Pseudo-Acoustic Wavefield Propagation in Anisotropic Media

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Overview

- Exact constant velocity solutions for the acoustic wave equation and interpolation
- Dispersion relations
- Anisotropic wavefield propagators
- A few examples

Pseudospectral

$$\Delta = \frac{\partial^2}{\partial x^2} + \ldots + \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial^2 U}{\partial t^2}(t, \vec{x}) = c^2 \Delta U(t, \vec{x})$$

- Calculated Laplacian in Fourier Domain

$$\frac{\partial^2 U}{\partial t^2}(t, \vec{x}) = -c^2 \mathrm{FT}_{\vec{k}}^{-1}\left\{ |\vec{k}|^2 \ \mathrm{FT}_{\vec{x}}\{U(t, \vec{x})\} \right\}$$

- Centered finite difference

$$U(\Delta t + t, \vec{x}) = -U(-\Delta t + t, \vec{x}) + 2U(t, \vec{x})$$
$$-(c\Delta t)^2 \mathrm{FT}_{\vec{k}}^{-1}\left\{ |\vec{k}|^2 \ \mathrm{FT}_{\vec{x}}\{U(t, \vec{x})\} \right\}$$

Exact solution of constant velocity wave equation

$$U(\Delta t + t, \vec{x}) = -U(-\Delta t + t, \vec{x})$$

$$+2\mathrm{FT}_{\vec{k}}^{-1}\left\{\cos(2\pi c \,|\, \vec{k} \,|\, \Delta t)\mathrm{FT}_{\vec{x}}\{U(t, \vec{x})\}\right\}$$

Constant velocity to variable velocity

$$U(\Delta t, \vec{x}) = -U(-\Delta t, \vec{x}) +$$

$$2\mathrm{FT}_{\vec{k}}^{-1}\left\{\cos(2\pi c(\vec{x}) \,|\, \vec{k} \,|\, \Delta t)\mathrm{FT}_{\vec{x}}\{U(0, \vec{x})\}\right\}$$

Cosine Interpolation
(Etgen 2009)

$$\cos(v(x)\,|\,k\,|\,dt) \approx$$

$$\left(\frac{v_{\max}^2 - v^2(x)}{v_{\max}^2 - v_{\min}^2}\right)\cos(v_{\min}\,|\,k\,|\,dt) + \left(\frac{v^2(x) - v_{\min}^2}{v_{\max}^2 - v_{\min}^2}\right)\cos(v_{\max}\,|\,k\,|\,dt)$$

$$\approx 1 - \frac{(v\ dt\,|\,k\,|)^2}{2!} + \frac{(v\ dt\,|\,k\,|)^4}{4!} - \frac{\left(v^2 - v_{\max}^2\right)\left(v^2 - v_{\min}^2\right)}{4!} dt^4\,|\,k\,|^4 + \ldots$$

$$\cos(v(x)\,|\,k\,|\,dt) \approx \sum_{n=1}^{N} F_n(\vec{x}) G_n(\vec{k})$$

Song, X., and Fomel, S., 2010
Fowler 2009
Yu Zhang 2008

- One-way depth step Extrapolation:

$$U(z+\Delta z,k_x,\omega)=e^{i\Delta z\sqrt{\frac{\omega^2}{v^2}-k_x^2}}U(z,k_x,\omega)$$

- Two-way phase-shift timestepping extrapolator:

$$U(t+\Delta t,k_x,k_z)$$
$$=2\left(e^{+i\Delta tv\sqrt{k_x^2+k_z^2}}+e^{-i\Delta tv\sqrt{k_x^2+k_z^2}}\right)U(t,k_x,k_z)-U(t-\Delta t,k_x,k_z)$$

- One-way phase-shift timestepping extrapolator:

$$U(t+\Delta t,k_x,k_z)=\left(e^{\pm i\Delta tv\sqrt{k_x^2+k_z^2}}\right)U(t,k_x,k_z)$$

Pseudo-acoustic

- Elastic wave equation is computationally expensive
- Elastic RTM imaging condition requires wavefield separation at each propagation step
- Alternatively, Propagate each wave mode (P,SV) separately
- TTI

Dispersion Relations

$$\det\left[c_{ijkl}n_j n_l - \rho V^2 \delta_{ik}\right] = 0$$

In TTI media

$$\omega^4 = \left[\left(v_{px}^2 + v_{sz}^2\right)\left(k_x^2 + k_y^2\right) + \left(v_{pz}^2 + v_{sz}^2\right)k_z^2\right]\omega^2$$
$$- v_{px}^2 v_{sz}^2 \left(k_x^2 + k_y^2\right)^2 - v_{pz}^2 v_{sz}^2 k_z^4$$
$$+ \left[v_{pz}^2\left(v_{pn}^2 - v_{px}^2\right) - v_{sz}^2\left(v_{pn}^2 + v_{pz}^2\right)\right]\left(k_x^2 + k_y^2\right)k_z^2$$

$$v_{pn} = v_{pz}\sqrt{1+2\delta}, v_{px} = v_{pz}\sqrt{1+2\varepsilon}$$

Fletcher, 2009

Dispersion Relations

- setting $v_{sz} = 0$

$$\omega^4 = \left[\left(v_{px}^2\right)\left(k_x^2 + k_y^2\right) + \left(v_{pz}^2\right)k_x^2\right]\omega^2$$
$$+ \left[v_{pz}^2\left(v_{pn}^2 - v_{px}^2\right)\right]\left(k_x^2 + k_y^2\right)k_z^2$$

- Can create a couple system of PDEs to solve the pseudo-acoustic equation.
- There are still two solutions of the dispersion relation so still shear wave propagation.

Solving the square root

$$\omega_{v_p}^2 = v_{P_z}^2 \left(k^2 \left(1 - \frac{f}{2}\right) + \right.$$

$$\left. \varepsilon k_x^2 + \frac{f}{2} \sqrt{k^4 + \frac{4k_x^2}{f}\left(2\delta k_z^2 - \varepsilon\left(k_z^2 - k_x^2\right)\right) + \frac{4\varepsilon^2 k_x^4}{f^2}} \right)$$

$$f = 1 - \frac{v_{S_0}^2}{v_{P_0}^2}$$

Tsvankin, 2001

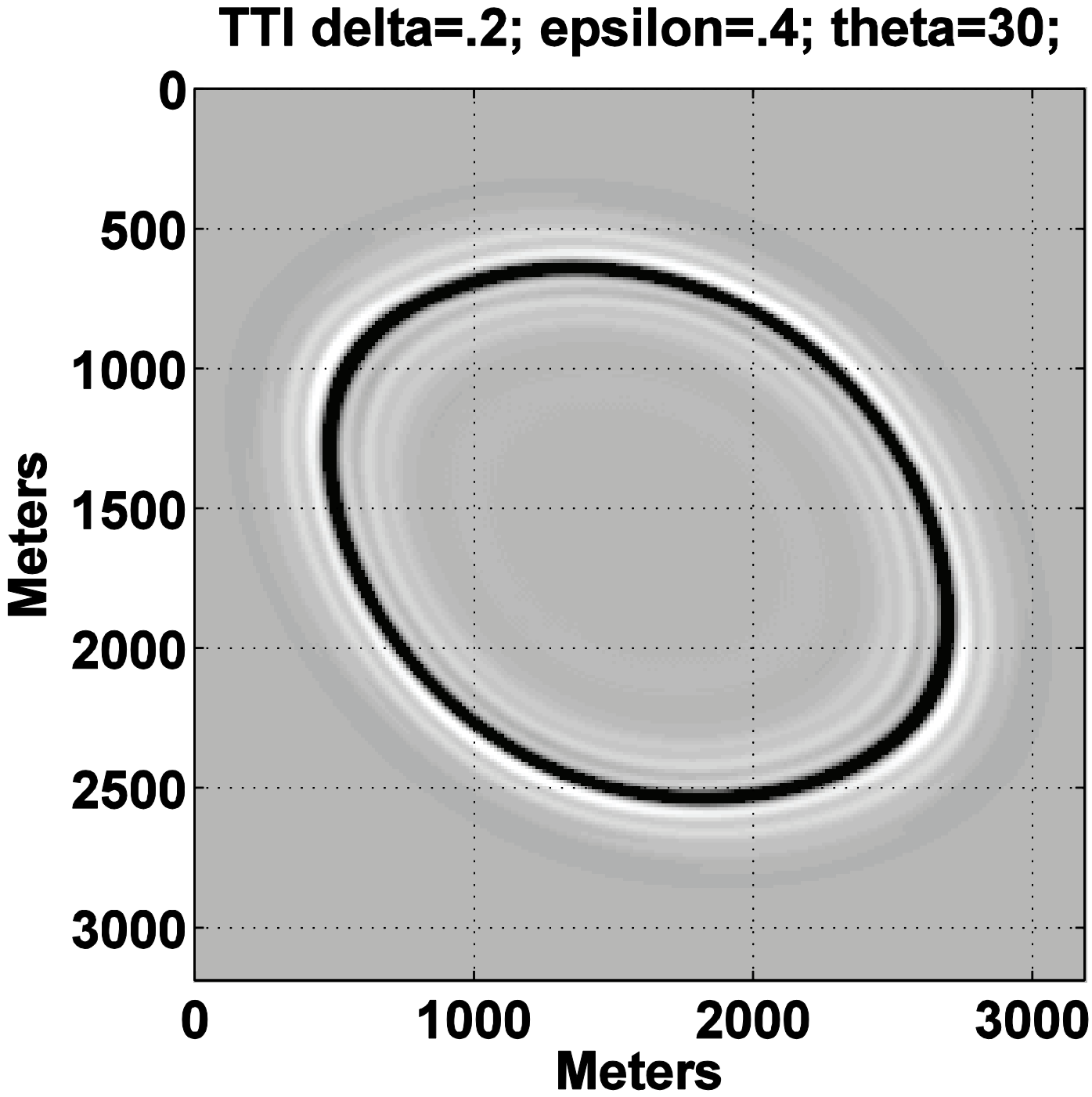
Weak anisotropy

$$\omega_{v_p}^2 \cong v_x^2 k_x^2 + v_n^2 k_z^2 + \left(v_n^2 - v_x^2\right) \frac{k_z^2 k_z^2}{k_x^2 + k_z^2}$$

Pseudo-acoustic wave propagation for any dispersion relationship

$$\frac{\partial^2 U}{\partial t^2} = -FT_{\vec{k}}^{-1}\left\{\omega^2(\vec{k},\vec{x})FT\left\{U(t,\vec{x})\right\}\right\}$$

$$U(\Delta t,\vec{x}) = U(-\Delta t,\vec{x}) +$$
$$2FT_{\vec{k}}^{-1}\left\{\cos\left(2\pi\omega(\vec{k},\vec{x})\Delta t\right)FT_{\vec{x}}\left\{U(0,\vec{x})\right\}\right\}$$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3000
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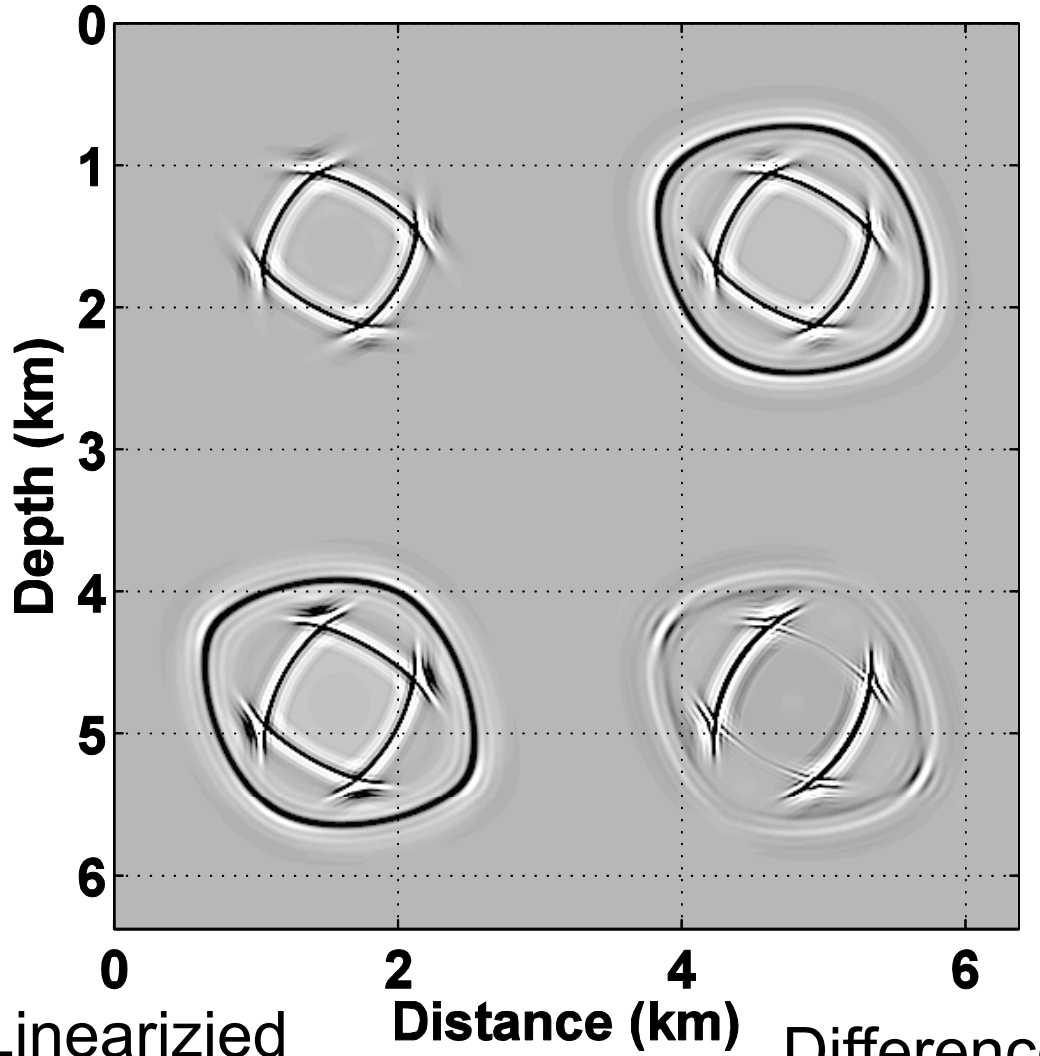
Meters

S-wave Exact
dispersion relation

P&S-wave Exact
dispersion relation

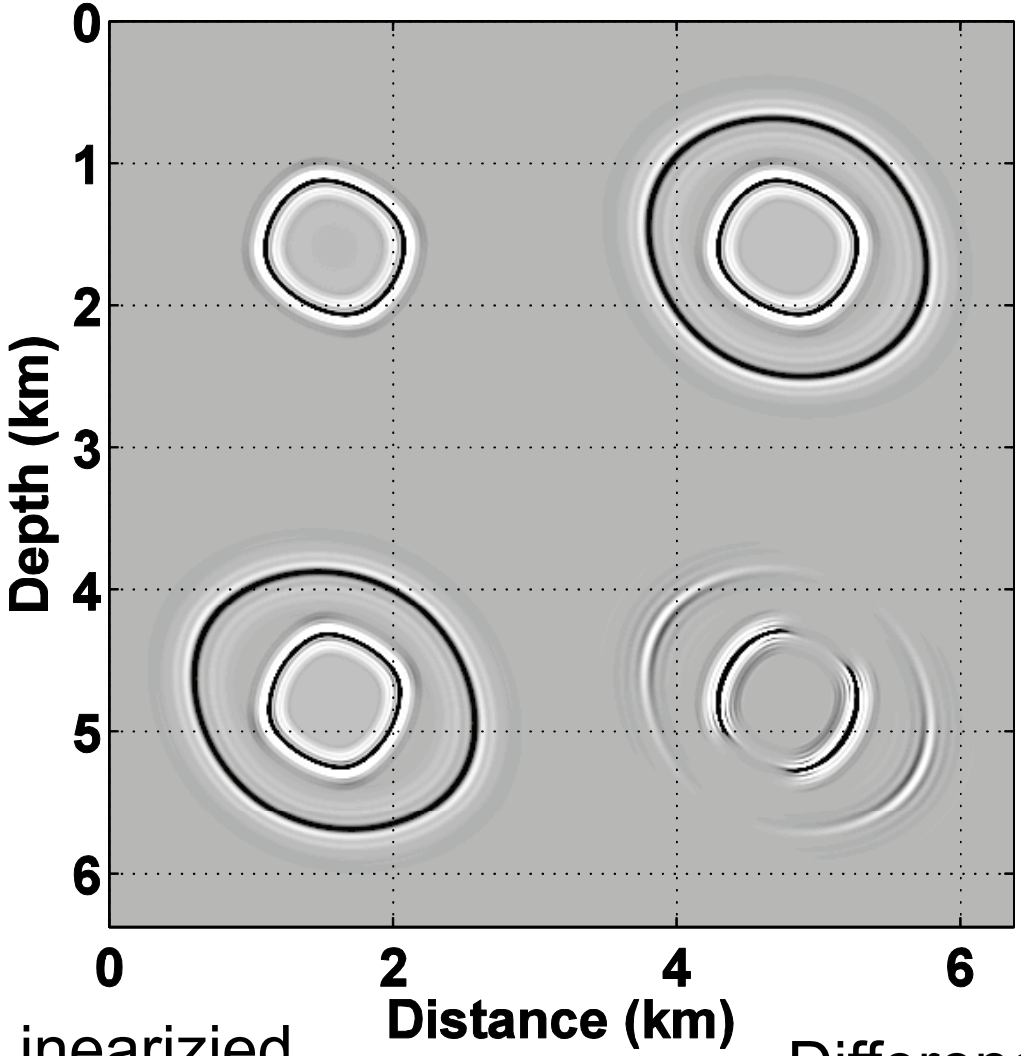


P&S-wave Linearizied
dispersion relation

Difference between
Linearizied & exact

S-wave Exact
dispersion relation

P&S-wave Exact
dispersion relation



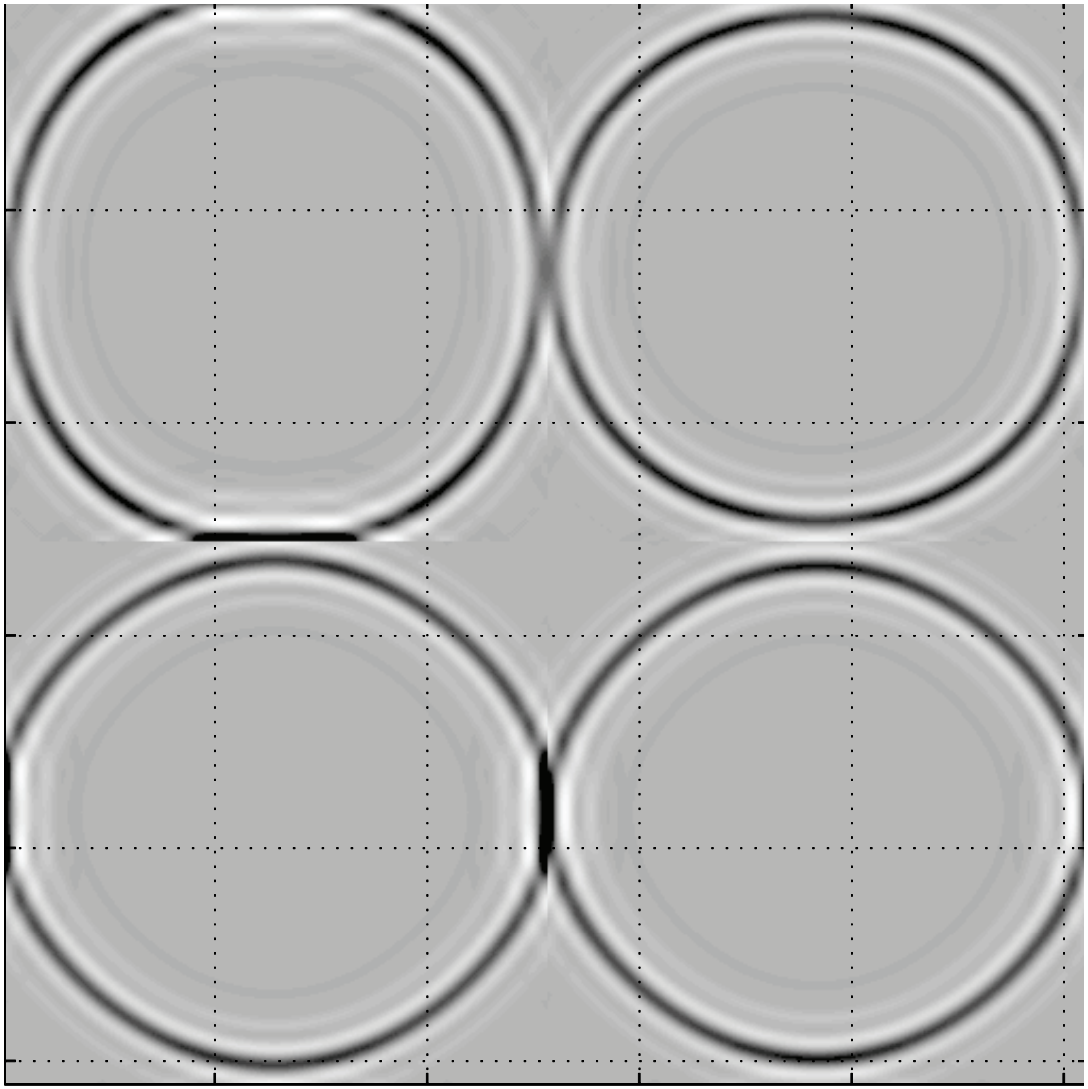
P&S-wave Linearizied
dispersion relation

