3D anisotropic phase shift operators

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Introduction

- Through Rayleigh-Sommerfeld modelling and imaging we can:
  - Control up ↑ and down ↓ and mode conversion:
  - Cause reflection and transmission.
  - Ensure high frequency stability and reduced runtime.
- Extrapolate wavefields with phaseshift operators one planewave at a time.
- Handle heterogeneous media with Gabor windows [Margrave et al., 2002].
- Anisotropy?

Phaseshift operators in TTI media

- Phaseshift operators are defined relative to the recording surface.
- TTI media are defined relative to the axis of symmetry.
- To reconcile, build operators in the TI coordinate system, map to the data coordinates, then apply.

Planewave vs. angle, parameters vs. coefficients

- Monochromatic spectrum φ is phaseshifted to a new depth Δz according to

$$\varphi_{\Delta z} = \varphi\, e^{i\,\omega\, q\, \Delta z}. \tag{1}$$

- In TI media q is [Daley and Hron, 1977]:

$$q \rightarrow q\left(p_I, C_{11}, C_{12}, C_{13}, C_{33}, C_{44}\right). \tag{2}$$

- Alternatively, q is [Thomsen, 1986]:

$$q \rightarrow q\left(\theta_I, \alpha_0, \beta_0, \varepsilon, \delta^*, \gamma\right). \tag{3}$$

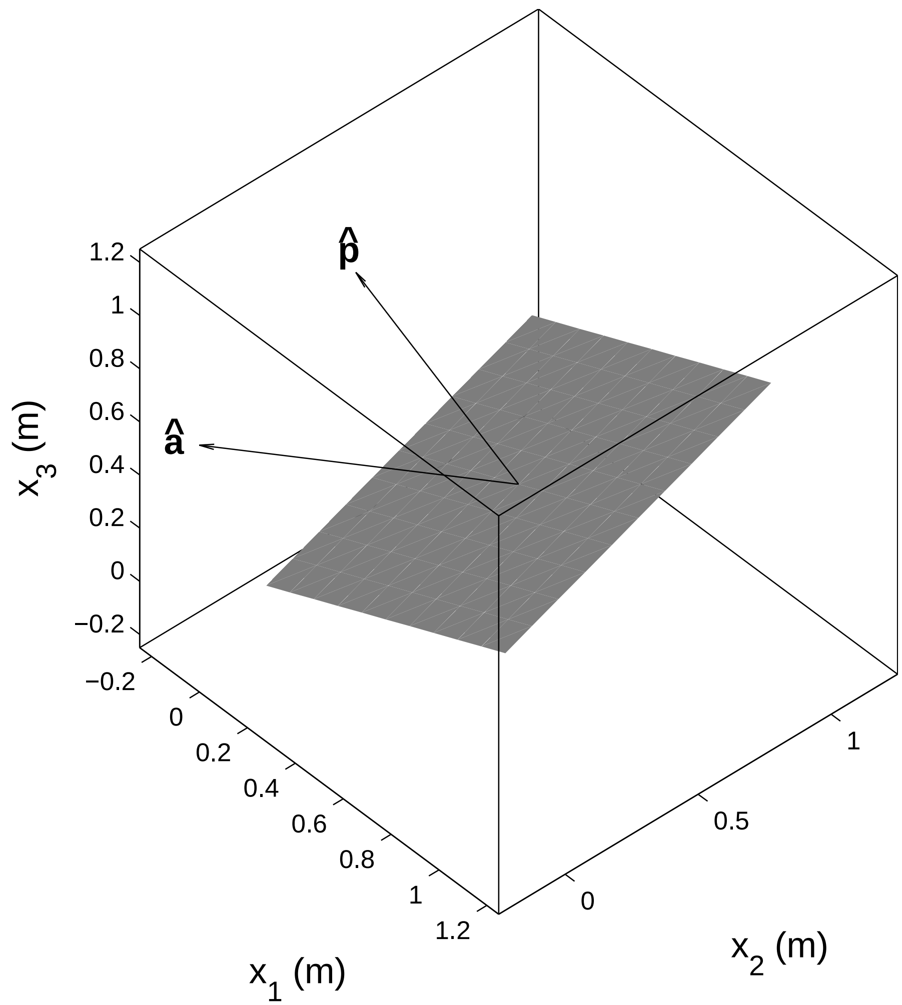
- For RS, we want to use planewaves, but $\alpha_0, \beta_0, \varepsilon, \delta^*$, and γ are ubiquitous - reconcile notation [Ferguson and Sen, 2004] so:

$$q \rightarrow q\left(p_I, \alpha_0, \beta_0, \varepsilon, \delta^*, \gamma\right). \tag{4}$$

  – Matlab's symbolic math toolbox helps.

What is p_I in a TTI medium?

