Elastic FWI with rock physics constraints

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Sponsors Meeting

NSERC
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UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
Department of Geoscience

Motivation

- Key challenges of EFWI:
- Highly nonlinear and ill-posed: objective function is multi-modal, solution is non-unique
- Parameter crosstalk: errors in one parameter are mapped into the updates of another

  → increase the nonlinearity and uncertainty of the inverse problem

- Regularization: imposing constraints in the solution

  → stabilize the inversion and obtain more plausible model

Objective function:

$$E(\mathbf{m}) = \underset{\substack{\uparrow \\ \text{Data misfit}}}{E_d}(\mathbf{m}) + \lambda \underset{\substack{\nwarrow \\ \text{Model penalty}}}{E_c}(\mathbf{m})$$

Motivation

Objective function: $E(\mathbf{m}) = E_d(\mathbf{m}) + \lambda E_c(\mathbf{m})$

$$\downarrow$$

$$\left\|\mathbf{W}_d(\mathbf{d}_{obs} - \mathbf{d}(\mathbf{m}))\right\|^2$$

smoothness: $E_c(\mathbf{m}) = \left\|\mathbf{D}_x\mathbf{m}\right\|^2 + \left\|\mathbf{D}_z\mathbf{m}\right\|^2$ Tikhonov and Arsenin, 1977

prior estimate: $E_c(\mathbf{m}) = \left\|\mathbf{W}_m(\mathbf{m} - \mathbf{m_0})\right\|^2$ Asnaashari et al, 2003

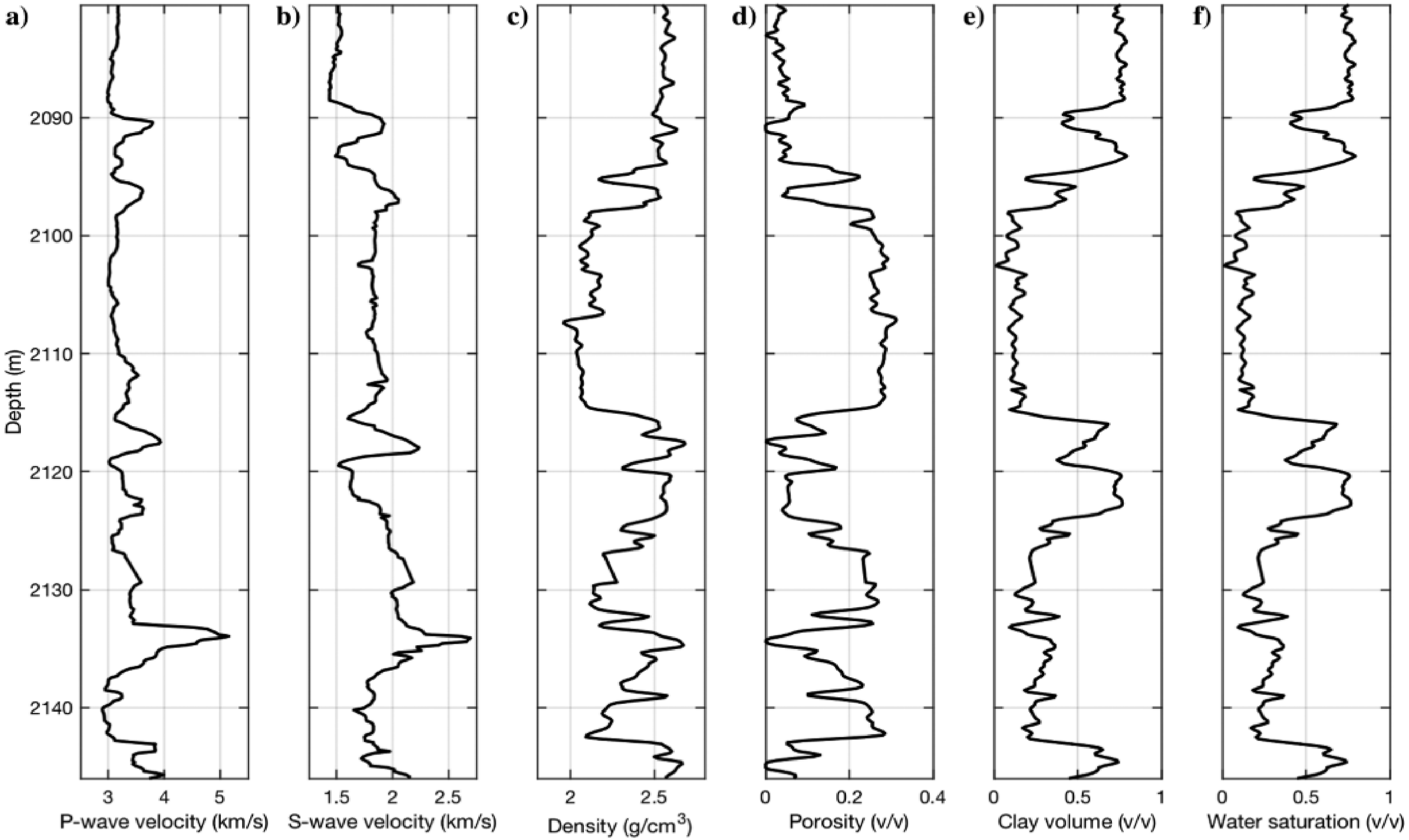
barriers: $E_c(\mathbf{m}) = -\sum_{\mathbf{x}}[\log(h_u) + \log(h_l)]$ Rocha and Sava, 2018

