Determining time-lapse variations in the Earth

directly from differenced seismic reflection data

Kris Innanen & Mostafa Naghizadeh

Banff, AB
2 Dec 2010

CREWES

Elements of the TL problem, terminology, and goals of inversion

Defining and interpreting the difference model

Scattering formulation & a mysterious initial finding

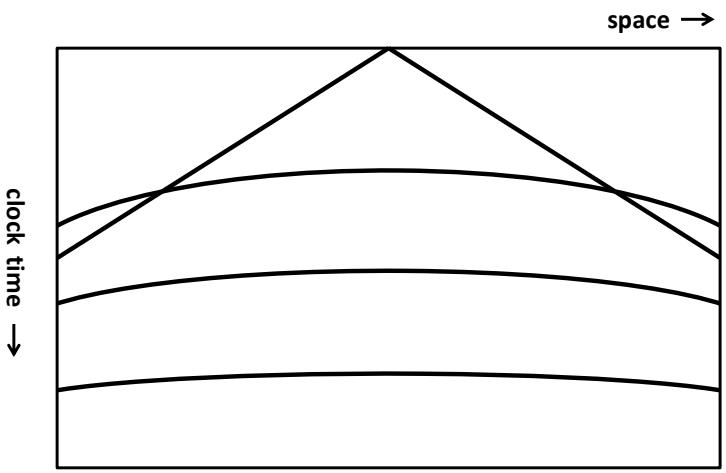
Results emerging from "TL scattering theory"

Onward

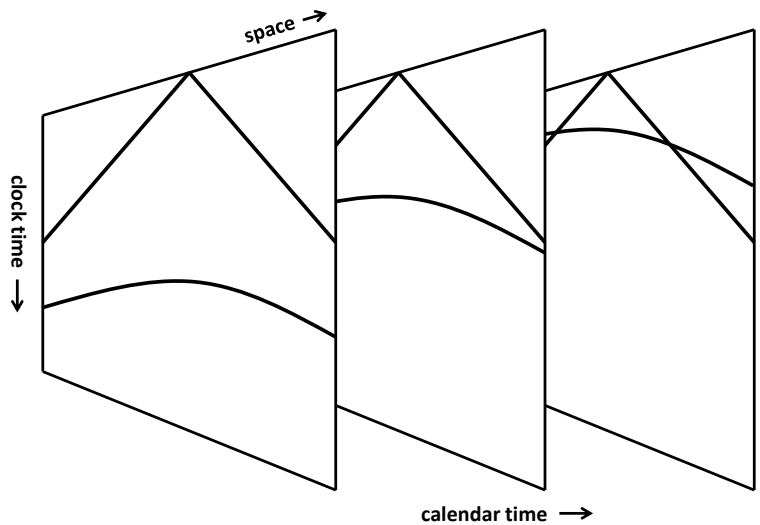
Elements of the time-lapse seismic reflection problem

Introduce 'calendar time' axis:

standard survey

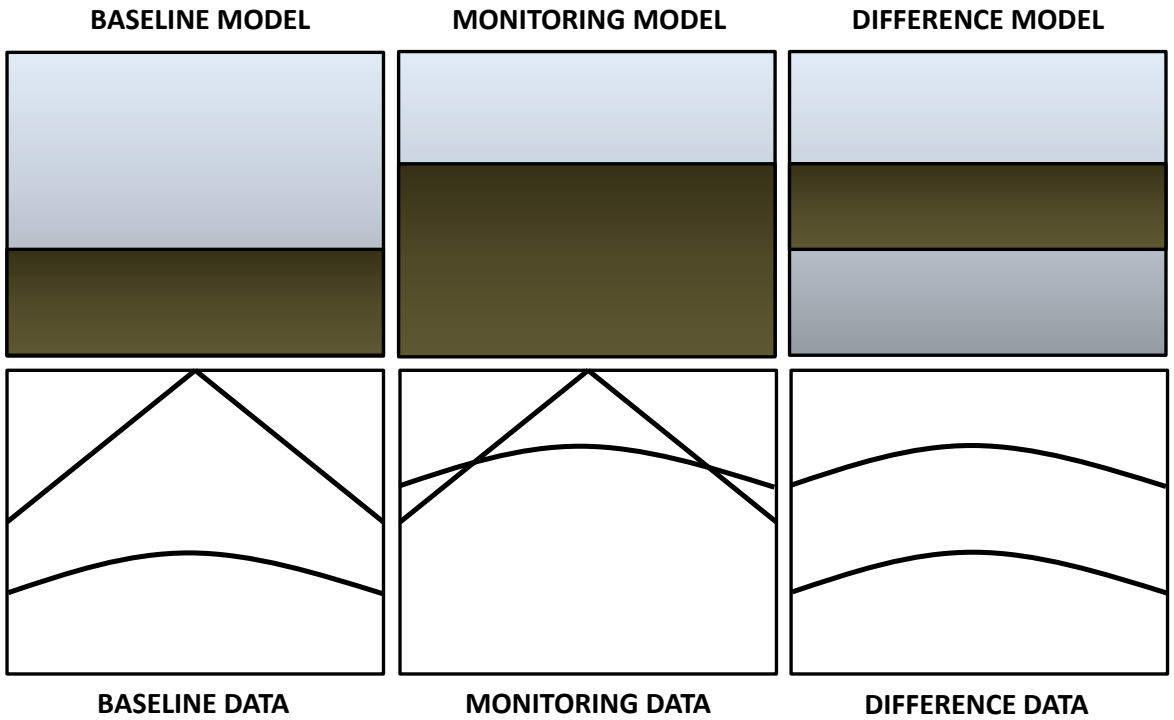


time-lapse survey



- Production: reservoir monitoring, EOR
- CO_2 Storage: injection, long-term monitoring, failure detection

Elements of the time-lapse seismic reflection problem



- Interest is in what has *changed* – "difference model"
- Focus therefore on the "difference data", which is, we presume,
  1. relatively insensitive to the static portion of Earth
  2. relatively sensitive to the dynamic portion

Goals of a framework for time-lapse inversion

1. Based on a direct relationship between difference model & difference data

2. Maximally wave-theoretic
   - two-way wave equations
   - amplitudes & phases/traveltimes
   - multidimensional, multiparameter
   - inclusive of acoustic, elastic, anelastic, anisotropic, etc.

3. Providing (as a meaningful theory should):
   - inversion algorithms
   - insight into the character of inverse problem

Defining and interpreting the difference model

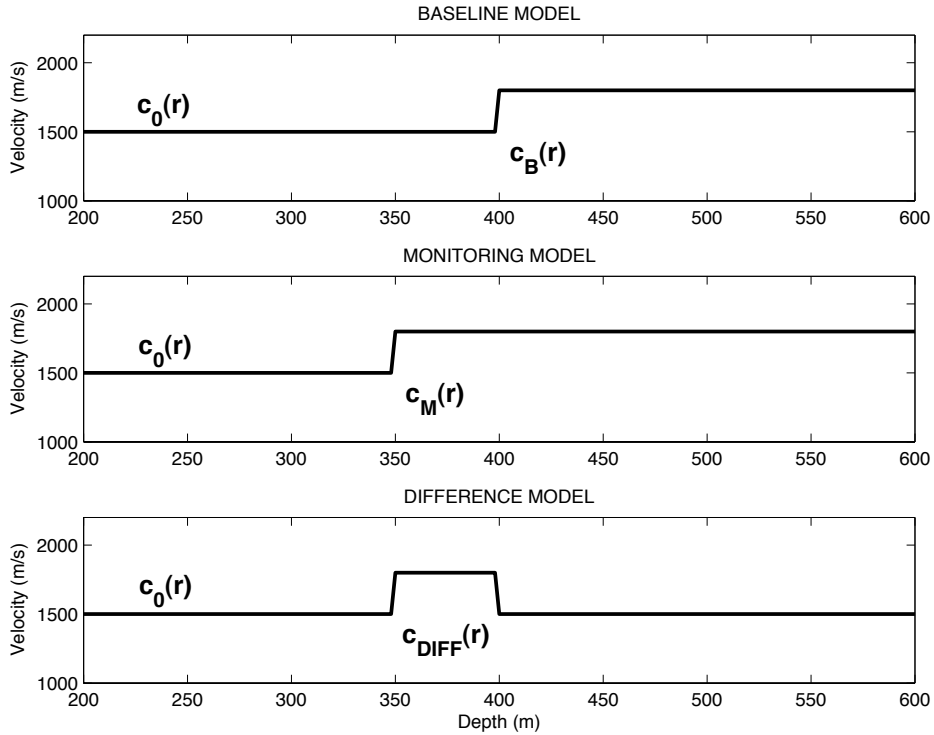
BL $\rightarrow$ M perturbation: $$\frac{1}{c_{\mathrm{M}}^2(\mathbf{r})} = \frac{1}{c_B^2(\mathbf{r})}\left[1 - \alpha_{TL}(\mathbf{r})\right]$$

