Full wavefield migration in the frequency-wavenumber domain

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Outline

- Motivation
- Theory
- Numerical examples
- Conclusion and future work

Motivation

Multiples can provide additional information for subsurface structures

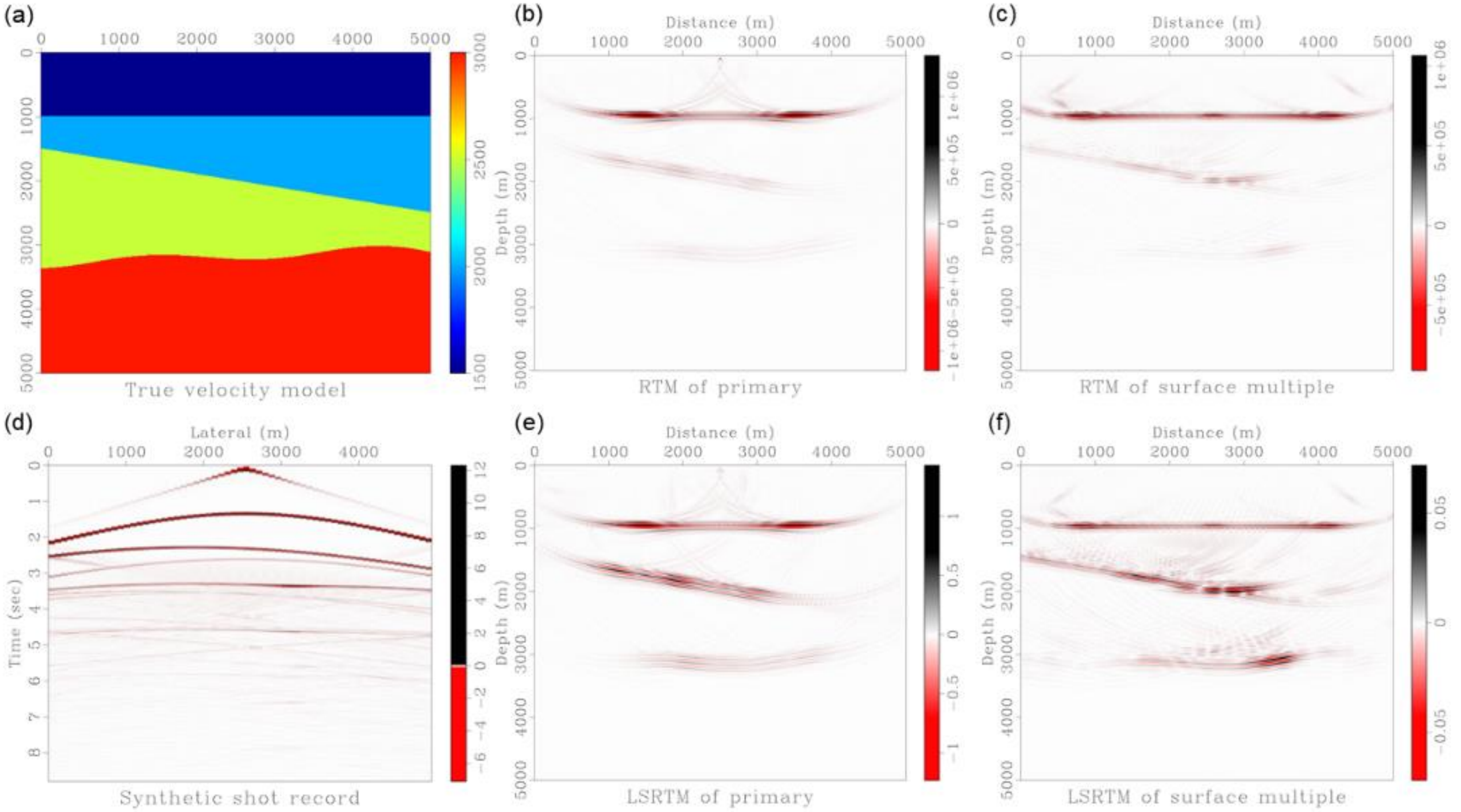

Figure from Huang and Trad (2019)

Motivation

Multiples can provide additional information for subsurface structures

FWM (Berkhout, 2014; Verschuur and Berkhout, 2015; Davydenko and Verschuur, 2016):

- Inversion-based method
- Frequency-space domain
- Cross-correlation imaging condition

Motivation

Multiples can provide additional information for subsurface structures

FWM (Berkhout, 2014; Verschuur and Berkhout, 2015; Davydenko and Verschuur, 2016):

- Inversion-based method
- Frequency-space domain
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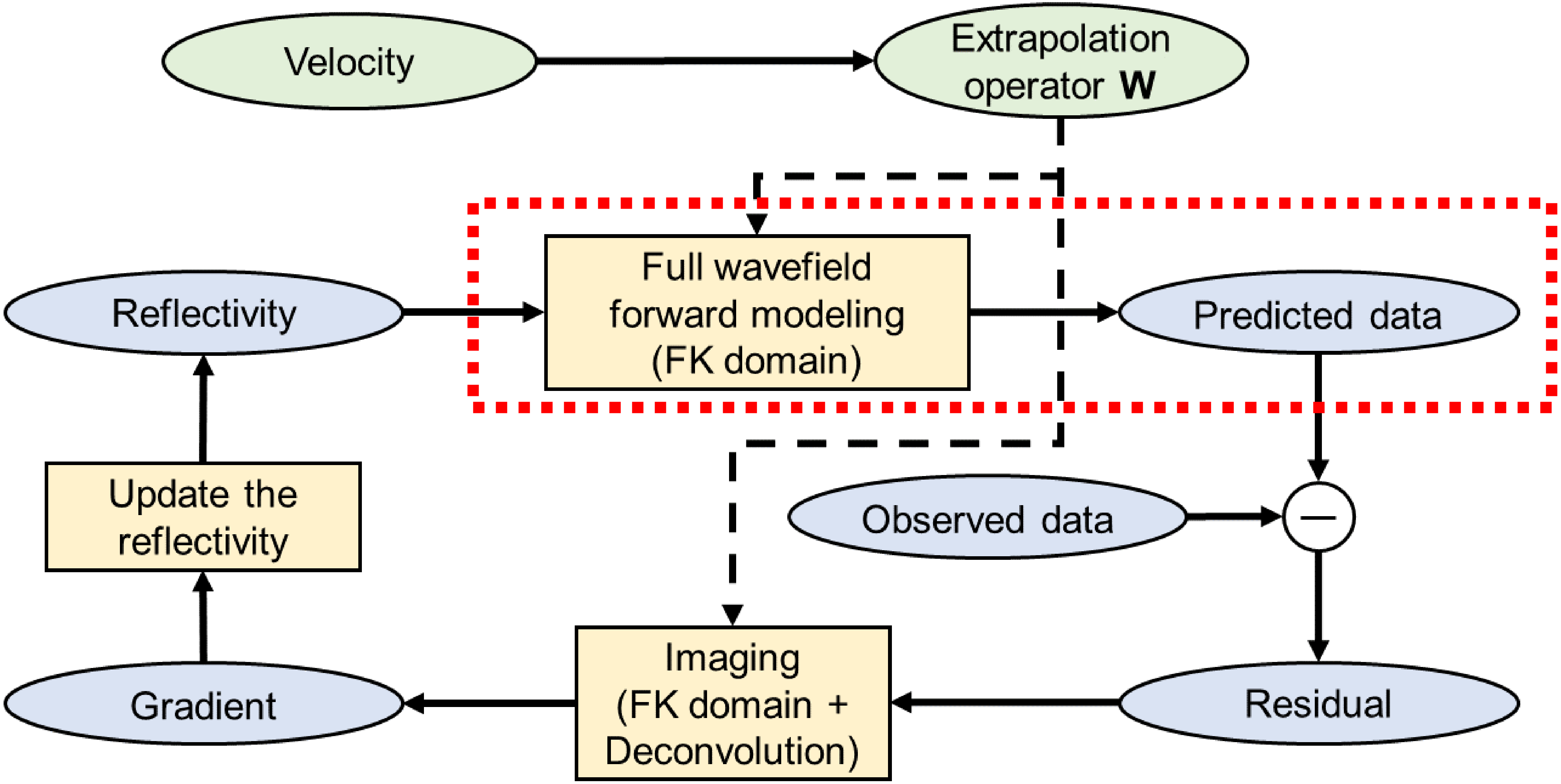
In this project:

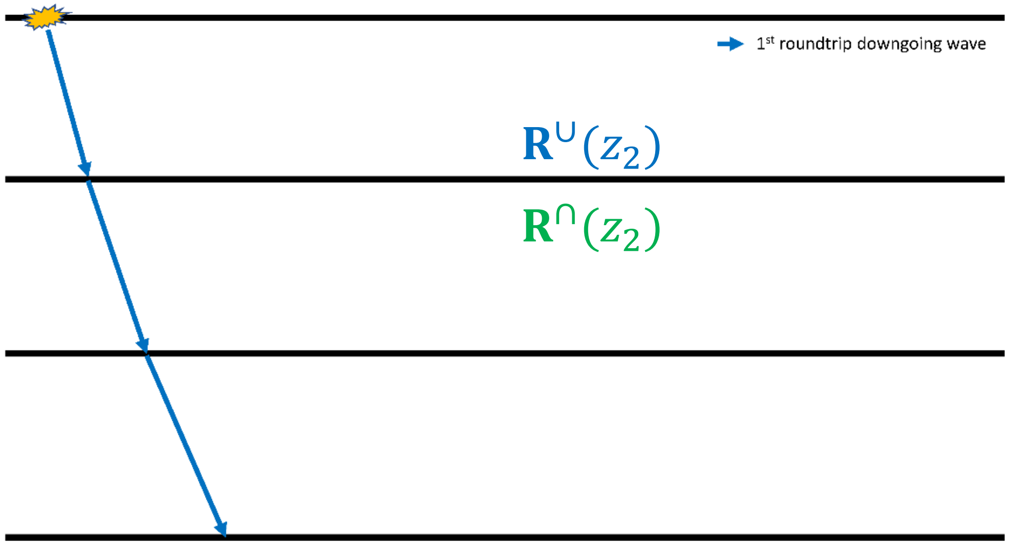
- Inversion-based
- Frequency-wavenumber domain
- Deconvolution imaging condition

Full-wavefield migration (FWM) workflow



Workflow adapted from Davydenko and Verschuur (2016)

