Frequency domain elastic FWI for VTI media

Junxiao Li*

Kris Innanen, Wenyong Pan, Geng Yu

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Outline

- 1 Introduction
- 2 Frequency domain forward modeling
- 3 Gradient direction
- 4 Examples
- 5 Conclusions

Introduction

- Frequency domain finite difference method, fast approach for multi-source and multi-receiver acquisition.
- Step length calculation for multi-parameter FWI.
- Low convergence rate of inversion.
- Simultaneous elastic constants reconstruction.

Frequency domain forward modeling

Second-order wave equation:

$$-\rho\omega^2 u_x = \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} + f_x$$

$$-\rho\omega^2 u_y = \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} + f_y$$

$$-\rho\omega^2 u_z = \frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} + f_z$$

$$c_{VTI} = \begin{bmatrix} c_{11} & c_{13} & 0 \\ c_{13} & c_{33} & 0 \\ 0 & 0 & c_{44} \end{bmatrix}$$

The 2D elastic wave equations for VTI medium:

$$-\rho\omega^2 \tilde{u}_x = \frac{\partial}{\partial x}\left(c_{11}\frac{\partial u_x}{\partial x} + c_{13}\frac{\partial u_z}{\partial z}\right) + \frac{\partial}{\partial z}\left(c_{44}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\right) + \tilde{f}_x(\omega)$$

$$-\rho\omega^2 \tilde{u}_z = \frac{\partial}{\partial z}\left(c_{13}\frac{\partial u_x}{\partial x} + c_{33}\frac{\partial u_z}{\partial z}\right) + \frac{\partial}{\partial x}\left(c_{44}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\right) + \tilde{f}_z(\omega)$$

Frequency domain forward modeling

The 2D elastic wave equations for VTI medium:

$$\begin{bmatrix} W_{xx}(\mathbf{x},\omega) & W_{xz}(\mathbf{x},\omega) \\ W_{zx}(\mathbf{x},\omega) & W_{zz}(\mathbf{x},\omega) \end{bmatrix} \begin{bmatrix} \tilde{u}_x(\mathbf{x},\omega) \\ \tilde{u}_z(\mathbf{x},\omega) \end{bmatrix} = \begin{bmatrix} \tilde{f}_x(\mathbf{x},\omega) \\ \tilde{f}_z(\mathbf{x},\omega) \end{bmatrix}$$

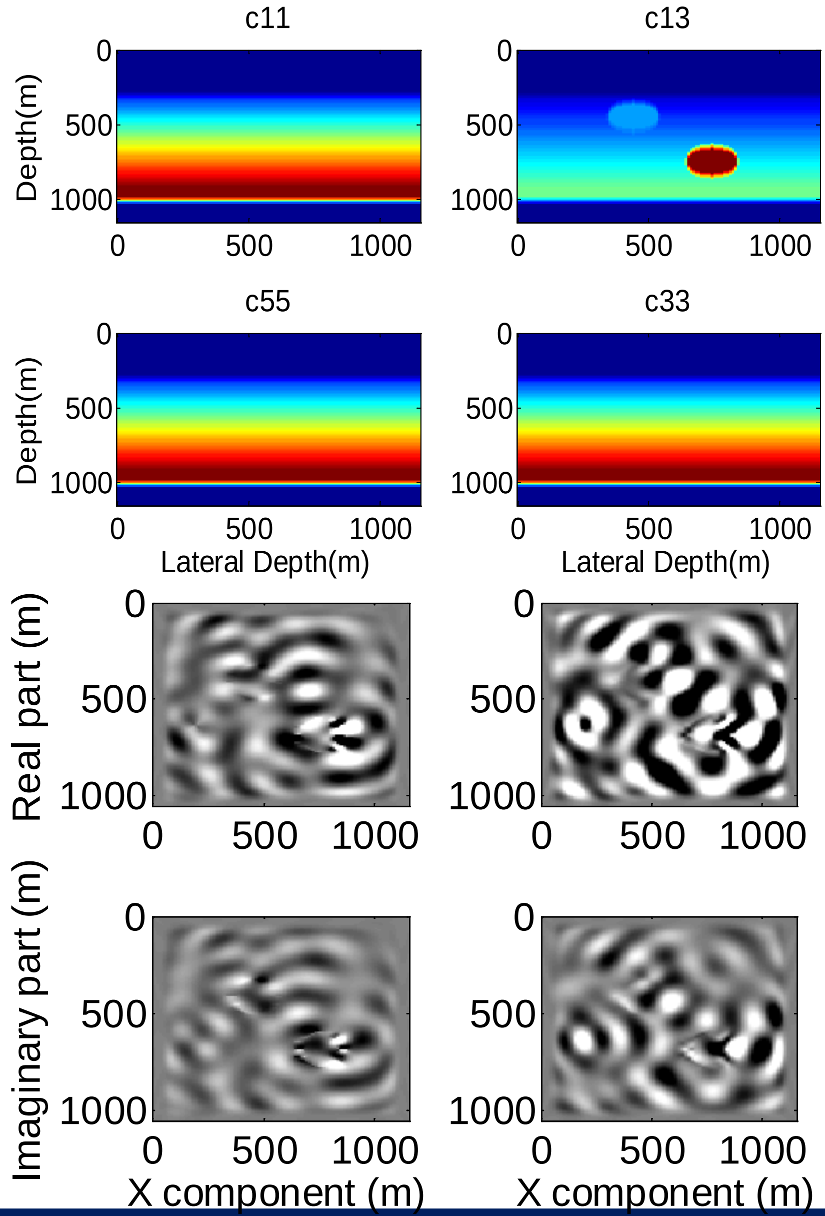
$$W_{xx}(\mathbf{x},\omega) = -\rho(\mathbf{x})\omega^2 - \frac{\partial}{\partial x}c_{11}\frac{\partial}{\partial x} - \frac{\partial}{\partial z}c_{44}\frac{\partial}{\partial z}$$

