Strategies for efficient multiparameter frequency domain QFWI

Scott Keating and Kris Innanen

Motivations

- Full waveform inversion (FWI) is becoming a powerful tool for velocity model building.
- There is significant potential for FWI considering additional parameters, such as Q.
- Cross-talk becomes a concern in these implementations.
- Cross-talk is often expensive to eliminate.
- More efficient approaches to preventing cross-talk make multiparameter FWI more feasible.

Full Waveform Inversion

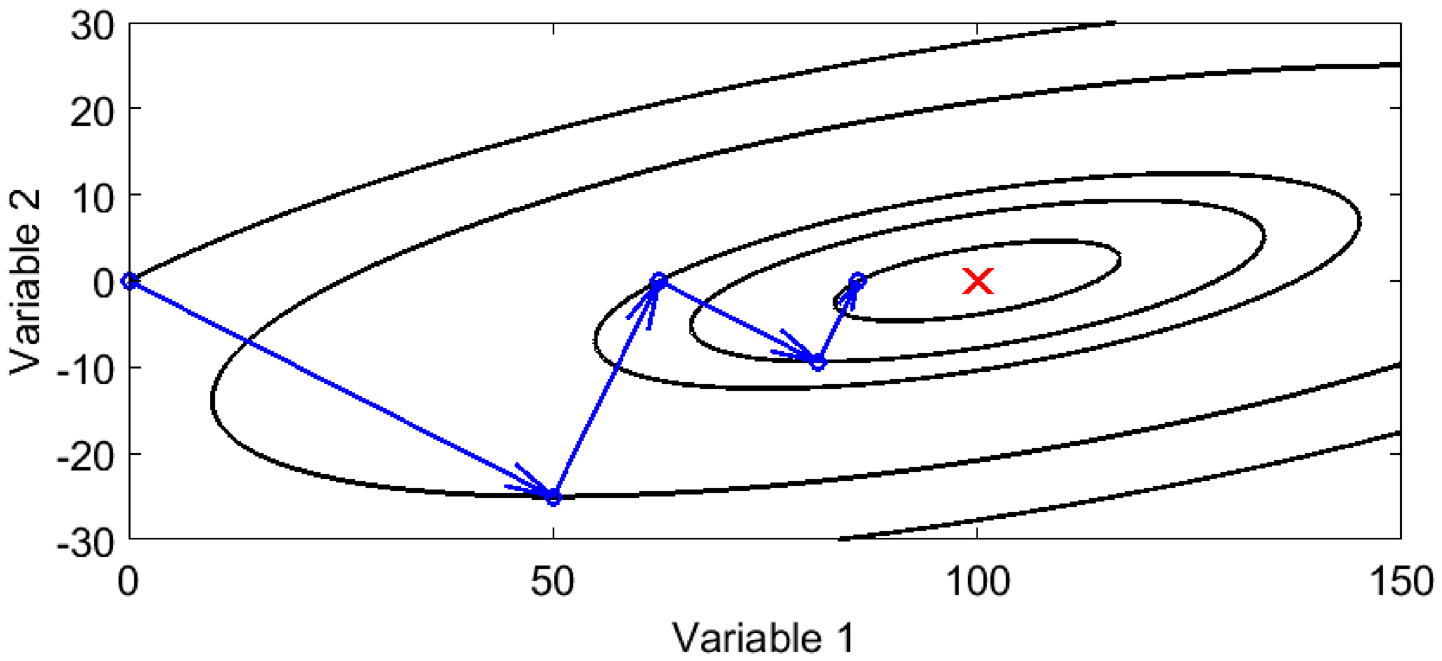
- FWI is an optimization problem which seeks to minimize discrepancy between measured data and synthetic data generated using the current model estimate.
- Ideally, the more similar the measured and synthetic data are, the closer the model estimate will be to the true subsurface.
- This discrepancy is usually characterized by a scalar objective function.

Optimization in FWI

- Data residuals introduced by an error in the estimate of one variable can be mistakenly attributed to another.
- When these variables represent the same physical parameter, this results in reduced resolution.
- When these variables represent different parameters, the result is cross-talk.
- These effects can be mitigated by considering more appropriate optimization techniques.

Cross-talk and optimization

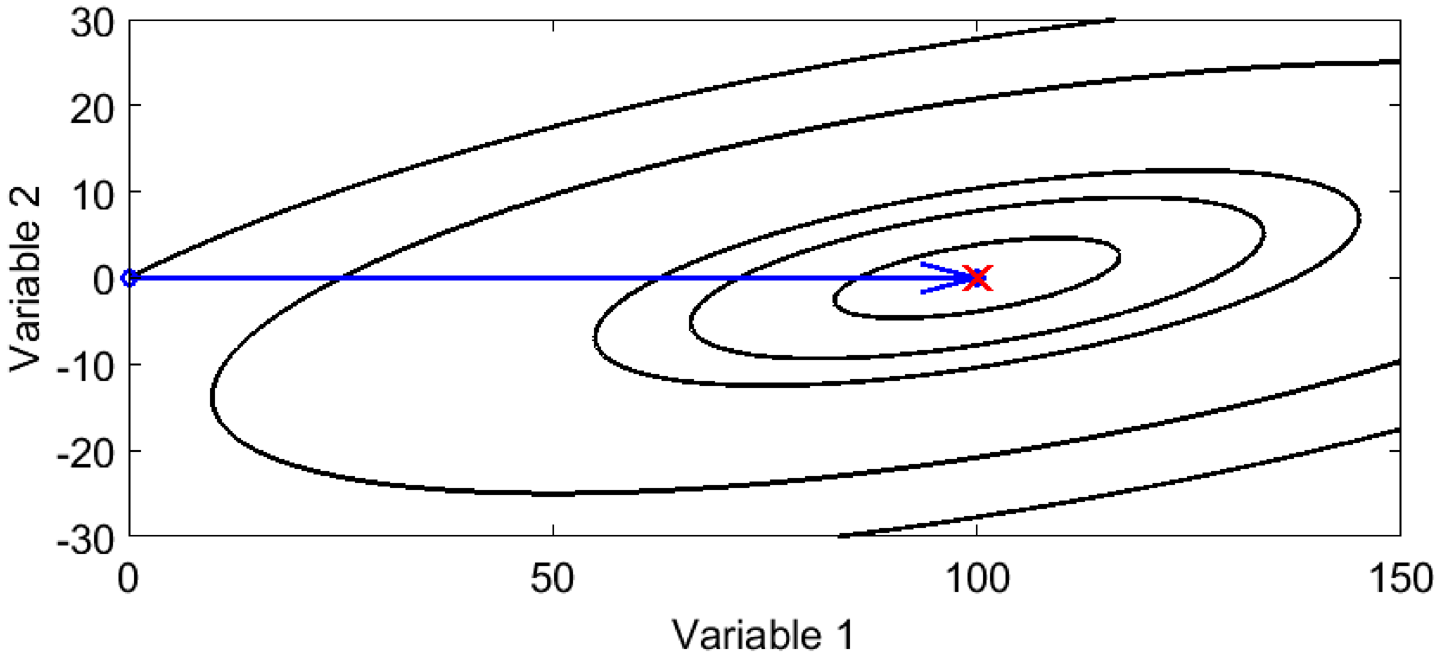
- Cross-talk is closely related with the way we update the model.
- Steepest descent optimization considers only the gradient (first derivatives) of the objective function.
  - If changes in several different variables can reduce the same part of the data residual, all will be changed in a steepest descent update.



$$m_k = m_{k-1} + \alpha * g$$

Cross-talk and optimization

- Cross-talk is closely related with the way we update the model.
- Newton optimization considers the gradient and Hessian (second derivatives).
  - By considering how the derivative with respect to one variable changes as another variable changes, we can avoid updating several variables to account for the residual introduced by one.



$$m_k = m_{k-1} + H^{-1} g$$

Cross-talk and optimization

- Newton optimization suppresses cross-talk, but requires that we solve the system