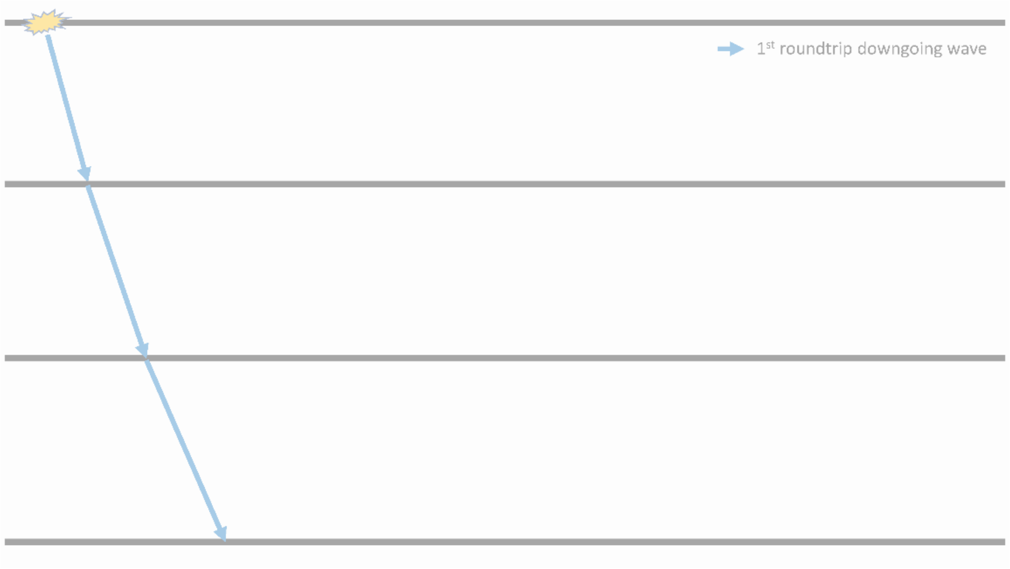
First roundtrip downgoing wavefield

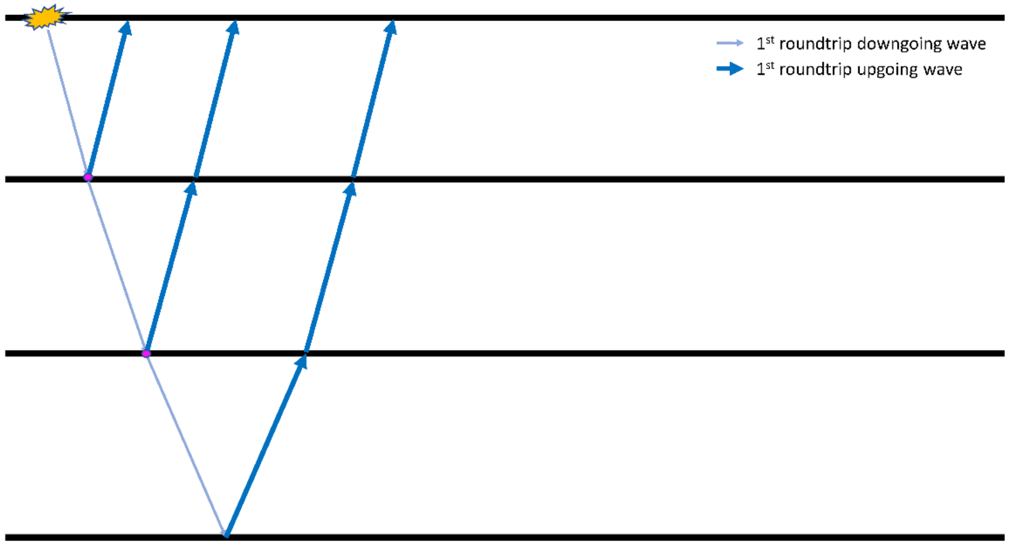


$$\vec{P}^{+}(z_m) = \sum_{n<m} \mathbf{W}(z_m, z_n)[\vec{S}^{+}(z_n) + \delta\vec{S}(z_n)] \qquad (1)$$

$$\delta\vec{S}(z_m) = \mathbf{R}^{\cup}(z_m)\vec{P}^{+}(z_m) + \mathbf{R}^{\cap}(z_m)\vec{P}^{-}(z_m) \qquad (2)$$

First roundtrip upgoing wavefield

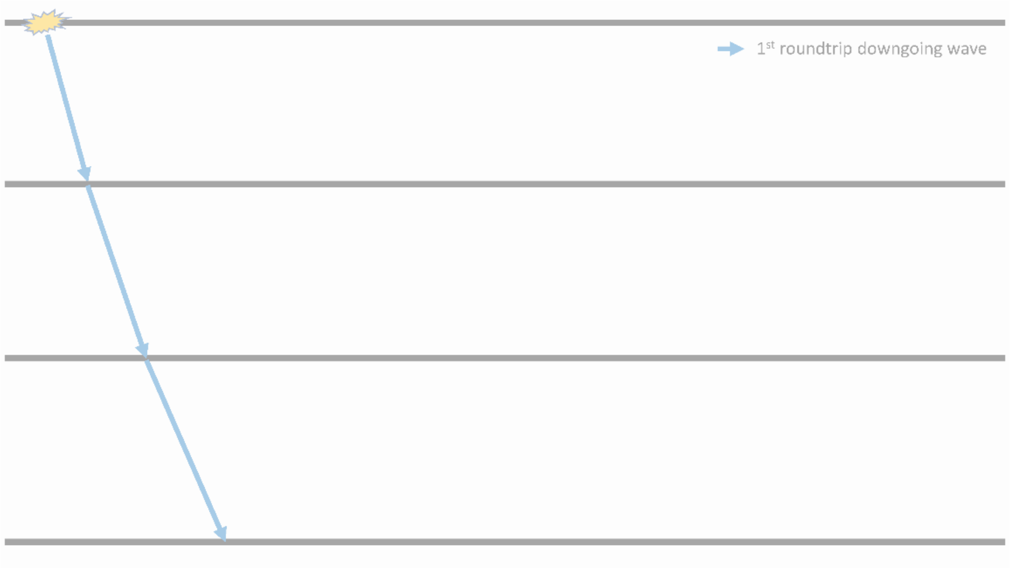


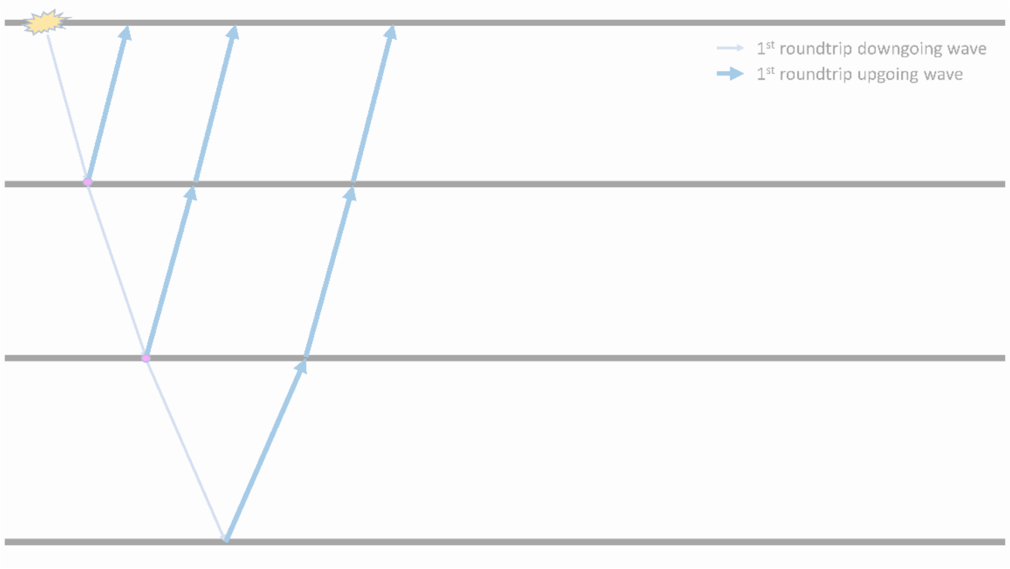


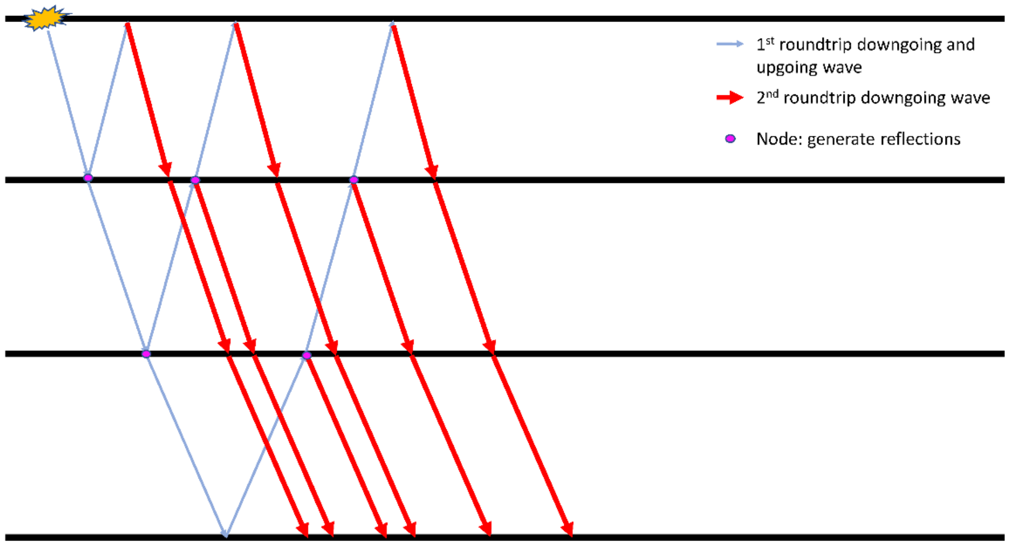
$$\vec{P}^{-}(z_m) = \sum_{n>m} \mathbf{W}\,(z_m, z_n)\delta\vec{S}(z_n) \tag{3}$$

$$\delta\vec{S} = \mathbf{R}^{\cup}(z_m)\vec{P}^{+}(z_m) + \mathbf{R}^{\cap}(z_m)\vec{P}^{-}(z_m) \tag{2}$$