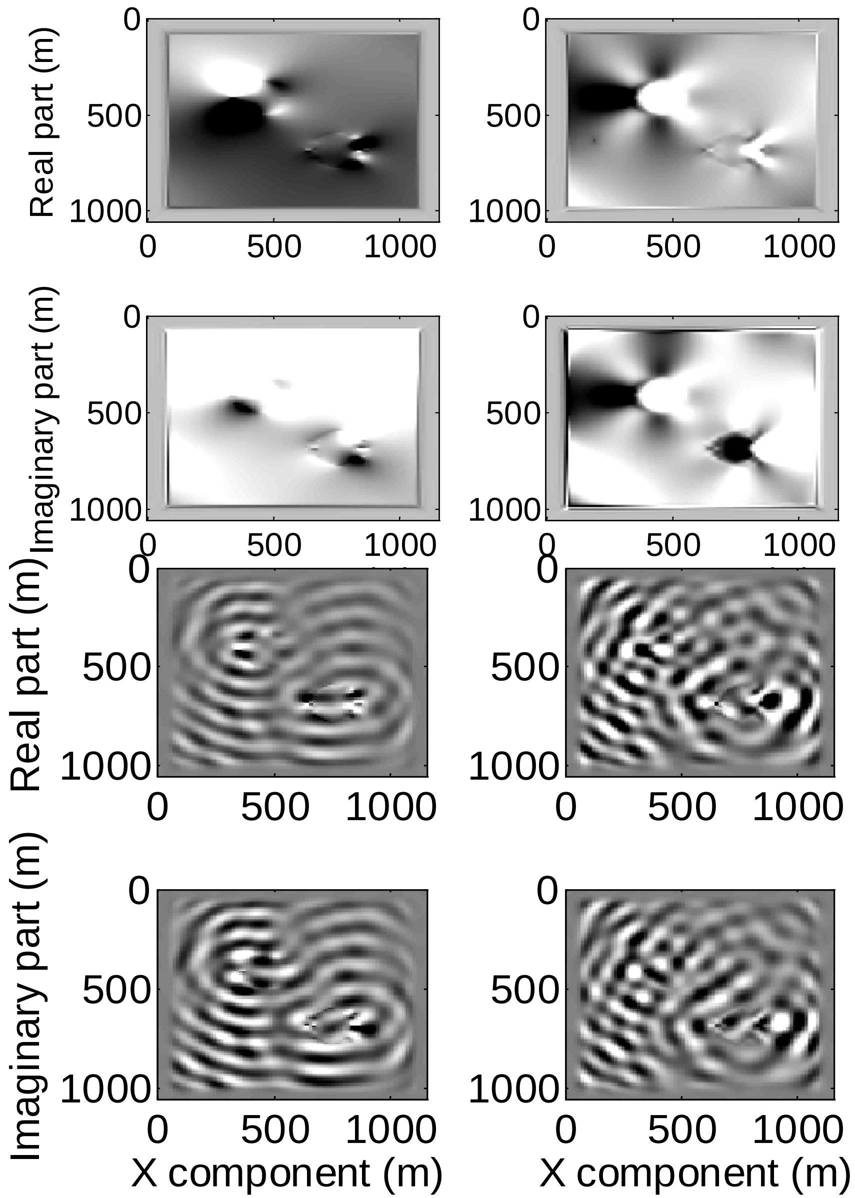
$$W_{xz}(\mathbf{x},\omega) = -\frac{\partial}{\partial x}c_{13}\frac{\partial}{\partial z} - \frac{\partial}{\partial z}c_{44}\frac{\partial}{\partial x}$$

$$W_{zx}(\mathbf{x},\omega) = -\frac{\partial}{\partial z}c_{13}\frac{\partial}{\partial x} - \frac{\partial}{\partial x}c_{44}\frac{\partial}{\partial z}$$

$$W_{zz}(\mathbf{x},\omega) = -\rho(\mathbf{x})\omega^2 - \frac{\partial}{\partial z}c_{33}\frac{\partial}{\partial z} - \frac{\partial}{\partial x}c_{44}\frac{\partial}{\partial x}$$

Frequency domain forward modeling





Gradient direction

The general relation between the model **m** and data **u**: $\mathbf{u} = g(\mathbf{m})$

The objective function: $E(\mathbf{m}) = \frac{1}{2}\sum_{\omega}\sum_{\mathbf{s}}[\mathbf{u}-\mathbf{d}]^T[\mathbf{u}-\mathbf{d}]^*$

For a given initial model $\mathbf{m}_0$:

$$E(\mathbf{m}_0 + \delta\mathbf{m}) = E(\mathbf{m}_0) + \nabla_{\mathbf{m}} E(\mathbf{m}_0)^T \delta\mathbf{m} + 1/2\delta\mathbf{m}^T \nabla^2_{\mathbf{m}} E(\mathbf{m}_0)\delta\mathbf{m} + O(\delta\mathbf{m}^2)$$

$$\nabla_{\mathbf{m}} E(\mathbf{m}_0) = \nabla^2_{\mathbf{m}} E(\mathbf{m}_0)\delta\mathbf{m} = H(\mathbf{m}_0)\delta\mathbf{m}$$

$$\nabla_{m_k} E(m_k) = \sum_{\omega}\sum_{\mathbf{s}} Re\left[\left(\frac{\partial\tilde{\mathbf{u}}}{\partial m_k}\right)^T \left(\tilde{\mathbf{u}}-\tilde{\mathbf{d}}\right)^*\right]$$

Gradient direction

Derivative of parameter m_k: $$\frac{\partial \tilde{\mathbf{u}}}{\partial m_k} = \mathbf{W}^{-1}\tilde{\mathbf{f}}_k$$

Virtual source: $$\tilde{\mathbf{f}}_k = -\frac{\partial \mathbf{W}}{\partial m_k}\tilde{\mathbf{u}}$$

Gradient for each parameter **m_k** :

$$\begin{aligned}\nabla_{m_k} E(m_k) &= \sum_\omega \sum_\mathbf{s} Re\left[(\tilde{\mathbf{f}}_k)^T(\mathbf{W}^{-1})^T\left(\tilde{\mathbf{u}}-\tilde{\mathbf{d}}\right)^*\right] \\ &= -\sum_\omega \sum_\mathbf{s} Re\left[\tilde{\mathbf{u}}^T \frac{\partial \mathbf{W}^T}{\partial m_k}(\mathbf{W}^{-1})^T\left(\tilde{\mathbf{u}}-\tilde{\mathbf{d}}\right)^*\right]\end{aligned}$$

$$\Delta W_{xx}(\mathbf{x},\omega)/\Delta c_{11} = -\frac{\partial}{\partial x}\frac{\partial}{\partial x}$$

$$\Delta W_{xz}(\mathbf{x},\omega)/\Delta c_{11} = 0$$

$$\Delta W_{zx}(\mathbf{x},\omega)/\Delta c_{11} = 0$$

$$\Delta W_{zz}(\mathbf{x},\omega)/\Delta c_{11} = 0$$

Pseudo-Hessian matrix

$$\nabla_{m_k} E(m_k) = \sum_\omega \left(\frac{\sum_\mathbf{s} Re\left[(\tilde{\mathbf{f}}_k)^T(\mathbf{W}^{-1})^T\left(\tilde{\mathbf{u}}-\tilde{\mathbf{d}}\right)^*\right]}{\sum_\mathbf{s}\left[diag\left((\tilde{\mathbf{f}}_k)^T(\tilde{\mathbf{f}}_k)^*\right)+\lambda\mathbf{I}\right]}\right)$$

