Transfer functions of geophones and accelerometers and their effects on frequency content and wavelets

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Outline

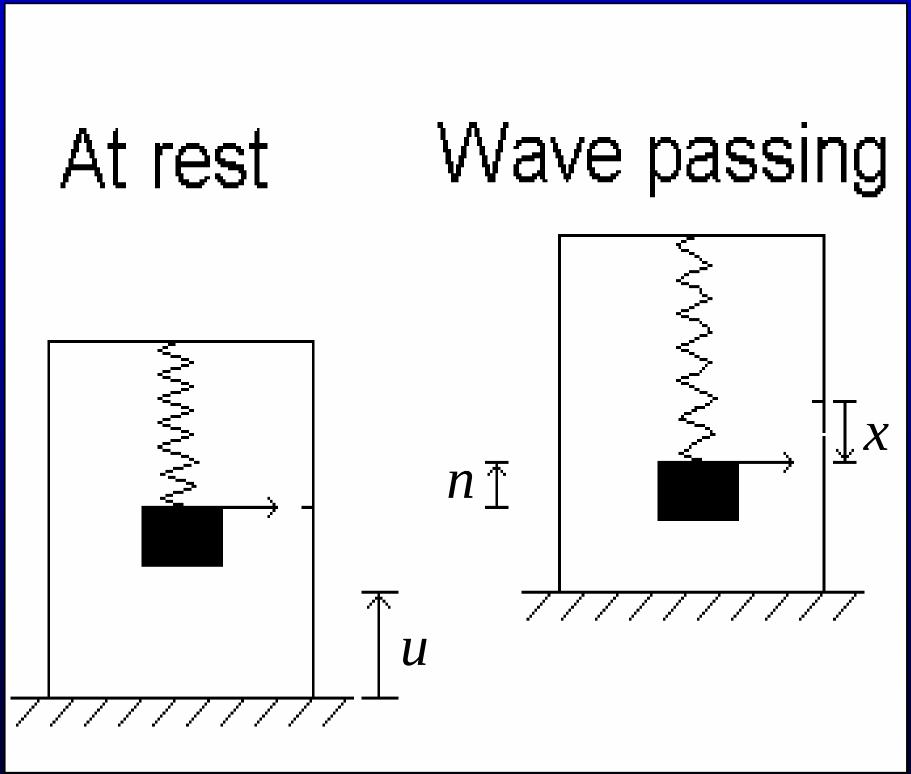
- Intro to transfer functions
- Deriving transfer functions
- Implications in the derivation
- Examples
- Conclusions

Transfer Functions

$$\frac{B}{A} = H$$

- A is input
- B is output
- H is transfer function

Deriving transfer functions



- u is ground displacement
- x is proof mass displacement relative to the case
- n is the net motion, used earlier in the derivation

Deriving Transfer Functions

- Must represent output divided by input
- Seismic sensors are "single degree of freedom" systems, or damped simple harmonic oscillators

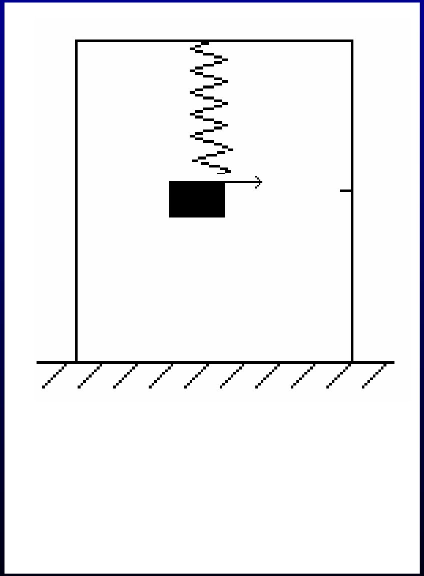
$$\frac{\partial^2 x}{\partial t^2} + 2\lambda\omega_0 \frac{\partial x}{\partial t} + \omega_0^2 x = -\frac{\partial^2 u}{\partial t^2}$$