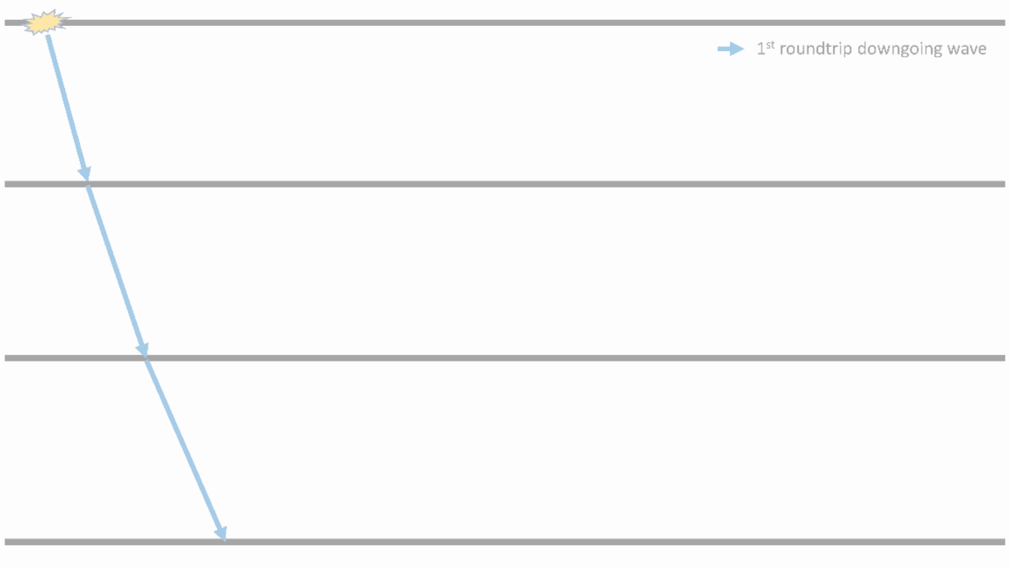
Second roundtrip downgoing wavefield

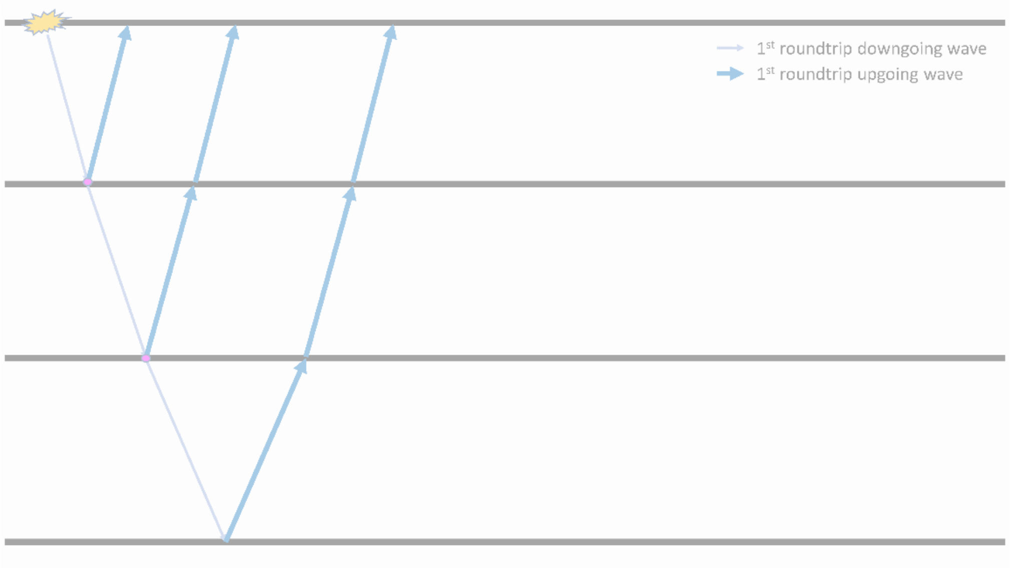


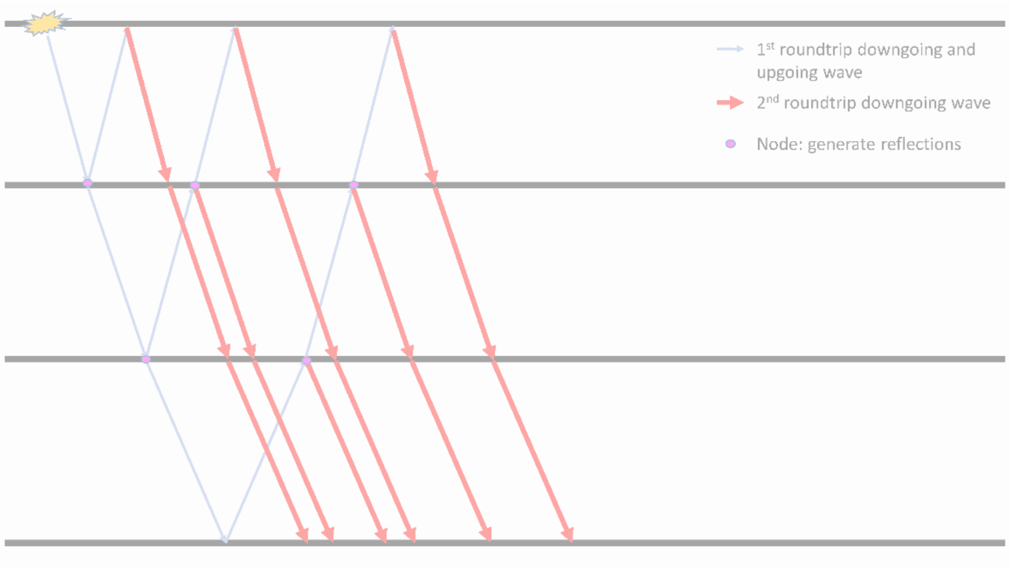


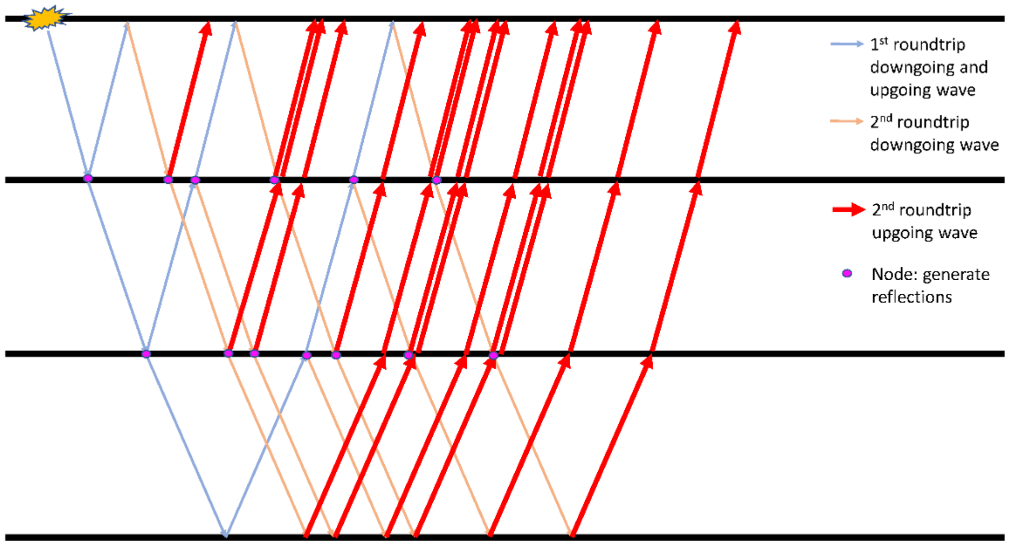


Second roundtrip upgoing wavefield





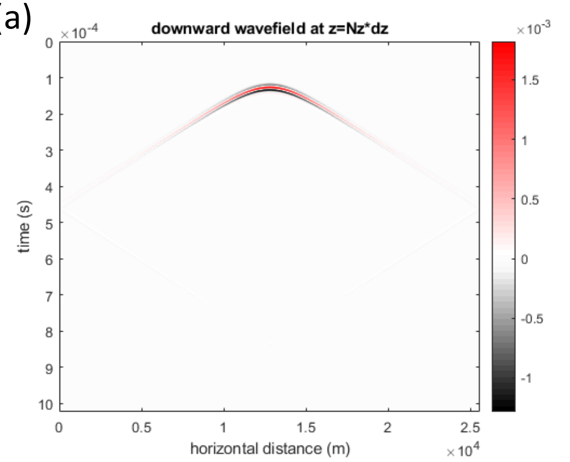




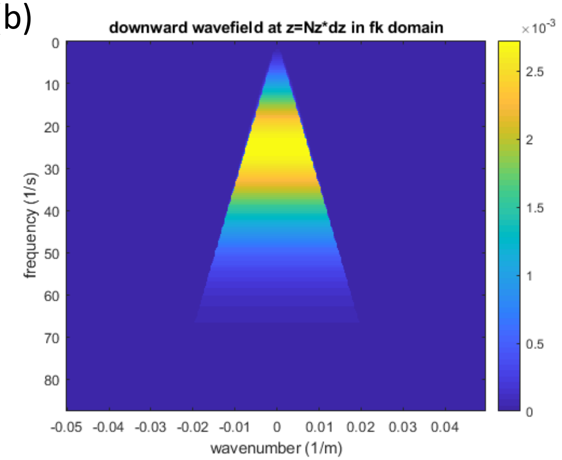
Advantage of using F-K domain

Downgoing wavefield (at z=Nz*dz) after the first and second iteration:

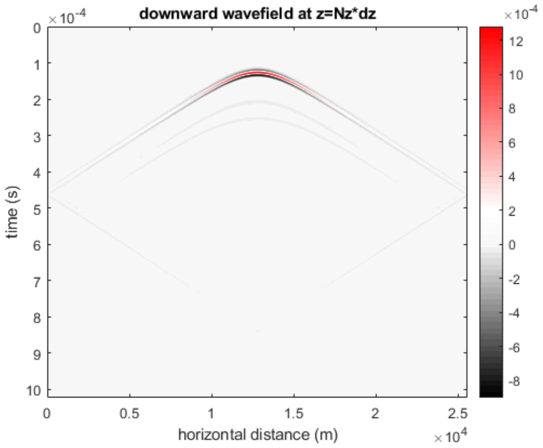
(a)



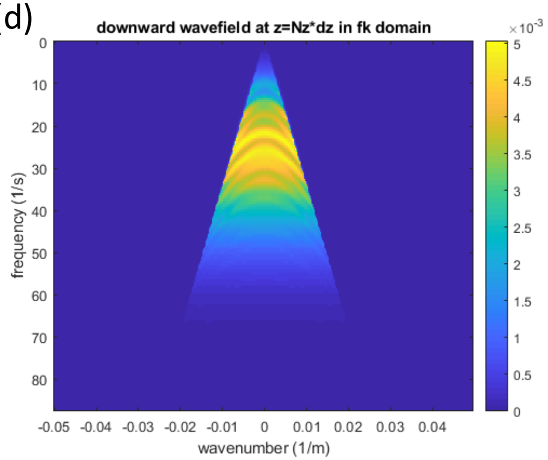
(b)



(c)

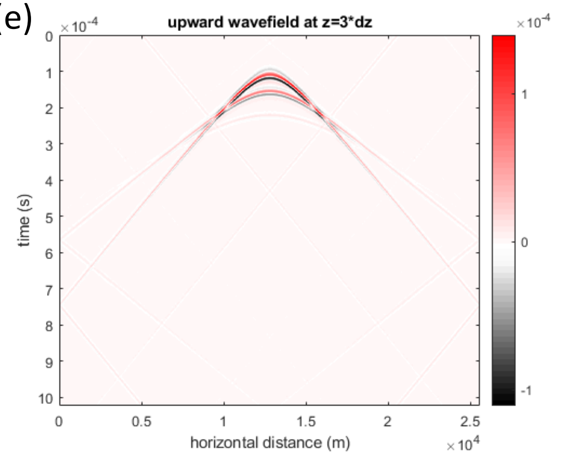


(d)

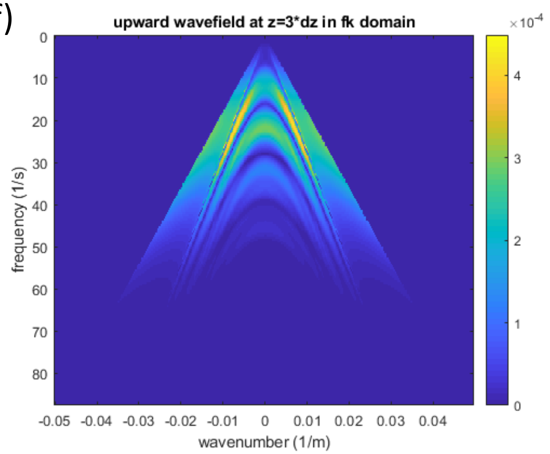


Upgoing wavefield (at z=3*dz) after the first and second iteration:

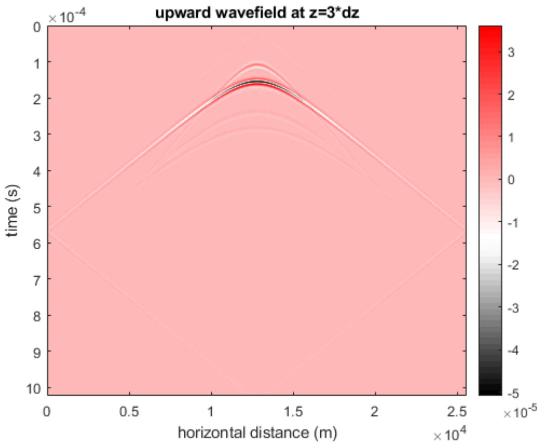
(e)