- For TTI, $p_I = \frac{\sin\theta_I}{v}$ where θ_I is the angle between planewave unit normal $\hat{\mathbf{p}}$ and normal $\hat{\mathbf{a}}$ to the symmetry plane.



p̂
â
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
x3 (m)
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
x1 (m)
0
0.5
1
x2 (m)

- Use Snell's Law to eliminate raytracing:

$$p_I = \frac{\sin \theta_I}{v} = |\hat{\mathbf{p}} \times \hat{\mathbf{a}}| \, |\hat{\mathbf{p}}| \, . \qquad (5)$$

- Recall, $p_1 = \frac{k_x}{\omega}$, $p_2 = \frac{k_y}{\omega}$, and $p_3 = \frac{1}{v}\sqrt{1 - (v\,p_1)^2 - (v\,p_2)^2}$, so given α_0, β_0, ε, δ^*, and γ, q_α, $q_{\beta_{SH}}$ and $q_{\beta_{SV}}$ are computable for TTI media.

- Note, q are referenced to $\hat{\mathbf{a}}$ - map $q\,(\hat{\mathbf{a}} \rightarrow \hat{\mathbf{p}})$ to extrapolate $\varphi\,(\hat{\mathbf{p}})$ so invoke Snell's Law again so that

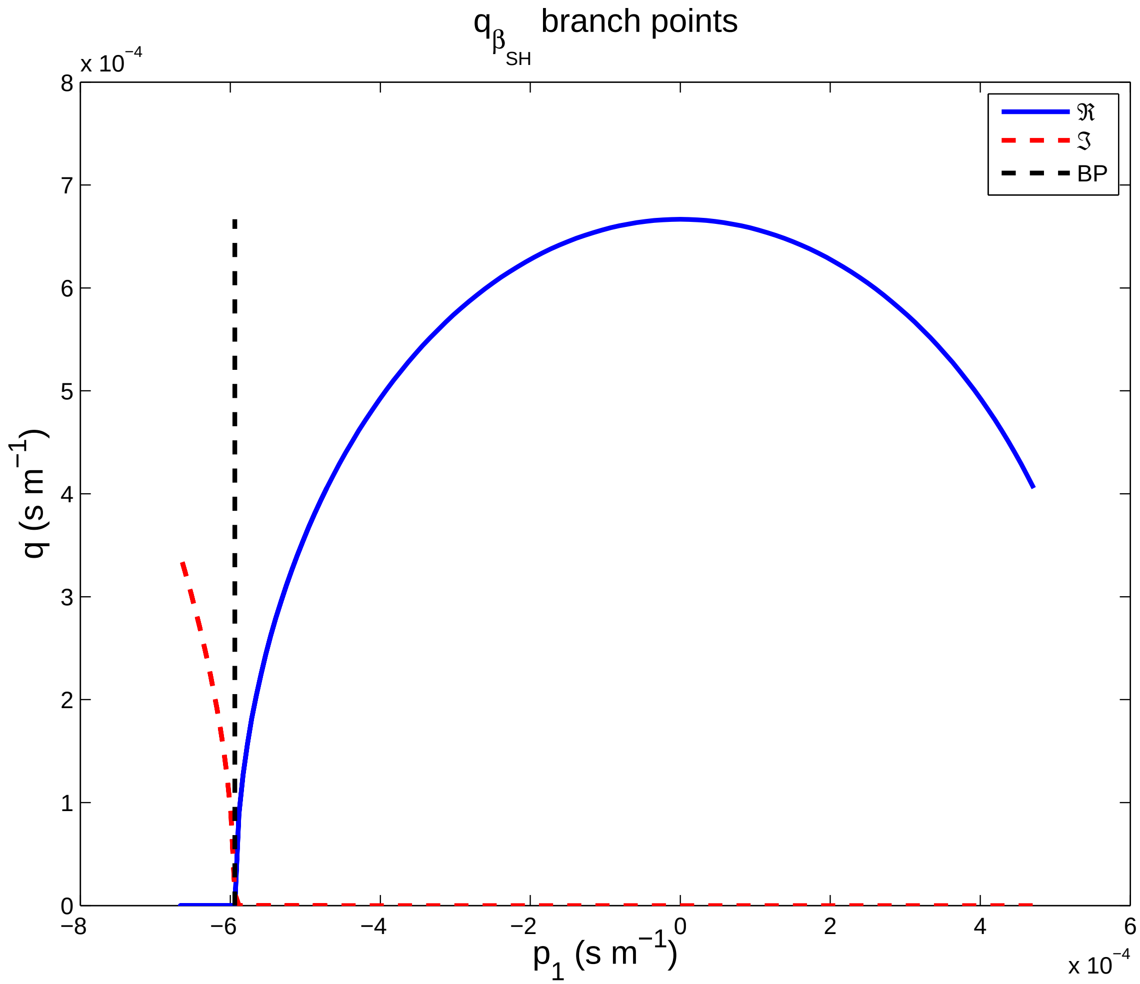
$$q_r^2 = q^2 + p_I^2 - p_1^2 - p_2^2, \qquad (6)$$

is the general form for the three modes.