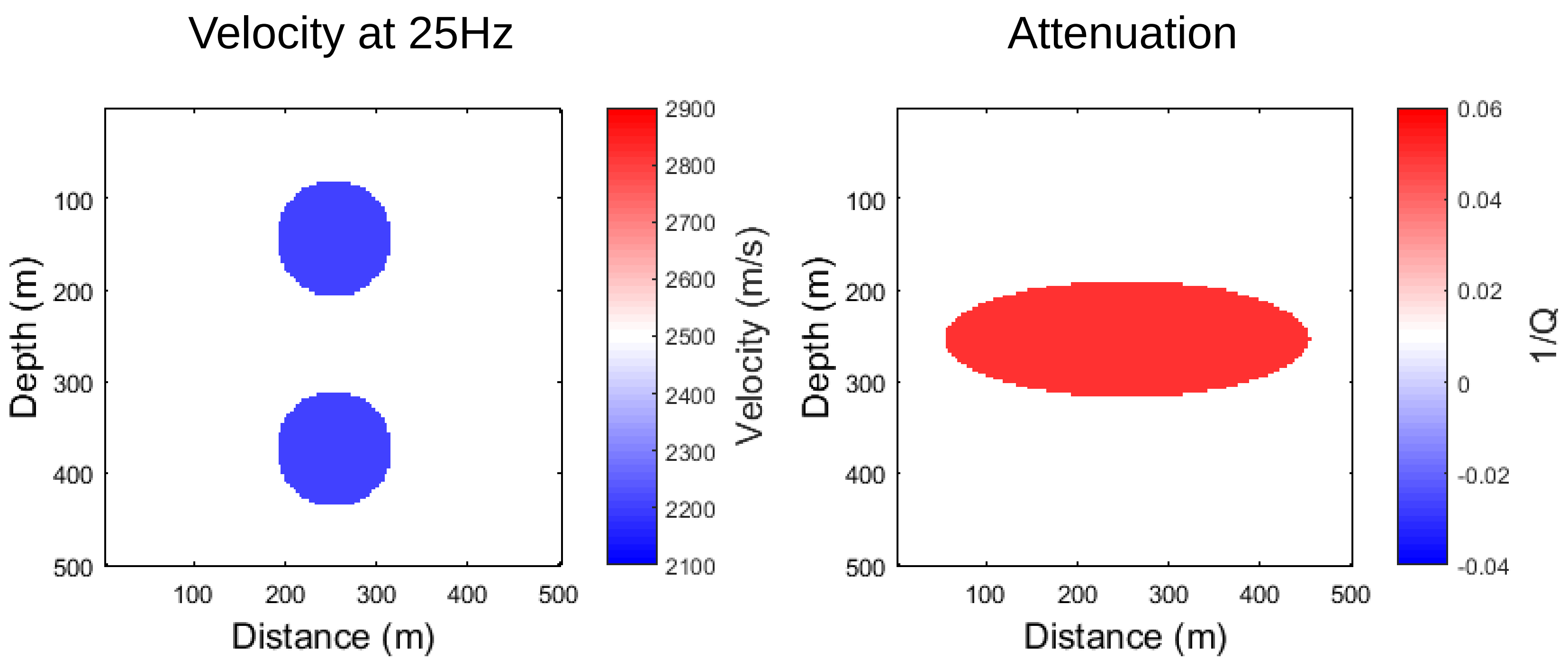
$$\mathbf{H}\mathbf{u} = -\mathbf{g}$$

- The cost of solving this system for N variables is about $O(N^3)$ computations.
- This is typically not achievable at the level of resolution desired in FWI.
- More practical are methods which approximate the Newton update, such as the truncated Newton method.

Challenges in reducing cross-talk

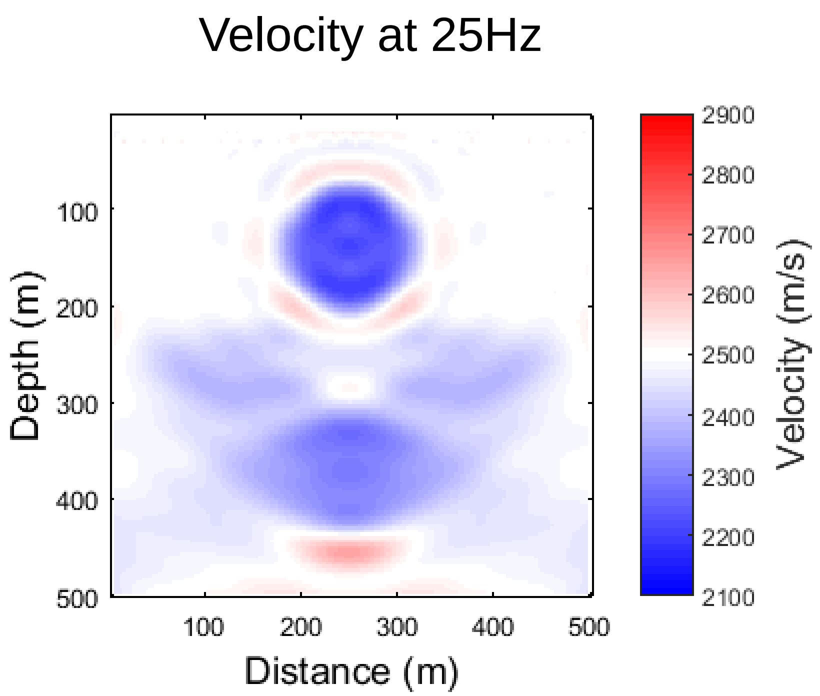
- Many optimization methods efficiently approximate the effect of the Hessian matrix.
- Much of the information introduced in this way affects resolution, but not cross-talk.
- This can be wasteful, as there are other ways to improve resolution.
- Introducing sufficient second order information to eliminate cross-talk may incur extreme expense.

