AVAZ inversion for fracture orientation and intensity: A physical modeling study

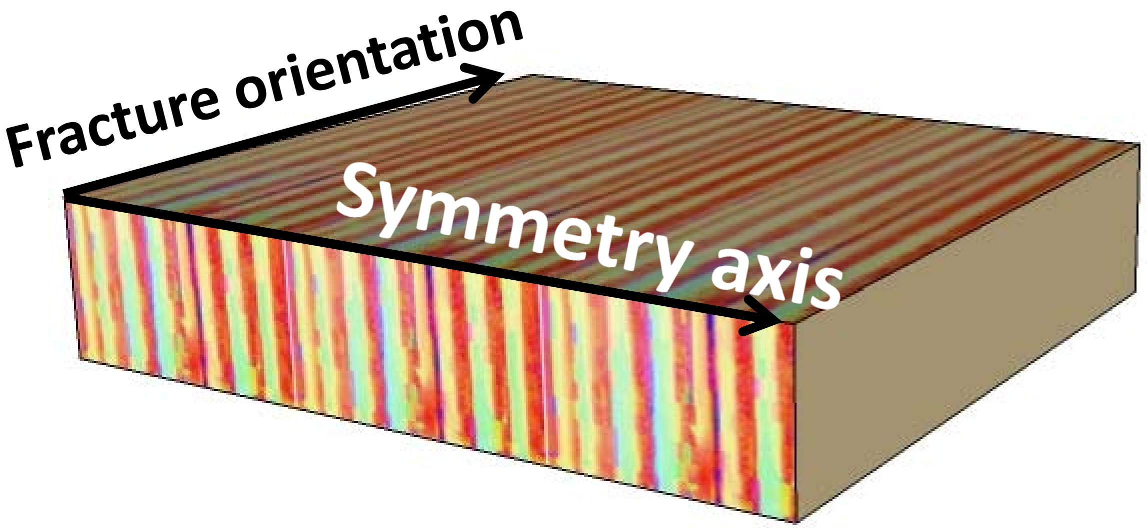
Faranak Mahmoudian

Gary Margrave

UNIVERSITY OF CALGARY

CREWES

Objective



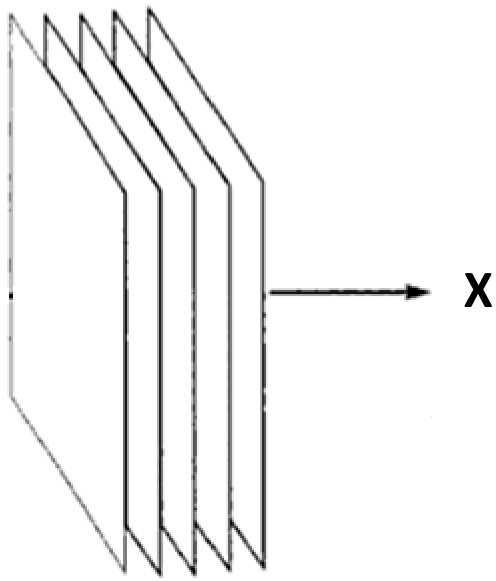
Fracture orientation: direction of fracture planes
Fracture intensity: number of fractures in unit volume times (mean diameter)^3

P-wave AVAZ inversion for fracture orientation and intensity
Fracture orientation (Jenner, 2002)
AVAZ inversion using Rüger's equation for ($\varepsilon^{(V)}$, $\delta^{(V)}$, γ),
γ is directly related to fracture intensity

Outline

- HTI model
- Previous work on physical modeling
- Theory of AVAZ inversion
- Implementation on physical model data
- Conclusions
- Acknowledgements

HTI (horizontal transverse isotropy)
Simple model to describe vertical fractures



- Vertical isotropic plane
- Horizontal symmetry axis
- $(\alpha, \beta, \varepsilon^{(V)}, \delta^{(V)}, \gamma)$ to describe the medium

$\alpha =$ P-vertical velocity

$\beta =$ S-vertical velocity ($S^{||}$)

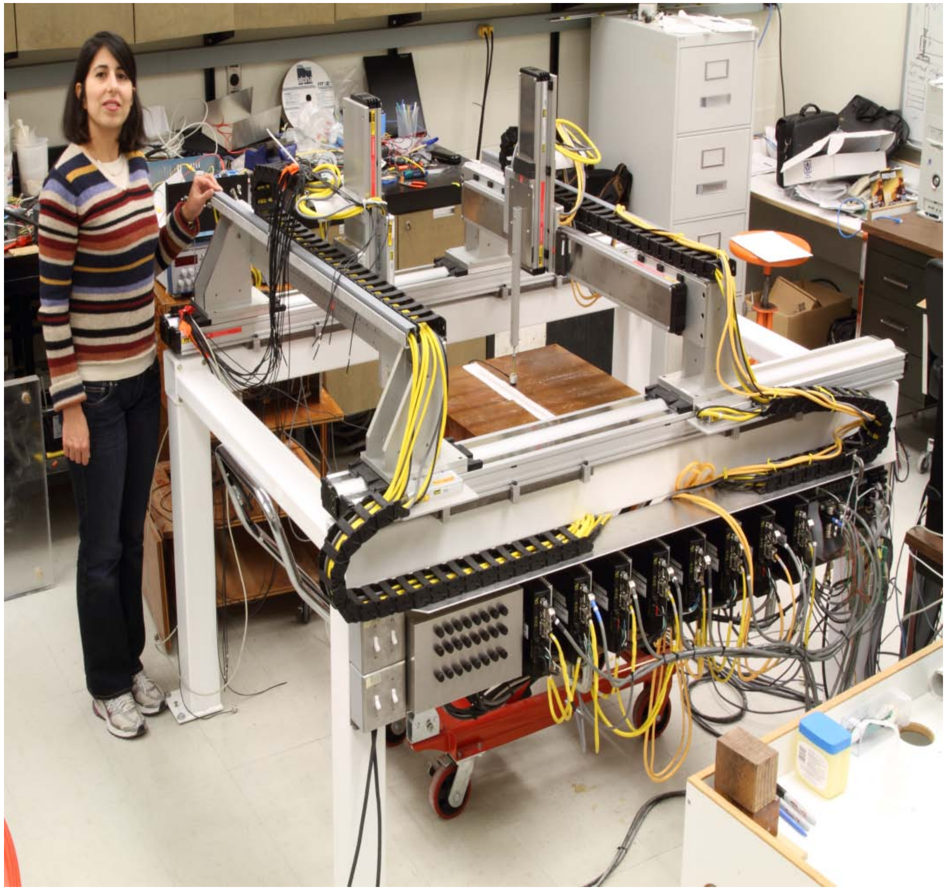
$$\varepsilon^{(V)} = \frac{V_{Px} - V_{Pz}}{V_{Pz}}$$