Branch points

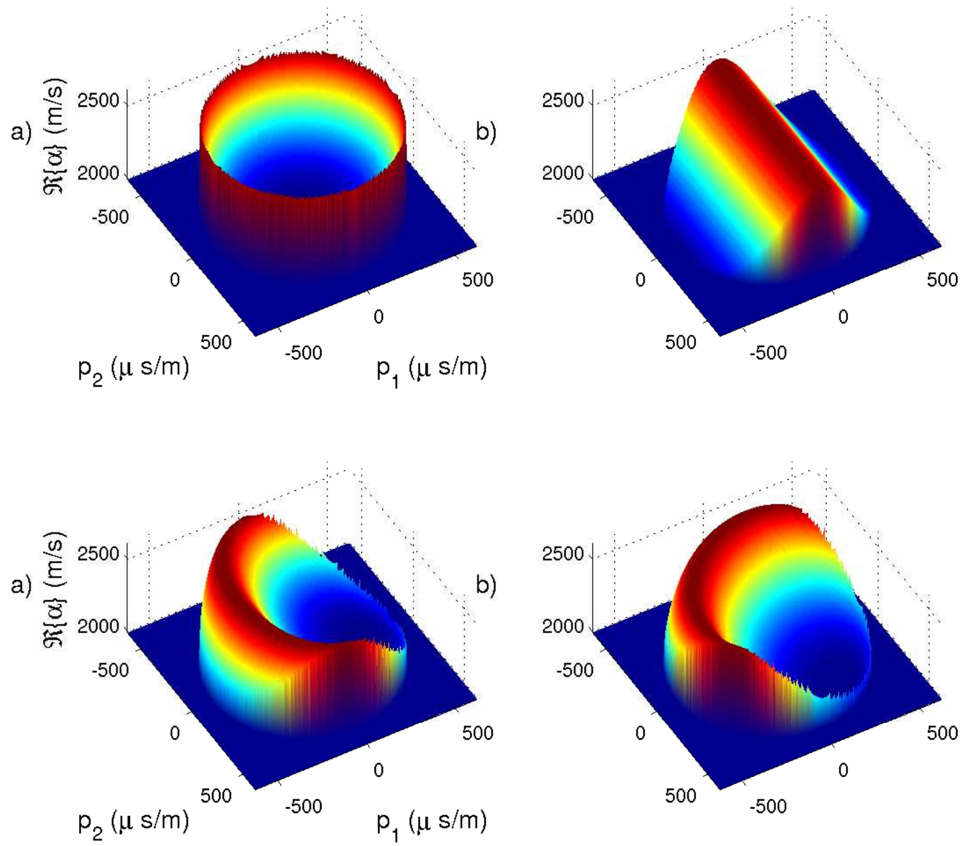
- For VTI and HTI, the evanescent region is defined.
- TTI is more complicated, and branchpoints must be determined, for example, set $q_{\beta_{SH}} = 0$ to get [Bale, 2006]

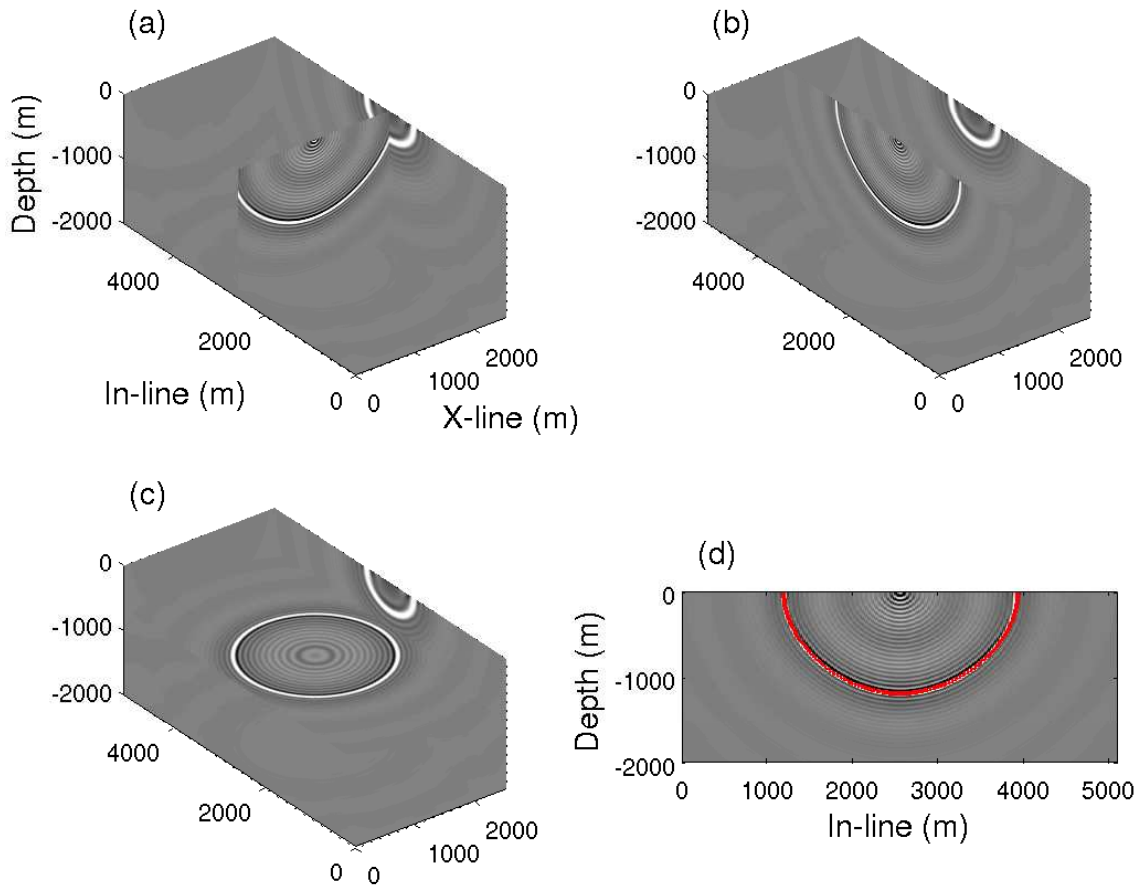
$$1 = \beta_0^2 \left[p_1^2 + p_2^2\right] \left[2\,\gamma + 1\right], \tag{7}$$



