Direct elastic FWI updating of rock physics properties

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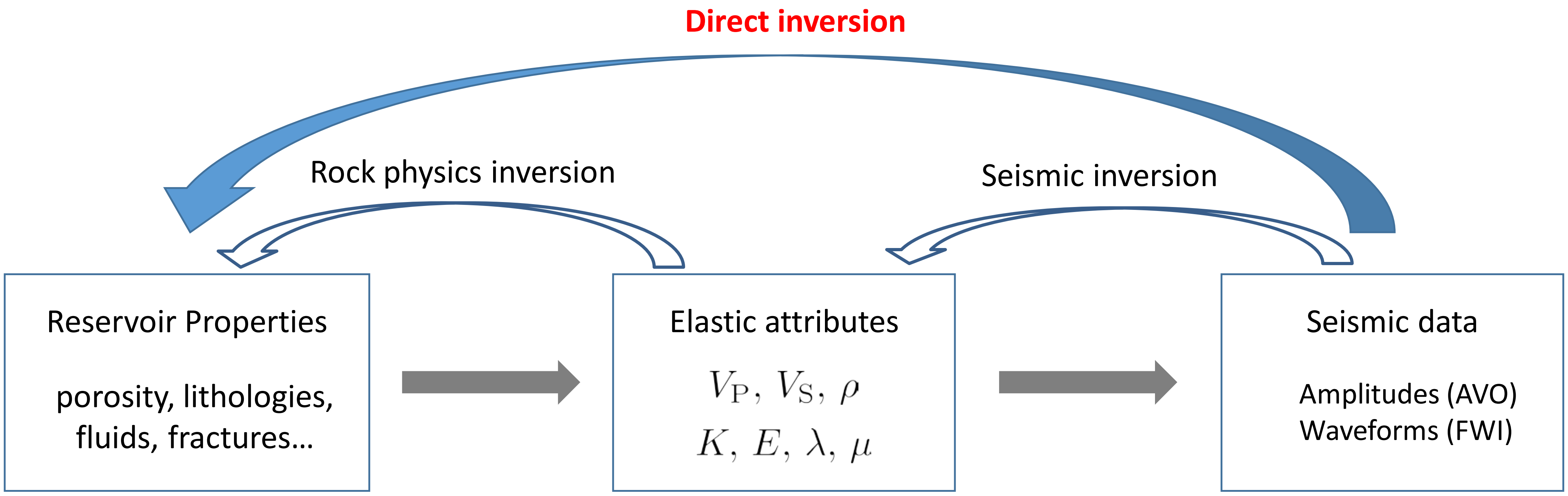
Quantitative seismic reservoir characterization



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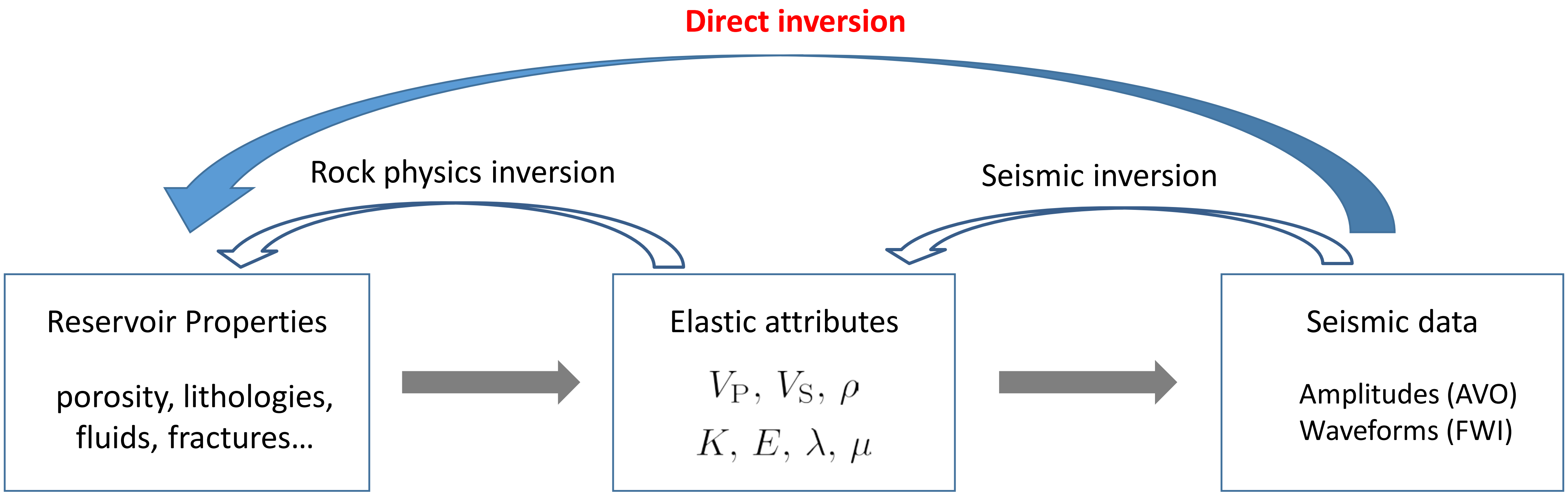

	Forward engine/Data source	**Workflow**	**Inversion method**
Current	AVO, Convolution/ Amplitudes	Direct or Indirect	Deterministic (Optimization) Or Stochastic (Sampling)
Our approach	**Wave equation/ Waveforms**	**Direct**	**Deterministic**

Parameterizations in AVO

Linearized approximations to the Zoeppritz equations:

Aki & Richards (1980)

$$R_{\mathrm{PP}}(\theta) = \left(\frac{1}{2}sec^2\theta\right)\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}} + \left(-\frac{4sin^2\theta}{\gamma^2}\right)\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}} + \left(\frac{1}{2} - \frac{2sin^2\theta}{\gamma^2}\right)\frac{\Delta\rho}{\rho}$$

Gray et al. (1999)

$$R_{\mathrm{PP}}(\theta) = \left[\left(\frac{1}{4} - \frac{1}{2\gamma^2}\right) sec^2\theta\right]\frac{\Delta\lambda}{\lambda} + \left[\frac{1}{\gamma^2}\left(\frac{1}{2}sec^2\theta - 2sin^2\theta\right)\right]\frac{\Delta\mu}{\mu} + \left(\frac{1}{2} - \frac{1}{4}sec^2\theta\right)\frac{\Delta\rho}{\rho}$$