(Shear-wave splitting parameter) directly related to fracture intensity →

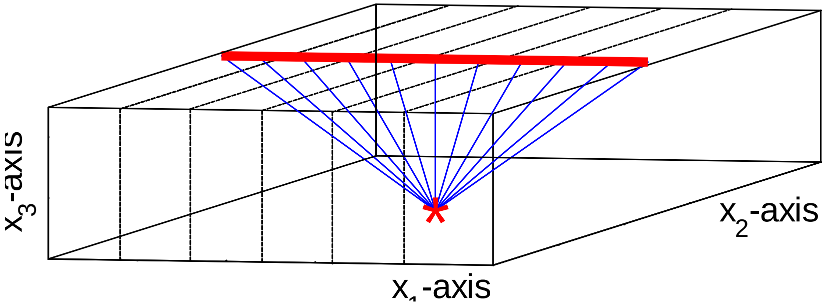
$$\gamma = \frac{V_{Sx} - V_{Sz}}{V_{Sz}}$$

$$\delta^{(V)} = \frac{(A_{13} + A_{55})^2 - (A_{33} - A_{55})}{2A_{33}(A_{33} - A_{55})}$$

Simulated fractured layer
(2010 work)
Phenolic layer ≈ HTI

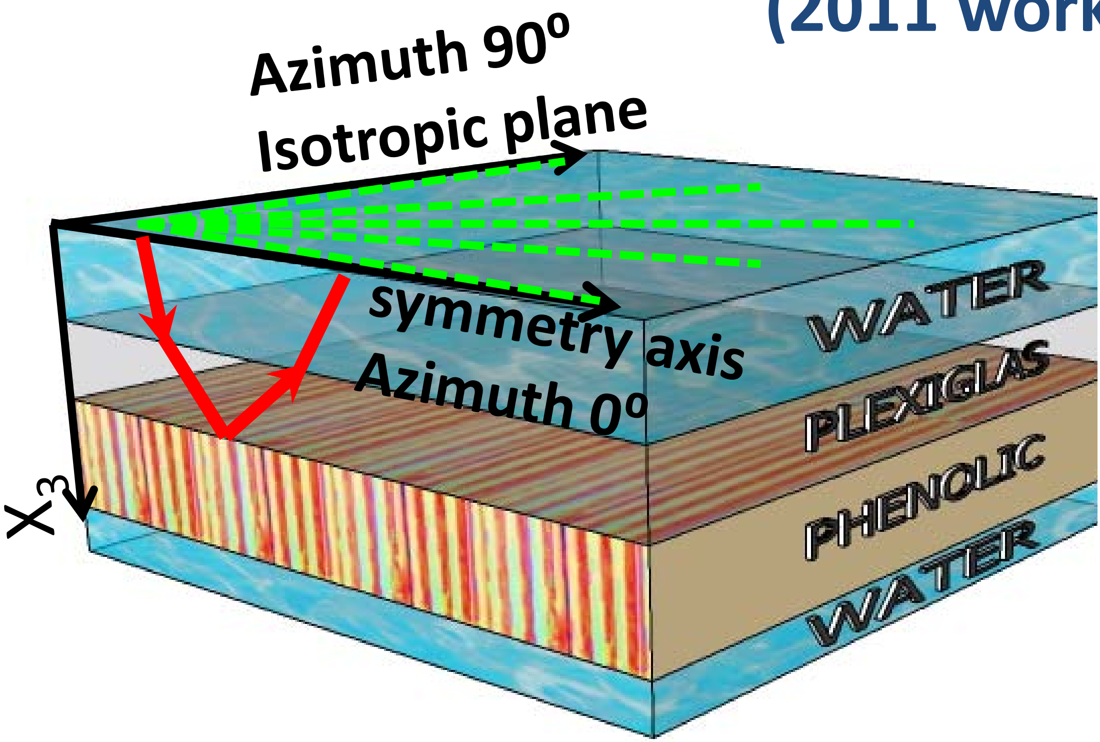


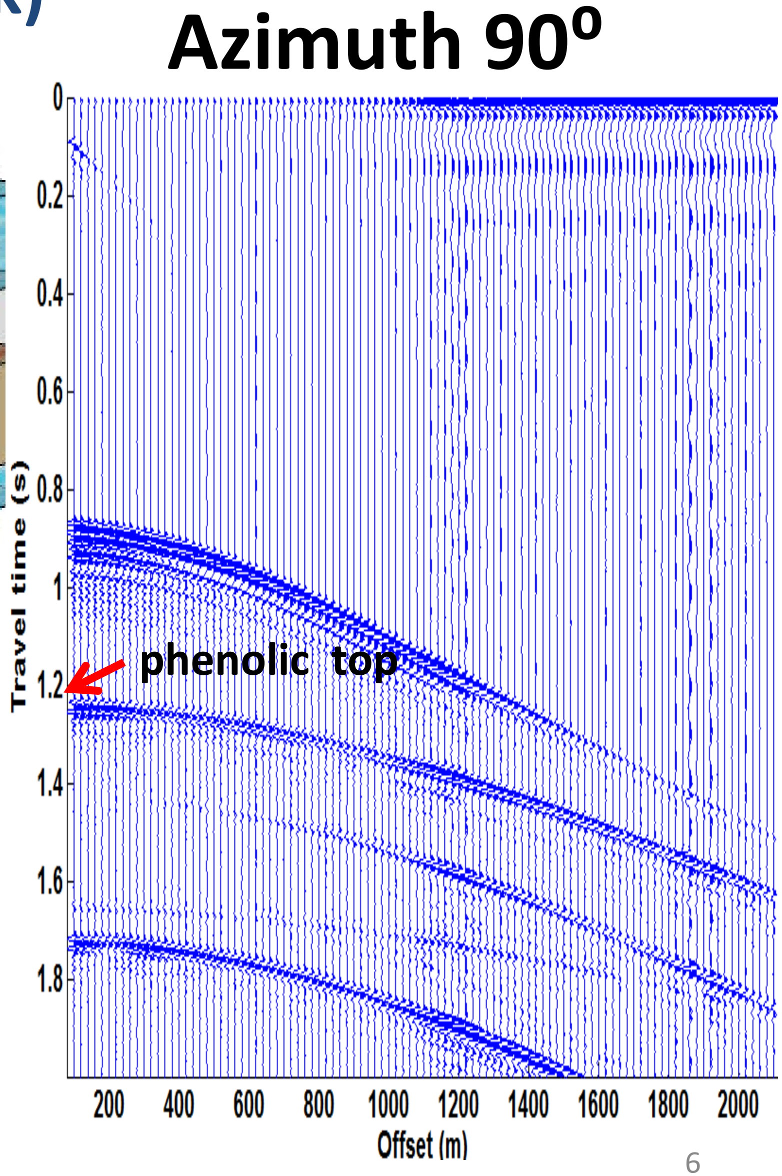
- Transmission shot gathers on single layer
- Traveltime inversion
- True ($\varepsilon^{(V)}$, $\delta^{(V)}$, γ)