Difference between
Linearizied & exact

P Wave 3D Orthorhombic

XZ plane

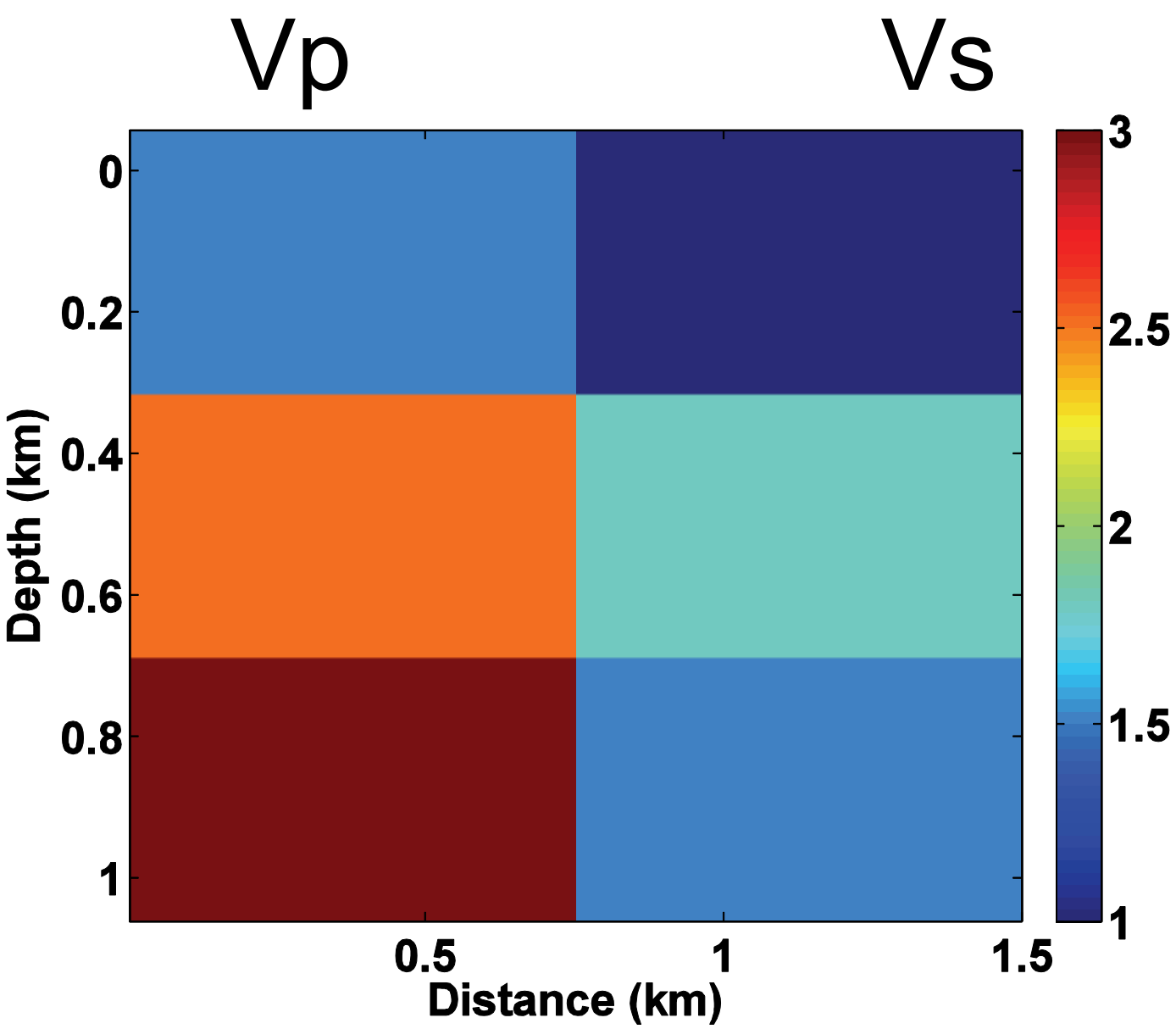
XZ plane



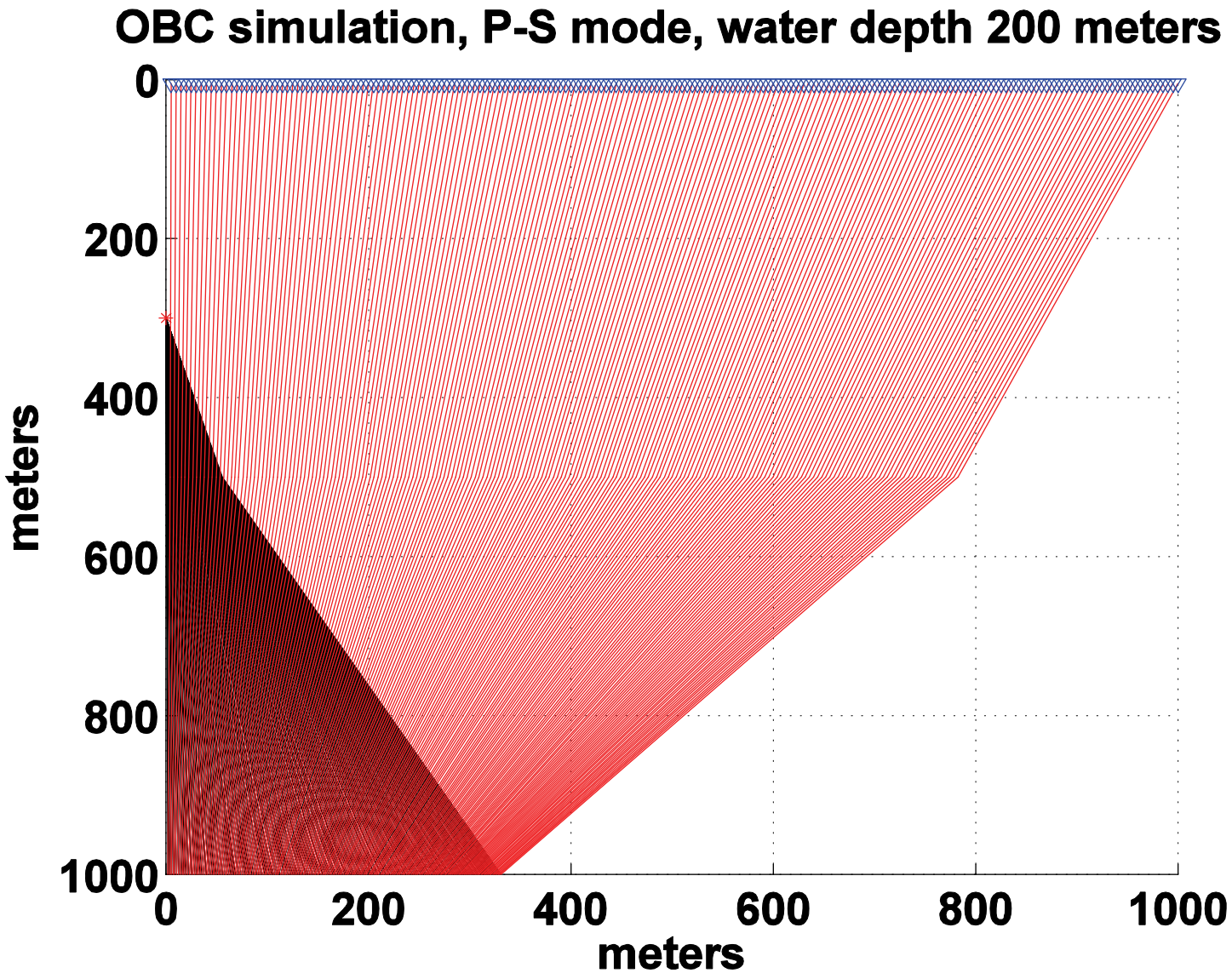
XY plane

XY plane-
280m

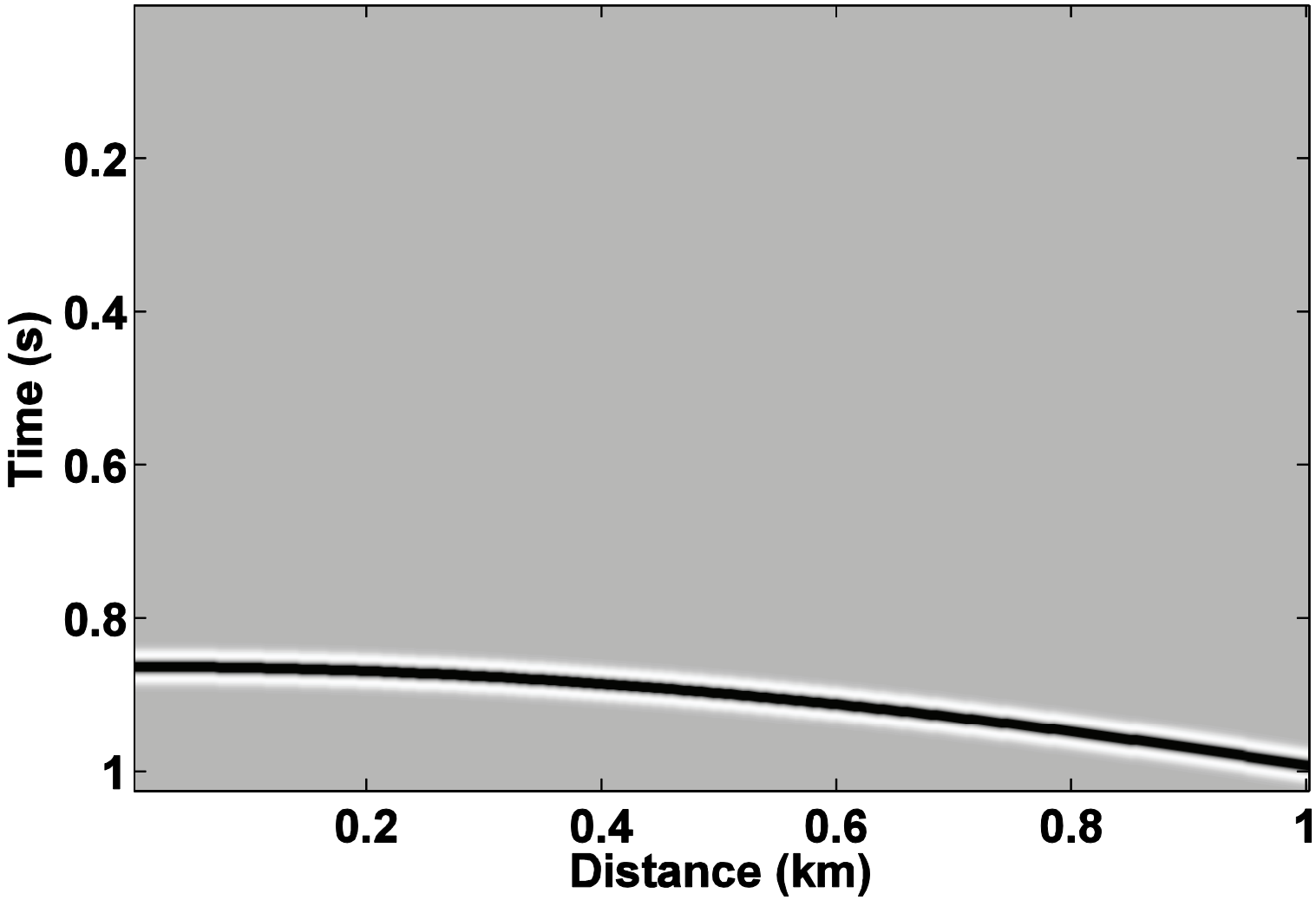
Simple Model



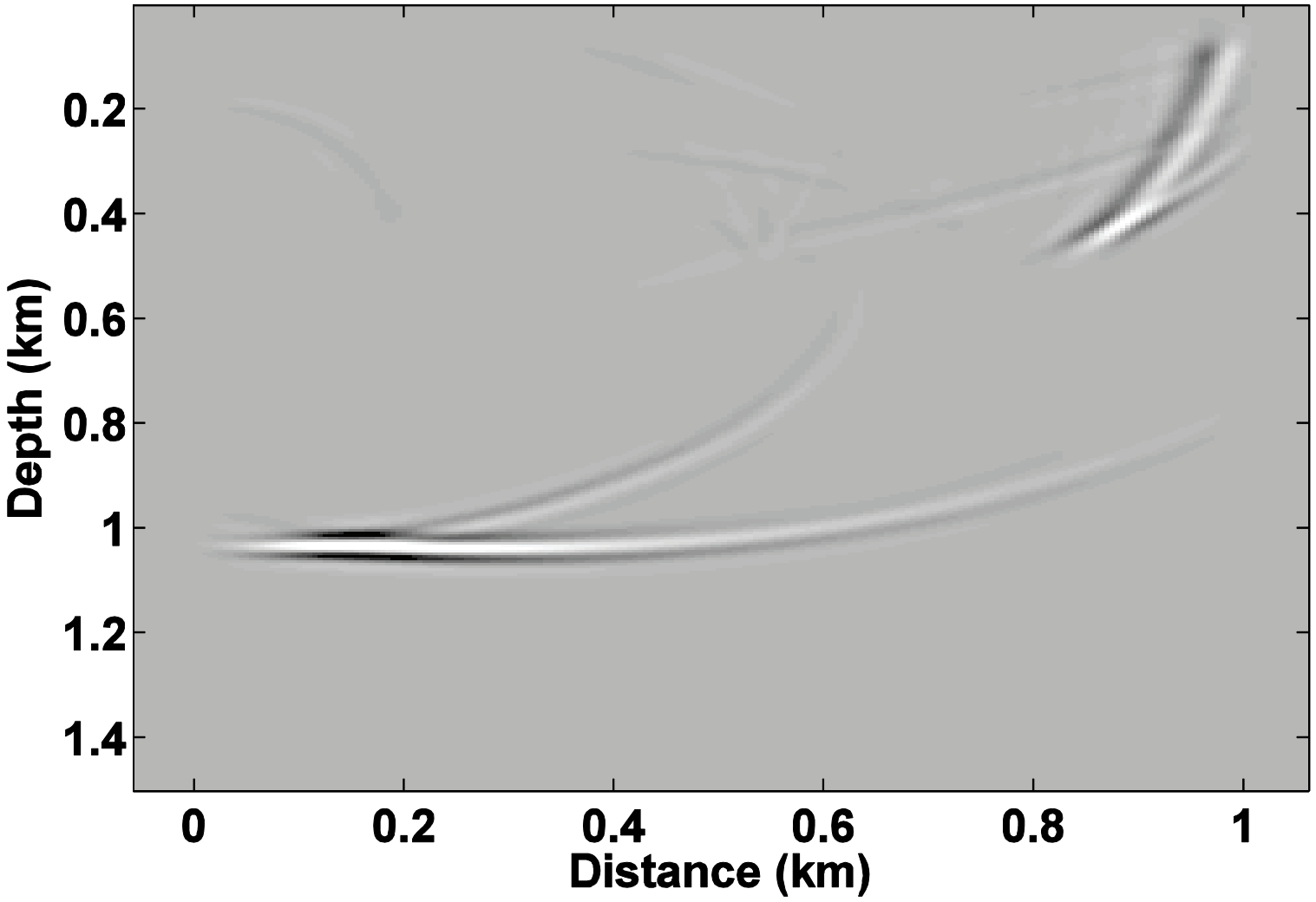
Geometry


OBC simulation, P-S mode, water depth 200 meters
0
200
400
600
800
1000
meters
0
200
400
600
800
1000
meters

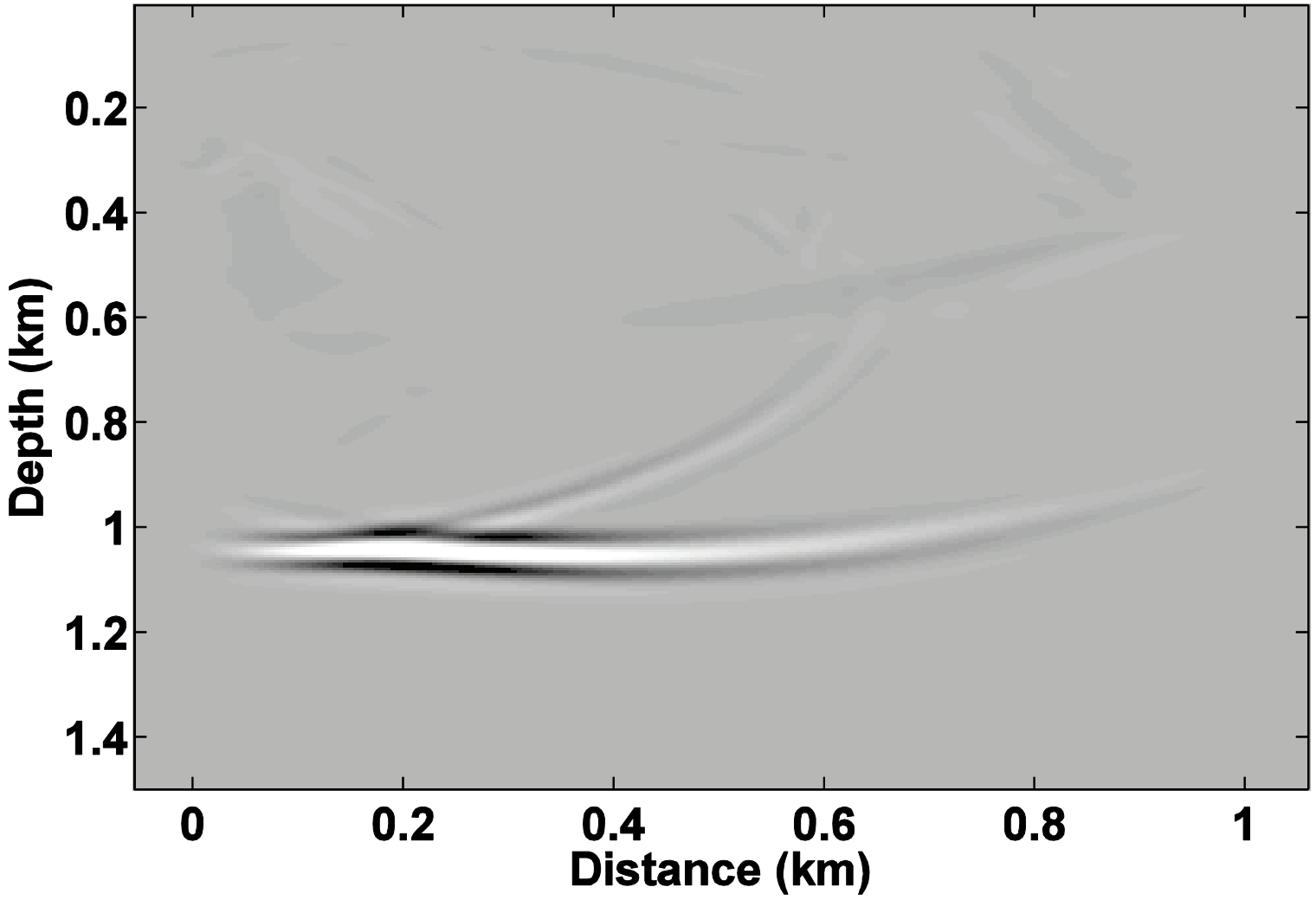
OBC PS Node gather
Constant amplitude



Migrated single shot PS



Migrated single shot PP



Cost

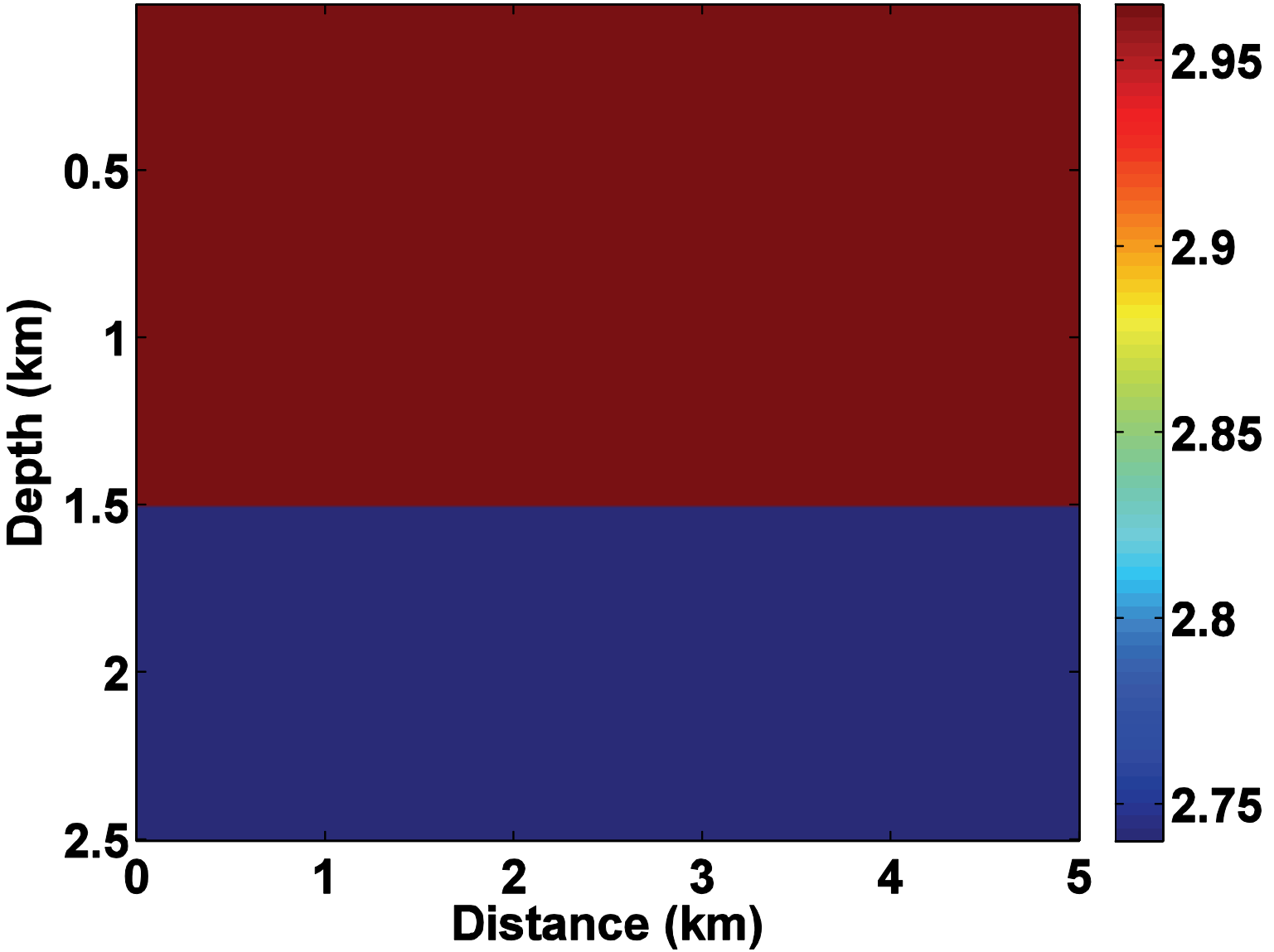
	PP	PS
Max vel	5500m/s	2500m/s
Min vel	1500m/s	500m/s
fmax	55Hz	55Hz
dx	12.5m	4m
dt		
Computation cost	1	3^4 (3D)

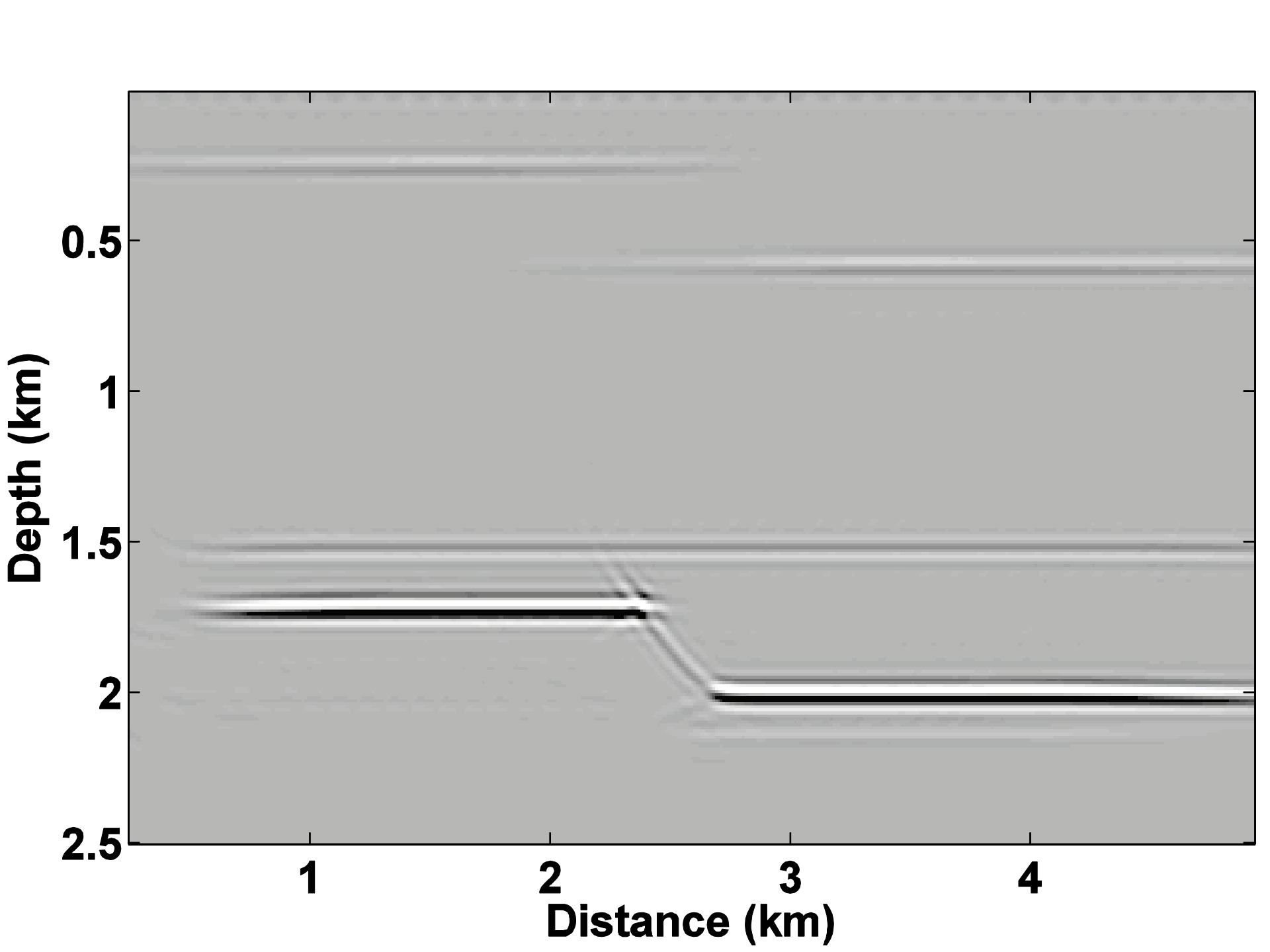
$$\lambda_{\min} = v_{\min} / f_{\max}$$

