Grid scaling 2-D acoustic full-waveform inversion

Vladimir Zubov

Michael Lamoureux

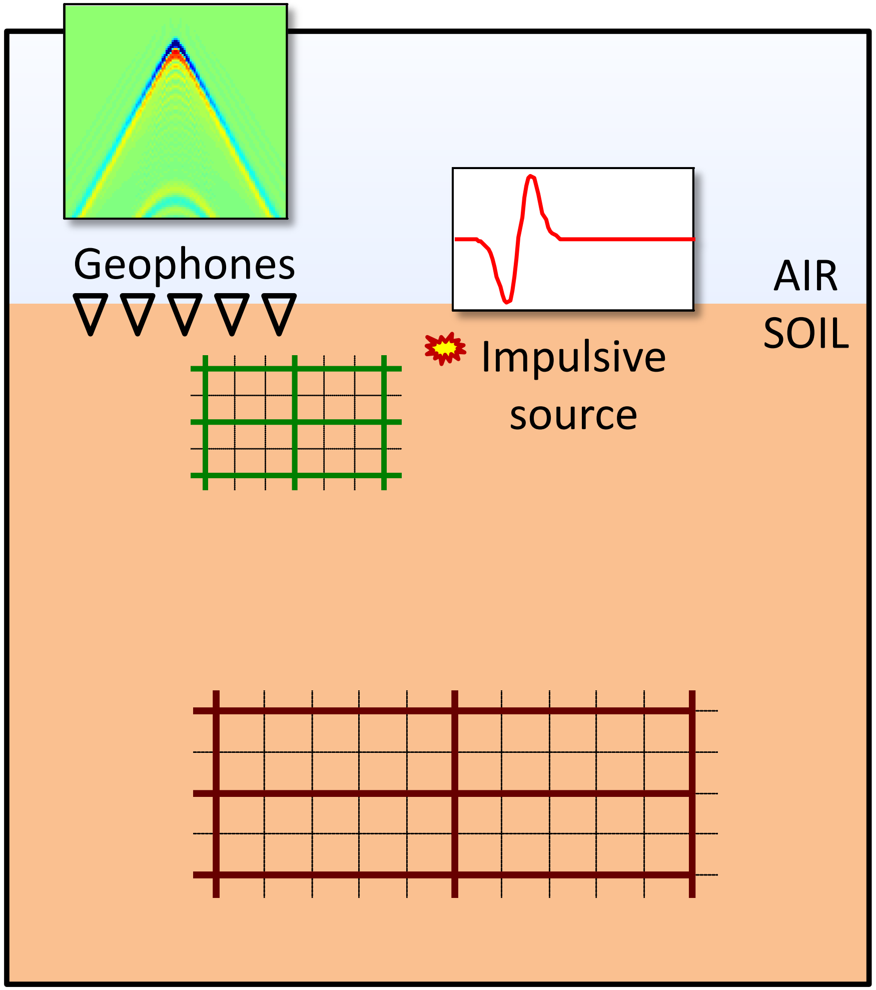
Gary Margrave

Outline

- Mathematical model
  - Acoustic equation
  - Inversion algorithm
- Multiscaling approach
- Domain decomposition
- Numerical experiments
- Conclusion

Mathematical problem

- FWI problem
  - Forward propagation
  - Back propagation
  - Source estimation
  - Residual minimizing
- Grid scaling
- Domain decomposition

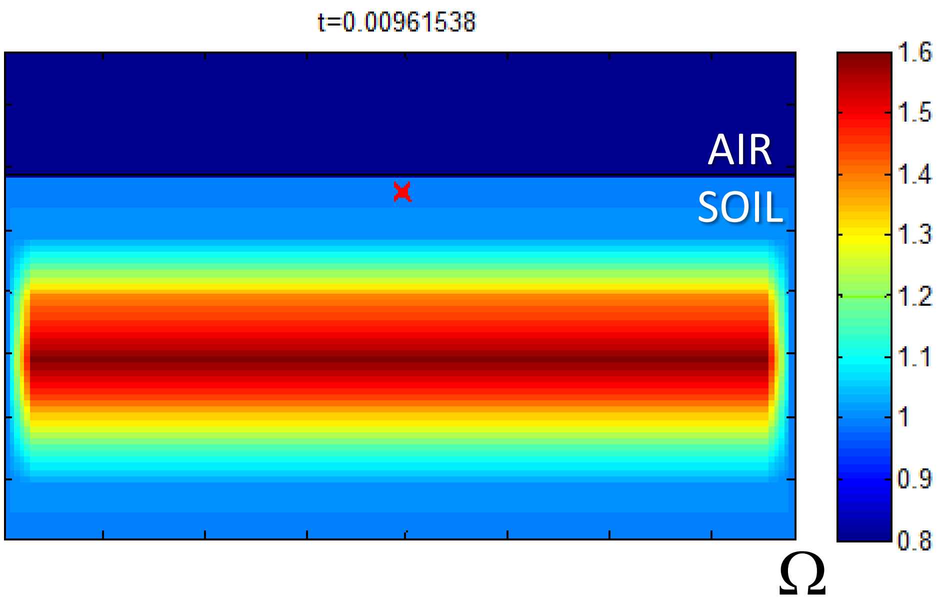


Forward 2D acoustic problem

$$\begin{cases} u_{tt} = div\left(c^2 \nabla u\right) + f \\ u\big|_{t=0} = 0,\ u_t\big|_{t=0} = 0 \\ u\big|_{\partial\Omega} = 0 \end{cases}$$

where

$$\left(\vec{x}, t\right) \in \Omega \times [0, T]$$



$$f = \delta\left(x - x_0\right) \cdot e^{-[\lambda(t - t_0)]^2} \sin \omega\left(t - t_0\right)$$