Azimuthal AVO from reflection data

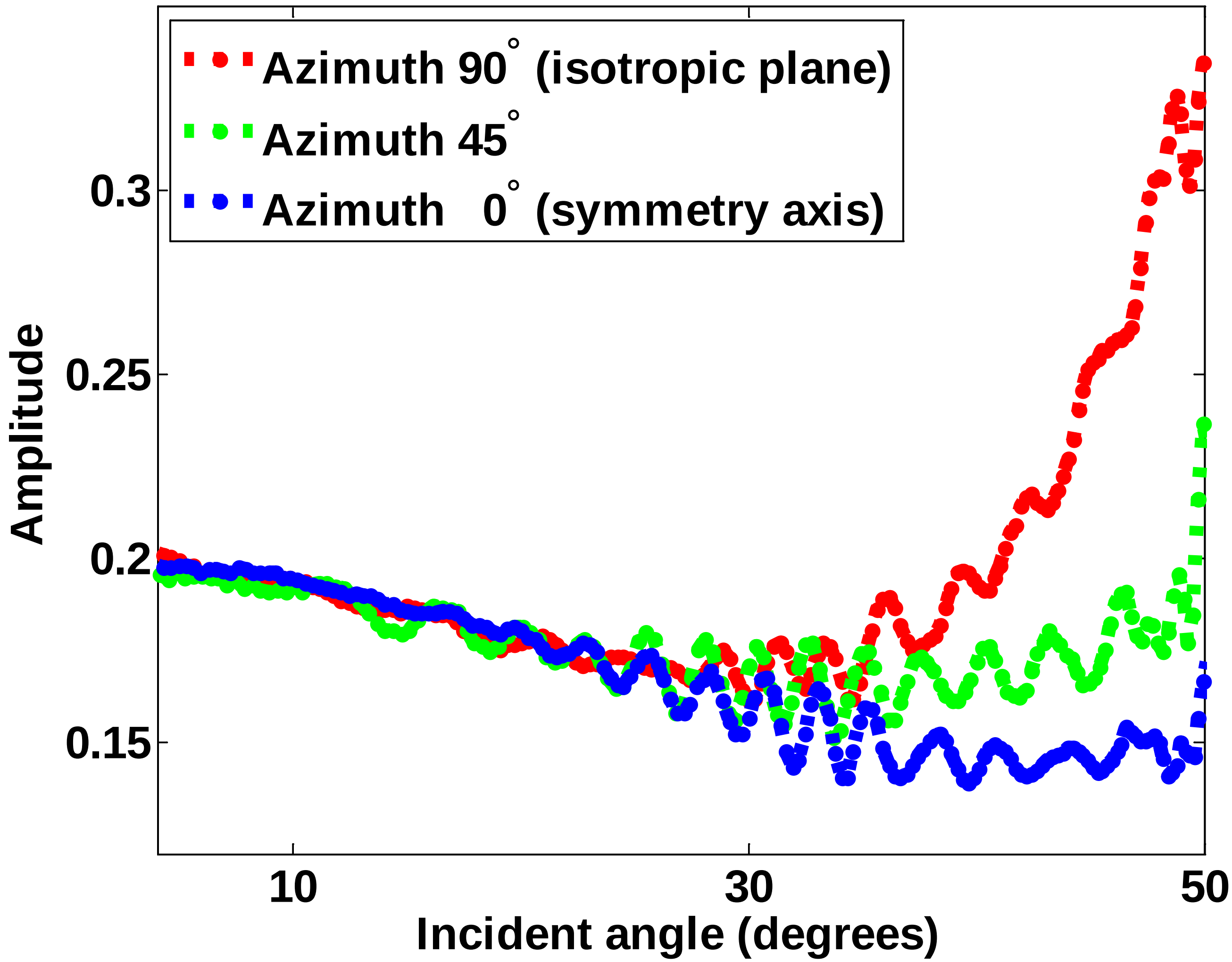
(2011 work)



