

Discriminating elastic wave modes with shaped DAS fibres

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DAS-fibreoptic projects @ **CREWES**

- **Directivity & 6C sensing** with shaped fibres (2016)
- Full waveform DAS response: **coupling the FGSM with 3D elastic FD simulations** (Matt Eaid)
- Acquisition: **DAS installations** at CaMI (Don Lawton)
- **Processing**: straight, HWC, vertical, trenched, thumper, vibe (Heather Hardeman, Adriana Gordon, Kevin Hall)
- **Elastic wave response**: in the context of DAS fibre layout and design : moment tensor ID and discrimination, up vs down, P- vs S-, body vs surface, etc.

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Acknowledgments

 **CREWES** industrial sponsors

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MacDonald

CREWES Fibre Geometry and Sensing Model (FGSM)

INPUT FIBRE PARAMETERS

Cable axis Λ_n , $n=\{1,...,N\}$
 HWC radius r , turning rate θ
 Gauge length γ
 Channel spacing ζ

PRE-HWC FIBRE DATA

(non HWC fibre or HWC axis)

Axis position vectors $\mathbf{x}_1^0, \mathbf{x}_2^0, \mathbf{x}_3^0$
 Axis geometry vectors $\mathbf{t}_0, \mathbf{n}_0, \mathbf{b}_0$
 Arc-length vector $\mathbf{s}_0$

HWC FIBRE DATA

Fibre position vectors $\mathbf{x}_1^1, \mathbf{x}_2^1, \mathbf{x}_3^1$
 Axis geometry vectors $\mathbf{t}_1, \mathbf{n}_1, \mathbf{b}_1$
 Arc-length vector $\mathbf{s}_1$

INPUT 3D Elastic Wave Field

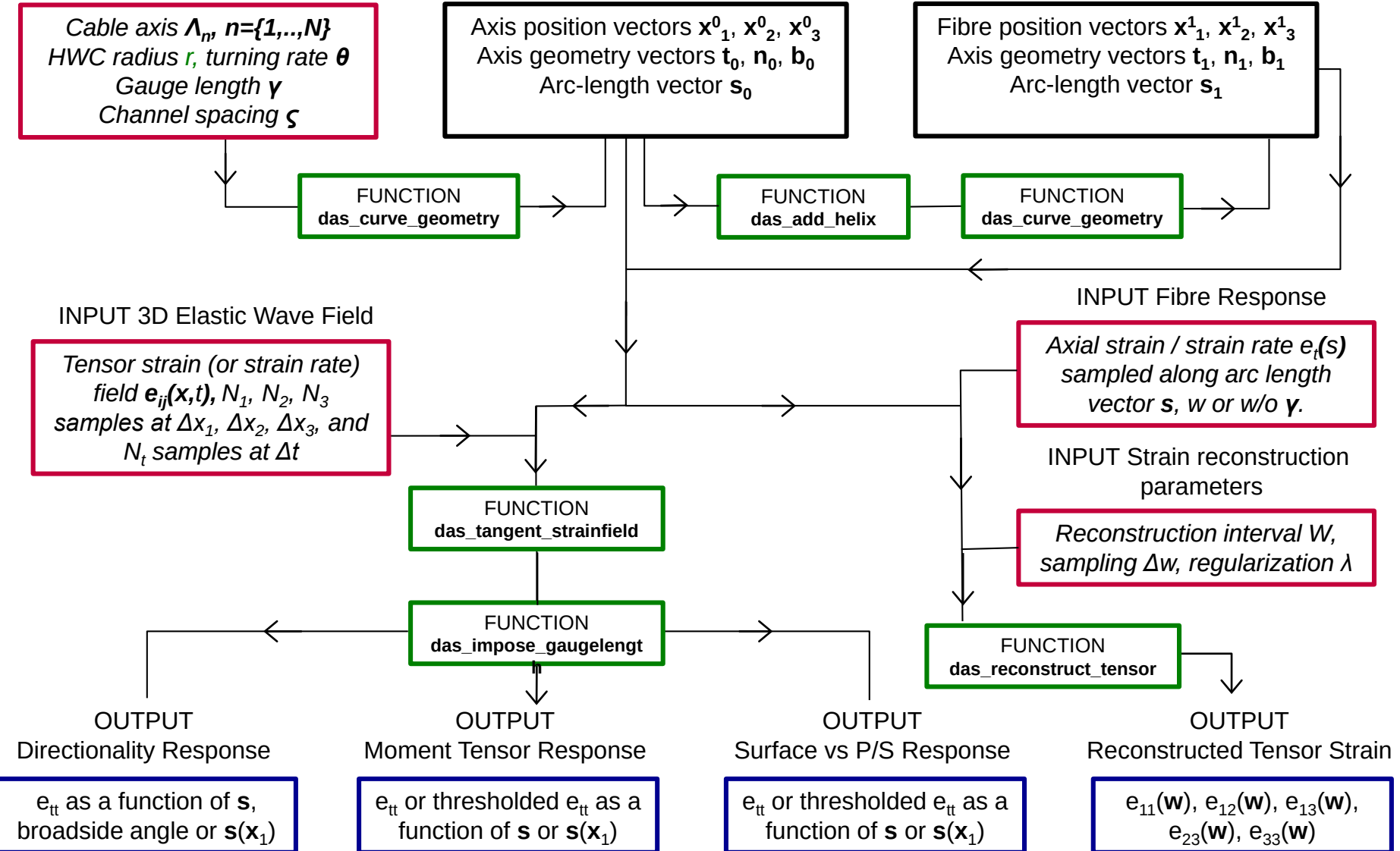
Tensor strain (or strain rate)
 field $\mathbf{e}_{ij}(\mathbf{x}, t)$, N_1, N_2, N_3
 samples at $\Delta x_1, \Delta x_2, \Delta x_3$, and
 N_t samples at Δt