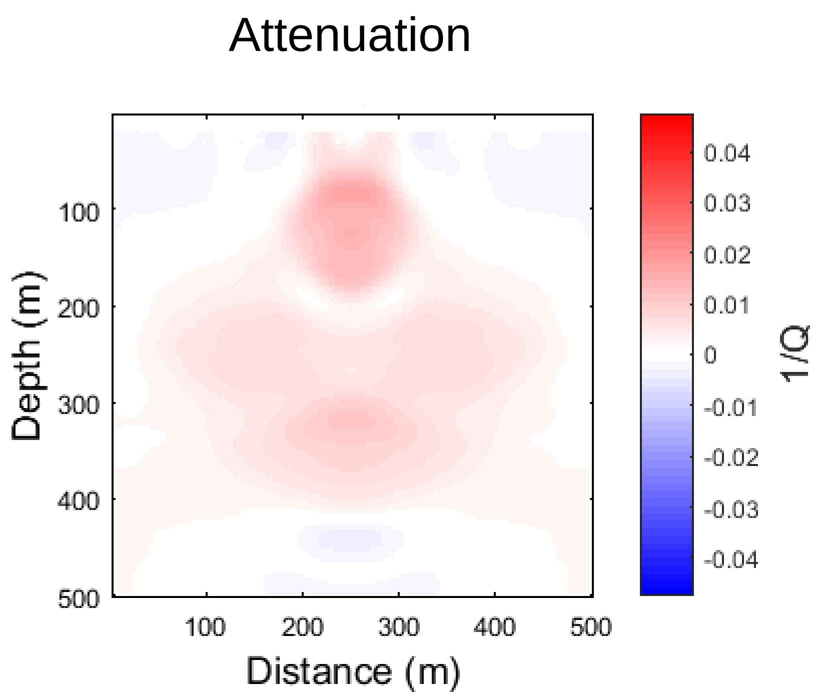
Numerical Examples



- Surface acquisition geometry
- Two dimensions
- Initial model constant

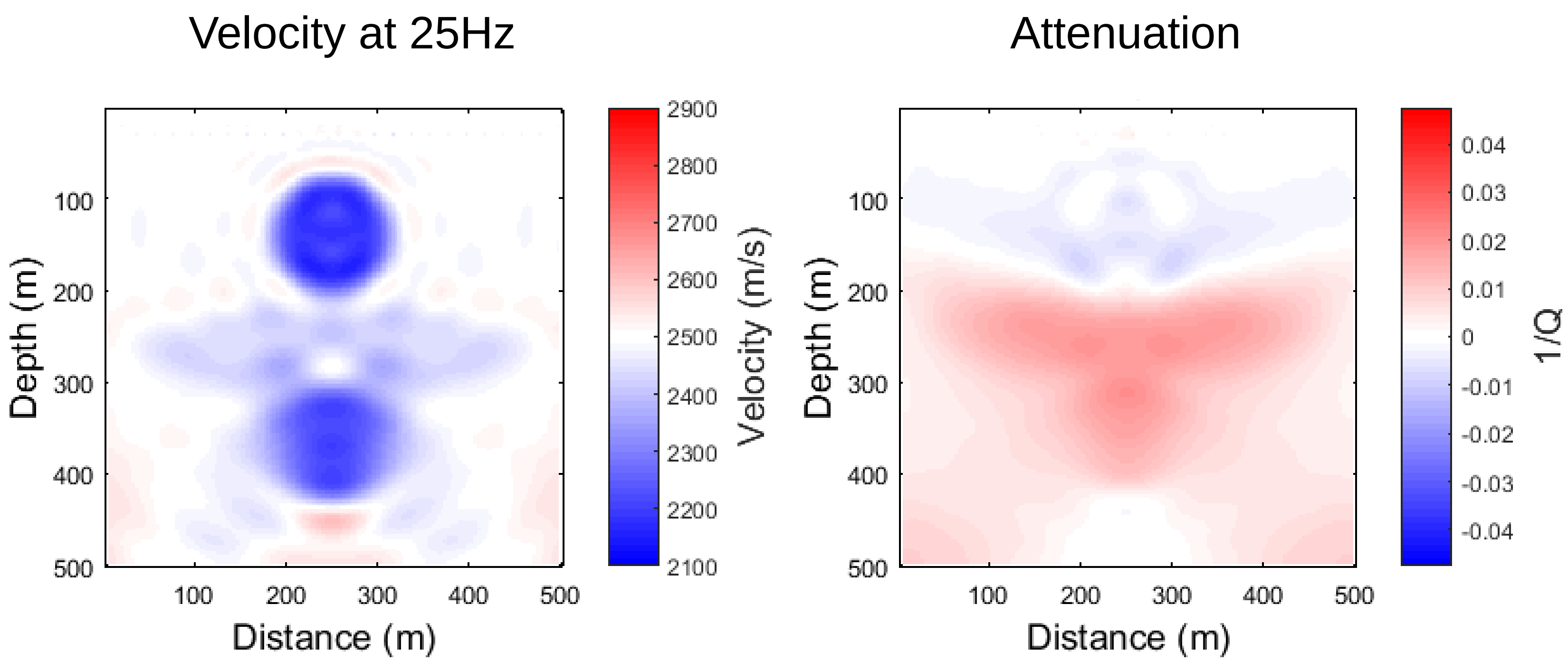
Steepest descent optimization





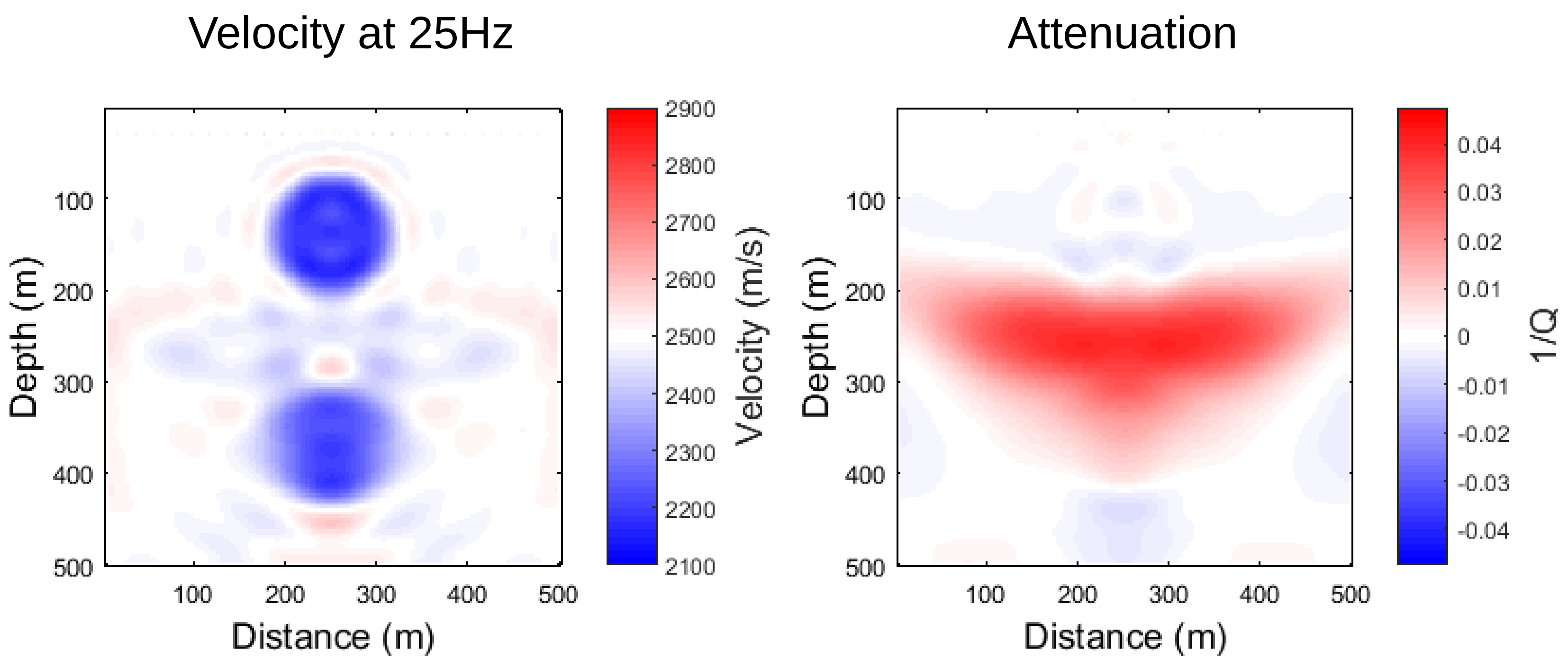
- 575 model updates
- No second derivative information considered

Truncated Newton optimization



- 25 model updates
- Approximate solution to Newton update used
- Increased cost per iteration

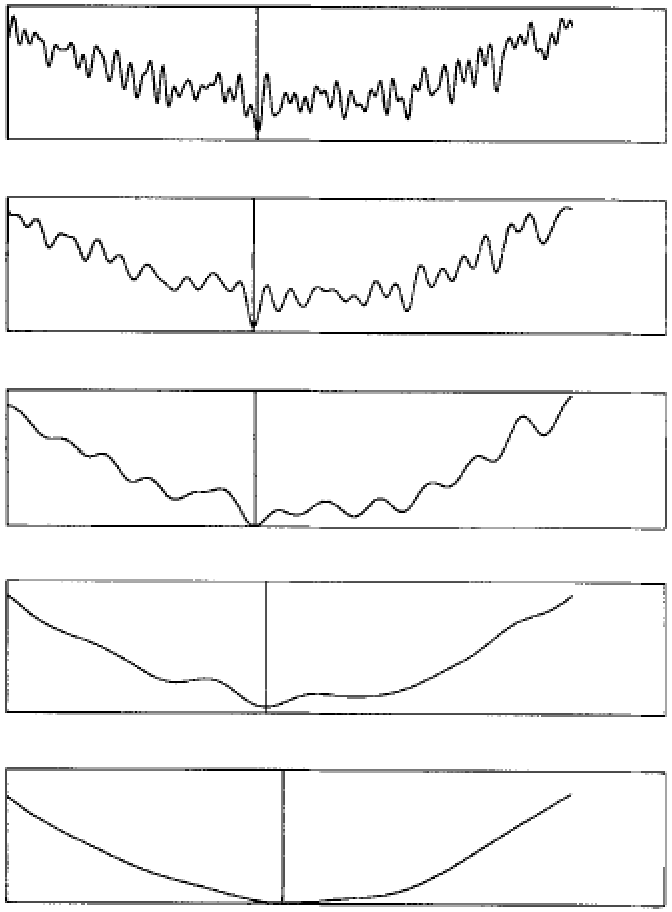
Truncated Newton optimization



- 50 model updates
- Approximate solution to Newton update used
- Further increased cost per iteration

