# The variable factor S-Transform deconvolution and noise attenuation

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The CREWES Project

SUNCOR ENERGY

# Outline

Introduction

Time-frequency signal representation

- Fourier transform
- Gabor transform
- S-transform
- Variable factor (VF) S-transform

VF S-transform deconvolution

F-T-X noise attenuation

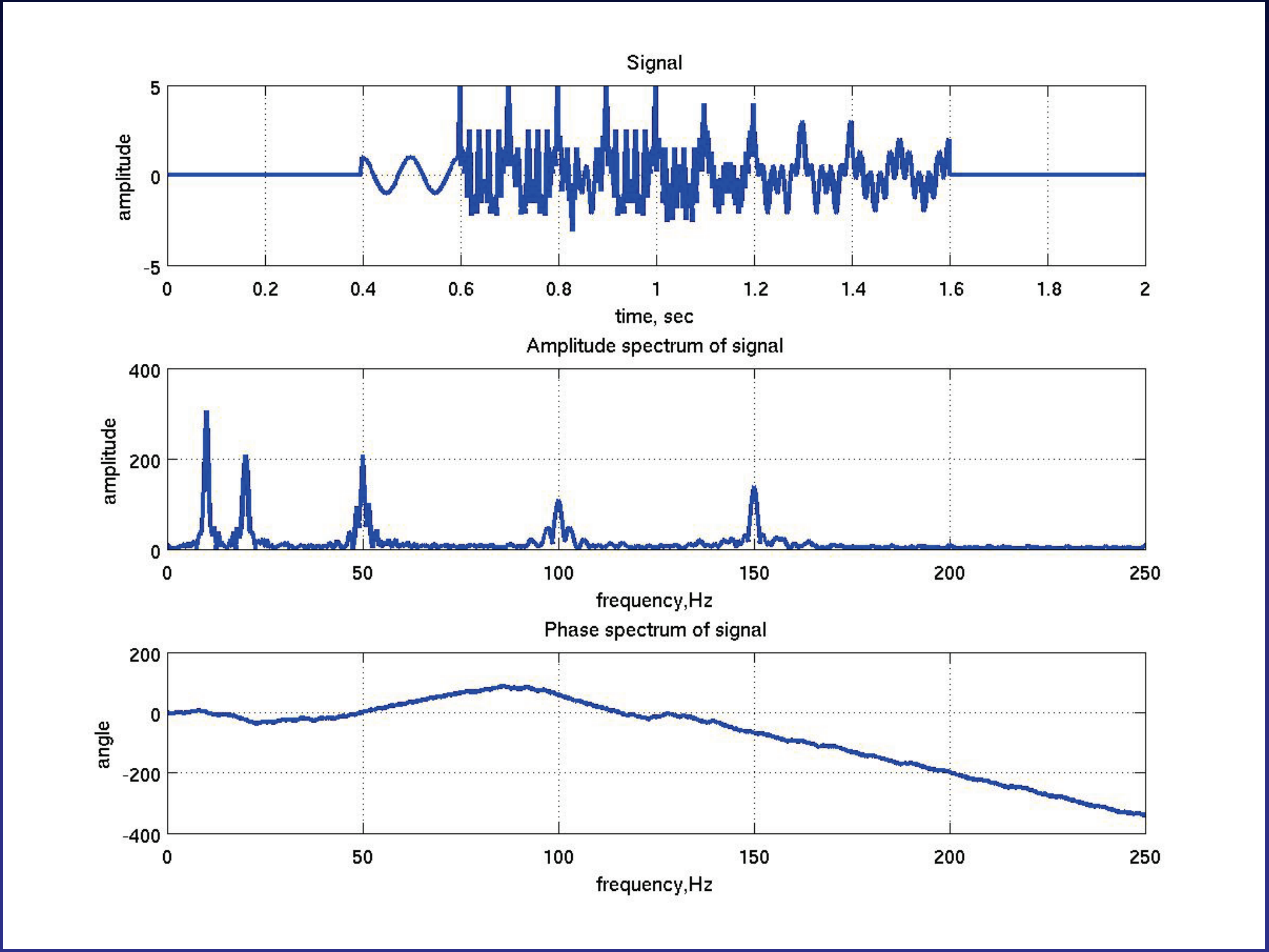
Conclusions

Future work

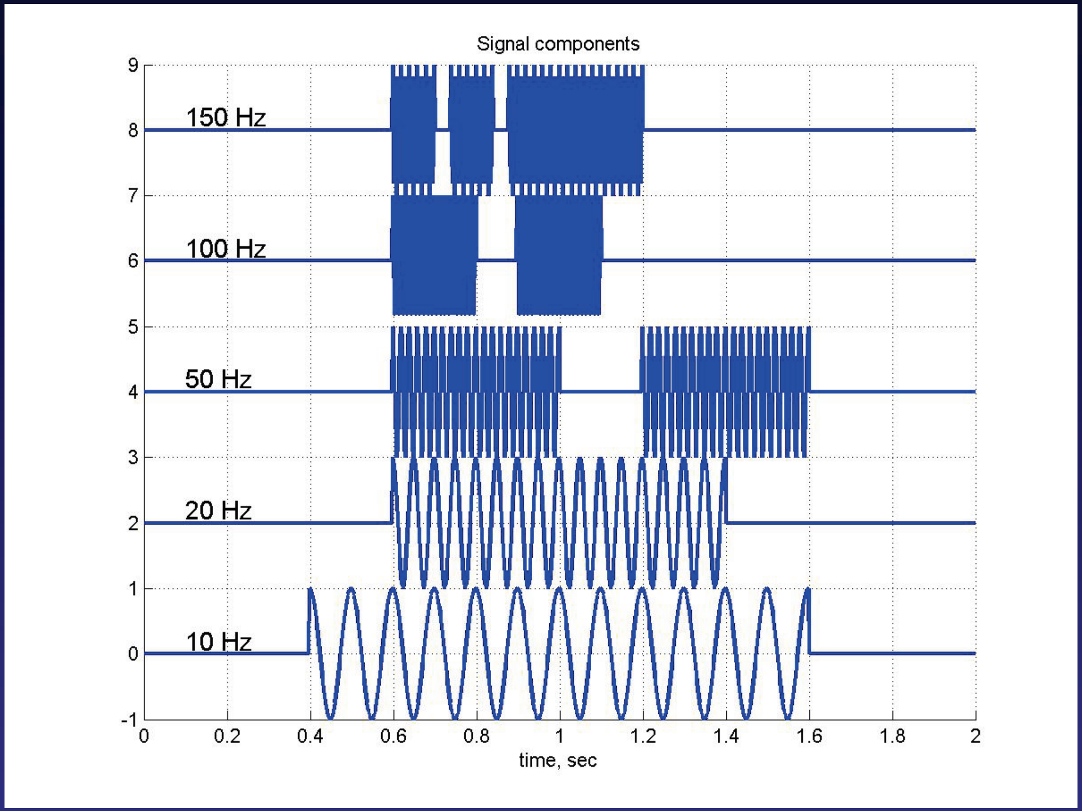
# Introduction

- current processing: stationary, Fourier transform
- seismic trace: attenuation → nonstationary
- some seismic noise: nonstationary
- Margrave (1998): nonstationary linear filtering
- Margrave and Lamoureux (2001): Gabor decon