Gradient direction

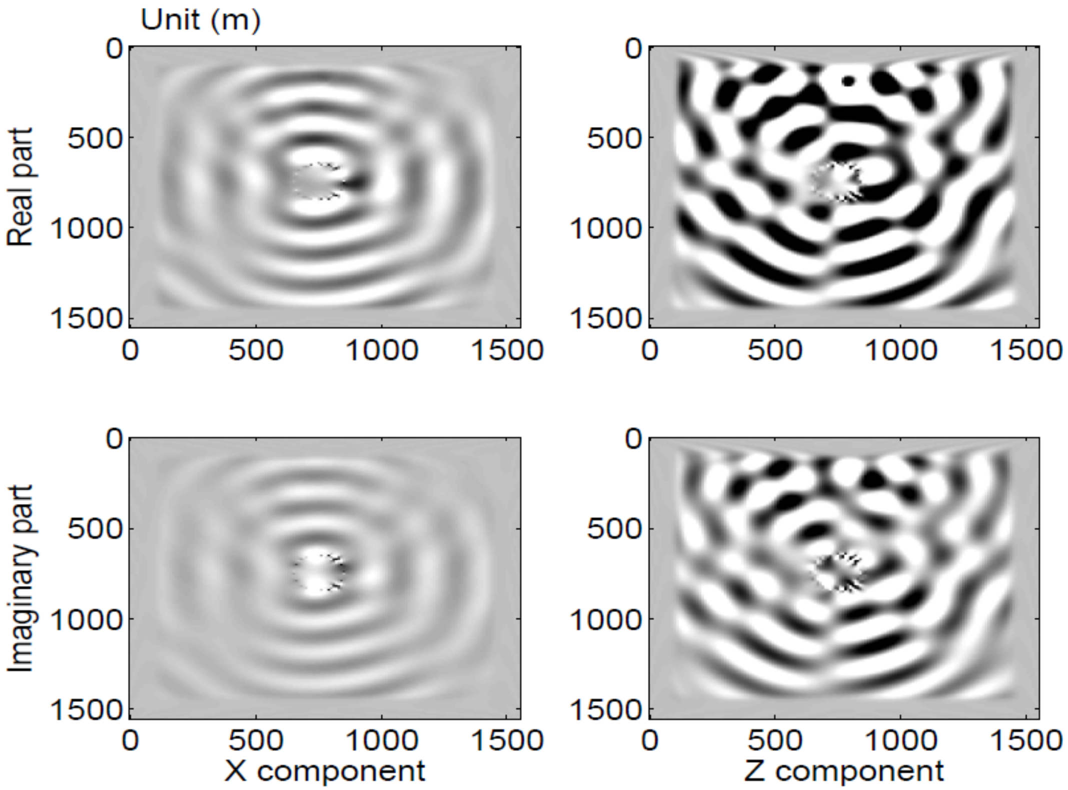


FIG. 1. The wavefields of this true model with a P-wave anomaly in the middle.

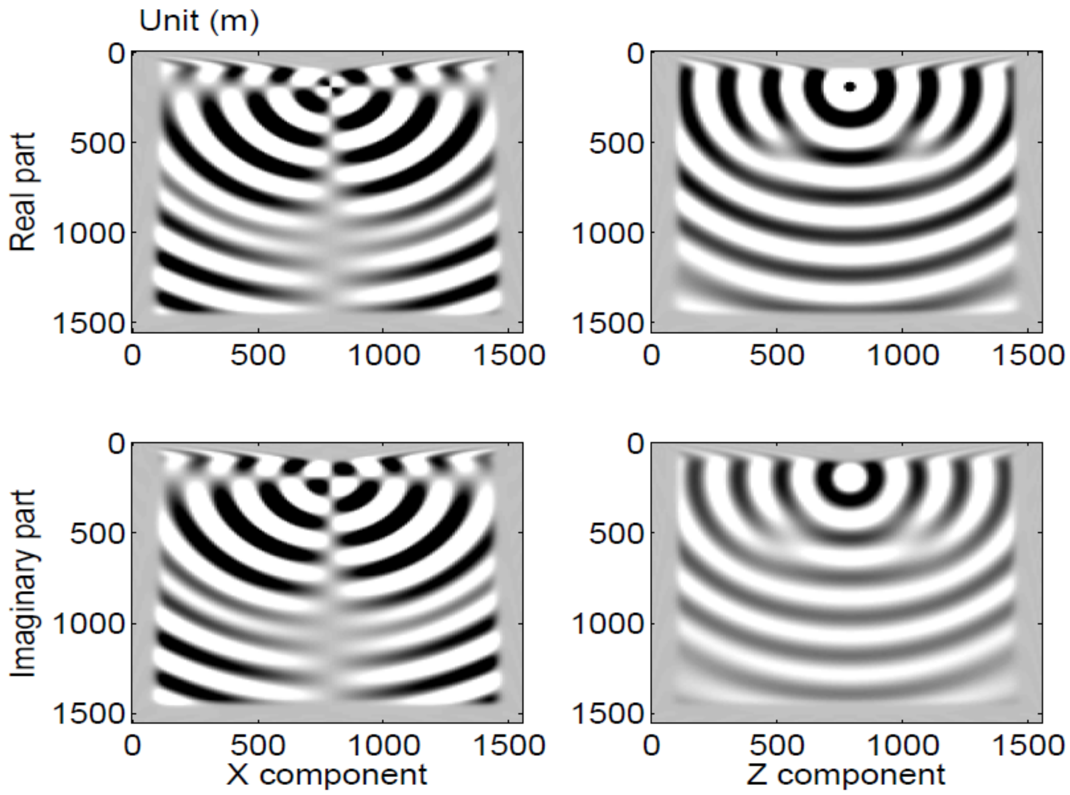


FIG. 2. The wavefields of a homogeneous initial model.

