



Azimuthal AVO and Curvature

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Azimuthal AVO

What is azimuthal AVO?:

- Analysis of incidence angle and azimuthal amplitude variations of reflection coefficients;
- Measures change in elastic properties at an interface.

Why is azimuthal AVO used?:

- Better vertical resolution than propagation methods (e.g. VVAZ, S-wave splitting);
- Usually used to attempt to characterize natural fractures or differential stress in a reservoir.

Motivation

Want an azimuthal AVO technique that

- Works for general anisotropy (no rock physics assumptions in the initial stage);
- Has no ambiguities (solutions are unique);
- Is easy to understand.

Voigt notation

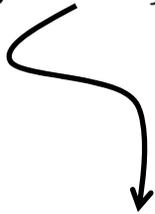
Hooke's Law:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}$$

Voigt notation

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General Anisotropy

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix}$$

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Orthotropy

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix}$$

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$\curvearrowright A = C/\rho$

$$\begin{bmatrix} (V_{P1})^2 & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & (V_{P2})^2 & A_{23} & 0 & 0 & 0 \\ A_{13} & A_{32} & (V_{P3})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & (V_{S23})^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (V_{S13})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (V_{S12})^2 \end{bmatrix}$$

Isotropic AVO

Shuey (1985) wrote the linearized weak contrast PP reflection coefficient in the form

$$R_{PP}^{iso}(\theta) = A + B \sin^2 \theta + C \tan^2 \theta \sin^2 \theta$$

And Thomsen (1990) used this form to describe the effect of common seismic parameters V_P, V_S, ρ , and μ on the reflection coefficient:

$$R_{PP}^{iso}(\theta) = \frac{1}{2} \left[\frac{\Delta V_P}{\bar{V}_P} + \frac{\Delta \rho}{\bar{\rho}} \right] + \frac{1}{2} \left[\frac{\Delta V_P}{\bar{V}_P} - \left(\frac{2\bar{V}_S}{\bar{V}_P} \right) \frac{\Delta \mu}{\bar{\mu}} \right] \sin^2 \theta + \frac{1}{2} \frac{\Delta V_P}{\bar{V}_P} \tan^2 \theta \sin^2 \theta$$

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Intercept Gradient Curvature

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Intercept Gradient Curvature

Anisotropic reflection coefficient

Thomsen (1993) derived the linearized PP reflection coefficient for small contrast weak anisotropy along a vertical plane:

$$\begin{aligned}
 R_{PP}(\theta) = & \frac{1}{2} \left[\frac{\Delta Z_0}{\bar{Z}_0} \right] \\
 & + \frac{1}{2} \left[\frac{\Delta V_{P_0}}{\bar{V}_{P_0}} - \left(\frac{2\bar{V}_{S_0}}{\bar{V}_{P_0}} \right) \frac{\Delta \mu_0}{\bar{\mu}_0} + (\delta_2 - \delta_1) \right] \sin^2 \theta \\
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 \end{aligned}$$

as did Vavryčuk and Pšenčík (1998) using perturbations from background P-wave velocities, α , and S-wave velocities β :

$$\begin{aligned}
 R_{PP}(\theta_P) = & \frac{\rho \Delta A'_{33} + 2\alpha^2 \Delta \rho}{4\rho\alpha^2} \\
 & + \frac{1}{2} \left[\frac{\Delta A'_{33}}{2\alpha^2} - \frac{4(\rho \Delta A'_{55} + \beta^2 \Delta \rho)}{\rho\alpha^2} + \Delta \delta^{*'} \right] \sin^2 \theta_P \\
 & + \frac{1}{2} \left(\frac{\Delta A'_{33}}{2\alpha^2} + \Delta \epsilon^{*' } \right) \sin^2 \theta_P \tan^2 \theta_P
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Where

$$\delta^{*(I)'} = \frac{A_{13}^{(I)'} + 2A_{55}^{(I)'} - A_{33}^{(I)'}}{\alpha^2}, \quad \epsilon^{*(I)'} = \frac{A_{11}^{(I)'} - A_{33}^{(I)'}}{2\alpha^2}$$

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Curvature term

Substituting $\Delta\varepsilon^{*}$ into the equation for a reflection coefficient from Vavryčuk and Pšenčík (1998) results in a curvature of

$$\frac{1}{2} \left(\frac{\Delta A'_{33}}{2\alpha^2} + \frac{\Delta A'_{11} - \Delta A'_{33}}{2\alpha^2} \right) = \frac{\Delta A'_{11}}{4\alpha^2} \approx \frac{1}{2} \frac{\Delta V_{P'H}}{V_{P'H}}$$

