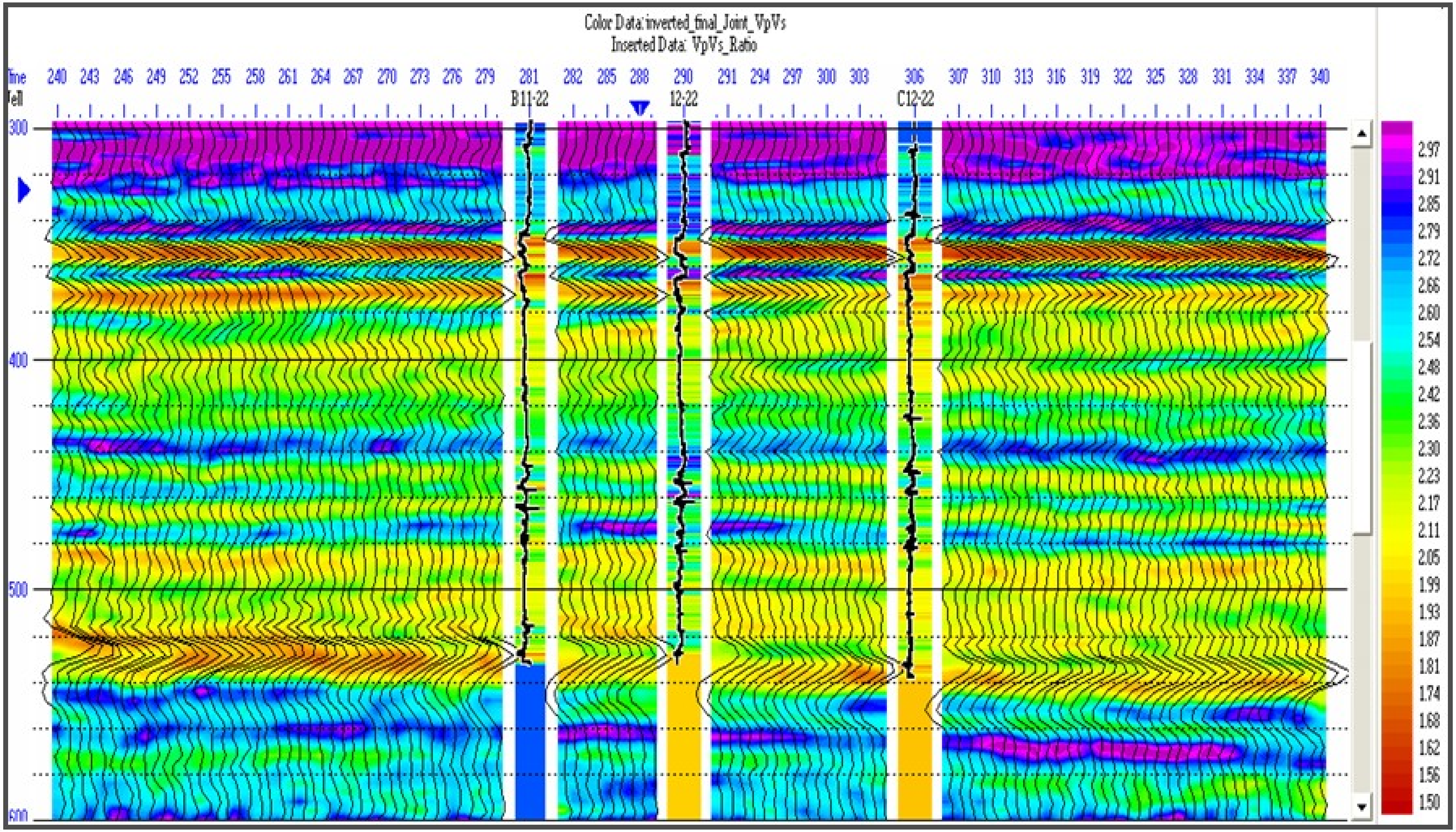
Joint Inverted P impedance



Independent Vp/Vs Result



Joint Vp/Vs Result



Conclusions

We have discussed a new simultaneous PP/PS inversion approach. The assumptions and results were:

- We use the Aki-Richards linearized equations.
- We use the small reflectivity assumption.
- We assume that a linear relationship between ln(Zs), ln(Zp) and ln(ρ) is reasonable.
- We showed that we could get good results on a model dataset.
- Initial application on a heavy oil dataset show the joint inversion is an improvement over independent inversion.

Acknowledgements

- We wish to thank our colleagues at both CREWES and at Veritas/Hampson-Russell for their input to this study.
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