Discriminating elastic wave modes
with shaped DAS fibres

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DAS-fibreoptic projects @ CREWES

- **Directivity** & **6C sensing** with shaped fibres (2016)
- Full waveform DAS response: **coupling the FGSM with 3D elastic FD simulations** (Matt Eaid)
- Acquisition: **DAS installations** at CaMI (Don Lawton)
- **Processing**: straight, HWC, vertical, trenched, thumper, vibe (Heather Hardeman, Adriana Gordon, Kevin Hall)
- **Elastic wave response:** in the context of DAS fibre layout and design : moment tensor ID and discrimination, up vs down, P- vs S-, body vs surface, etc.

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Acknowledgments

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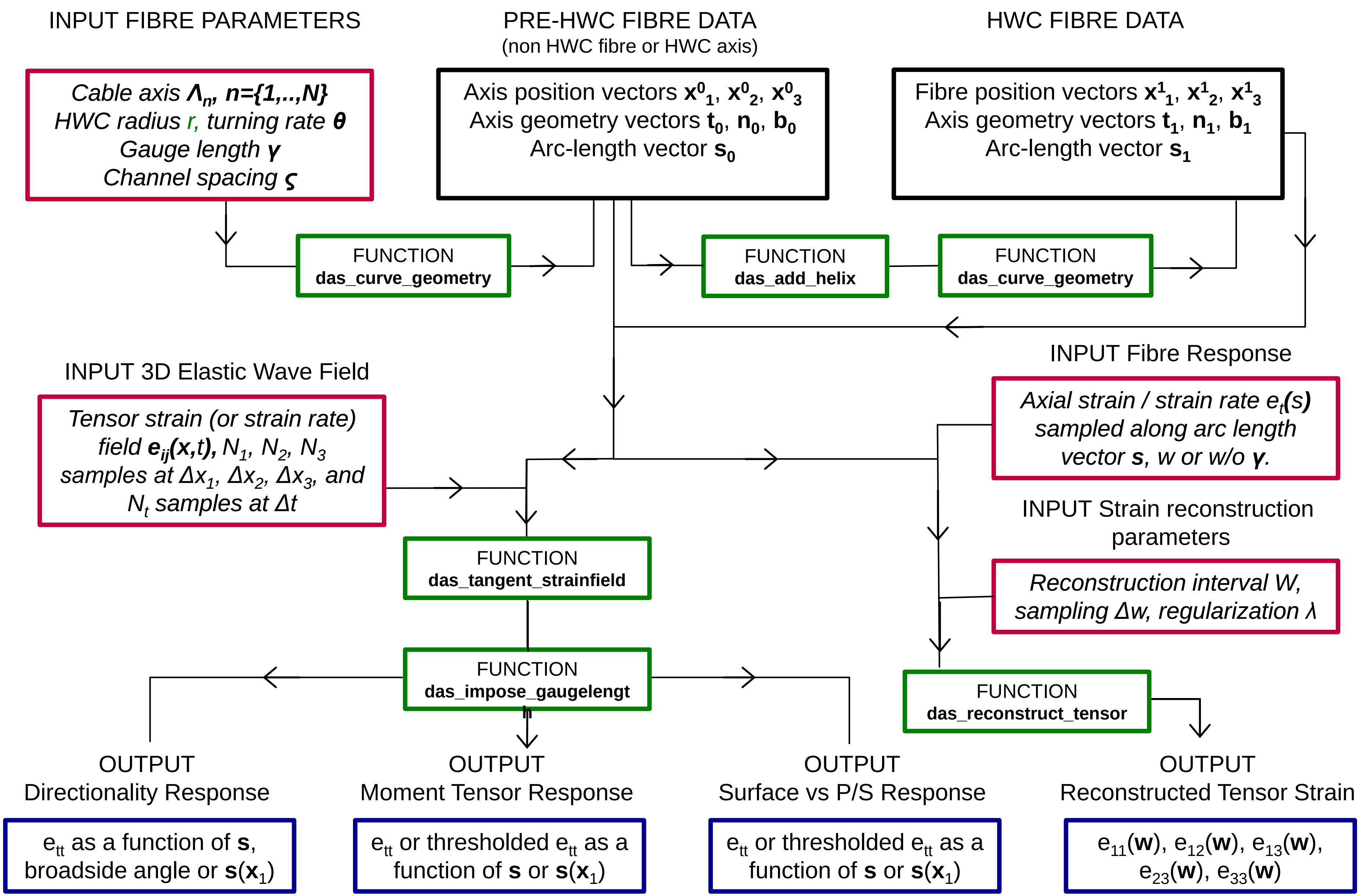
D. Lawton, S. Leaney, T. Daley, A. Mateeva, E. Martin, B. Biondi, M. MacDonald

CREWES Fibre Geometry and Sensing Model (FGSM)