Ref $\rightarrow$ BL perturbation: $$\frac{1}{c_{\mathrm{B}}^2(\mathbf{r})} = \frac{1}{c_0^2(\mathbf{r})}\left[1 - \alpha_S(\mathbf{r})\right]$$

Mixture: $$\frac{1}{c_{\mathrm{M}}^2(\mathbf{r})} = \frac{1}{c_0^2(\mathbf{r})}\left[1 - \alpha_S(\mathbf{r})\right] \times \left[1 - \alpha_{TL}(\mathbf{r})\right]$$

Difference: $$\frac{1}{c_{\mathrm{DIFF}}^2(\mathbf{r})} \equiv \frac{1}{c_0^2(\mathbf{r})}\left[1 - \alpha_{TL}(\mathbf{r})\right]$$

Defining and interpreting the difference model



$$\frac{1}{c^2_{\text{DIFF}}(\mathbf{r})} \equiv \frac{1}{c^2_0(\mathbf{r})}\left[1 - \alpha_{TL}(\mathbf{r})\right], \qquad \text{perturbation form}$$

$$\frac{1}{c^2_{\text{DIFF}}(\mathbf{r})} \equiv \frac{1}{c^2_0(\mathbf{r})} + \left(\frac{1}{c^2_M(\mathbf{r})} - \frac{1}{c^2_B(\mathbf{r})}\right), \qquad \text{explicit form}$$

Defining and interpreting the difference model

Goals revisited:

1. Difference data $\leftrightarrow$ difference model
2. Fully wave-theoretic

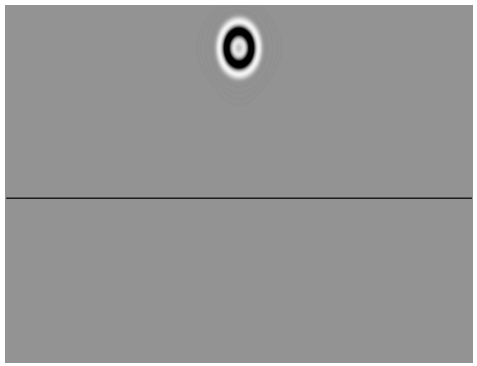
Does our choice of difference model align with these goals?

Initial evaluation: propagate a 2-way scalar wave field through a difference model, and compare it with the difference between fields propagating in BL and M models...

Response of the difference model to a 2-way wave

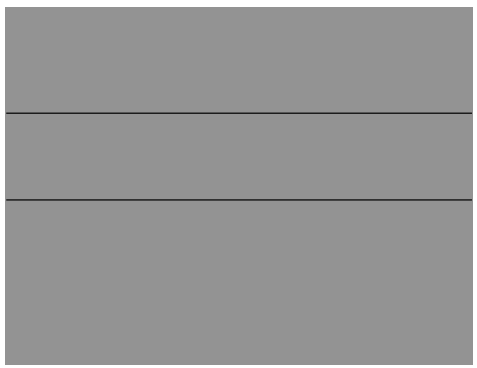
Propagate a scalar wave through three media:

BASELINE (BL)

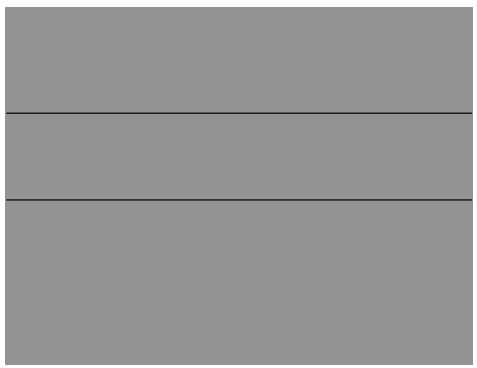


MONITORING (M)

DIFFERENCE OF BL, M FIELDS



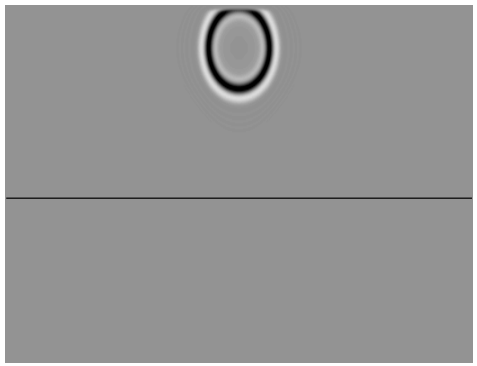
FIELD IN DIFFERENCE MODEL



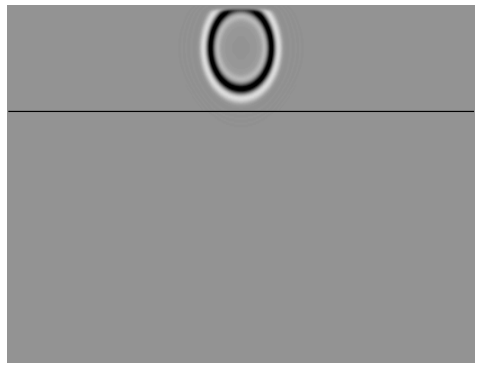
Response of the difference model to a 2-way wave

While the wave is in region common to M, BL, both fields are nil.

BASELINE (BL)



MONITORING (M)



DIFFERENCE OF BL, M FIELDS

FIELD IN DIFFERENCE MODEL

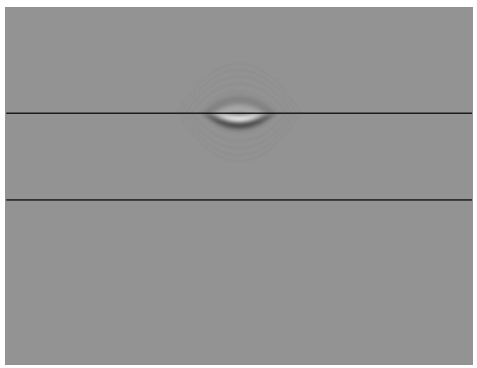
Response of the difference model to a 2-way wave

Onset of nonzero amplitudes: fields track each other well.

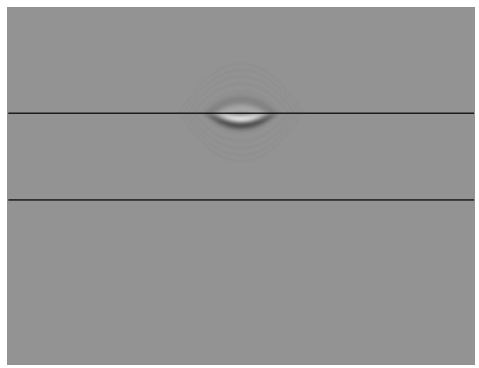
BASELINE (BL)

MONITORING (M)

DIFFERENCE OF BL, M FIELDS



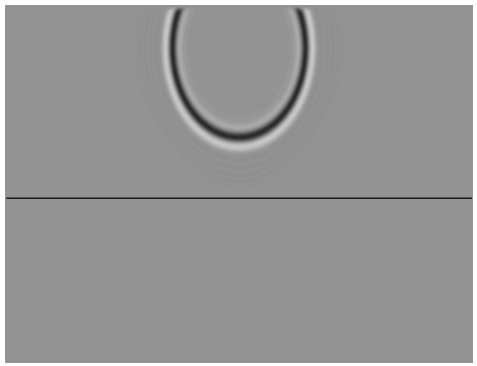
FIELD IN DIFFERENCE MODEL



Response of the difference model to a 2-way wave

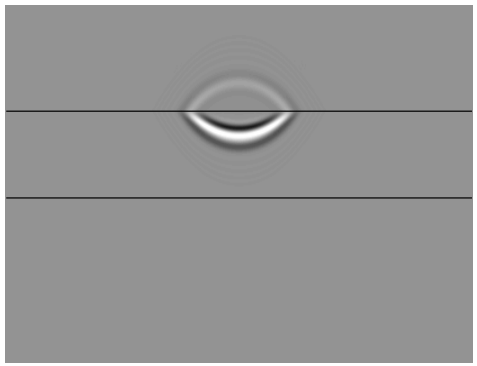
Onset of nonzero amplitudes: fields track each other well.