Gradient direction

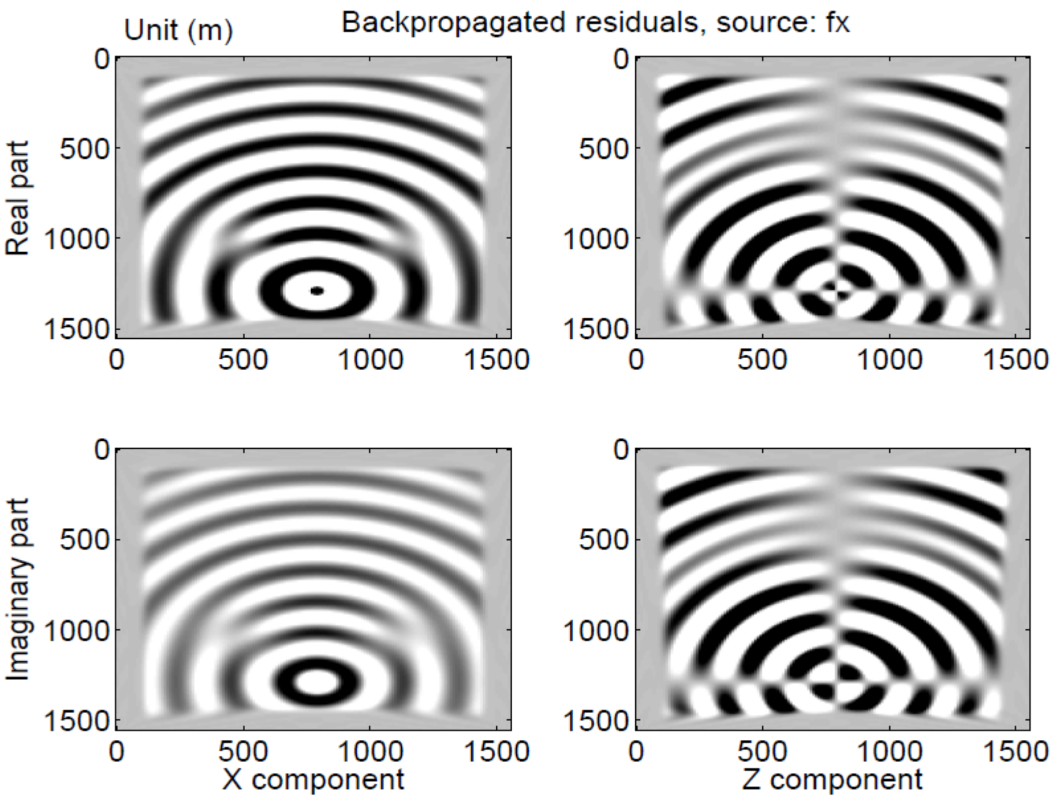


FIG. 3. The wavefields of an X-component back propagated source with a P-wave anomaly in the middle.

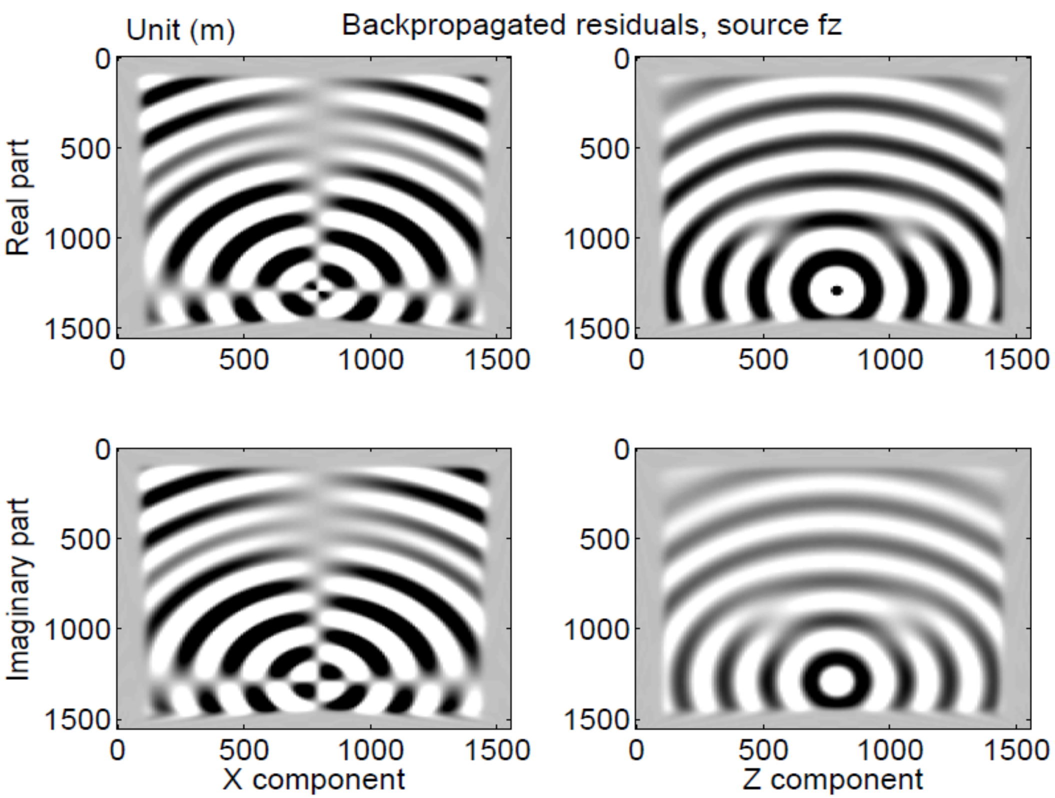
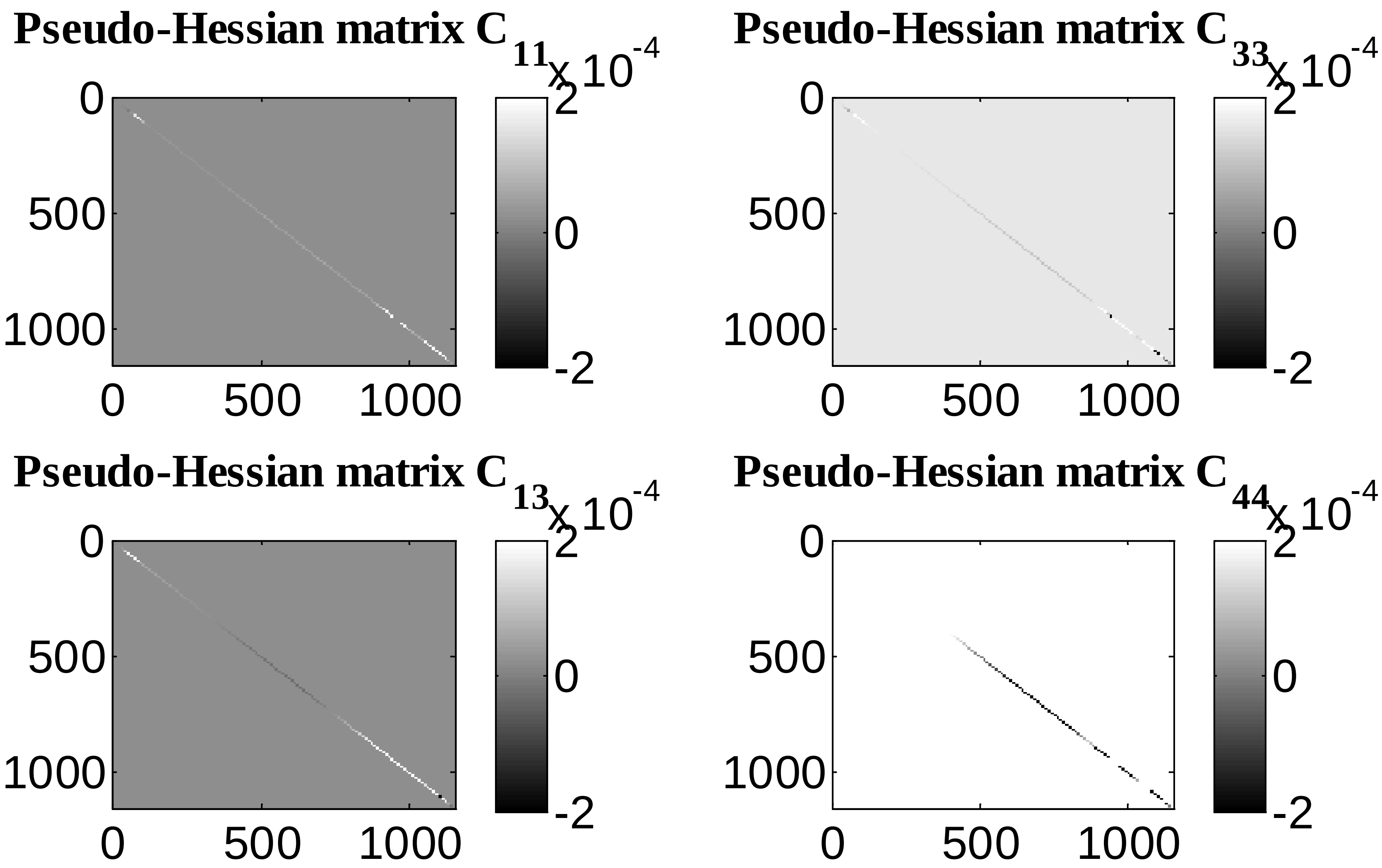