Multi-resolution inversion

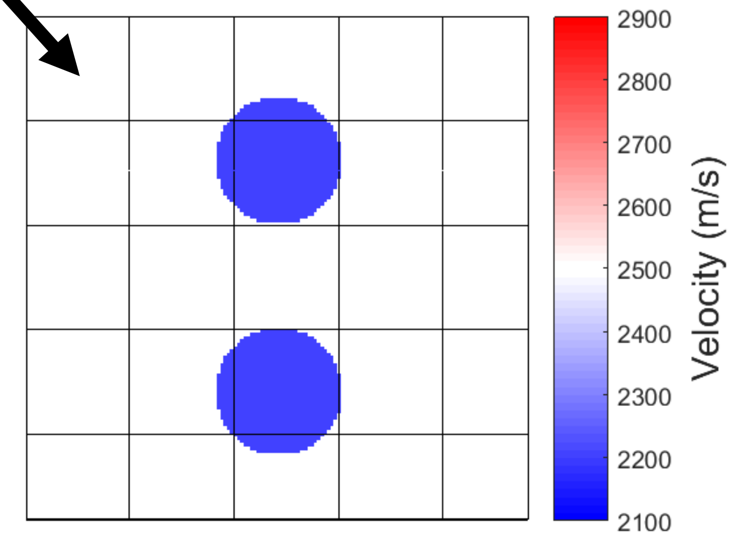
- Most formulations of FWI begin with low frequency data, then consider progressively higher frequencies.
- This helps to mitigate nonlinearity.
- Consequently, many more variables are usually defined than can be recovered.
- By eliminating unnecessary variables, more powerful optimization techniques become feasible.