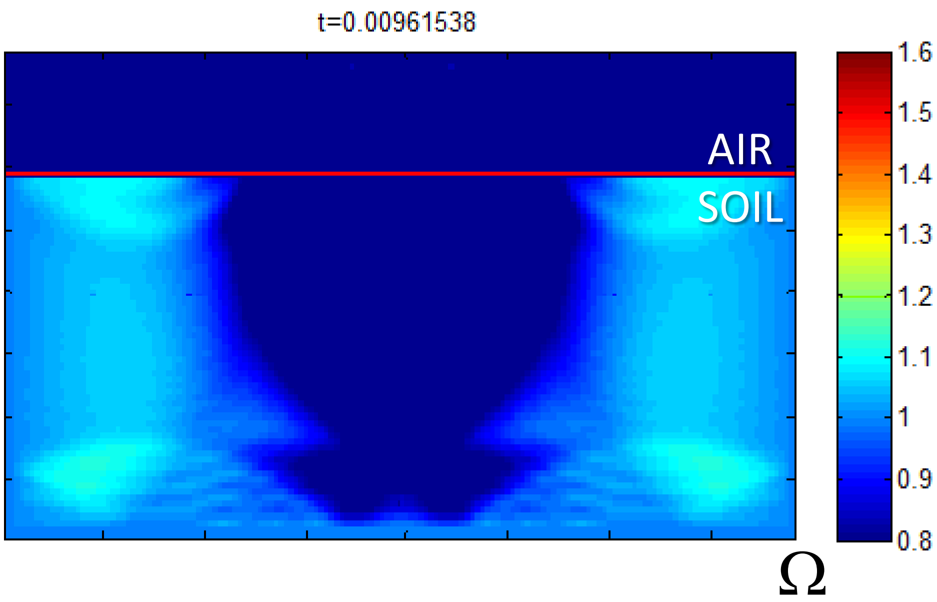
Adjoint 2D acoustic problem

$$\begin{cases} \varphi_{tt} = div\left(c^2 \nabla \varphi\right) + g \\ \varphi\big|_{t=T} = 0,\ \varphi_t\big|_{t=T} = 0 \\ \varphi\big|_{\partial\Omega} = 0 \end{cases}$$

where

$$\left(\vec{x}, t\right) \in \Omega \times [0, T]$$

$$g = \begin{cases} \Delta d = d_{obs} - d_{cal}, \quad t < t_0 \\ 0,\ t \geq t_0 \end{cases}$$



Finite-difference approximation

$$u_{tt} = \underbrace{\left(k \cdot u_x\right)_x}_{\Lambda_x u} + \underbrace{\left(k \cdot u_y\right)_y}_{\Lambda_y u} + f$$

Factorization scheme

$$\left(I - \tau^2 \sigma \Lambda_x\right)\left(I - \tau^2 \sigma \Lambda_y\right) u^{n+1} =$$

$$\left(2 + \tau^2 \left(1 - 2 \cdot \sigma\right)\left(\Lambda_x + \Lambda_y\right)\right) u^n - \left(1 - \tau^2 \sigma \left(\Lambda_x + \Lambda_y\right)\right) u^{n-1}$$

$$+ \tau^2 \sigma \cdot f^{n-1} + \tau^2 \left(1 - 2\sigma\right) \cdot f^n + \tau^2 \sigma \cdot f^{n+1}$$

Unconditional stability : $0.25 \le \sigma \le 0.5$

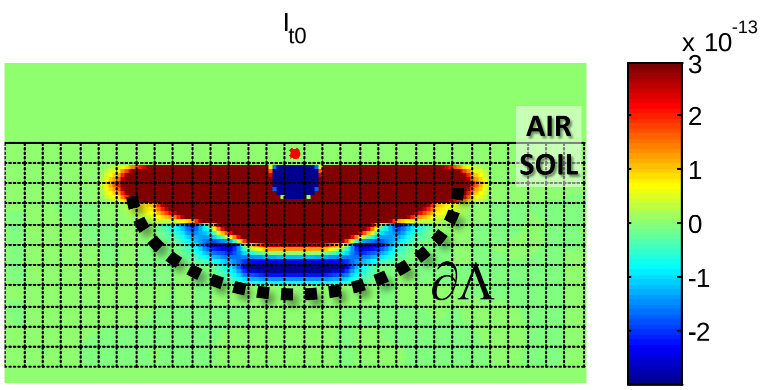
Kalitkin N.N. Numerical methods: Moscow, Nauka, 1978. – PP. 437-438