Russell et al. (2011)

$$R_{\mathrm{PP}}(\theta) = \left[\left(\frac{1}{4} - \frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^2}\right) sec^2\theta\right]\frac{\Delta f}{f} + \left(\frac{\gamma_{\mathrm{dry}}^2 sec^2\theta - 8sin^2\theta}{4\gamma_{\mathrm{sat}}^2}\right)\frac{\Delta\mu}{\mu} + \left(\frac{1}{2} - \frac{1}{4}sec^2\theta\right)\frac{\Delta\rho}{\rho}$$

Fluid term

Reparameterization: From elastic to rock physics

Parameterizations in FWI

Frequency-domain elastic wave equations:

$$\omega^2 \rho u + \frac{\partial}{\partial x}\left[(\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial z}\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\right)\right] + f = 0,$$

$$\omega^2 \rho v + \frac{\partial}{\partial z}\left[(\lambda + 2\mu)\frac{\partial v}{\partial z} + \lambda \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\right)\right] + g = 0,$$

$$\mathbf{A}\mathbf{u} = \mathbf{f},$$

Object function: $E(\mathbf{m}) = \frac{1}{2}\Delta\mathbf{d}^t \Delta\mathbf{d}^*,$

$$\nabla_{m_i} E = \Re\left\{\mathbf{u}^t \left[\frac{\partial \mathbf{A}}{\partial m_i}\right]^t \mathbf{A}^{-1}\Delta\mathbf{d}^*\right\}.$$

where parameterization matters (radiation pattern)

Reparameterization in FWI

Traditional parameterization: **p** $(p_1 - p_2 - p_3)$

⬇

New parameterization: **q** $(q_1 - q_2 - q_3)$

$$\frac{\partial \mathbf{A}}{\partial q_1} = \frac{\partial \mathbf{A}}{\partial p_1} \cdot \frac{\partial p_1}{\partial q_1} + \frac{\partial \mathbf{A}}{\partial p_2} \cdot \frac{\partial p_2}{\partial q_1} + \frac{\partial \mathbf{A}}{\partial p_3} \cdot \frac{\partial p_3}{\partial q_1};$$

$$\frac{\partial \mathbf{A}}{\partial q_2} = \frac{\partial \mathbf{A}}{\partial p_1} \cdot \frac{\partial p_1}{\partial q_2} + \frac{\partial \mathbf{A}}{\partial p_2} \cdot \frac{\partial p_2}{\partial q_2} + \frac{\partial \mathbf{A}}{\partial p_3} \cdot \frac{\partial p_3}{\partial q_2};$$

$$\frac{\partial \mathbf{A}}{\partial q_3} = \frac{\partial \mathbf{A}}{\partial p_1} \cdot \frac{\partial p_1}{\partial q_3} + \frac{\partial \mathbf{A}}{\partial p_2} \cdot \frac{\partial p_2}{\partial q_3} + \frac{\partial \mathbf{A}}{\partial p_3} \cdot \frac{\partial p_3}{\partial q_3}.$$

$$(p_1,\ p_2,\ p_3) = f(q_1,\ q_2,\ q_3)$$

Reparameterization in FWI

Traditional parameterization: **p** $(p_1 - p_2 - p_3)$

⬇

New parameterization: **q** $(q_1 - q_2 - q_3)$

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$$(p_1,\, p_2,\, p_3) = f(q_1,\, q_2,\, q_3)$$

Current Study

Elastic Parameterizations

p, q:

$V_\mathrm{P} - V_\mathrm{S} - \rho$	$I_\mathrm{P} - I_\mathrm{S} - \rho$
$K - G - \rho$	$\lambda - \mu - \rho$
$V_\mathrm{P} - V_\mathrm{S} - I_\mathrm{P}$	....

Reparameterization in FWI

Traditional parameterization: **p** $(p_1 - p_2 - p_3)$

⬇

New parameterization: **q** $(q_1 - q_2 - q_3)$

$$\frac{\partial \mathbf{A}}{\partial q_1} = \frac{\partial \mathbf{A}}{\partial p_1} \cdot \frac{\partial p_1}{\partial q_1} + \frac{\partial \mathbf{A}}{\partial p_2} \cdot \frac{\partial p_2}{\partial q_1} + \frac{\partial \mathbf{A}}{\partial p_3} \cdot \frac{\partial p_3}{\partial q_1};$$

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$$(p_1,\ p_2,\ p_3) = f(q_1,\ q_2,\ q_3)$$

Current Study

Elastic Parameterizations

p, q: $V_P - V_S - \rho$, $I_P - I_S - \rho$, $K - G - \rho$, $\lambda - \mu - \rho$, $V_P - V_S - I_P$ …….