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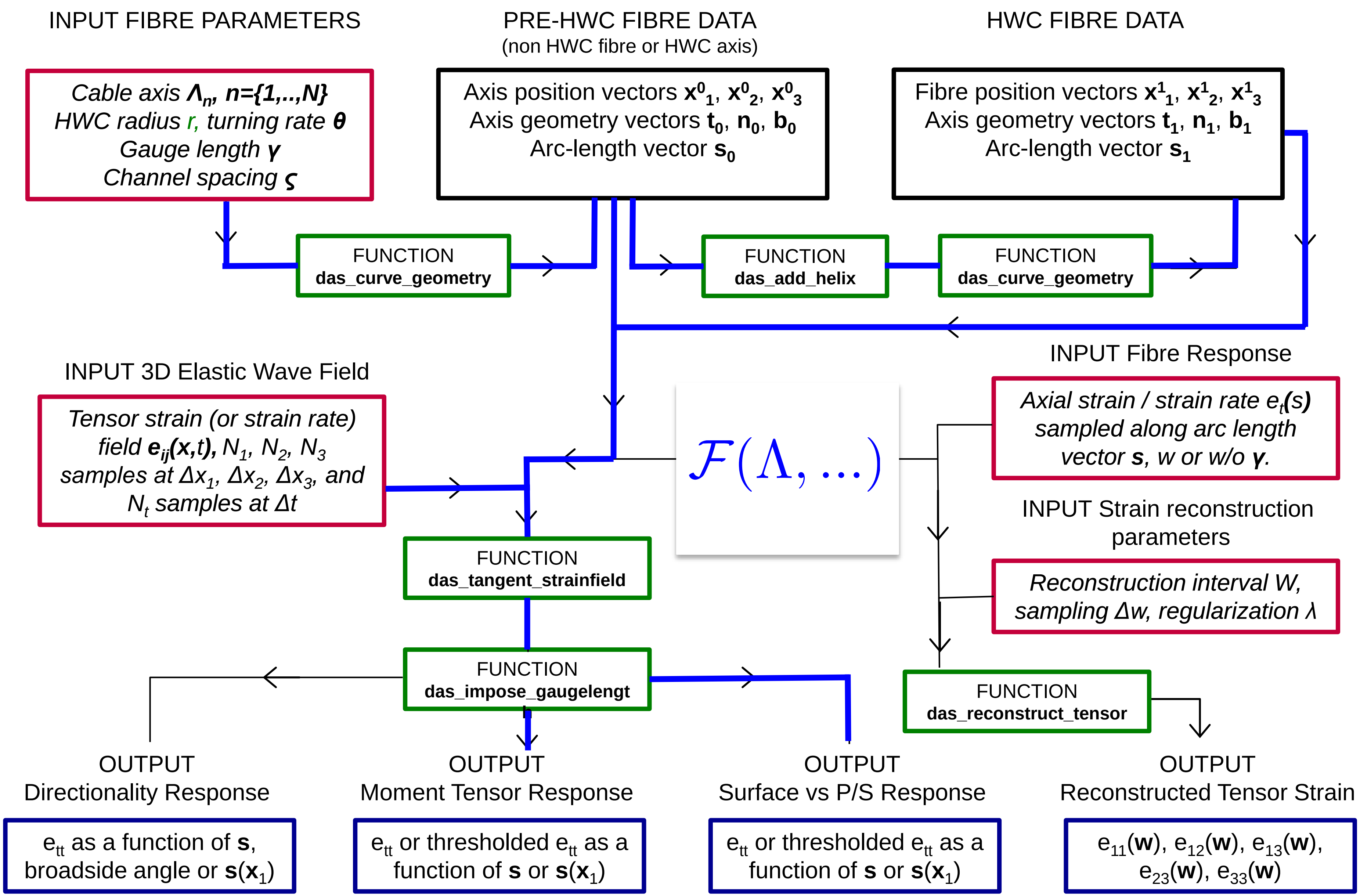
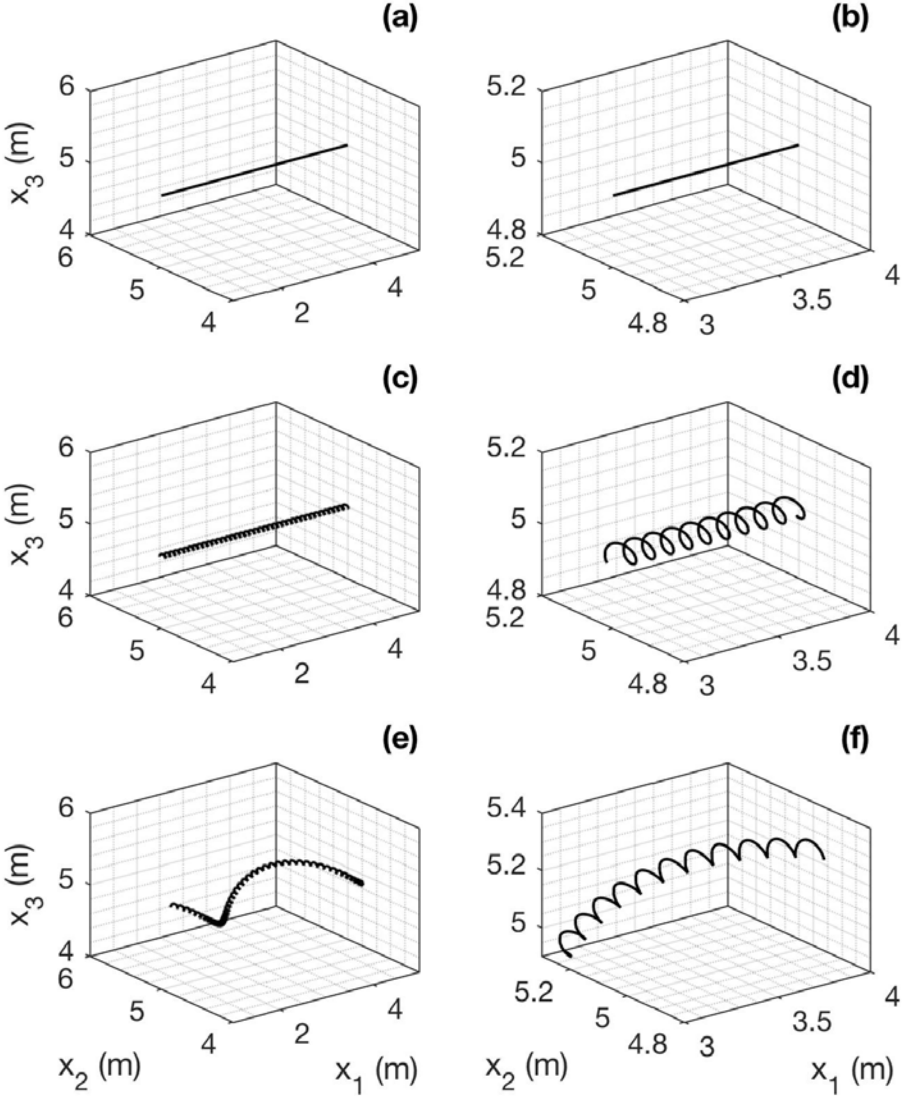