$q_{\beta_{SH}}$ branch points
x 10^{-4}
8
7
6
5
4
3
2
1
0
q (s m^{-1})
−8
−6
−4
−2
0
2
4
6
p$_1$ (s m^{-1})
x 10^{-4}
ℜ
ℑ
BP

Example: P-waves




(a)
0
-1000
-2000
Depth (m)
4000
2000
0
In-line (m)
0
1000
2000
X-line (m)
(b)
0
-1000
-2000
4000
2000
0
0
1000
2000
(c)
0
-1000
-2000
4000
2000
0
0
1000
2000
(d)
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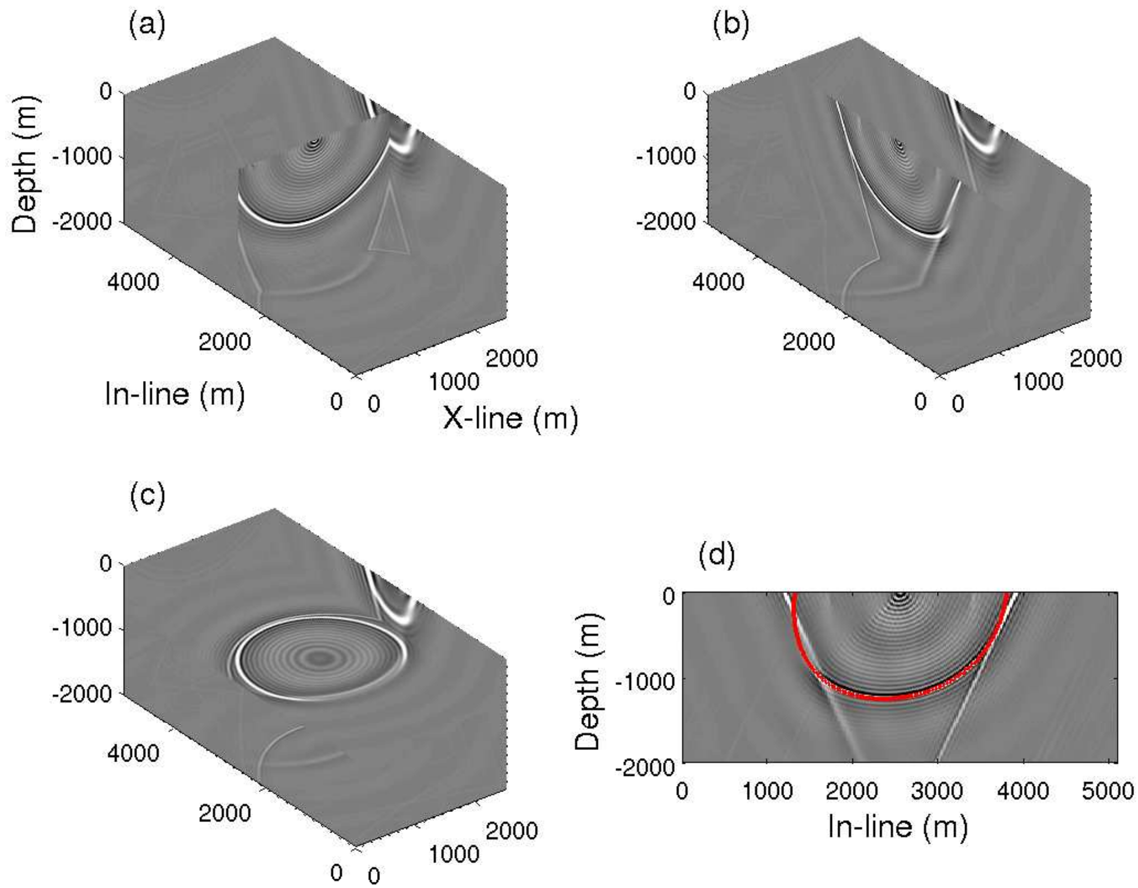
-1000
-2000
Depth (m)
0
1000
2000
3000
4000
5000
In-line (m)