Reservoir-Oriented

Elastic to Rock physics Parameterization

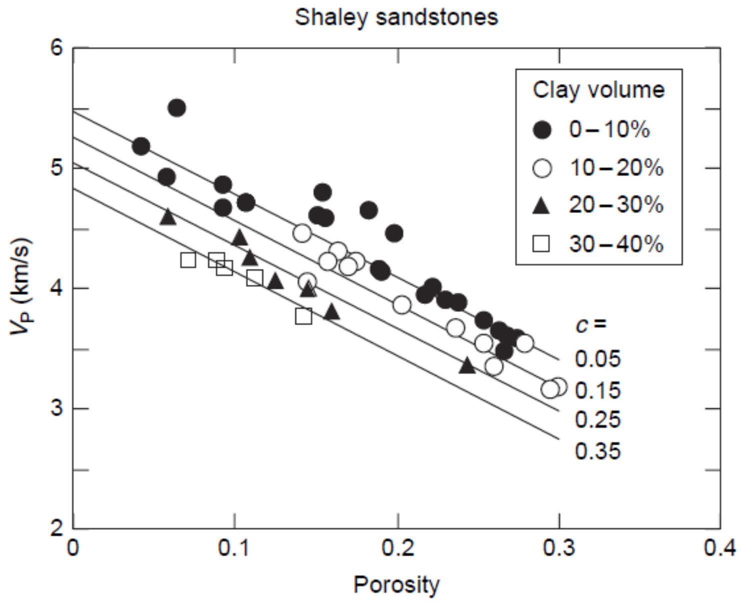
p: $V_P - V_S - \rho$ **(D-V)**

q: $\phi - C - S_w$ **(P-C-S)**

f: **Rock Physics Model**

- **Empirical: Han's relations (Han)**

$$V_P = a_1 - a_2\phi - a_3C,$$
$$V_S = b_1 - b_2\phi - b_3C,$$



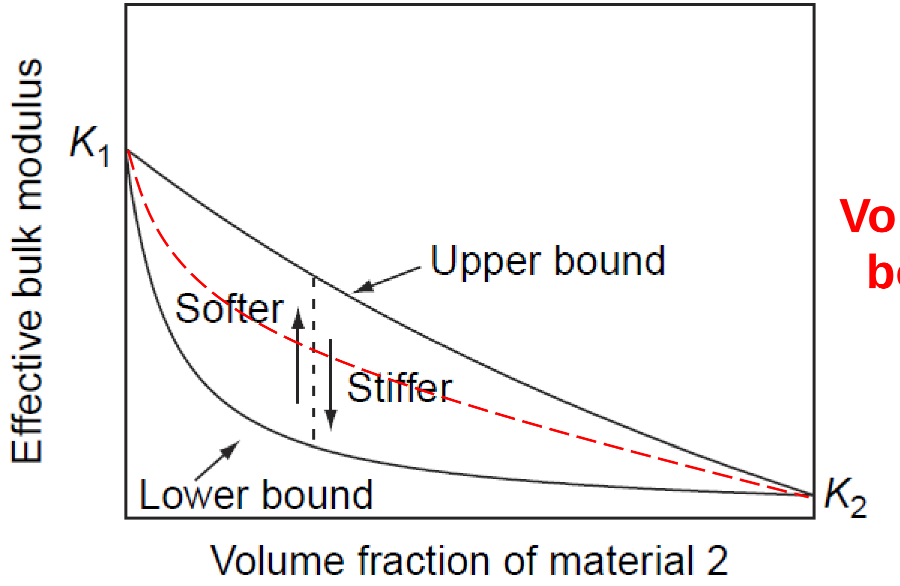
Han (1986)

- **Boundary model: Voigt-Reuss-Hill average (VRH)**

$$M_V = \sum_{i=1}^{N} f_i M_i, \quad \frac{1}{M_R} = \sum_{i=1}^{N} \frac{f_i}{M_i}.$$
$$M_{VRH} = \frac{M_V + M_R}{2}.$$

- **Inclusion model: Kuster-Toksoz model (KT)**

$$(K_{sat} - K_m)\frac{K_m + \frac{4}{3}G_m}{K_{sat} + \frac{4}{3}G_m} = \phi(K_f - K_m)P,$$
$$(G_{sat} - G_m)\frac{G_m + \xi}{G_{sat} + \xi} = -\phi G_m Q.$$



Voigt & Reuss boundaries

Rock physics parameterization

Han

VRH

KT

$$(V_{\mathrm{P}},\, V_{\mathrm{S}},\, \rho) = f(\phi,\, C,\, S_w)$$

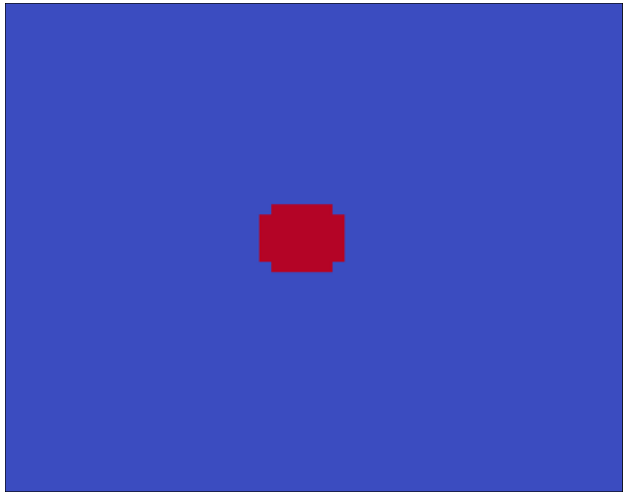
Direct Inversion

$\phi - C - S_w$

Rock physics parameterization

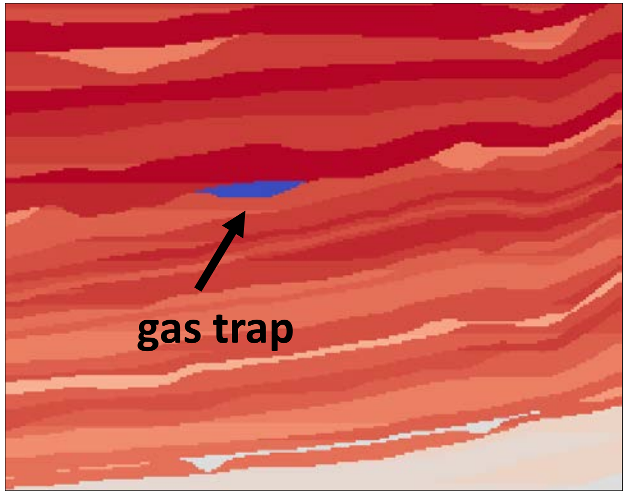
Synthetic Experiments

Toy model



Three-layer model

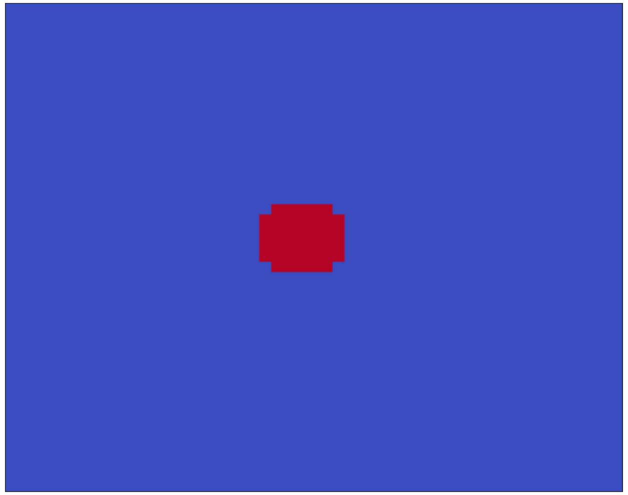
Modified small part of Marmousi



Geologically meaningful

Synthetic Experiments

Toy model



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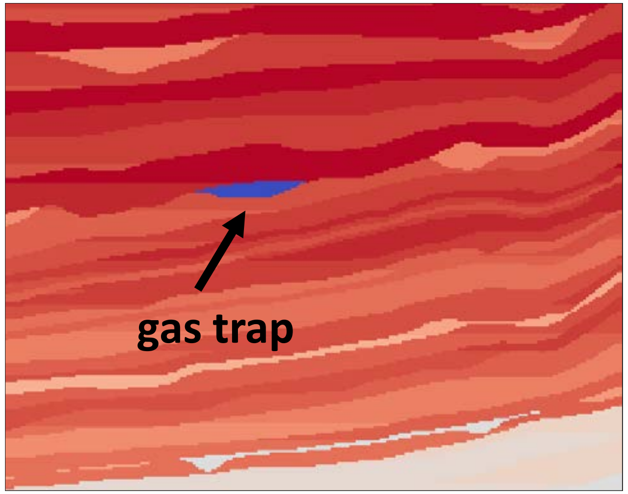