Benchmark cable shapes



Straight

HWC

Complex

Idealized waveforms (displacement)

$$\mathbf{u}(\mathbf{x}, t) = \begin{bmatrix} u_1(x_1, x_3, t) \\ u_3(x_1, x_3, t) \end{bmatrix}$$

Harmonic plane P wave
angle θ to vertical

$$u_1(x_1, x_3, t) = -\sin\theta \cos k\,(x_3 - \alpha t)$$
$$u_3(x_1, x_3, t) = \cos\theta \cos k\,(x_3 - \alpha t)$$

Plane Rayleigh wave
inline propagation

$$u_1(x_1, x_3, t) = c_1 \left(c_2 e^{c_3 k x_3} + c_4 e^{c_5 k x_3}\right) \cos k\,(x_1 - v_R t)$$
$$u_3(x_1, x_3, t) = c_1 \left(c_3 e^{c_3 k x_3} + c_6 e^{c_5 k x_3}\right) \sin k\,(x_1 - v_R t)$$

Idealized waveforms (displacement)

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_P + \mathbf{u}_S$$

$$\mathbf{u}_P = \left(\frac{c_\alpha}{r}\right) m\mathbf{g} \qquad \mathbf{u}_S = \left(\frac{c_\beta}{r}\right) (\mathbf{M}\mathbf{g} - m\mathbf{g})$$

$$\mathbf{g} = \left[\begin{array}{c} \cos\theta_1 \\ \cos\theta_3 \\ \cos\theta_3 \end{array} \right]$$

$$m = \mathbf{g}^T \mathbf{M} \mathbf{g}$$

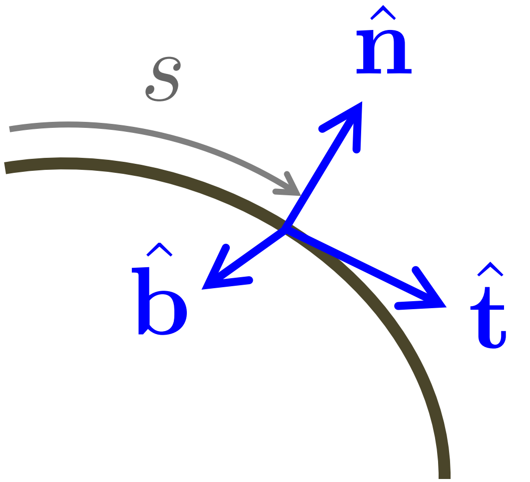
$$\mathbf{M} = \left[\begin{array}{ccc} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{array} \right]$$

Maximum displacement from source with arbitrary focal mechanism

Idealized waveforms (strain)

$$\mathbf{u} \rightarrow \mathbf{e} = e_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

$$= \mathbf{e}_{\mathrm{icd}} \rightarrow \mathbf{e}_{\mathrm{tnb}} = \mathbf{R}^T(s)\mathbf{e}_{\mathrm{icd}}\mathbf{R}(s)$$