(a)
(b)
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(d)
Depth (m)
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In-line (m)
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X-line (m)
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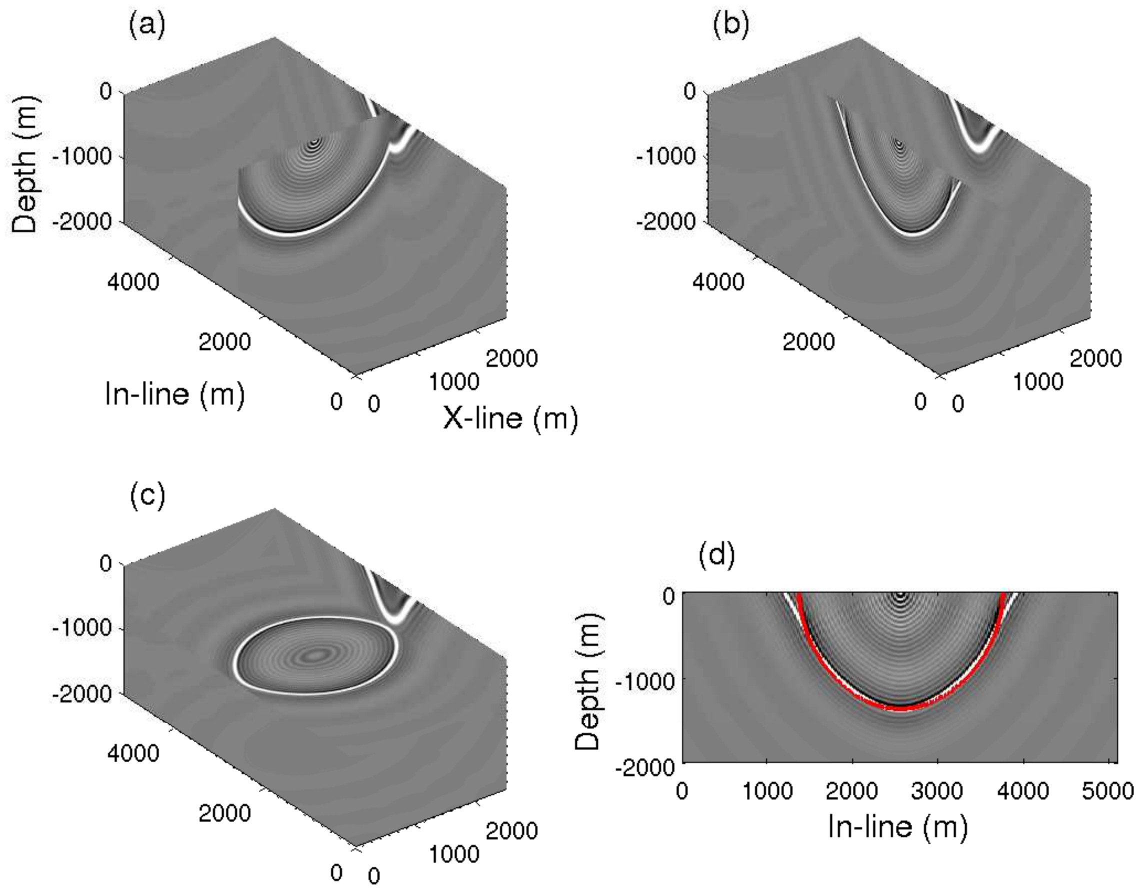
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Depth (m)