BASELINE (BL)

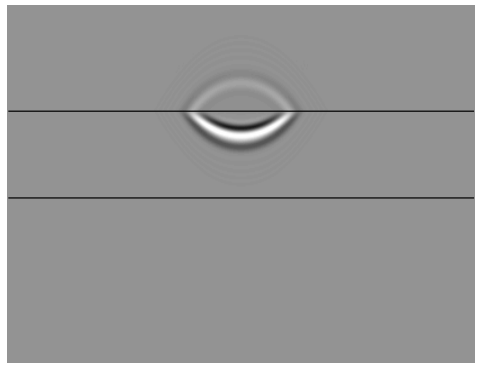


MONITORING (M)

DIFFERENCE OF BL, M FIELDS



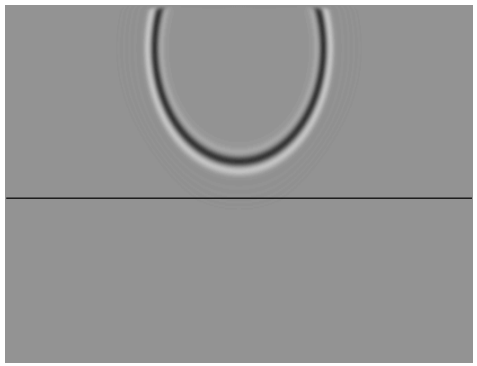
FIELD IN DIFFERENCE MODEL



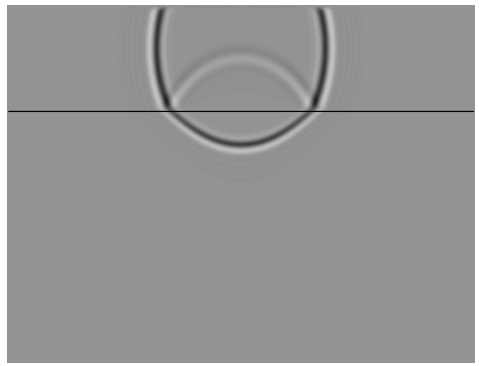
Response of the difference model to a 2-way wave

Upgoing reflection: fields have same polarity.

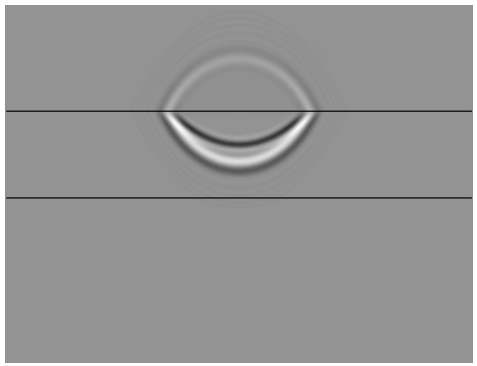
BASELINE (BL)



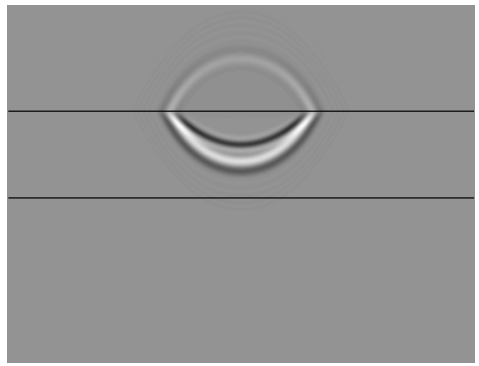
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



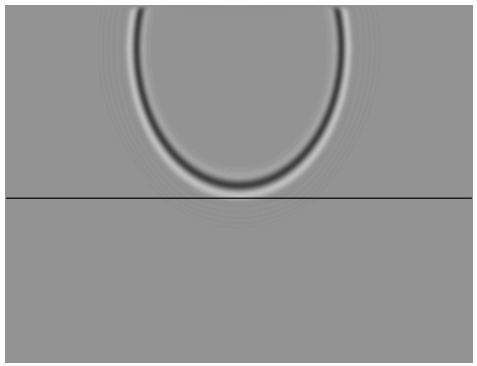
FIELD IN DIFFERENCE MODEL



Response of the difference model to a 2-way wave

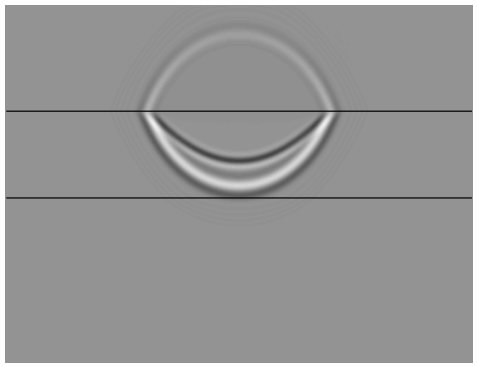
Upgoing reflection: fields have same polarity.

BASELINE (BL)

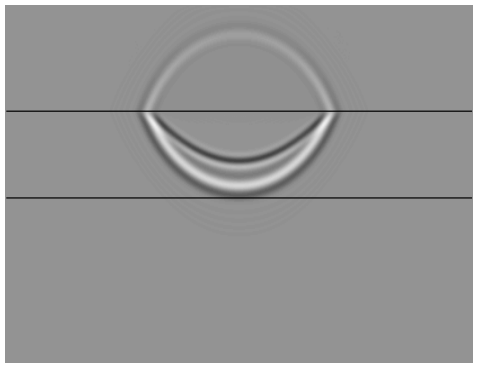


MONITORING (M)

DIFFERENCE OF BL, M FIELDS



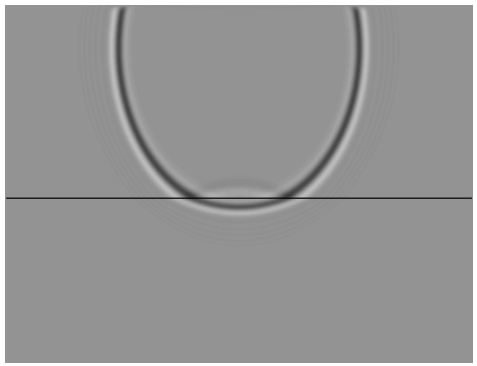
FIELD IN DIFFERENCE MODEL



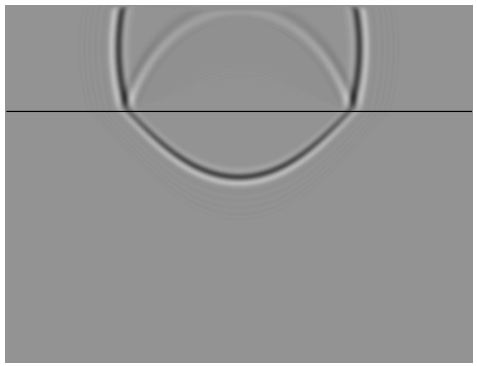
Response of the difference model to a 2-way wave

BL-M field: "deeper" wavefront reflects first.

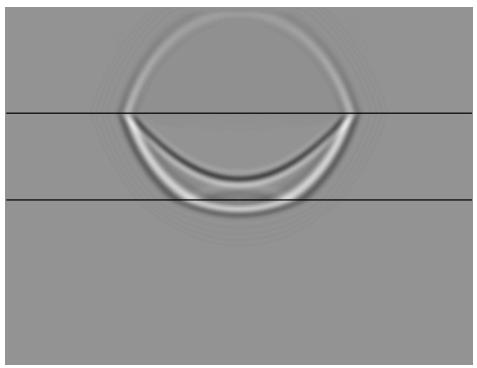
BASELINE (BL)



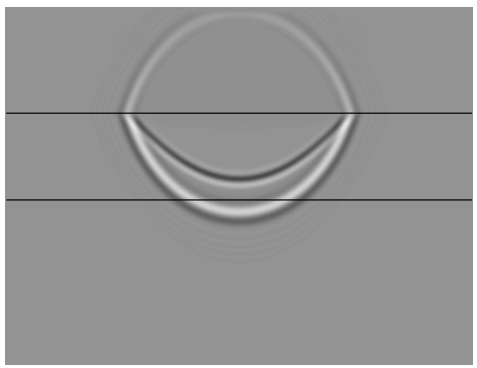
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



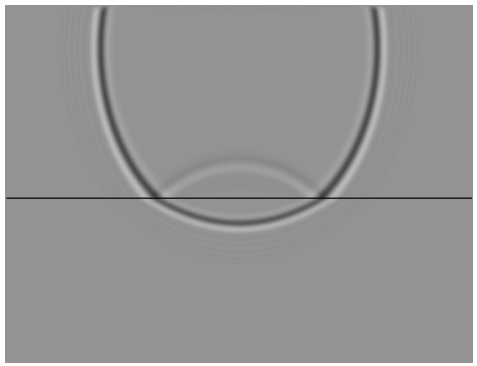
FIELD IN DIFFERENCE MODEL



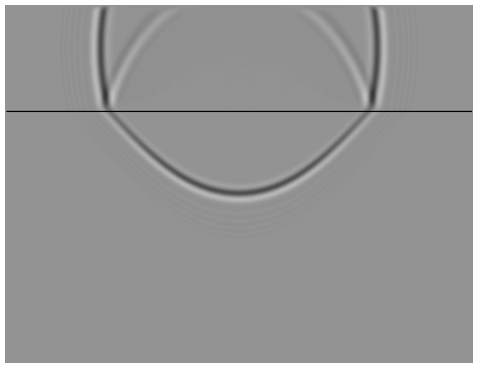
Response of the difference model to a 2-way wave

BL-M field: "deeper" wavefront reflects first.