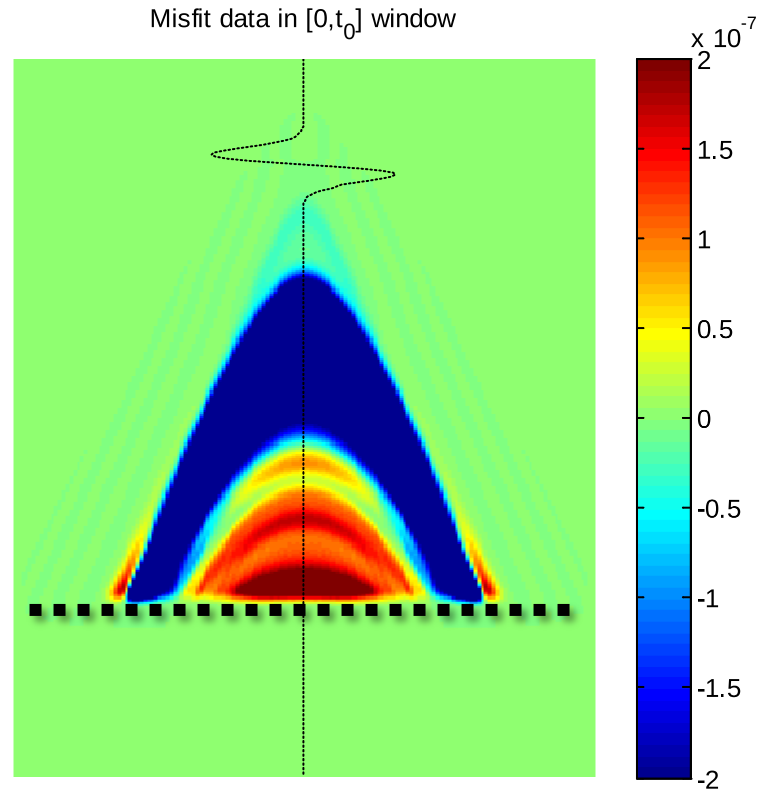
Minimization problem

Tarantola A. approach:

$$\frac{\partial \Delta d}{\partial c^2} \approx I_{t_0} = \int_0^{t_0} (\nabla u, \nabla \varphi) dt$$

$$I_{t_0}\Big|_{\partial \Lambda} = 0 \quad \text{or} \quad \min_{t_0,\, \partial\Lambda} \left\| I_{t_0} \right\|^2$$





Tarantola A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, 74, No. 8, 1259-1266.

Minimization strategy

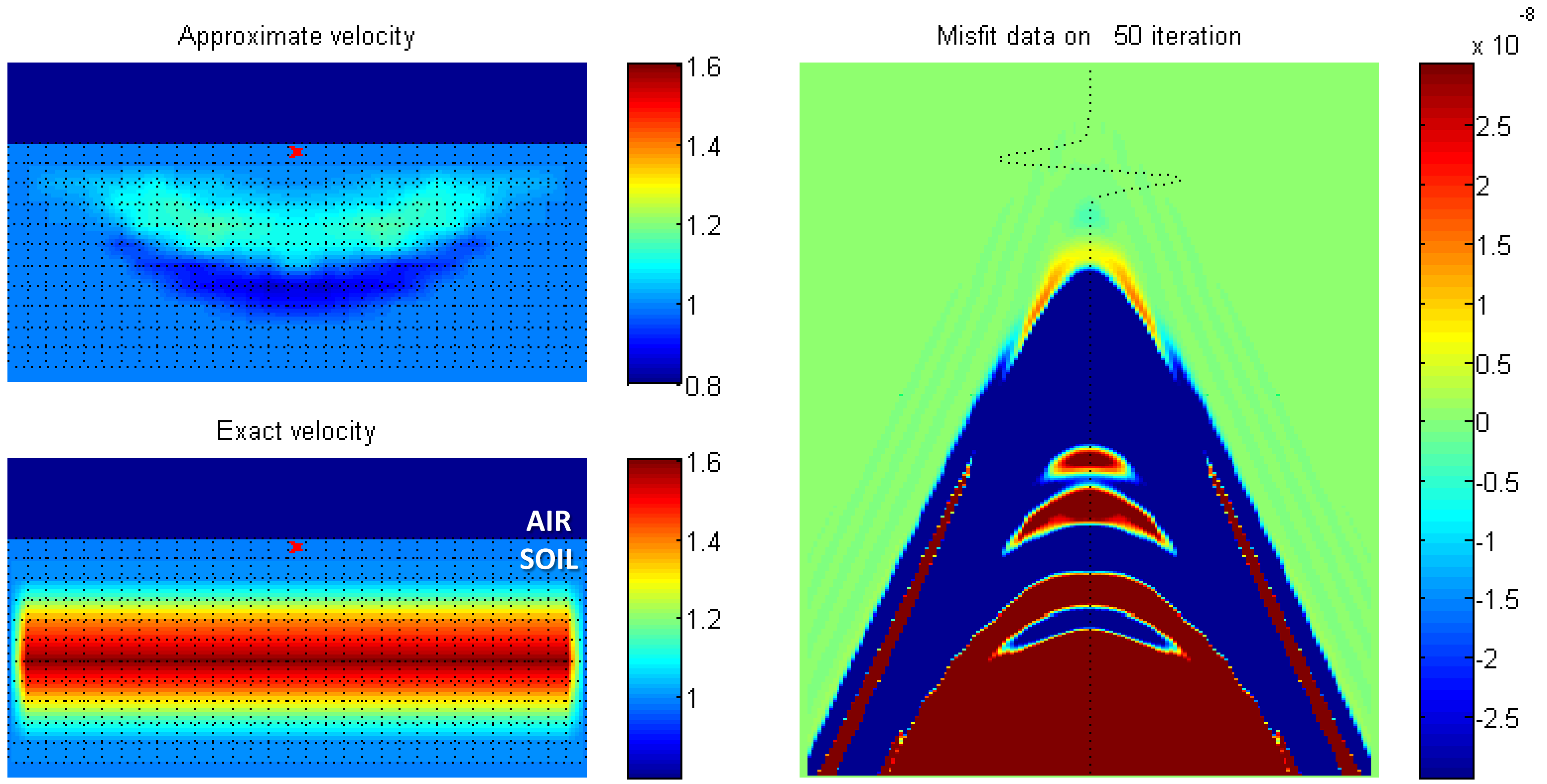
Root search: $I_{t_0}\Big|_{\partial\Lambda} = 0$