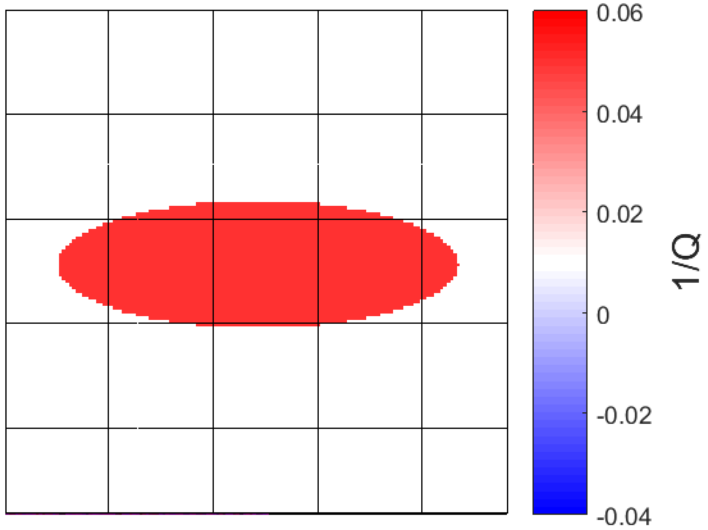


From Bunks et al., 1995

Seismic resolution at 3Hz

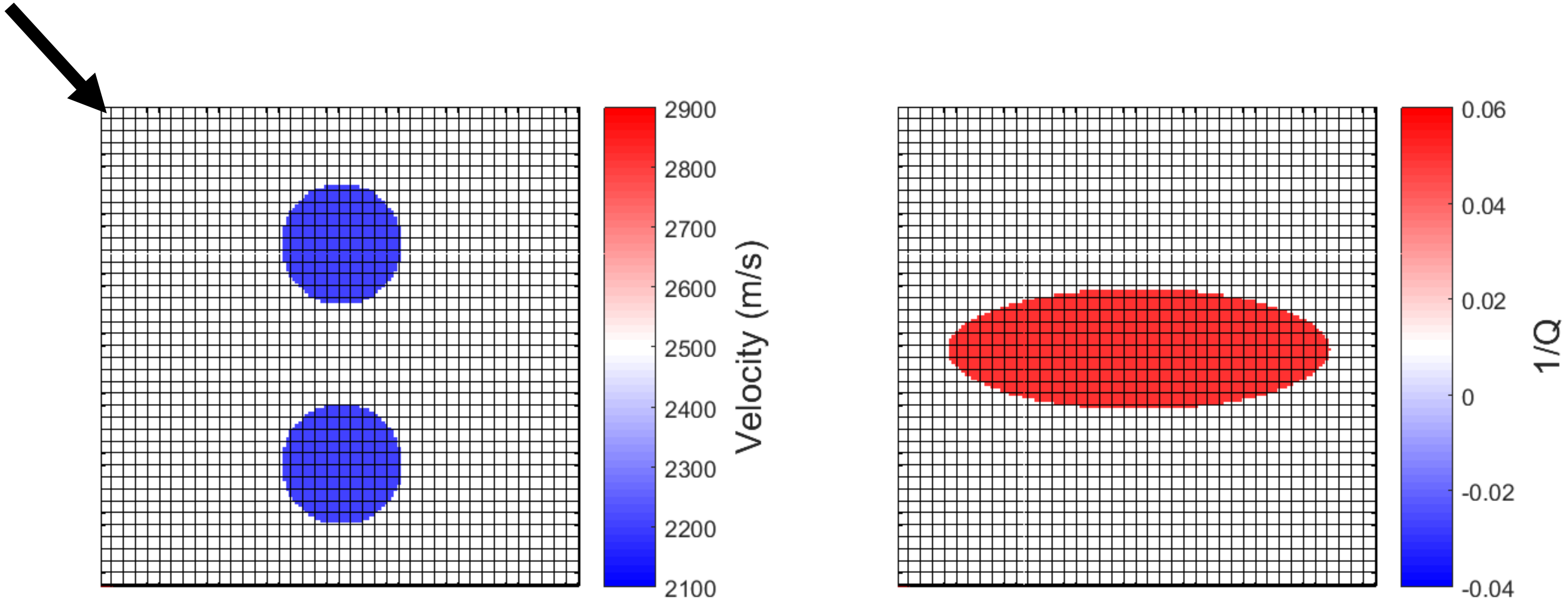
$\frac{\lambda}{8}$ resolution at 3Hz





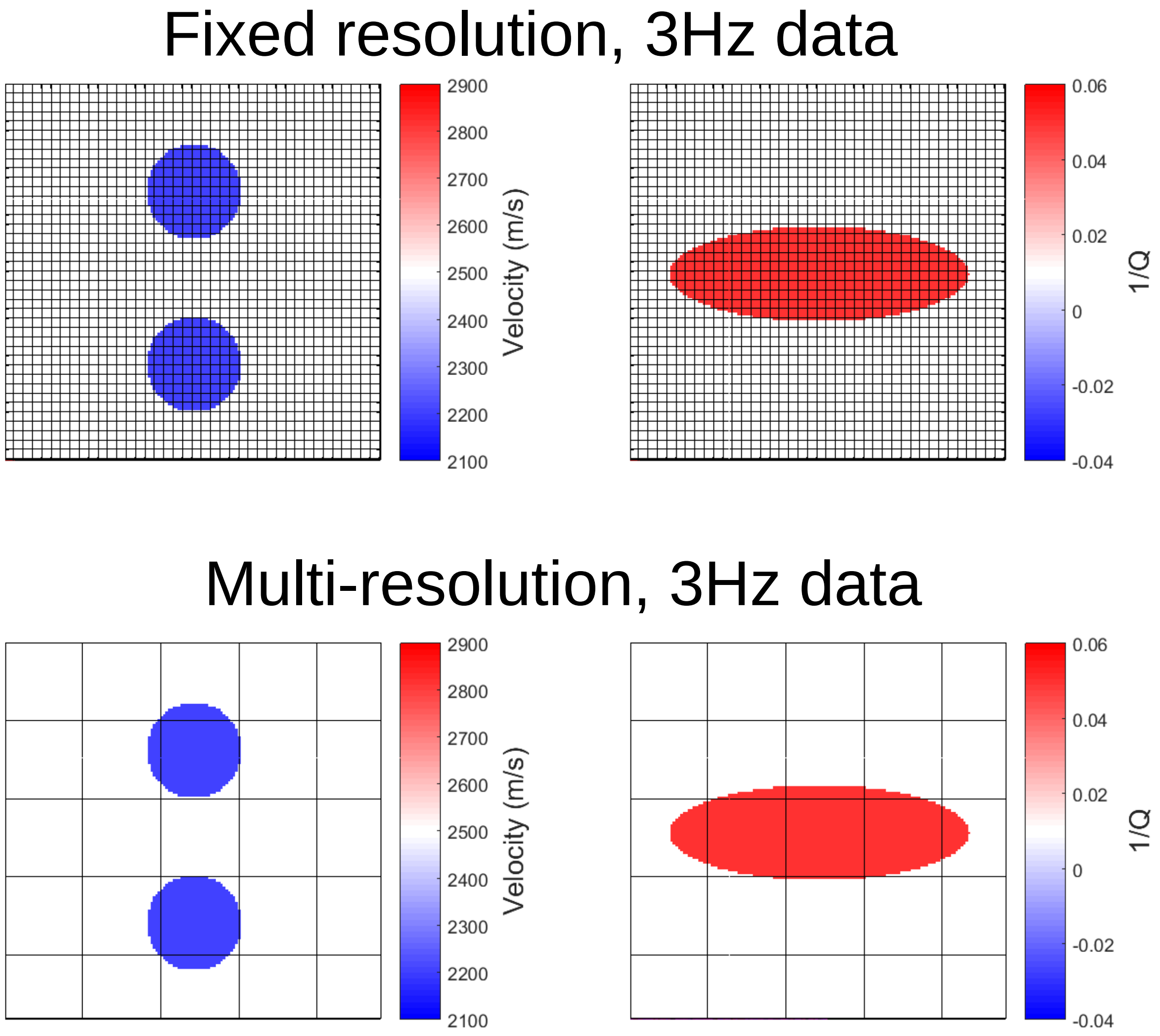
Seismic resolution at 25Hz

$\frac{\lambda}{8}$ resolution at 25Hz



Multi-resolution inversion

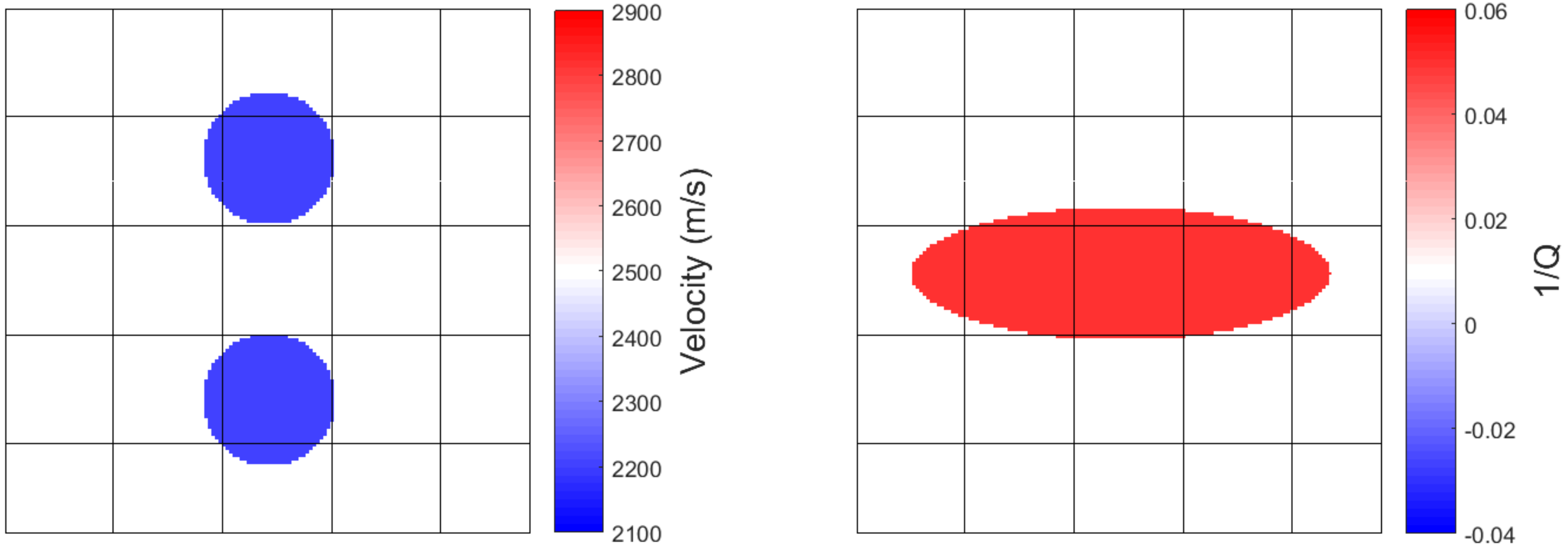
- Usually in FWI the inverted model is defined on a single grid.
- In a multi-resolution approach, the grid on which updates are calculated is allowed to change.
- This allows for more intensive approaches to be used at lower frequencies.