Geologically meaningful

Rock type assumed:
- Gas-bearing shaly sand
- Solid phase: quartz + clay
- Fluid phase: water + gas

Acquisition Geometry:
- Surface Seismic + VSP

Optimization:
- Multiscale: low to high frequencies (2 – 25 Hz)
- Truncated Newton

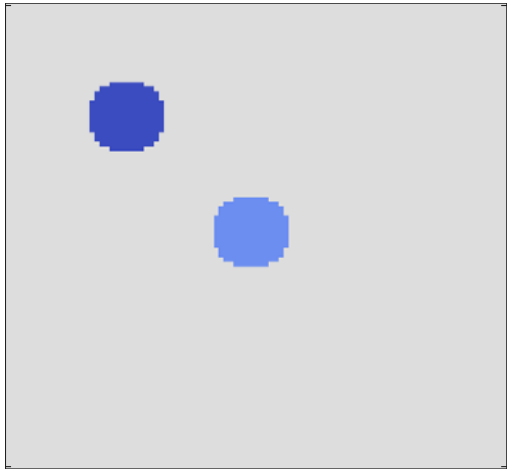
Inversion experiments with the Han model

Direct: FWI $\xrightarrow{\text{P-C-S by Han}}$ $\phi,\ C,\ S_w$

Indirect: FWI $\xrightarrow{\text{D-V}}$ $V_{\mathrm{P}},\ V_{\mathrm{S}},\ \rho$ $\xrightarrow{\text{Han}}$ $\phi,\ C,\ S_w$