INPUT Fibre Response

Axial strain / strain rate $e_t(s)$
 sampled along arc length
 vector $\mathbf{s}$, w or $w/o \gamma$.

INPUT Strain reconstruction parameters

Reconstruction interval W ,
 sampling Δw , regularization λ



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$\mathcal{F}(\Lambda, \dots)$

FUNCTION
das_curve_geometry

FUNCTION
das_add_helix

FUNCTION
das_curve_geometry

FUNCTION
das_tangent_strainfield

FUNCTION
das_impose_gaugelengt

FUNCTION
das_reconstruct_tensor

OUTPUT
 Directionality Response

e_{tt} as a function of $\mathbf{s}$,
 broadside angle or $\mathbf{s}(\mathbf{x}_1)$

OUTPUT
 Moment Tensor Response

e_{tt} or thresholded e_{tt} as a
 function of $\mathbf{s}$ or $\mathbf{s}(\mathbf{x}_1)$

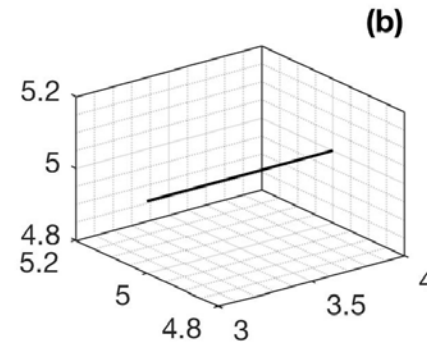
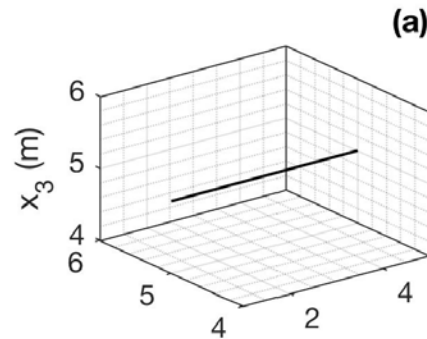
OUTPUT
 Surface vs P/S Response

e_{tt} or thresholded e_{tt} as a
 function of $\mathbf{s}$ or $\mathbf{s}(\mathbf{x}_1)$

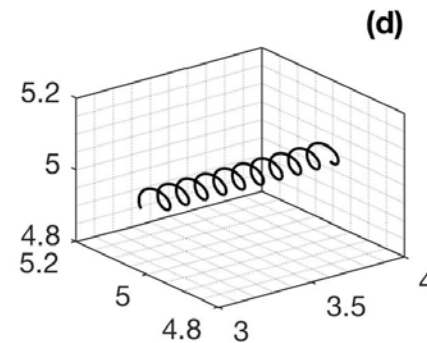
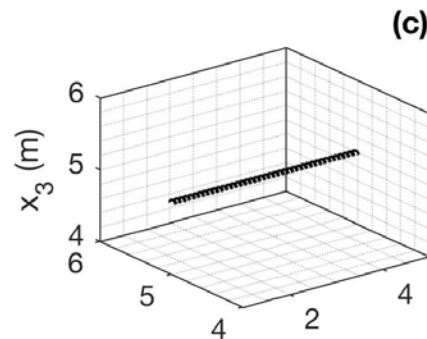
OUTPUT
 Reconstructed Tensor Strain

$e_{11}(\mathbf{w}), e_{12}(\mathbf{w}), e_{13}(\mathbf{w}),$
 $e_{23}(\mathbf{w}), e_{33}(\mathbf{w})$