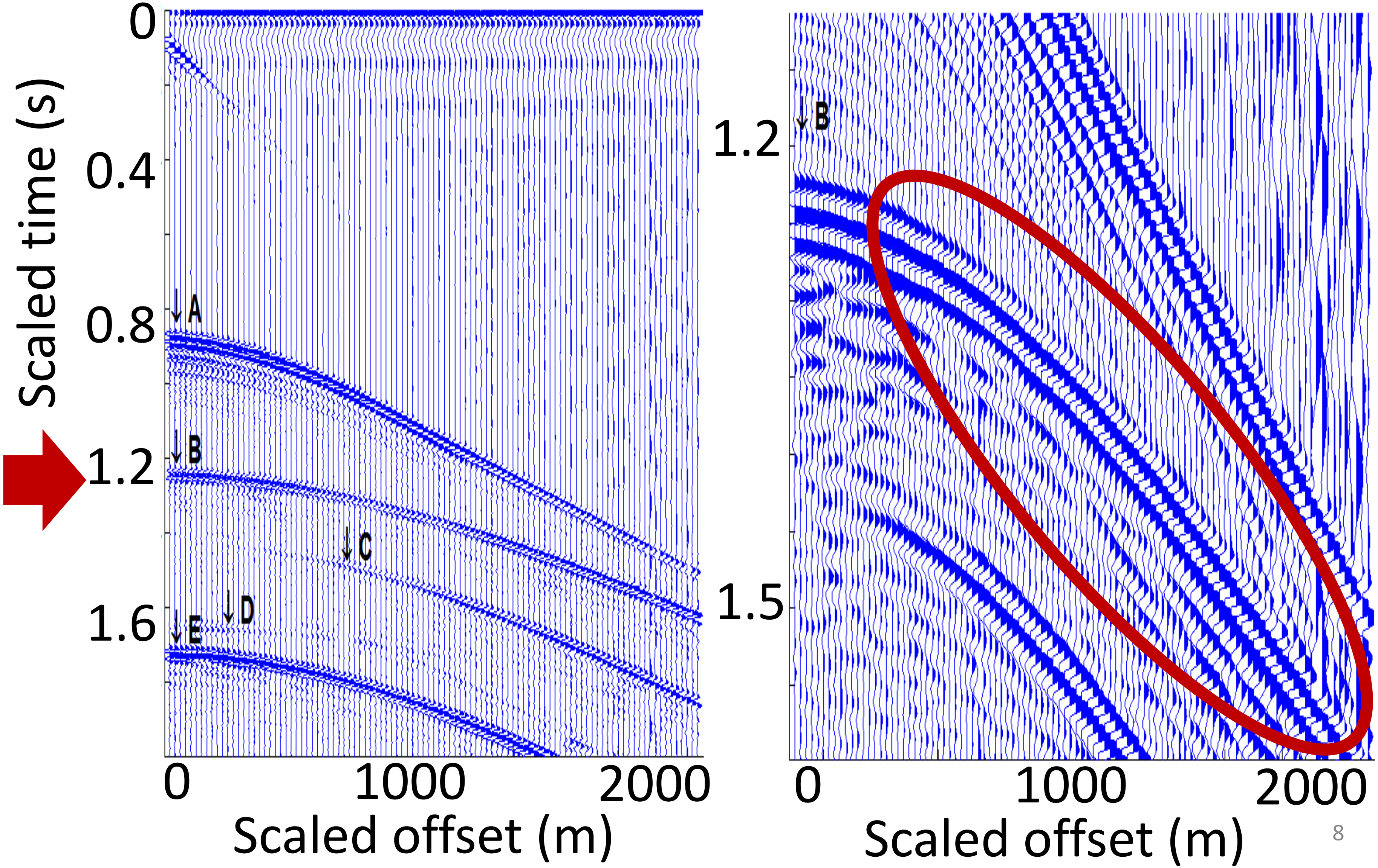


- Acquisition coordinate system along fracture system
- Azimuth lines: 0^o to 90^o
- Large offset data

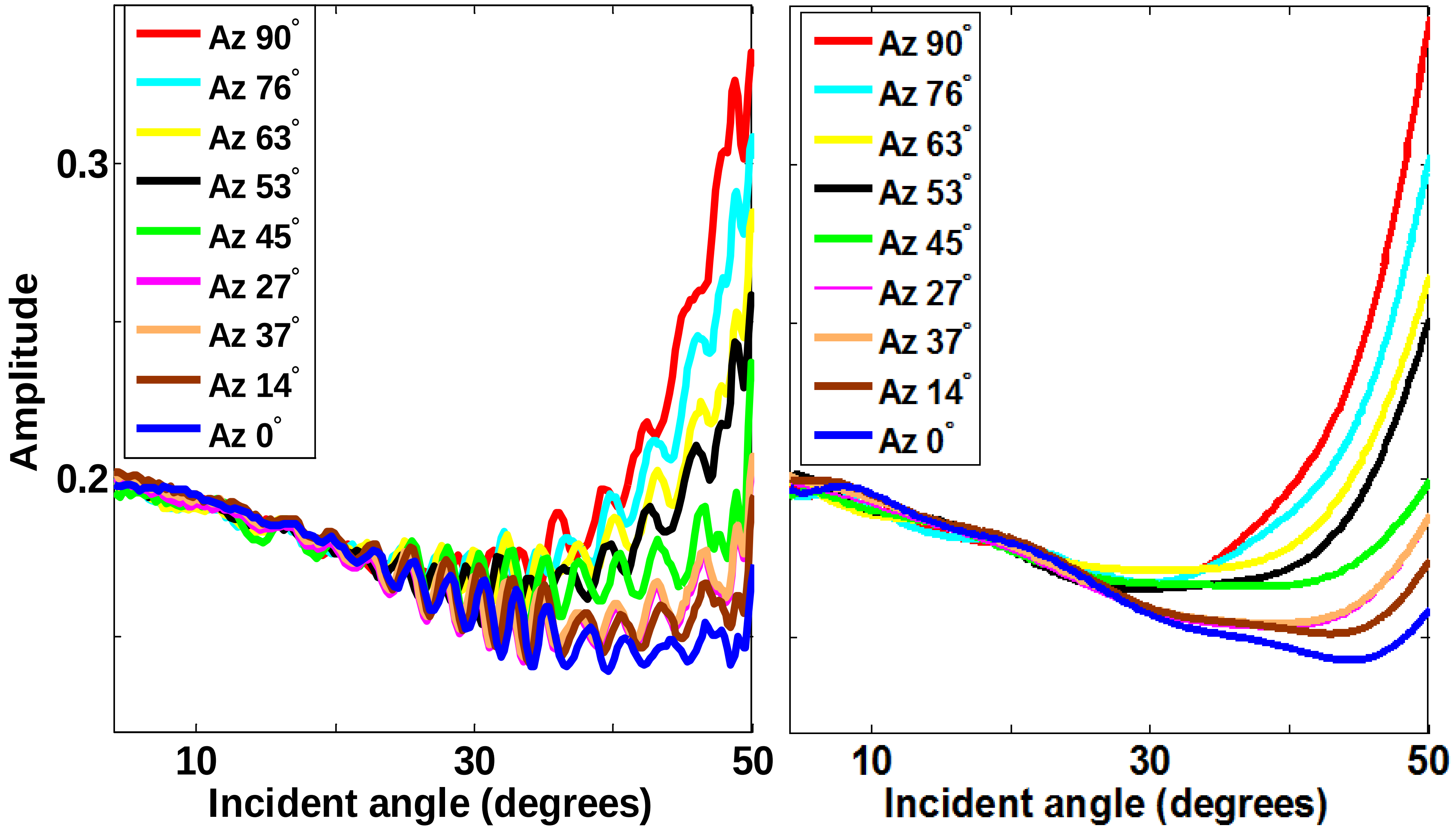
Amplitudes top fracture
corrected amplitudes



Oscillations on amplitude data

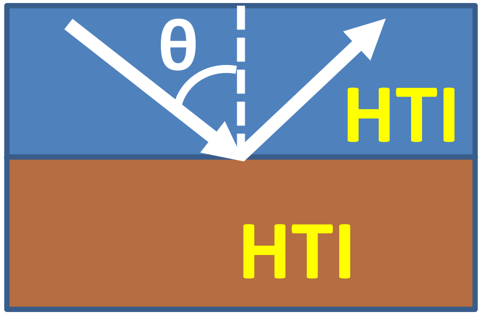


Smoothing the amplitude data



HTI: PP reflection coefficient (Rüger, 1997)

(known fracture orientation)



HTI: PP reflection coefficient (Rüger, 1997)
(known fracture orientation)

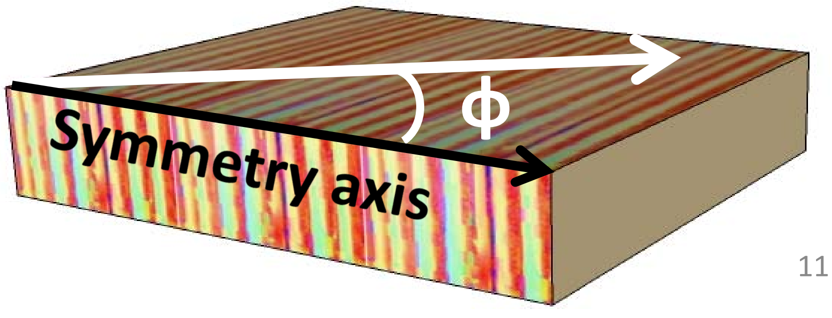
$$R_{PP}^{HTI}(\theta,\phi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} +$$

$$\frac{1}{2}\left(\cos^4\phi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon^{(V)} + \left(\frac{4\beta^2}{\alpha^2}\cos^2\phi\sin^2\theta\right)\Delta\gamma +$$

$$\frac{1}{2}\left(\cos^2\phi\sin^2\theta+\cos^2\phi\sin^2\phi\sin^2\theta\tan^2\theta\right)\Delta\delta^{(V)}$$