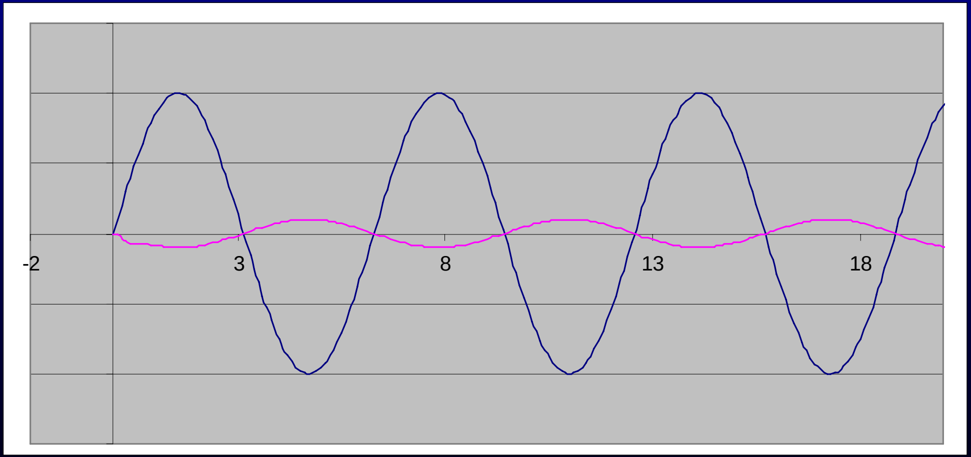
Deriving Transfer Functions

- The transducer
  - Detects the displacement of the proof mass relative to the case (x)
    - x is the input
  - Outputs an electrical signal
    - Accelerometer: capacitor responds to proof mass displacement
    - Geophone: magnetic induction responds to proof mass velocity

Deriving Transfer Functions

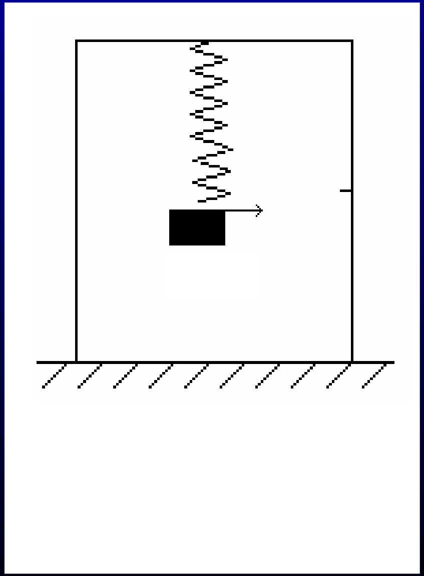
- At low frequencies, then proof mass displacement is directly proportional to acceleration
- When $\omega << \omega_0$ then $$x \propto \frac{\partial^2 u}{\partial t^2}$$