Newton method: $c^2_{New}\Big|_{\partial\Lambda} = c^2_{Old}\Big|_{\partial\Lambda} - \alpha\Big|_{\partial\Lambda}$

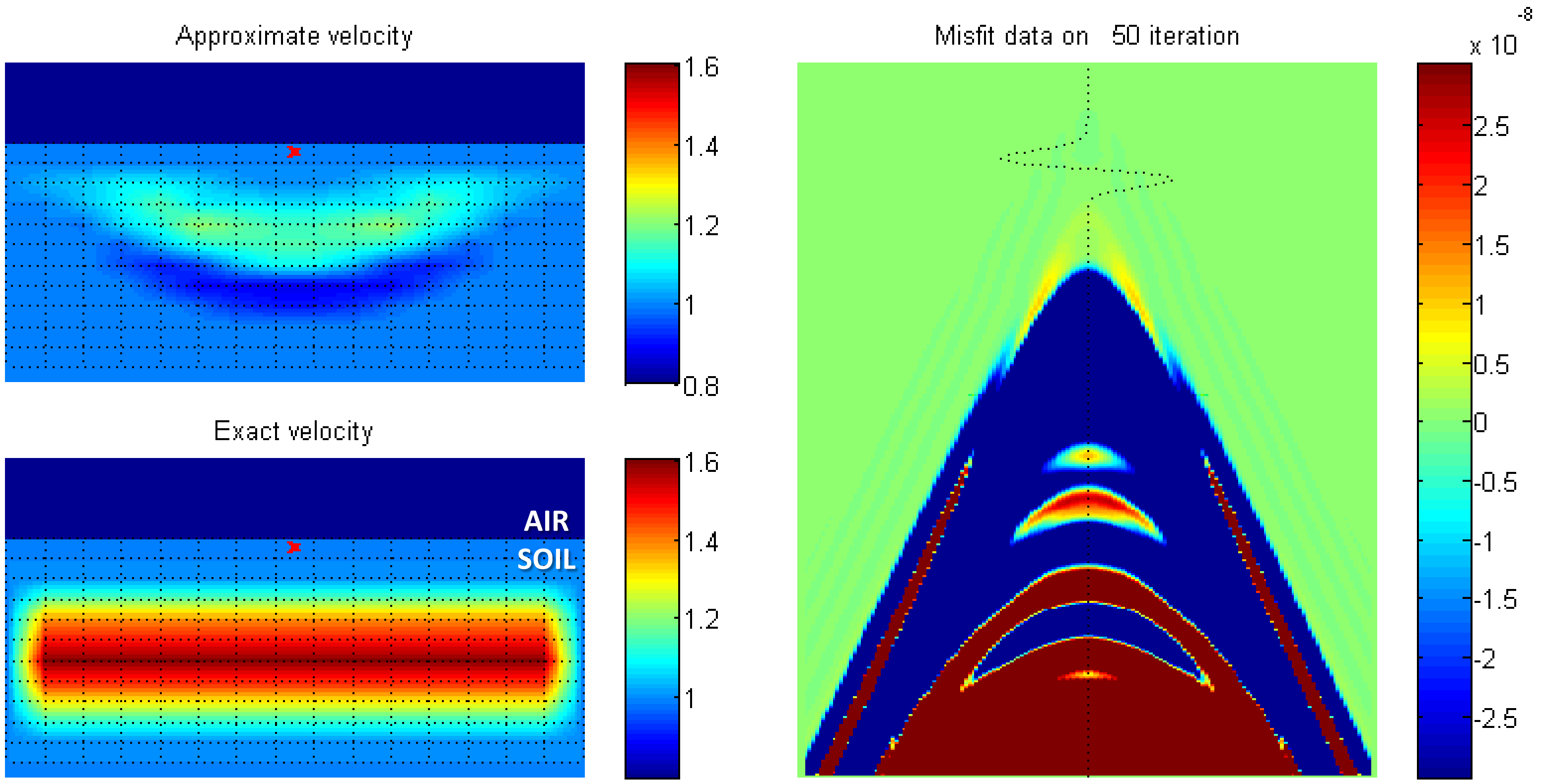
$$\left.\frac{\partial I_{t_0}}{\partial c^2}\right|_{\partial\Lambda} \approx \left.\frac{I_{t_0}\left(c^2_{Old} + \Delta\right) - I_{t_0}\left(c^2_{Old}\right)}{\Delta}\right|_{\partial\Lambda}, \ \alpha = \left(\frac{I_{t_0}}{\partial I_{t_0} / \partial c^2}\right)$$