Inversion Experiments: Toy model

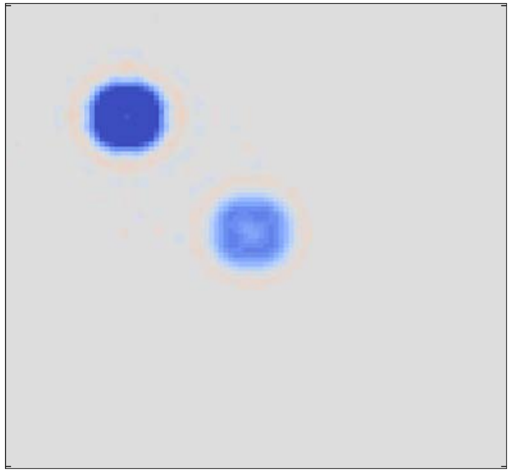
True V_{P}



True models

Inversion Experiments: Toy model

Inverted V_{P}

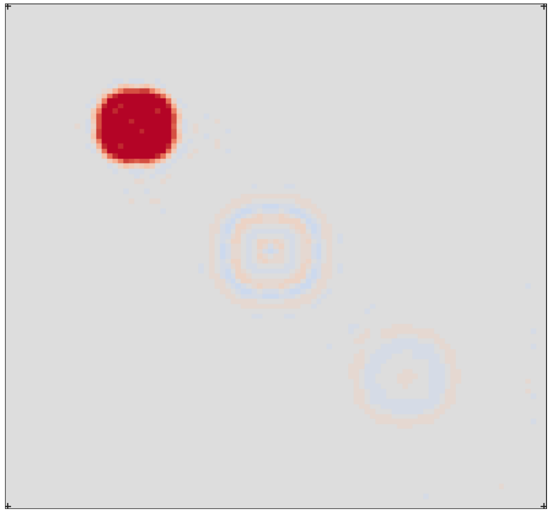


Indirect Inversion

Inversion Experiments: Toy model

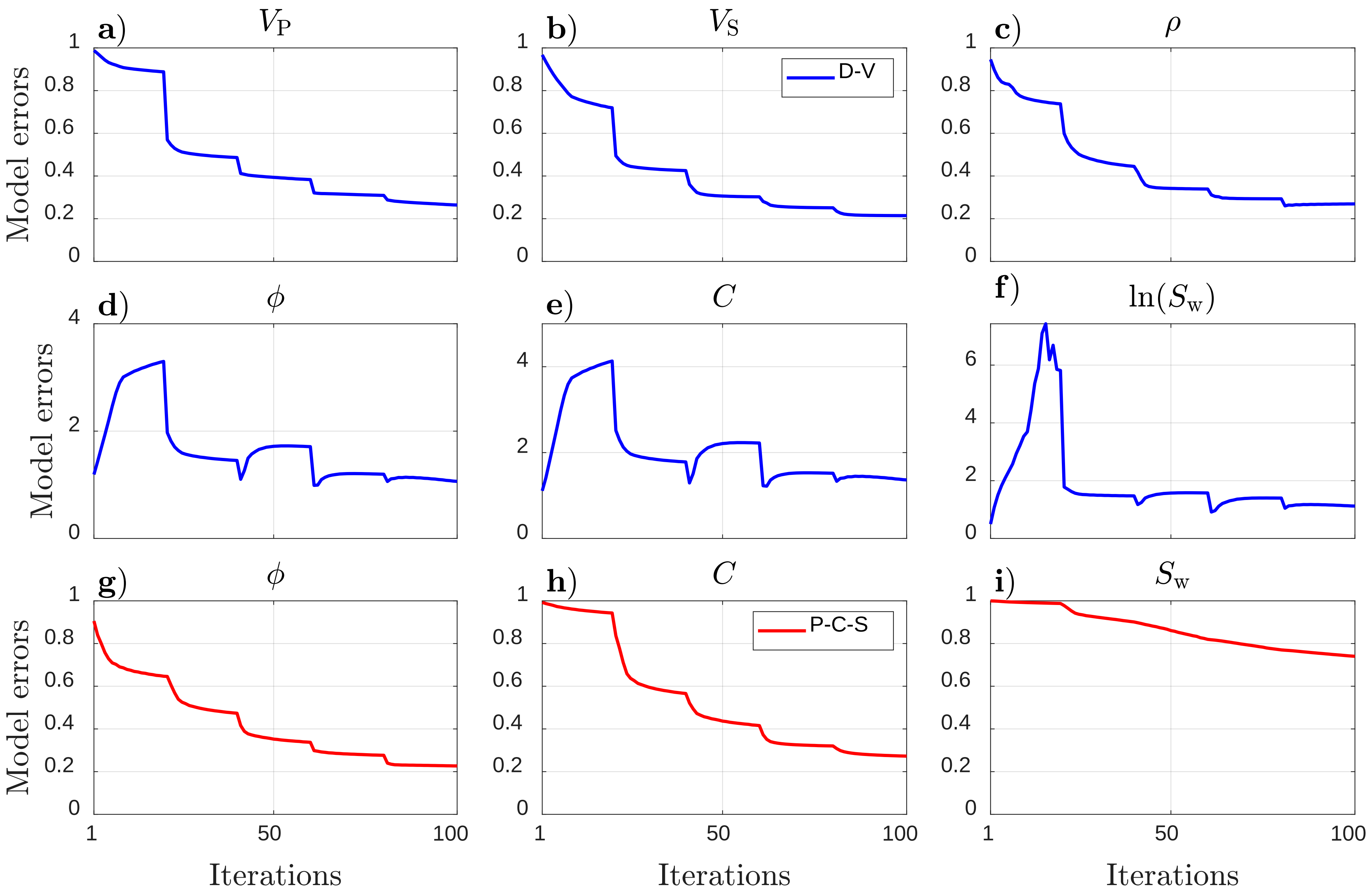
Direct Inversion

Inverted ϕ



Inversion Experiments: Toy model

History of model error reductions



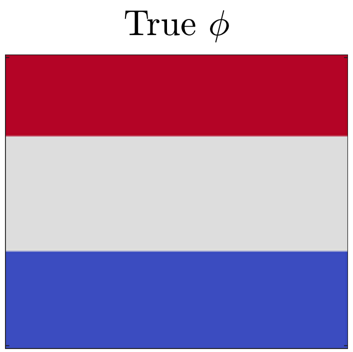
Inversion Experiments: layered model

Rock physics properties of each layer:

$\phi = 0.3, \quad C = 0.1, \quad S_w = 0.2$
$\phi = 0.2, \quad C = 0.3, \quad S_w = 0.5$
$\phi = 0.1, \quad C = 0.5, \quad S_w = 0.8$