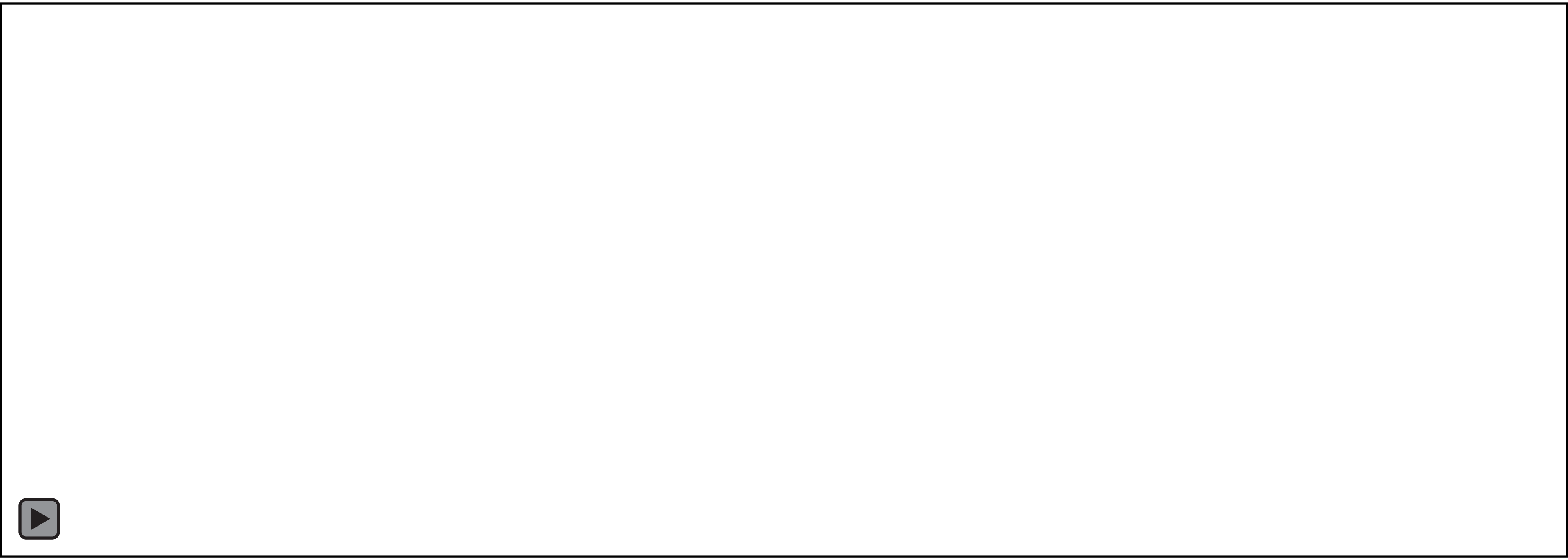
$$d^P_{t_1}(s, \theta_1) = \mathbf{e}_{\mathrm{tnb}}(1,1) = \mathcal{F}(\Lambda, \ldots)\mathbf{u}$$

1 DAS datum

Modeling DAS responses

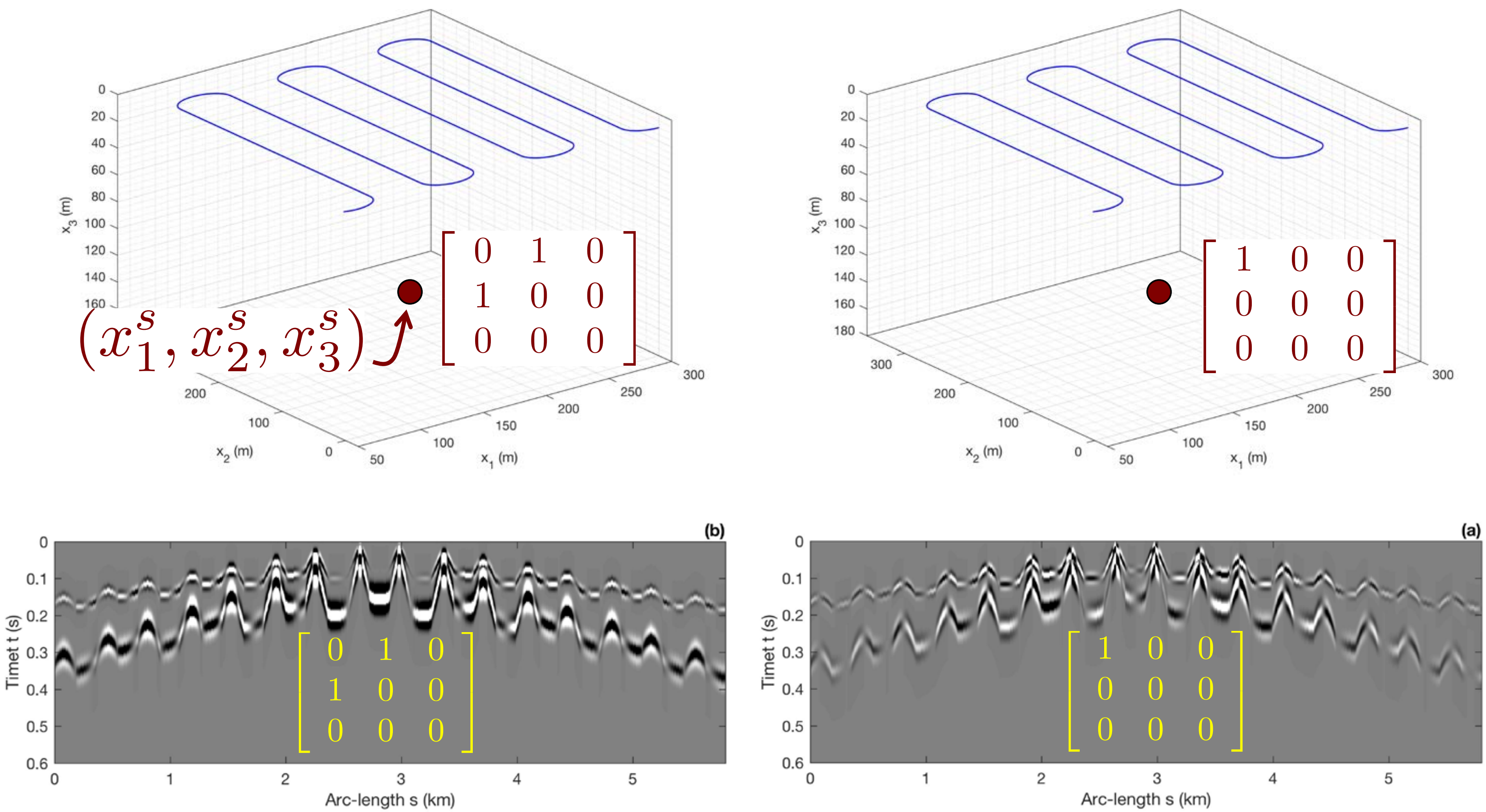
Dynamic results: obliquely incident P-wave