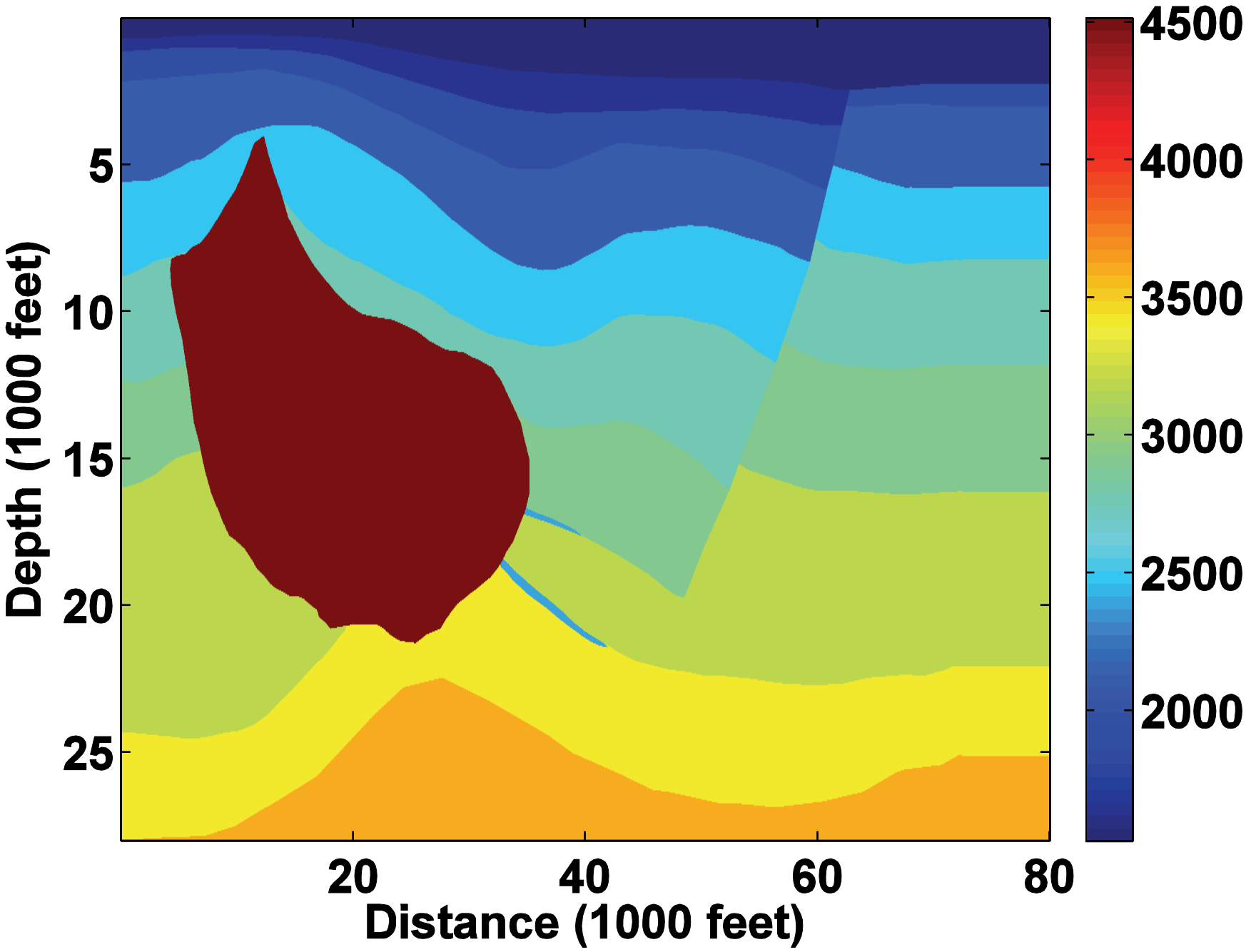
+ 2 samples/ wavelength

$$\frac{c\Delta t}{\Delta x} < \frac{1}{\sqrt{2}}$$

FRP TTI delta=0, epsilon=0.196,theta=45




0.5
1
1.5
2
2.5
Depth (km)
1
2
3
4
Distance (km)


Depth (1000 feet)
5
10
15
20
25
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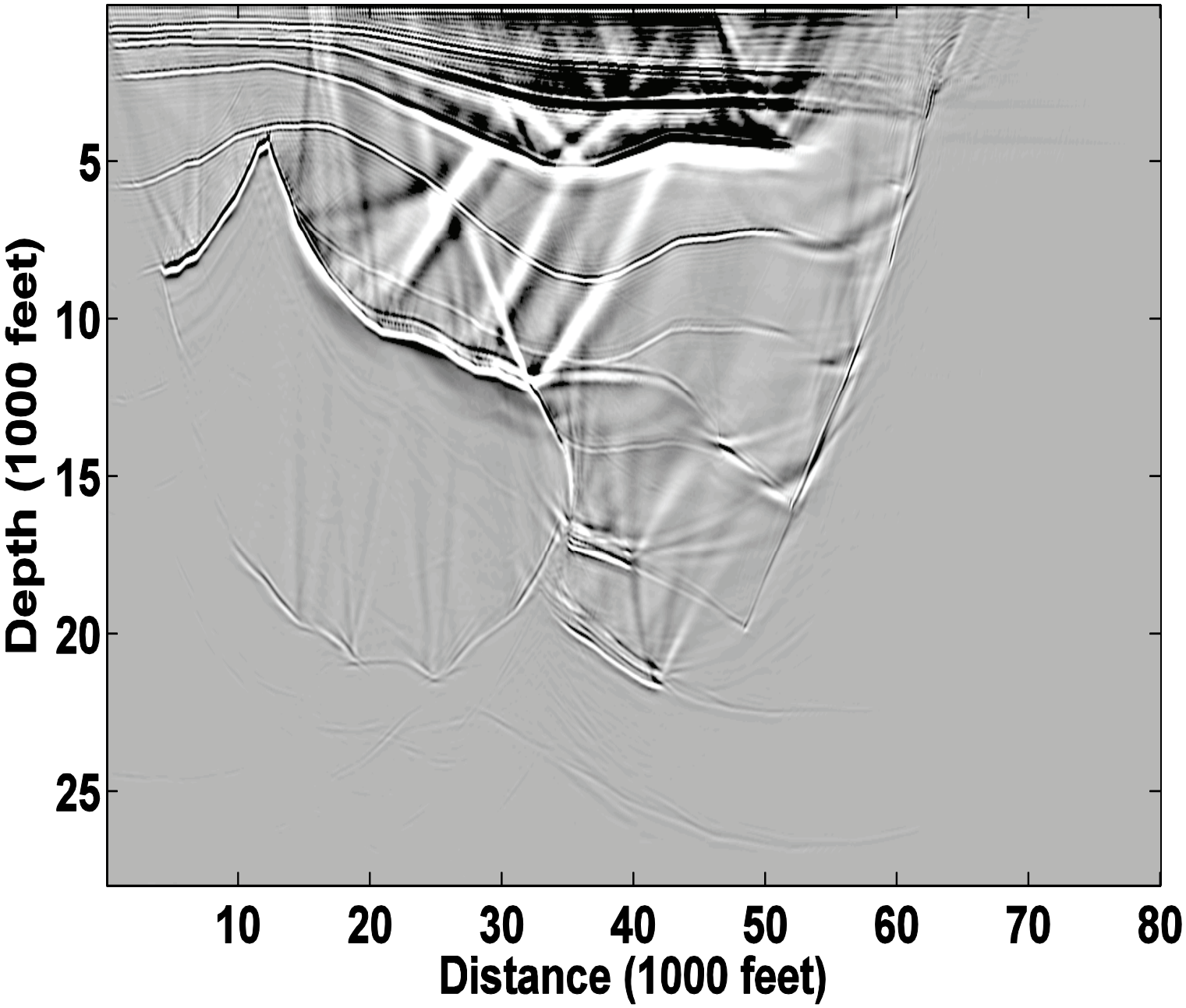
40
60
80
Distance (1000 feet)
4500
4000
3500
3000
2500
2000


Depth (1000 feet)
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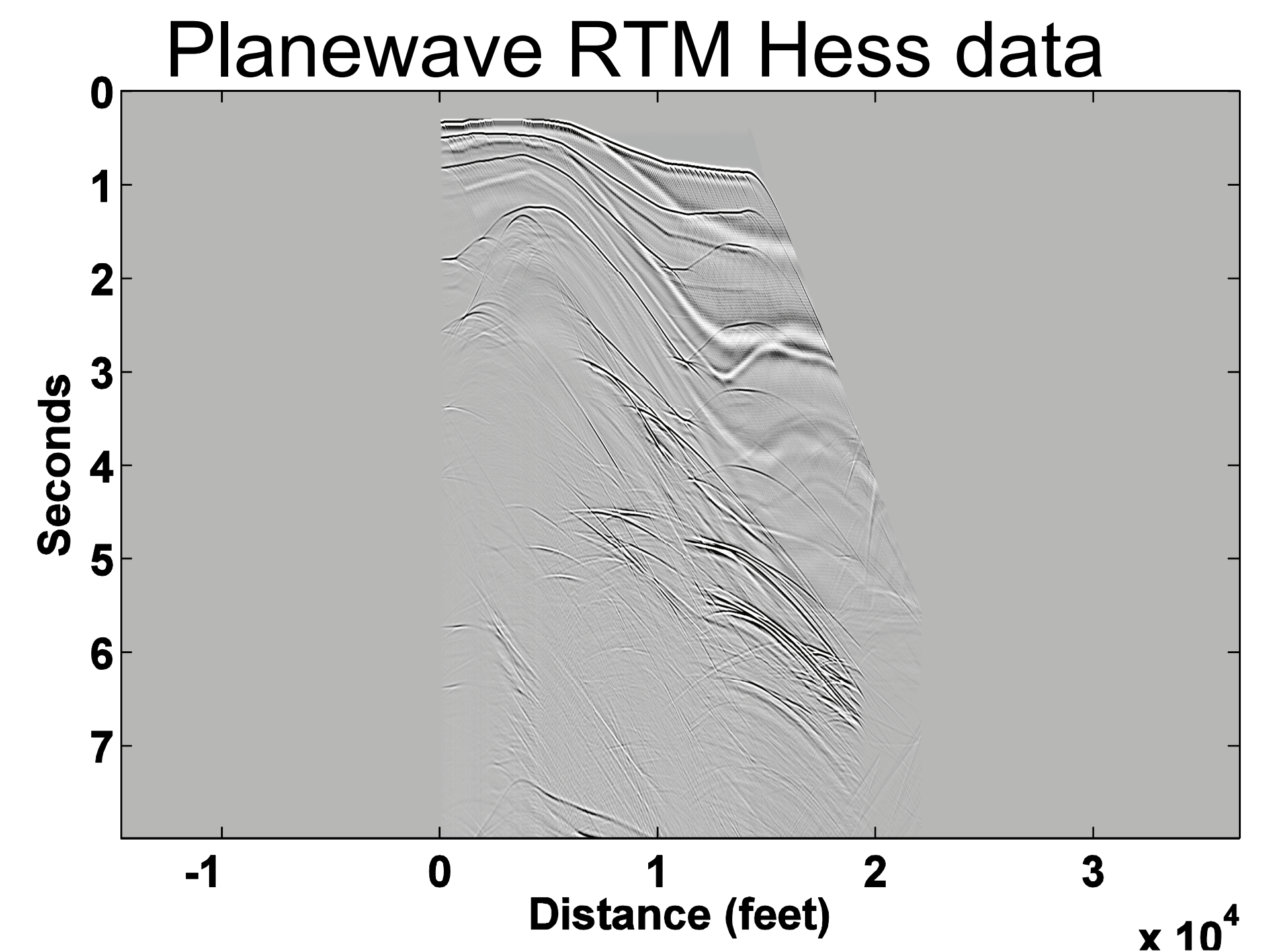
Distance (1000 feet)


Planewave RTM Hess data
Seconds
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1
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5
6
7
-1
0
1
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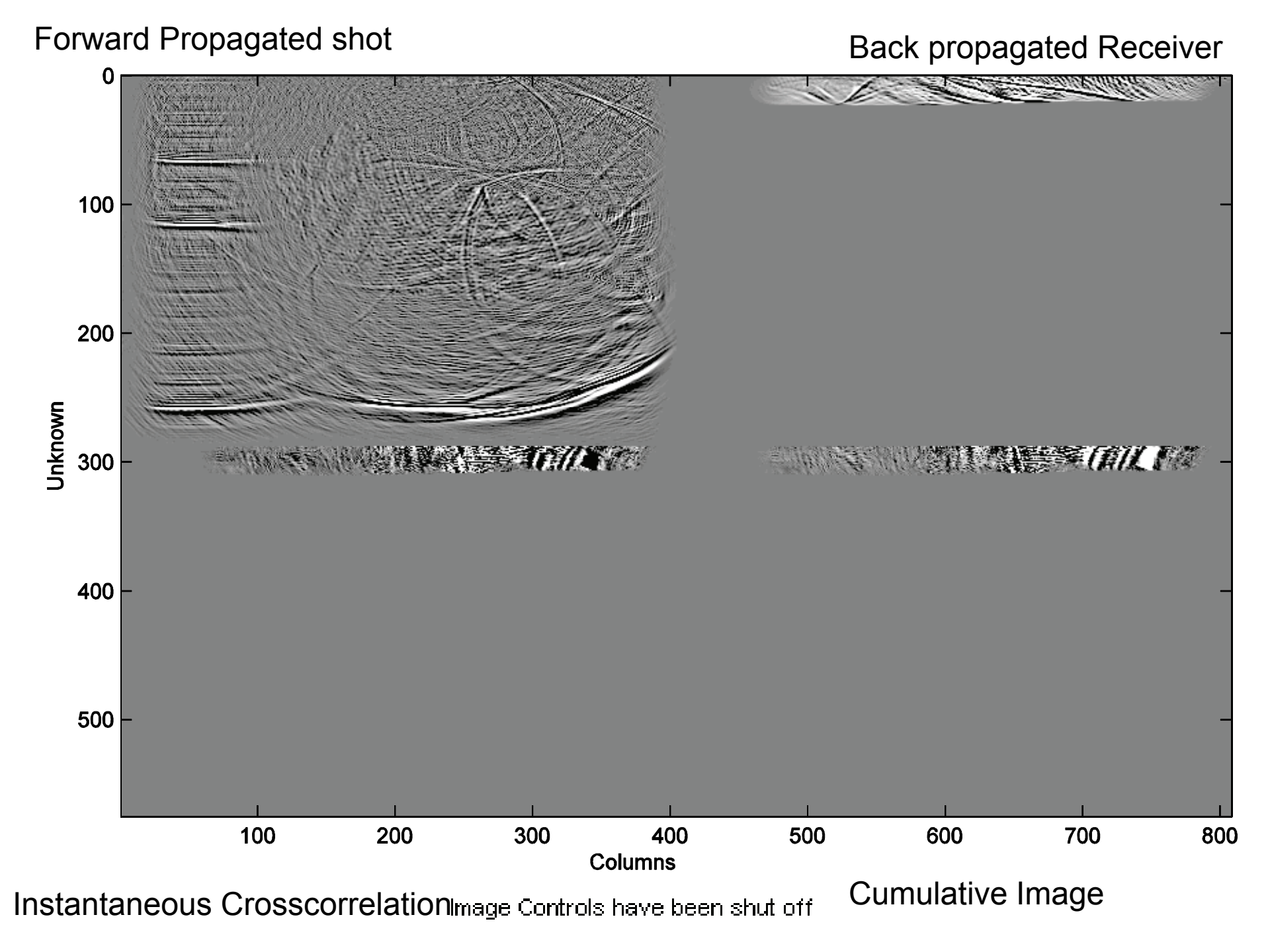
3
Distance (feet)
x 10^4