explicit relations: $E_c(\mathbf{m}) = \left\|\mathbf{m}_1 - f(\mathbf{m}_2)\right\|^2 \xrightarrow{\text{forcing}} \mathbf{m}_1 = f(\mathbf{m}_2)$

(Our Study)

Motivation

Physical parameters are often correlated

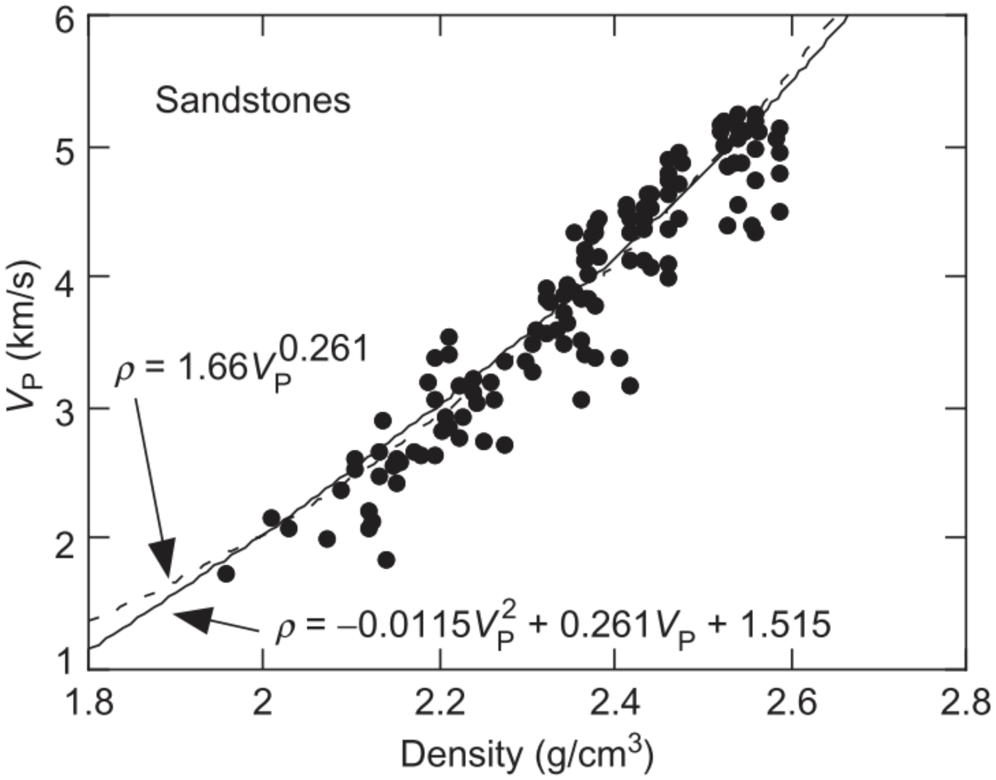


Well-log data from Grana(2016)