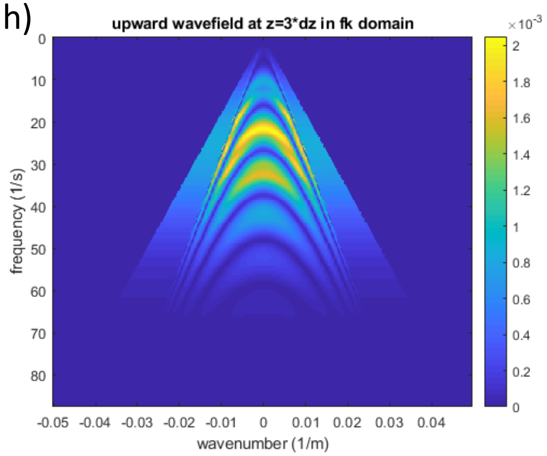
(f)



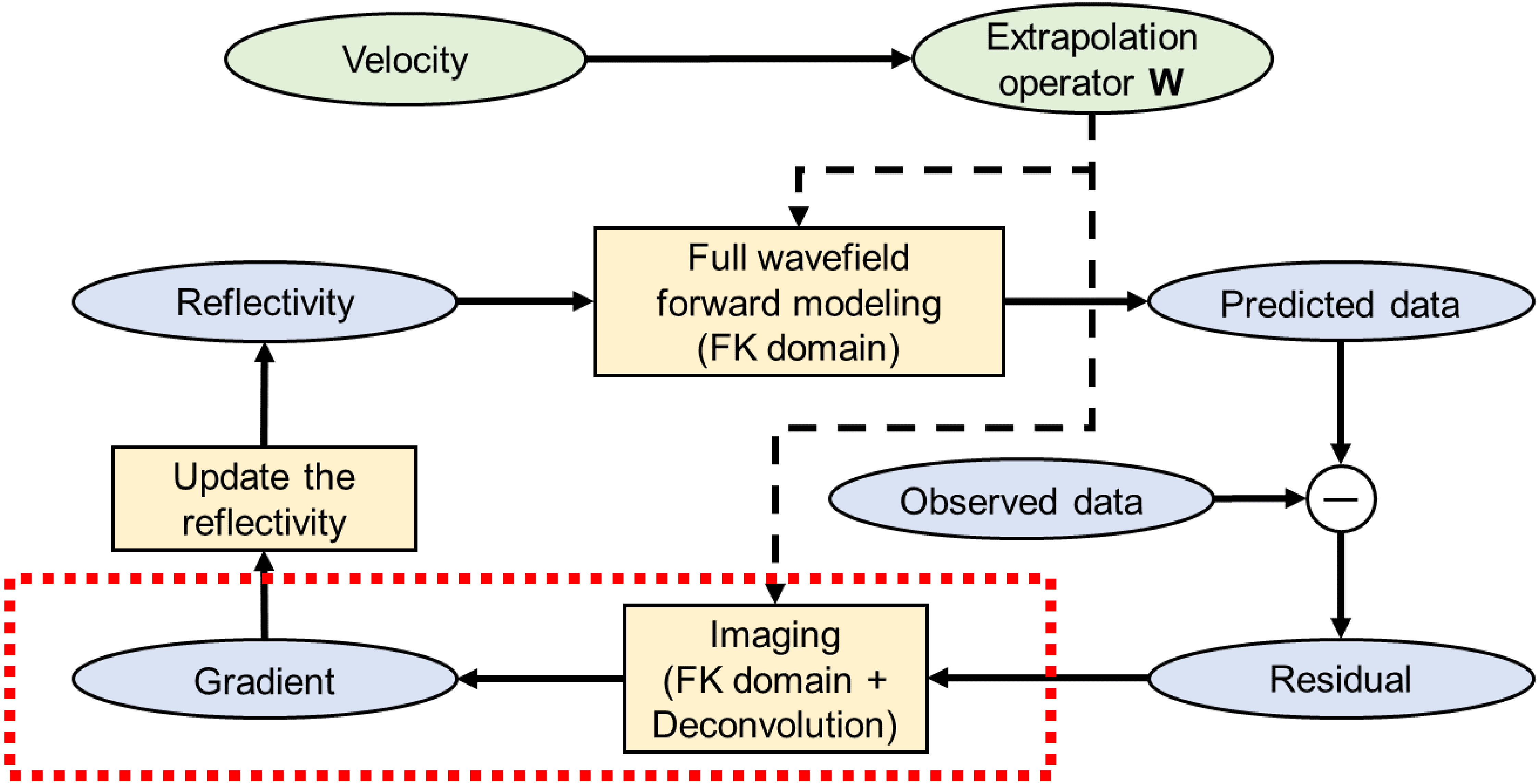
(g)



(h)



Full-wavefield migration (FWM) workflow



Workflow adapted from Davydenko and Verschuur (2016)

Imaging in full-wavefield migration (FWM)

- Objective function for FWM (Davydenko and Verschuur, 2016) but in the F-K domain:

$$J = ||\Delta \mathbf{P}||_2^2 + f(\mathbf{R}) = ||\mathbf{P}_{obs} - \mathbf{P}_{mod}||_2^2 + f(\mathbf{R}) \qquad (4)$$

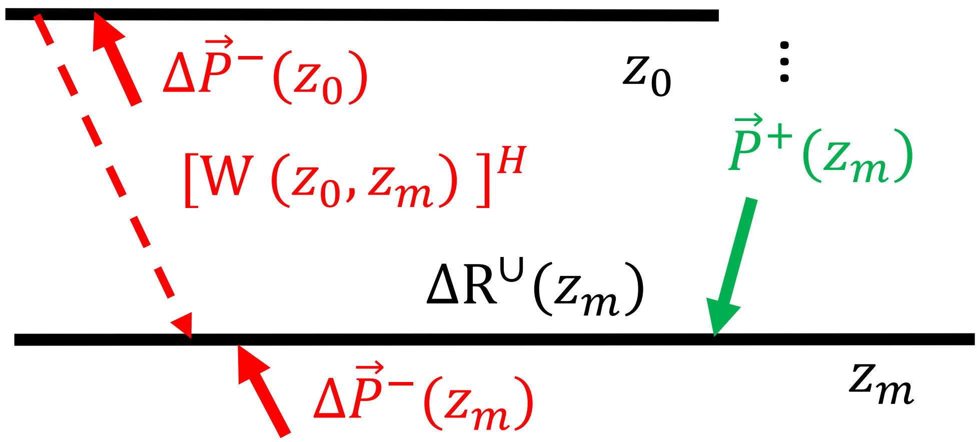


Fig 4. Reflectivity updates of both sides can be projected by cross-correlation between forward-modelled wavefield (green lines) and backward residuals (red lines).

- The gradient of objective function (Valenciano and Biondi, 2003)

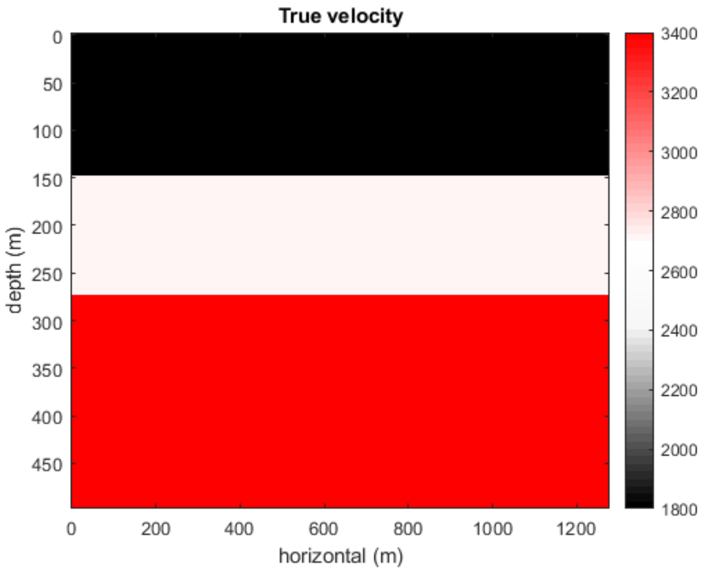
$$\mathbf{C}^\cup(z_m) = [\Delta\mathbf{P}^-(z_m)][\mathbf{P}^+(z_m)]^H / ([\mathbf{P}^+(z_m)][\mathbf{P}^+(z_m)]^H + \varepsilon^2)$$
$$\mathbf{C}^\cap(z_m) = [\Delta\mathbf{P}^+(z_m)][\mathbf{P}^-(z_m)]^H / ([\mathbf{P}^-(z_m)][\mathbf{P}^-(z_m)]^H + \varepsilon^2) \qquad (5)$$

- Update reflectivity matrix

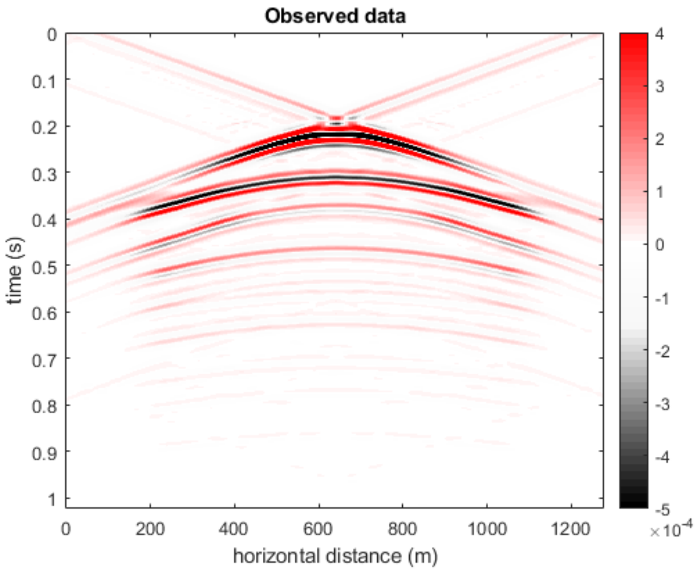
$$\Delta\mathbf{R}^\cup(z_m) = \left(\sum_{k_x}\sum_{\omega}\mathbf{C}^\cup(z_m)\right) + f'(\mathbf{R}^\cup(z_m))$$
$$\Delta\mathbf{R}^\cap(z_m) = \left(\sum_{k_x}\sum_{\omega}\mathbf{C}^\cap(z_m)\right) + f'(\mathbf{R}^\cap(z_m)) \qquad (6)$$