This term

- Is only influenced by a single stiffness coefficient
- Easily relates to the isotropic AVO equations
- Is simple to understand

Curvature compared to Ruger's equation

The coordinate transformation (Bond, 1943) for A'_{11} along an arbitrary vertical plane at an angle ϕ from the original plane is

$$A'_{11} = A_{11} \cos^4 \phi + 4A_{16} \cos^3 \phi \sin \phi + 2(A_{12} + 2A_{66}) \cos^2 \phi \sin^2 \phi + 4A_{26} \sin^3 \phi \cos \phi + A_{22} \sin^4 \phi$$

Ruger's curvature is

$$\frac{1}{2} \left[\frac{\Delta\alpha}{\bar{\alpha}} + \Delta\epsilon^{(V)} \cos^4 \phi + \Delta\delta^{(V)} \sin^2 \phi \cos^2 \phi \right]$$

and inserting the weak anisotropy parameters and assuming $\alpha^2 = A_{33}$ it becomes

$$\frac{1}{2} \left[\left(\frac{\Delta A_{33}}{2\alpha^2} \right) + \left(\frac{\Delta A_{11} - \Delta A_{33}}{2\alpha^2} \right) \cos^4 \phi + \left(\frac{\Delta A_{13} + 2\Delta A_{55} - \Delta A_{33}}{\alpha^2} \right) \sin^2 \phi \cos^2 \phi \right] =$$

$$\frac{1}{4\alpha^2} \left[\Delta A_{11} \cos^4 \phi + 2(\Delta A_{13} + 2\Delta A_{55}) \cos^2 \phi \sin^2 \phi + \Delta A_{33} \sin^4 \phi \right]$$

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Gradient along a plane

Substituting the weak anisotropy parameters into Vavryčuk and Pšenčík's gradient along a vertical plane results in

$$\frac{1}{2\alpha^2} \left[-2\Delta A'_{55} - 4\beta^2 \frac{\Delta\rho}{\rho} + \Delta A'_{13} - \frac{\Delta A'_{33}}{2} \right]$$

The only terms in this gradient that change with azimuth are $\Delta A'_{55}$ and $\Delta A'_{13}$ under the following relations:

$$A'_{55} = A_{55} \cos^2 \phi + 2A_{45} \cos \phi \sin \phi + A_{44} \sin^2 \phi,$$

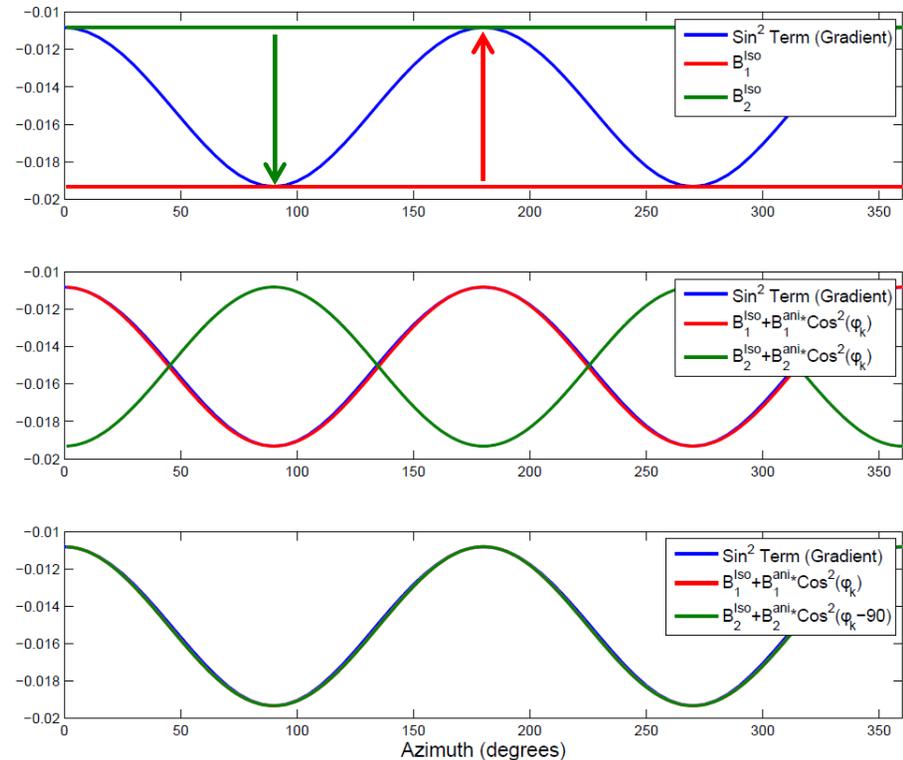
$$A'_{13} = A_{13} \cos^2 \phi + 2A_{36} \cos \phi \sin \phi + A_{23} \sin^2 \phi.$$

90-degree ambiguity

Rüger writes the gradient as

$$B(\phi_k) = B^{iso} + B^{ani} \cos^2(\phi_k - \phi_{sym})$$

which has 3 variables and is nonunique with 2 solutions.