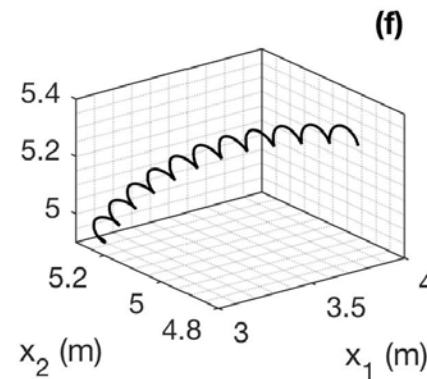
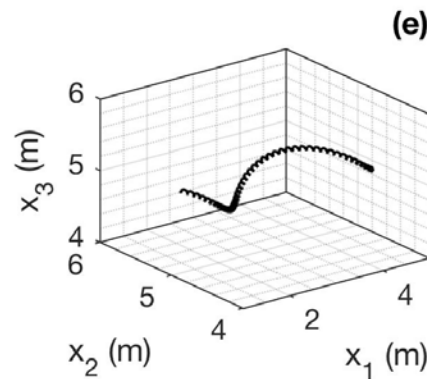
Benchmark cable shapes



Straight



HWC



Complex

Idealized waveforms (displacement)

$$\mathbf{u}(\mathbf{x}, t) = \begin{bmatrix} u_1(x_1, x_3, t) \\ u_3(x_1, x_3, t) \end{bmatrix}$$

Harmonic plane P wave
angle θ to vertical

$$u_1(x_1, x_3, t) = -\sin \theta \cos k(x_3 - \alpha t)$$
$$u_3(x_1, x_3, t) = \cos \theta \cos k(x_3 - \alpha t)$$

Plane Rayleigh wave
inline propagation

$$u_1(x_1, x_3, t) = c_1 \left(c_2 e^{c_3 k x_3} + c_4 e^{c_5 k x_3} \right) \cos k(x_1 - v_R t)$$

$$u_3(x_1, x_3, t) = c_1 \left(c_3 e^{c_3 k x_3} + c_6 e^{c_5 k x_3} \right) \sin k(x_1 - v_R t)$$

Idealized waveforms (displacement)

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_P + \mathbf{u}_S$$

$$\mathbf{u}_P = \left(\frac{c_\alpha}{r}\right) m \mathbf{g} \quad \mathbf{u}_S = \left(\frac{c_\beta}{r}\right) (\mathbf{M} \mathbf{g} - m \mathbf{g})$$

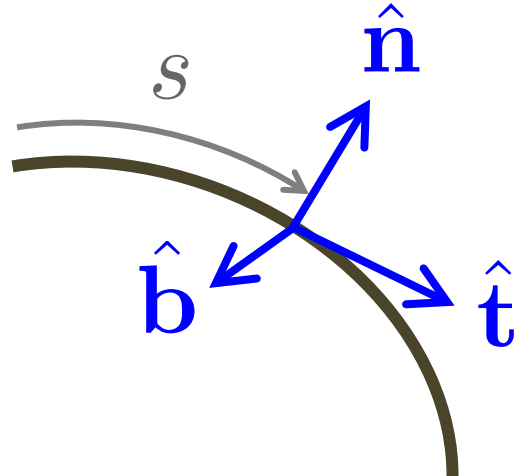
$$\mathbf{g} = \begin{bmatrix} \cos \theta_1 \\ \cos \theta_3 \\ \cos \theta_3 \end{bmatrix}$$

$$m = \mathbf{g}^T \mathbf{M} \mathbf{g}$$

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

Maximum
displacement
from source with
arbitrary focal
mechanism

Idealized waveforms (strain)

$$\mathbf{u} \rightarrow \mathbf{e} = e_{ij} = \frac{1}{2} \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]$$
$$= \mathbf{e}_{\text{icd}} \rightarrow \mathbf{e}_{\text{tnb}} = \mathbf{R}^T(s) \mathbf{e}_{\text{icd}} \mathbf{R}(s)$$
A diagram showing a curved surface, likely representing a seismic sensor or a geological feature. A gray arc labeled 's' indicates a segment of the surface. At a point on this arc, a local coordinate system is defined by three blue vectors: $\hat{\mathbf{n}}$ (normal to the surface, pointing upwards and to the right), $\hat{\mathbf{b}}$ (back-sight, pointing downwards and to the left), and $\hat{\mathbf{t}}$ (tangent, pointing downwards and to the right).

$$d_{t_1}^P(s, \theta_1) = \mathbf{e}_{\text{tnb}}(1, 1) = \mathcal{F}(\Lambda, \dots) \mathbf{u}$$