Example 1 – Horizontal-layered model

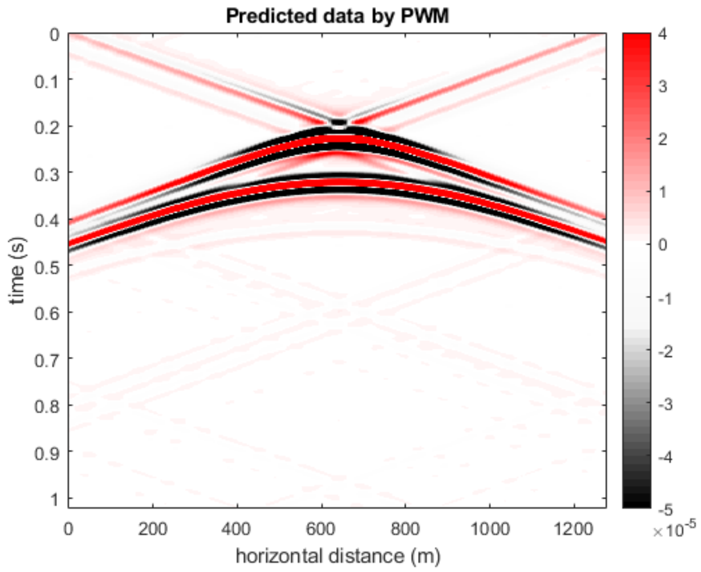
(a) True velocity model



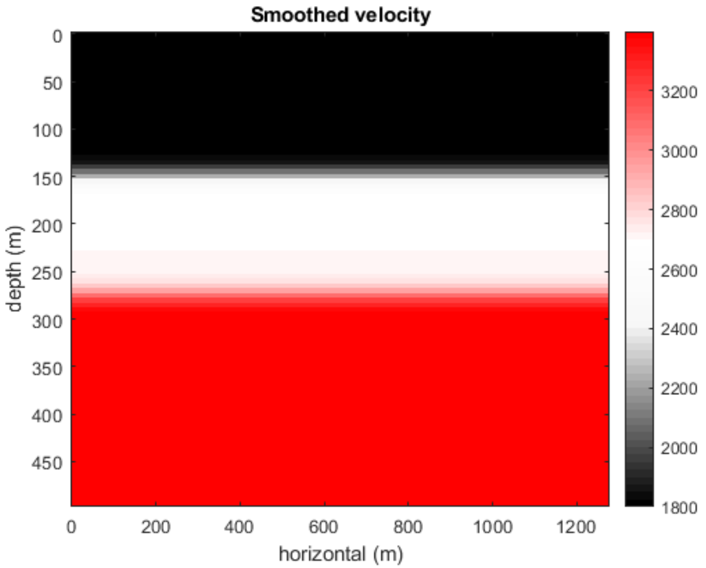
(b) Observed data



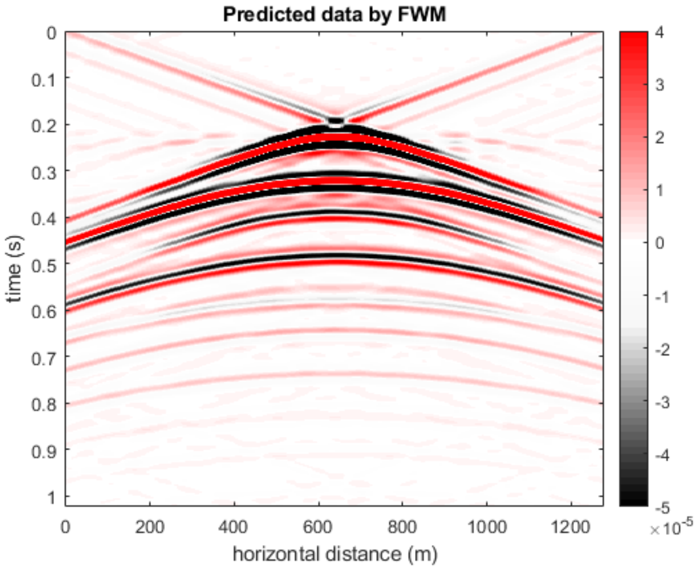
(c) Forward modeling in PWM



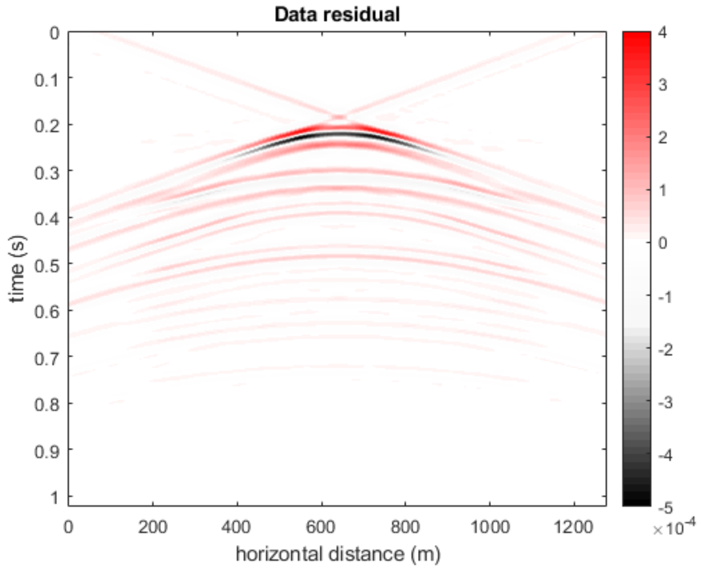
(d) Smoothed velocity model



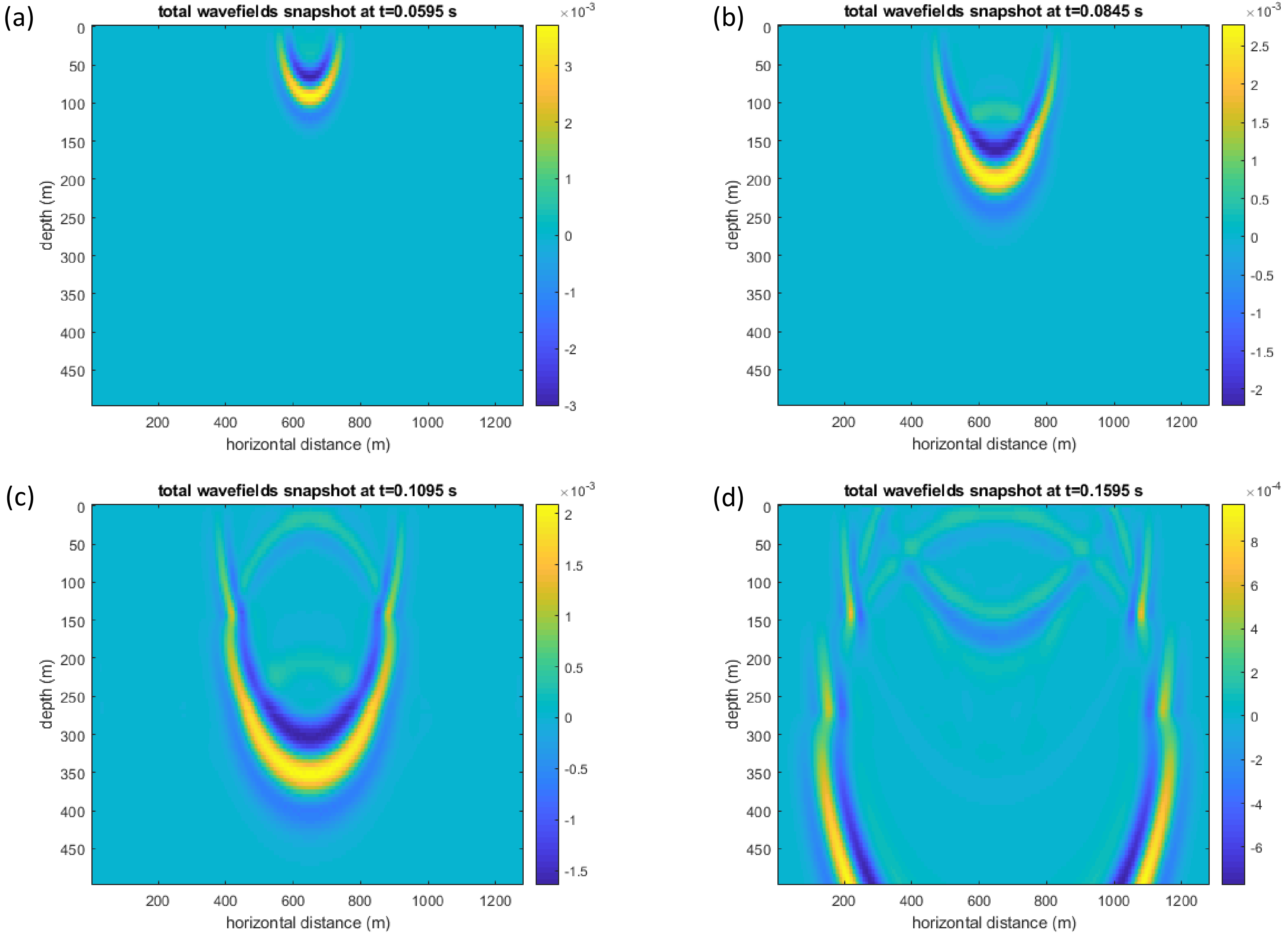
(e) Forward modeling in FWM



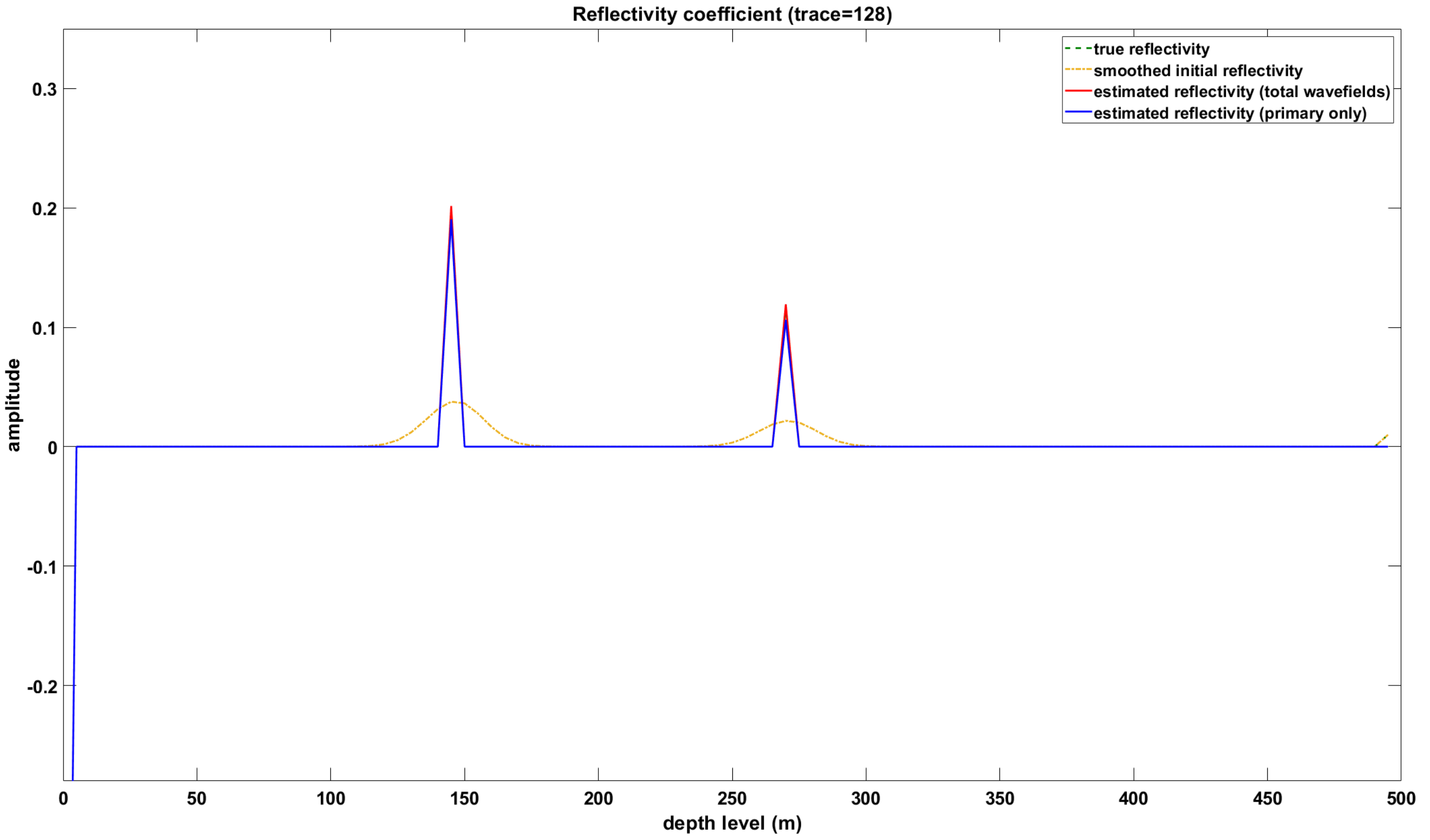
(f) Difference between (b) and (e)