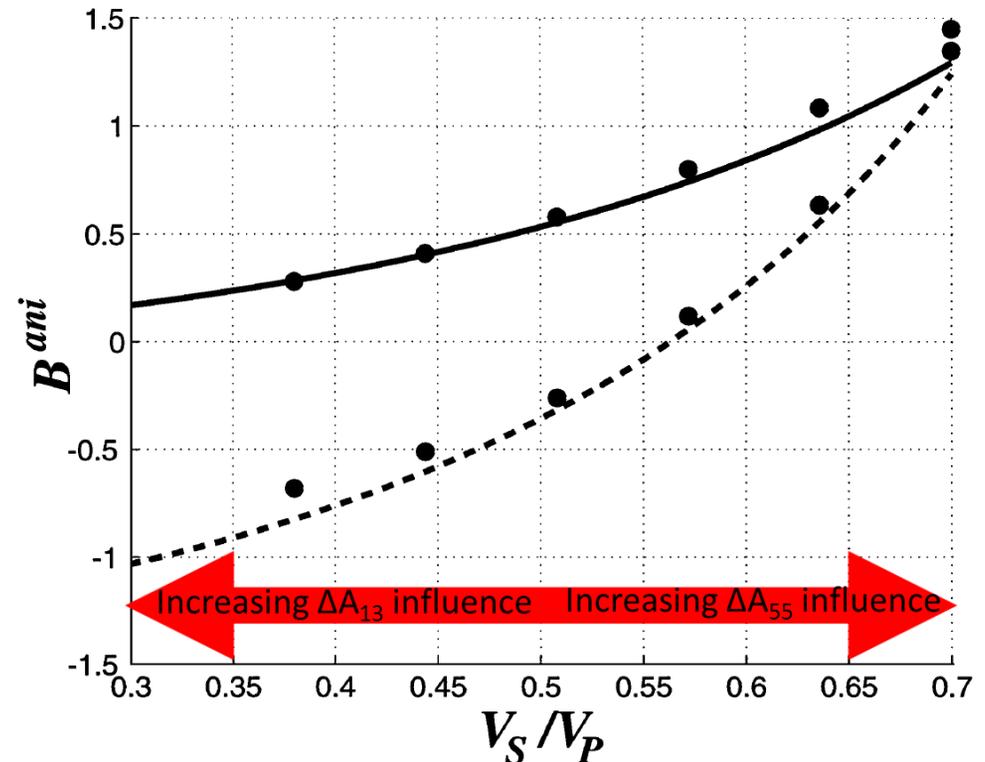


Azimuthal AVO gradient (blue) with two possible solutions (green and red). It is unclear if minima or maxima correspond to the symmetry axis.

Ambiguity as minima/maxima

The azimuthal gradient is a combination of gradients along individual vertical planes so its minima and maxima are minima and maxima of the quantity