FIG. 4. The wavefields of n Z-component back propagated source in z-direction with a P-wave anomaly in the middle.

Gradient direction



Gradient direction

$$\nabla_{m_k} E(m_k) = \left(\begin{bmatrix} \frac{\Delta W_{xx}}{\Delta m_k} & \frac{\Delta W_{xz}}{\Delta m_k} \\ \frac{\Delta W_{zx}}{\Delta m_k} & \frac{\Delta W_{zz}}{\Delta m_k} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}_x \\ \tilde{\mathbf{u}}_z \end{bmatrix} \right)^T \begin{bmatrix} W_{xx}(\mathbf{x},\omega) & W_{xz}(\mathbf{x},\omega) \\ W_{zx}(\mathbf{x},\omega) & W_{zz}(\mathbf{x},\omega) \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{u}}_x - \tilde{\mathbf{d}}_x \\ \tilde{\mathbf{u}}_z - \tilde{\mathbf{d}}_z \end{bmatrix}^*$$

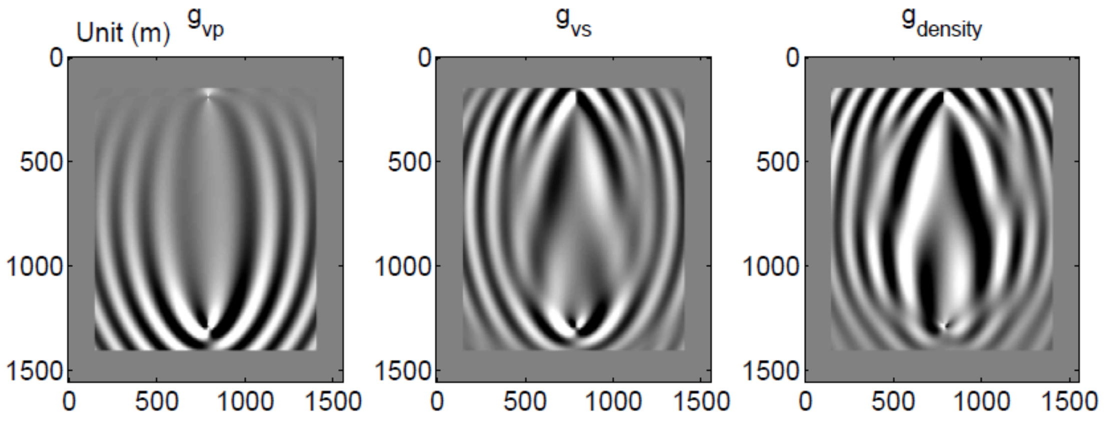


FIG. 5. Sensitivyty kernels for V_P, V_S and density in isotropic media.

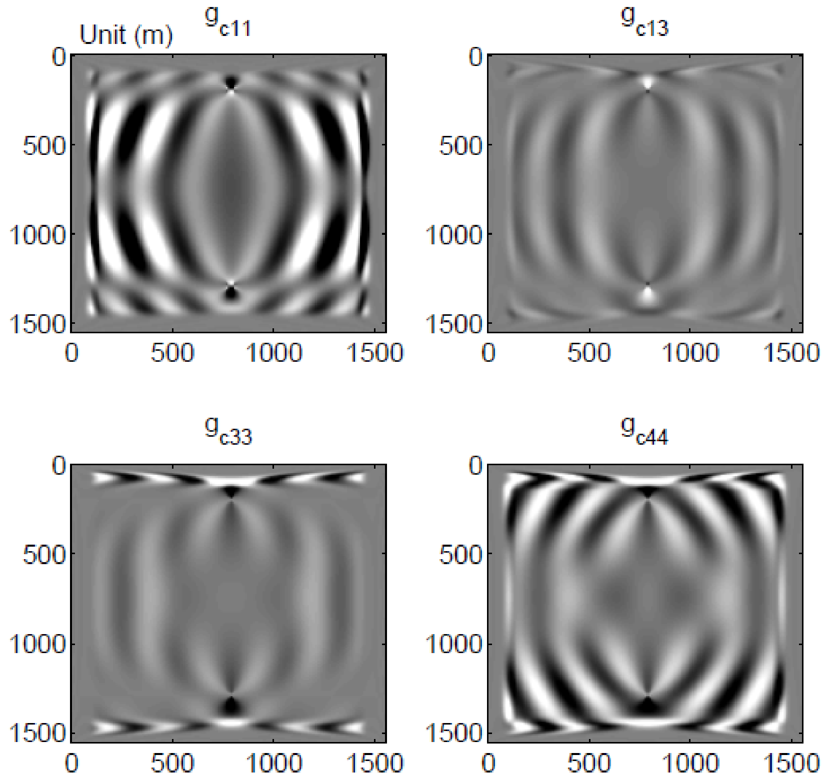


FIG. 6. Sensitivyty kernels for stiffness tensors in VTI media.