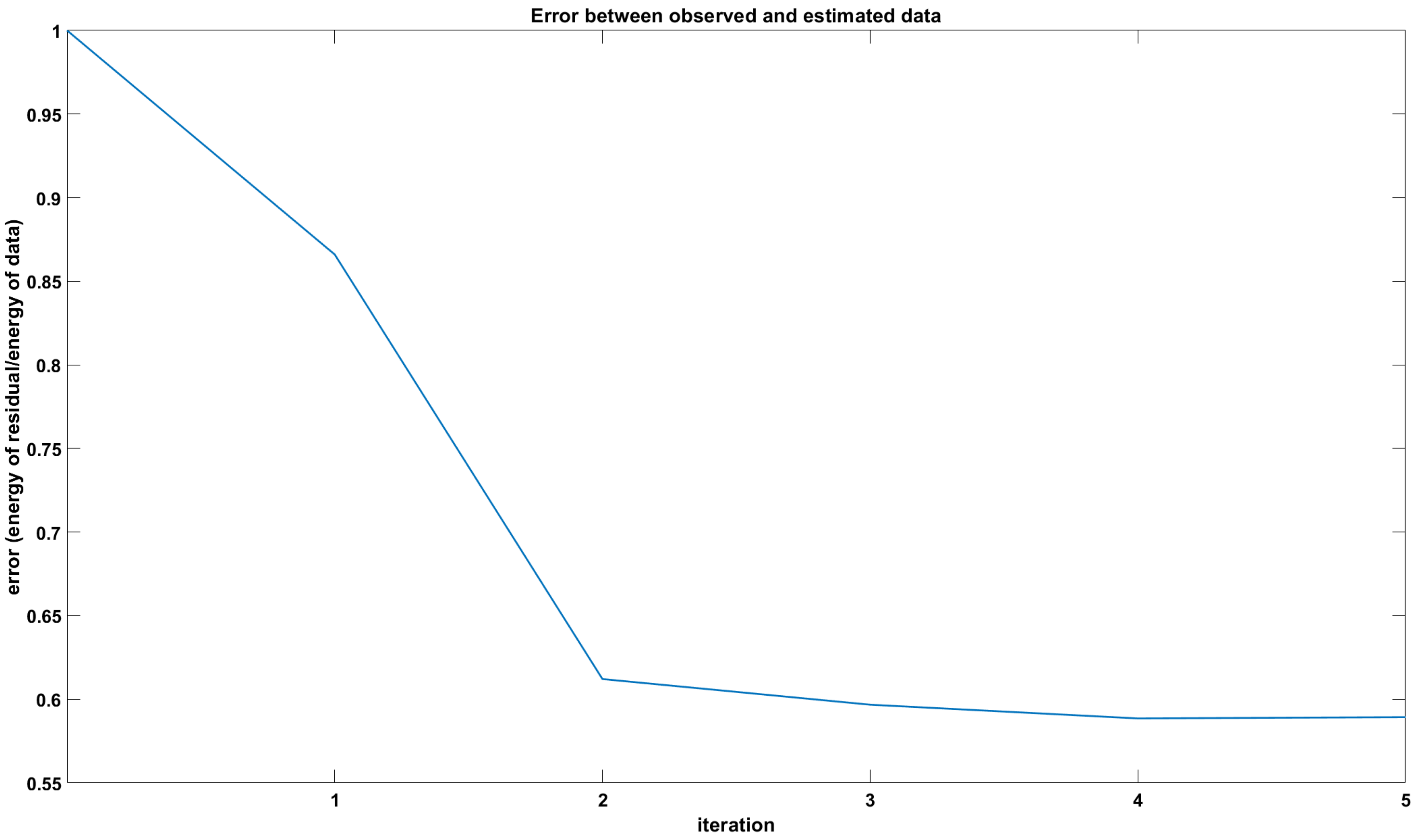
Example 1 – Total wavefields snapshots



Example 1 – Reflectivity coefficient comparison

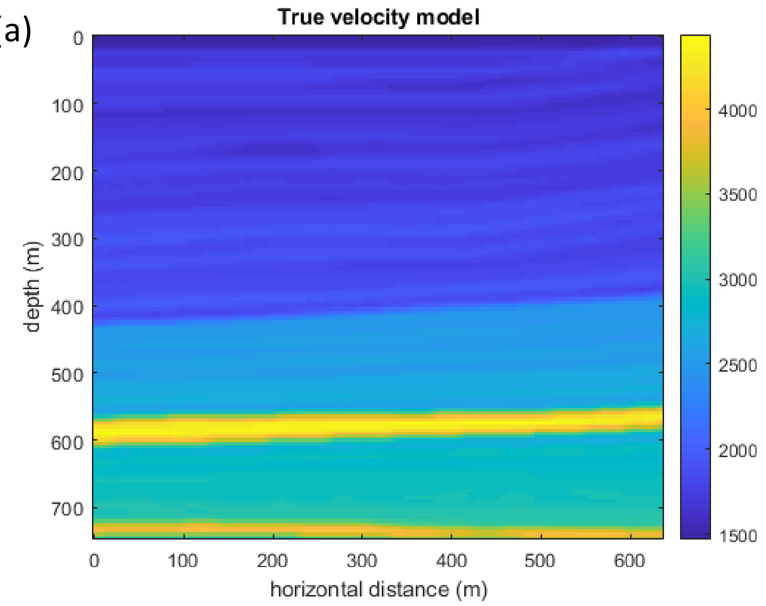


Example 1 – Error analysis

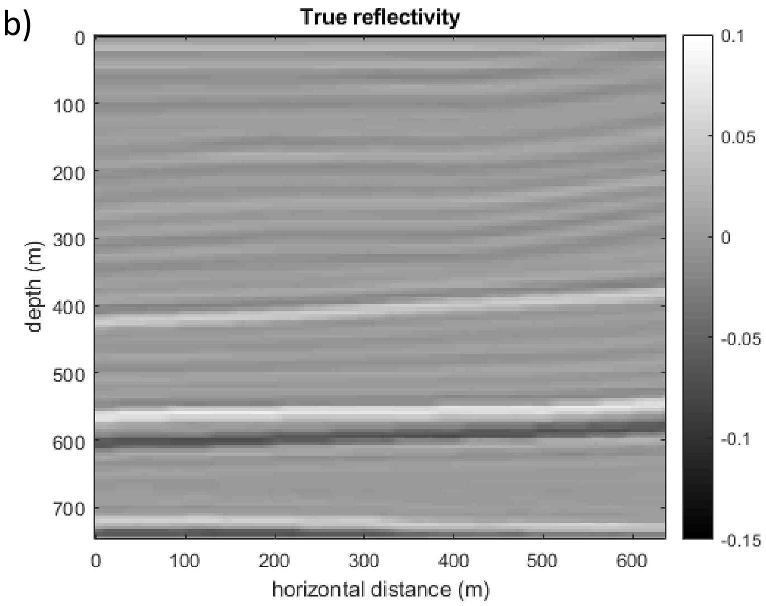


Example 2 – Left part of Marmousi model

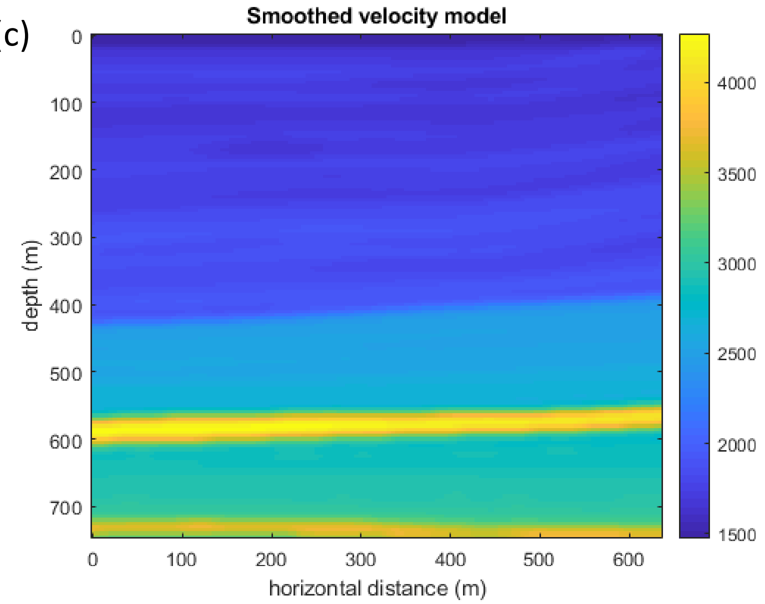
(a)



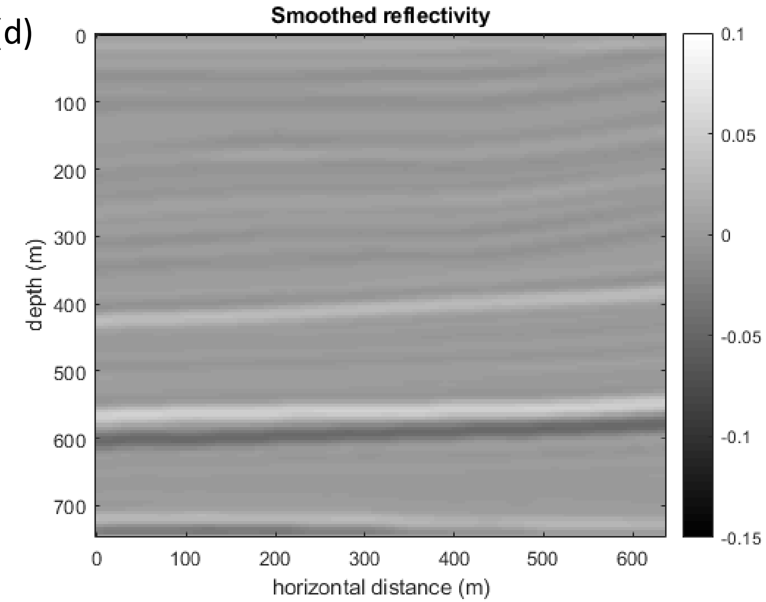
(b)



(c)



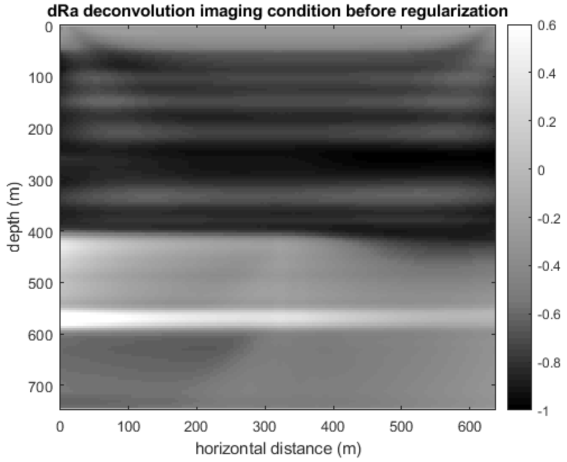
(d)