Forward Propagated shot
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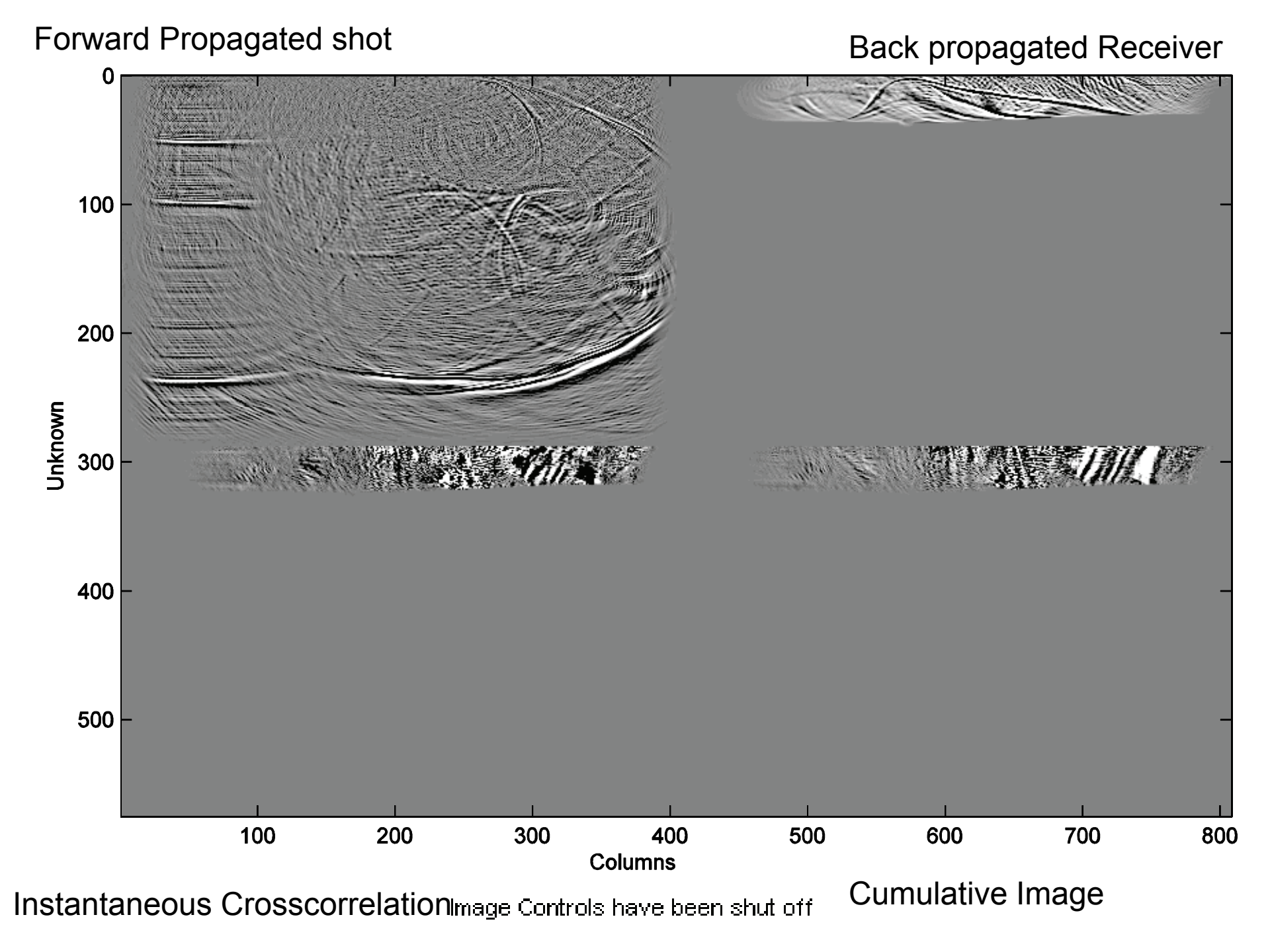
Instantaneous Crosscorrelation
Image Controls have been shut off
Cumulative Image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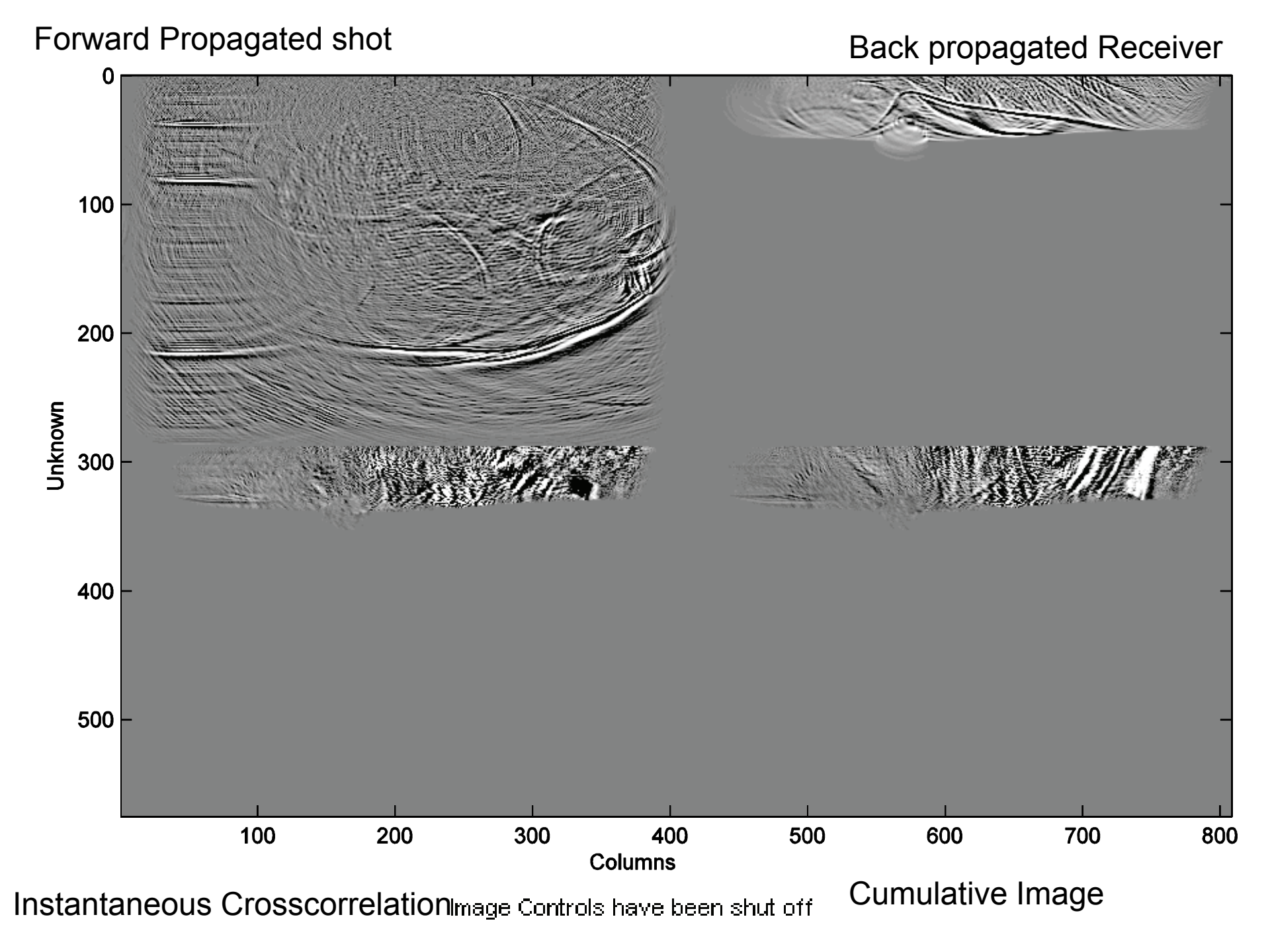
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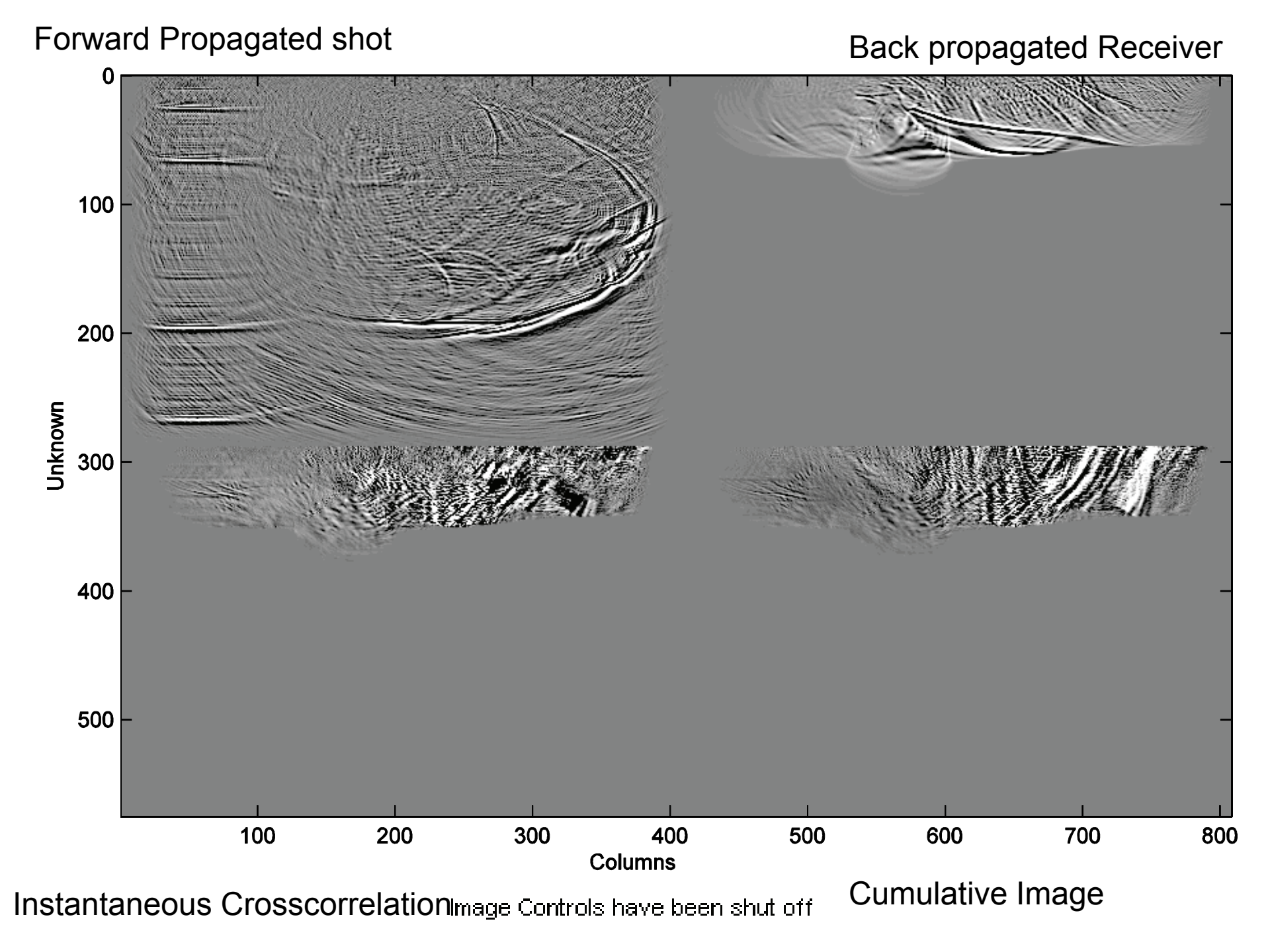
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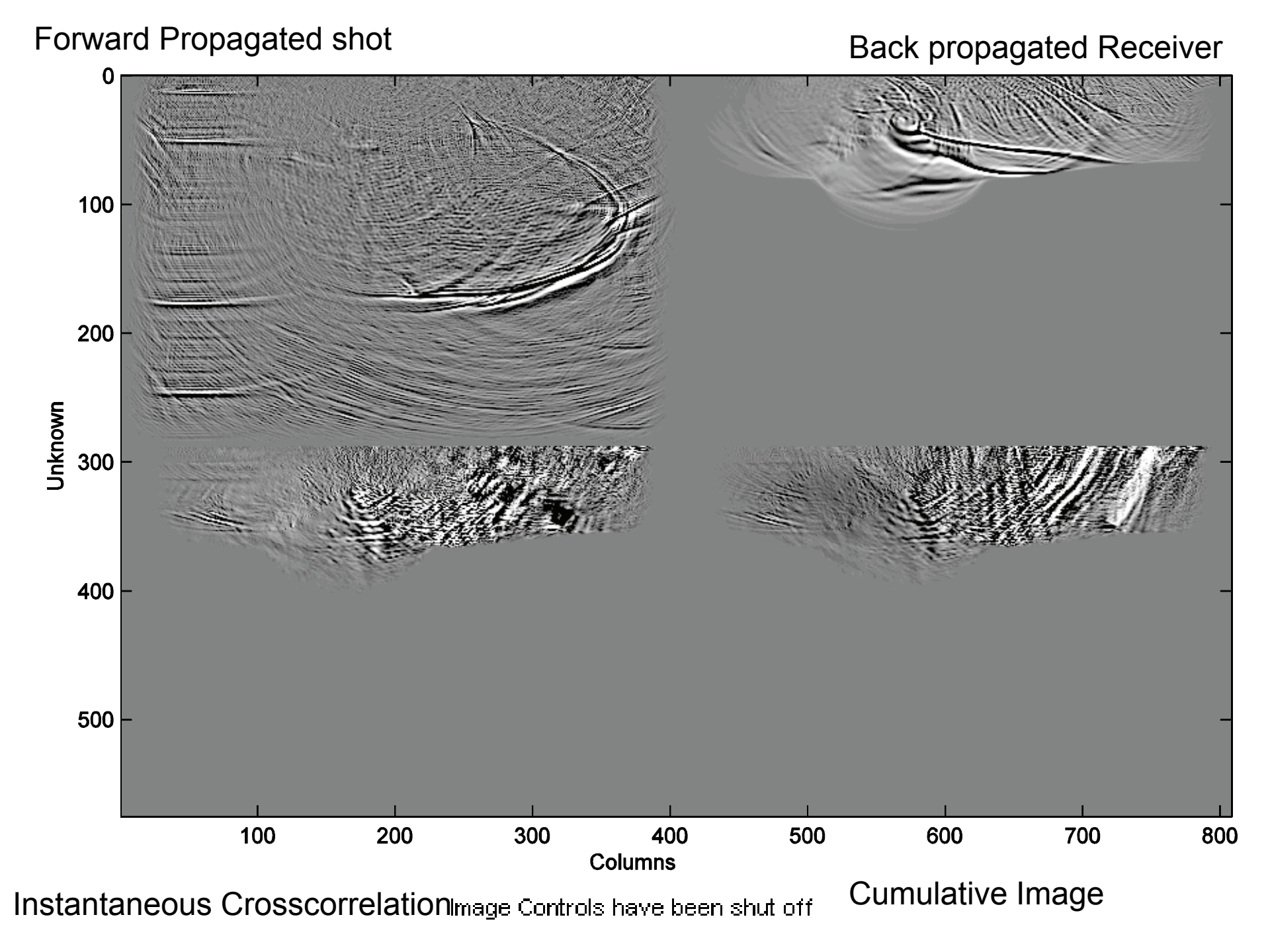
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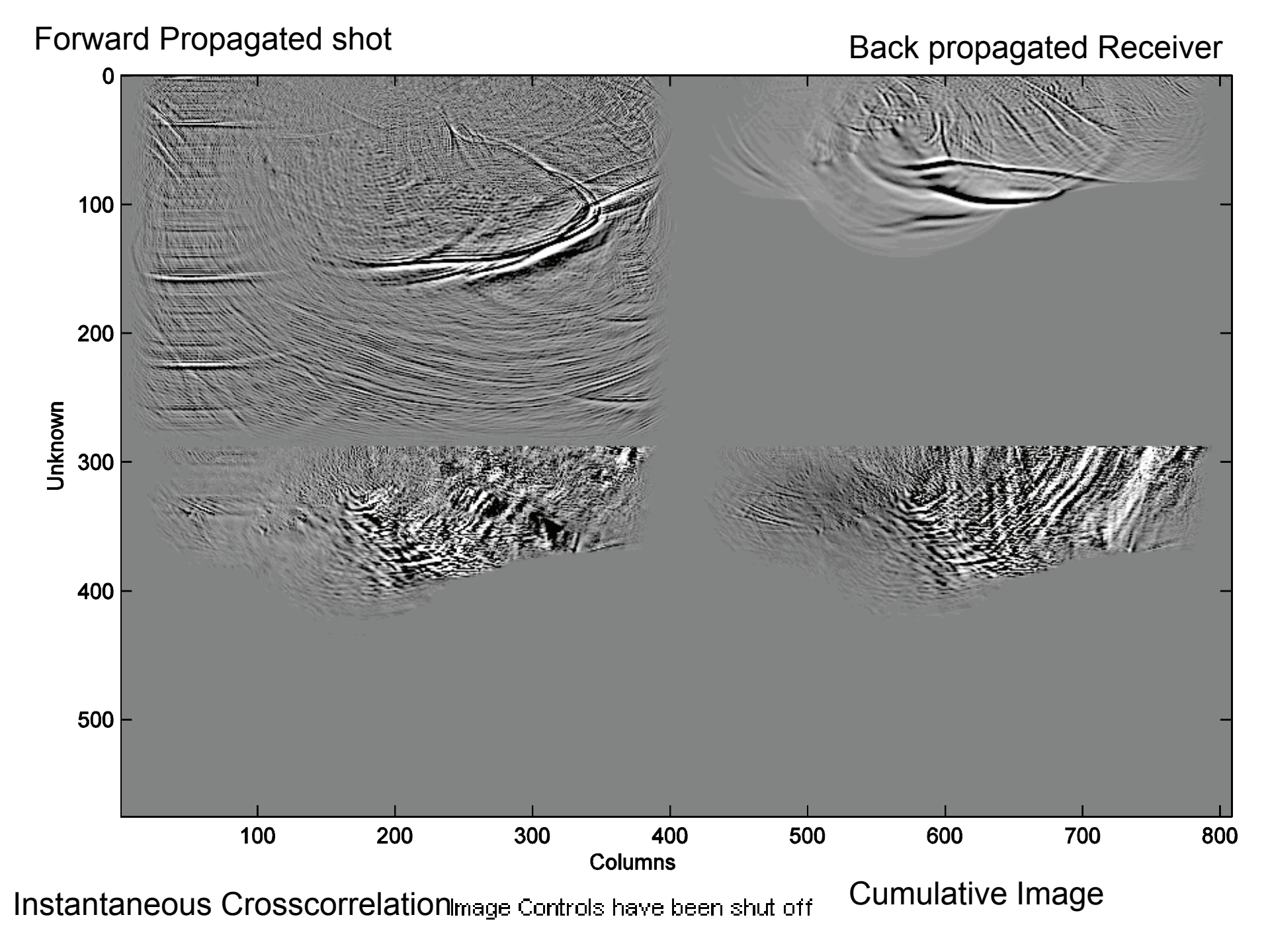
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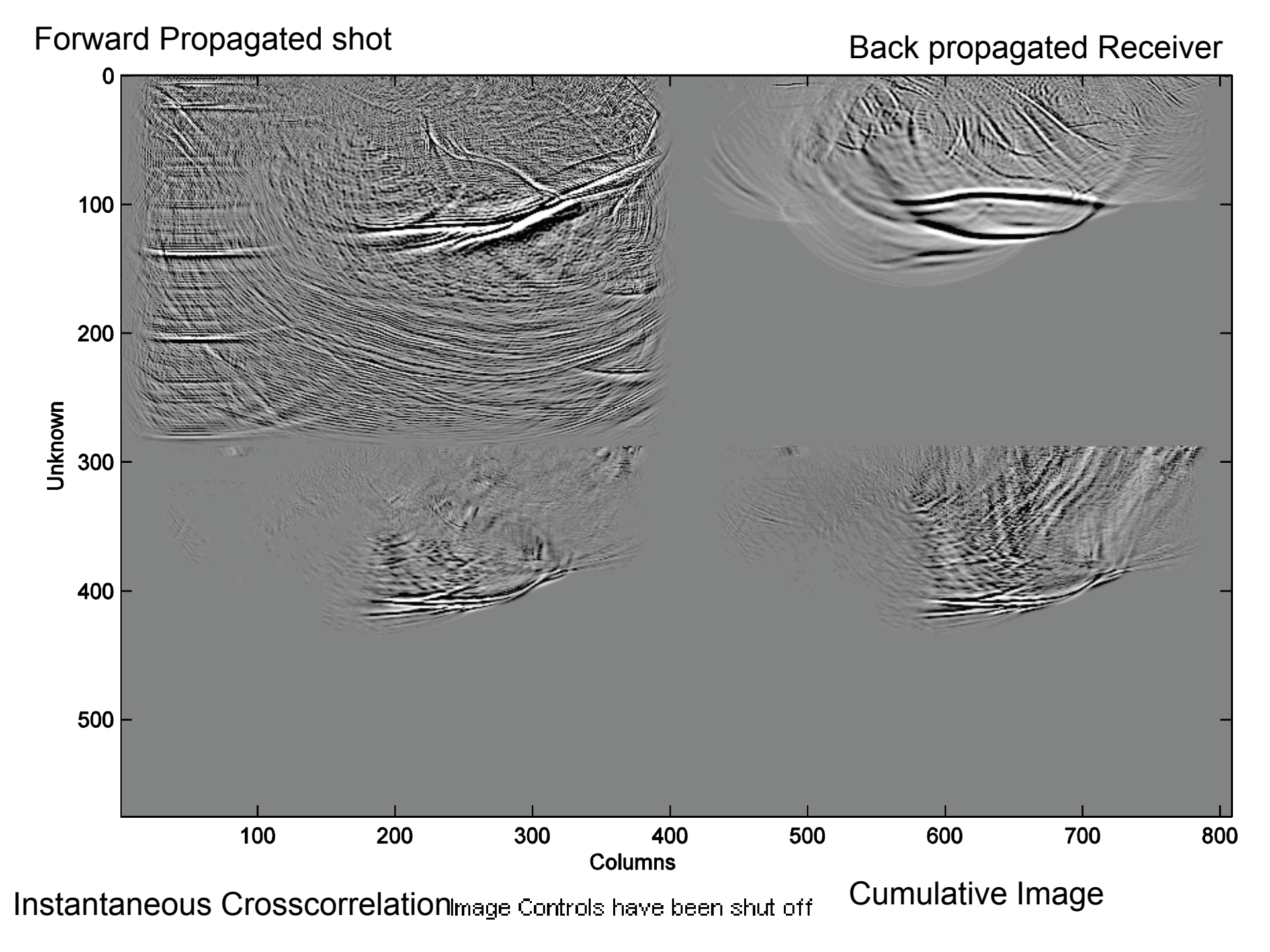
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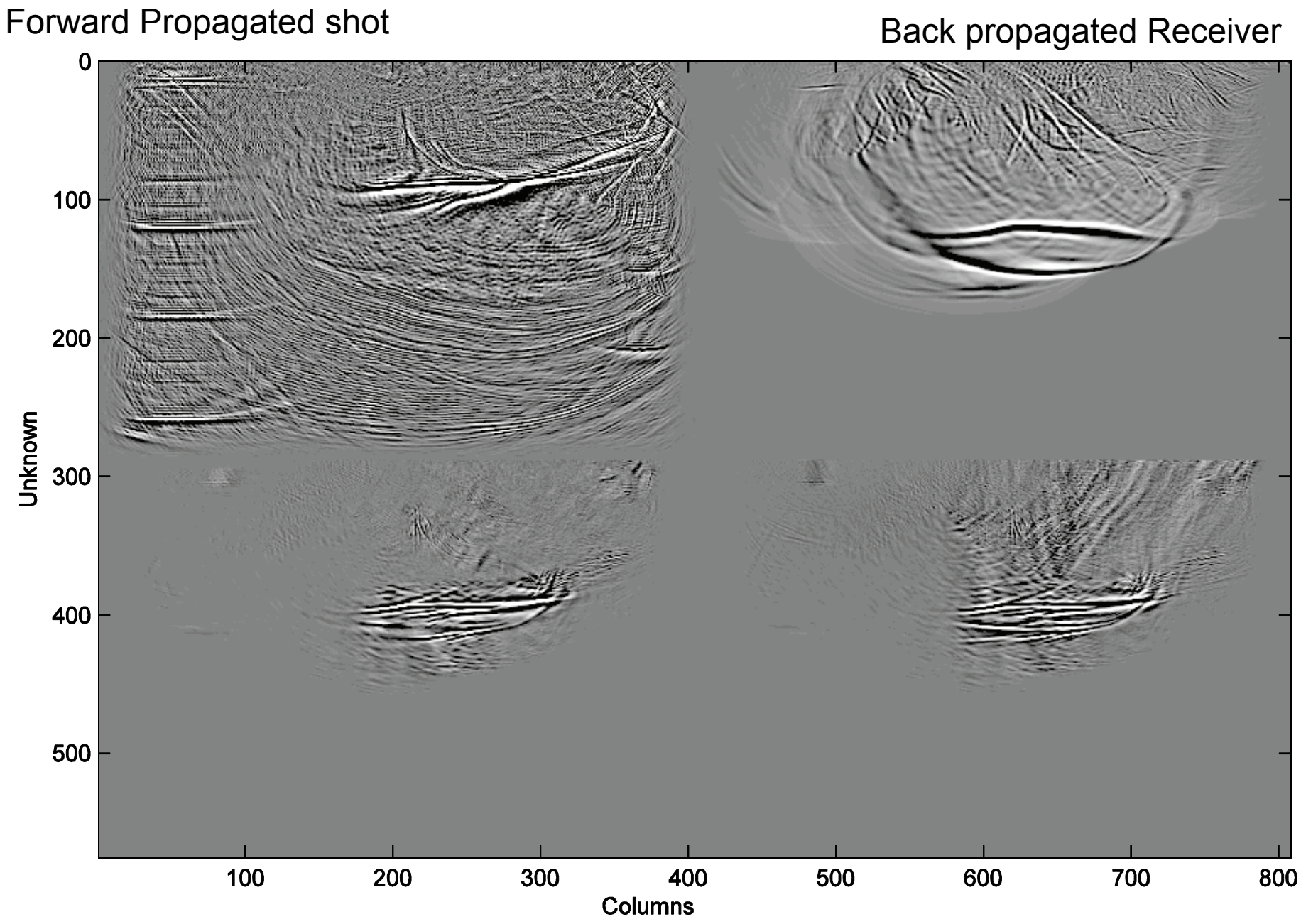