Modified quadratic interpolation method

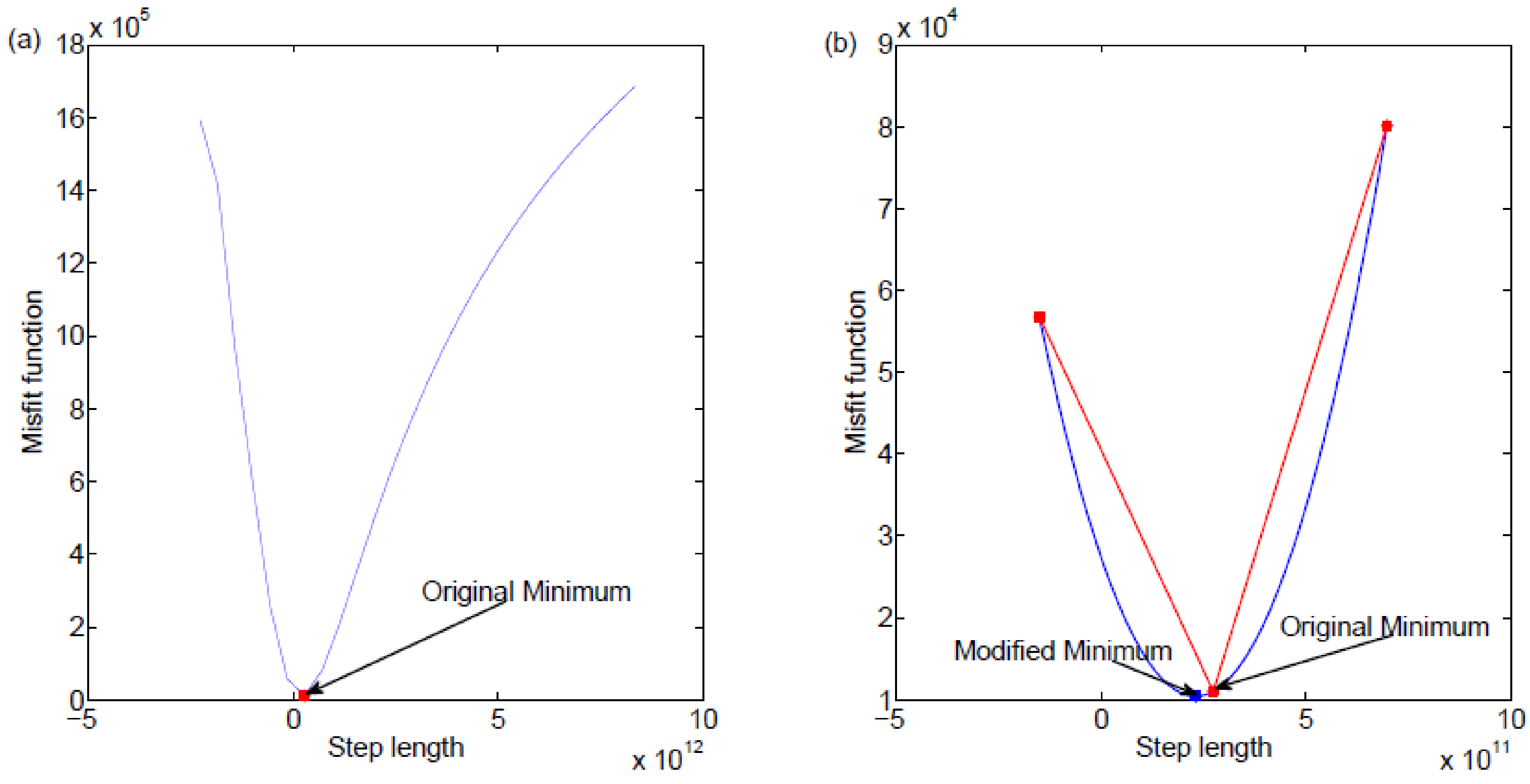


FIG. 7. Illustration that schematically outlines the principle of the modified quadratic interpolation step-length formula. (a) Original step length obtained by line search method and (b) Modified step length after interpolation.

Examples

$$c_{VTI} = \begin{bmatrix} 23.87 & 9.79 & 0 \\ 9.79 & 15.33 & 0 \\ 0 & 0 & 2.77 \end{bmatrix} \times 10^9 N/m^2,$$

$$c_{VTI} = \begin{bmatrix} 33.18 & 13.71 & 0 \\ 13.71 & 21.46 & 0 \\ 0 & 0 & 3.88 \end{bmatrix} \times 10^9 N/m^2$$

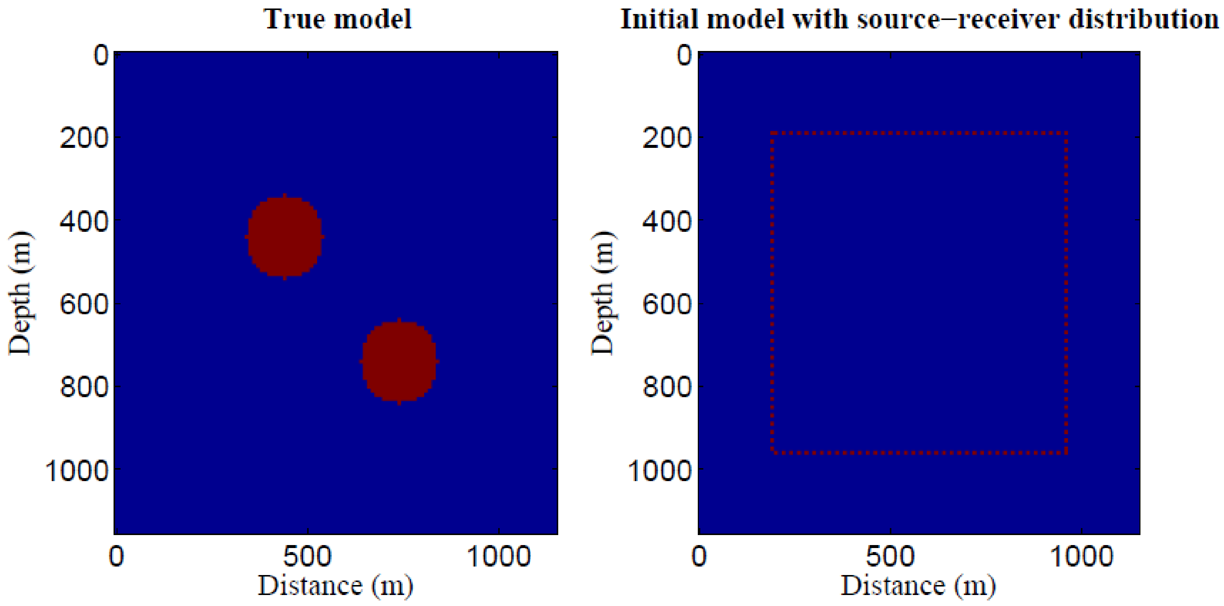


FIG. 8. True model (Left) and initial model with source-receiver distribution (Right).