$$[-2\Delta A'_{55} + \Delta A'_{13}]$$

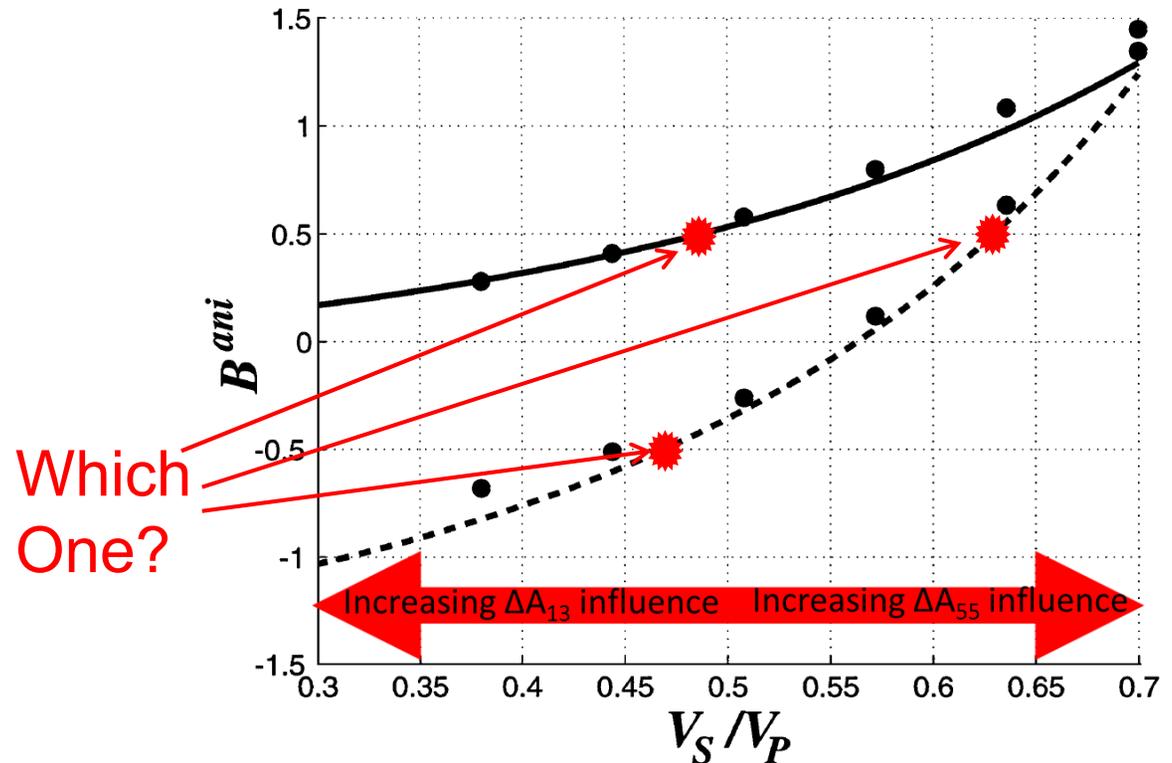


Change in anisotropic gradient with V_S/V_P ratio for aligned vertical dry (dashed line) and wet (solid line) fractures. After Bakulin et al. (2000).

Ambiguity example

Here is an example of an ambiguity that would be caused if B^{ani} had a measured magnitude of 0.5.

Values of 0.5 and -0.5 are both possibilities.



Change in anisotropic gradient with V_s/V_p ratio for aligned vertical dry (dashed line) and wet (solid line) fractures. After Bakulin et al. (2000).

Ways to constrain gradient

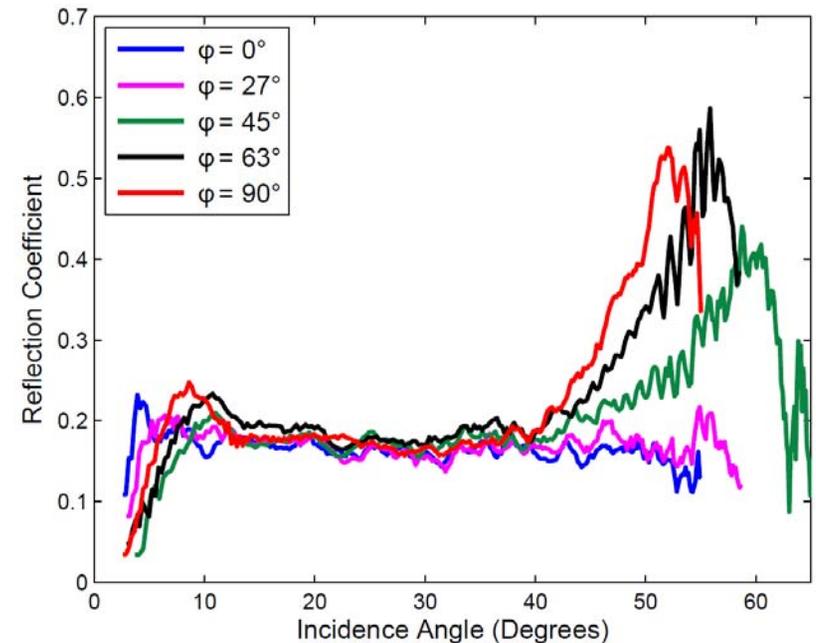
- Knowledge of liquid presence or lack of liquid presence + V_p/V_s ratio
- Approximate fracture direction from another method or from geology
- Azimuthal change in critical angle
- Azimuthal AVO curvature

Physical modeling data

Mahmoudian (2013)

collected and processed azimuthal reflection data using a model with the following approximate stiffnesses:

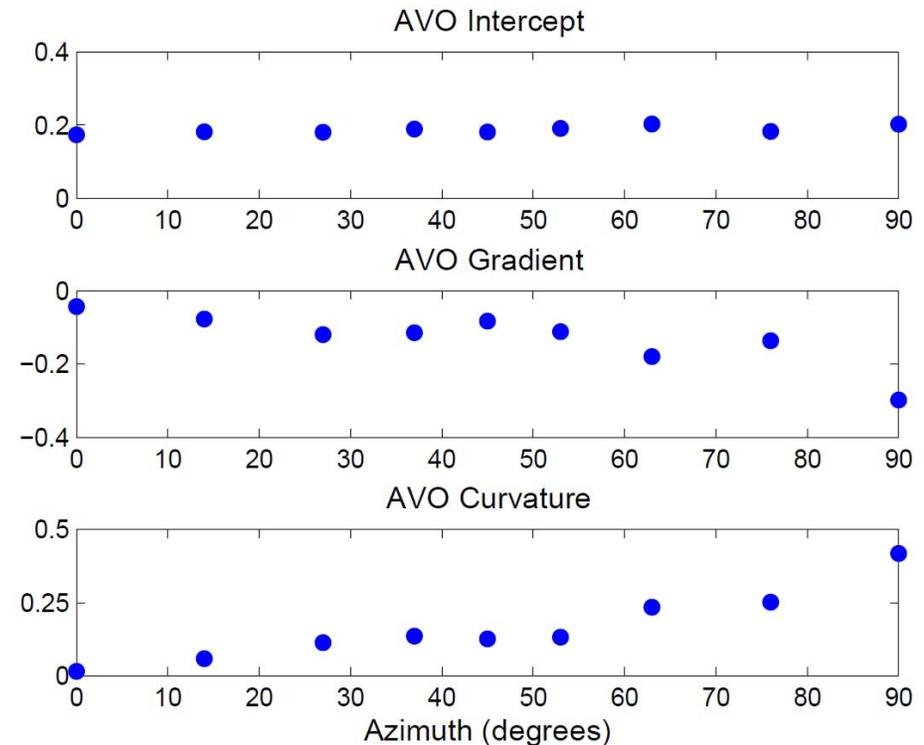
8.70 ± 0.49	4.68 ± 0.21	5.07 ± 0.21	0	0	0
	13.25 ± 0.49	5.13 ± 0.23	0	0	0
		12.25 ± 0.49	0	0	0
			2.89 ± 0.12	0	0
				2.34 ± 0.12	0
					2.28 ± 0.12



Azimuthal physical-modeling reflection data from Mahmoudian (2013).

Intercept, gradient, and curvature

- Intercept, gradient, and curvature were estimated.
- Gradient is more negative at 90 degrees but this is a combination of $-2\Delta A'_{55}$ and $\Delta A'_{13}$ and does not indicate fast and slow directions.

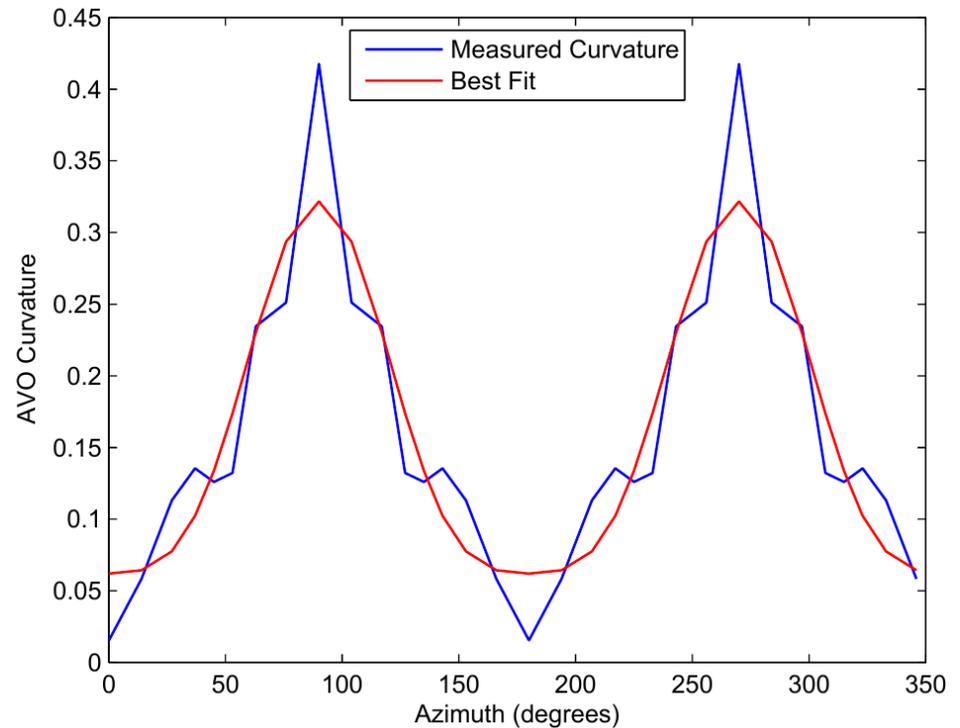


Azimuthal variations in the AVO intercept, gradient, and curvature for the physical modeling dataset.