# Signal and its Fourier spectrum

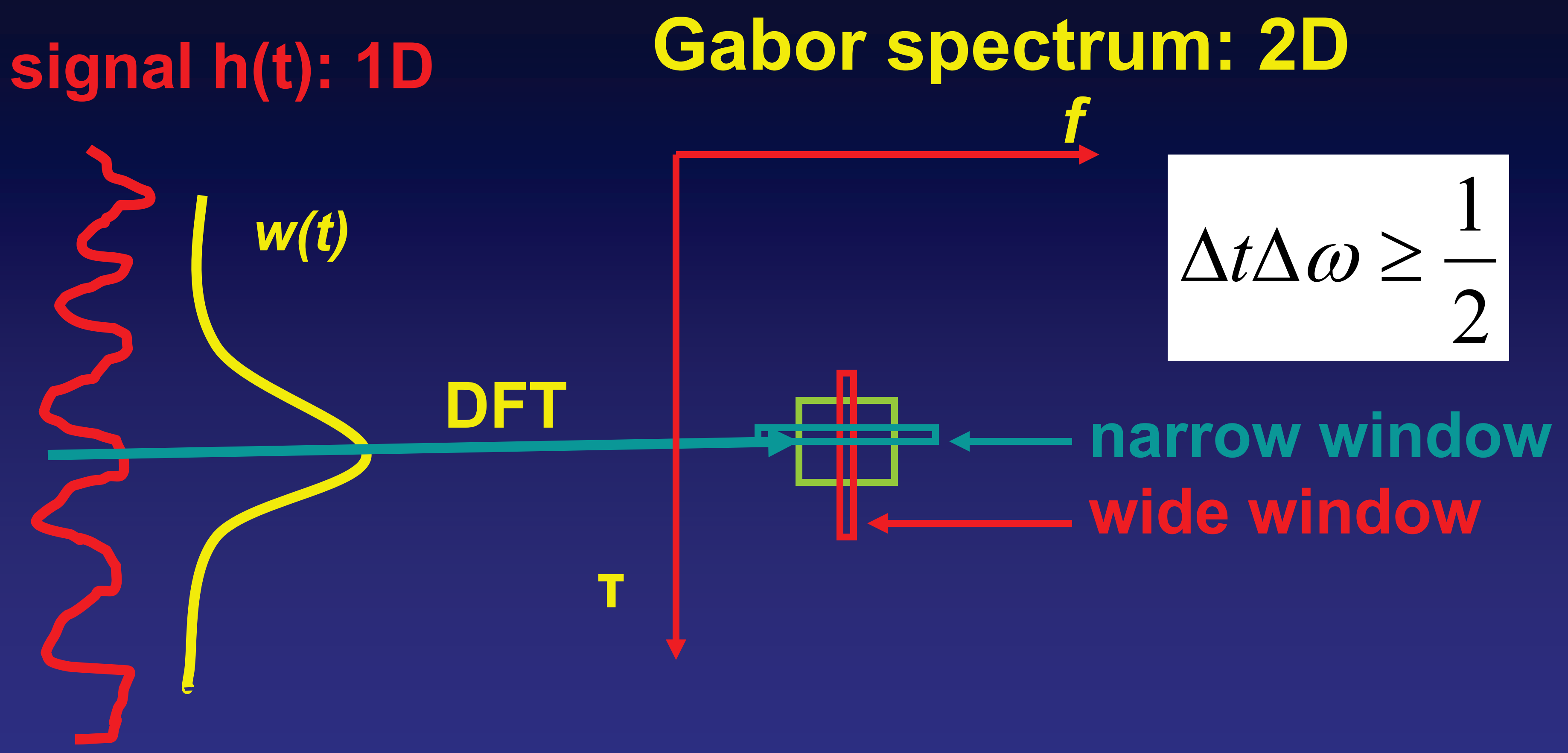


# Frequency components of the signal




- Fourier correctly tells us which frequency exist
- time information is lost
- good for stationary signals
- seismic trace is nonstationary
- we need time-frequency decomposition

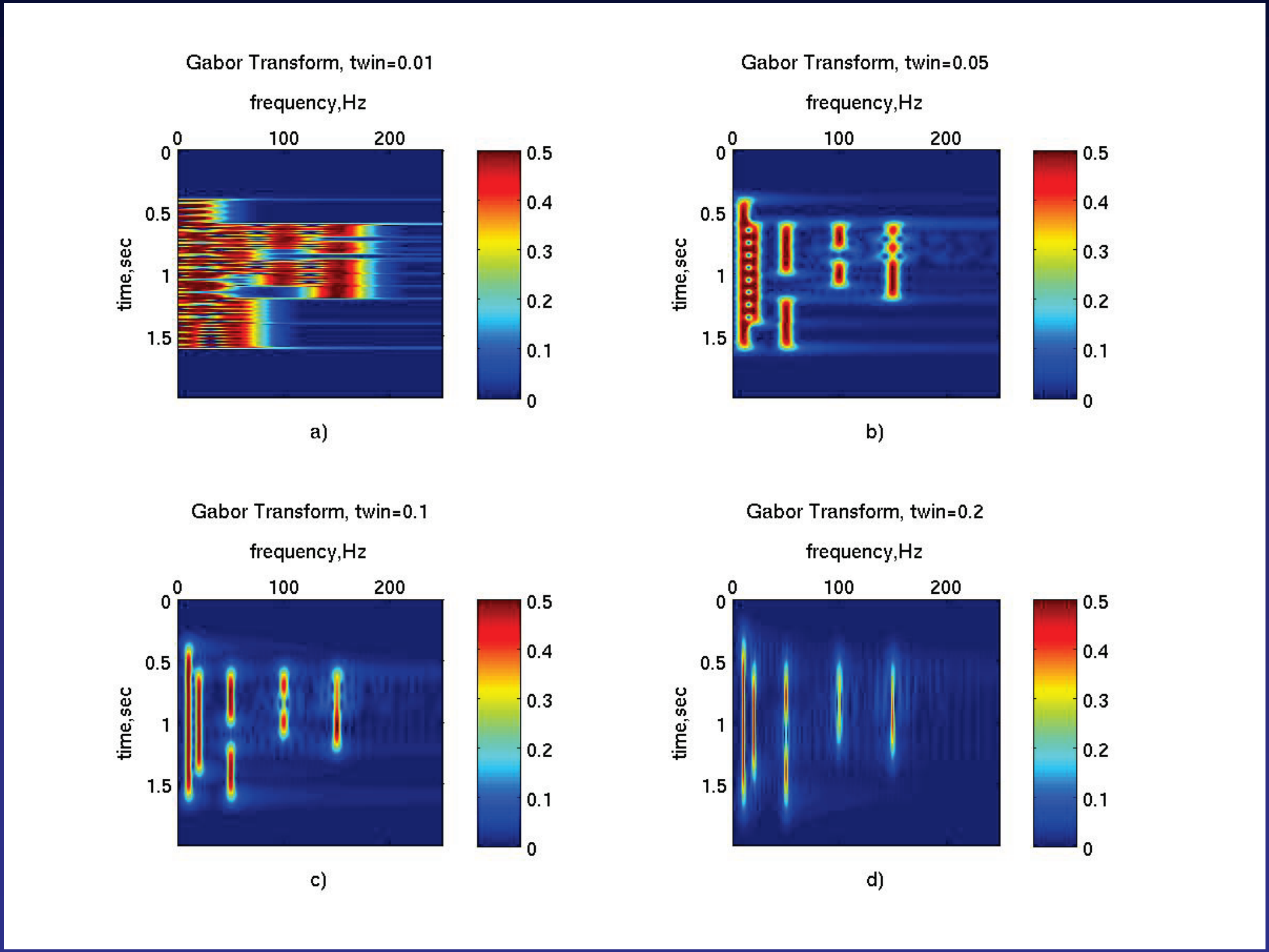
# Gabor Transform (Gabor, 1946)