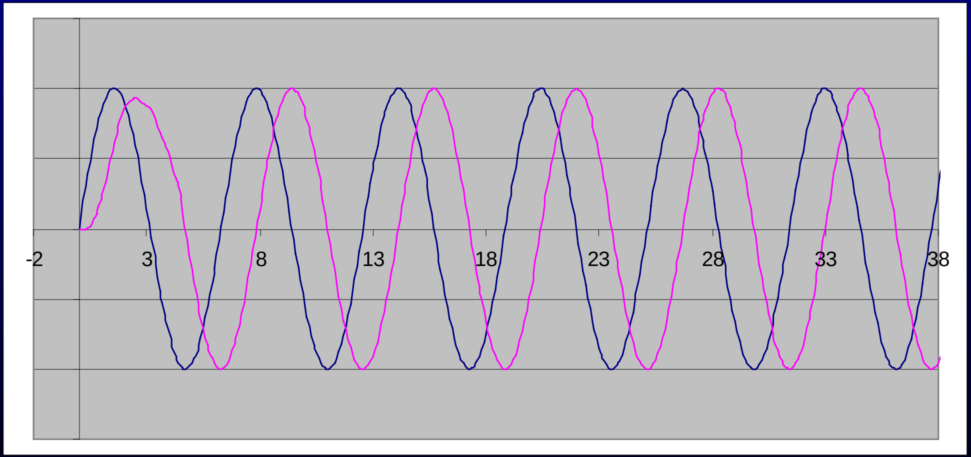




Deriving Transfer Functions

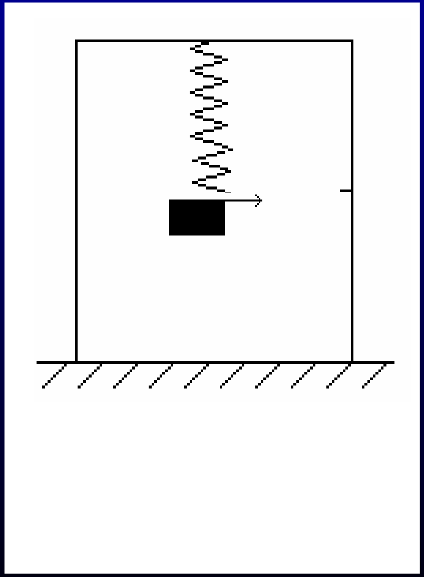
- At frequencies near resonance, the proof mass displacement is proportional to velocity
- When $\omega \cong \omega_0$ then $x \propto \dfrac{\partial u}{\partial t}$

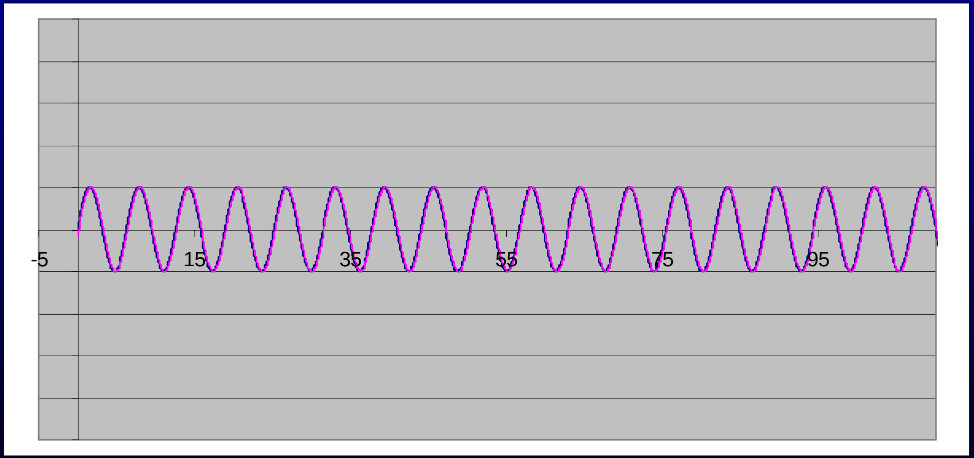




Deriving Transfer Functions

- At high frequencies, the proof mass displacement is directly proportional to ground displacement
- When $\omega \gg \omega_0$ then $x \propto u$





Deriving Transfer Functions

- Input:
  - Proof mass displacement relative to case, so A=x
    - x is proportional to some aspect of the ground motion, either displacement, velocity or acceleration
  - Thus $A \propto \frac{\partial^2 u}{\partial t^2}$ if $\omega \ll \omega_0$,

    or $A \propto \frac{\partial u}{\partial t}$ if $\omega \cong \omega_0$,

    or $A \propto u$ if $\omega \gg \omega_0$

Deriving Transfer Functions

- Output:
  - Related to some aspect of the motion of the proof mass relative to the case (either displacement or velocity), depending on the transducer used
  - For a geophone: $$B \propto \frac{\partial x}{\partial t}$$
  - For an accelerometer: $$B \propto x$$