θ : incident angle

ϕ: angle between source-receiver azimuth and fracture symmetry axis



Rüger's approximation

Aki and Richard approximation

$$R_{PP}^{HTI}(\theta,\varphi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} +$$

azimuthal dependent terms

$$\frac{1}{2}\left(\cos^4\phi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon^{(V)} + \left(\frac{4\beta^2}{\alpha^2}\cos^2\phi\sin^2\theta\right)\Delta\gamma +$$

$$\frac{1}{2}\left(\cos^2\phi\sin^2\theta + \cos^2\phi\sin^2\phi\sin^2\theta\tan^2\theta\right)\Delta\delta^{(V)}$$

$$R \cong A\frac{\Delta\alpha}{\bar{\alpha}} + B\frac{\Delta\beta}{\bar{\beta}} + C\frac{\Delta\rho}{\bar{\rho}} + D\Delta\varepsilon^{(V)} + \mathrm{E}\Delta\delta^{(V)} + F\Delta\gamma$$

At each offset, ray tracing using the overburden velocity model to obtain A, B, C, D, E, and F.

AVAZ inversion for six parameters

Azimuth ϕ_1

Azimuth ϕ_m

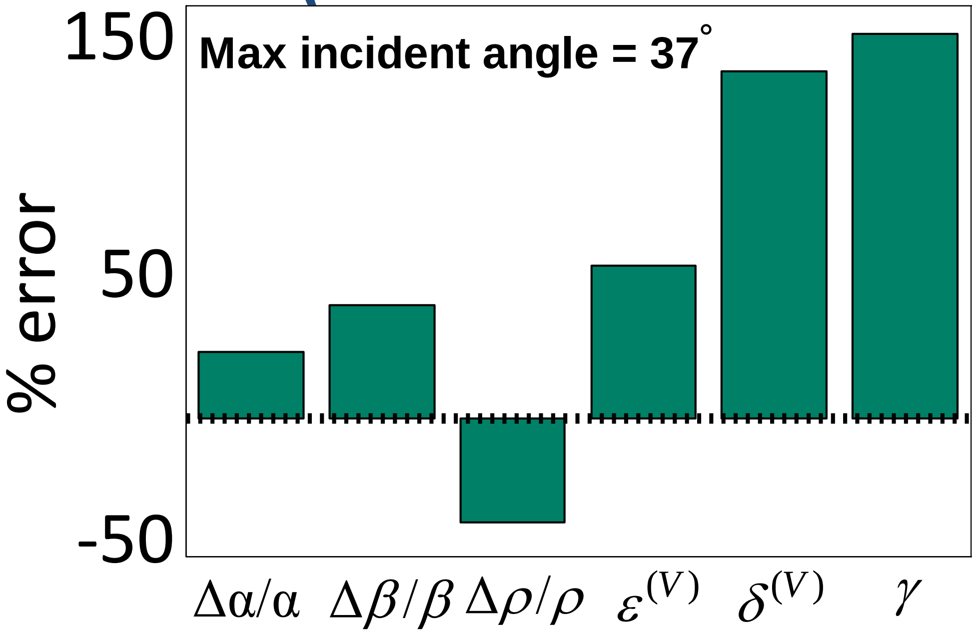
$$
\begin{bmatrix}
A_{1\varphi_1} & B_{1\varphi_1} & C_{1\varphi_1} & D_{1\varphi_1} & E_{1\varphi_1} & F_{1\varphi_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
A_{n\varphi_1} & B_{n\varphi_1} & C_{n\varphi_1} & D_{n\varphi_1} & E_{n\varphi_1} & F_{n\varphi_1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
A_{1\varphi_m} & B_{1\varphi_m} & C_{1\varphi_m} & D_{1\varphi_m} & E_{1\varphi_m} & F_{1\varphi_m} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
A_{n\varphi_m} & B_{n\varphi_m} & C_{n\varphi_m} & D_{n\varphi_m} & E_{n\varphi_m} & F_{n\varphi_m}
\end{bmatrix}_{(nm\times 6)}
\begin{bmatrix}
\Delta\alpha/\alpha \\
\Delta\beta/\beta \\
\Delta\rho/\rho \\
\Delta\varepsilon^{(V)} \\
\Delta\delta^{(V)} \\
\Delta\gamma
\end{bmatrix}_{(6\times 1)}
=
\begin{bmatrix}
R_{11} \\ \vdots \\ R_{n1} \\ \vdots \\ \vdots \\ R_{1m} \\ \vdots \\ R_{nm}
\end{bmatrix}_{(nm\times 1)}
$$