Motivation

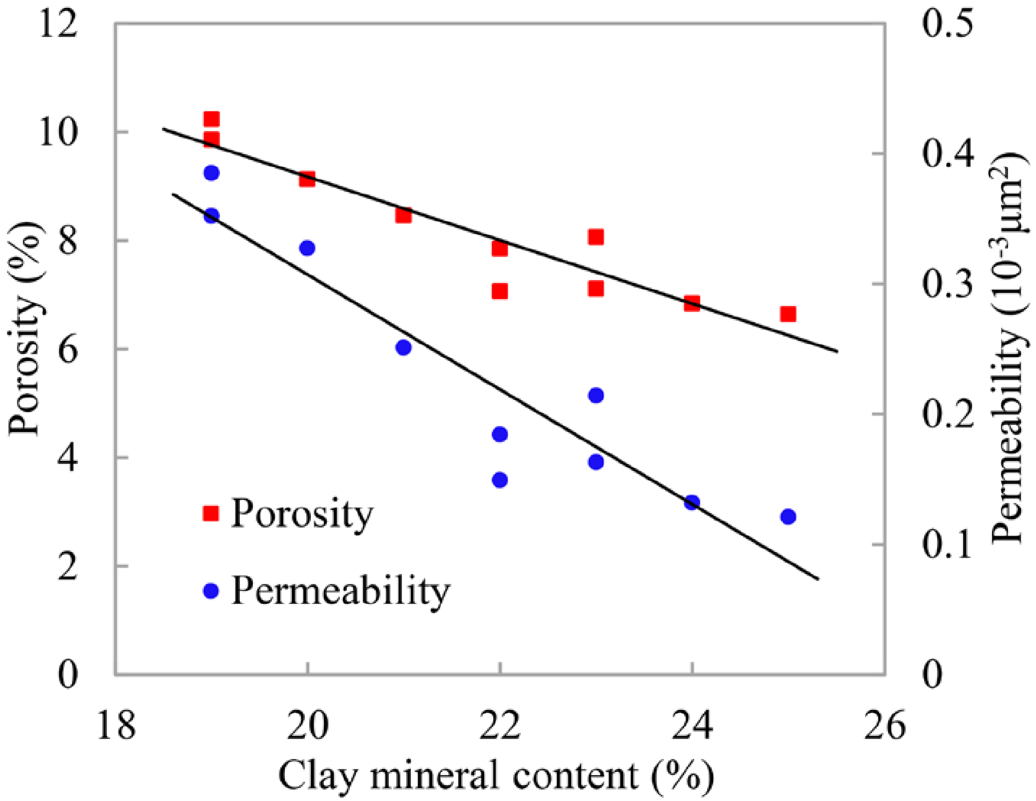
Explicit relations can be obtained if the two are highly correlated:

Vp-Den relation



Castagna et al. (1993).

P-C relation



Liu et al. (2016).