Deriving Transfer Functions

- Transform to frequency domain, rearrange according to velocity (geophone) output and assorted inputs yields:

$$\frac{\frac{\partial X}{\partial t}}{\frac{\partial^2 U}{\partial t^2}} = \frac{-j\omega}{-\omega^2 + 2j\lambda\omega_0\omega + \omega_0^2}, \quad \omega << \omega_0$$

$$\frac{\frac{\partial X}{\partial t}}{\frac{\partial U}{\partial t}} = \frac{\omega^2}{-\omega^2 + 2j\lambda\omega_0\omega + \omega_0^2}, \quad \omega \cong \omega_0$$

$$\frac{\frac{\partial X}{\partial t}}{U} = \frac{j\omega^3}{-\omega^2 + 2j\lambda\omega_0\omega + \omega_0^2}, \quad \omega >> \omega_0$$

Deriving Transfer Functions

- Arranging for displacement (accelerometer) output and various inputs yields:

$$\frac{X}{\frac{\partial^2 U}{\partial t^2}} = \frac{-1}{-\omega^2 + 2j\lambda\omega_0\omega + \omega_0^2}, \quad \omega << \omega_0$$

$$\frac{X}{\frac{\partial U}{\partial t}} = \frac{-j\omega}{-\omega^2 + 2j\lambda\omega_0\omega + \omega_0^2}, \quad \omega \cong \omega_0$$

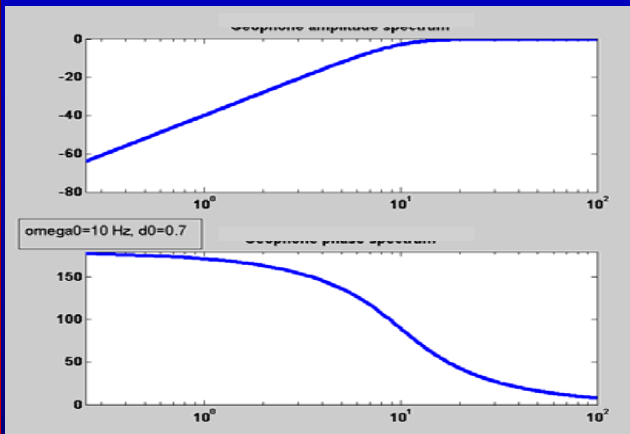
$$\frac{X}{U} = \frac{\omega^2}{-\omega^2 + 2j\lambda\omega_0\omega + \omega_0^2}, \quad \omega >> \omega_0$$

Implications

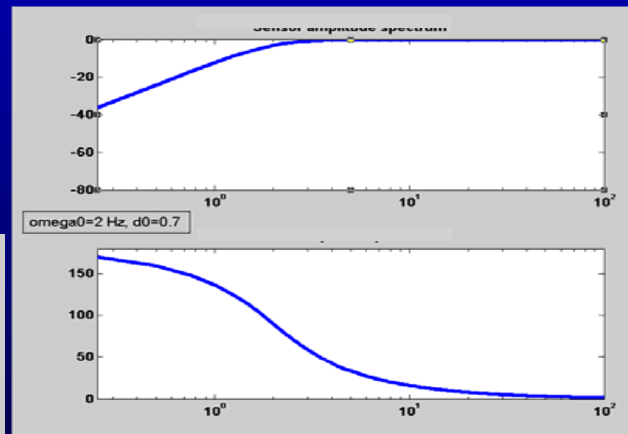
- In both cases, raw output is a double time derivative of ground displacement
- Geophone equation retains ω in the numerator, MEMS accelerometer equation does not
- Equations as solutions = no frequency limits
- Equations as transfer functions = frequency limits
- Geophone equation not a transfer function at low frequencies