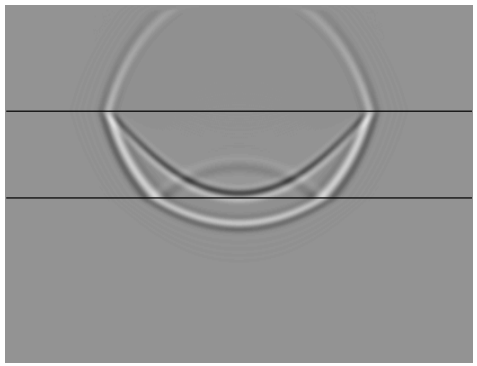
BASELINE (BL)



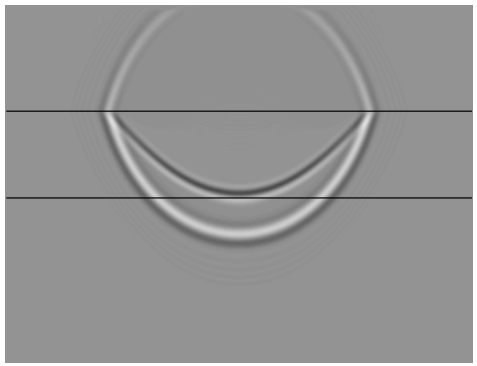
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



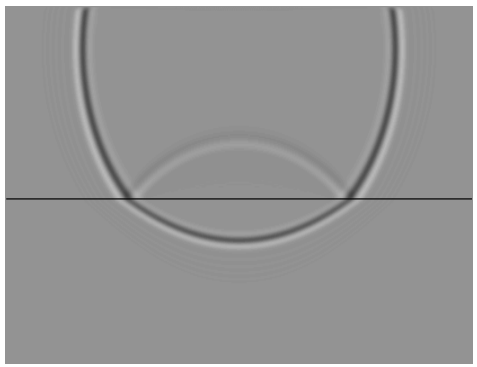
FIELD IN DIFFERENCE MODEL



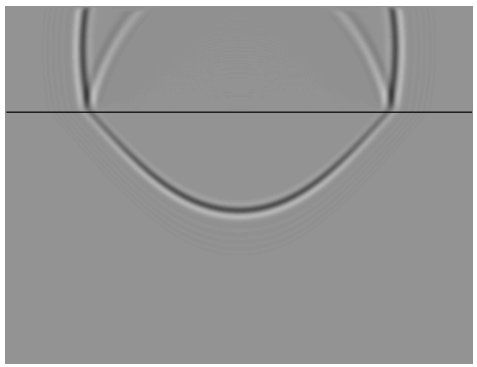
Response of the difference model to a 2-way wave

Field in difference model: "shallower" wavefront reflects instead.

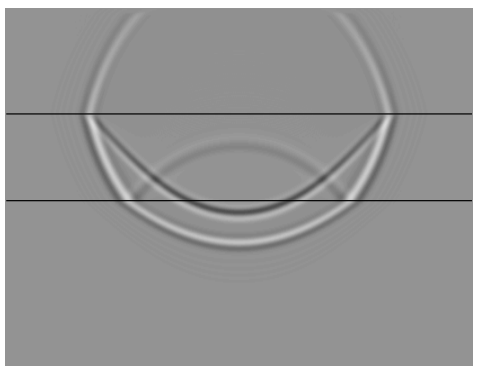
BASELINE (BL)



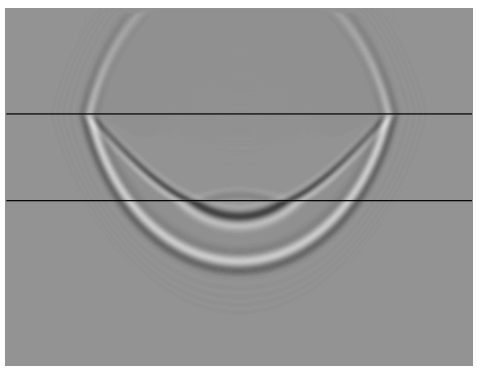
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



FIELD IN DIFFERENCE MODEL

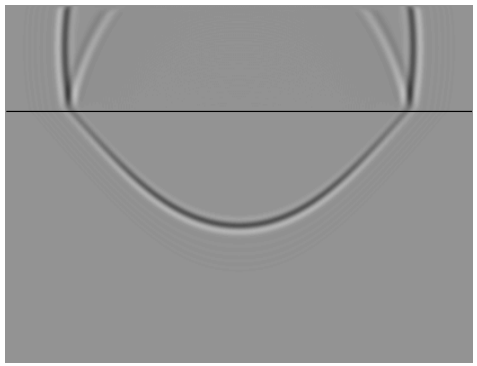


Response of the difference model to a 2-way wave

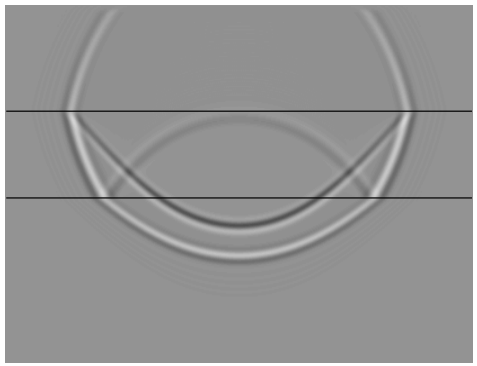
Reflections with paths *in perturbed medium* don't match.

BASELINE (BL)

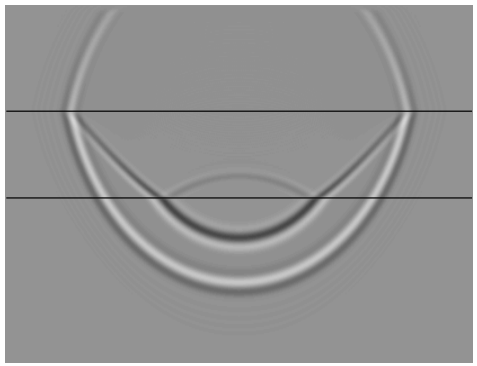
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



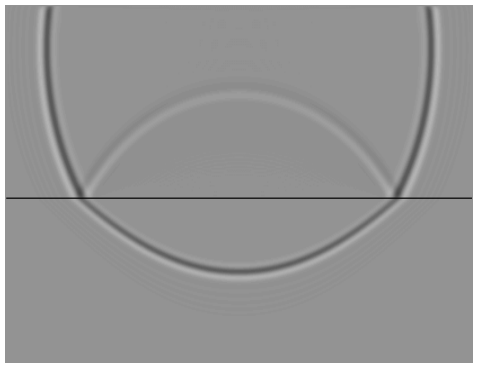
FIELD IN DIFFERENCE MODEL



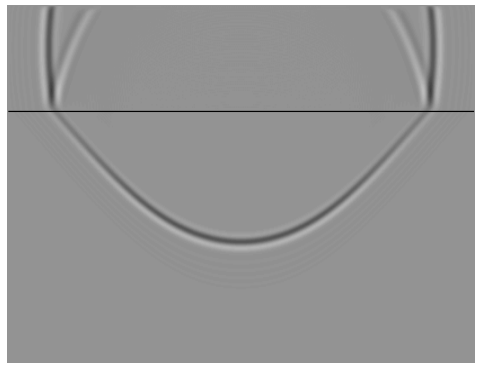
Response of the difference model to a 2-way wave

Difference model generates correct negative polarity on BL reflection.