Fitting the curvature

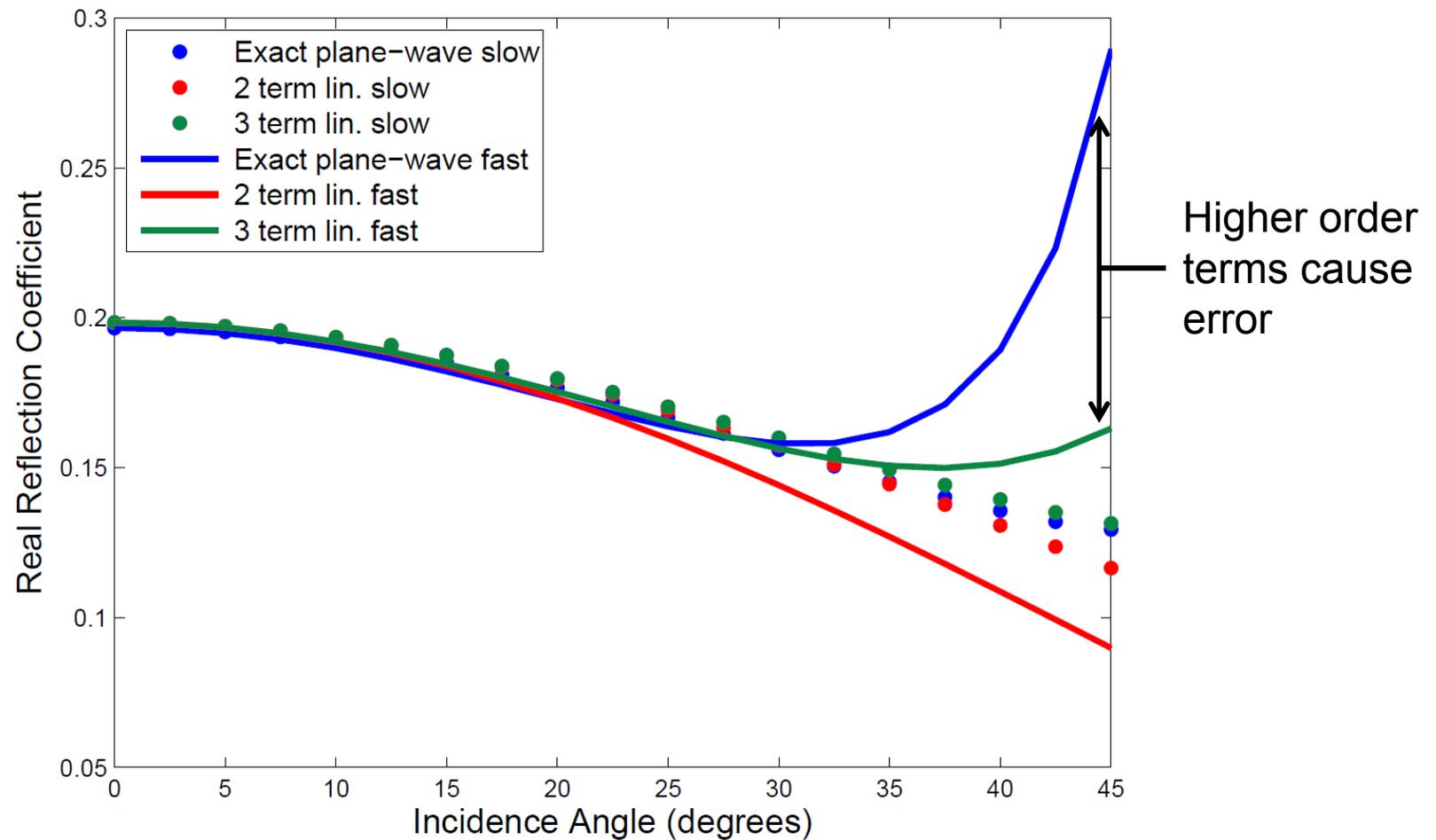
Fitting the curvature resulted in finding 0° and 90° as the azimuths having the biggest separation between ΔA_{11} and ΔA_{22} .