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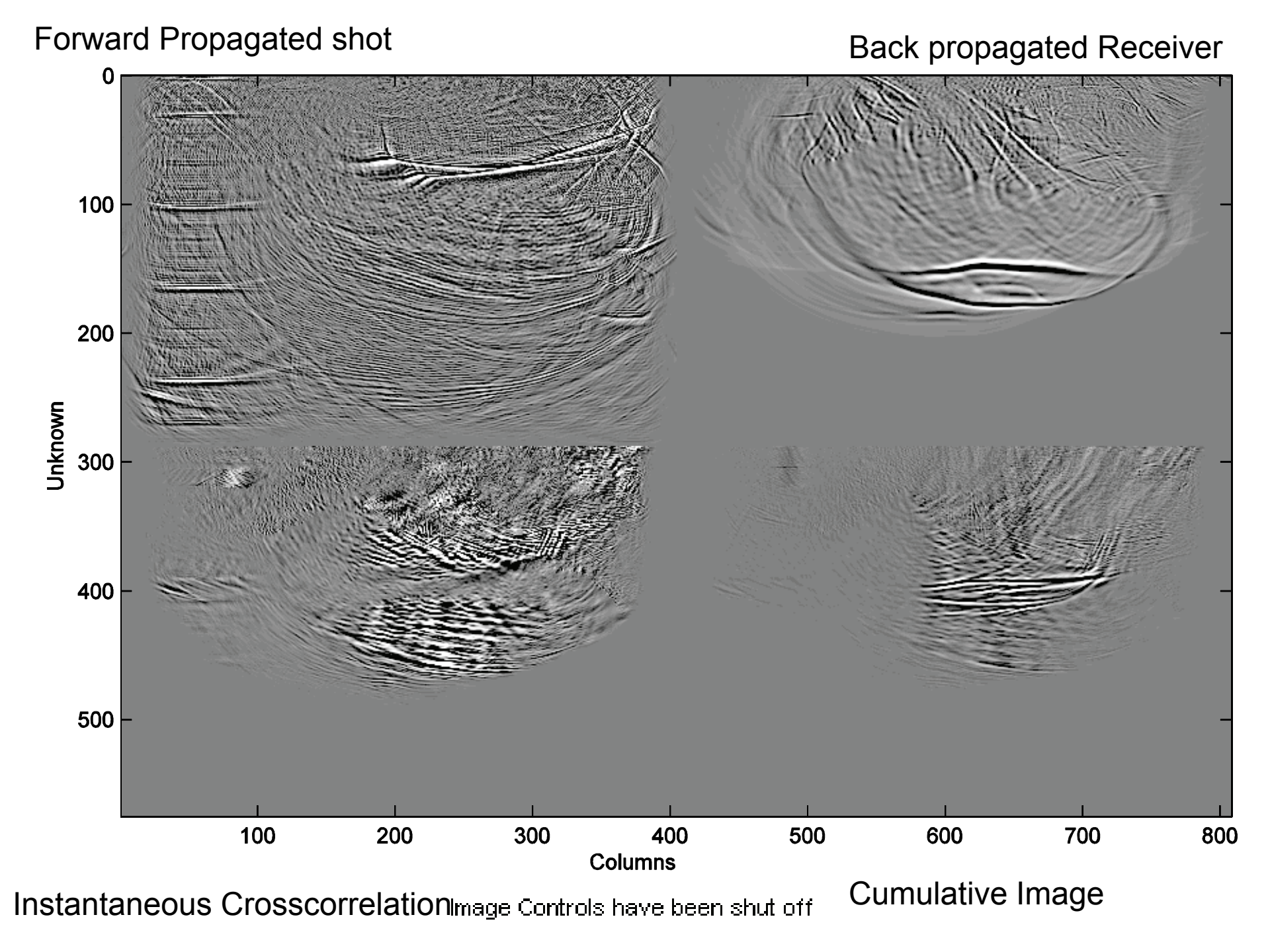
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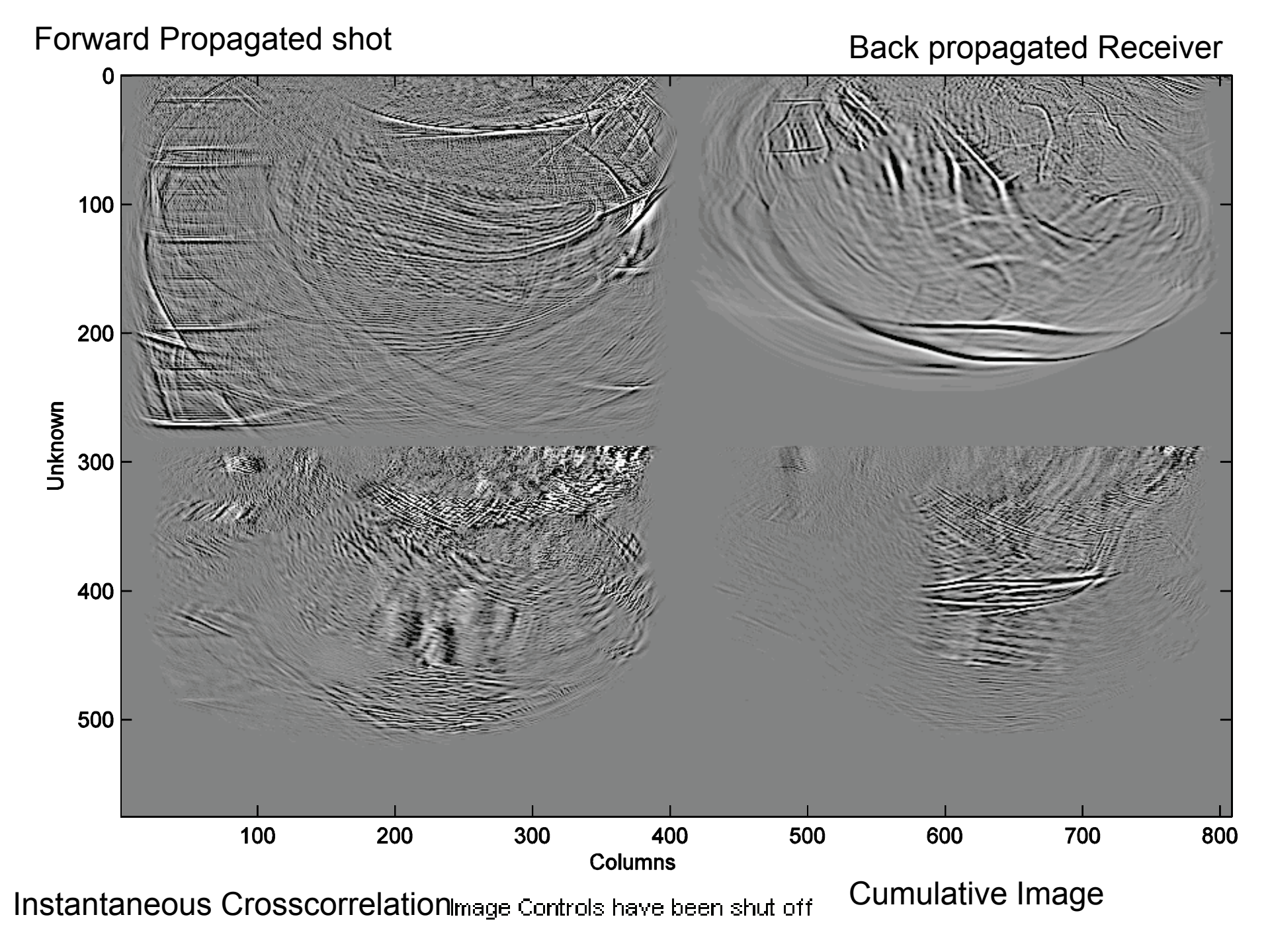
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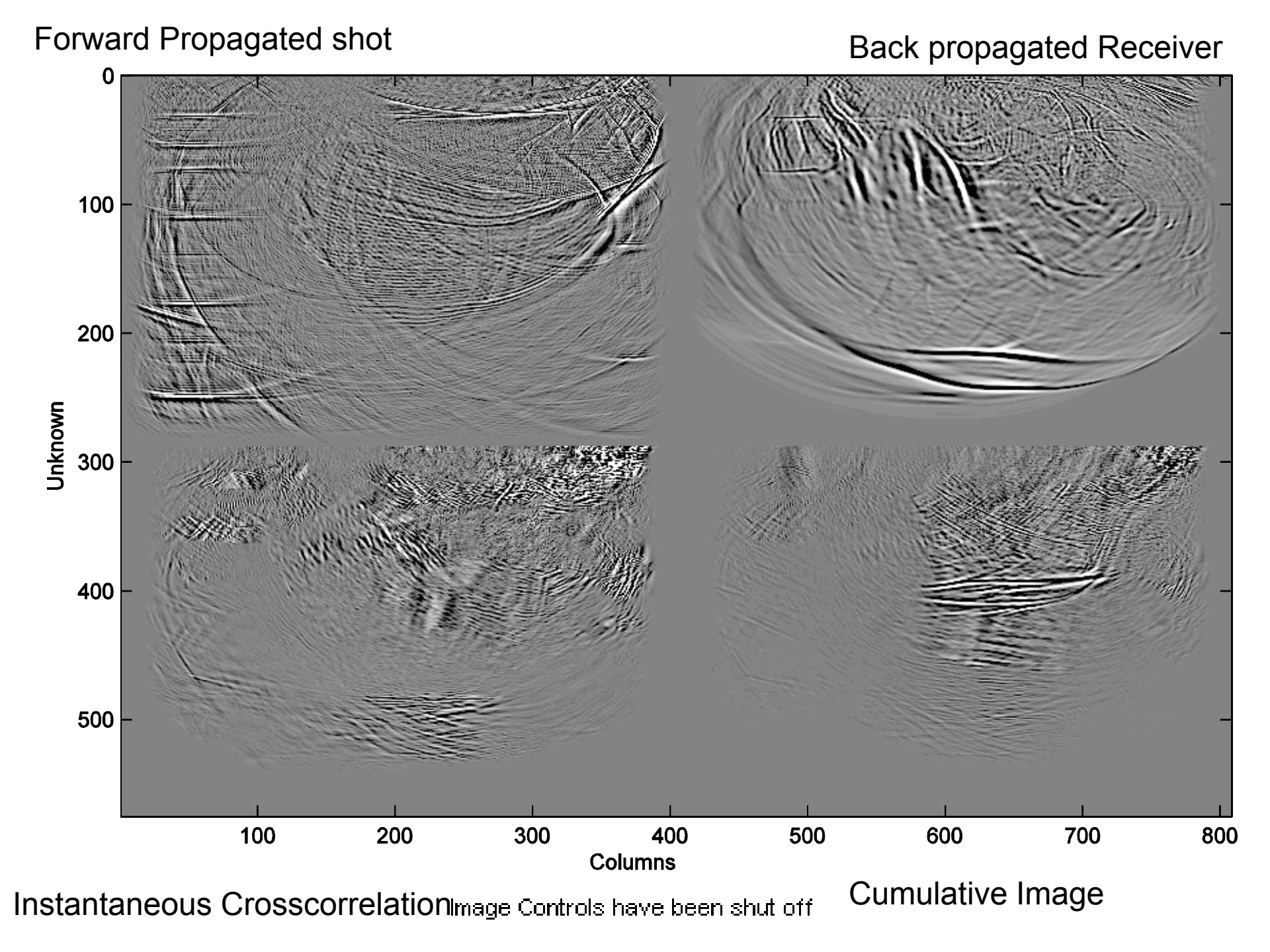
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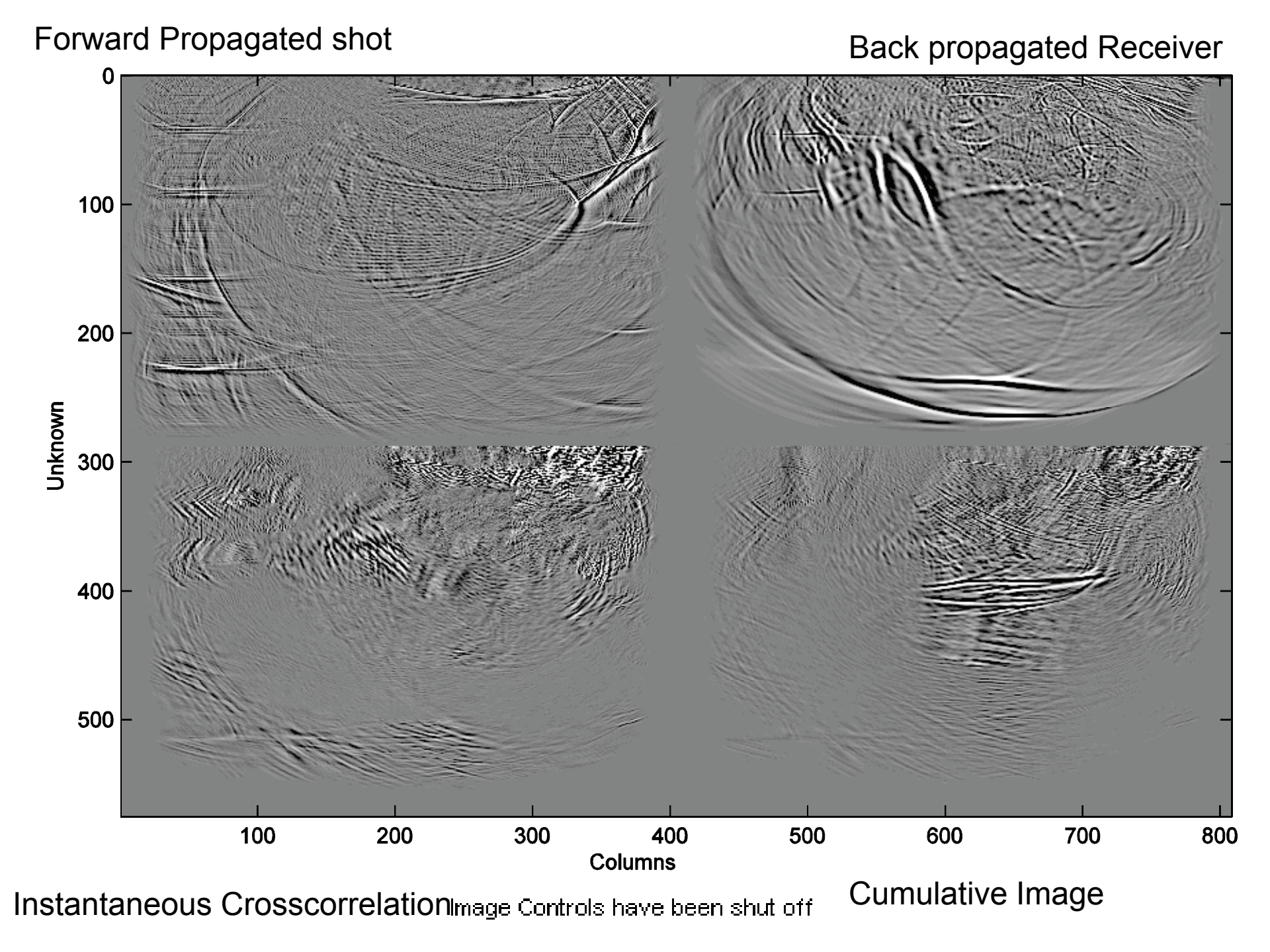
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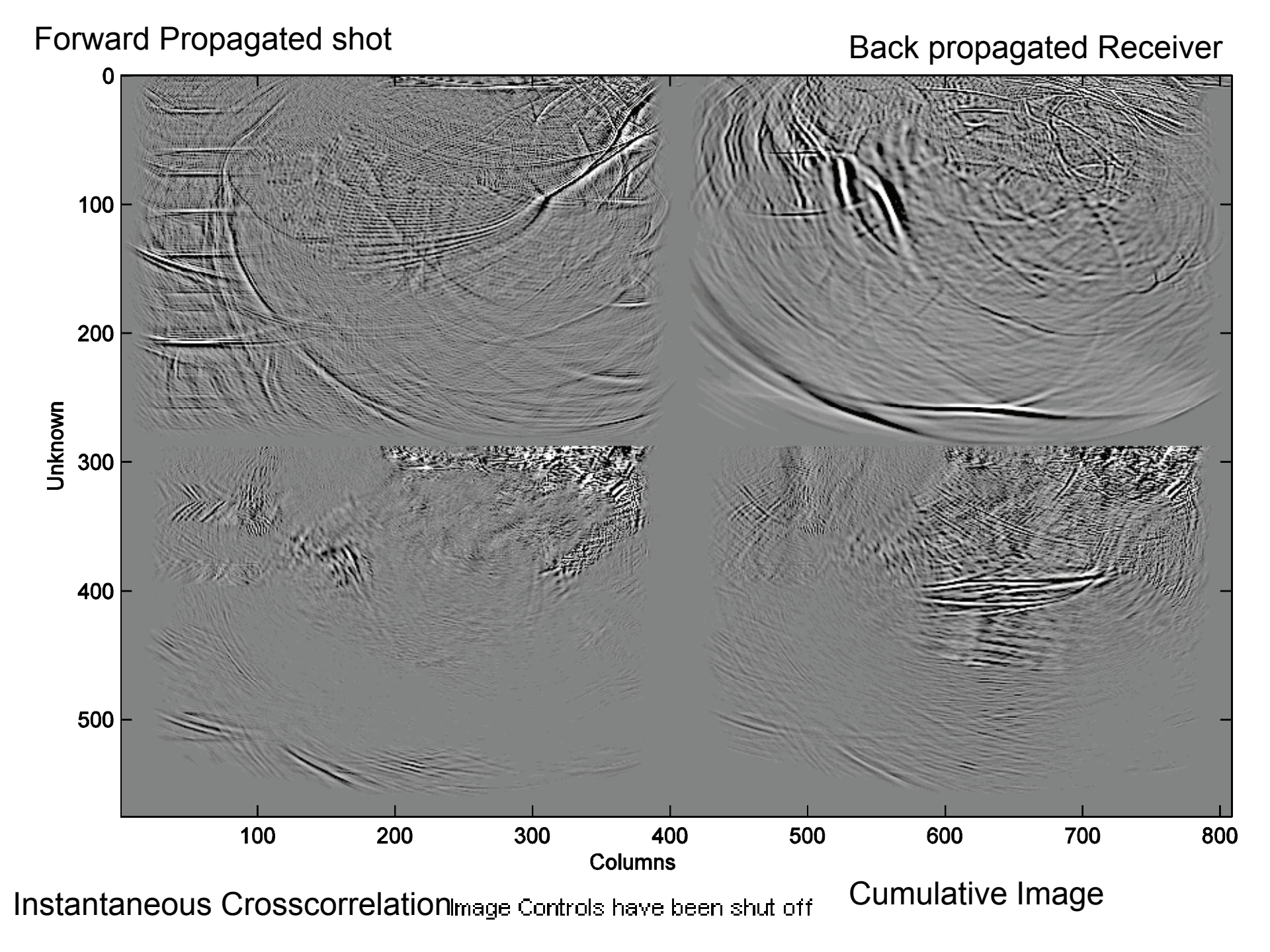
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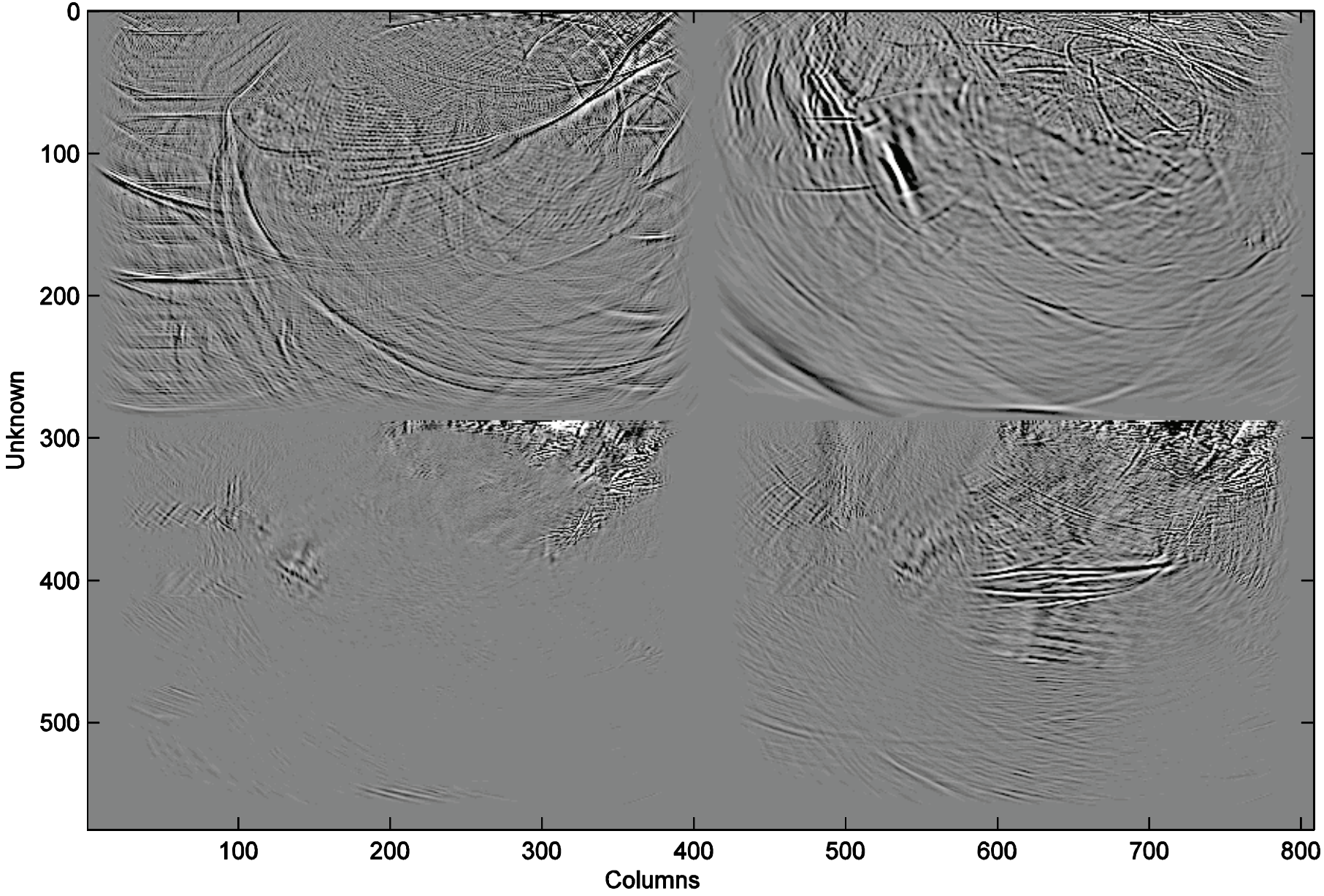
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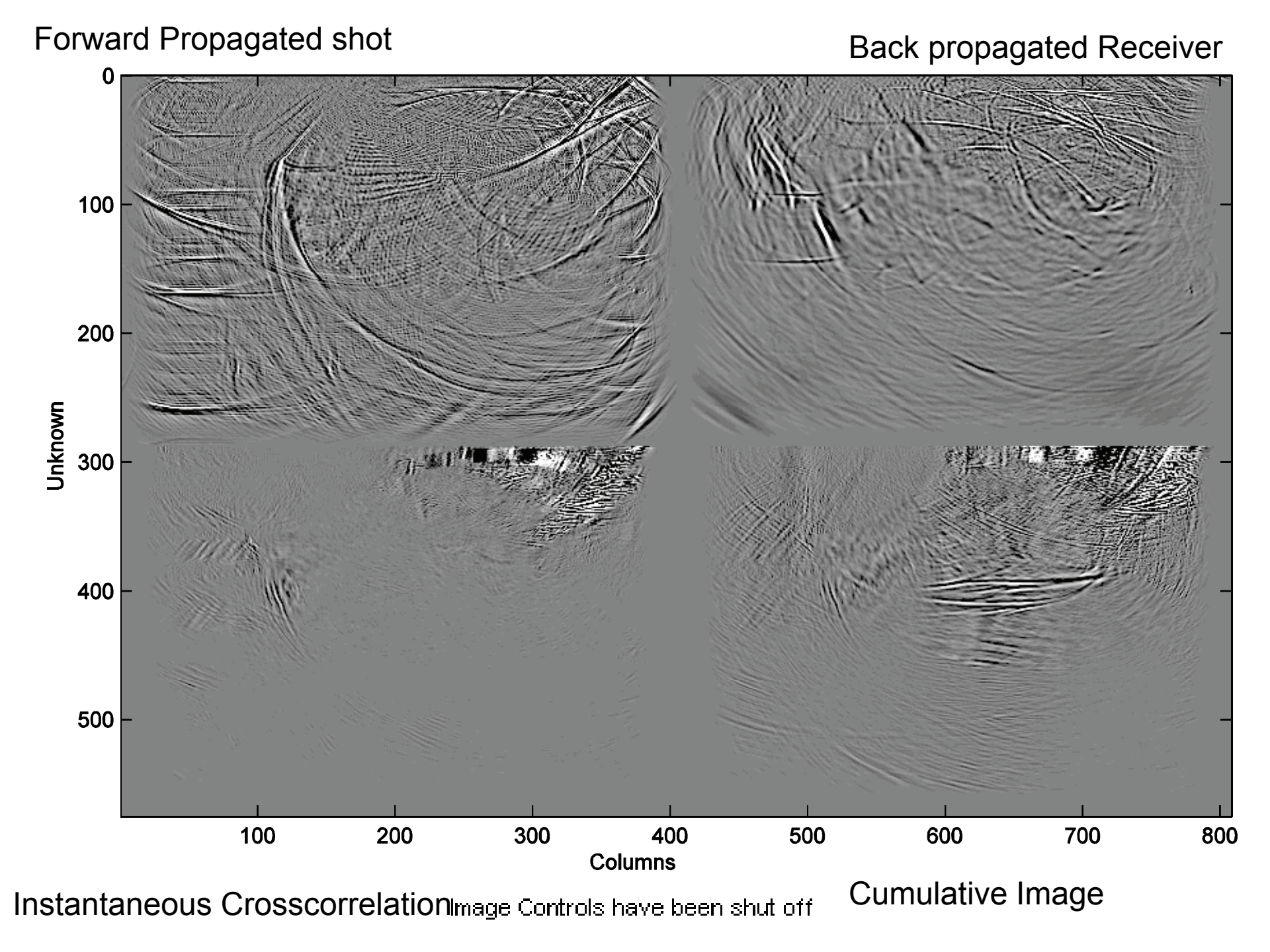