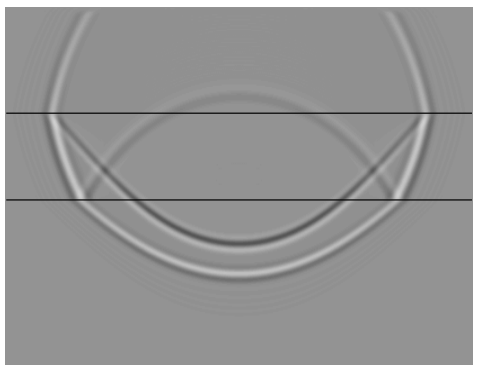
BASELINE (BL)



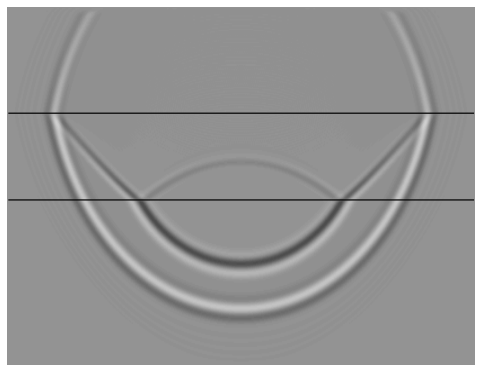
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



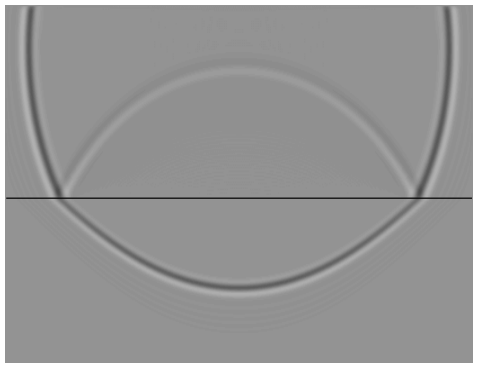
FIELD IN DIFFERENCE MODEL



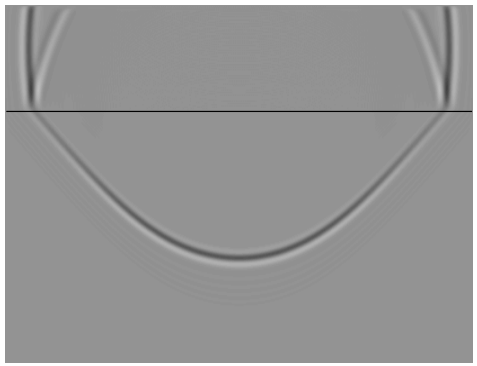
Response of the difference model to a 2-way wave

One more difference is coming...

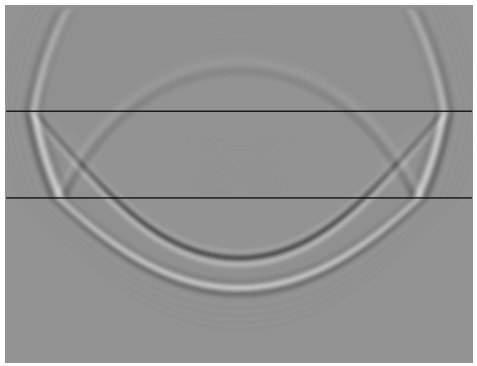
BASELINE (BL)



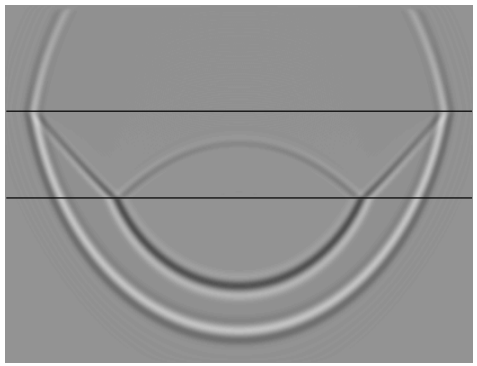
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



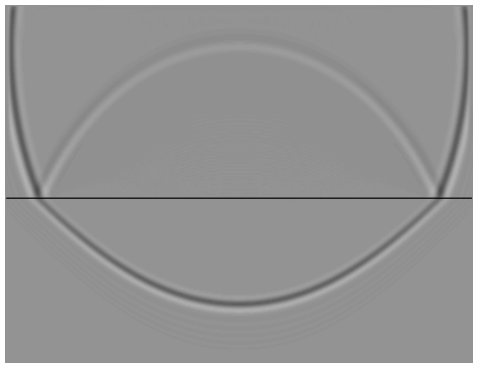
FIELD IN DIFFERENCE MODEL



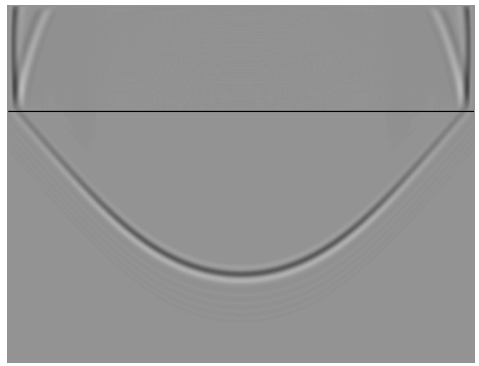
Response of the difference model to a 2-way wave

One more difference is coming...

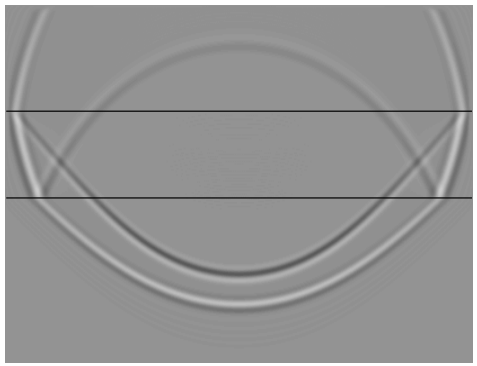
BASELINE (BL)



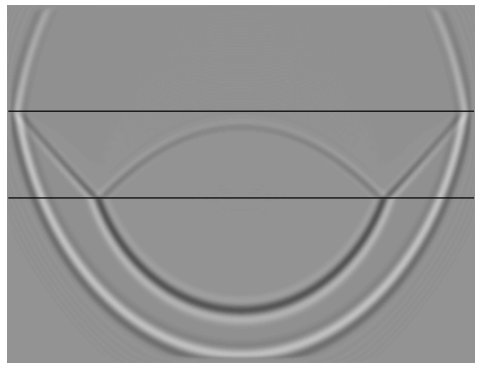
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



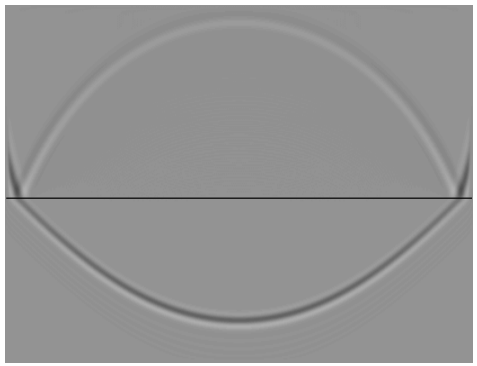
FIELD IN DIFFERENCE MODEL



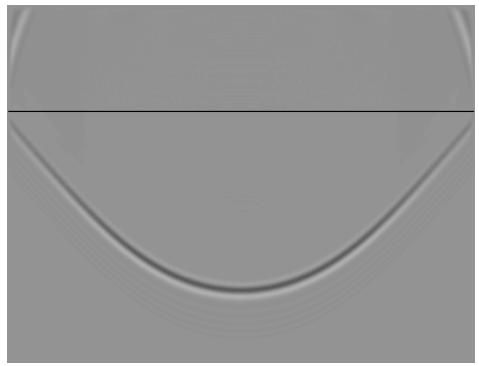
Response of the difference model to a 2-way wave

One more difference is coming...