This allows us to determine that 0° is the slow direction.

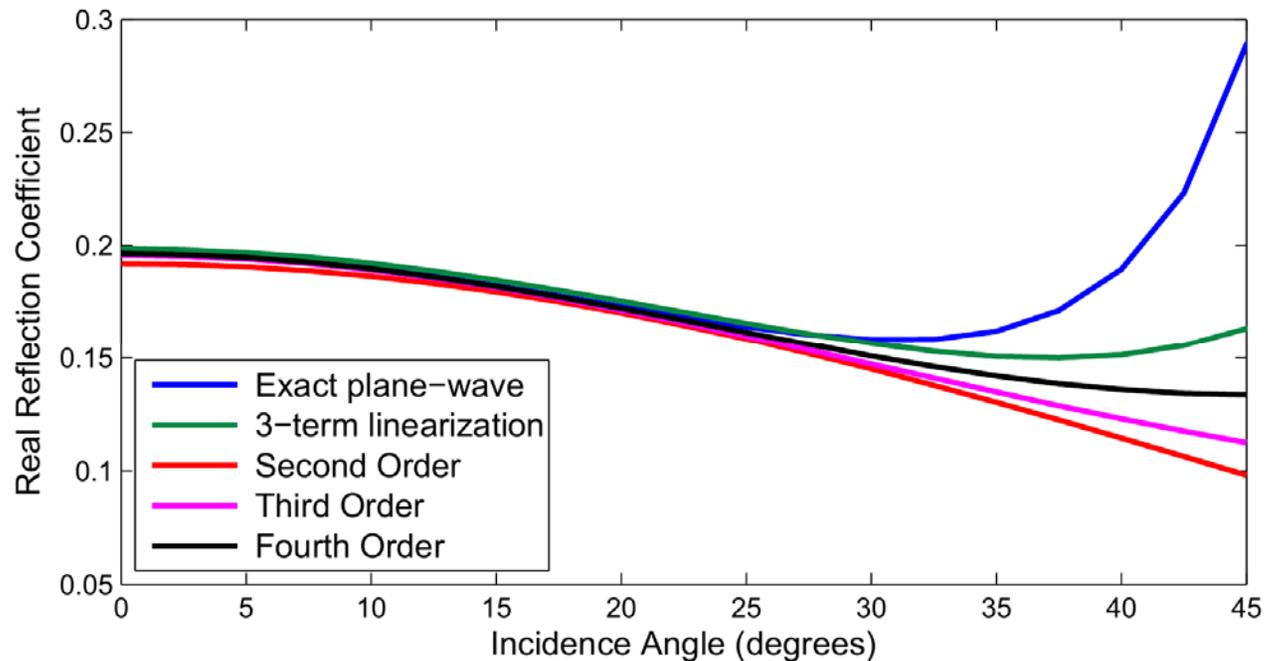


Best Fit of the coordinate transformation of ΔA_{11} to the curvature.