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(c)
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0
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(d)
Depth (m)
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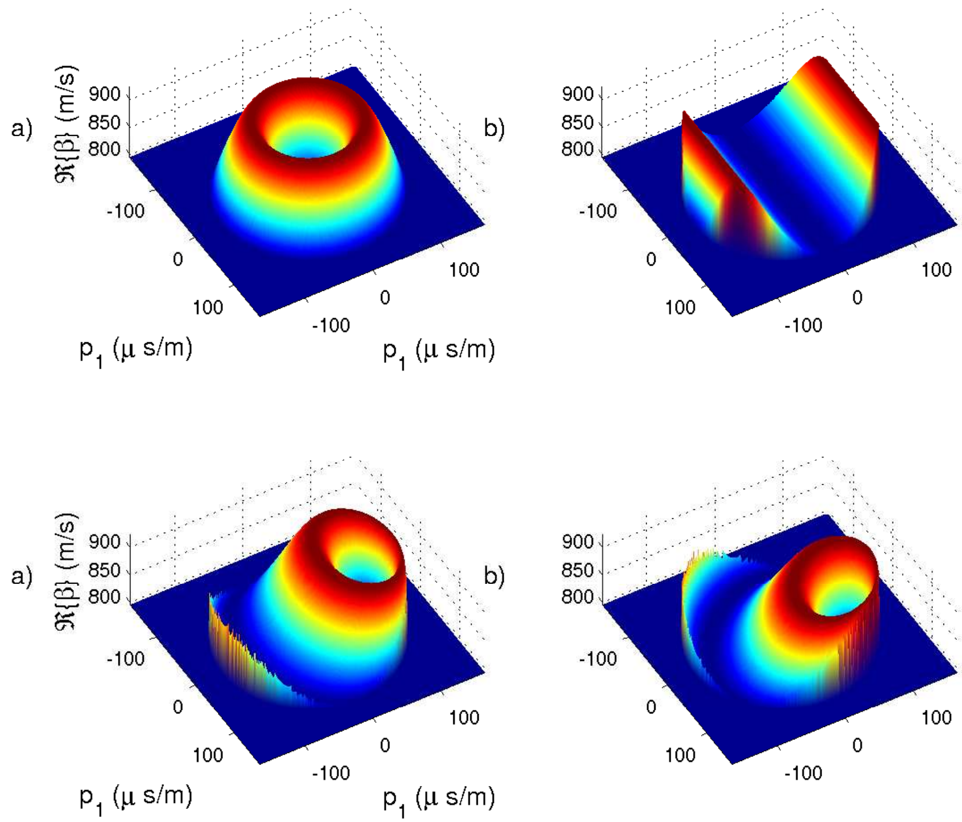
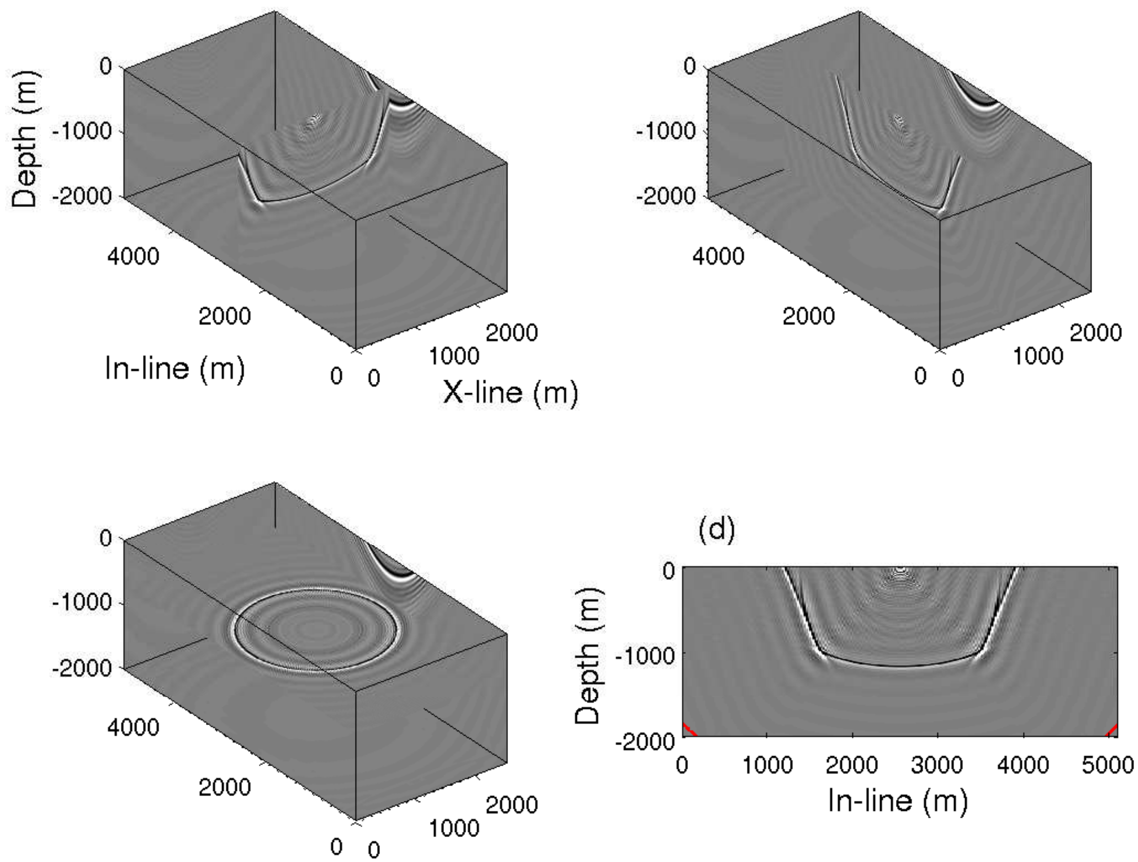
3000
4000
5000
In-line (m)