Strong limitation: $|\alpha| < \varepsilon$

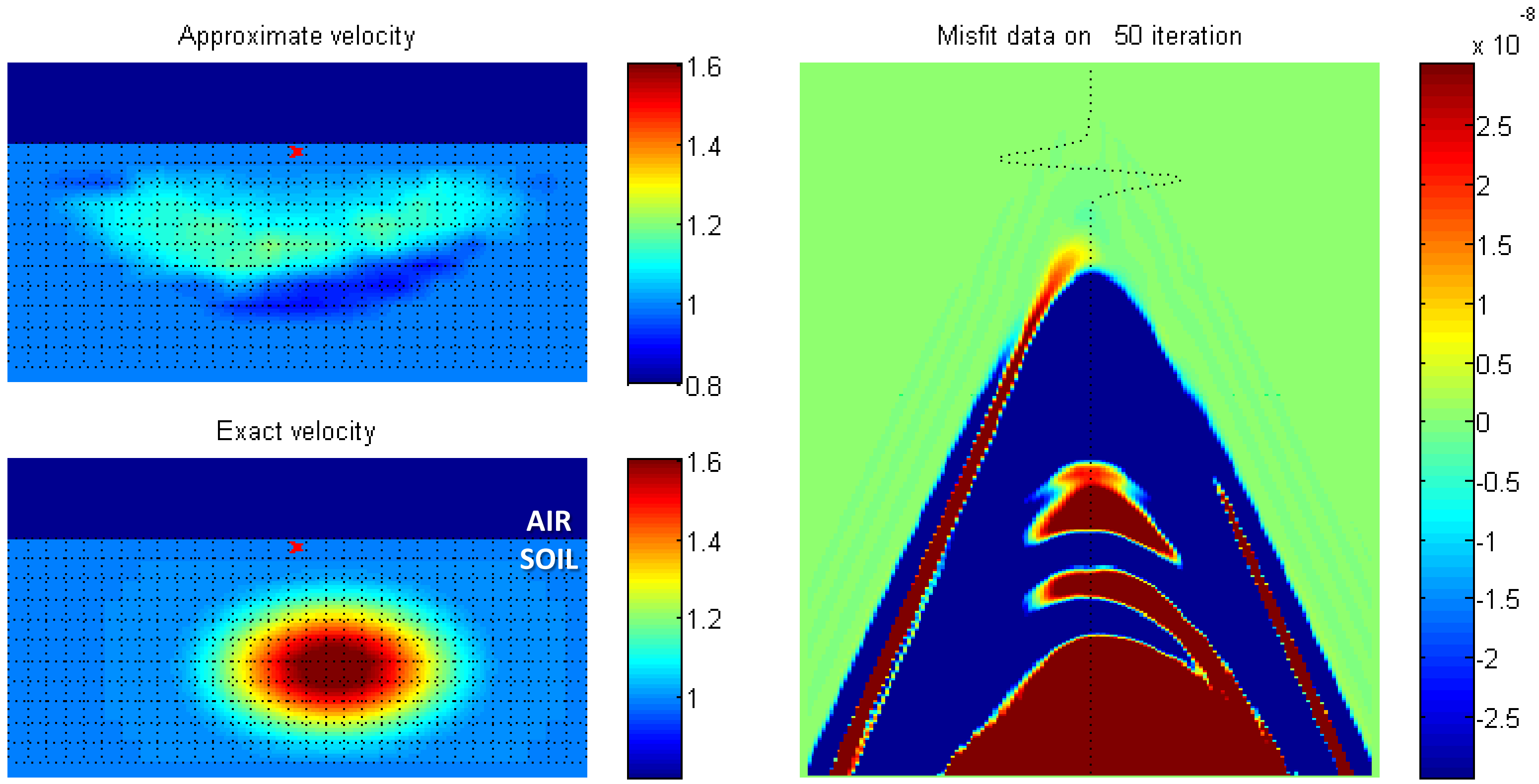
Horizontal layers
Scaling factor 5x5



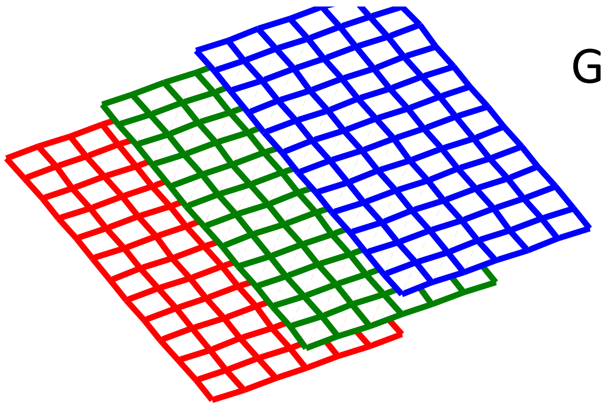
Horizontal layers
Scaling factor 11x5



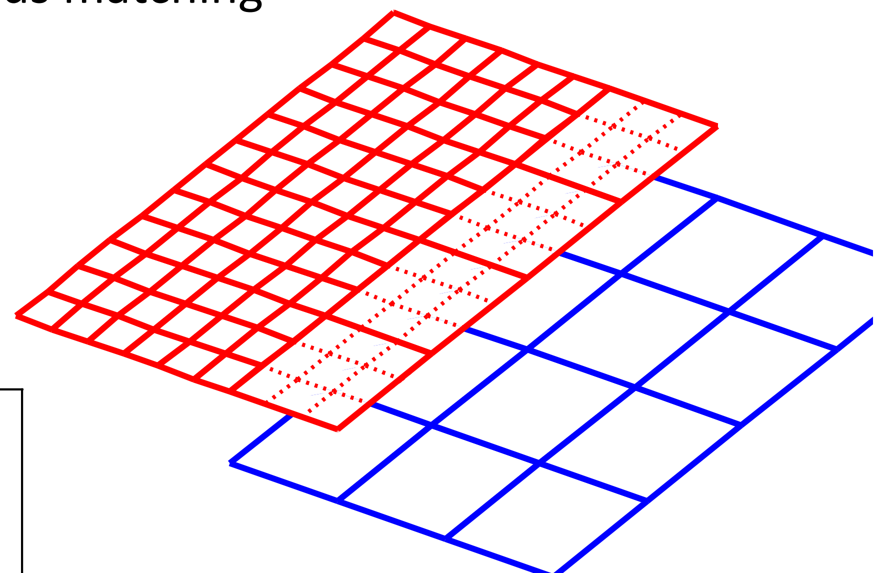
Elliptic reflector
Scaling factor 5x5