$$\Delta t \Delta \omega \geq \frac{1}{2}$$

$$w(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-t^2/2\sigma^2}$$

# Gabor Transform of the signal



# S-transform (Stockwell, 1996)

$$w(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-t^2/2\sigma^2}$$

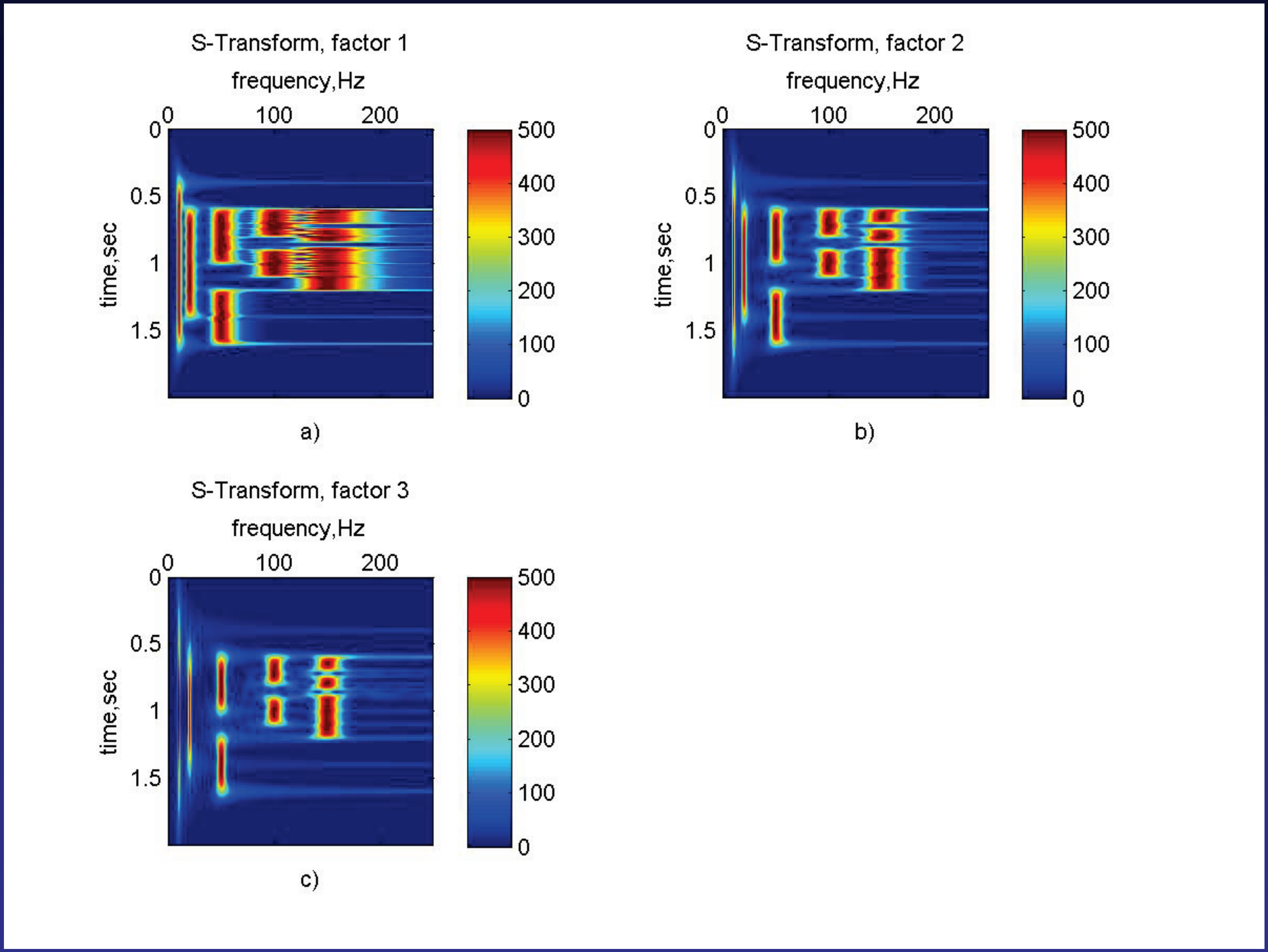
**Gabor transform**
**sigma = const**

$$\sigma(f) = \frac{1}{|f|}$$

**S-transform**
**sigma = inverse of frequency**

$$S(\tau, f) = \int_{-\infty}^{+\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau - t)^2 f^2}{2}} e^{-i2\pi f t} dt$$

# S-Transform of the signal




$$\sigma(f) = \frac{k}{|f|}$$

**k=const**

# Variable factor S-transform

$$\sigma(f) = \frac{k}{|f|}$$

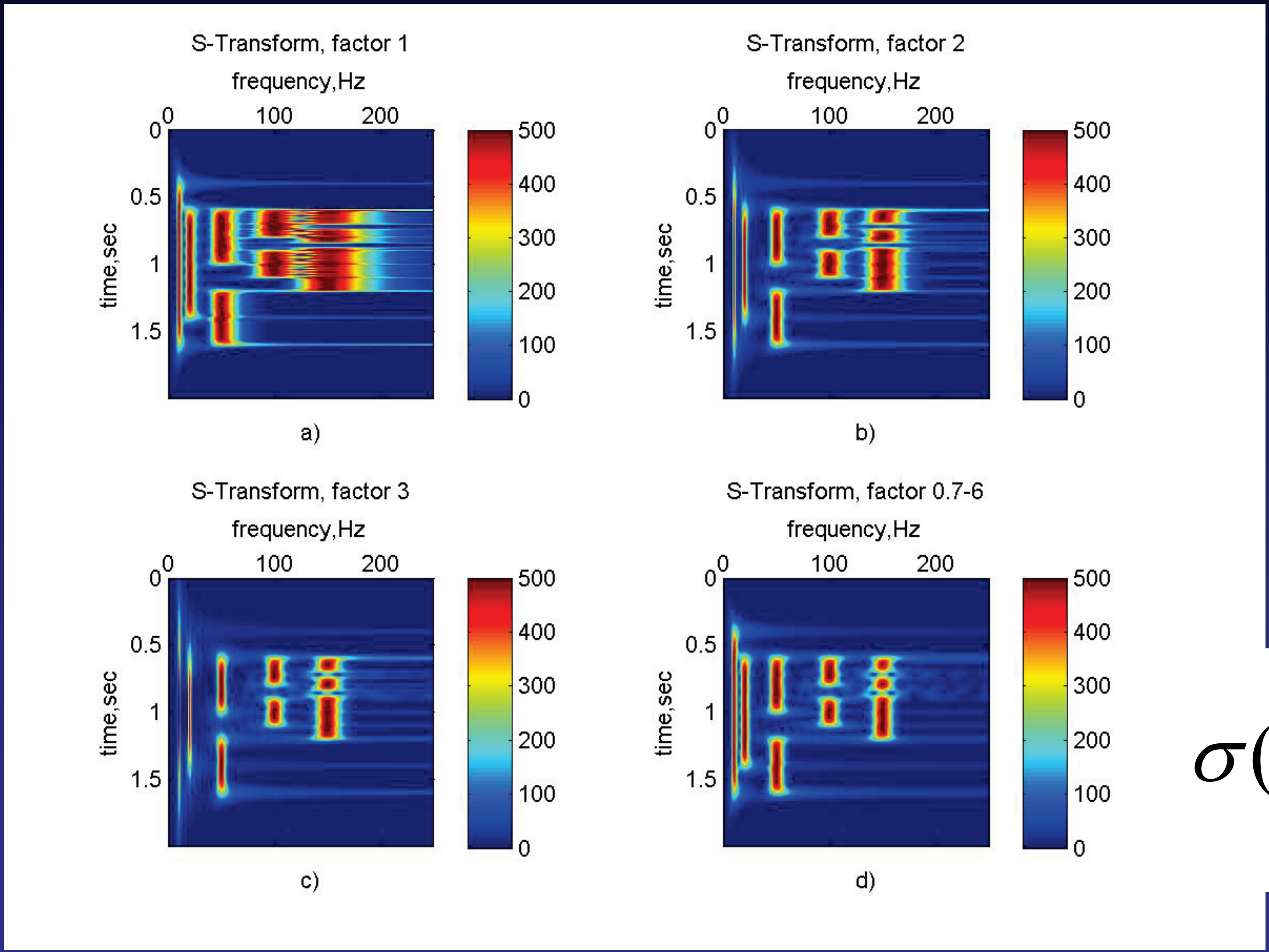
**S-transform**
**sigma = factor / frequency**
**Manshinha (1997)**

$$\sigma(f) = \frac{k(f)}{|f|}$$

**Variable Factor S-transform**
**sigma = factor(f)/frequency**

$$S(\tau, f) = \int_{-\infty}^{+\infty} h(t) \frac{|f|}{\sqrt{2\pi k(f)}} e^{-\frac{(\tau - t)^2 f^2}{2k^2(f)}} e^{-i2\pi ft} dt$$

# Variable factor S-Transform of the signal




$$\sigma(f) = \frac{k(f)}{|f|}$$

# The Wiener deconvolution

$$s(t) = w(t) * r(t) + n(t)$$

- s(t) – recorded seismic trace
- w(t) – embedded seismic wavelet
- r(t) – the earth reflectivity
- n(t) – white noise