Example: SV-waves

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Depth (m)
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2000
In-line (m)
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1000
2000
X-line (m)
(d)
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Depth (m)
0
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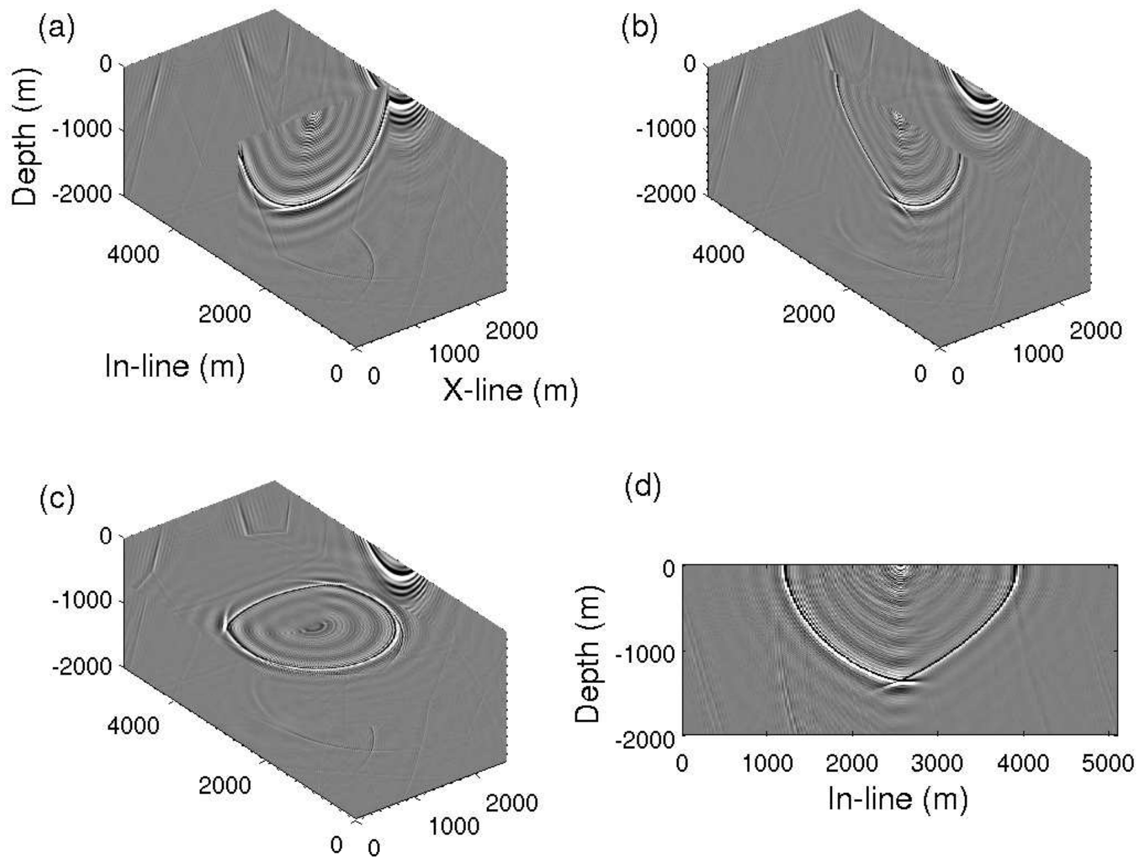
5000
In-line (m)


(a)
Depth (m)
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In-line (m)
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X-line (m)
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-2000
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(c)
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(d)
Depth (m)
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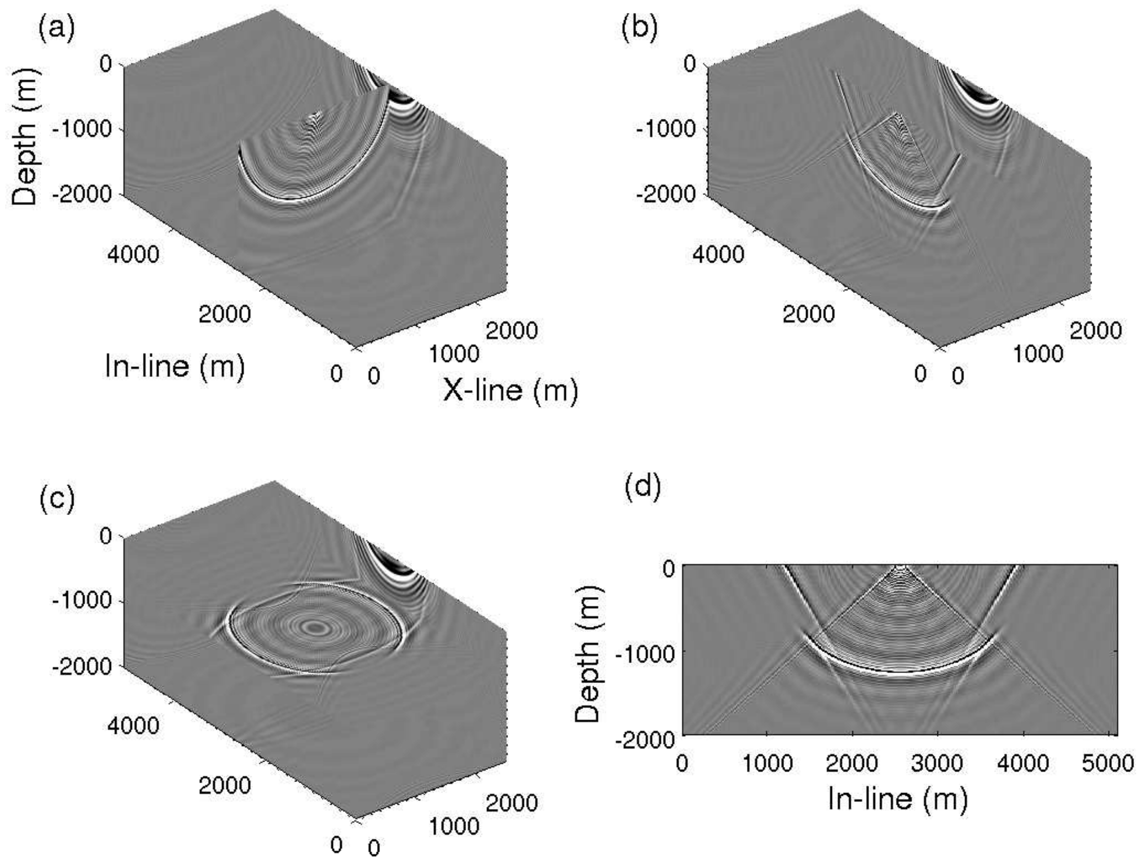
In-line (m)