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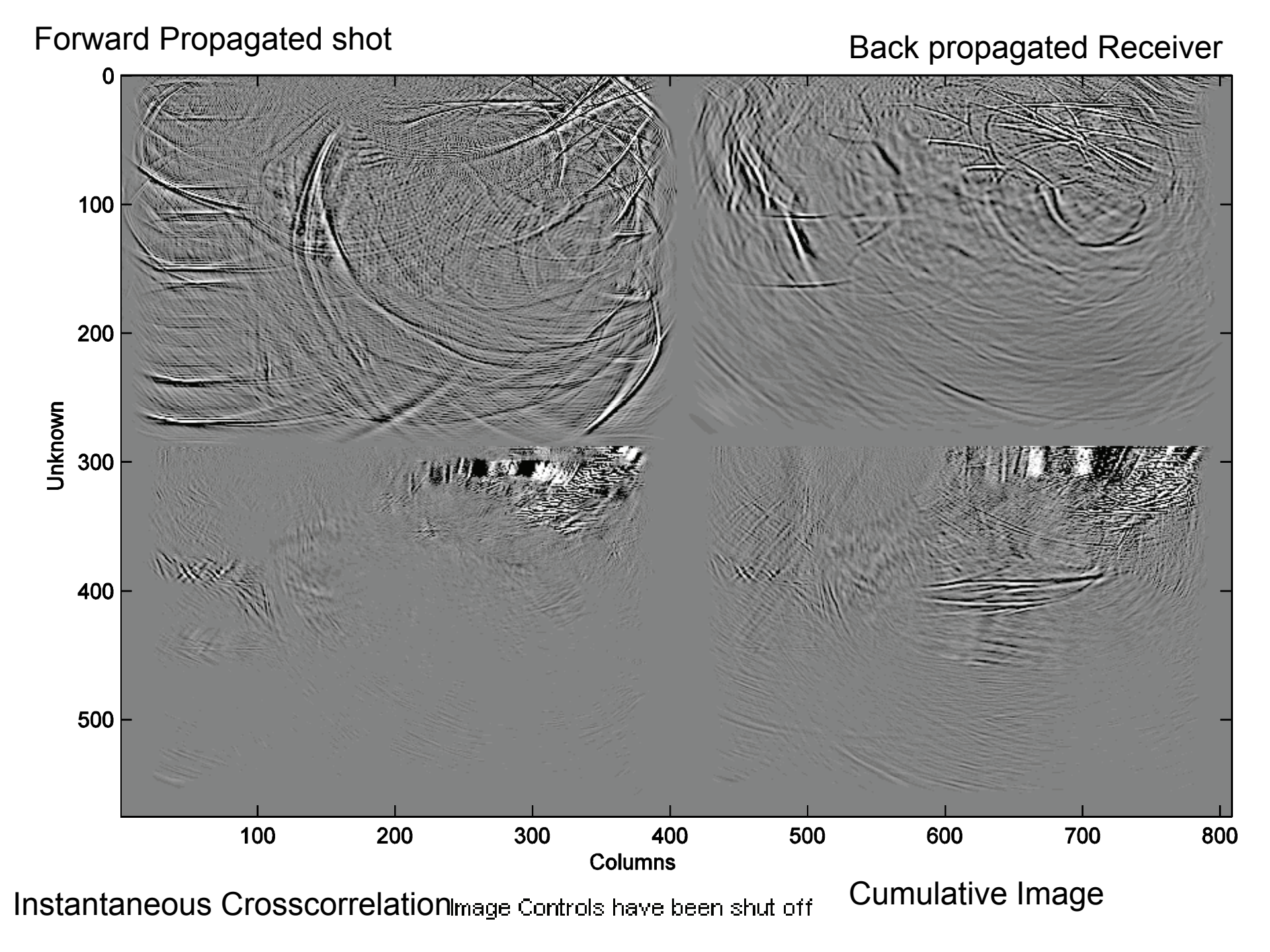
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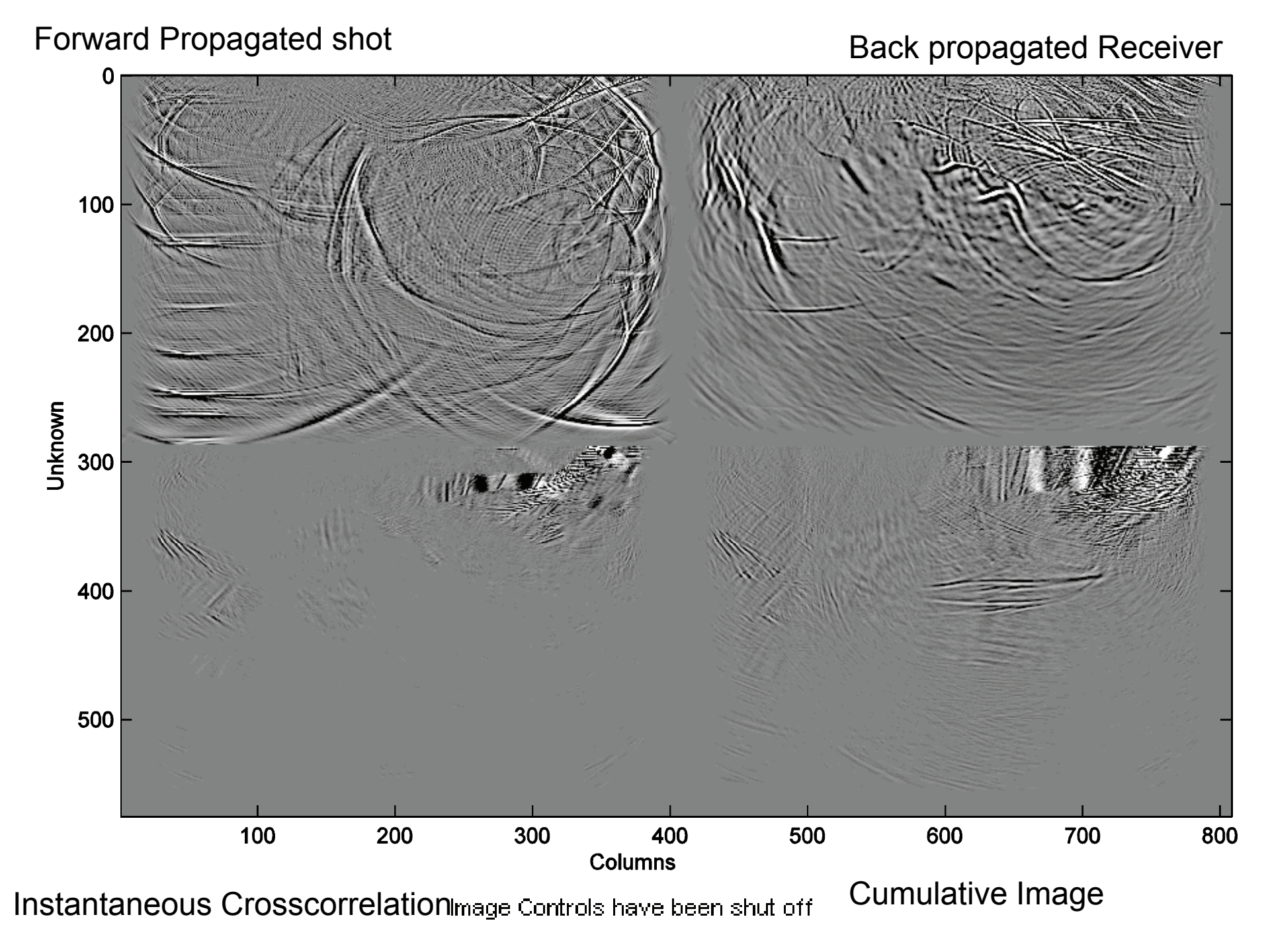
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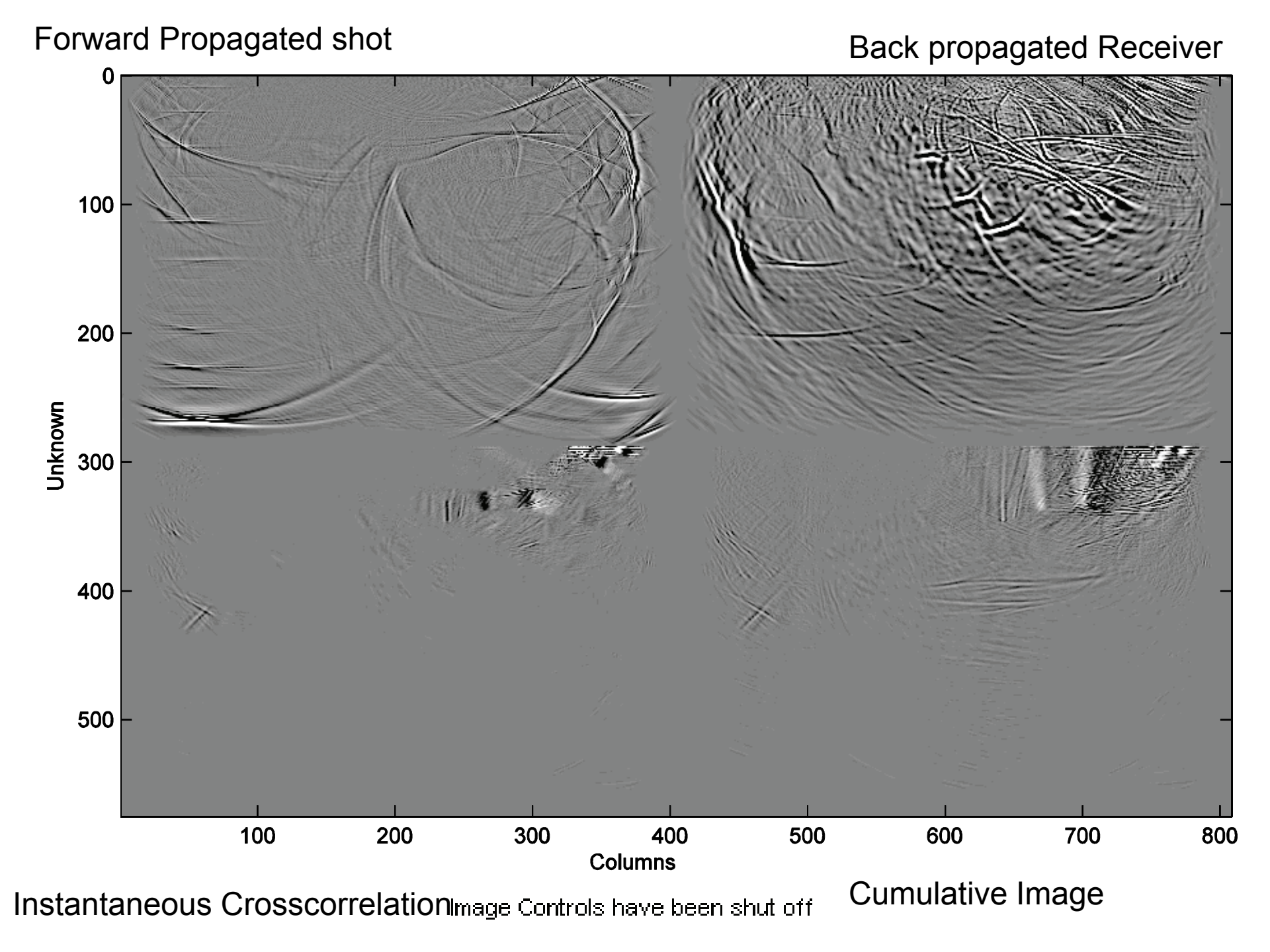
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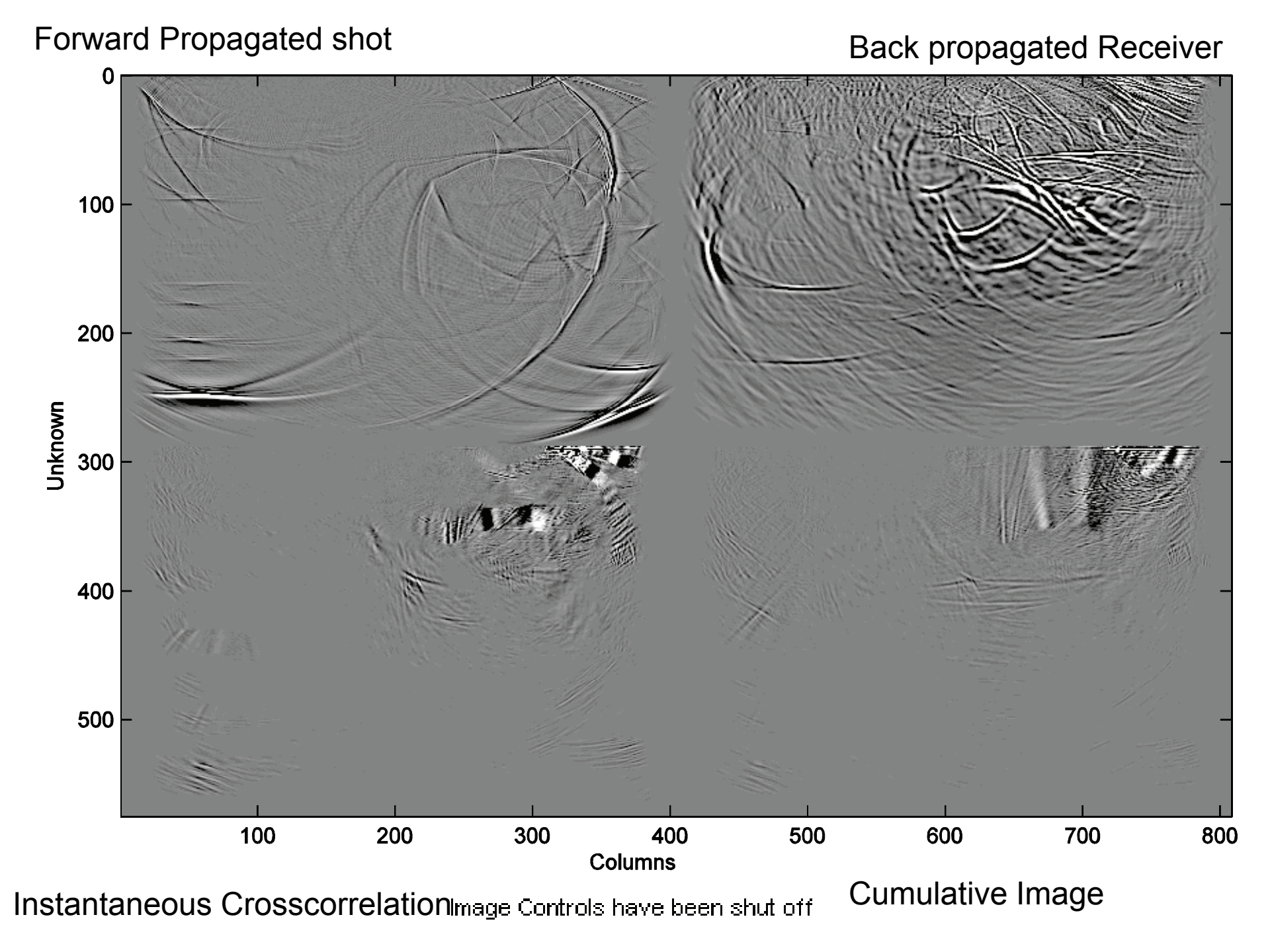
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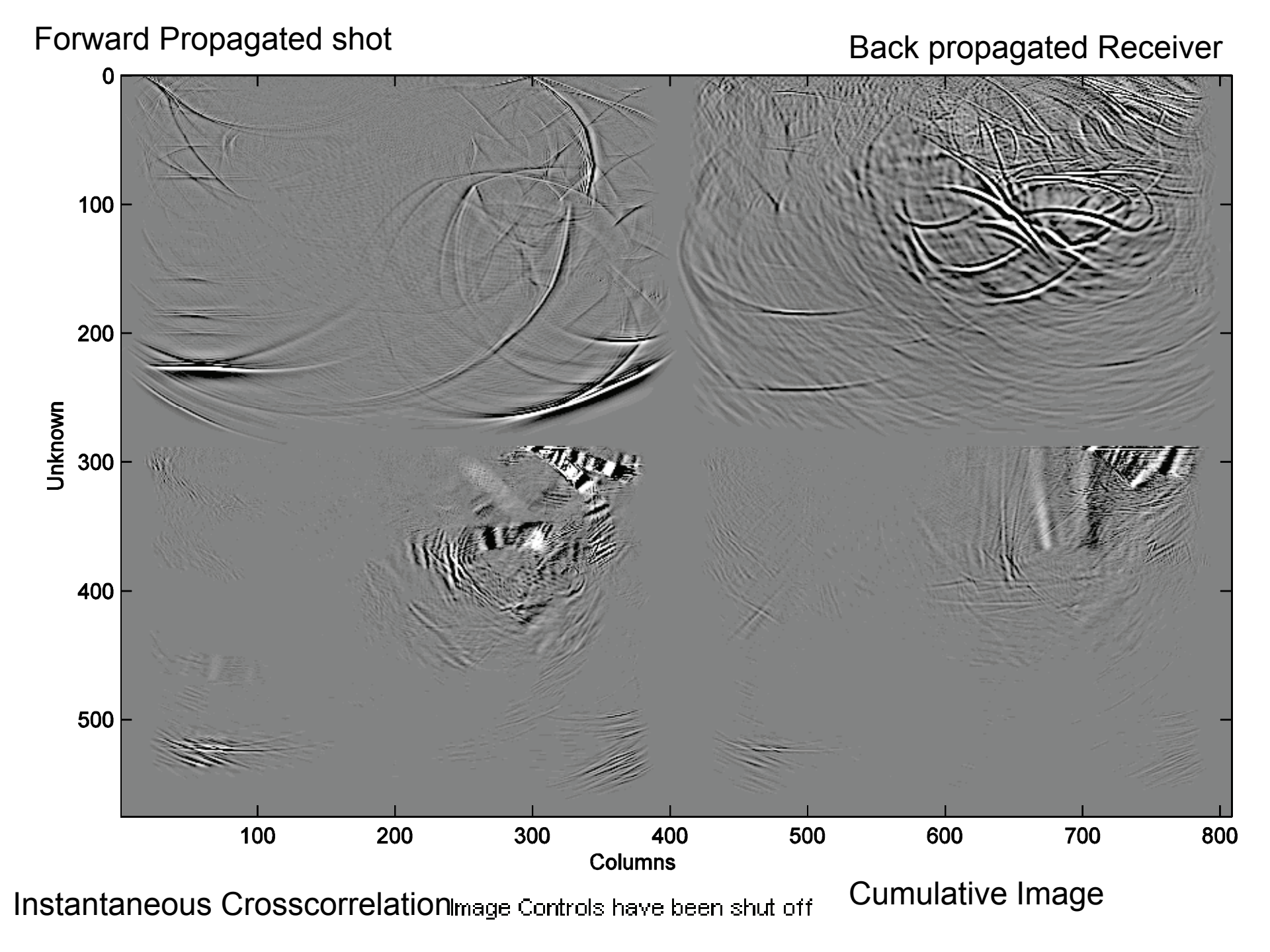
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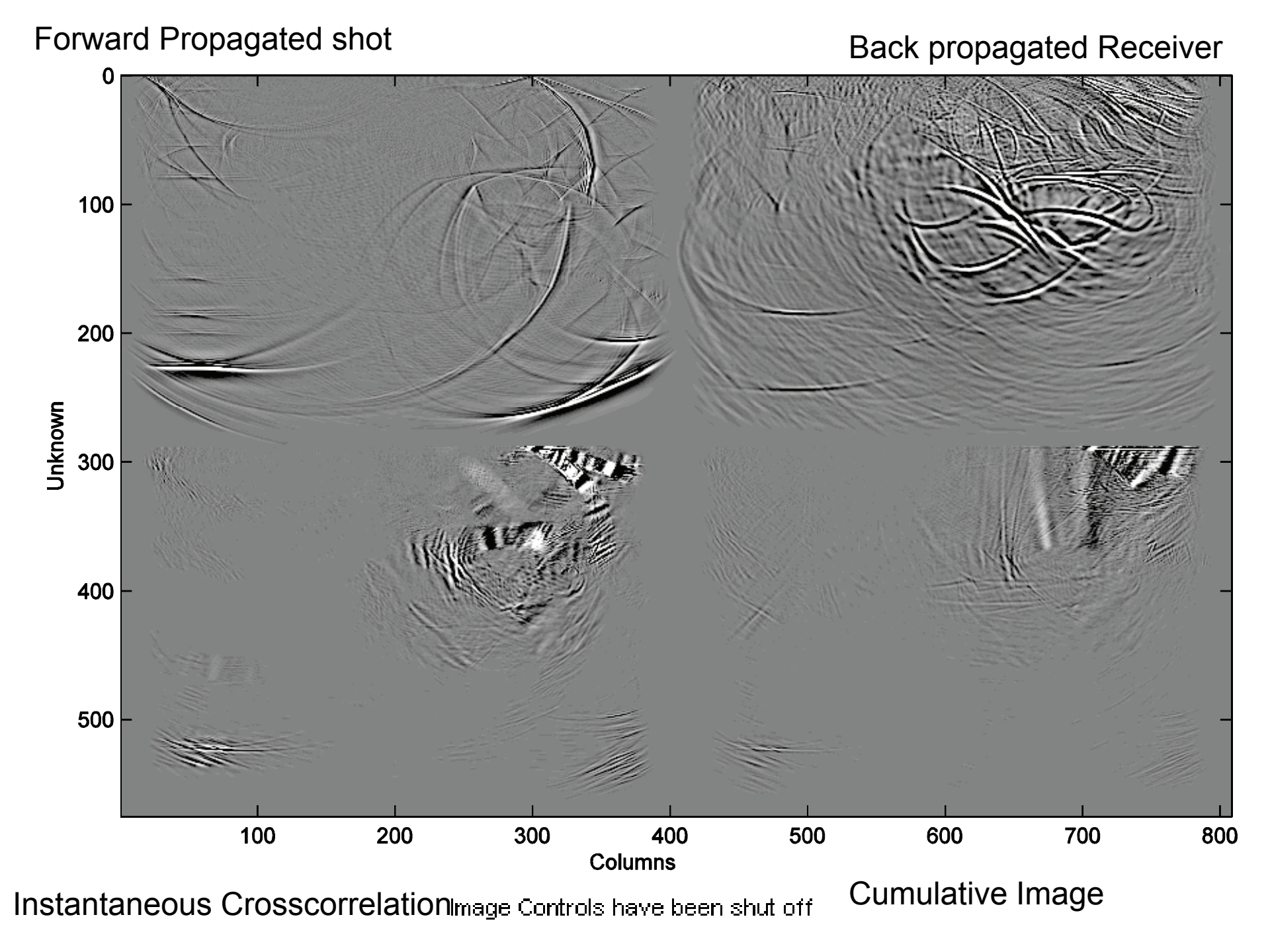
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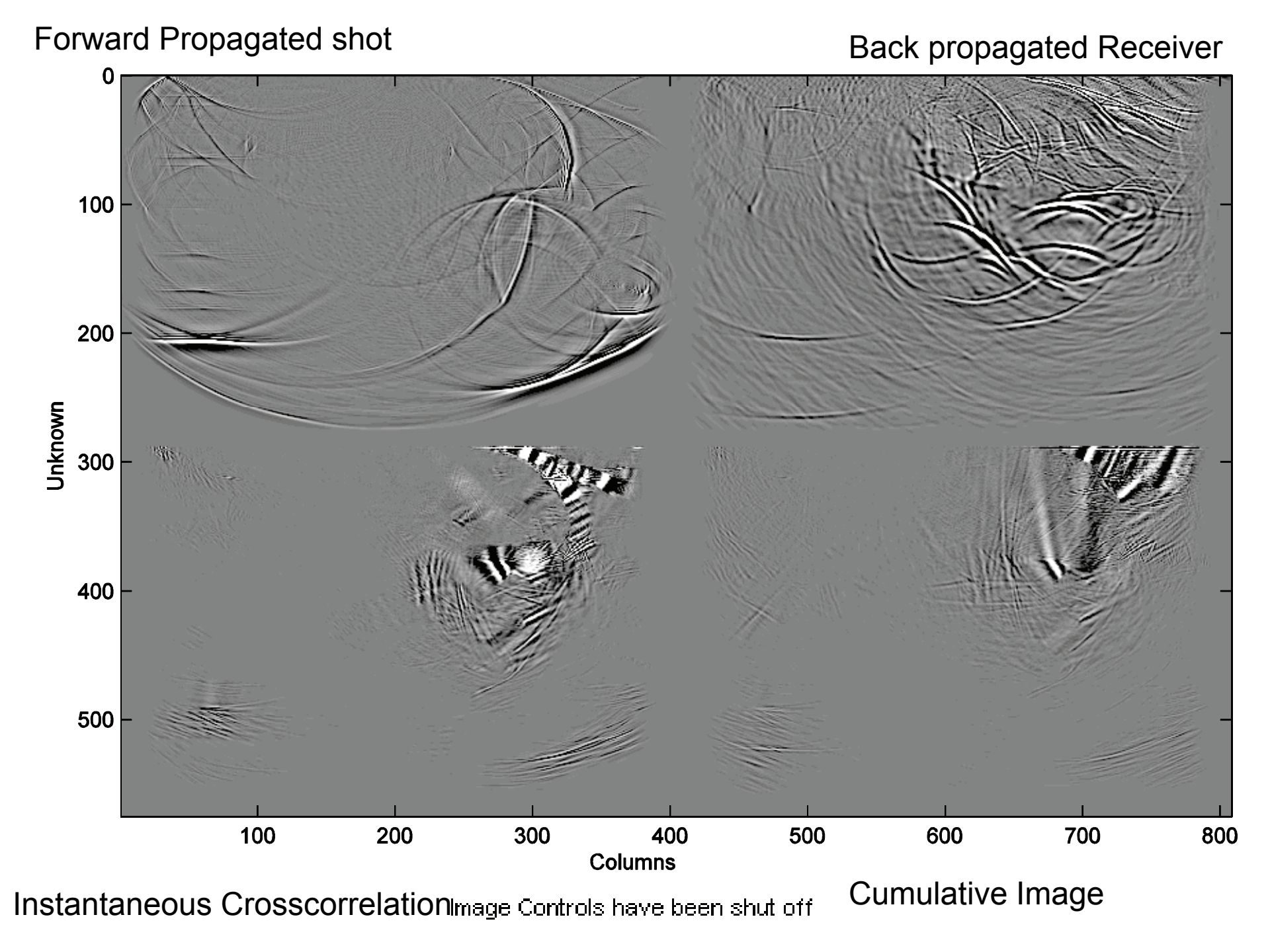
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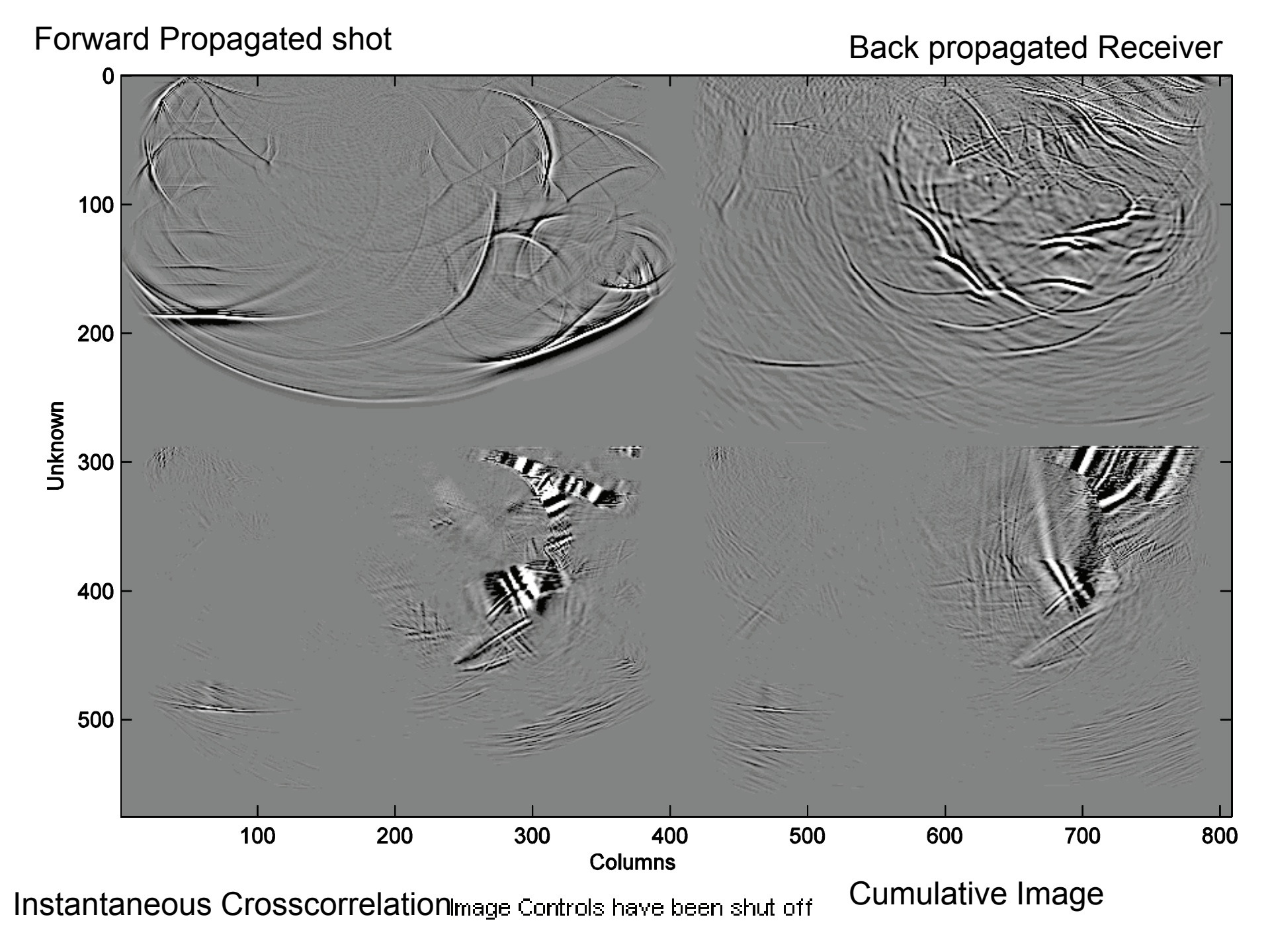