Need for higher order reflection coefficients



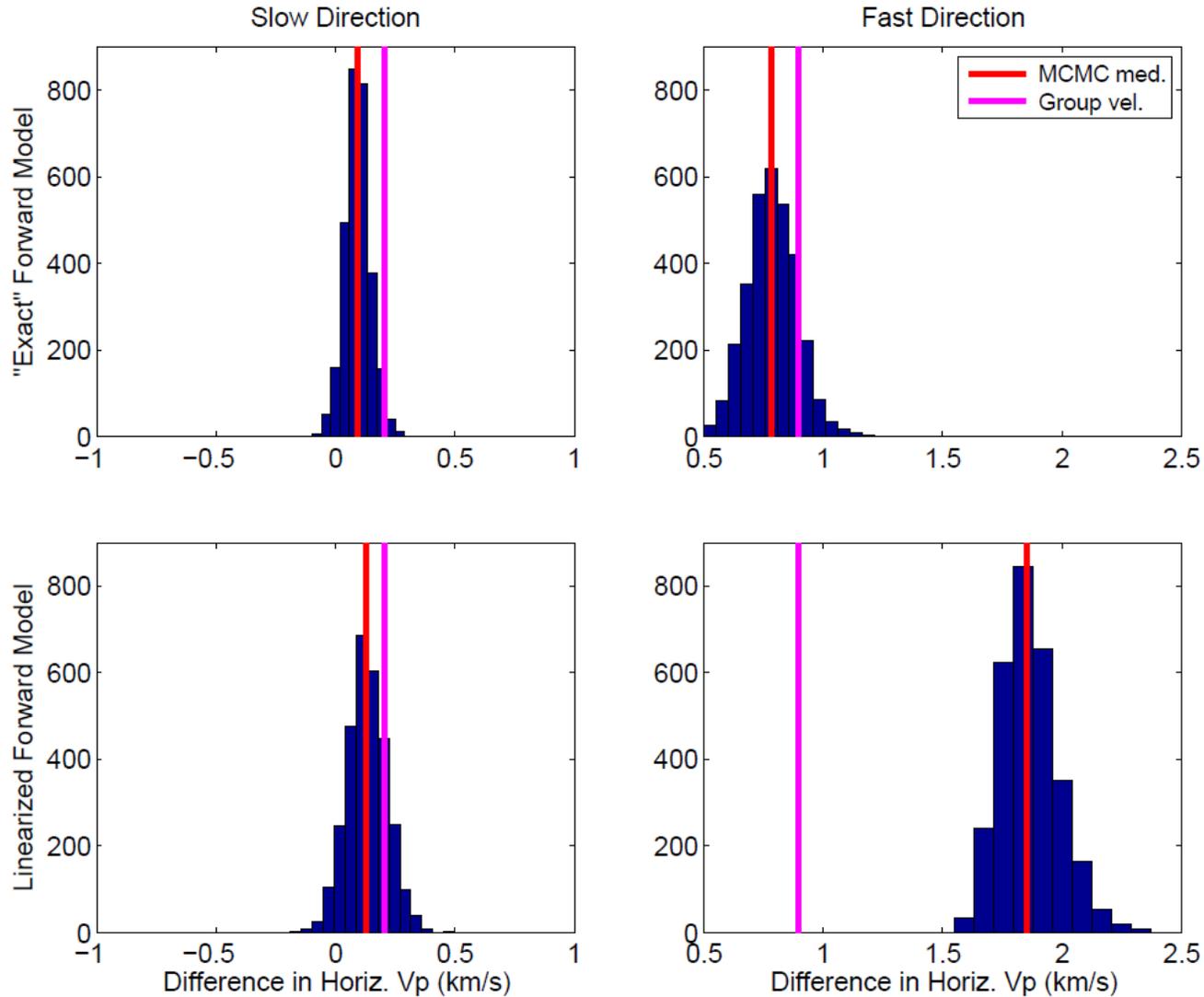
Series expansion of reflection coefficient



- Expansion in ray parameter and small contrasts, δx , replacing second layer parameters:

$$\delta x = 1 - \frac{x^{(1)}}{x^{(2)}} \quad \longrightarrow \quad x^{(2)} = \frac{x^{(1)}}{1 - \delta x}$$

MCMC results: exact vs. linearized



Conclusions

- Individual azimuths = simple representation of theory
- Assumptions about symmetry don't need to be made to analyze certain subsurface elastic properties.
- Curvature is only dependent on a single elastic stiffness, while gradient is dependent on two, leading to ambiguity in interpretation made from the gradient.
- Curvature can be used to determine azimuthal changes in horizontal P-velocity and can be a tool to constrain anisotropy orientation estimates from the gradient.
- Higher order terms important but complicated. Use exact formulas?

Possible Future Work

- Analyze AVO curvature / large offsets in real datasets;
- Continue testing nonlinear inversions;
- Incorporate attenuation anisotropy to determine if cracks contain liquid;
- Analyze azimuthal change in critical angles;

Acknowledgements

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References

- Bakulin, A., Grechka, V., and Tsvankin, I., 2000, Estimation of fracture parameters from reflection seismic data-Part I: HTI model due to a single fracture set: *Geophysics*, **65**, No. 6, 1788-1802.
- Bond, W. L., 1943, The mathematics of the physical properties of crystals. *Bell system technical journal*, 22(1), 1-72.
- Mahmoudian, F., 2013, Physical Modeling and analysis of seismic data from a simulated fractured medium: Ph.D. thesis, University of Calgary.
- Rüger, A., 1998, Variation of P-wave reflectivity with offset and azimuth in anisotropic media: *Geophysics*, **63**, No. 3, 935-947.
- Shuey, R., 1985, A simplification of the Zoeppritz equations: *Geophysics*, **50**, No. 4, 609-614.
- Thomsen, L., 1993, Weak anisotropic reflections: Offset-dependent reflectivity-Theory and practice of AVO analysis: *Soc. Expl. Geophys*, 103-111.
- Vavryčuk, V., and Pšenčík, I., 1998, PP-wave reflection coefficients in weakly anisotropic media: *Geophysics*, **63**, No. 6, 2129-2141.