Modeling DAS responses

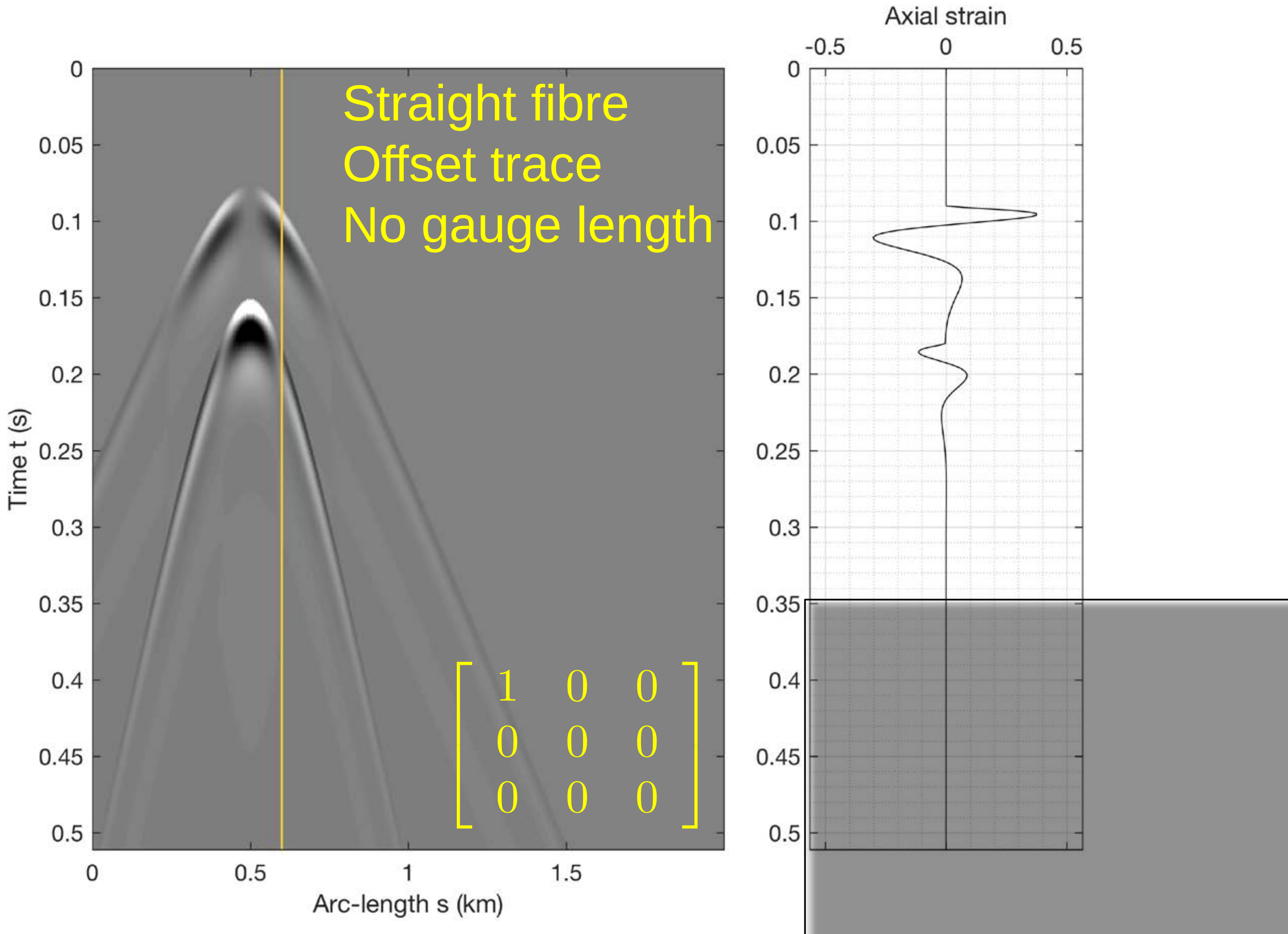


Dynamic results: plane Rayleigh wave

Modeling DAS responses

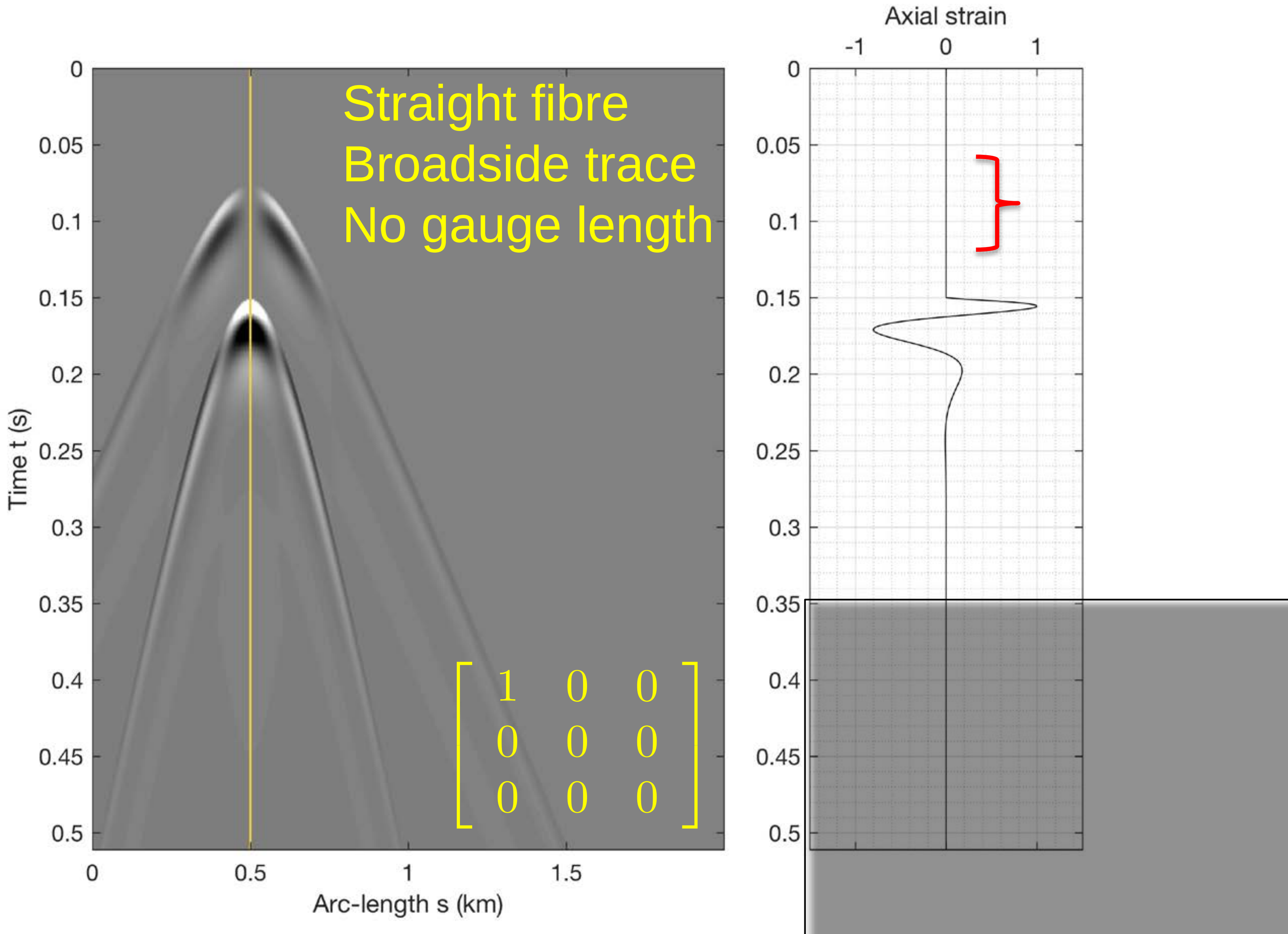


Modeling DAS responses

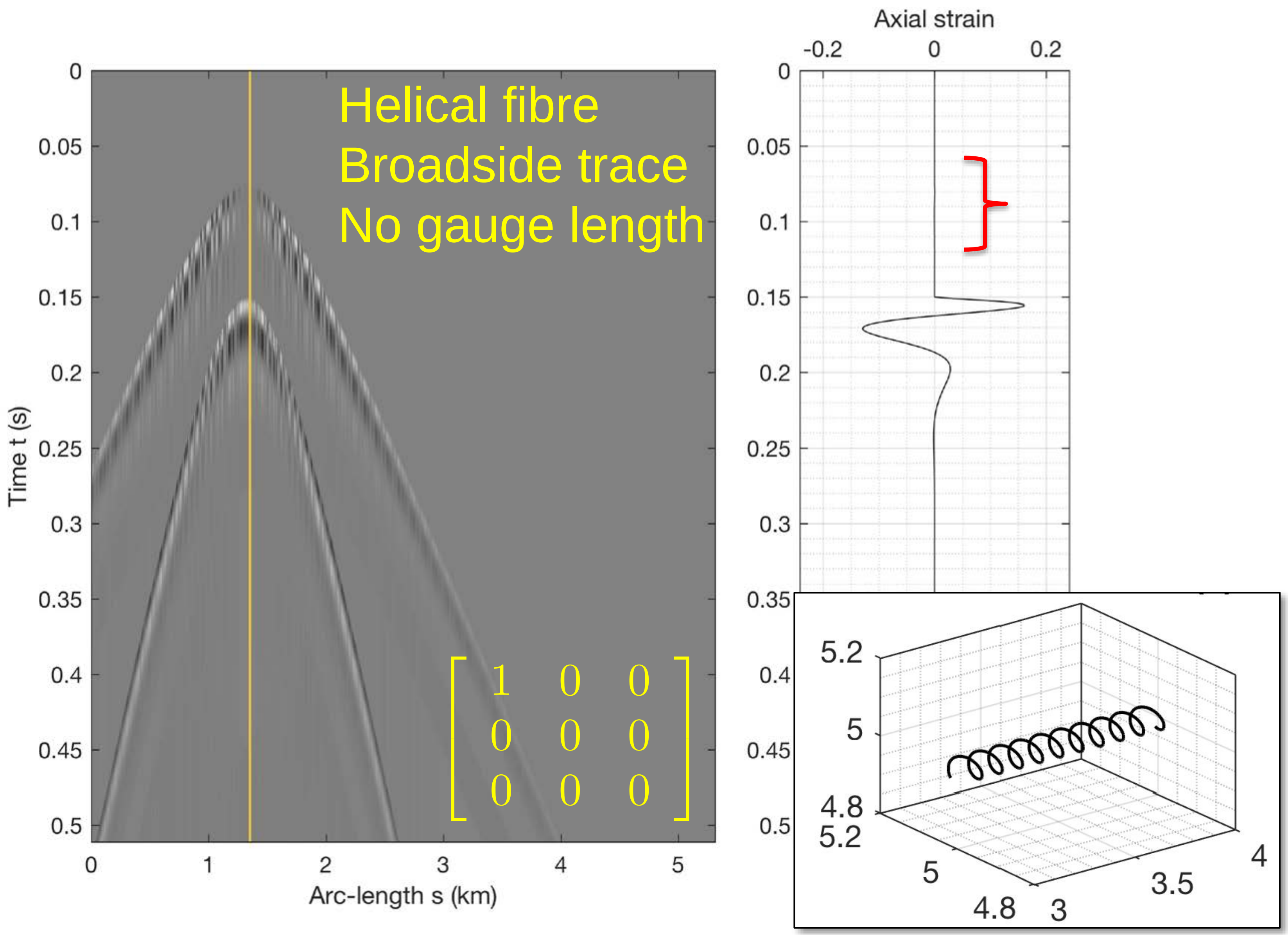


$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

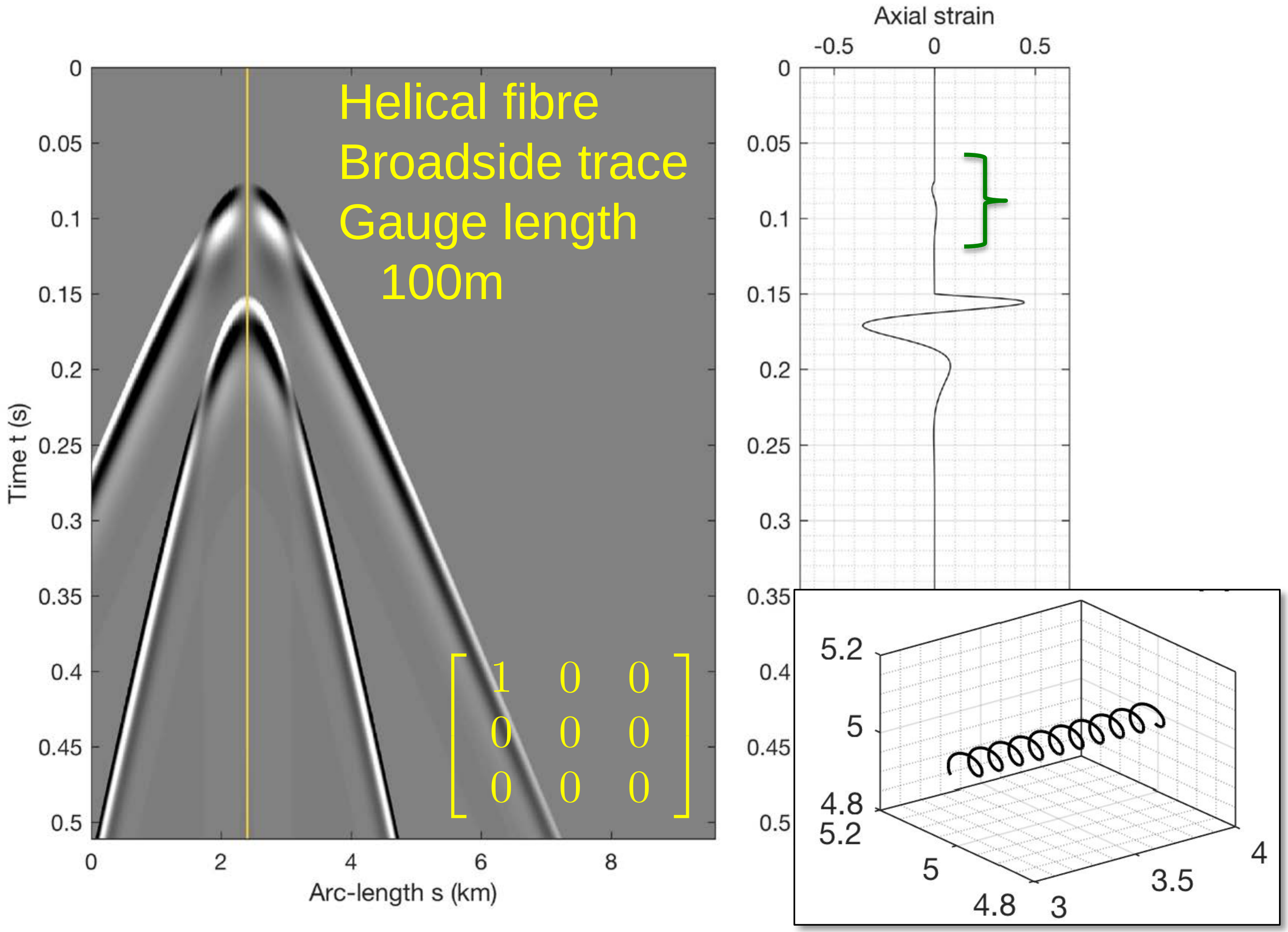
Modeling DAS responses



Modeling DAS responses



Modeling DAS responses



Design criteria

$$\mathbf{d}^P = \begin{bmatrix} \mathbf{d}^P(\theta_1) \\ \mathbf{d}^P(\theta_2) \\ \mathbf{d}^P(\theta_3) \\ \vdots \end{bmatrix} \quad \mathbf{d}^P(\theta_1) = \begin{bmatrix} \mathbf{d}^P_{t_1}(\theta_1) \\ \mathbf{d}^P_{t_2}(\theta_1) \\ \mathbf{d}^P_{t_3}(\theta_1) \\ \vdots \end{bmatrix} \quad \mathbf{d}^P_{t_1}(\theta_1) = \begin{bmatrix} d^P_{t_1}(s_1,\theta_1) \\ d^P_{t_1}(s_2,\theta_1) \\ d^P_{t_1}(s_3,\theta_1) \\ \vdots \end{bmatrix}$$

$$\chi_{\mathrm{BR}}(\text{fibre parameters}) = (\mathbf{d}^S)^T(\mathbf{d}^R) + \lambda(\mathbf{d}^P)^T(\mathbf{d}^R)$$

Penalizing similarity of response between ground roll and S-waves

Penalizing similarity of response between ground roll and P-waves

$$\chi_{\mathrm{RR}}(\text{fibre parameters}) = (\mathbf{d}^R)^T(\mathbf{d}^R)$$

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$$\chi_{\mathrm{BR}}(\text{fibre parameters}) = (\mathbf{d}^S)^T(\mathbf{d}^R) + \lambda(\mathbf{d}^P)^T(\mathbf{d}^R)$$

Penalizing strength of ground roll

$$\chi_{\mathrm{RR}}(\text{fibre parameters}) = (\mathbf{d}^R)^T(\mathbf{d}^R)$$

Conclusions

- **DAS data "character"** depends on fibre material, shape, source, gauge length, channel spacing, interrogator features, etc.; collect within a response model, CREWES-FGSM
- Uses: directionality, 6C sensing, **fibre design**
- **Library of characteristic responses** to ground roll, body waves (then, up- and down-going waves, separate P- and S-waves, etc.)
- Next: investigate a 2-step process:
  i. Design fibre (determine Λ_i) maximizing separability
  ii. Teach fibre to suppress or disciminate during processing (machine learning?), or real time?