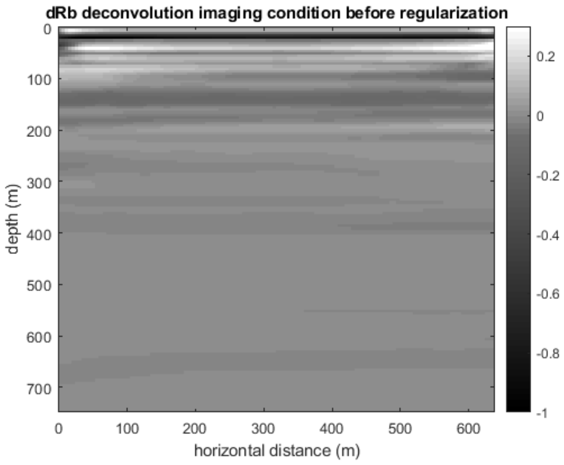
Example 2 – Using deconvolution imaging condition

Deconvolution imaging condition:

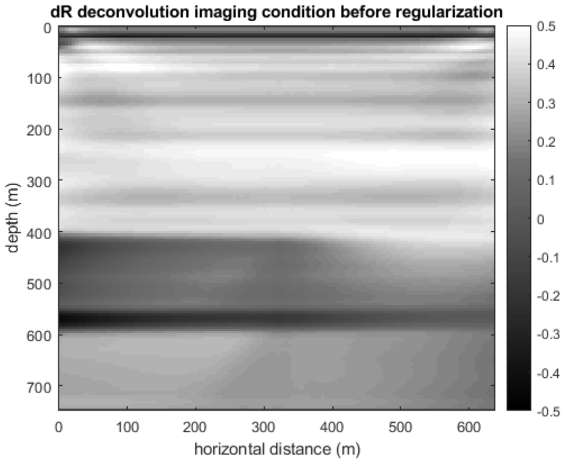
(a) $\mathbf{dR}^{\cup}$



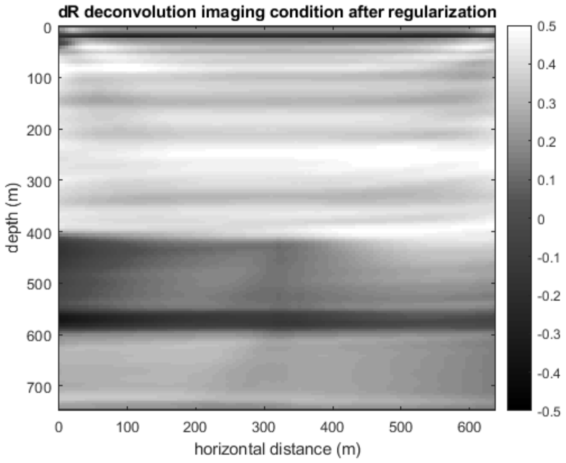
(b) $\mathbf{dR}^{\cap}$



(c) $\mathbf{dR}$

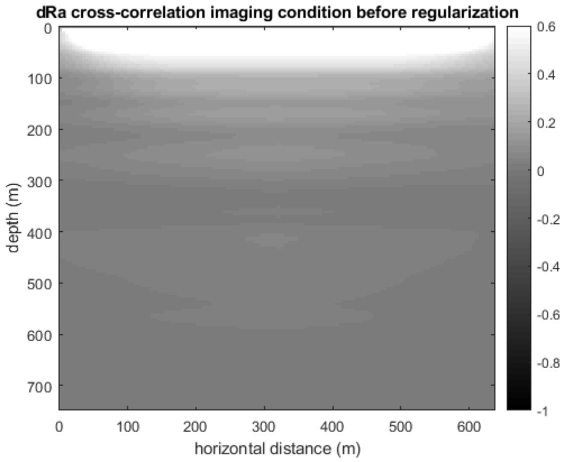


(d) $\mathbf{dR} + \mathbf{df}(\mathbf{R})$

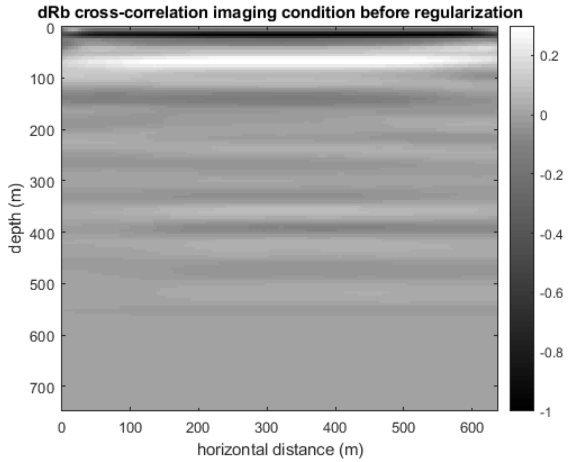


Cross-correlation imaging condition:

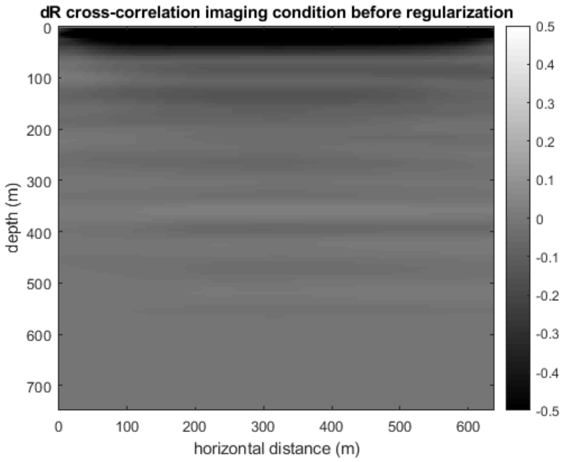
(e) $\mathbf{dR}^{\cup}$



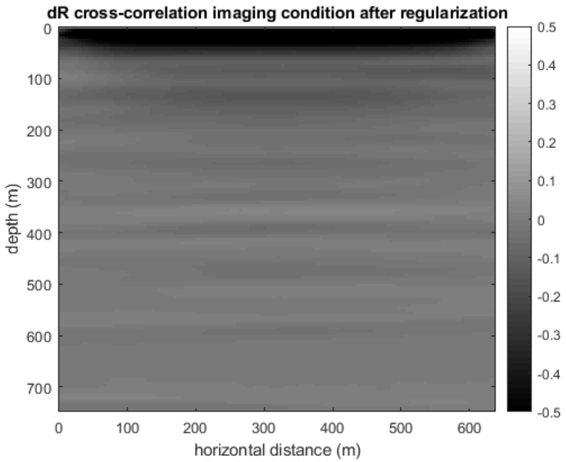
(f) $\mathbf{dR}^{\cap}$



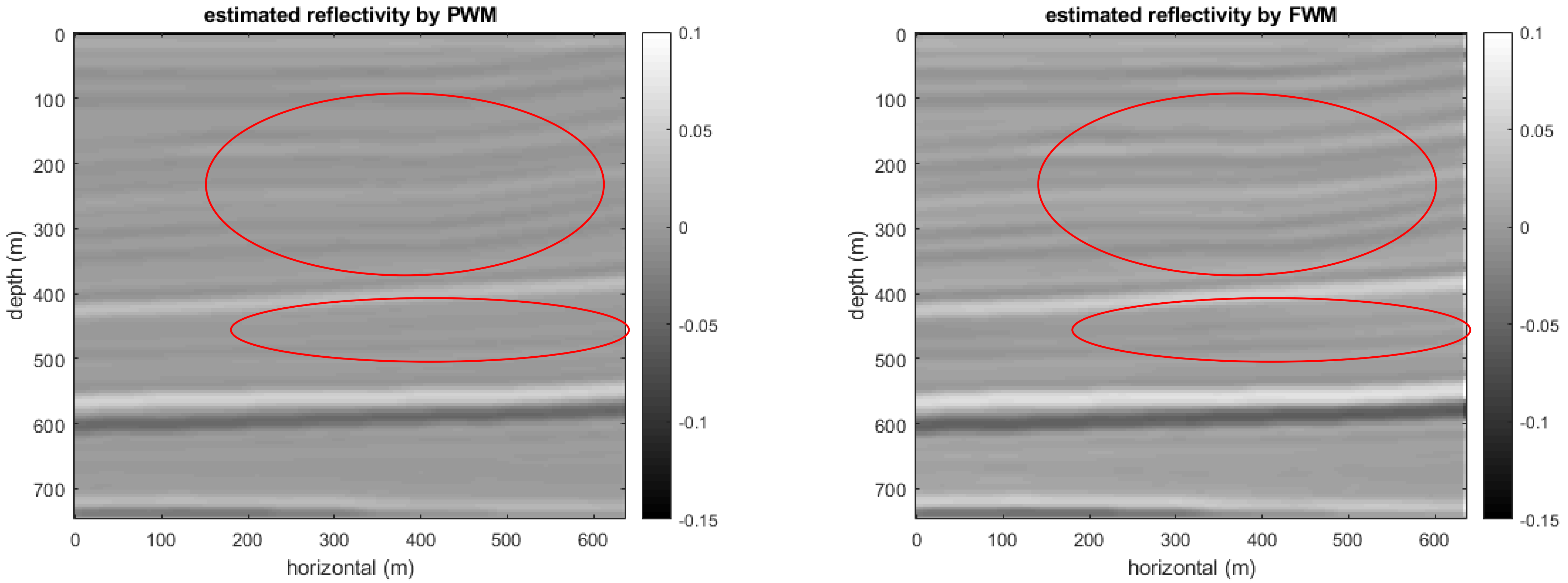
(g) $\mathbf{dR}$



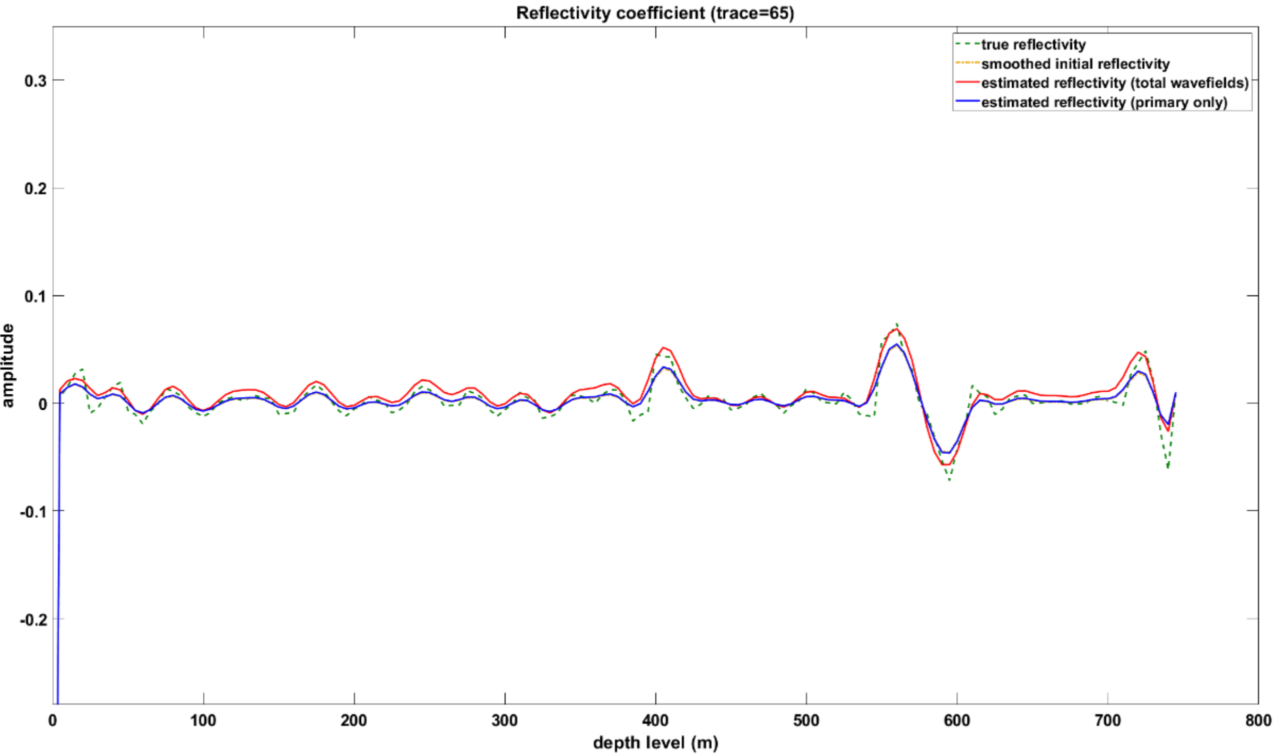
(h) $\mathbf{dR} + \mathbf{df}(\mathbf{R})$