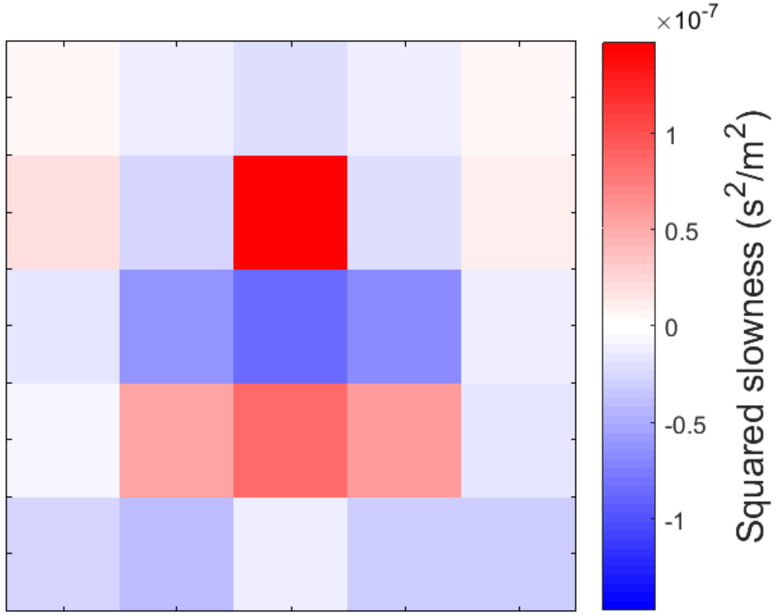
Multi-resolution effect on optimization

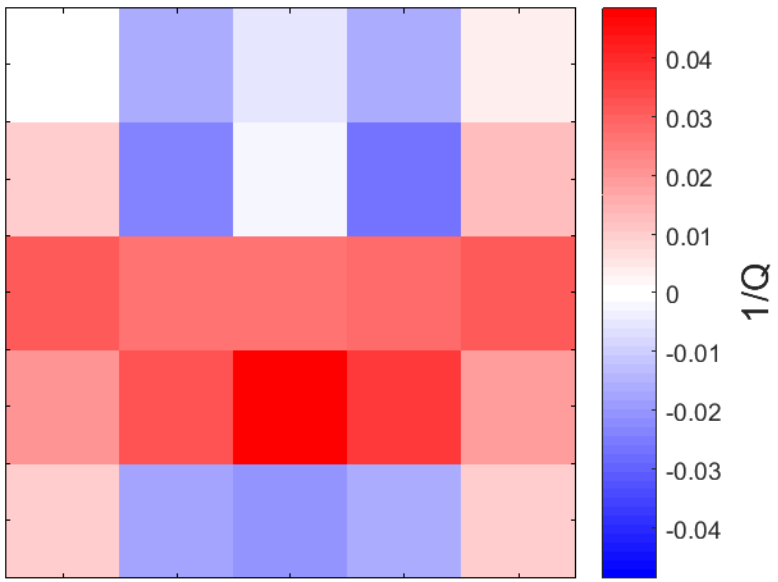
- The $O(N^3)$ cost of calculating the exact Newton update is dramatically reduced when the number of variables considered, N, is very small.
- Methods which attempt to approximate the Newton step, such as truncated Newton and Quasi-Newton methods also benefit from a reduction in the number of variables.
- This potentially simplifies cross-talk reduction on large scales.