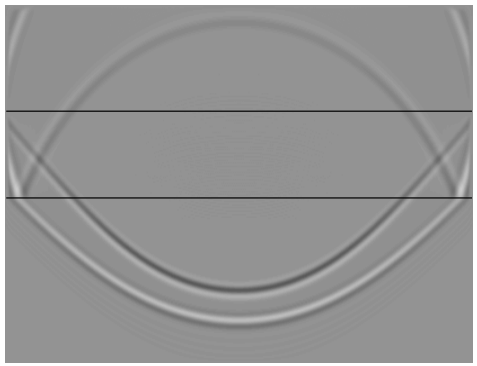
BASELINE (BL)



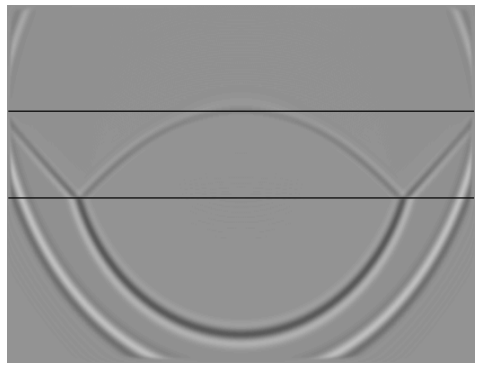
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



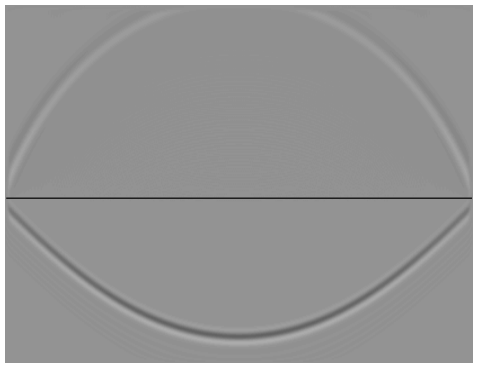
FIELD IN DIFFERENCE MODEL



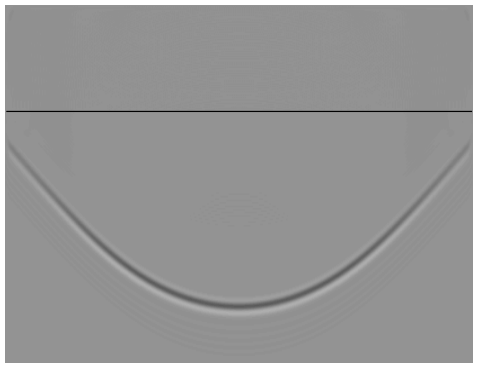
Response of the difference model to a 2-way wave

One more difference is coming...

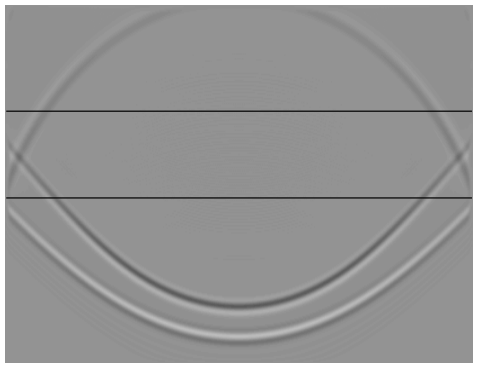
BASELINE (BL)



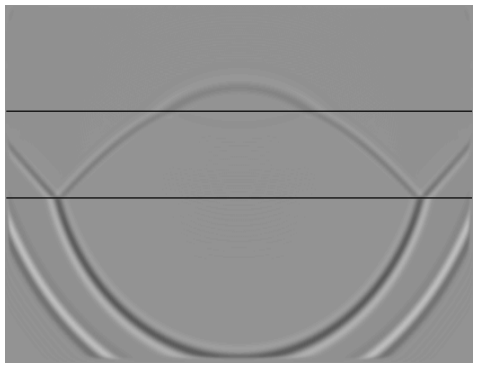
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



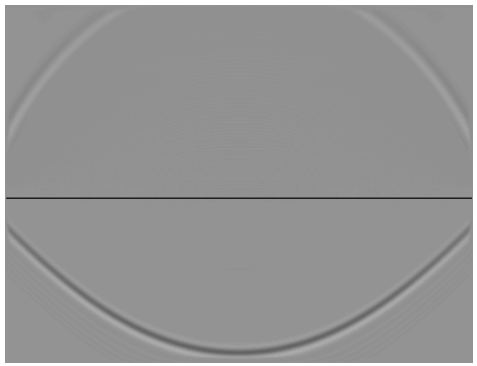
FIELD IN DIFFERENCE MODEL



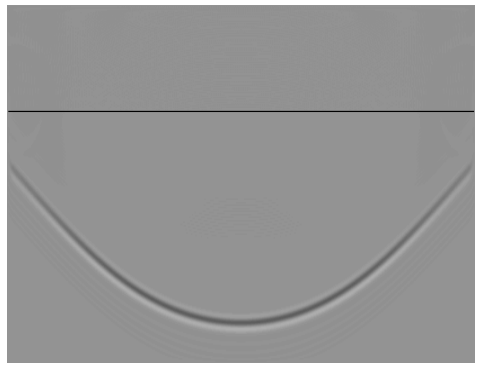
Response of the difference model to a 2-way wave

One more difference is coming...

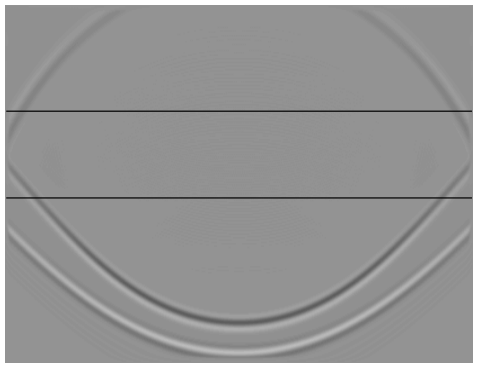
BASELINE (BL)



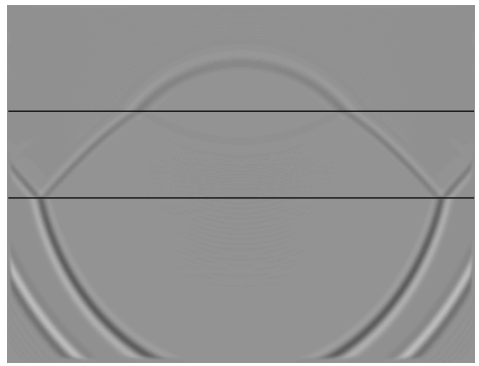
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



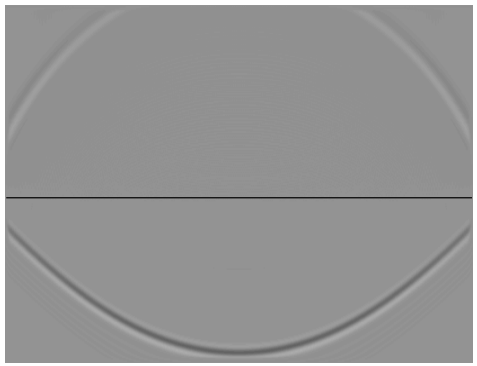
FIELD IN DIFFERENCE MODEL



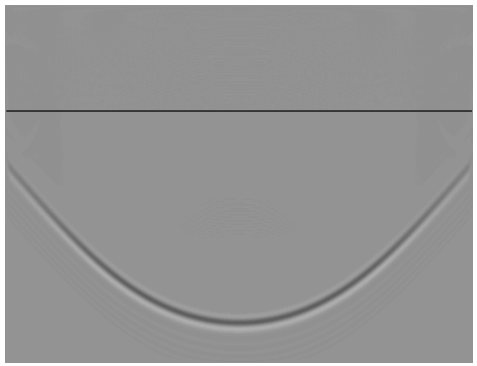
Response of the difference model to a 2-way wave

There!

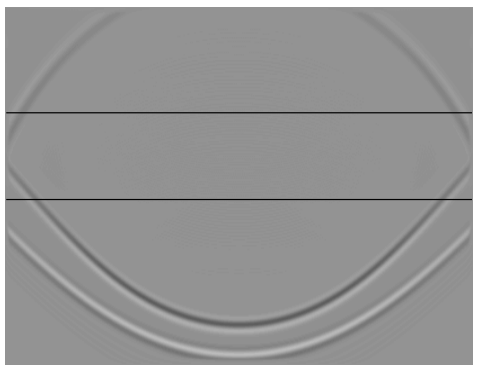
BASELINE (BL)



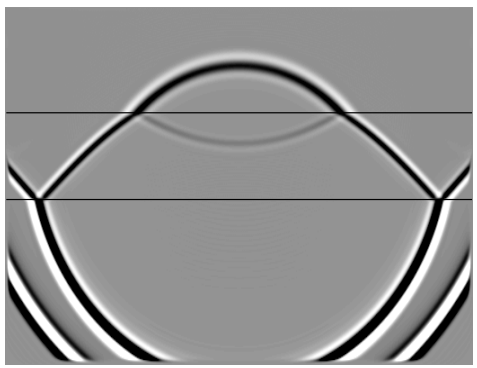
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



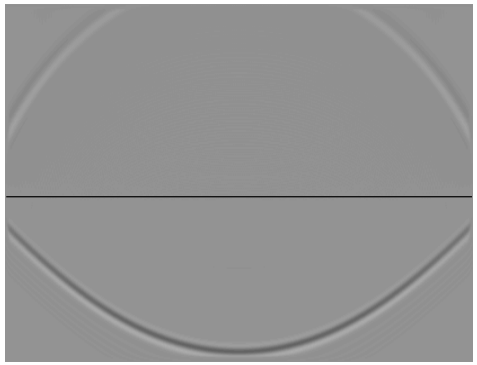
FIELD IN DIFFERENCE MODEL



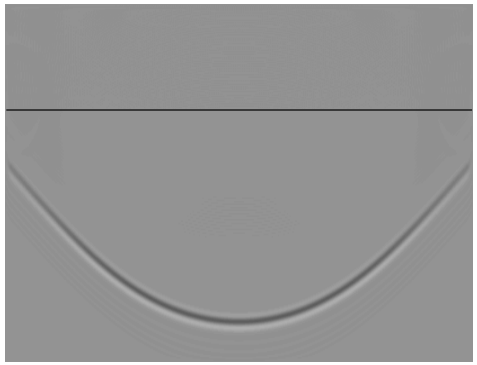
Response of the difference model to a 2-way wave

There! A wave in the difference model *reverberates* between an interface and itself at a later time. Spurious multiples...