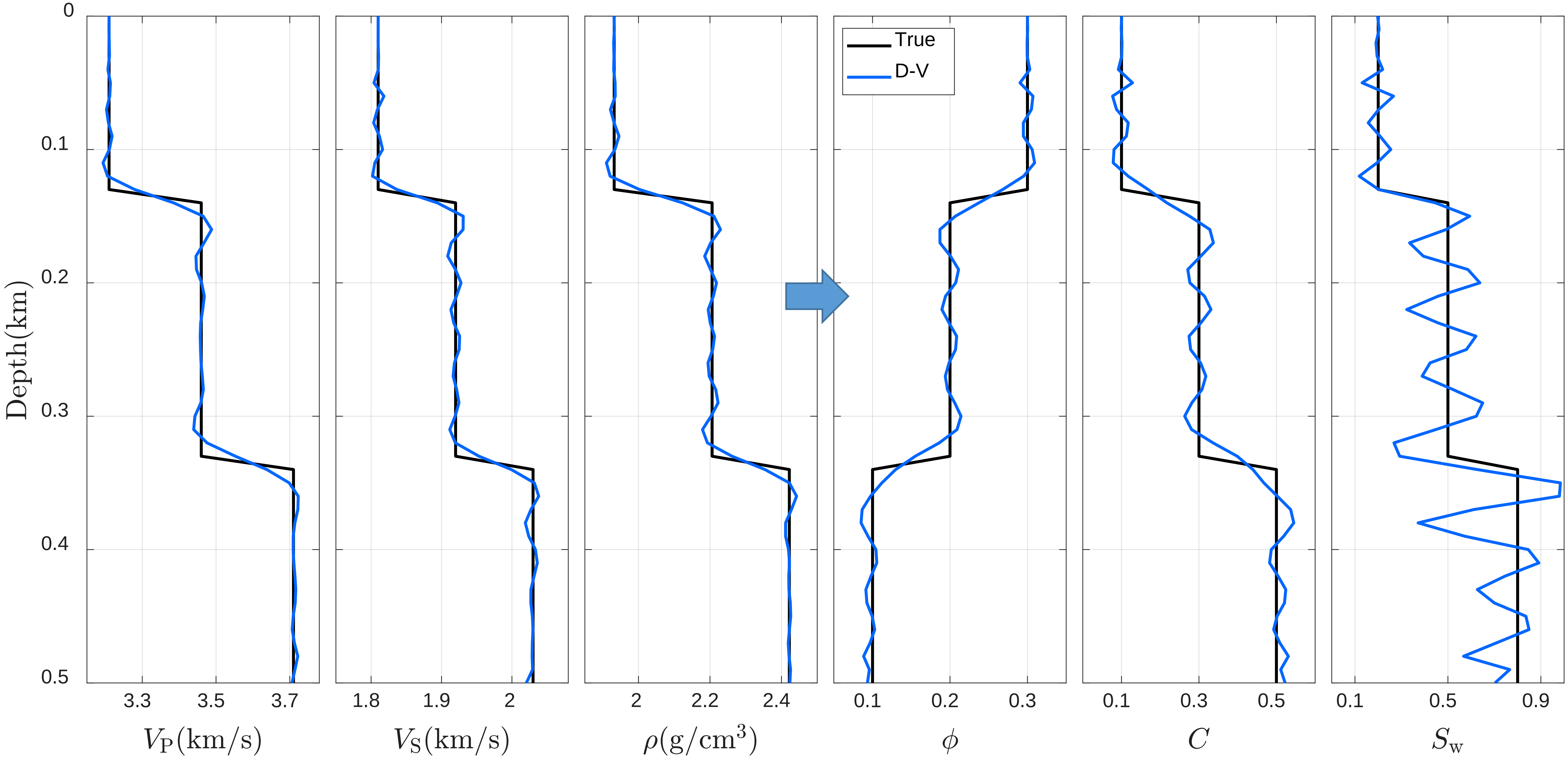
Inversion Experiments: layered model

	True	Initial	D-V	P-C-S

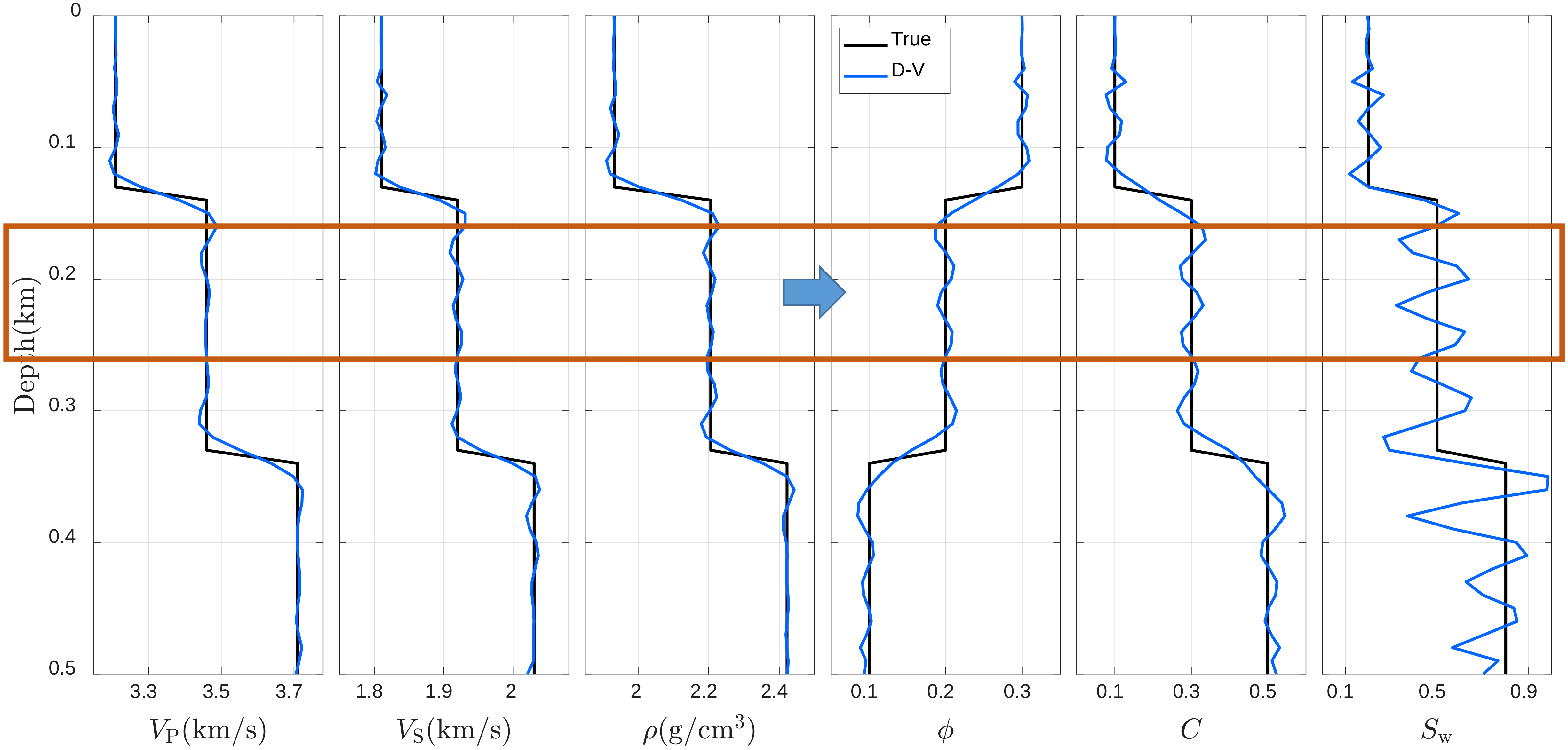


Direct
VS
Indirect

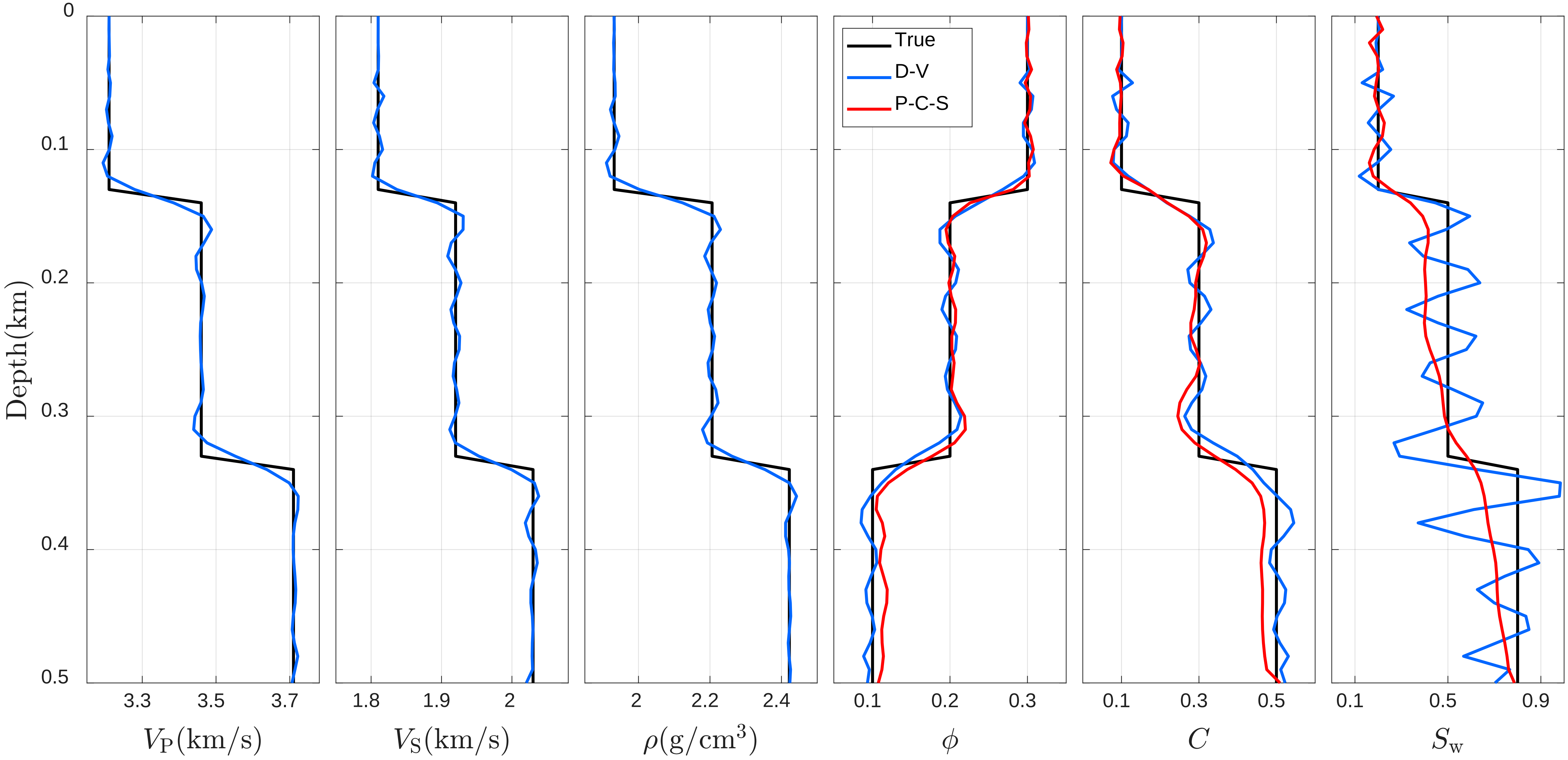
Vertical profile of inverted parameters