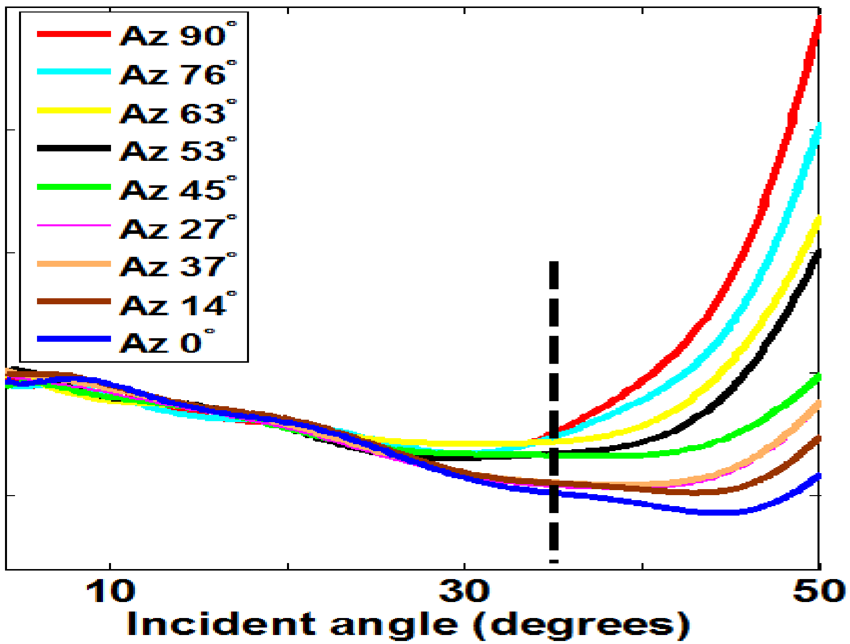
G ***m*** ***d***

$$m_{est} = (G^T G + \mu)^{-1} G^T d$$

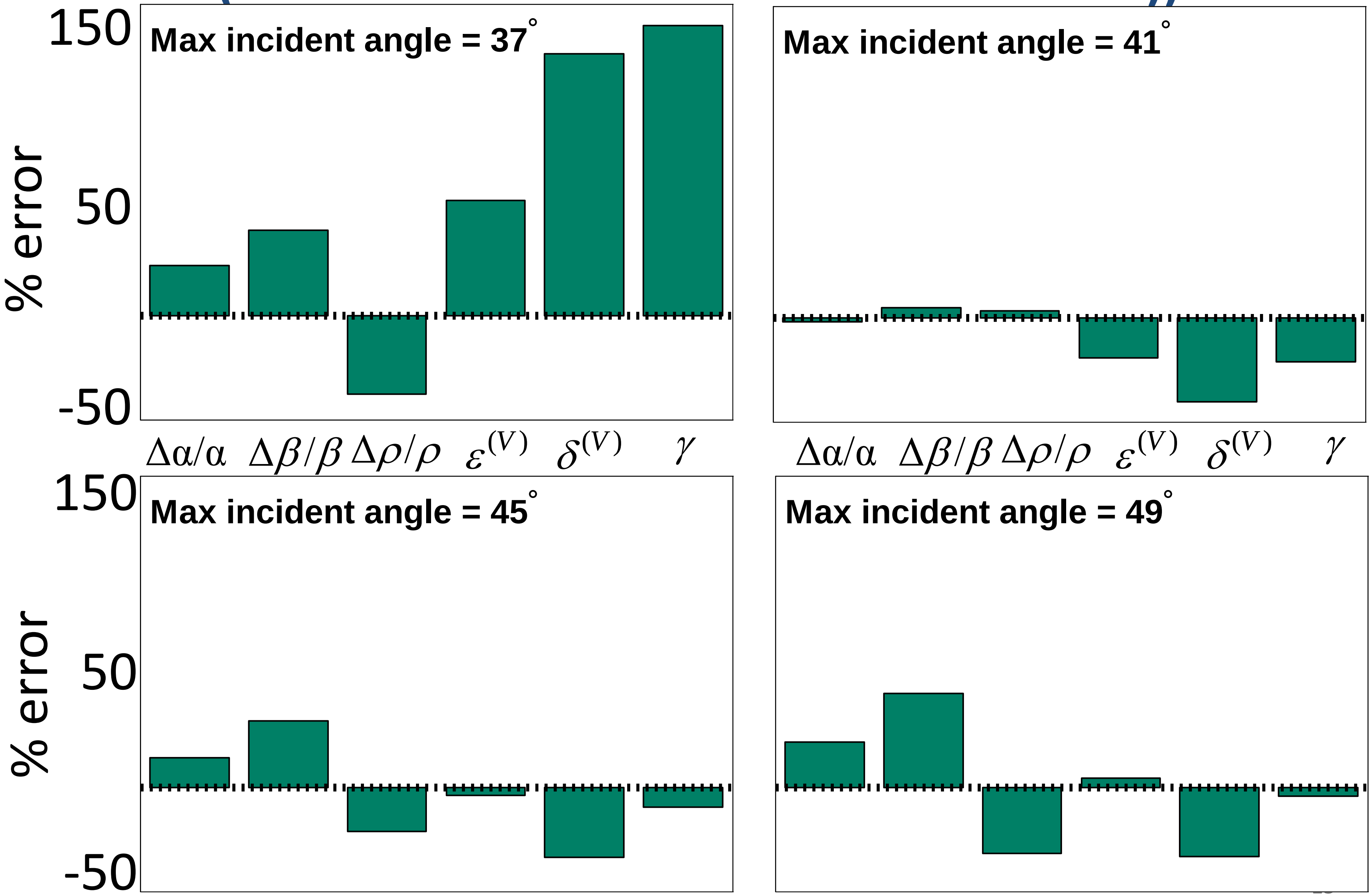
AVAZ inversion for six parameters

(errors WRT traveltime inversion results)





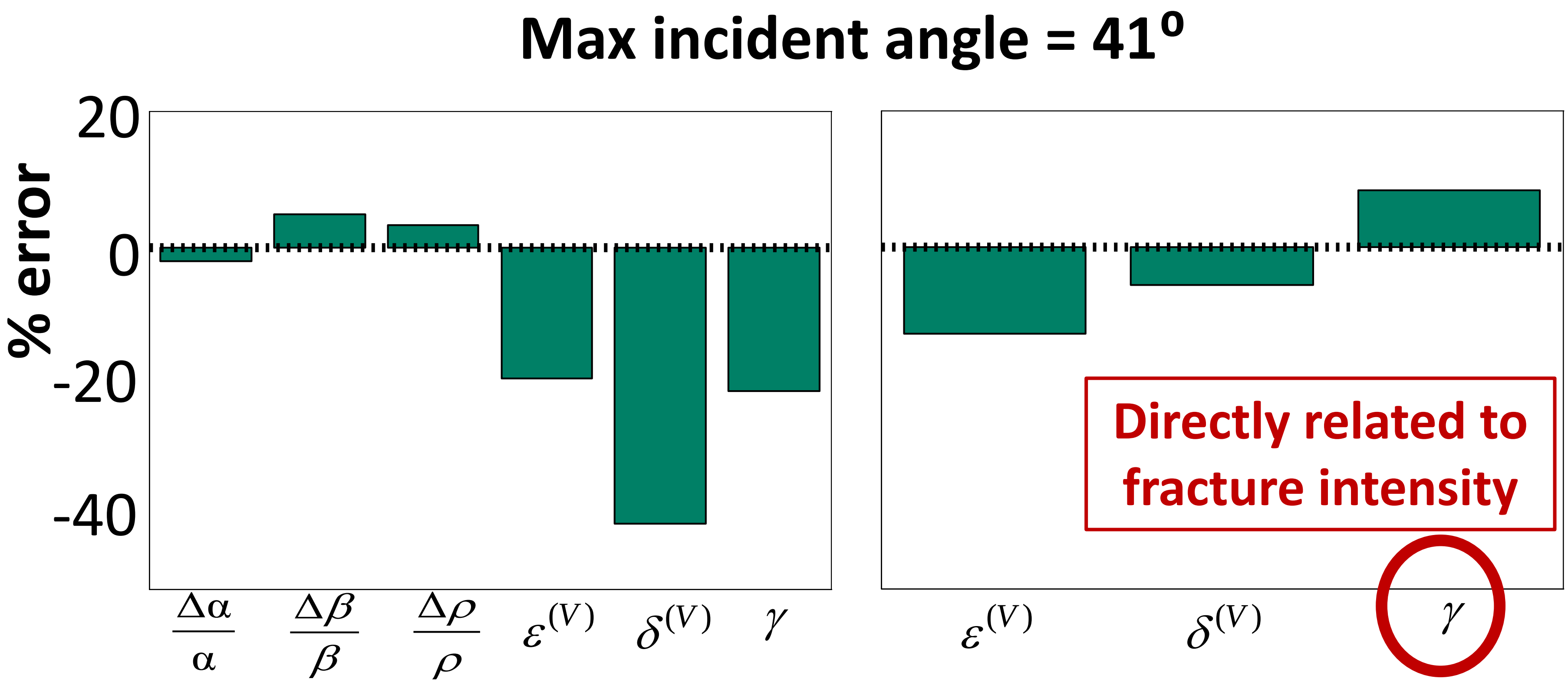
AVAZ inversion for six parameters
(errors WRT traveltime inversion results))