**Assumptions: stationary process; causal, minimum-phase wavelet; random reflectivity; white, stationary noise**

# The Gabor deconvolution

$$s(t) = \int_{-\infty}^{+\infty} w(t-\tau,\tau) r(\tau) d\tau$$

nonstationary convolution

$$S(f) = W(f) \int_{-\infty}^{+\infty} \alpha(t,f) r(t) e^{-i2\pi ft} dt$$

frequency domain

$$\alpha(t,f) = \exp\left( -\frac{\pi t}{Q(t)} (f + iH(f)) \right)$$

nonstationary term

$$S_G(\tau,f) = W(f)\alpha(\tau,f)R_G(\tau,f)$$

**Assumptions: causal, minimum-phase wavelet**

# The VF S-transform deconvolution

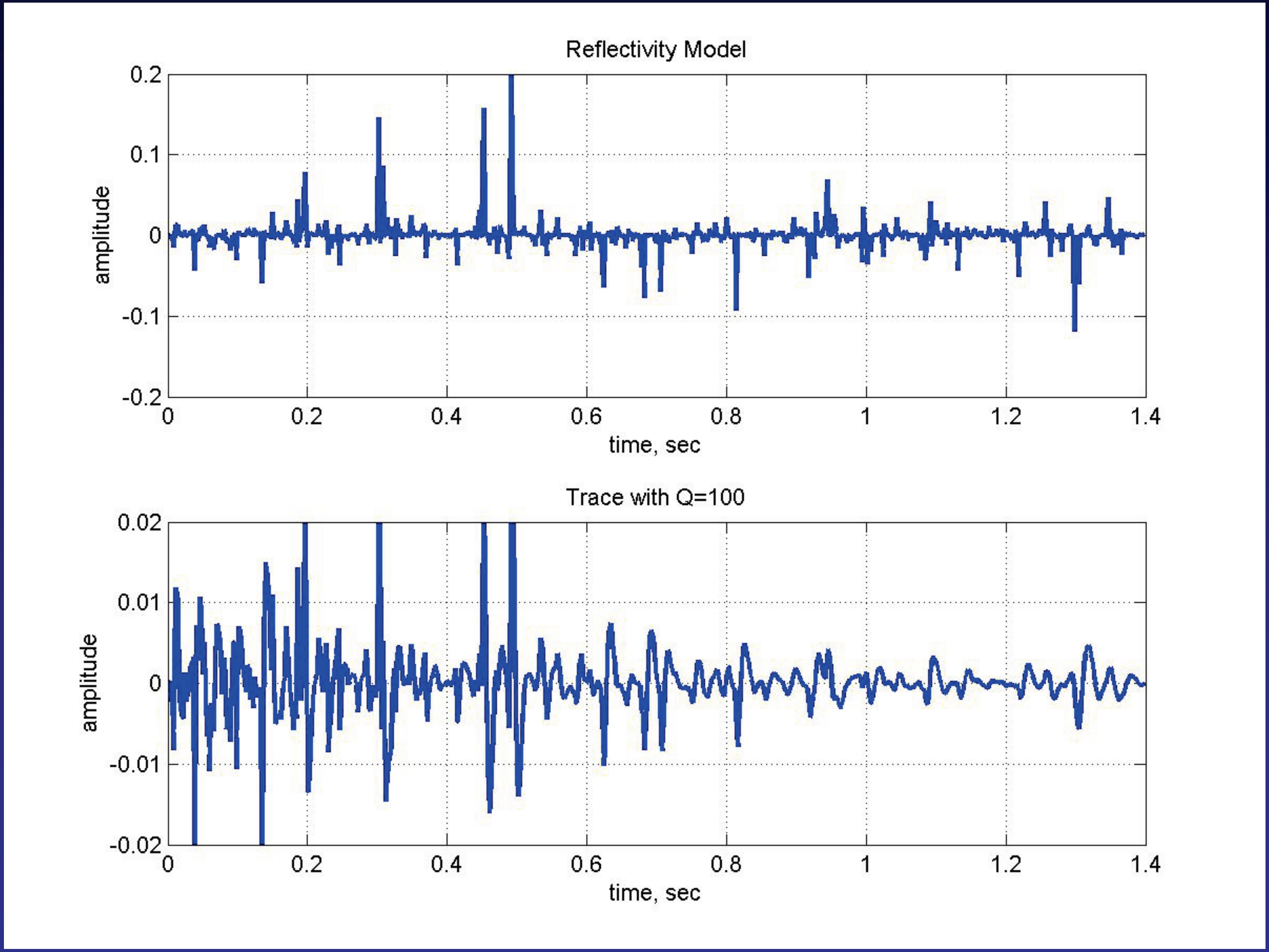
## - replace Gabor with VF S-transform

$$S_{ST}(\tau, f) = W(f)\alpha(\tau, f)R_{ST}(\tau, f)$$

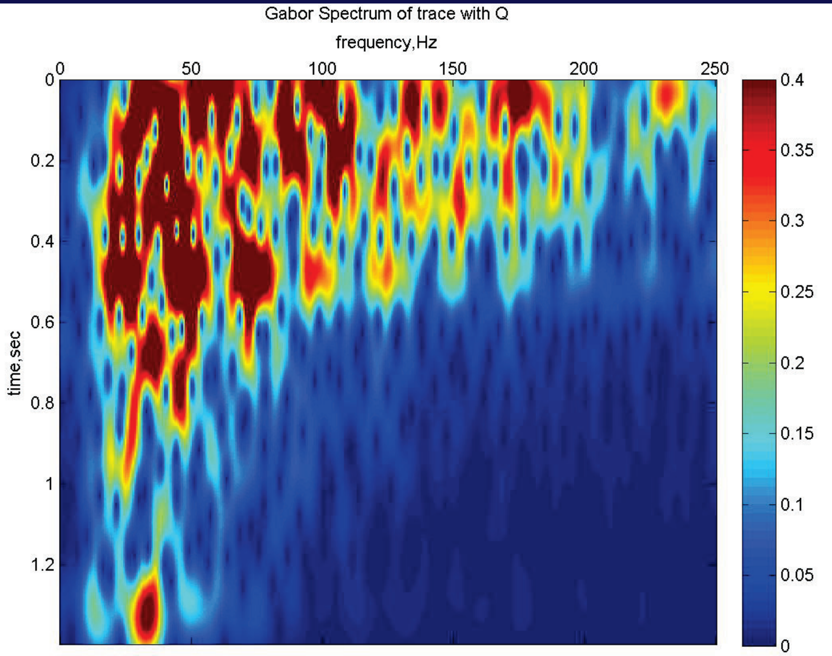
- compute VF $S_{ST}(\tau,f)$ of s(t)
- apply hyperbolic smoothing
- estimate inverse operator using minimum-phase
- multiply $S_{ST}(\tau,f)$ with the inverse operator
- inverse VF S-transform

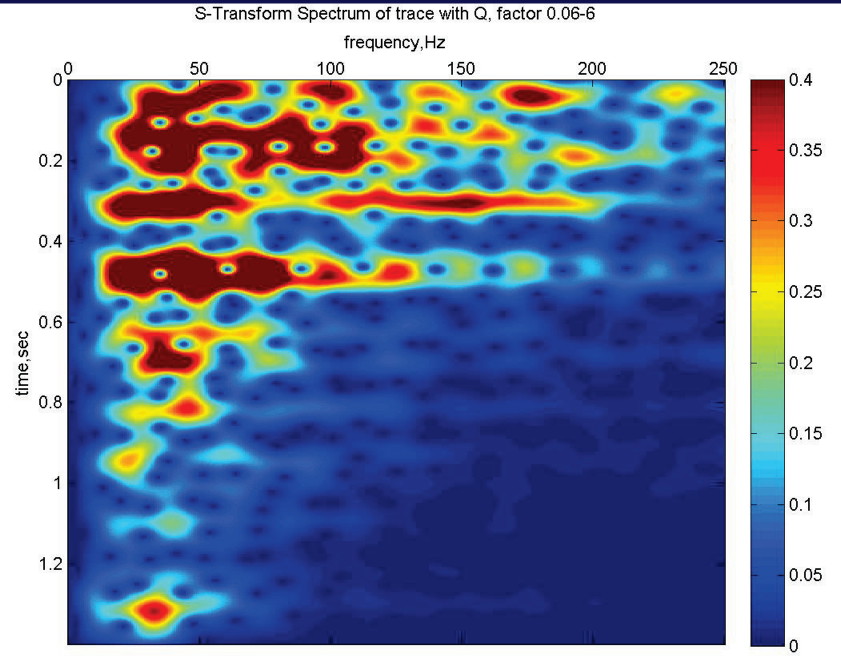
**Assumptions: causal, minimum-phase wavelet**