Vertical profile of inverted parameters

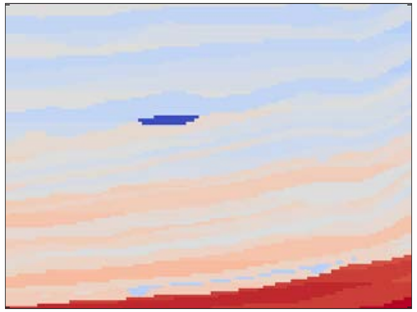


Vertical profile of inverted parameters



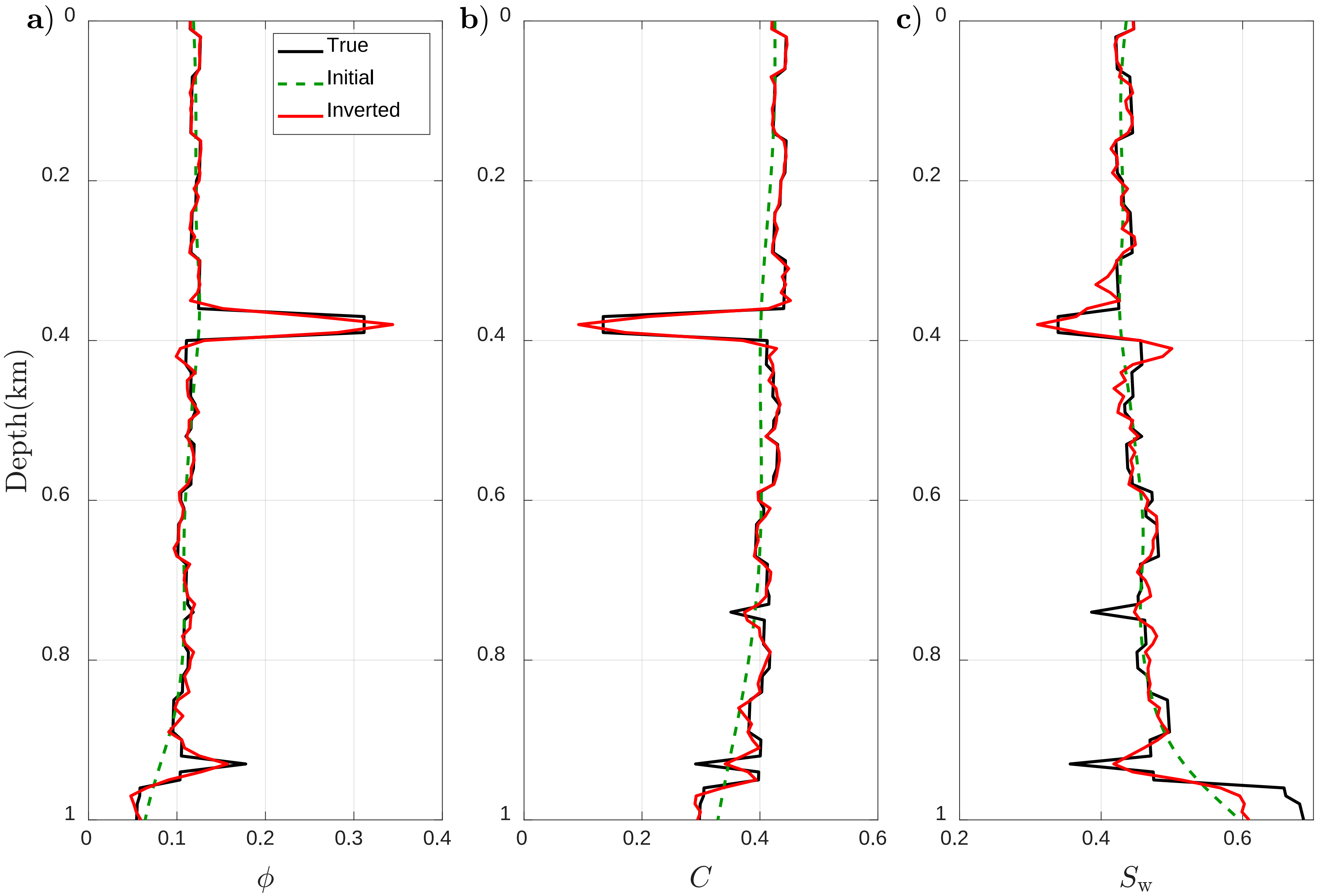
Inversion Experiments: Modified Marmousi model