AVAZ inversion for three-parameters
(anisotropy parameters)

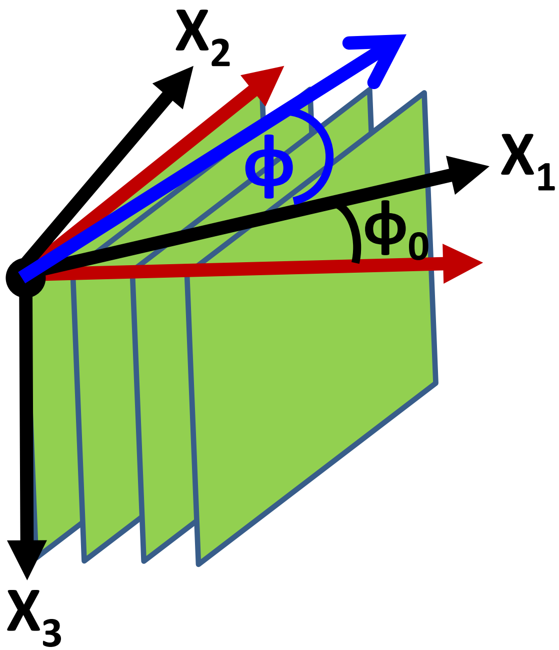
1. Determine ($\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$)
   from logs, or
   from conventional AVA inversion of
   isotropic plane direction.
2. Invert for ($\Delta\varepsilon^{(V)}$, $\Delta\delta^{(V)}$, $\Delta\gamma$)
   using Rüger's equation
   constrained by results from step 1.

AVAZ inversion
6-parameter vs. 3-parameter



Favourable results compared to those obtained previously by traveltime inversion

Fracture symmetry axis not known



- Red line: fracture system
- Black line: acquisition coordinate
- ϕ : source-receiver azimuth
- ϕ_0 : fracture symmetry direction

$$
\begin{aligned}
R_{PP}^{HTI}(\theta,\phi) \cong\ & \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} + \\
& \frac{1}{2}\left(\cos^4(\phi-\phi_0)\sin^2\theta\tan^2\theta\right)\Delta\varepsilon + \left(\frac{4\beta^2}{\alpha^2}\cos^2(\phi-\phi_0)\sin^2\theta\right)\Delta\gamma + \\
& \frac{1}{2}\left(\cos^2(\phi-\phi_0)\sin^2\theta + \cos^2(\phi-\phi_0)\sin^2(\phi-\phi_0)\sin^2\theta\tan^2\theta\right)\Delta\delta
\end{aligned}
$$

HTI: PP reflection coefficient

Small incident angle (θ < 35°)

$$R_{PP}^{HTI}(\theta,\phi) \cong \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\bar{\alpha}} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\bar{\beta}} + \frac{1}{2}\left(1-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\bar{\rho}} +$$

$$+\left(\frac{4\beta^2}{\alpha^2}\Delta\gamma + \frac{1}{2}\Delta\delta^{(V)}\right)\cos^2(\phi-\phi_0)\sin^2\theta$$

AVO intercept

Q = AVO gradient

$$R_{PP}^{HTI}(\theta,\varphi) \cong I + \left[G_1 + G_2\cos^2(\phi-\phi_0)\right]\sin^2\theta$$

Isotropic gradient

Anisotropic gradient

$$G_2 = \frac{4\beta^2}{\alpha^2}\Delta\gamma + \frac{1}{2}\Delta\delta^{(V)}$$

gradient non-linear with respect to (G_1,G_2, ϕ_0)

Estimate fracture orientation

(Grechka and Tsvankin (1998); Jenner (2002))

$$Q = G_1 + G_2 \cos^2(\varphi - \varphi_0)$$
$$= (G_1 + G_2)\cos^2(\varphi - \varphi_0) + G_1 \sin^2(\varphi - \varphi_0)$$
$$= W_{11}\cos^2\phi + 2W_{12}\cos\phi\sin\phi + W_{22}\sin^2\phi$$

$$Q = (G_1 + G_2)y_1^2 + G_1 y_2^2$$

Coordinate system aligned with fractures

$$\begin{cases} y_1 = r\cos(\phi - \phi_0) \\ y_2 = r\sin(\phi - \phi_0) \end{cases}$$