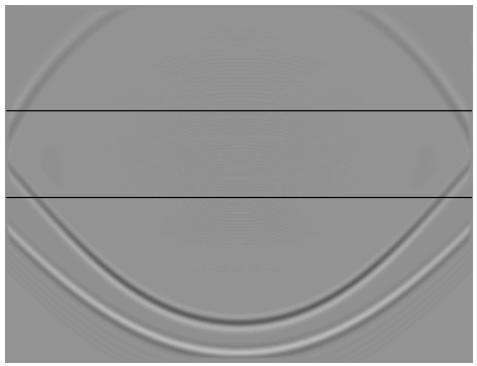
BASELINE (BL)



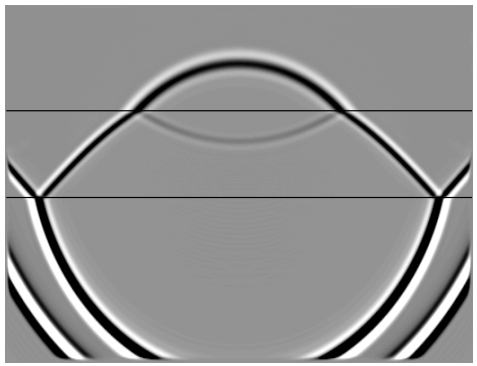
MONITORING (M)



DIFFERENCE OF BL, M FIELDS



FIELD IN DIFFERENCE MODEL



Evaluating our choice of difference model

Preliminary conclusions — the difference model as defined:

- Correctly generates "positive" monitoring reflections
- Correctly generates "negative" baseline reflections
- Phase/amplitude error for events propagating through large/extended regions of difference
- Nonlinearity — here seen in form of multiple reflections

...looks promising, but *difference model* $\leftrightarrow$ *difference data* mapping by straight propagation of a 2-way wave through the difference model generates artifacts & errors at large contrasts.

Scattering formulation

SCATTERING QUANTITIES		TIME-LAPSE QUANTITIES	
reference medium	$c_0(\mathbf{r})$	baseline (BL) medium	$c_{\mathrm{B}}(\mathbf{r})$
perturbed medium	$c(\mathbf{r})$	monitoring medium	$c_{\mathrm{M}}(\mathbf{r})$
reference field	$\left(\nabla^2 + \frac{\omega^2}{c_0^2(\mathbf{r})}\right) G_0 = \delta$	baseline data	D_{B}
perturbed field	$\left(\nabla^2 + \frac{\omega^2}{c^2(\mathbf{r})}\right) P = \delta$	monitoring data	D_{M}
scattered field	$P - G_0$	difference data	$D_{\mathrm{M}} - D_{\mathrm{B}}$
perturbation	$\alpha(\mathbf{r}) = 1 - \frac{c_0^2(\mathbf{r})}{c^2(\mathbf{r})}$	difference model	$\frac{1}{c_0^2} + \left(\frac{1}{c_{\mathrm{M}}^2} - \frac{1}{c_{\mathrm{B}}^2}\right)$

Scattering formulation

Options:

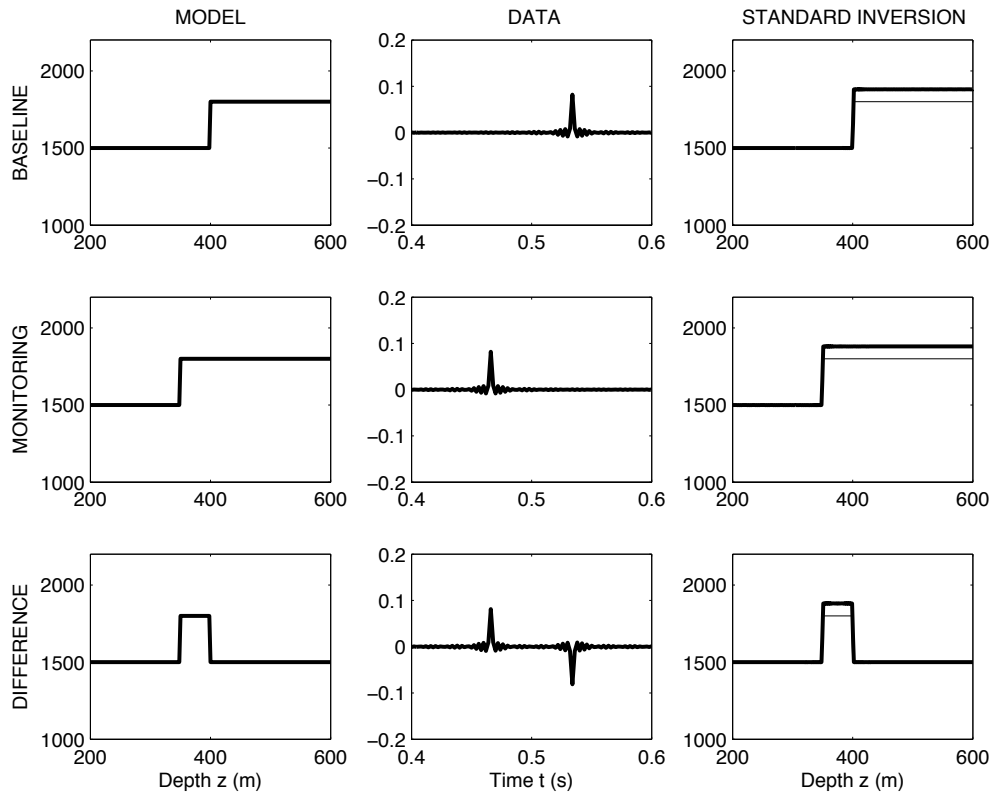
1. "Port" existing inverse scattering methods (Zhang, 2006), or
2. Reformulate problem entirely.

Either route will require us to contend with **complexity of reference medium**.

First route, on its face, seems impossible: **assumption of smooth or homogeneous reference medium** vs. **content of difference data**.

Scattering formulation

A simple test... and a mysterious result.



(Standard linear I.S. is trace integration: $\alpha_{TL}(z) \approx \int_0^z D(z')dz'$.)

Goals revisited... revisited

1. Based on a direct relationship between difference model & difference data
2. Maximally wave-theoretic
   - two-wave wave equations
   - amplitudes & phases/traveltimes
   - multidimensional, multiparameter
   - inclusive of acoustic, elastic, anelastic, anisotropic, etc.
3. A meaningful theory provides:
   - inversion algorithms
   - insight into the character of inverse problem
4. Does not encounter, or provides a means to avoid, problematic aspects of difference data $\leftrightarrow$ difference model relationship.
5. Predict when "non time-lapse" algorithms will work, in spite of contradictory assumptions, and why!