1 DAS datum

Modeling DAS responses



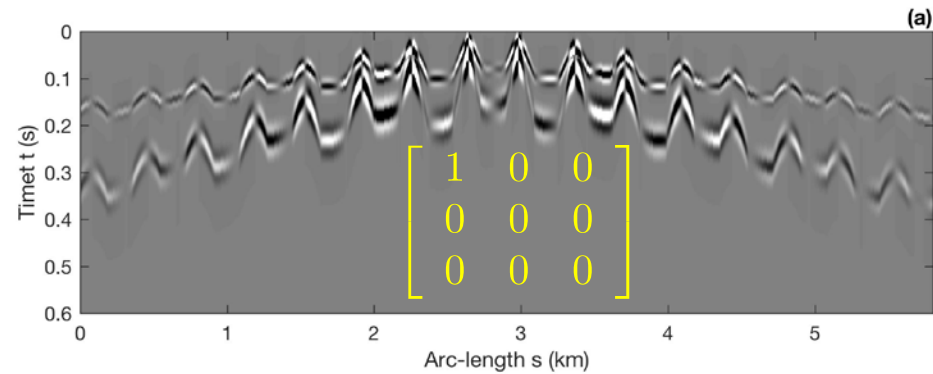
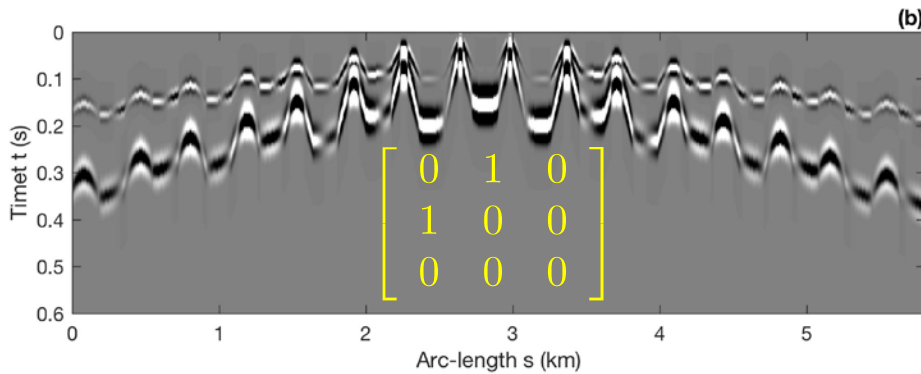
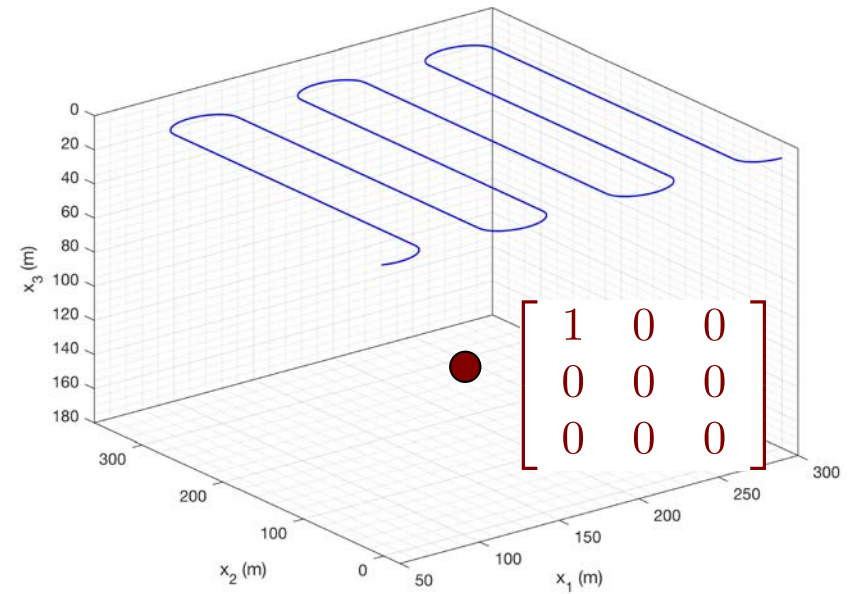
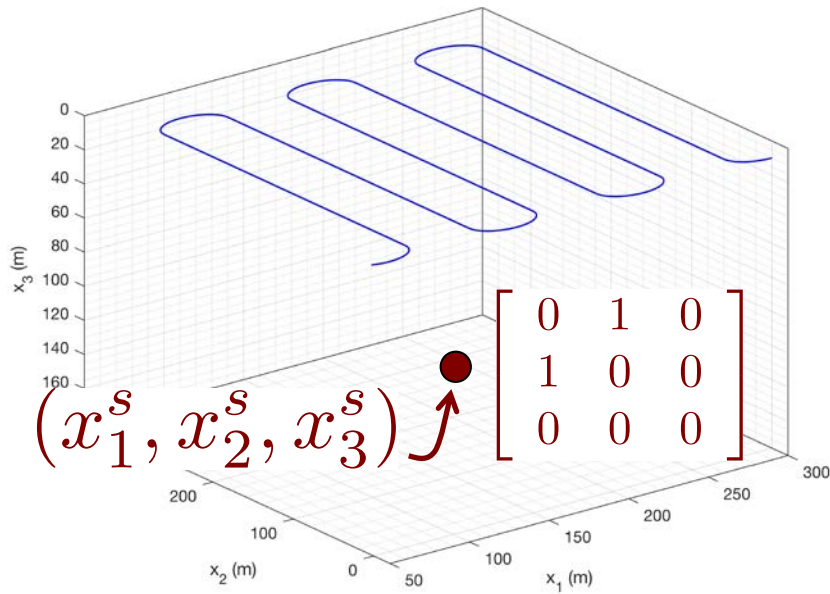
Dynamic results: obliquely incident P-wave