Examples

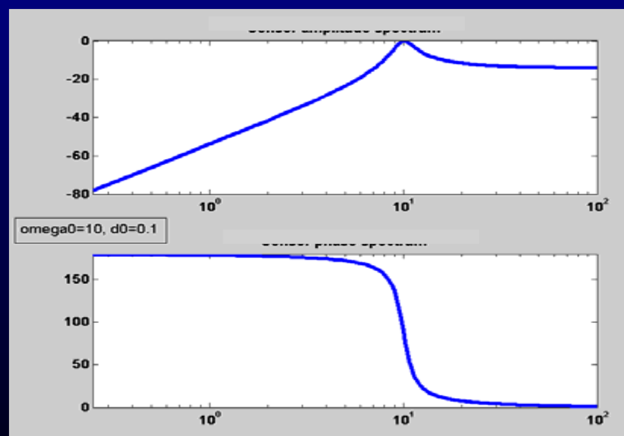
- Geophone response curves



10 Hz, 0.7 damping



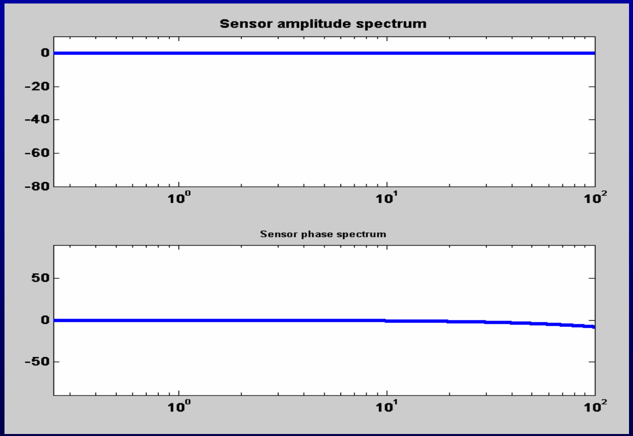
4 Hz, 0.7 damping



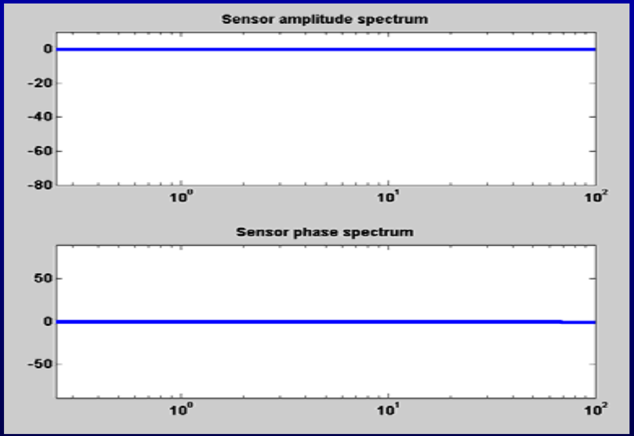
10 Hz, 0.1 damping

Examples

- Accelerometer response curves

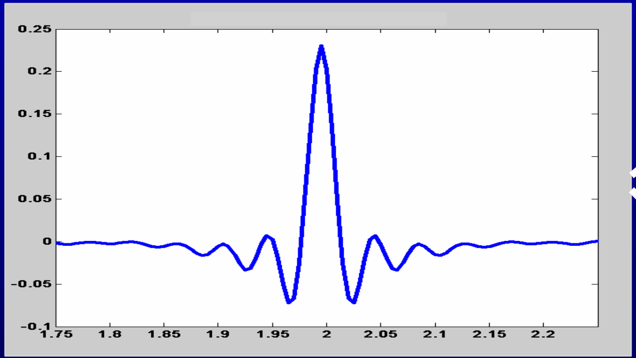


1000 Hz, 0.7 damping

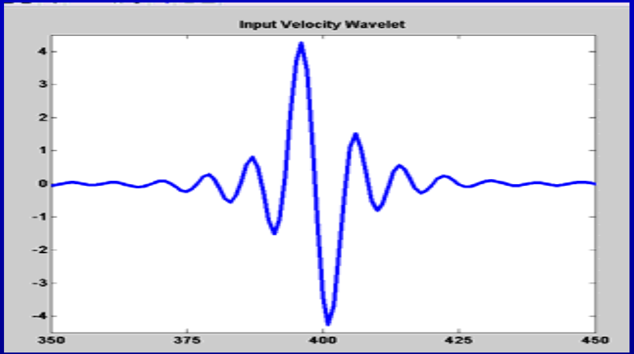


1000 Hz, 0.1 damping

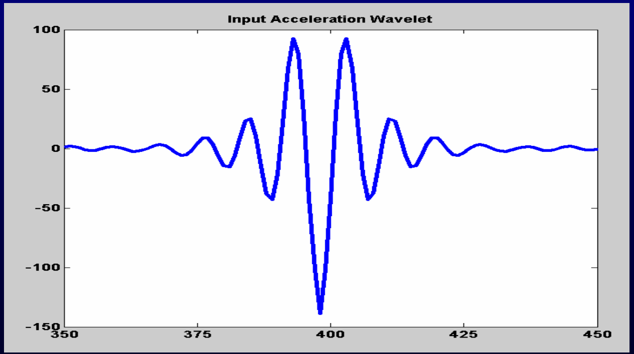
Examples



Input ground displacement
Bandpass 1-8-60-70 Hz

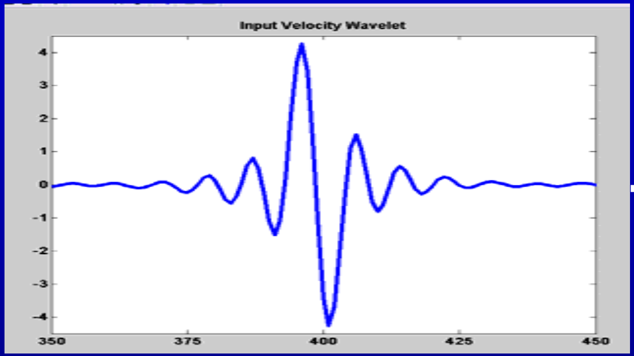


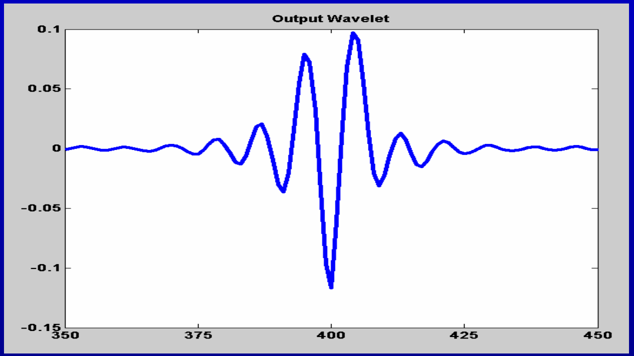