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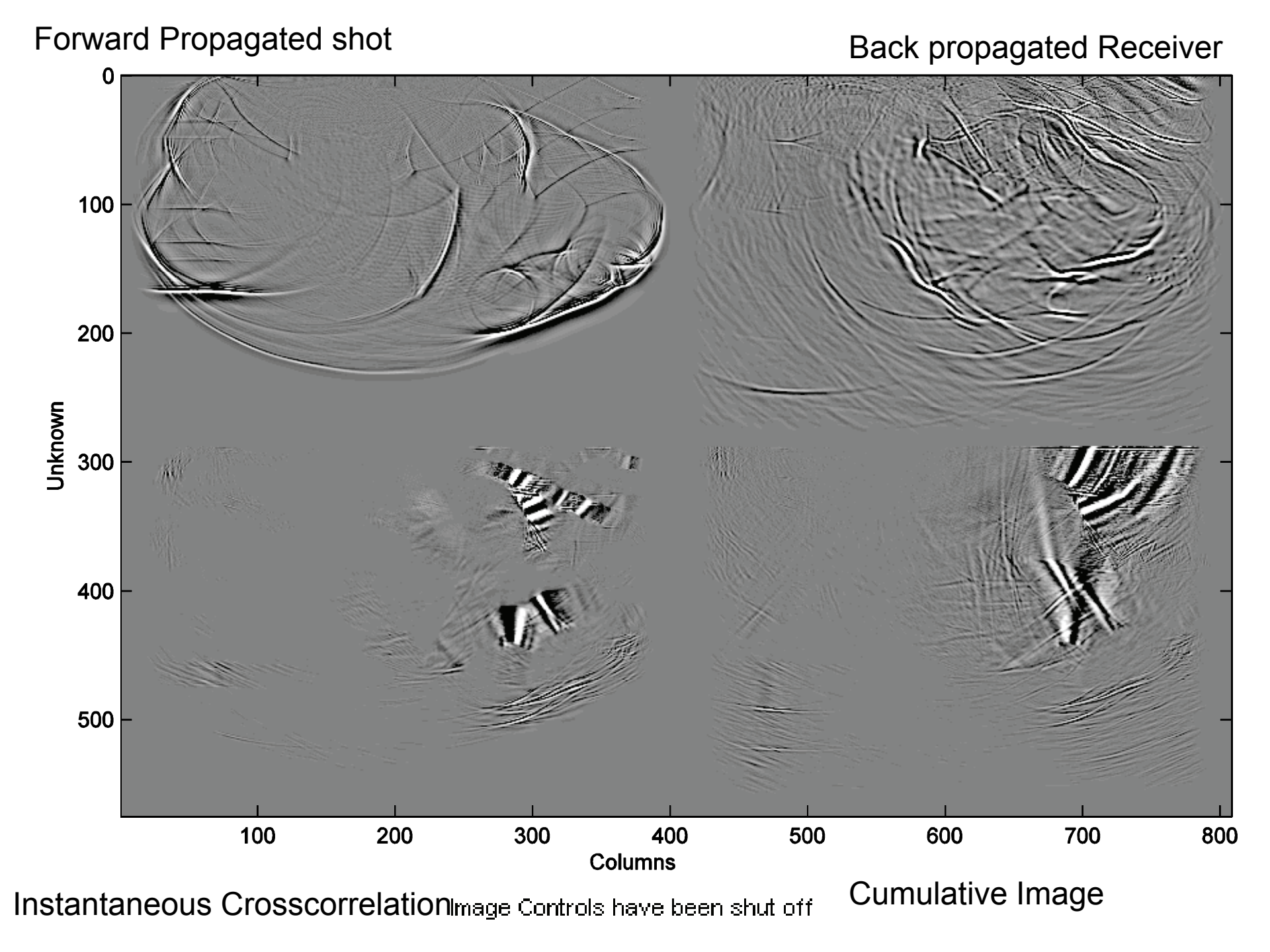
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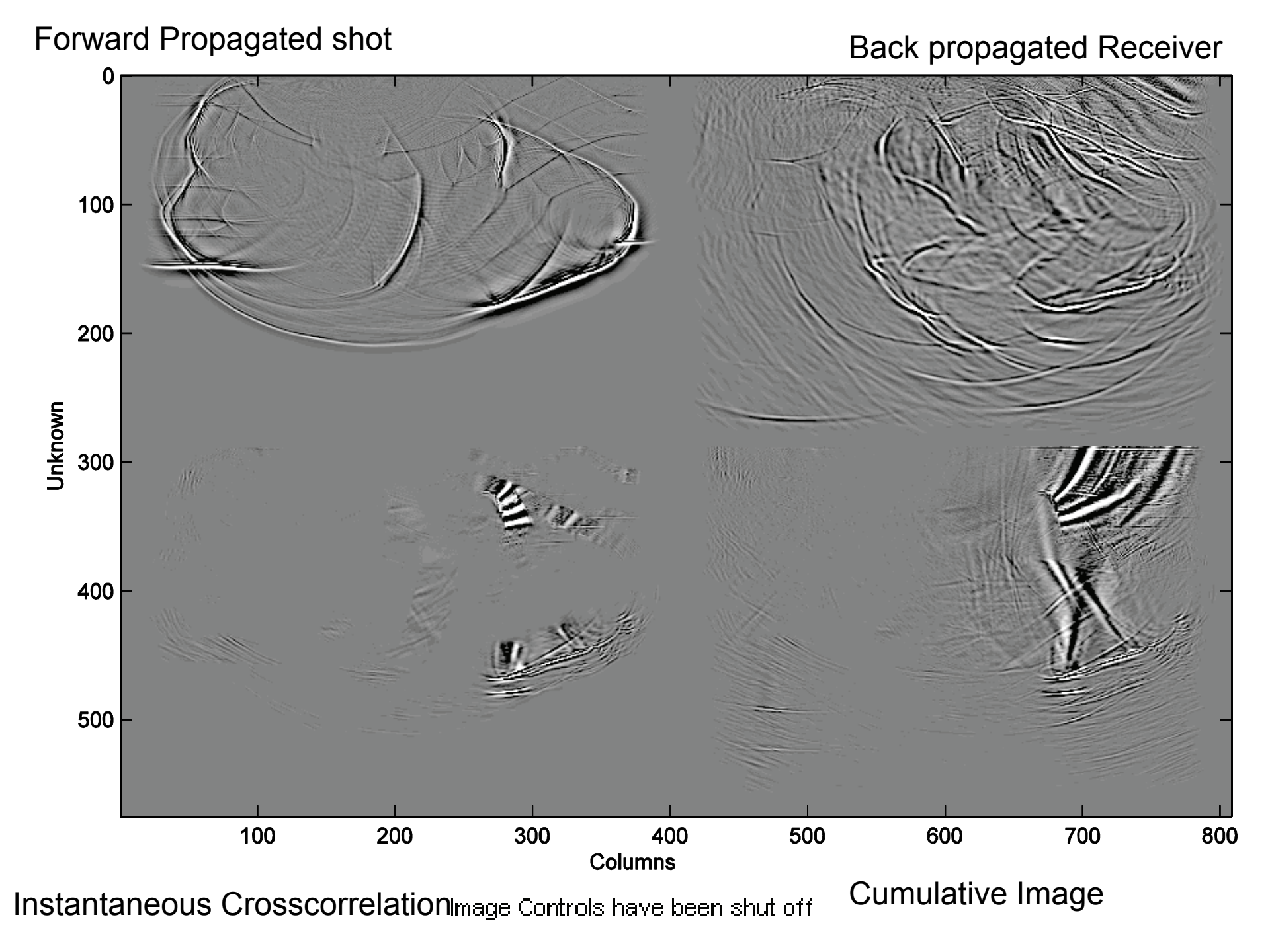
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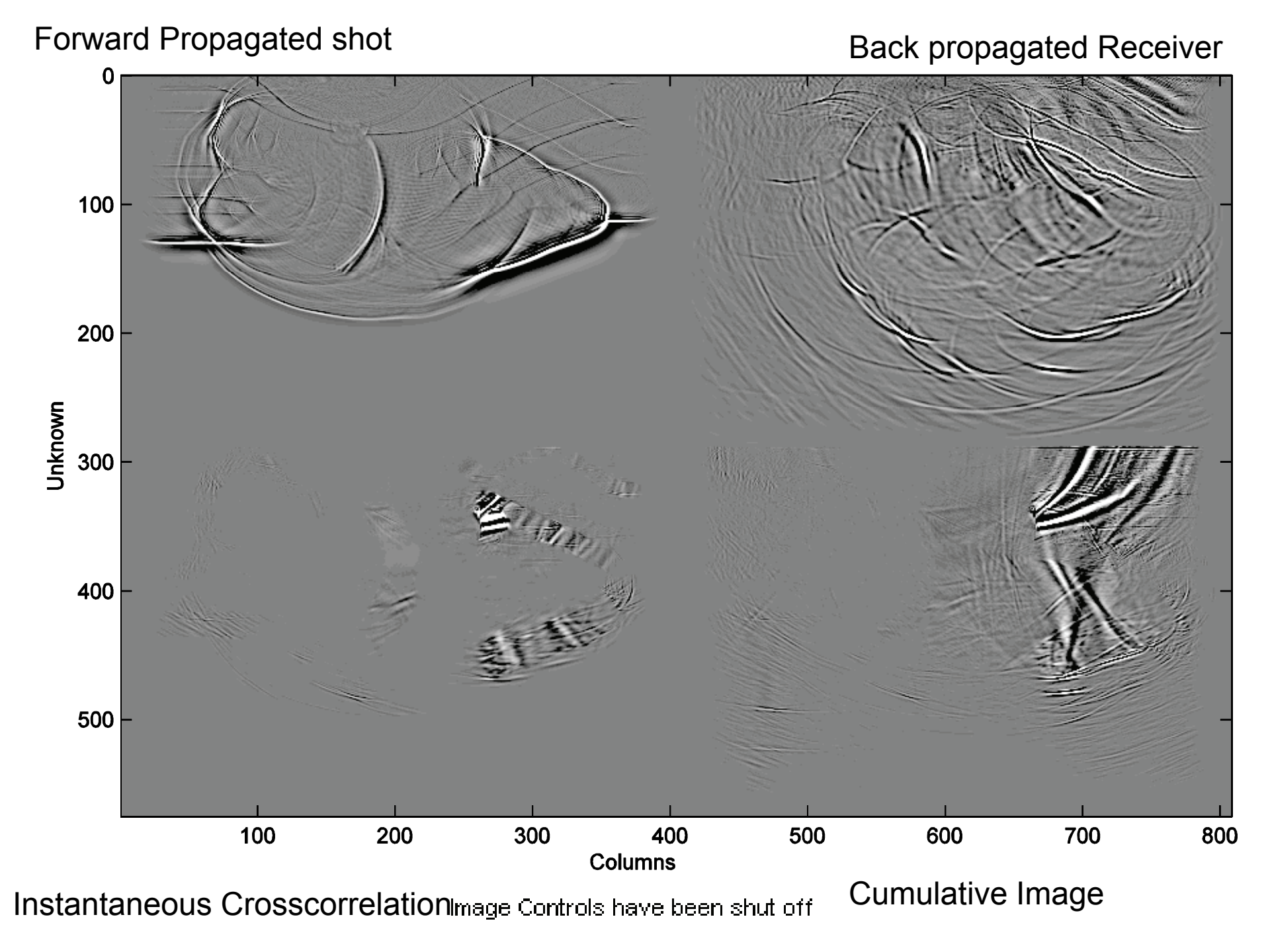
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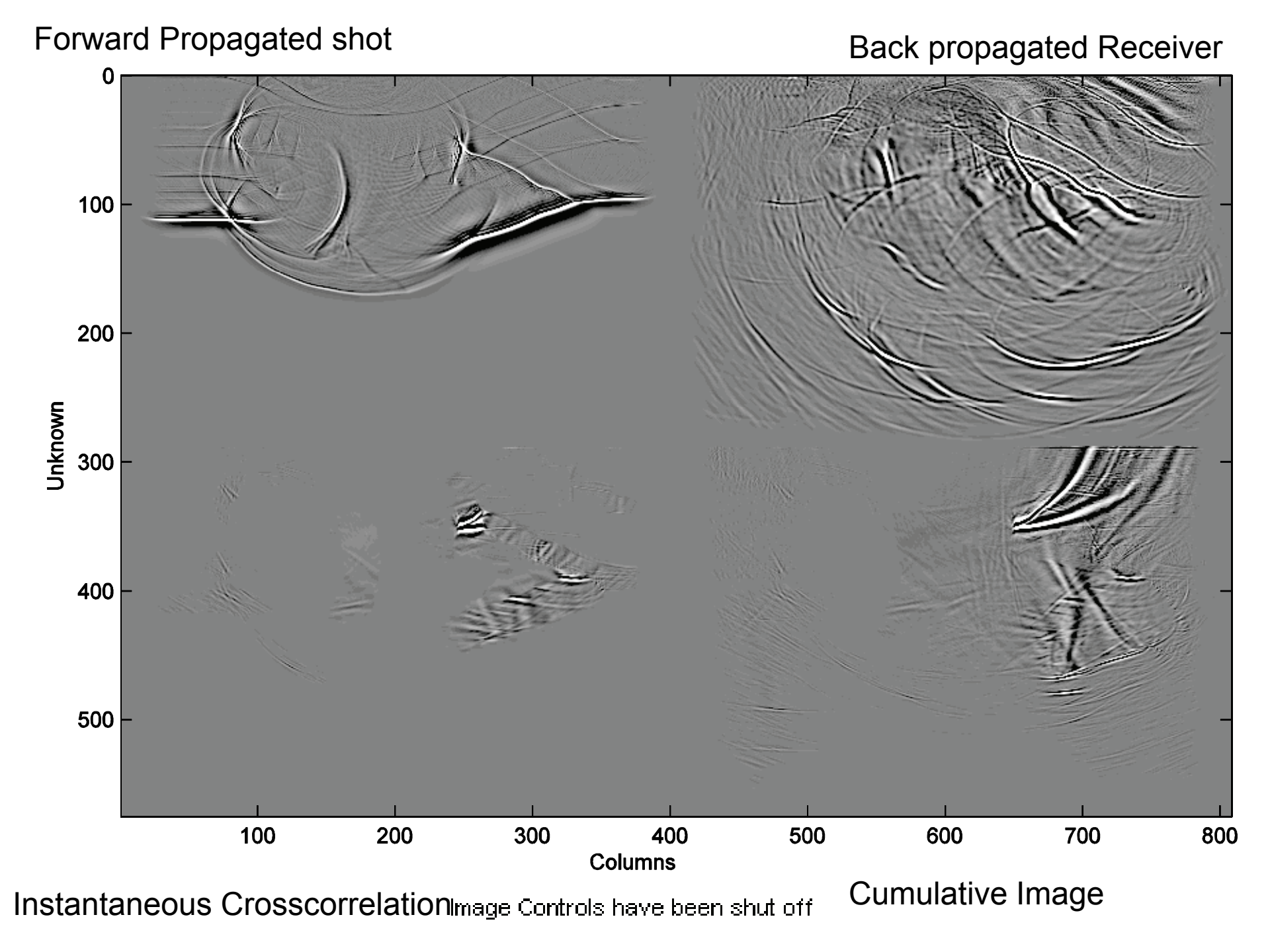
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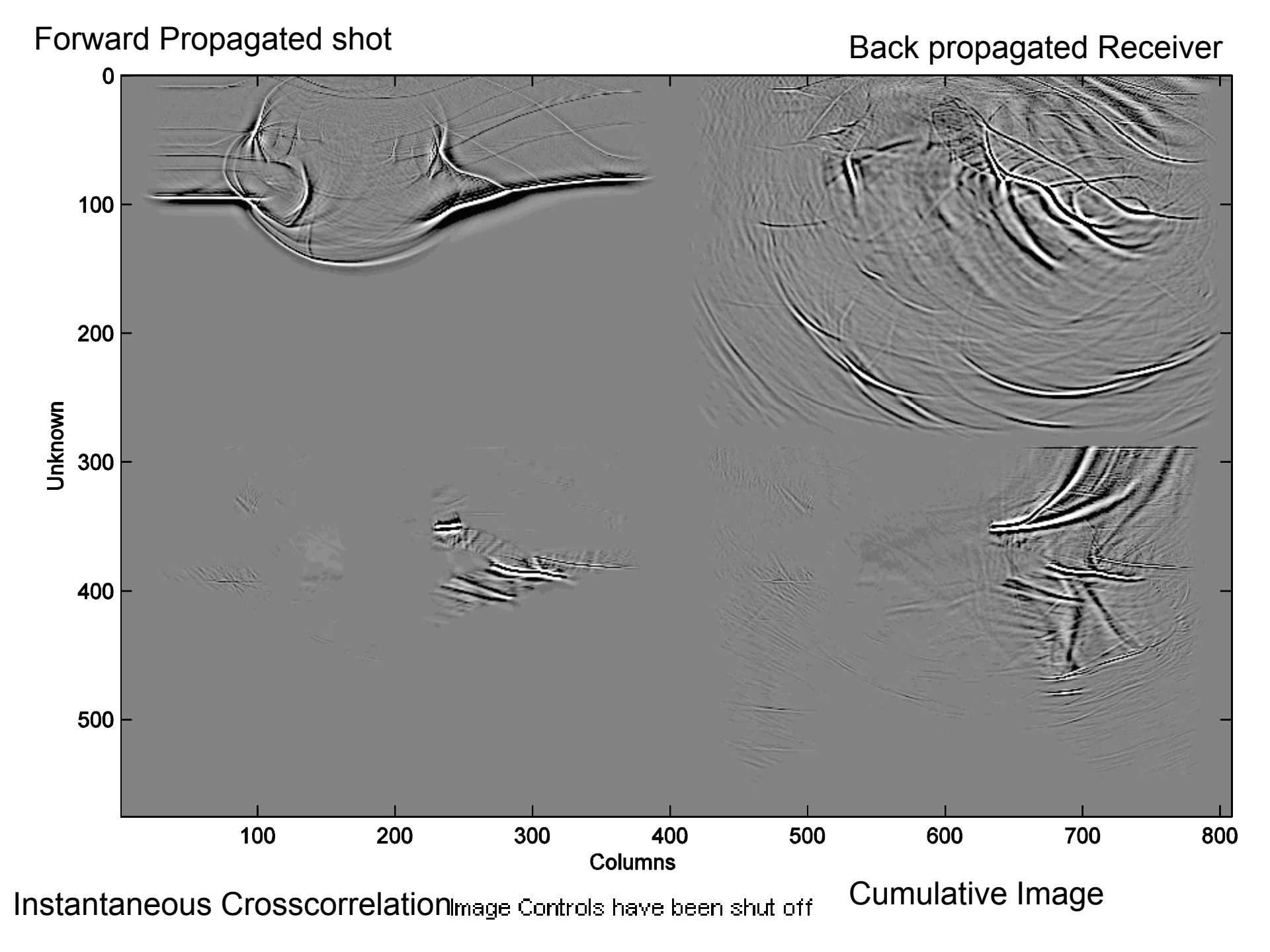
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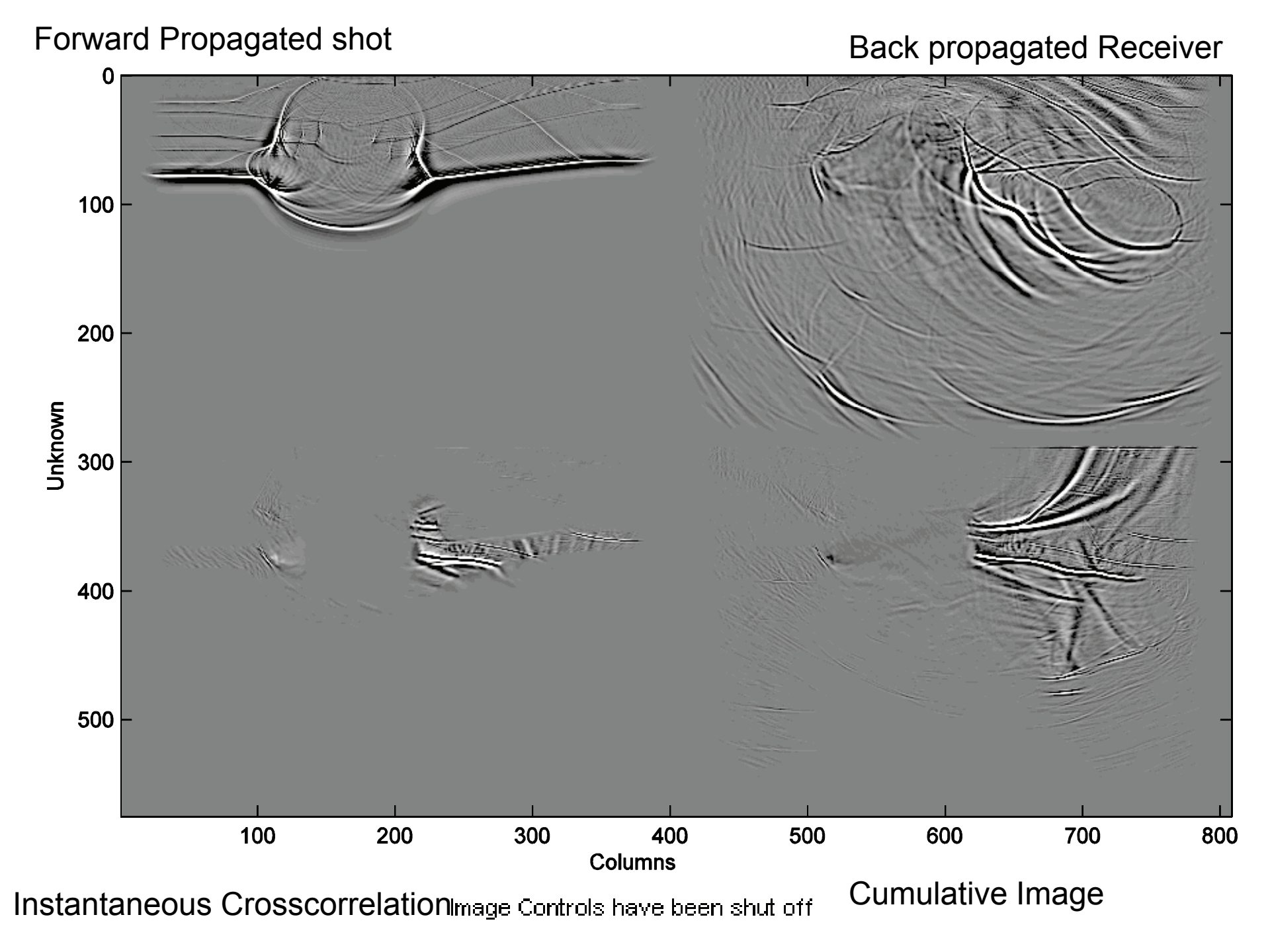
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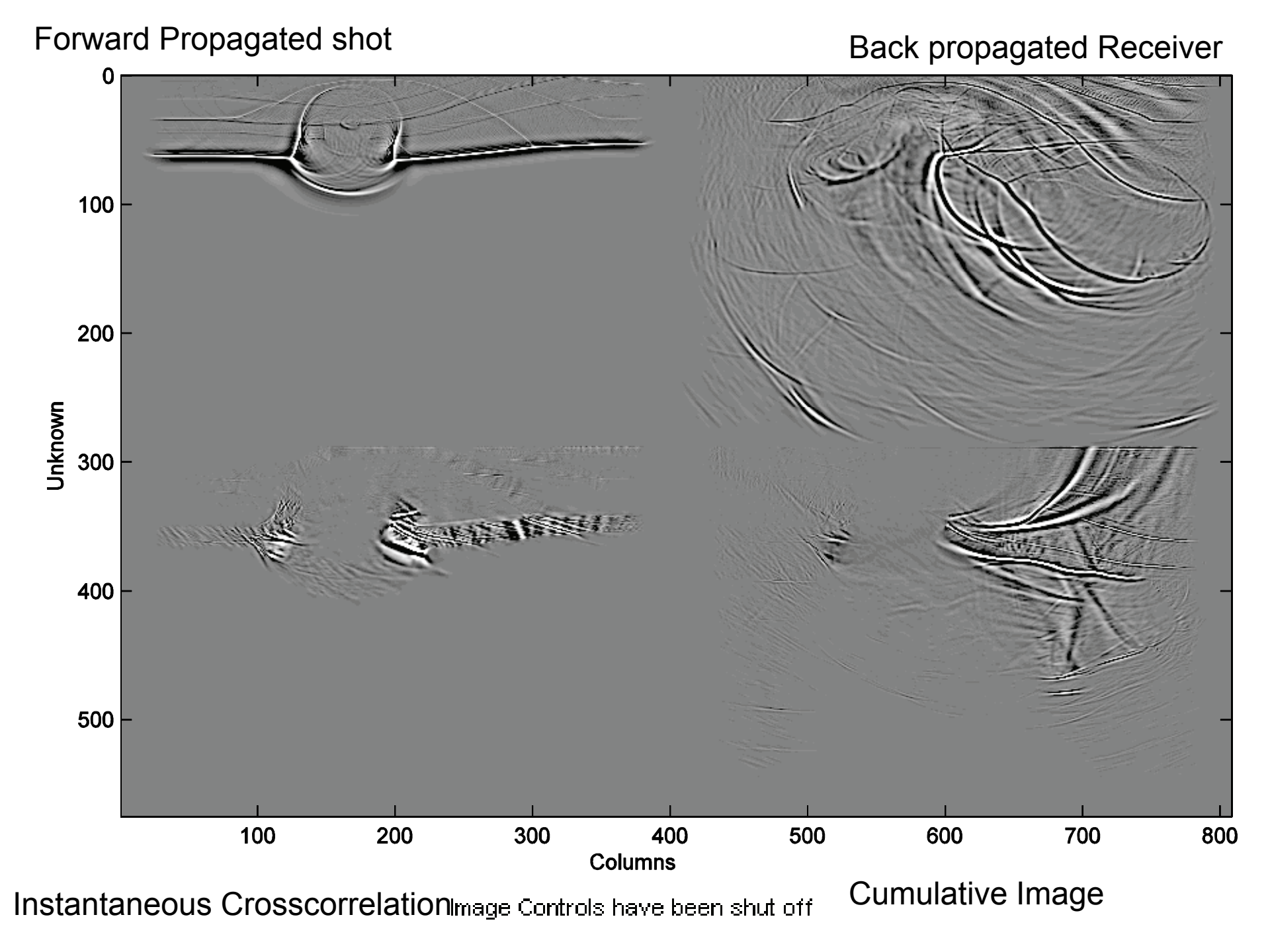