# Seismic trace, Q=100

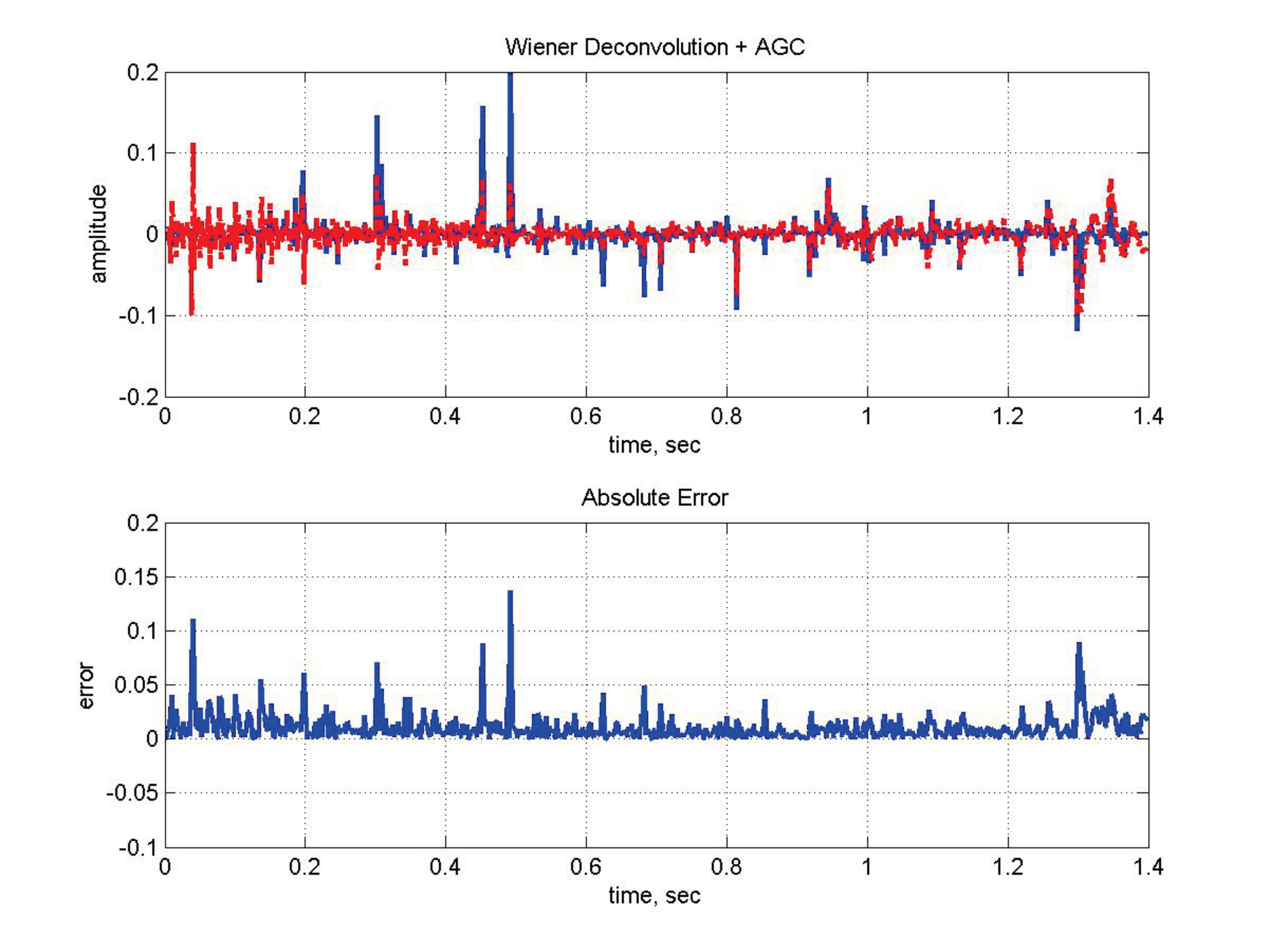


# Gabor / VF S-transform spectrums, Q=100

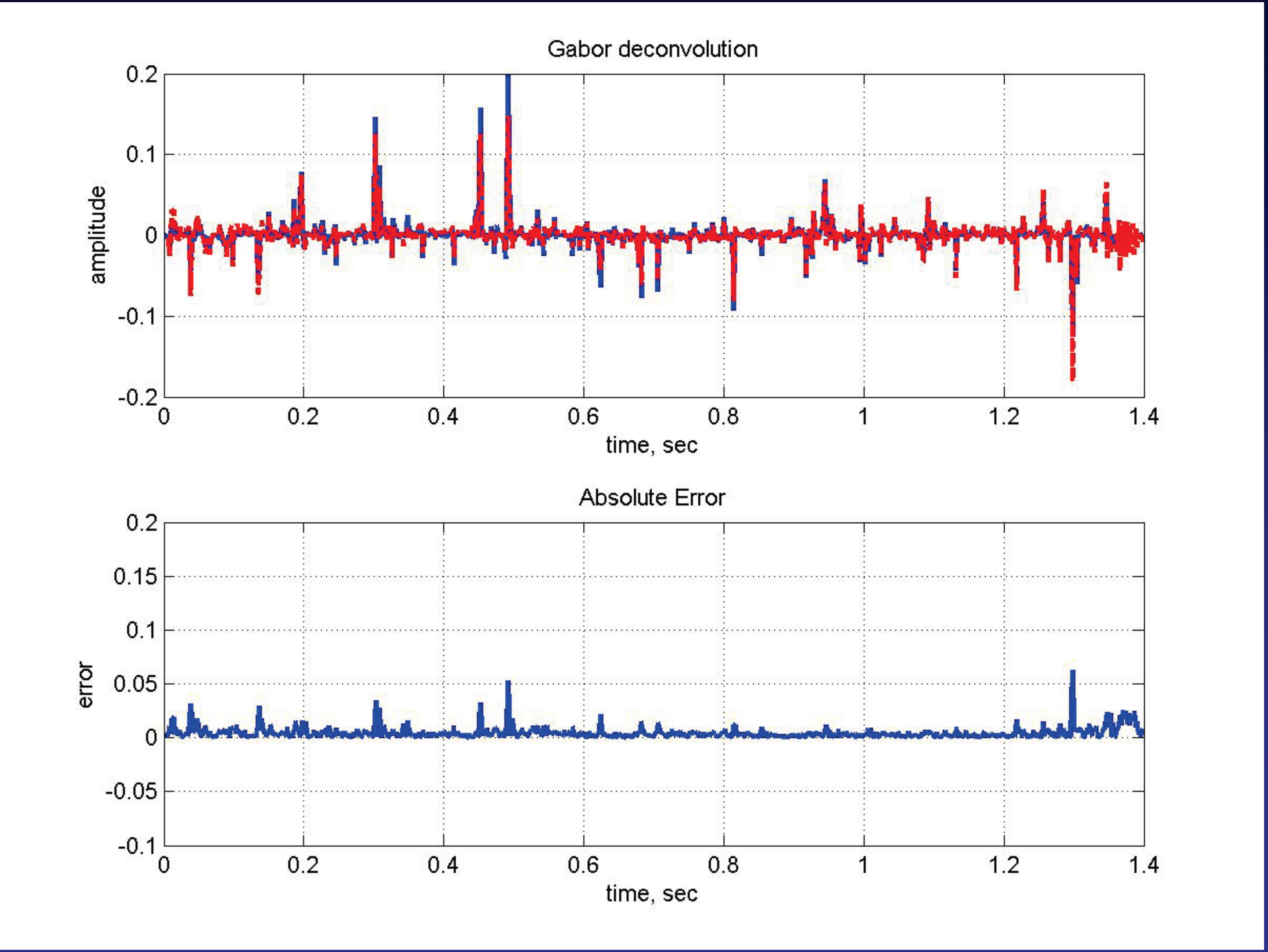


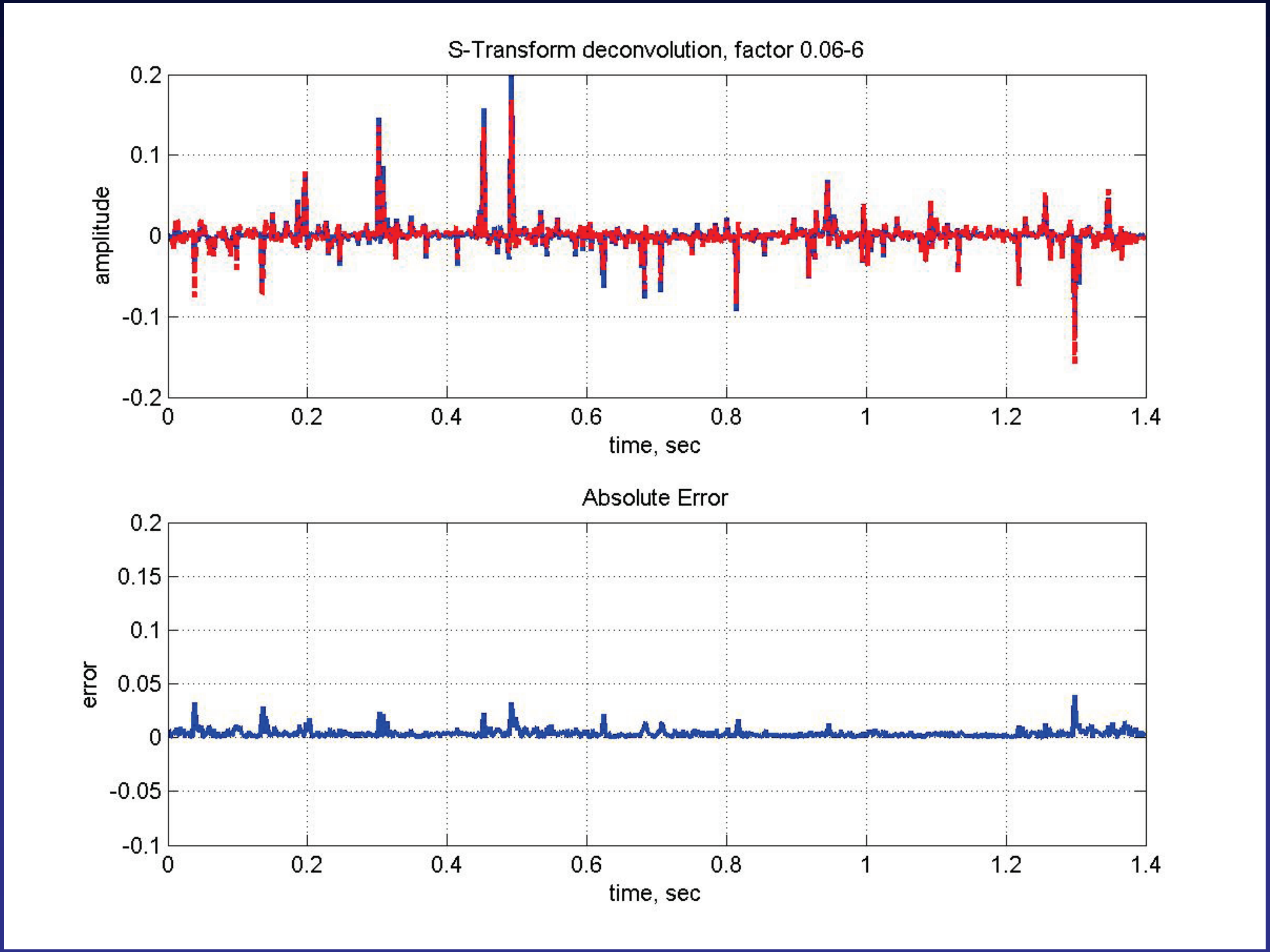
# Wiener deconvolution, Q=100



# Gabor deconvolution, Q=100



# VF S-transform deconvolution, Q=100



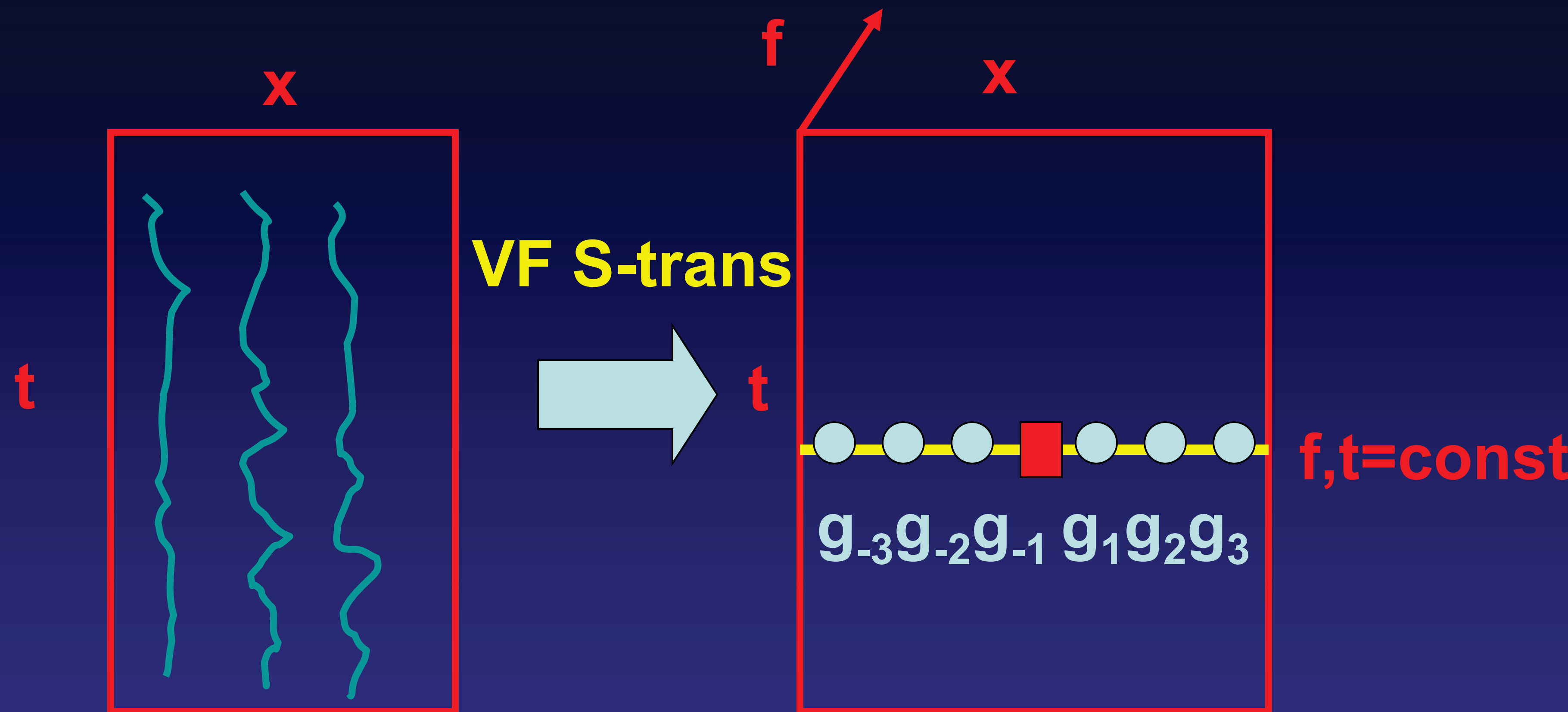
# Total Absolute error

| Method | Abs. Error Q=100 | Abs. Error Q=60 |
|---|---|---|
| Wiener | 7.2167 | 8.1997 |
| Gabor | 3.0356 | 3.0757 |
| S-transform | 2.6180 | 2.7349 |

# F-T-X noise attenuation

- f-x noise attenuation: Canales (1984)
- based on the Fourier transform
- seismic trace is not stationary
- noise is not stationary