Modeling DAS responses

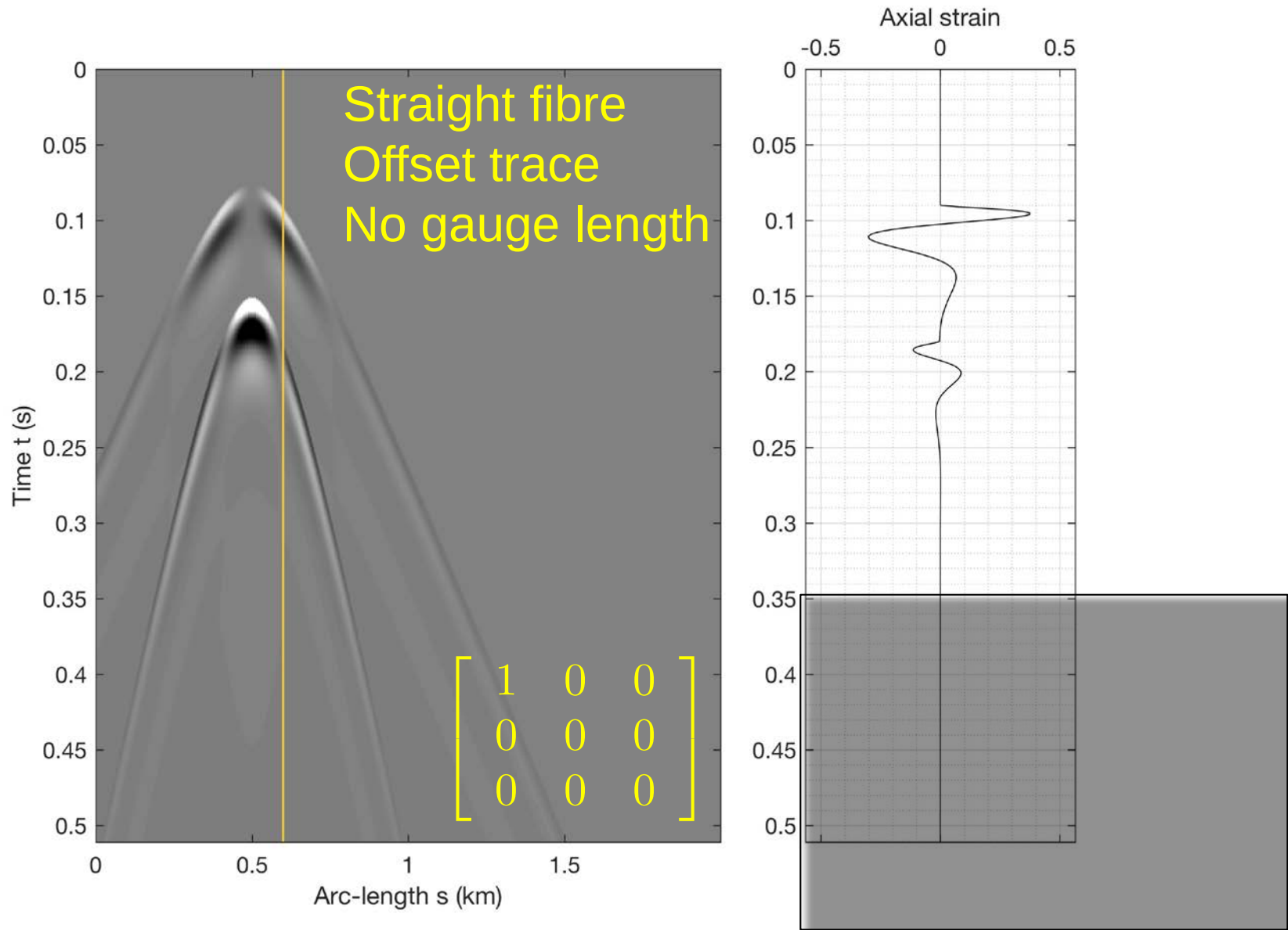


Dynamic results: plane Rayleigh wave

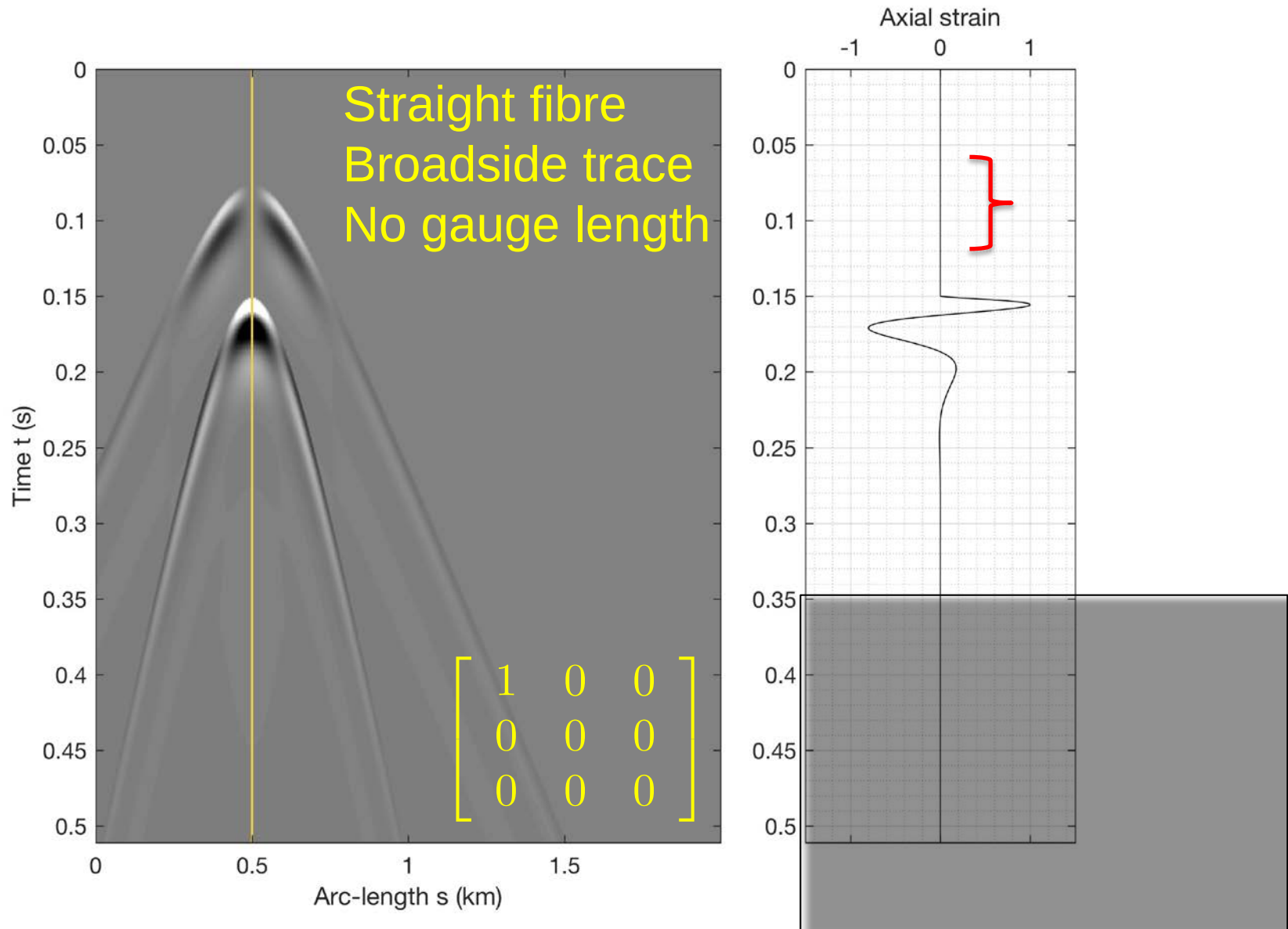
Modeling DAS responses



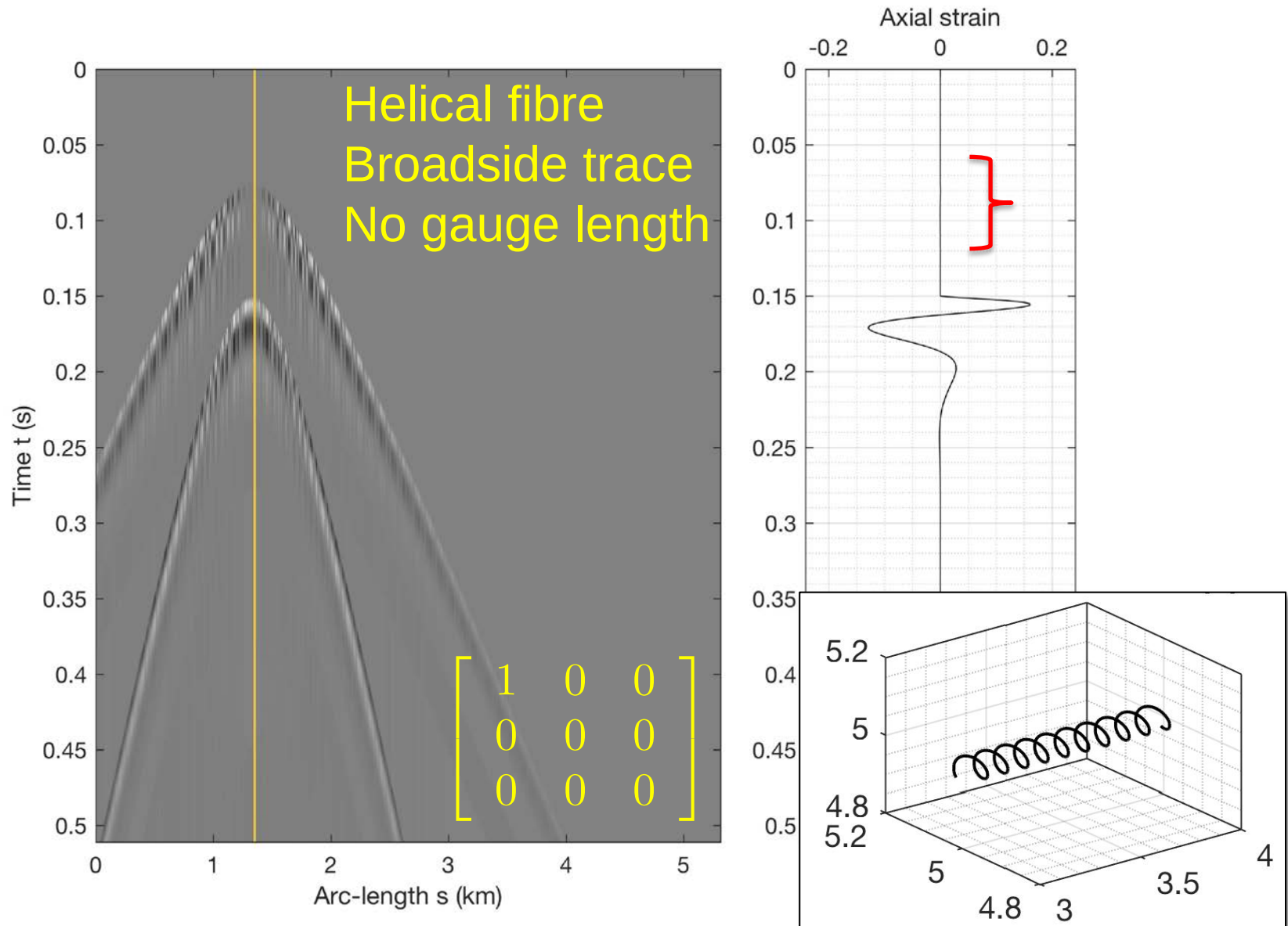
Modeling DAS responses



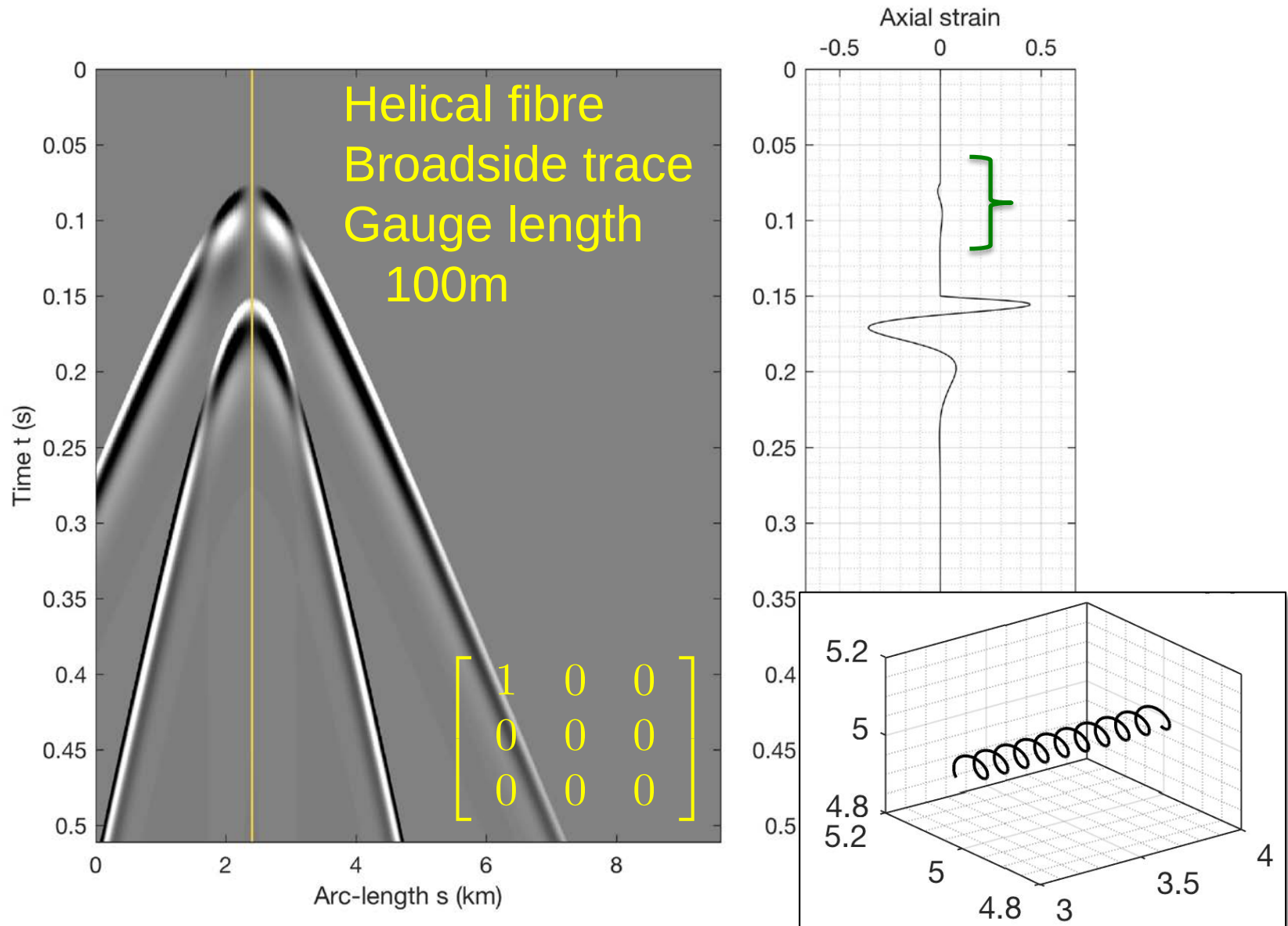
Modeling DAS responses



Modeling DAS responses



Modeling DAS responses



Design criteria

$$\mathbf{d}^P = \begin{bmatrix} \mathbf{d}^P(\theta_1) \\ \mathbf{d}^P(\theta_2) \\ \mathbf{d}^P(\theta_3) \\ \vdots \end{bmatrix} \quad \mathbf{d}^P(\theta_1) = \begin{bmatrix} \mathbf{d}_{t_1}^P(\theta_1) \\ \mathbf{d}_{t_2}^P(\theta_1) \\ \mathbf{d}_{t_3}^P(\theta_1) \\ \vdots \end{bmatrix} \quad \mathbf{d}_{t_1}^P(\theta_1) = \begin{bmatrix} d_{t_1}^P(s_1, \theta_1) \\ d_{t_1}^P(s_2, \theta_1) \\ d_{t_1}^P(s_3, \theta_1) \\ \vdots \end{bmatrix}$$

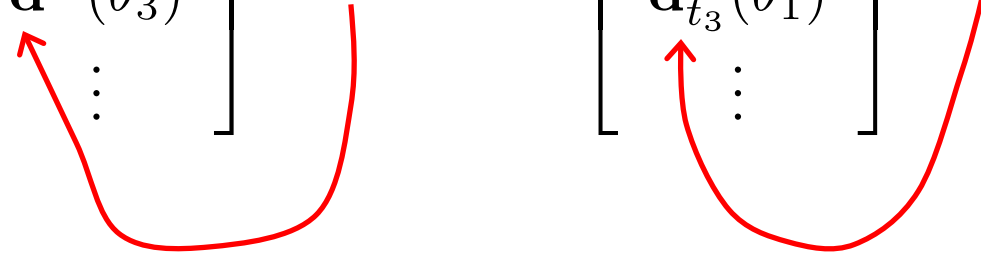
$$\chi_{\text{BR}}(\text{fibre parameters}) = (\mathbf{d}^S)^T (\mathbf{d}^R) + \lambda (\mathbf{d}^P)^T (\mathbf{d}^R)$$

Penalizing similarity of
response between
ground roll and S-waves

Penalizing similarity of
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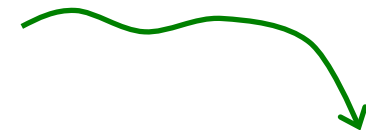
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$$\chi_{\text{BR}}(\text{fibre parameters}) = (\mathbf{d}^S)^T (\mathbf{d}^R) + \lambda (\mathbf{d}^P)^T (\mathbf{d}^R)$$

Penalizing strength of
ground roll



$$\chi_{\text{RR}}(\text{fibre parameters}) = (\mathbf{d}^R)^T (\mathbf{d}^R)$$

Conclusions

- **DAS data “character”** depends on fibre material, shape, source, gauge length, channel spacing, interrogator features, etc.; collect within a response model, CREWES-FGSM
- Uses: directionality, 6C sensing, **fibre design**
- **Library of characteristic responses** to ground roll, body waves (then, up- and down-going waves, separate P- and S-waves, etc.)
- Next: investigate a 2-step process:
 - i. Design fibre (determine λ_i) maximizing separability
 - ii. Teach fibre to suppress or discriminate during processing (machine learning?), or real time?