(a)
Depth (m)
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In-line (m)
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X-line (m)
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2000
(c)
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-1000
-2000
4000
2000
0
0
1000
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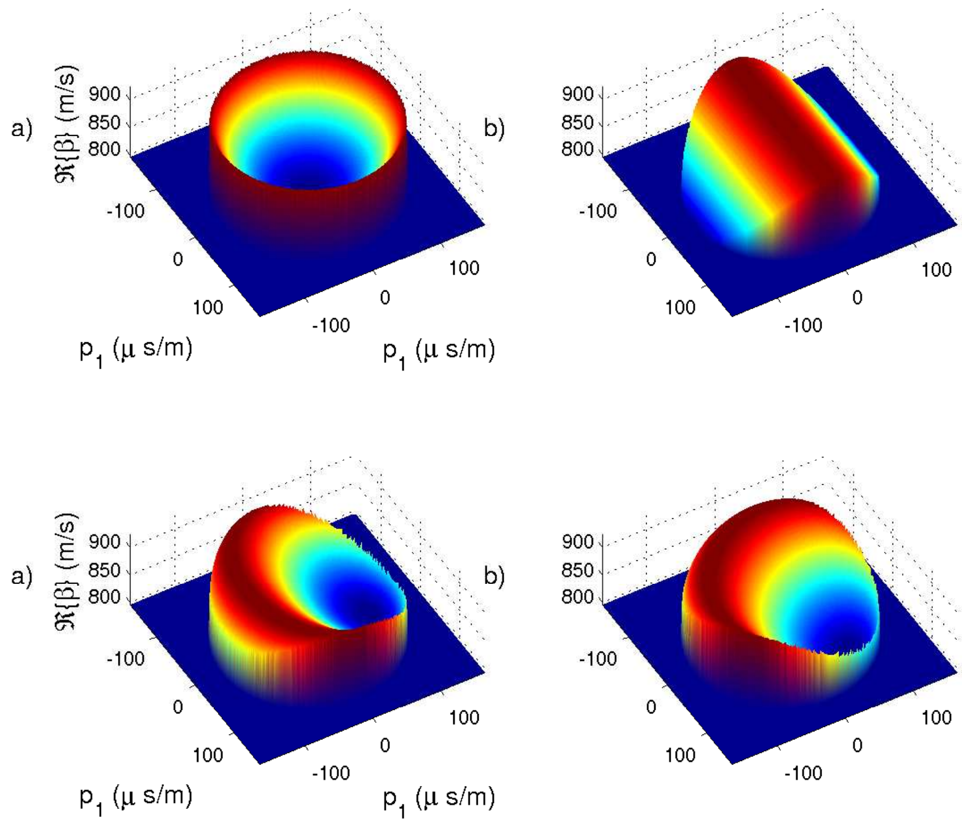
(d)
Depth (m)
0
-1000
-2000
0
1000
2000
3000
4000
5000
In-line (m)

Example: SH-waves



Conclusions and future work

- 3D phaseshift operators for homogeneous TTI media:
  - P, SV, and SH.
- Need group direction impulses for SV and SH.
- Compute branch points for P and SV.
- Rather than map $q_{p_I} \longrightarrow q_{\hat{\mathbf{p}}}$ then extrapolate $\varphi_{\hat{\mathbf{p}}}$, extrapolate φ_{p_I} then map $\varphi\left(p_I\right) \longrightarrow \varphi\left(\hat{\mathbf{p}}\right)$.

Acknowledgements

- CREWES sponsors and staff.

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