Inversion results for C11

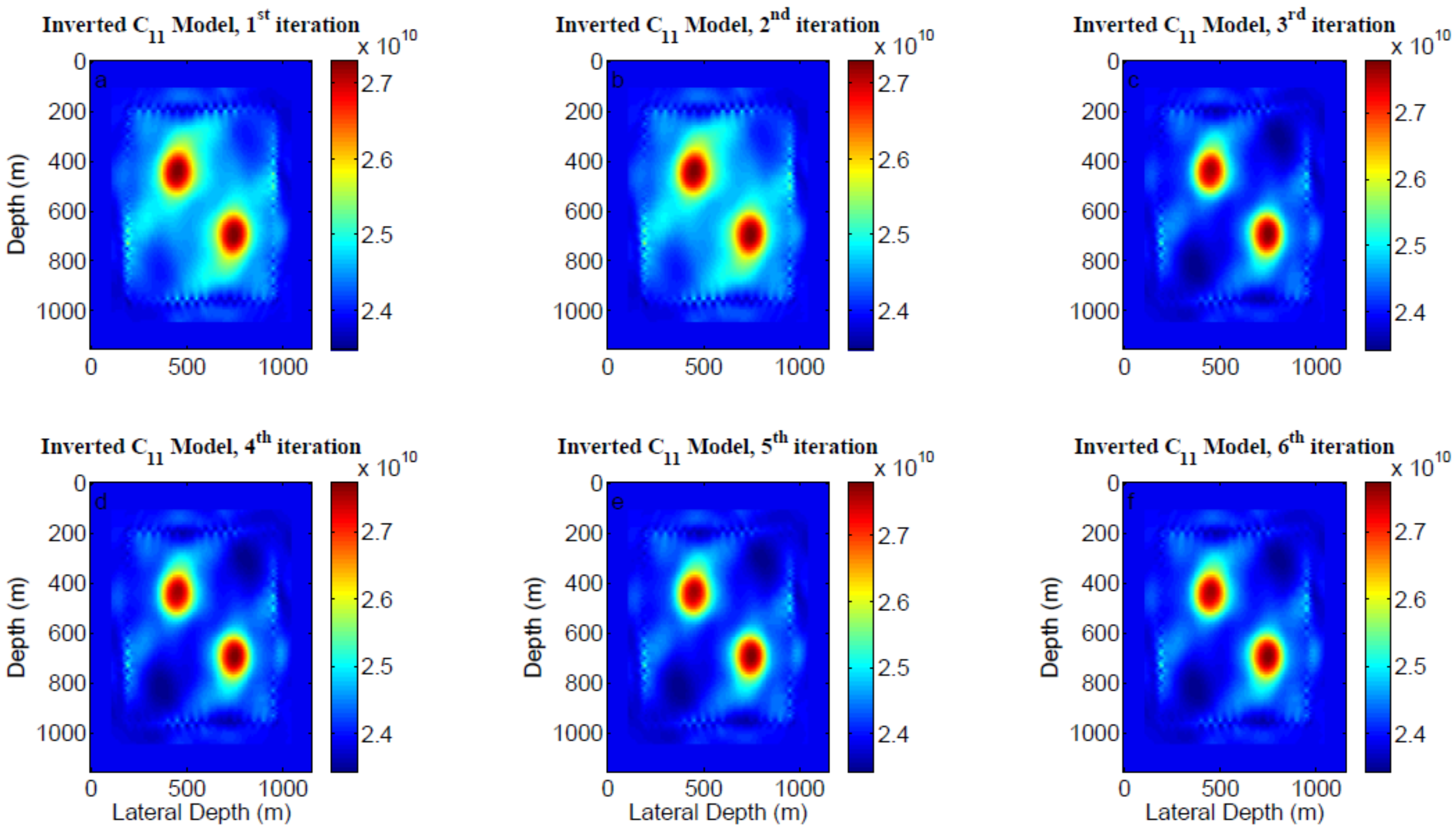


FIG. 9. Inversion results of c_{11} with the increase of iteration steps.