Input ground velocity

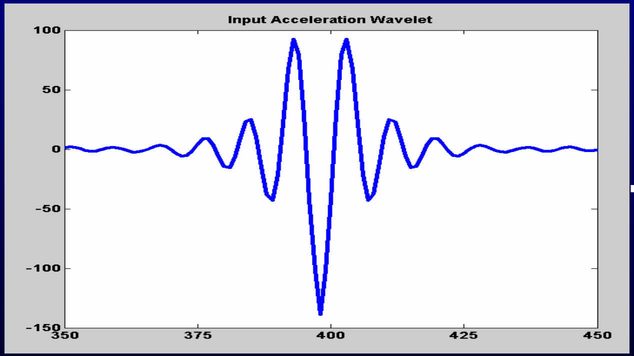


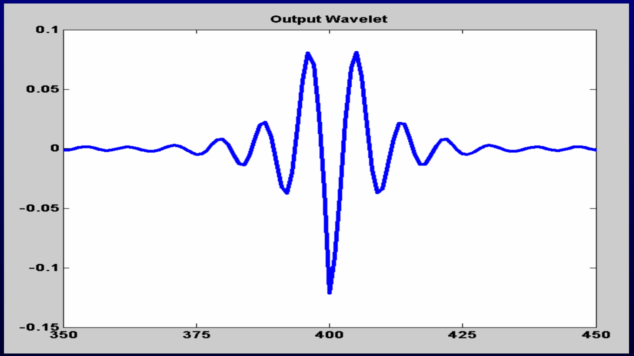
Input ground acceleration

Examples









Transducer input

Transducer output

Conclusions

- Equations governing proof mass motion in terms of ground motion become transfer functions when they represent transducer output/input
- Raw output from geophone and accelerometer is expected to be similar
- Geophone equation is not a valid transfer function for very low or very high frequencies

Acknowledgements

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