Seismic resolution at 3Hz

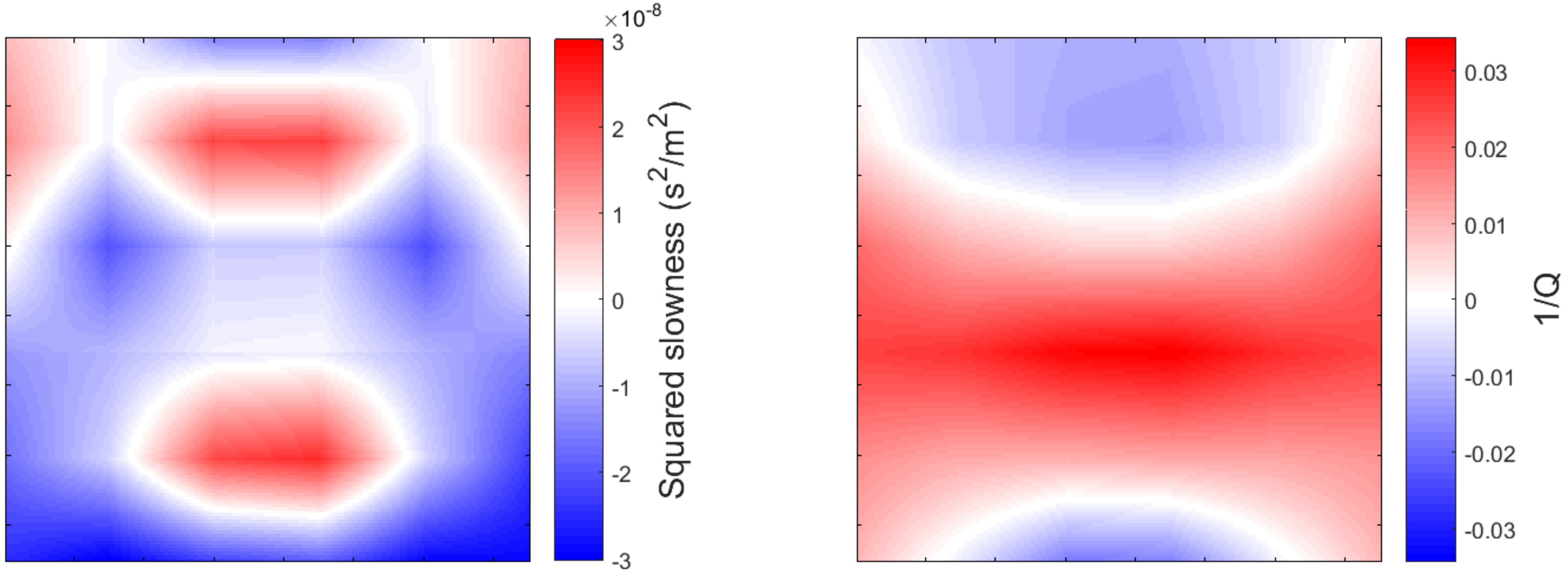


Calculated update at 3Hz

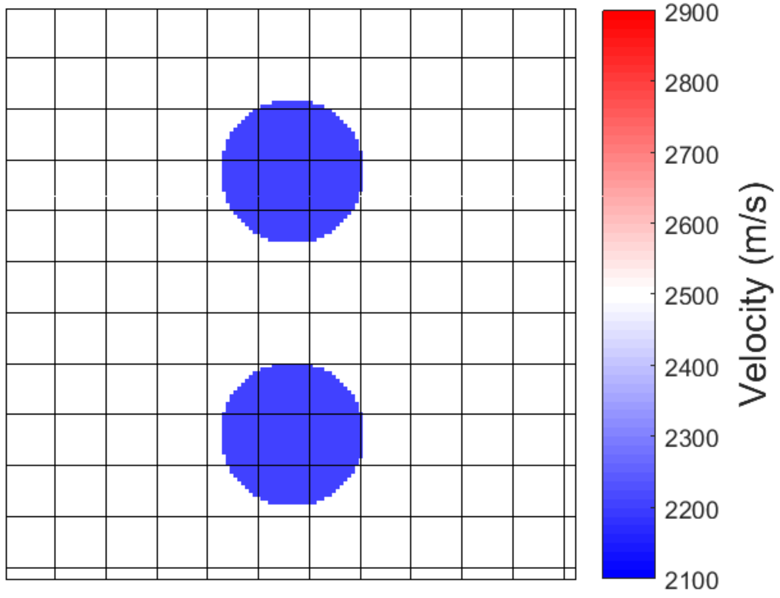


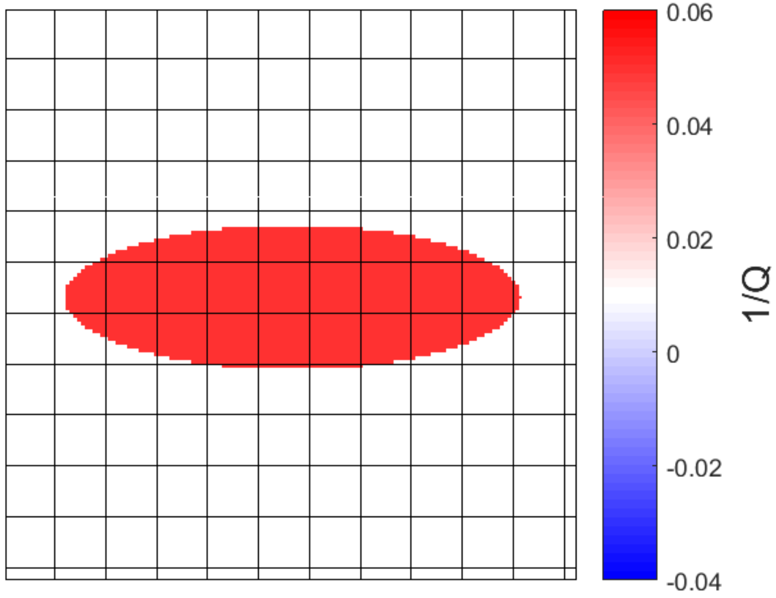


Applied update at 3Hz

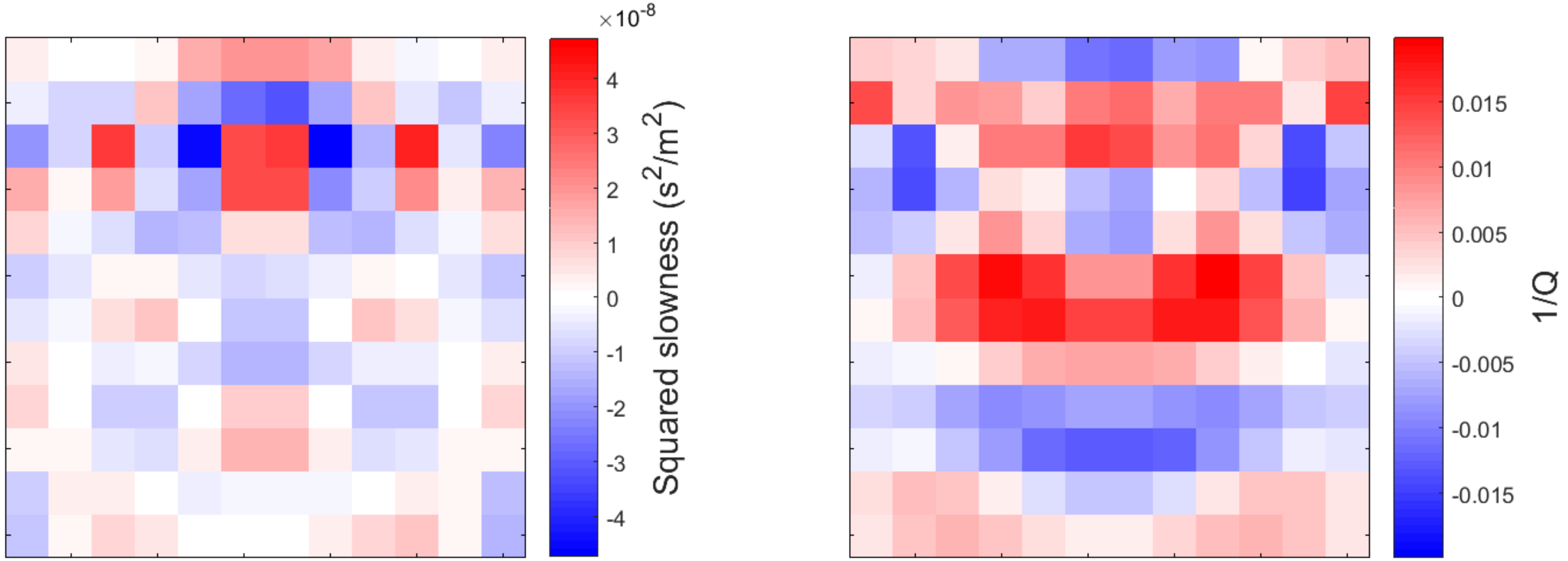


Seismic resolution at 7Hz

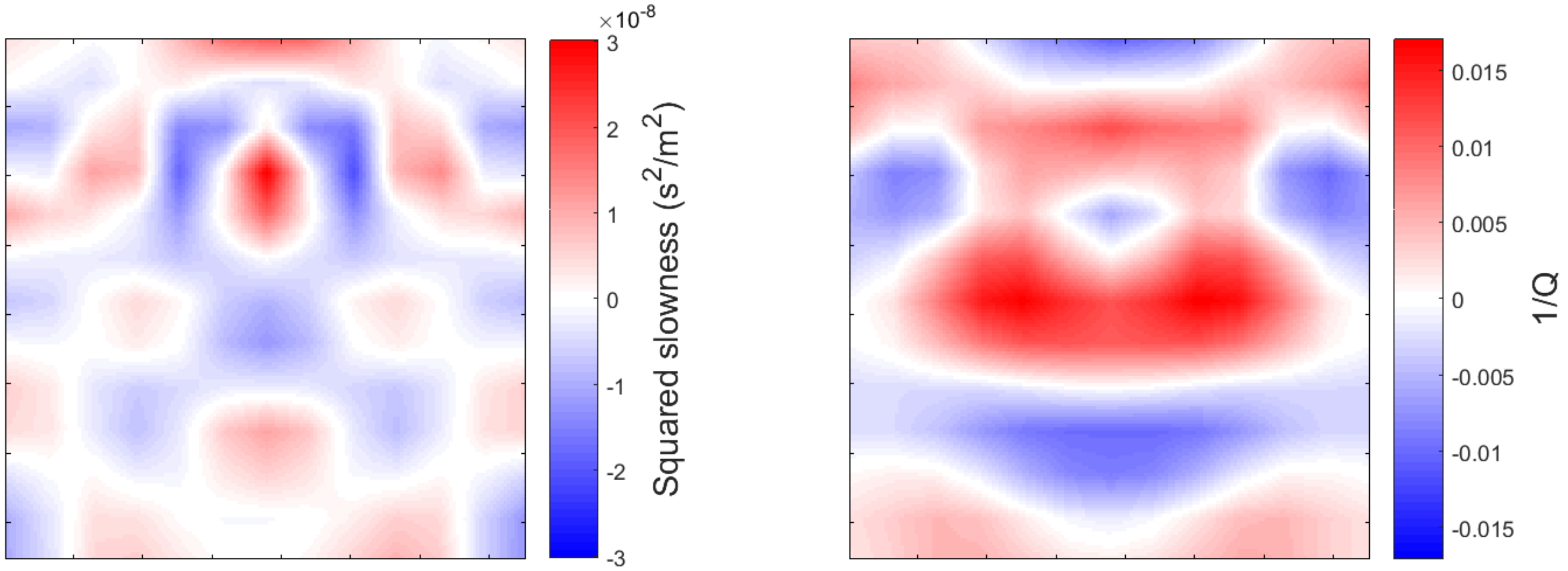




Calculated update at 7Hz

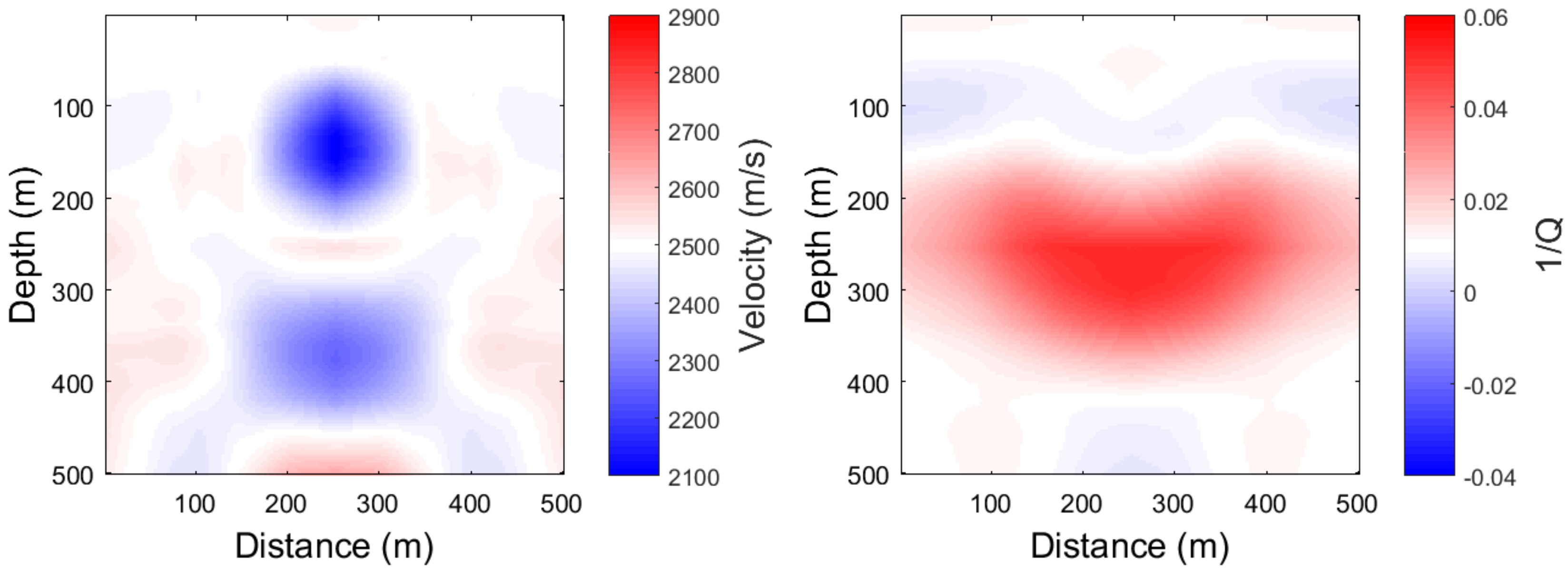


Applied update at 7Hz



Newton optimization

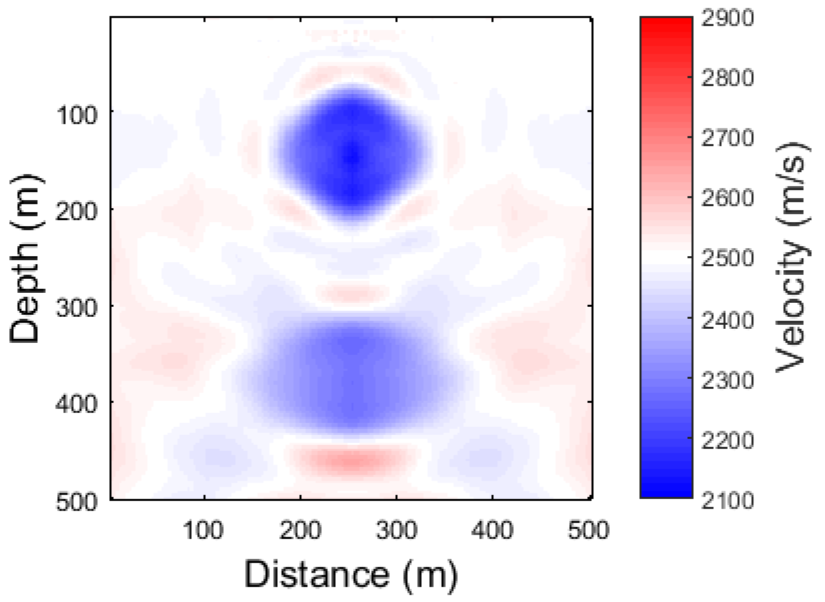
- The multi-resolution approach allows for a low cross-talk model to be recovered at large length scales.

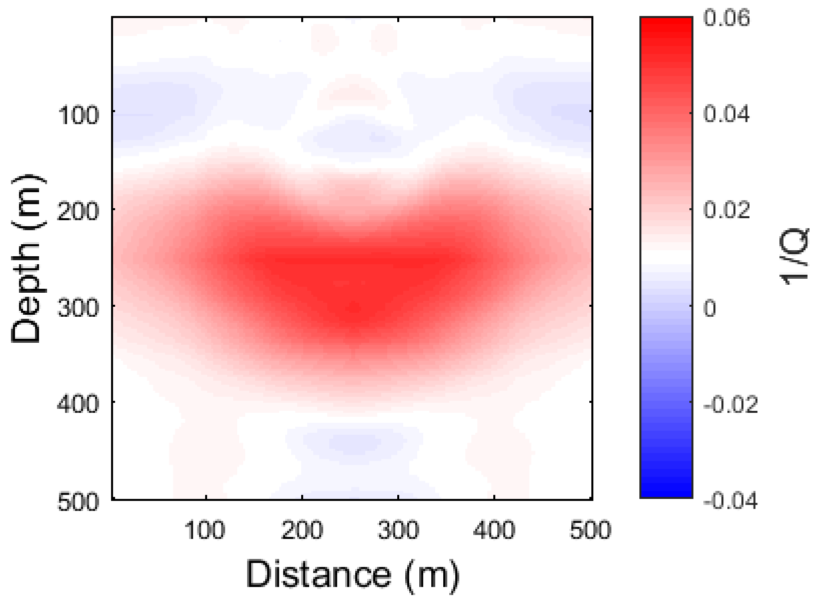


- 7 model updates using Newton step

Newton optimization + steepest descent

- This can serve as a starting point for more traditional FWI.
- Significantly less cross-talk is introduced this way.

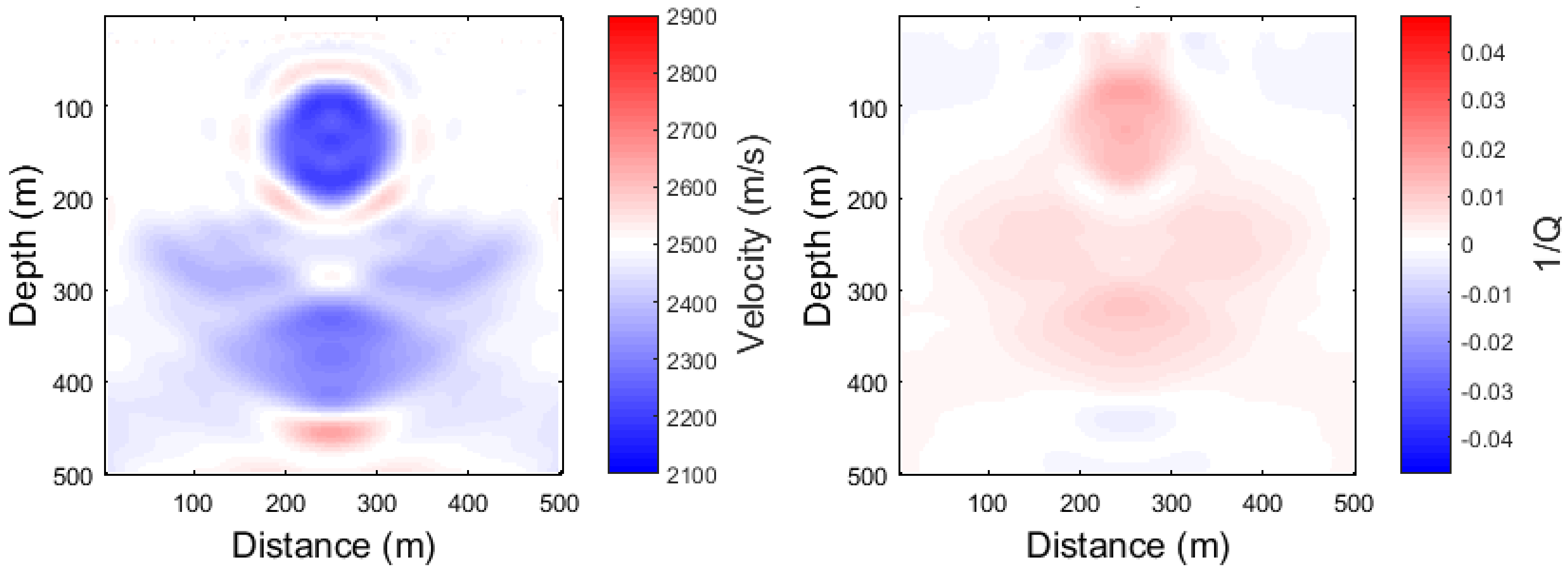




- 7 model updates using Newton step
- 180 model updates using steepest descent

Newton optimization + steepest descent

- This can serve as a starting point for more traditional FWI.
- Significantly less cross-talk is introduced this way.



- 575 model updates
- No second derivative information considered

Conclusions

- There are often far more variables defined at a given FWI iteration than can be realistically recovered.
- By defining a number of variables appropriate to the frequencies considered, cost savings can be made in certain FWI approaches.
- This opens the possibility of using more powerful techniques at lower frequencies, helping to mitigate cross-talk on large length scales.

Future work

- Studying efficiency improvements in a multi-resolution approach with other types of optimization.
  - Truncated and Quasi-Newton optimization methods should also see benefits from a model re-parameterization.
- Using low resolution estimates of the Hessian to provide a useful starting point for Quasi-Newton optimization.

Acknowledgments

- CREWES sponsors, staff and students

CREWES

NSERC-CRD (CRDPJ 461179-13)