Inversion results for C33

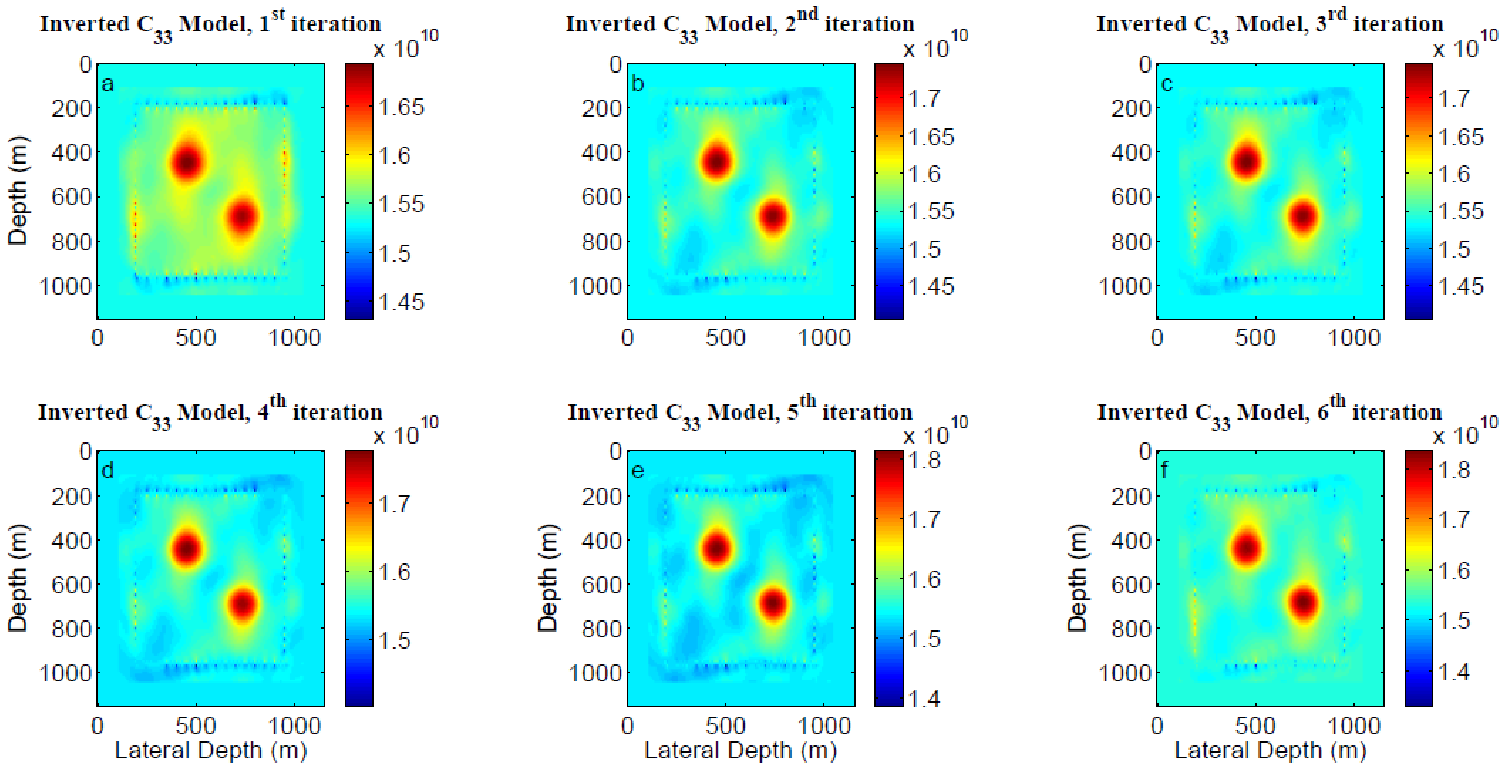
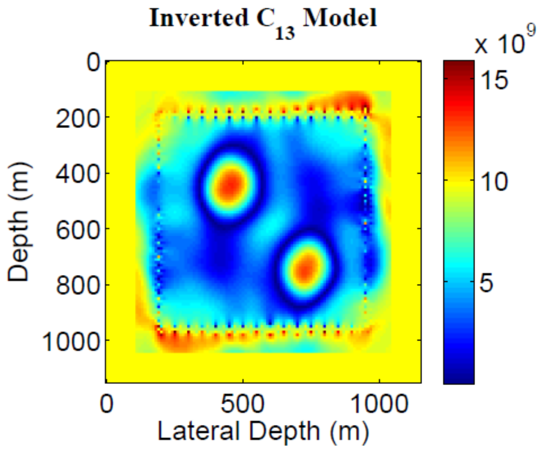
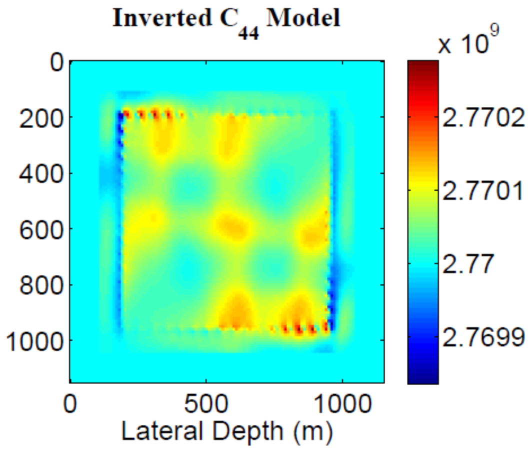
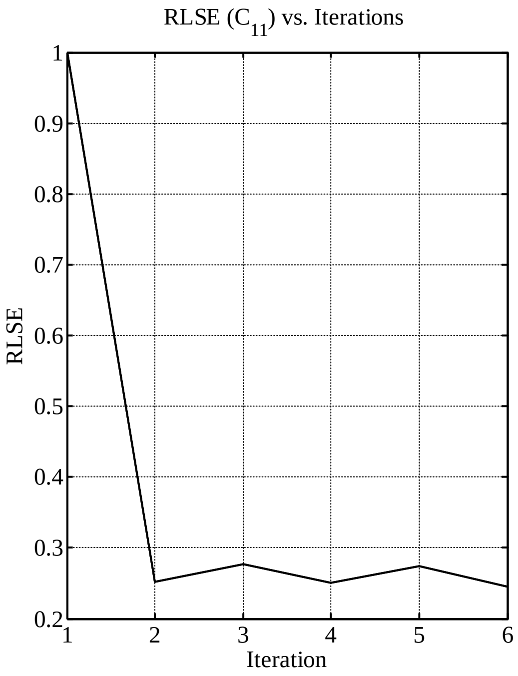


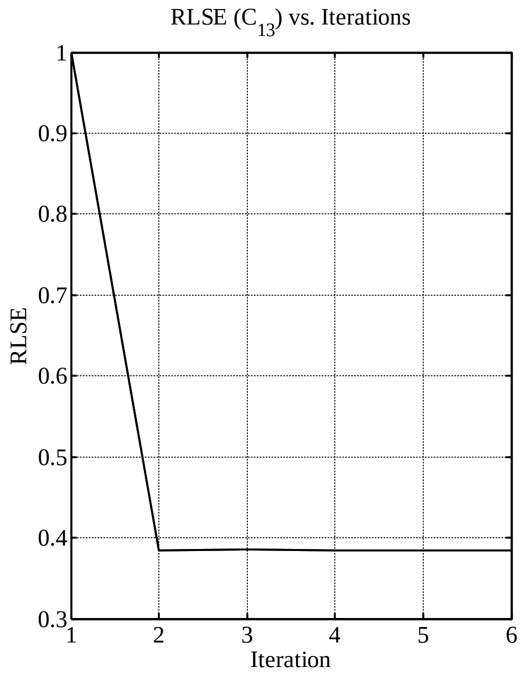
FIG. 10. Inversion results of c_{33} with the increase of iteration steps.

Inversion results for C44 and C13









Conclusions

- Gradient direction can be calculated in forms of matrix multiplication in frequency domain.
- To accelerate the convergence of inversion, the pseudo-Hessian matrix is applied to constrain the step length.
- The step length is calculated by a modified quadratic interpolation method.
- Simultaneous elastic constants reconstruction results show the parameter C_{11}, C_{33} and C_{13} can be obtained, yet the parameter C_{44} can not be inverted simultaneously.

Future work

- Step length method
- Different parameterizations
- Weighting misfit function
- Frequency VS. parameters

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Questions & Comments