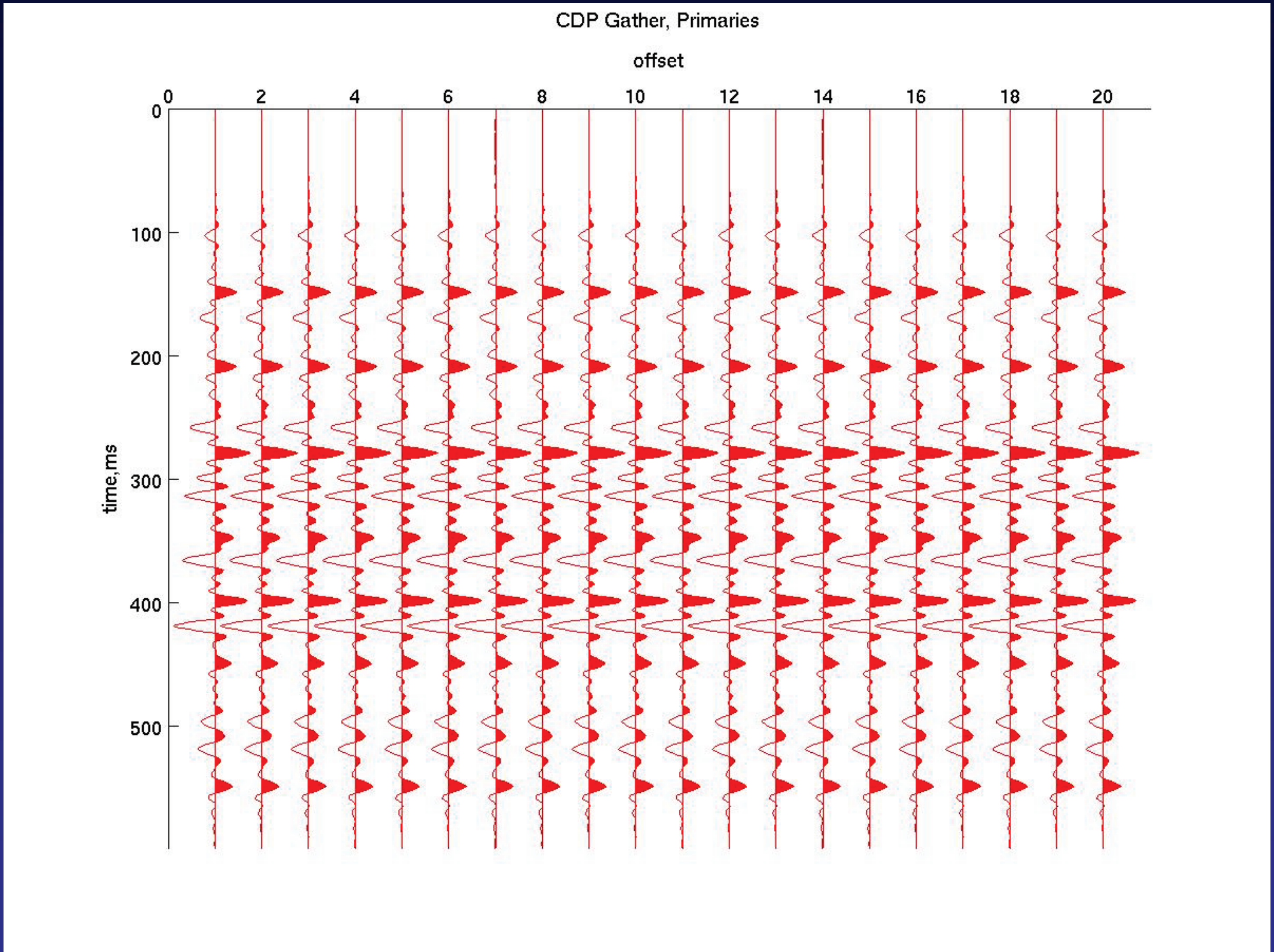
# CDP F-T-X noise attenuation



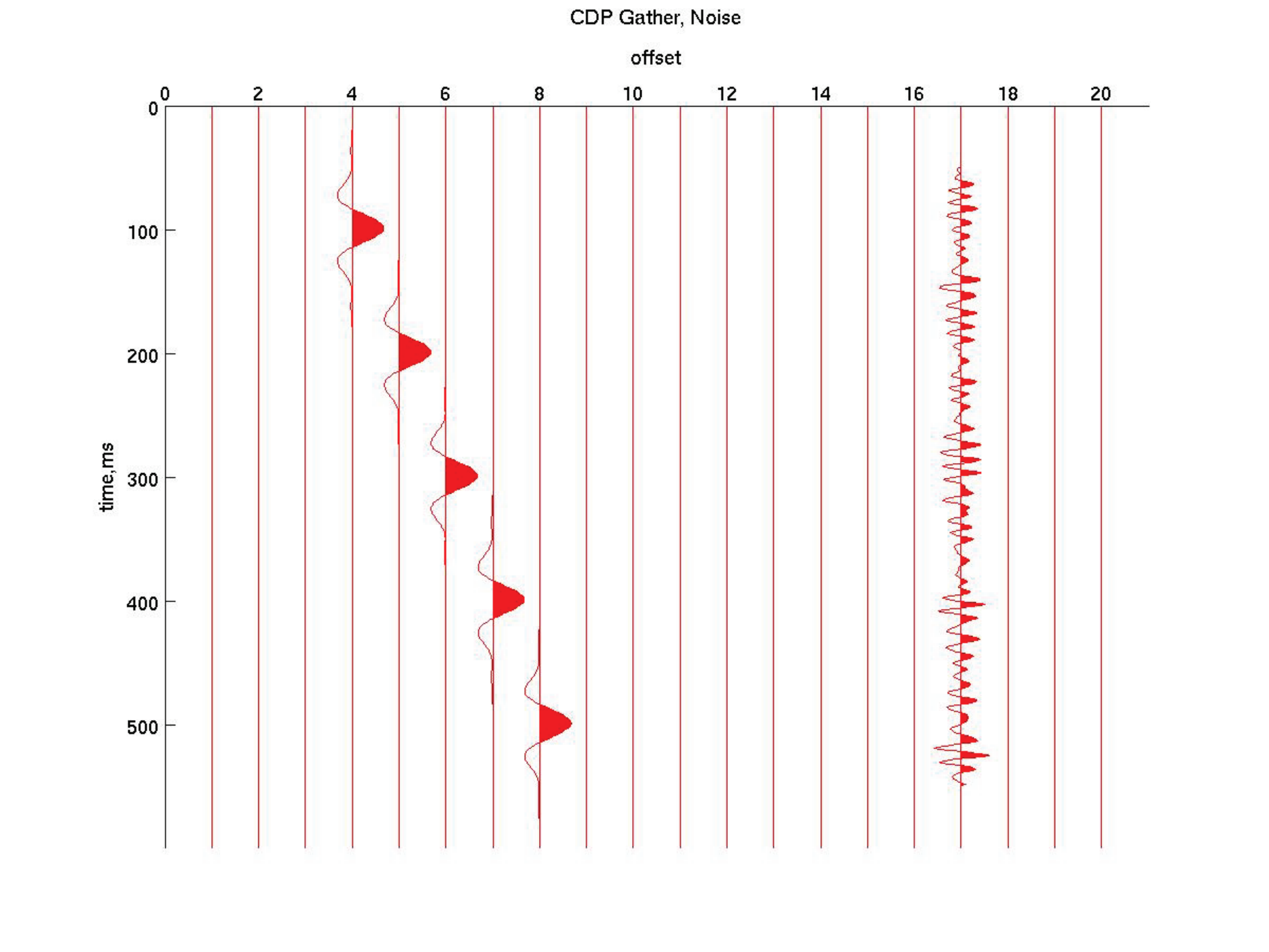

$$
\begin{bmatrix}
0 & d(x_{i-2}) & d(x_{i-1}) & d(x_{i+1}) & d(x_{i+2}) & d(x_{i+3}) \\
d(x_{i-3}) & 0 & d(x_{i-1}) & d(x_{i+1}) & d(x_{i+2}) & d(x_{i+3}) \\
d(x_{i-3}) & d(x_{i-2}) & 0 & d(x_{i+1}) & d(x_{i+2}) & d(x_{i+3}) \\
d(x_{i-3}) & d(x_{i-2}) & d(x_{i-1}) & 0 & d(x_{i+2}) & d(x_{i+3}) \\
d(x_{i-3}) & d(x_{i-2}) & d(x_{i-1}) & d(x_{i+1}) & 0 & d(x_{i+3}) \\
d(x_{i-3}) & d(x_{i-2}) & d(x_{i-1}) & d(x_{i+1}) & d(x_{i+2}) & 0
\end{bmatrix}
\begin{bmatrix}
g(x_{i-3}) \\ g(x_{i-2}) \\ g(x_{i-1}) \\ g(x_{i+1}) \\ g(x_{i+2}) \\ g(x_{i+3})
\end{bmatrix}
=
\begin{bmatrix}
d(x_{i-3}) \\ d(x_{i-2}) \\ d(x_{i-1}) \\ d(x_{i+1}) \\ d(x_{i+2}) \\ d(x_{i+3})
\end{bmatrix}
$$