Motivation

Relations between elastic parameters are lithology/facies dependent

→ rock physics constraint

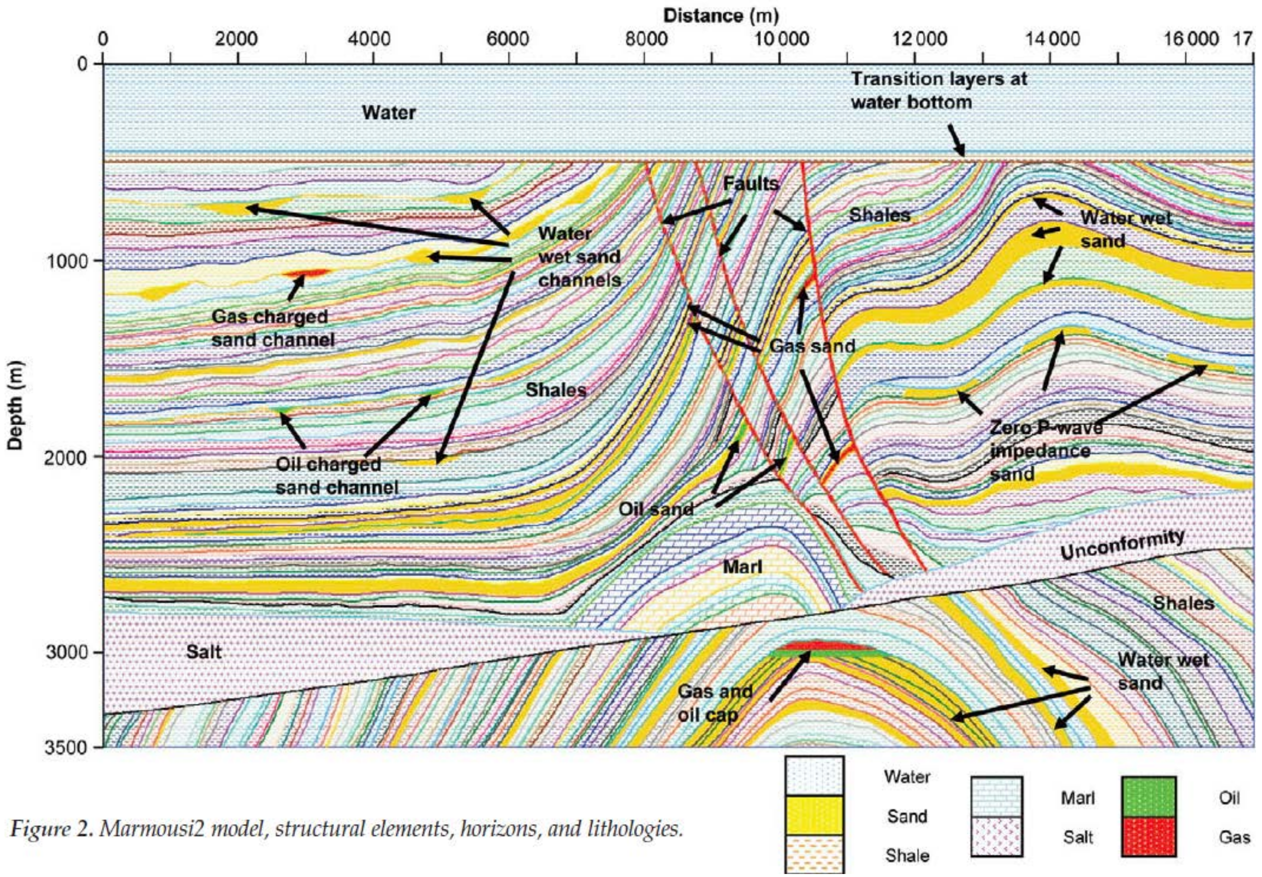


Figure 2. Marmousi2 model, structural elements, horizons, and lithologies.

The creation of Marmousi2 model

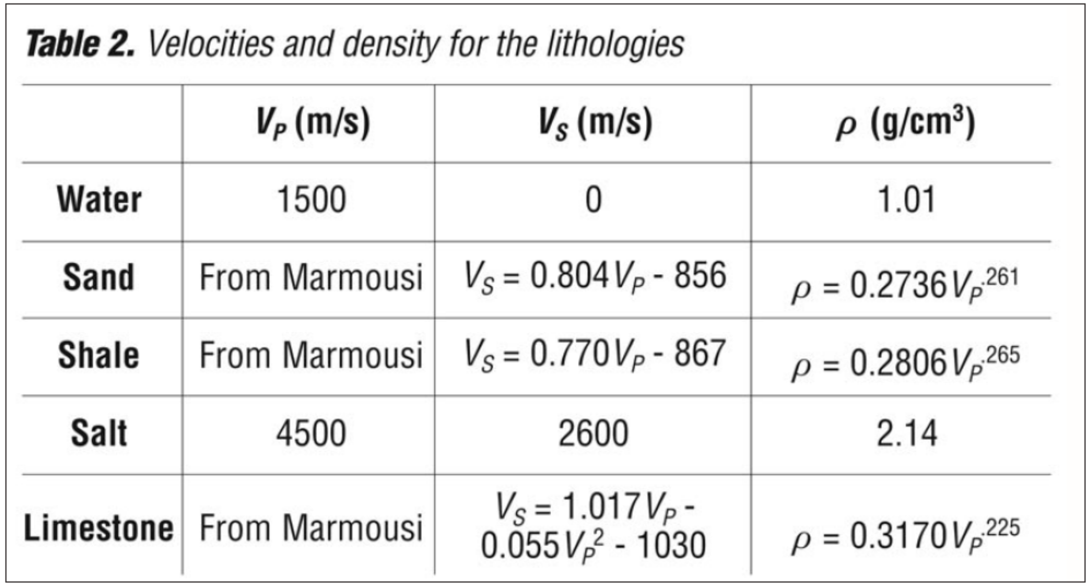


Table 2. *Velocities and density for the lithologies*

	V_P (m/s)	V_S (m/s)	ρ (g/cm^3)
Water	1500	0	1.01
Sand	From Marmousi	$V_S = 0.804 V_P - 856$	$\rho = 0.2736 V_P^{.261}$
Shale	From Marmousi	$V_S = 0.770 V_P - 867$	$\rho = 0.2806 V_P^{.265}$
Salt	4500	2600	2.14
Limestone	From Marmousi	$V_S = 1.017 V_P - 0.055 V_P^2 - 1030$	$\rho = 0.3170 V_P^{.225}$

Martin(2006).

Methods

Following:

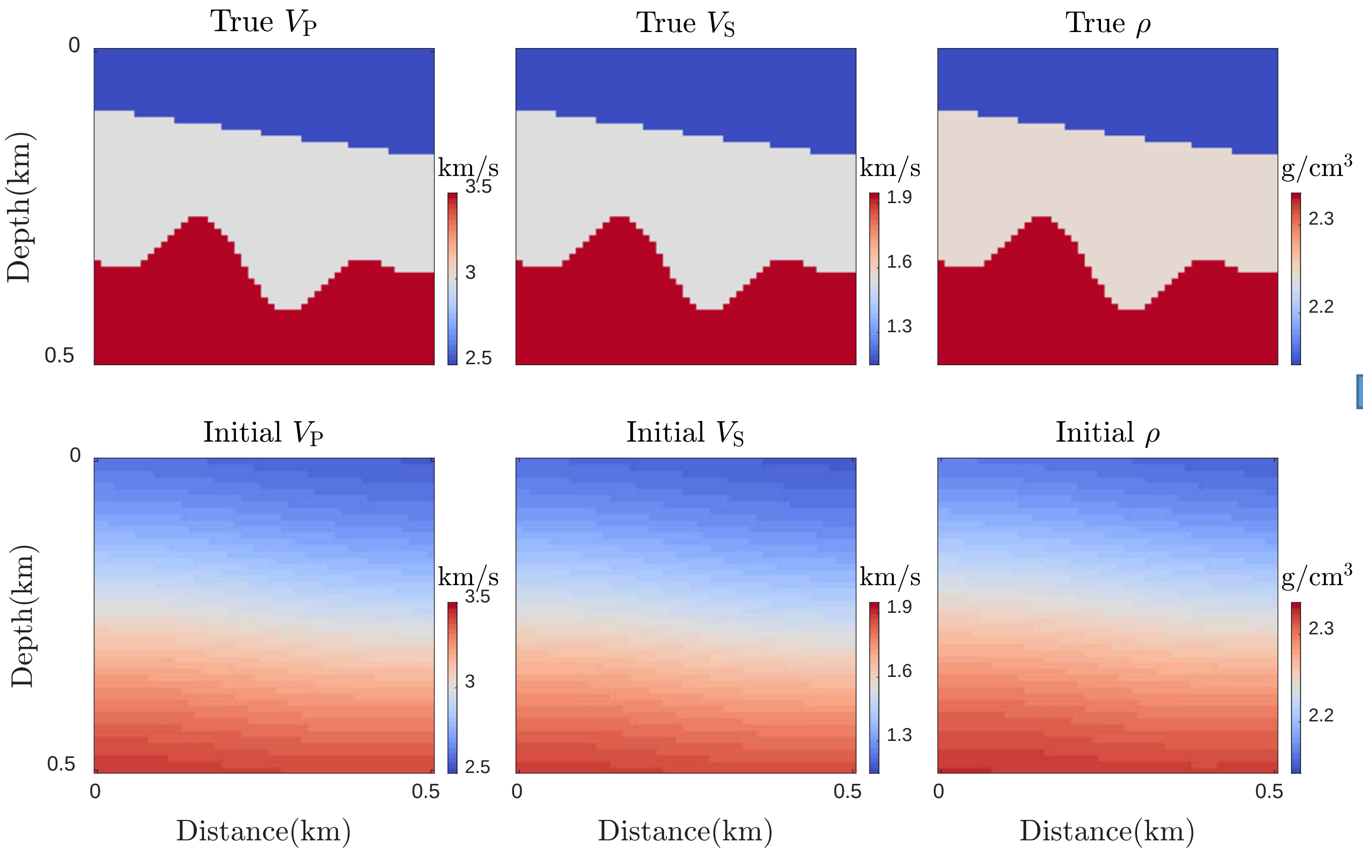
- EFWI parameterized by $V_{\mathrm{P}}, V_{\mathrm{S}}, \rho$

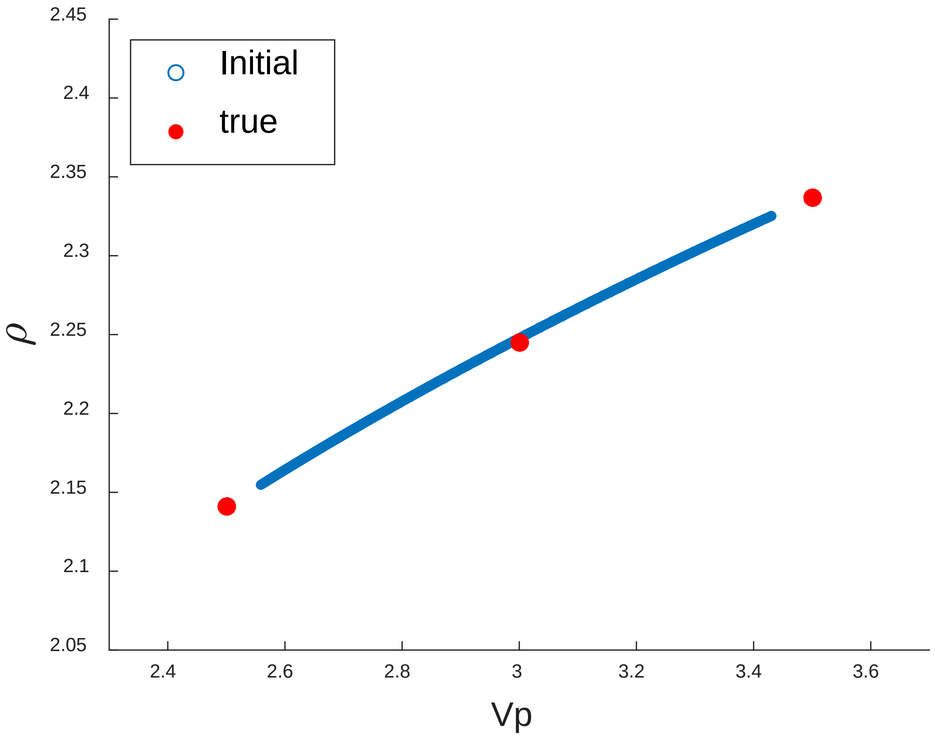
Model Constraint:

$$E_c(\mathbf{m}) = \|\underbrace{\rho}_{\text{Density: inverted}} - \underbrace{f(V_{\mathrm{P}})}_{\text{Density: estimated from inverted velocity}}\|^2 \xrightarrow{\text{forcing}} \rho = f(V_{\mathrm{P}})$$

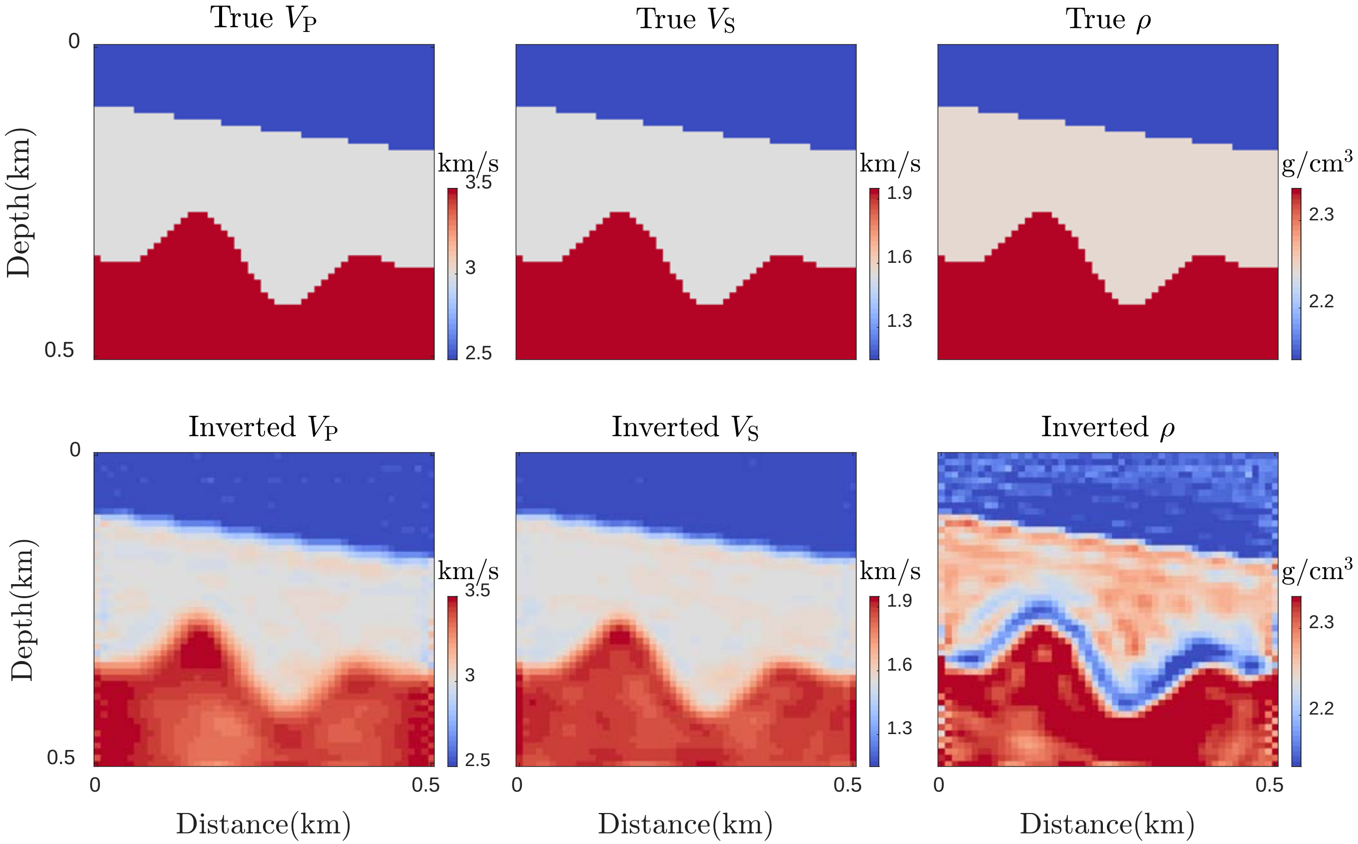
Numerical Examples

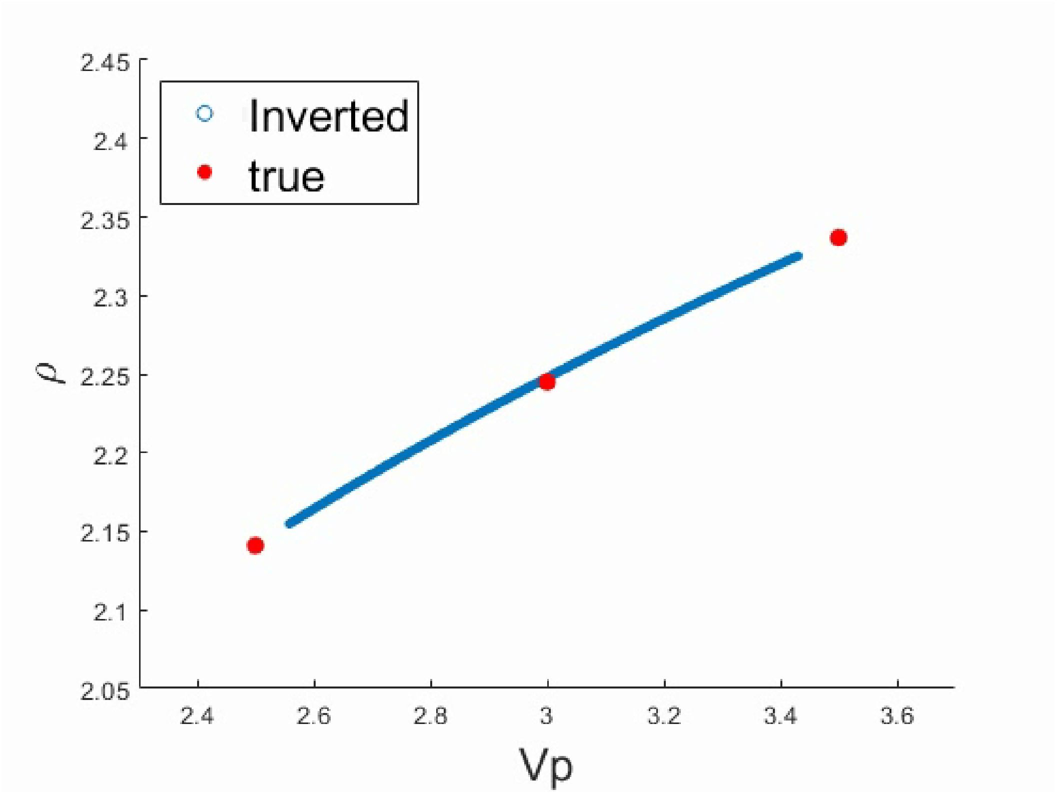