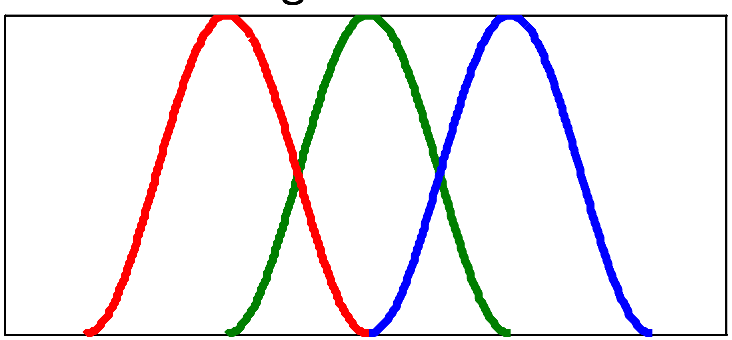
Domain decomposition



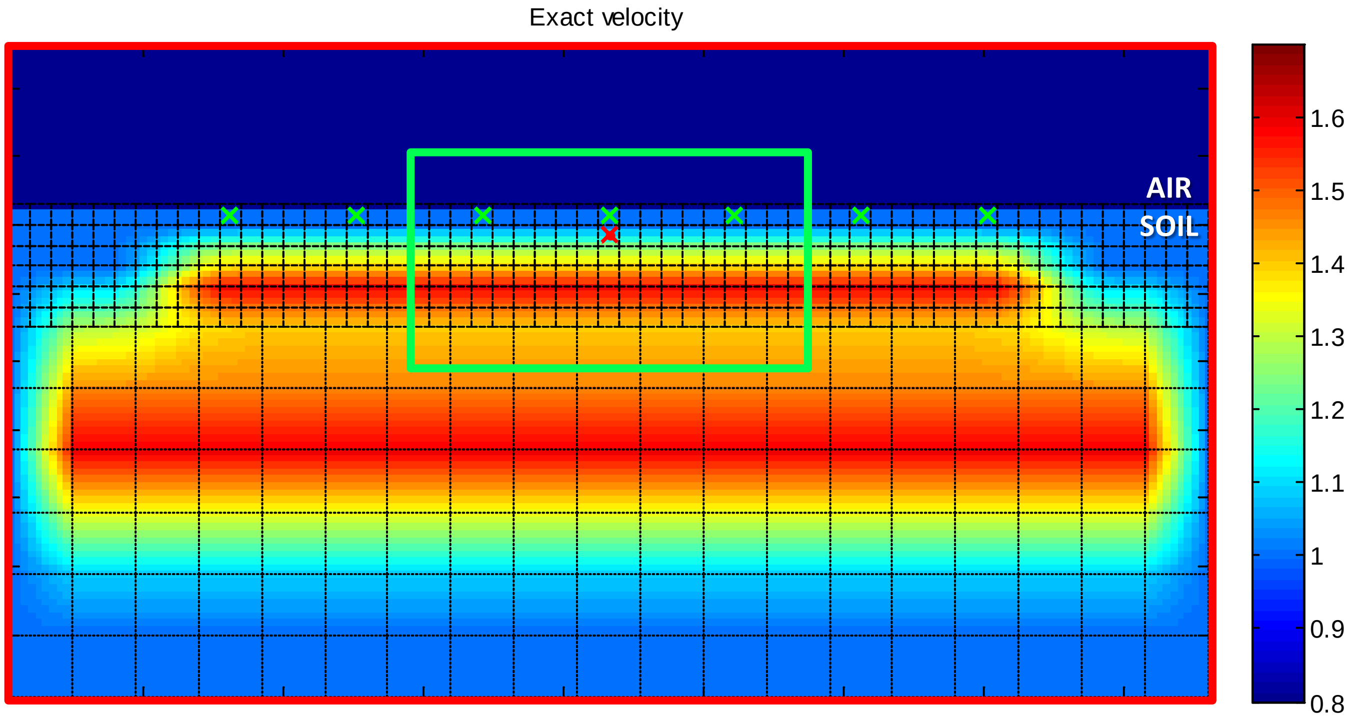
Grids matching



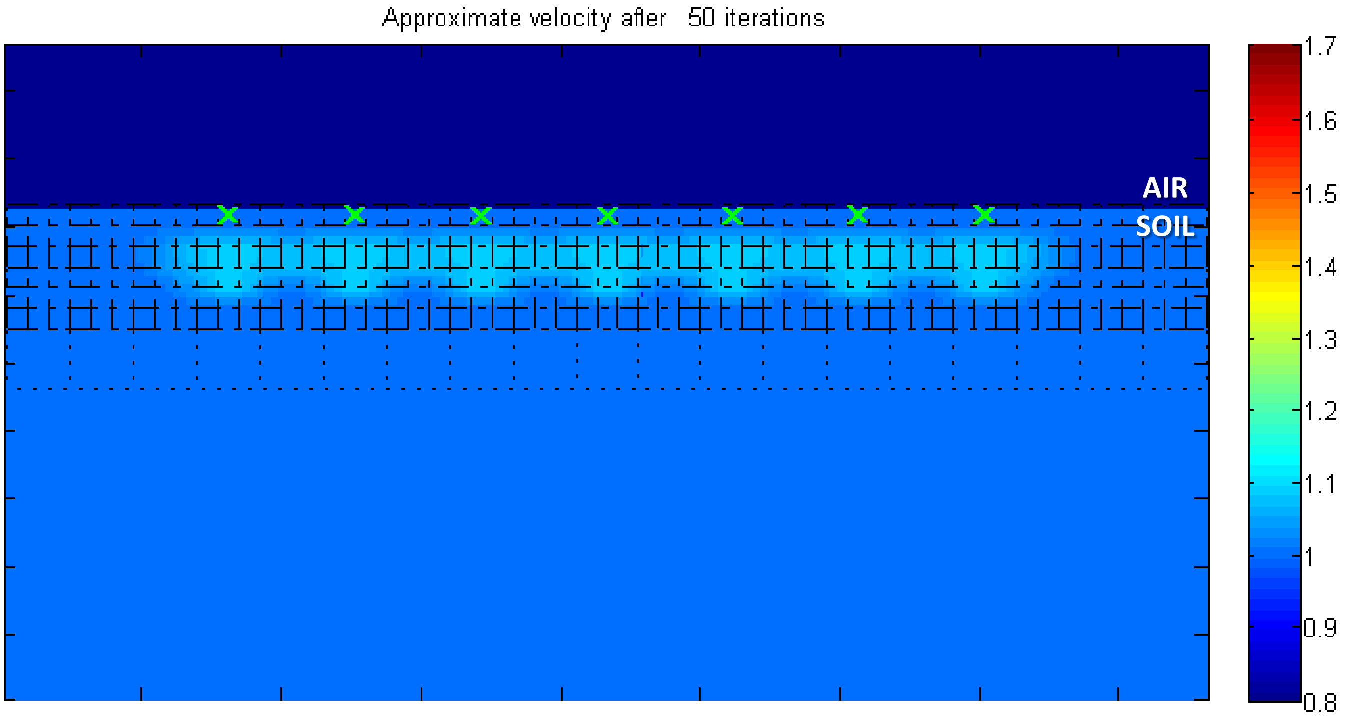
Weight function



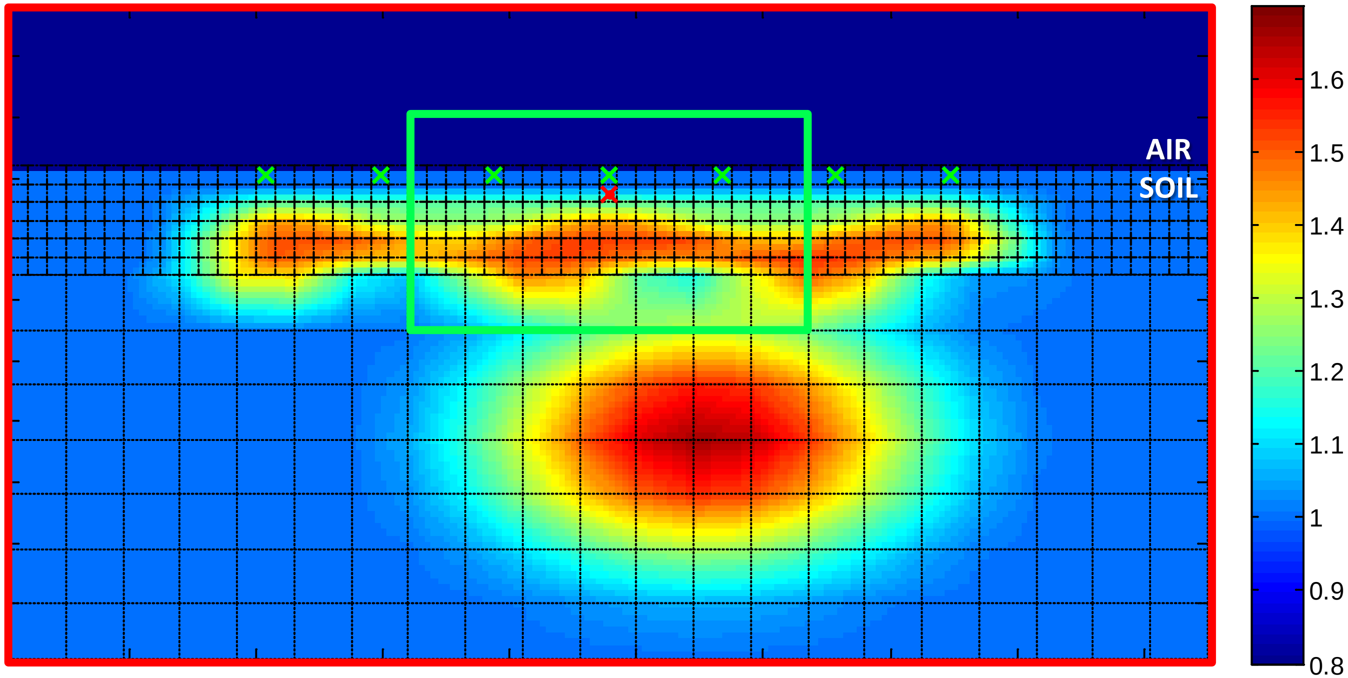
Domain decomposition
Exact velocity



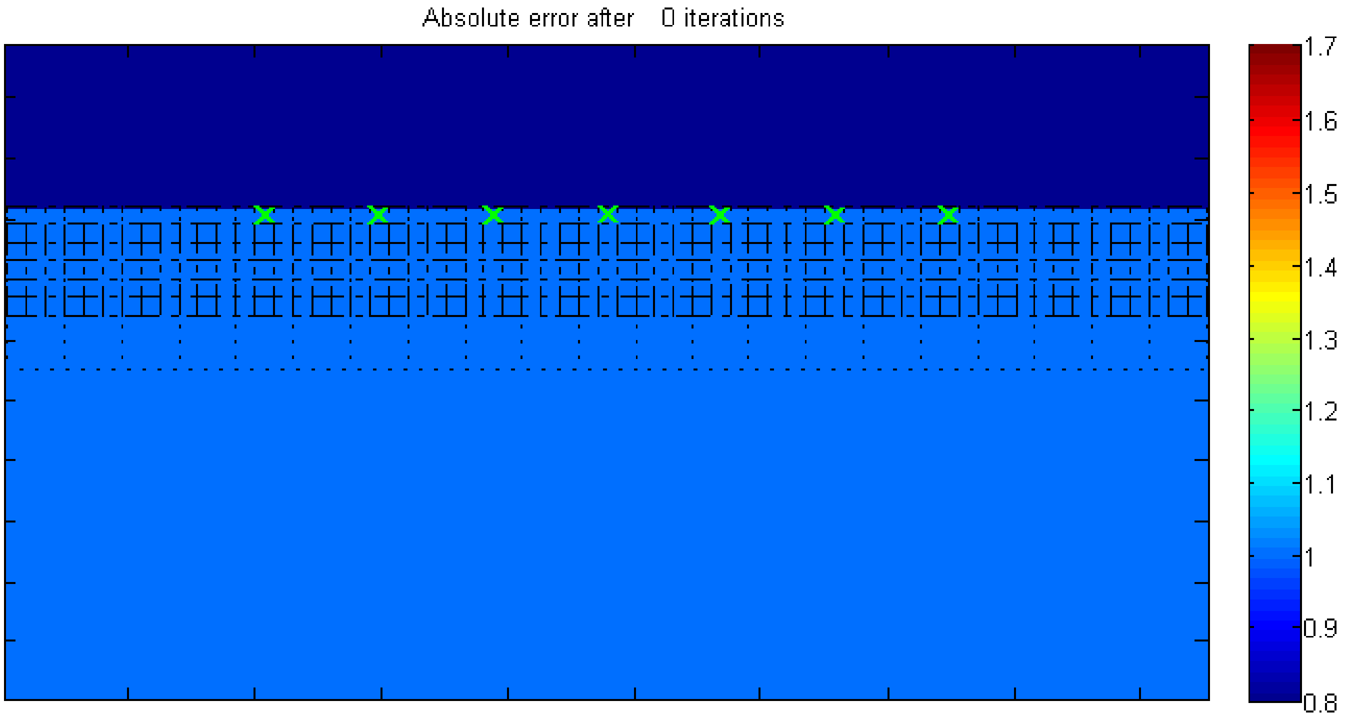
Domain decomposition
Approximate velocity



Domain decomposition
Exact velocity



Domain decomposition
Approximate velocity



Conclusion

- High frequency impulse source FWI is a good preconditioner for low frequency impulse source FWI
- Multiscaling significantly affect FWI convergence rate
- Domain decomposition increases risks of FWI instability but it also provides easy interface between high and low frequency source FWIs

Acknowledgments

- Supervisors
  - Dr. Michael Lamoureux
  - Dr. Gary Margrave
  - Dr. Cristian Rios
- CREWES staff and students
- CREWES industrial sponsors
- PIMS and POTSI support

Thank you for your attention