**Solution: Truncated Singular Value Decomposition**

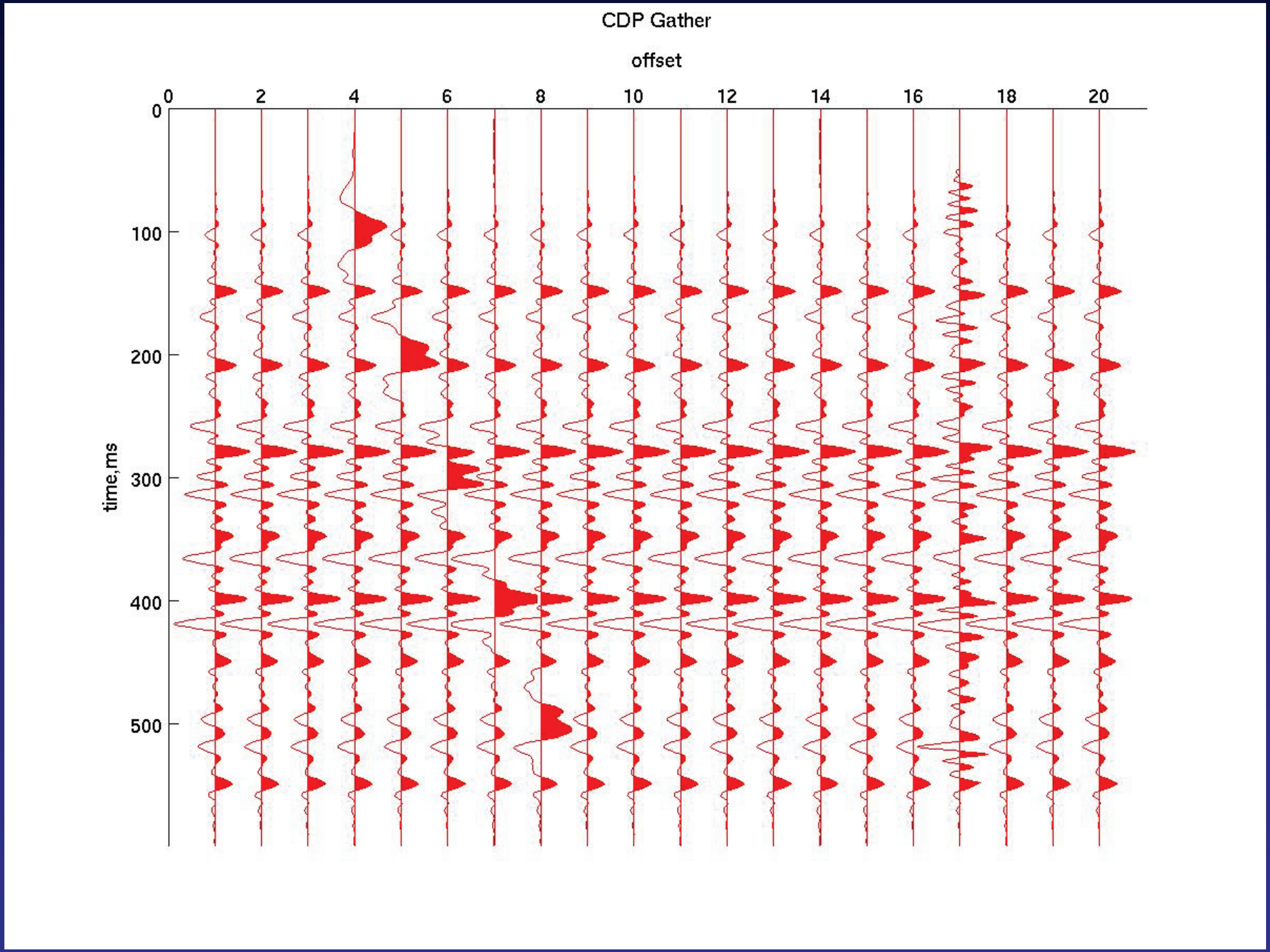
# CDP gather, primaries



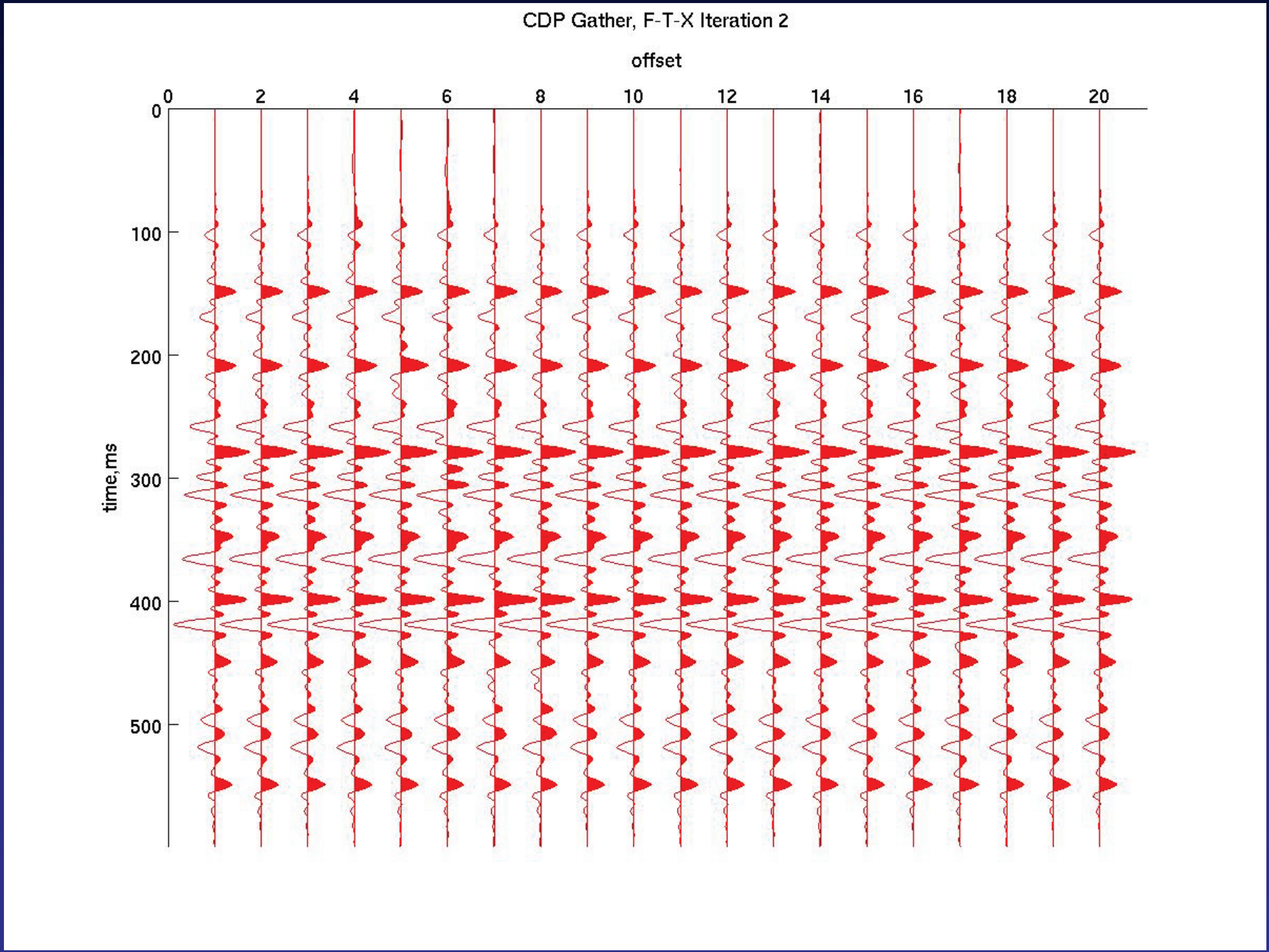
# CDP gather, noise



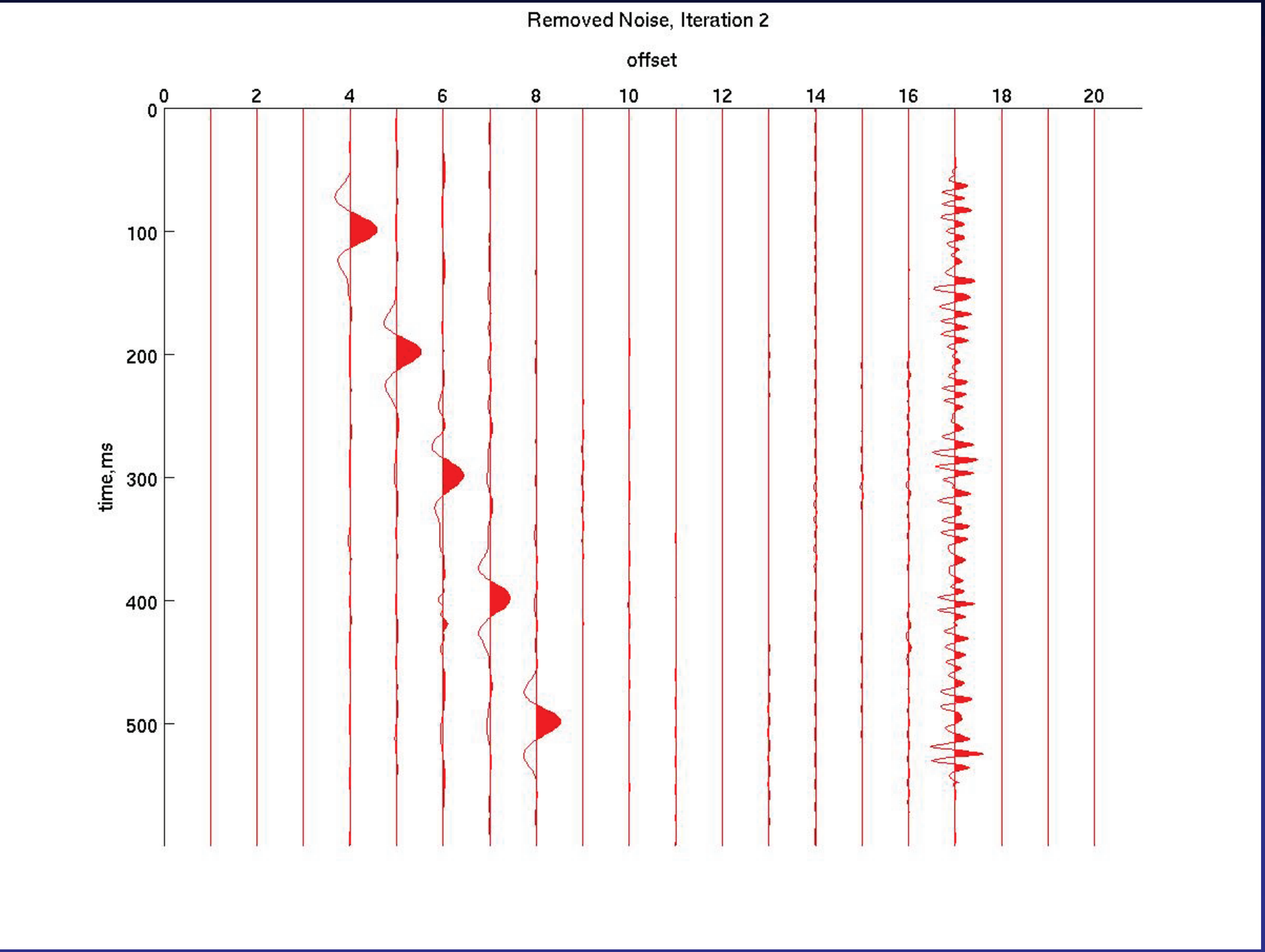
# CDP gather with noise



# CDP gather, F-T-X noise, second iteration



# Removed noise, two iterations



# Conclusions

- the variable factor S-transform shows better simultaneous time-frequency resolution over the Gabor and Stockwell's S-transform
- the VF S-transform deconvolution is superior over the Wiener and an improvement to Gabor
- F-T-X noise attenuation shows good potential

# Future Work

- Frequency domain implementation of the S-transform, redundancy for speed
- Surface-consistent VF S-transform deconvolution
- F-T-X noise attenuation on AVO model, NMO stretch, real data
- Q estimation

# Acknowledgements

## The CREWES Project sponsors

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