True V_P



Direct inversion

Vertical profiles across the gas sand

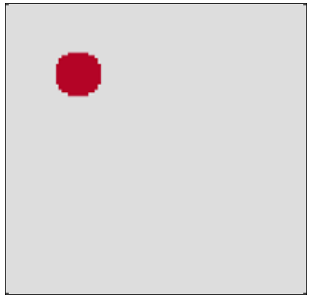


Inversion experiments with VRH and KT

Direct: **FWI** $\xrightarrow{\text{P-C-S}}$ $\phi,\ C,\ S_w$

Inversion results with Han, VRH, and KT

True ϕ



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True ϕ

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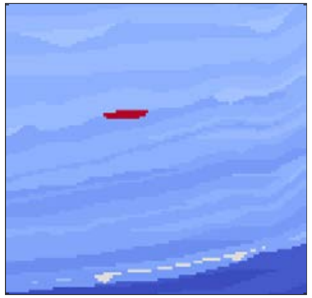
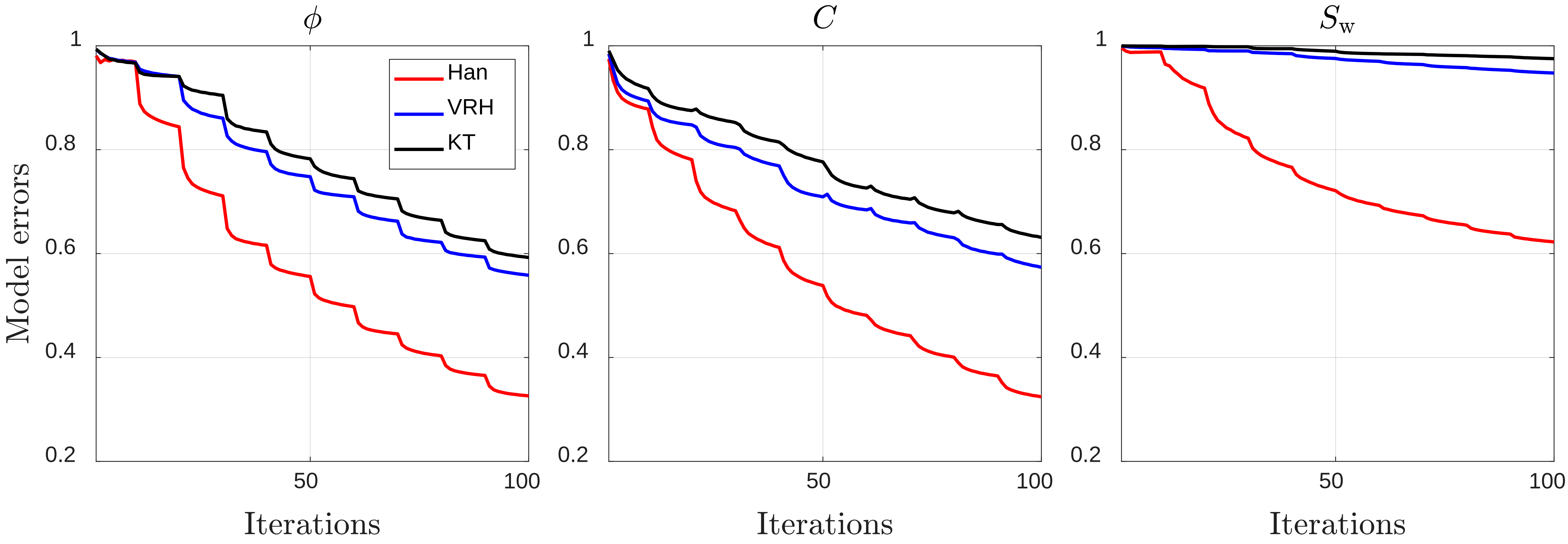
True ϕ

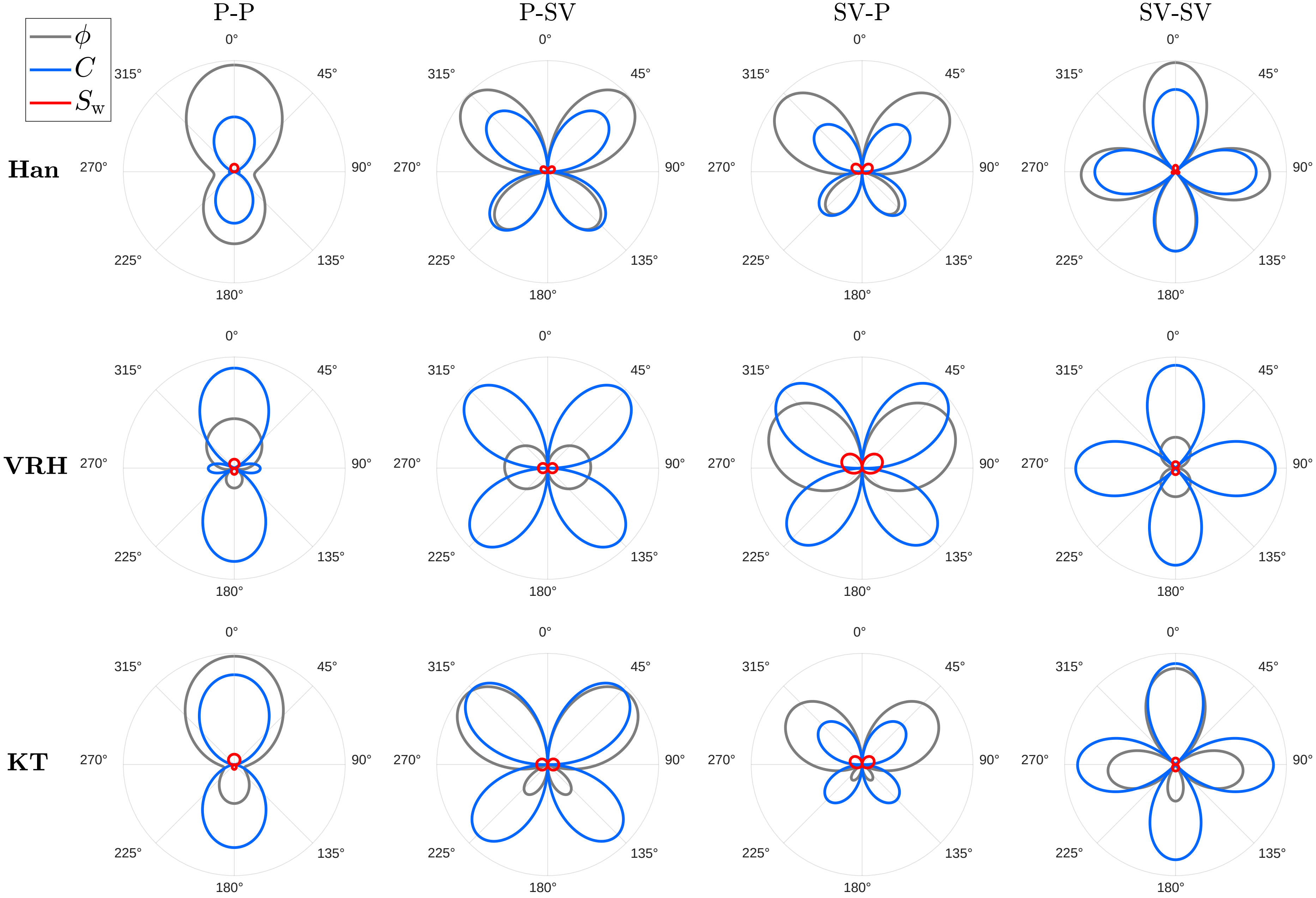


History of model error reductions



Comparing the performance of Han, VRH, KT

Sensitivity analysis: radiation patterns



Conclusions

- **Direct updating of rock physics properties using FWI shows promise.**
- **We demonstrate that the direct inversion is superior to the indirect one.**
- **Radiation patterns can be used as well for the sensitivity analysis of rock physics properties.**

Acknowledgments

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