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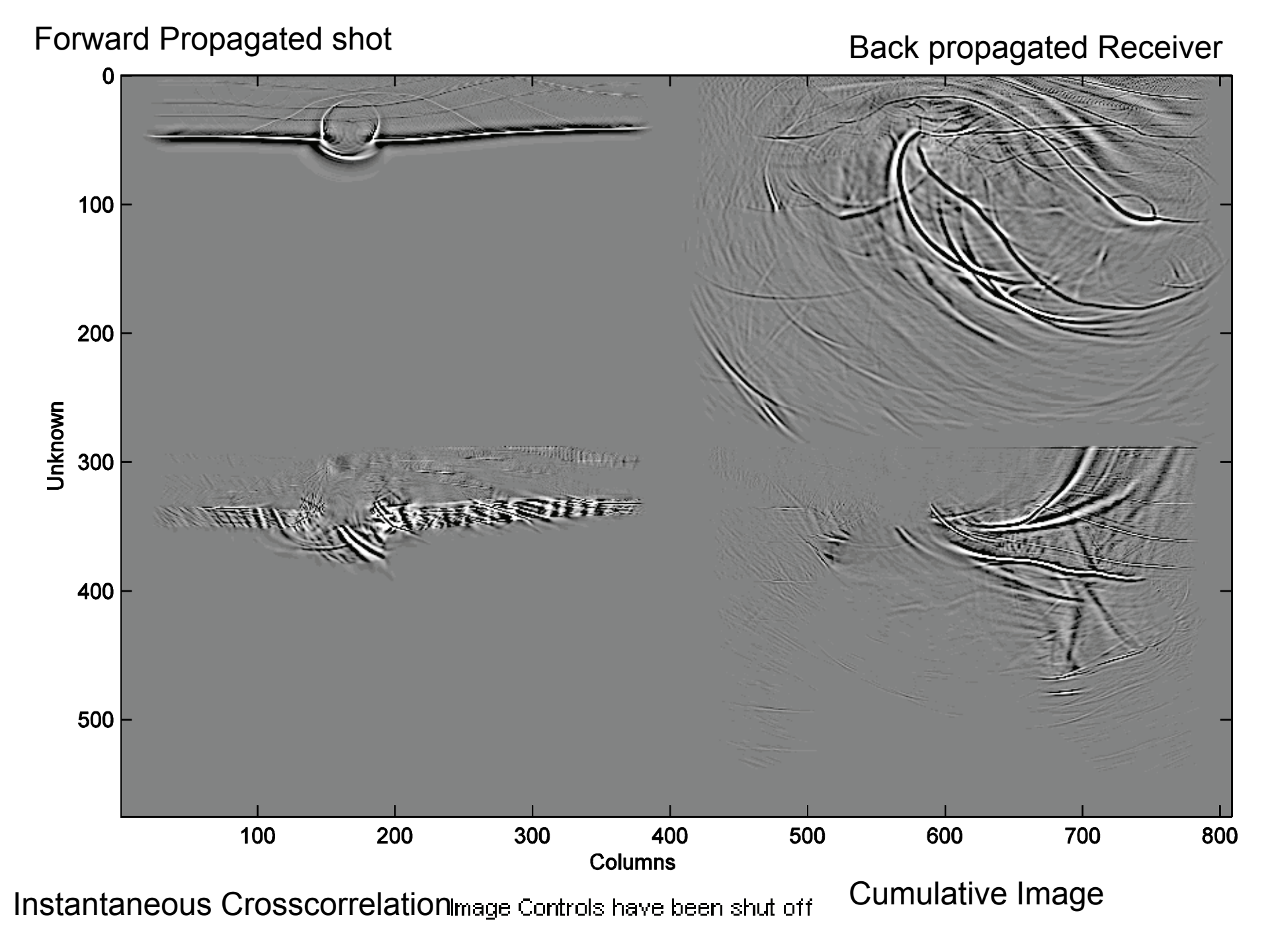
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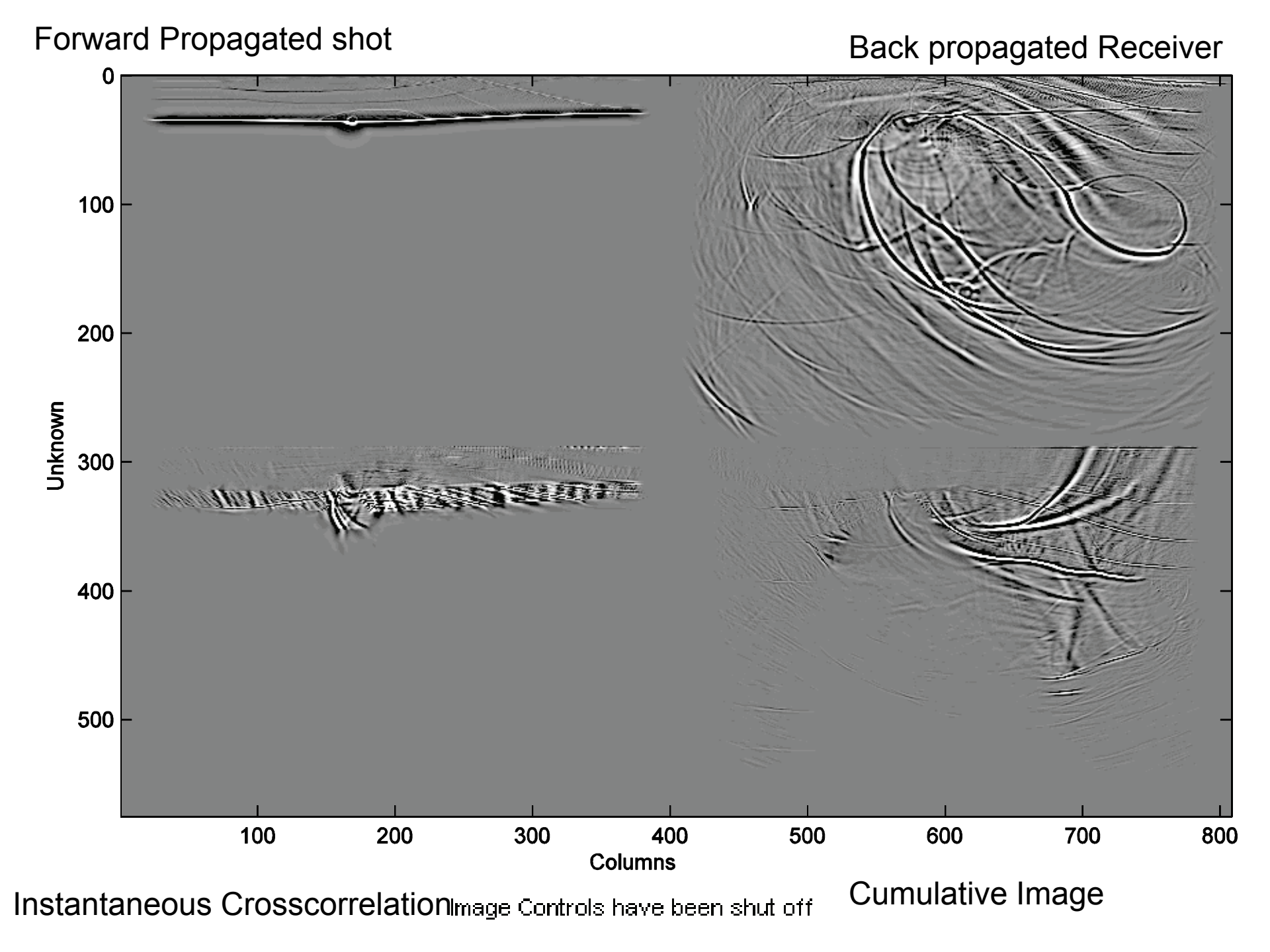
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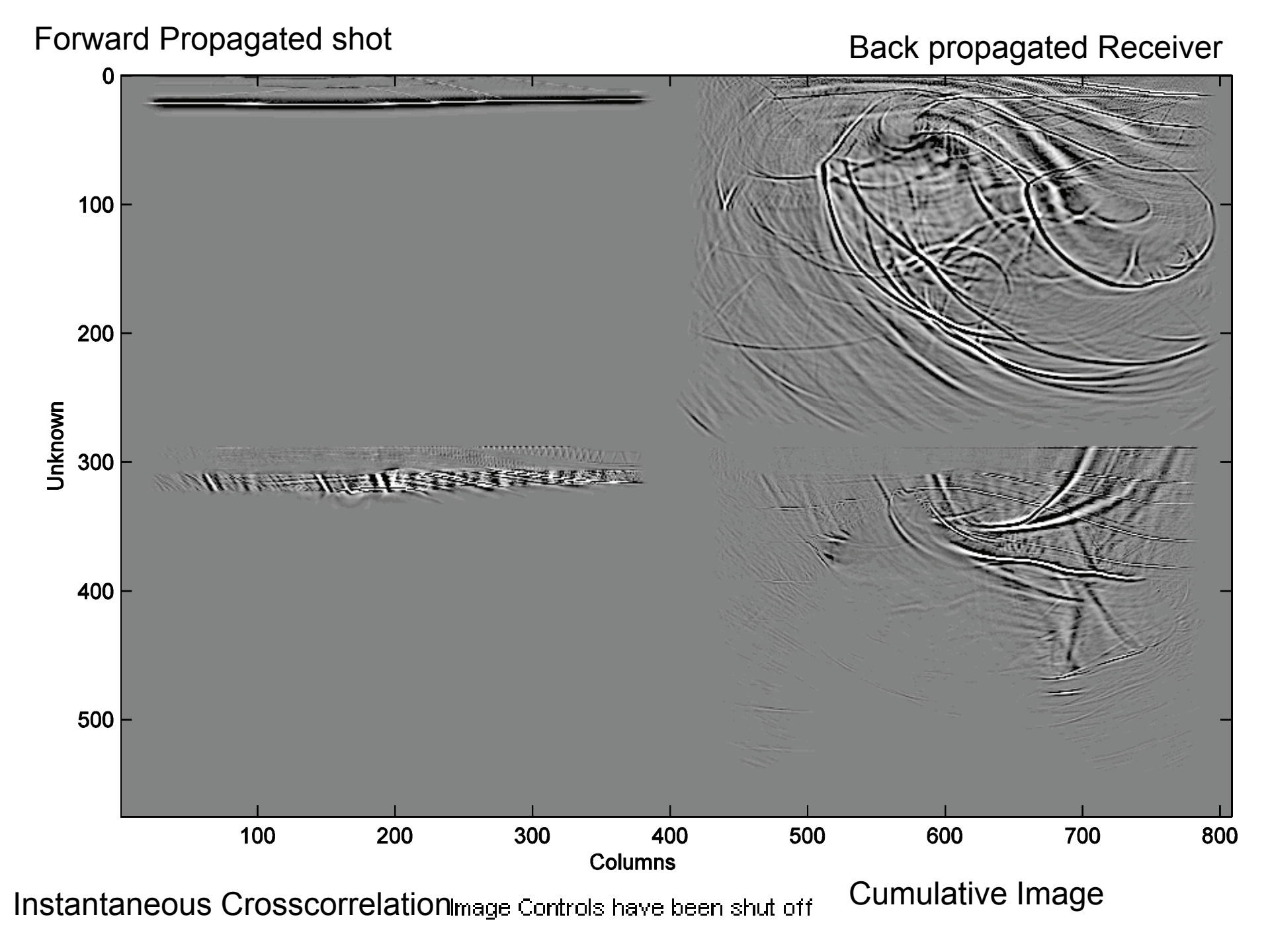
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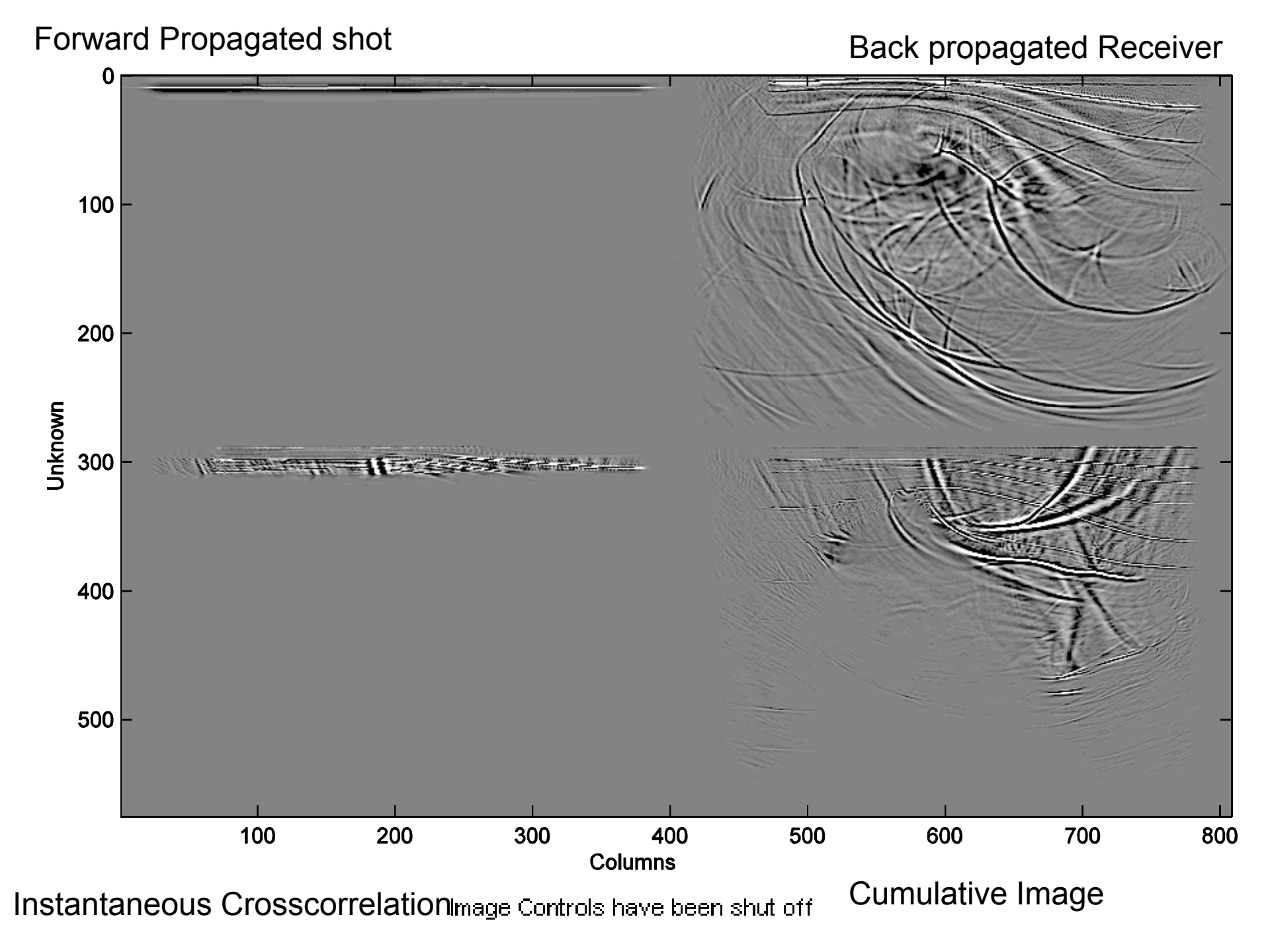
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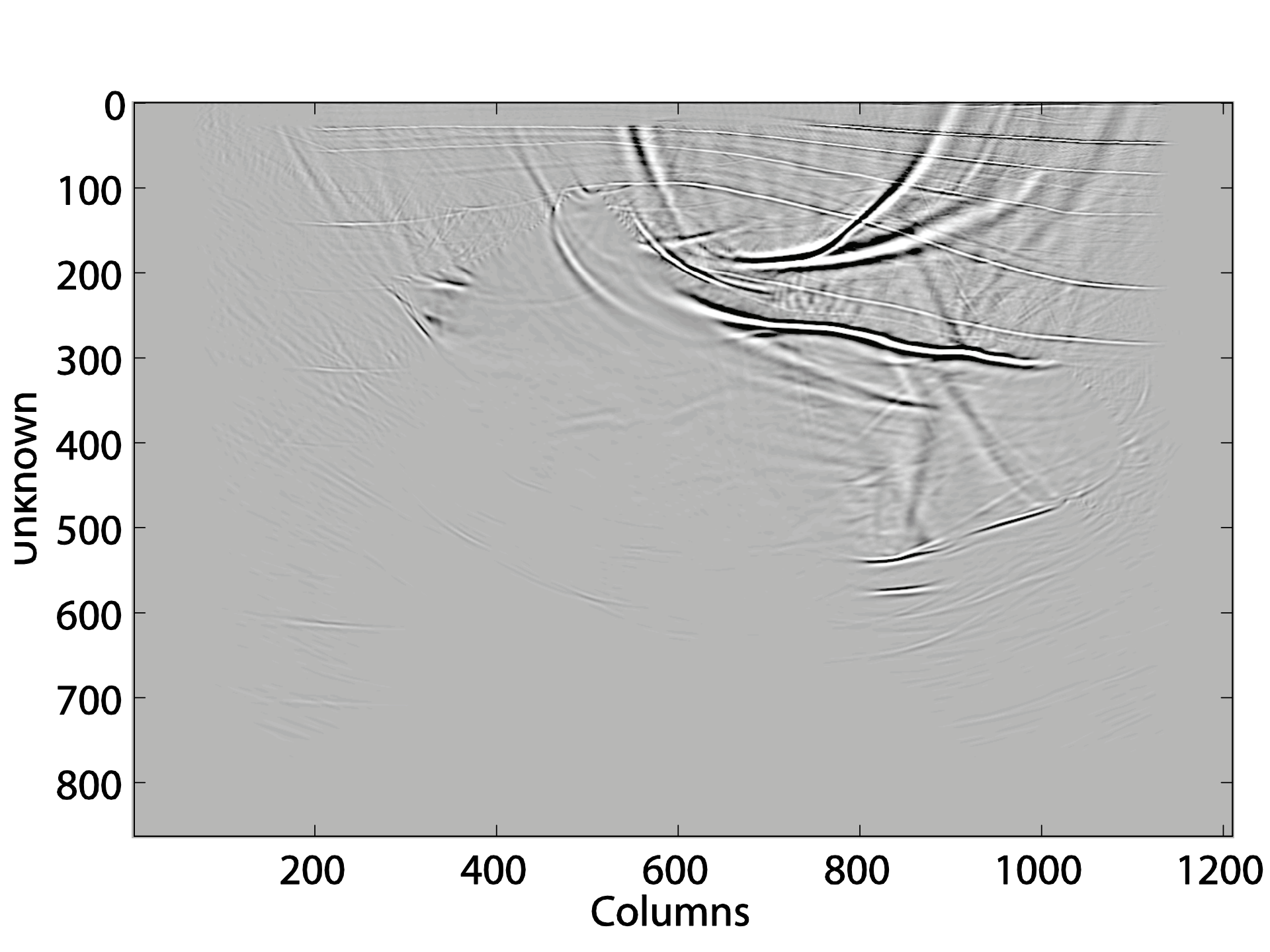
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Conclusions

- S and P wave propagation without coupling
- Pseudo acoustic wave equation
- Dispersion relations
- TTI RTM

Acknowledgements

- CREWES sponsors
- POTSI, University of Calgary
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