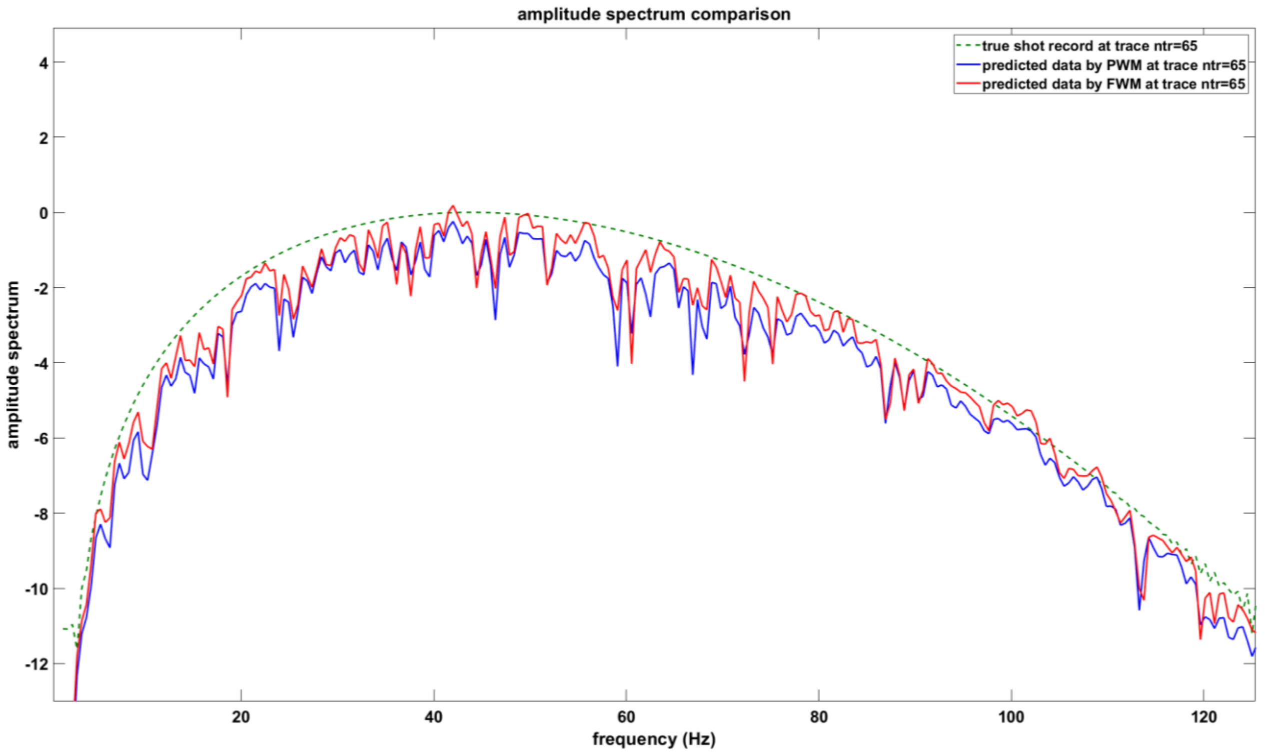


Migration results



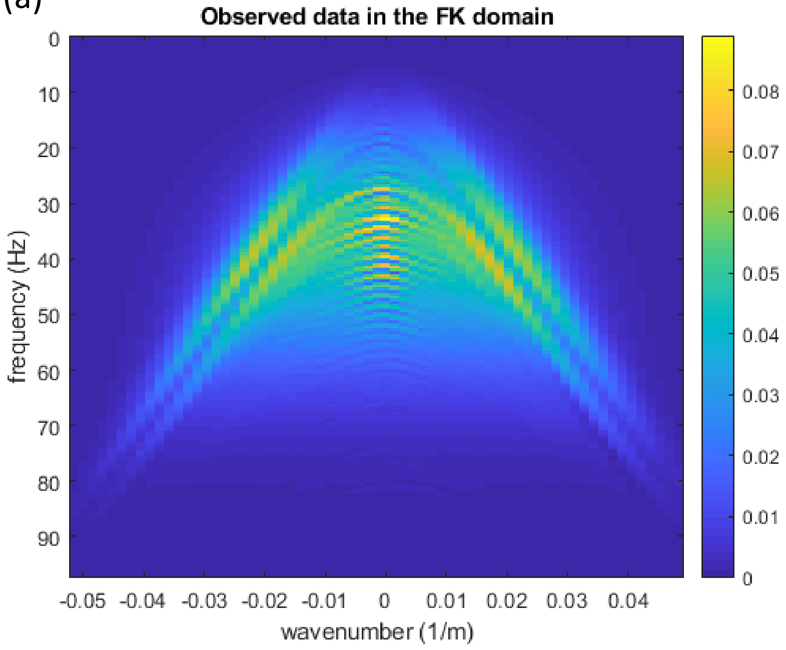
Example 2 – Reflection coefficient and amplitude spectrum comparison



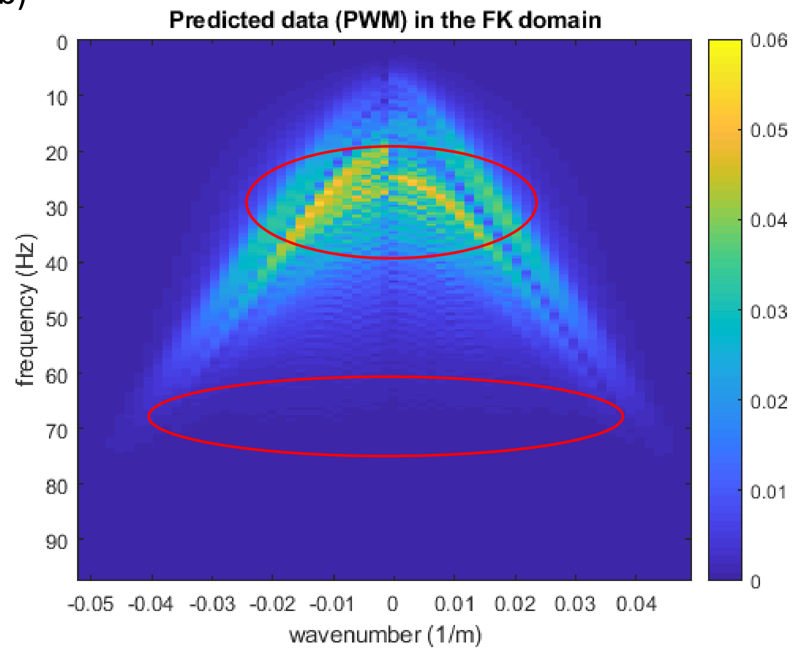


F-K domain comparison

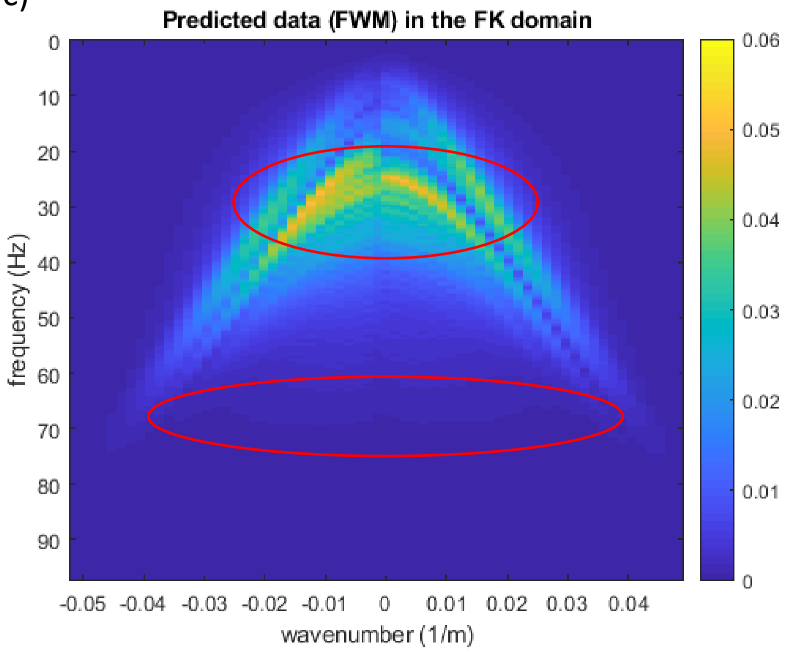
(a)



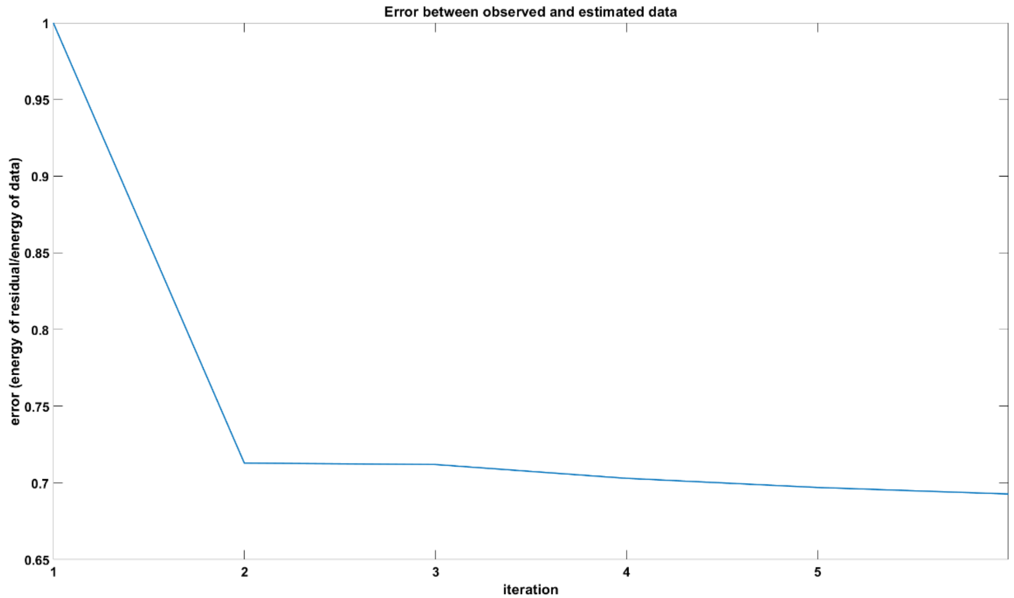
(b)



(c)



(d)



Conclusion

- Full wavefield modelling in the F-K domain can track the different orders of multiple reflections.
- Full wavefield migration result is more accurate than applying primary wavefield migration.
- Given wide offsets and a good initial background model, the deconvolution imaging condition can improve to predict subsurface layer locations with fewer artifacts.

Future work

- In future work, we should correct the amplitude and phase information showing in the F-K domain.
- The next step is trying to reduce the computational cost due to the three-dimensional data structure.
- Furthermore, we need to consider angle-dependent reflectivity or angle gathers into the migration process for better imaging results.

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Questions?

Thank you!