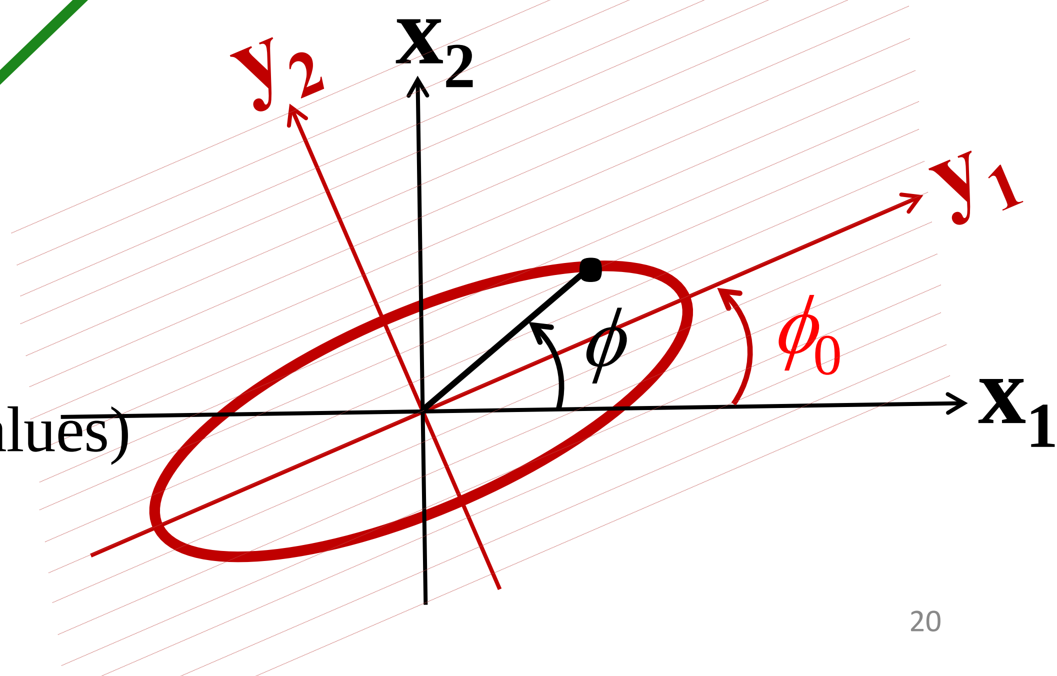
Acquisition coordinate system

$$\begin{cases} x_1 = r\cos\phi \\ x_2 = r\sin\phi \end{cases}$$

$$Q = W_{11}x_1^2 + 2W_{12}x_1x_2 + W_{22}x_2^2$$
$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Q = \lambda_1 y_1^2 + \lambda_2 y_2^2 \quad (\lambda_{1,2} : eigenvalues)$$

$$\tan(2\phi_0) = \frac{2W_{12}}{W_{11} - W_{22}}$$



Estimate fracture orientation

(Grechka and Tsvankin (1998); Jenner (2002))

$$\begin{cases} \phi_0^{(1)} = \tan^{-1}\left(\dfrac{W_{11} - W_{22} + \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}}{2W_{12}} \right) \\ \phi_0^{(2)} = \tan^{-1}\left(\dfrac{W_{11} - W_{22} - \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}}{2W_{12}} \right) \end{cases}$$

$$\phi_0^{(2)} = \phi_0^{(1)} + \frac{\pi}{2}$$

Accurate prediction of fracture orientation requires extra geological info, or azimuthal NMO velocity.

AVAZ inversion for fracture orientation
(small incident angle, θ < 35°)

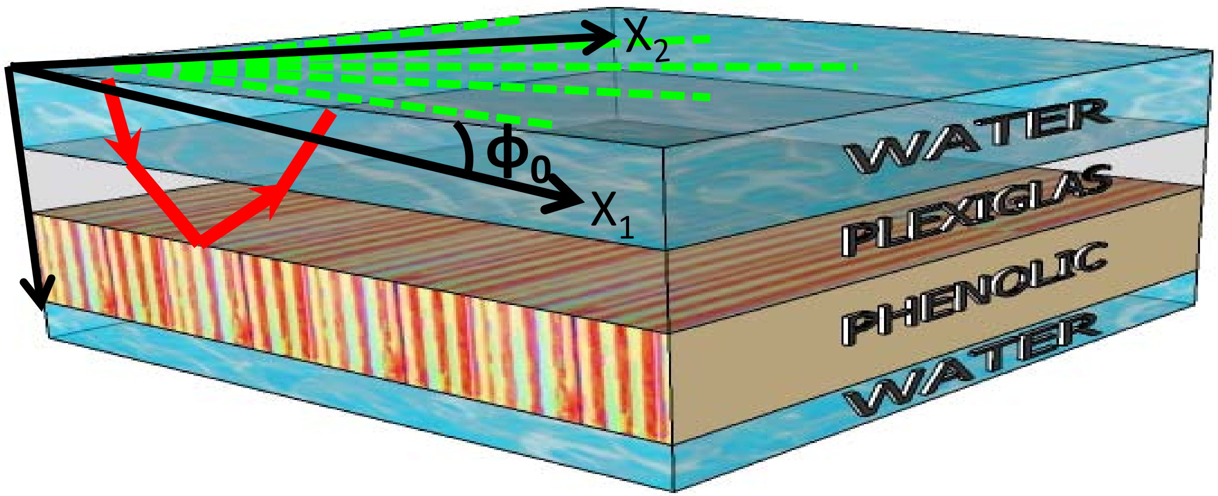
$$R(\theta,\phi) = I + \left(W_{11}\cos^2\phi + 2W_{12}\cos\phi\sin\phi + W_{22}\sin^2\phi\right)\sin^2\theta$$

$$\begin{bmatrix} \cos^2\phi_1\sin^2\theta_{11} & 2\cos\phi_1\sin\phi_1\sin^2\theta_{11} & \sin^2\phi_1\sin^2\theta_{11} \\ \vdots & \vdots & \vdots \\ \cos^2\phi_1\sin^2\theta_{n1} & 2\cos\phi_1\sin\phi_1\sin^2\theta_{n1} & \sin^2\phi_1\sin^2\theta_{n1} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \cos^2\phi_m\sin^2\theta_{1m} & 2\cos\phi_m\sin\phi_m\sin^2\theta_{1m} & \sin^2\phi_m\sin^2\theta_{1m} \\ \vdots & \vdots & \vdots \\ \cos^2\phi_m\sin^2\theta_{nm} & 2\cos\phi_m\sin\phi_m\sin^2\theta_{1m} & \sin^2\phi_m\sin^2\theta_{1m} \end{bmatrix}_{(nm\times 3)} \begin{bmatrix} W_{11} \\ W_{12} \\ W_{22} \end{bmatrix}_{(3\times 1)} = \begin{bmatrix} R_{11}-I \\ \vdots \\ R_{n1}-I \\ \vdots \\ \vdots \\ R_{1m}-I \\ \vdots \\ R_{nm}-I \end{bmatrix}_{(nm\times 1)}$$

Least squares inversion for (W_{11}, W_{22}, W_{33})

Test on physical model data, AVAZ inversion
ϕ_0 = 30°

ϕ_0: fracture symmetry axis azimuth



True ϕ_0	30°
Estimate ϕ_0	28.5° and 118.5°

Testing different fracture orientations

ϕ_0: fracture symmetry axis azimuth

True ϕ_0	0^o	10^o	20^o	40^o	60^o	80^o	90^o
Estimate ϕ_0	-1.5^o 88.5^o	8.5^o 98.5^o	18.5^o 108.5^o	38.5^o 128.5^o	58.5^o 148.5^o	78.5^o 168.5^o	88.5^o 178.5^o

Conclusions

- Rüger equation for HTI, PP reflection coefficient, can be used in an inversion for anisotropy parameters, but fracture orientation must be known.
- Knowing the fracture orientation, fracture intensity can be estimated from AVAZ inversion of large-offset data .
- Fracture orientation can be determined from AVAZ inversion of small incident angle data, but with 90^{o} ambiguity.
- Implementation on physical model data gives results consistent with these theories.

Acknowledgments

- CREWES sponsors
- Dr. Joe Wong
- Xinxiang Li
- Dr. P.F. Daley
- Dr. Kris Innanen
- David Henley
- Marcus Wilson
- Mahdi Al-Mutlaq