Results emerging from scattering TL theory I: algorithms

1. Scalar, multidimensional imaging of difference model structure:

$$D_{\mathrm{DIFF}}(\mathbf{r}, \mathbf{r}_S) \approx \int d\mathbf{r}' G_0(\mathbf{r}, \mathbf{r}') \frac{\omega^2}{c_l^2(\mathbf{r}')} \alpha_{\mathrm{TL}}(\mathbf{r}') G_0(\mathbf{r}', \mathbf{r}_S)$$

$$\alpha_{\mathrm{TL}}(k_m, k_z) \approx -4 \frac{q_g q_s c_0^2}{\omega^2} D_{\mathrm{DIFF}}(k_g, k_s, \omega).$$

2. Determination of difference parameters from difference amplitudes:

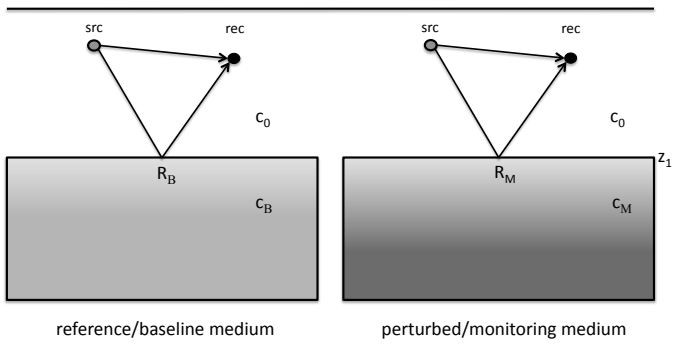
$$R_{\mathrm{DIFF}}(\omega) = \left(\frac{1}{4} \alpha_c^{TL} - \frac{1}{2} F(\omega) \alpha_Q^{TL} \right) (1 - R_B^2) + \ldots$$

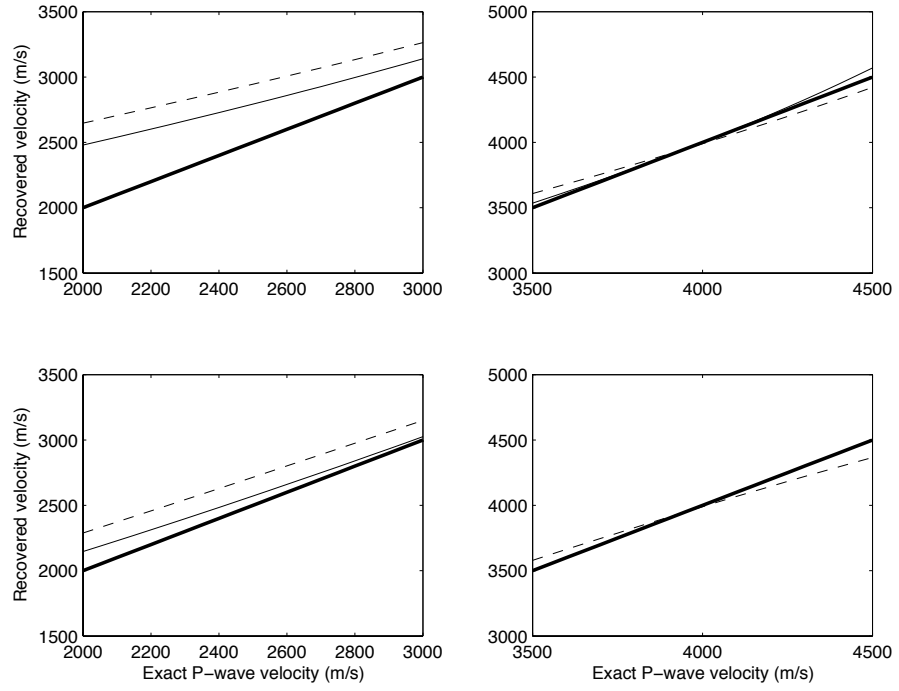
$$\alpha_{c_1}^{TL} = -4 \frac{F(\omega_2) \mathcal{R}_{\mathrm{DIFF}}(\omega_1) - F(\omega_1) \mathcal{R}_{\mathrm{DIFF}}(\omega_2)}{F(\omega_2) - F(\omega_1)},$$

$$\alpha_{Q_1}^{TL} = -2 \frac{\mathcal{R}_{\mathrm{DIFF}}(\omega_1) - \mathcal{R}_{\mathrm{DIFF}}(\omega_2)}{F(\omega_2) - F(\omega_1)}.$$

Results II: "difference reflectivity" analysis

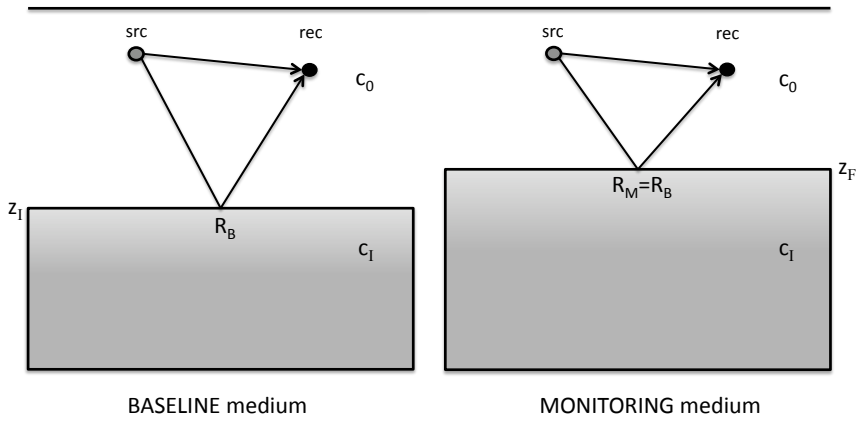
Scalar problem: determine α_{TL} directly from R_{DIFF} and R_{B}:





$$\alpha_{TL} = 4R_{\mathrm{DIFF}}\left(\frac{1}{1-R_{\mathrm{B}}^2}\right) - 8R_{\mathrm{DIFF}}^2\left[\frac{1-(1/2)R_{\mathrm{B}}-R_{\mathrm{B}}^2}{(1-R_{\mathrm{B}}^2)^3}\right] + \ldots$$

Results III: "the mystery of the working algorithm"

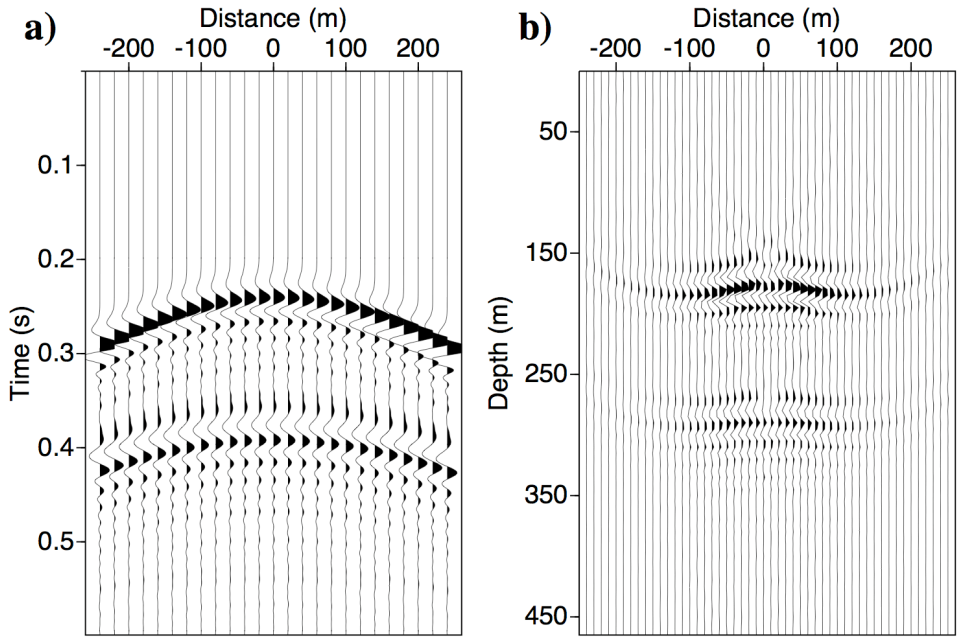


$$\text{MONITORING FIELD} = \underbrace{\frac{1}{i2k_0} + R_B \frac{e^{i2k_0 z_I}}{i2k_0}}_{\text{BASELINE FIELD}} + \underbrace{\frac{\alpha_{TL}}{4}\left[\frac{e^{i2k_0 z_F}}{i2k_0} - \frac{e^{i2k_0 z_I}}{i2k_0}\right]}_{\text{1st order correction}} + \ldots$$

- Origin of terms
- Destructive interference of "un-shared BL quantities"
- Correct (though linear) construction of negative of BL quantities
- Door open for approximate use of standard inverse scattering methods

Results IV: Least-squares

- Numerous practical issues: repeatability, image or event registration
- Linear data model $D_{DIFF} \approx \int G \alpha_{TL} G$
  - Least-squares/shot-profile framework (Kaplan et al., 2010)
- Implemented on TL data (Naghizadeh, this report/poster presentation)



Onward

- Extensions
  - Elastic PP, PS
  - Anelastic, Q_P, Q_S
  - Anisotropic, HTI
- Refine least-squares formulation, address particular TL data issues

Acknowledgments

- CREWES sponsors & personnel