True and Initial models



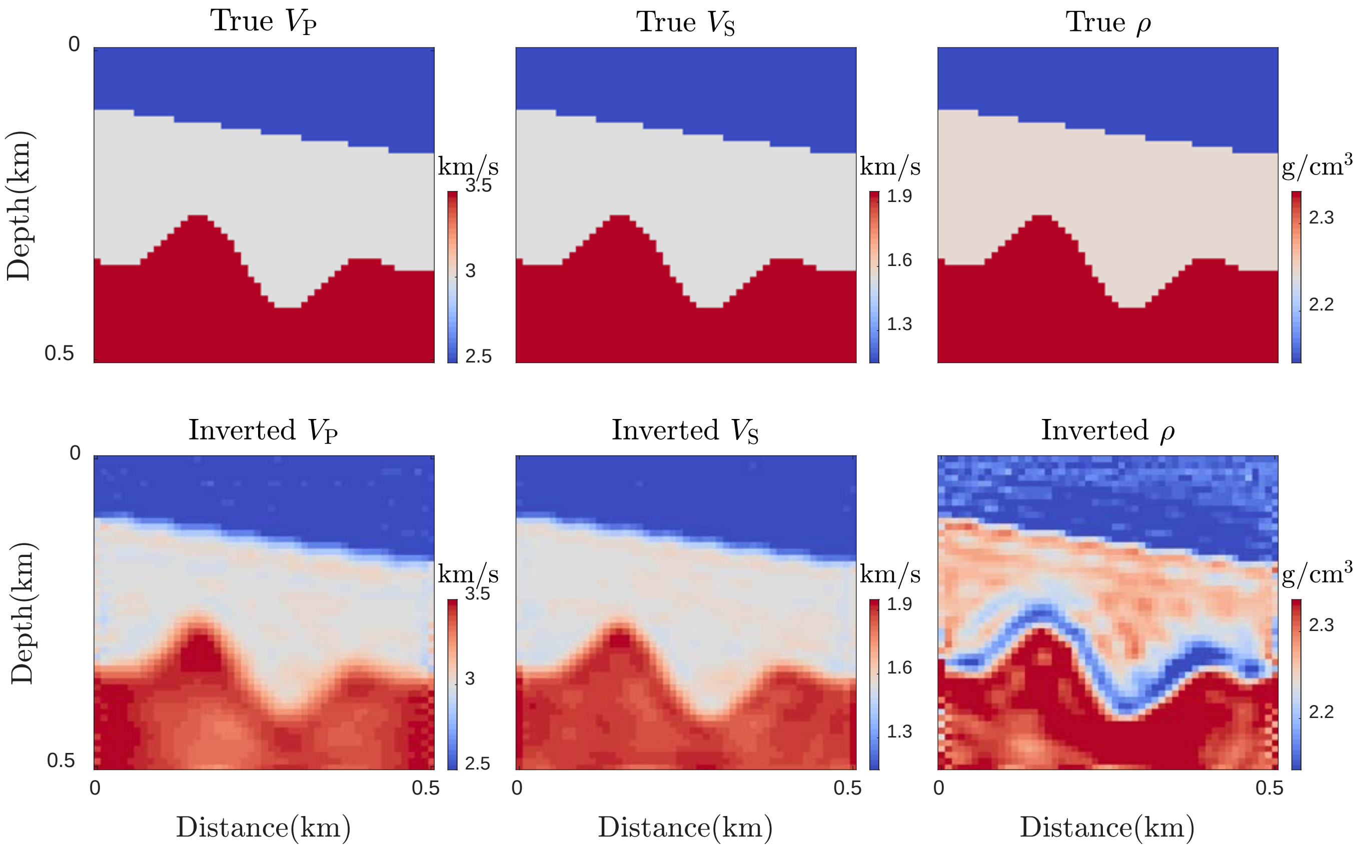


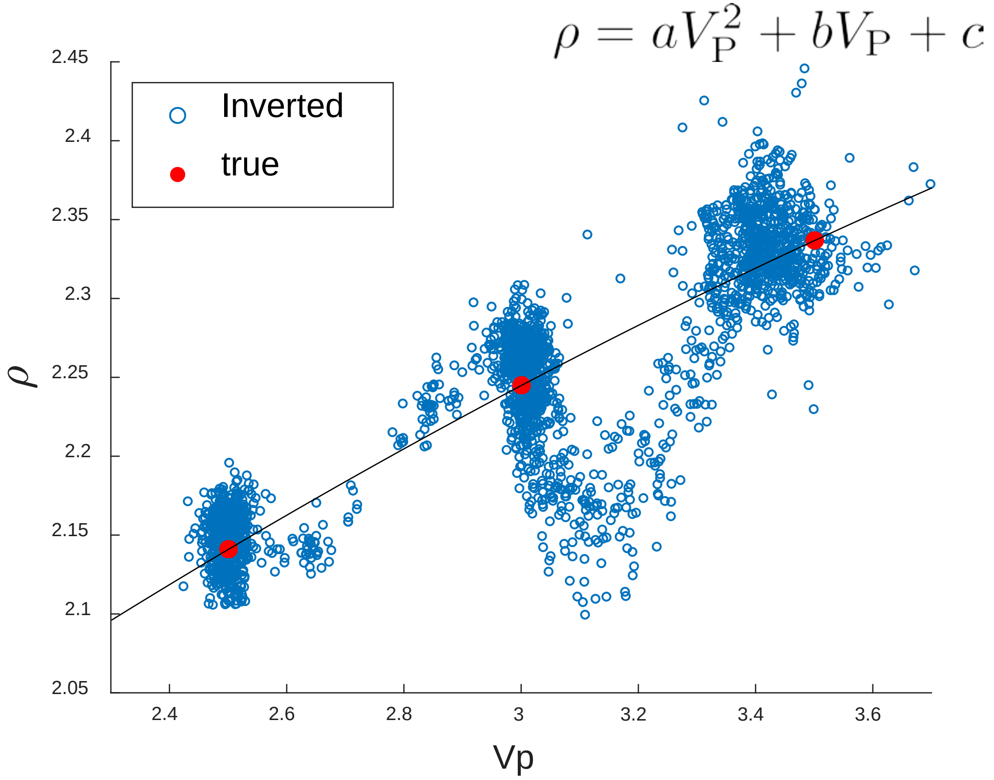
Inversion result: unconstrained





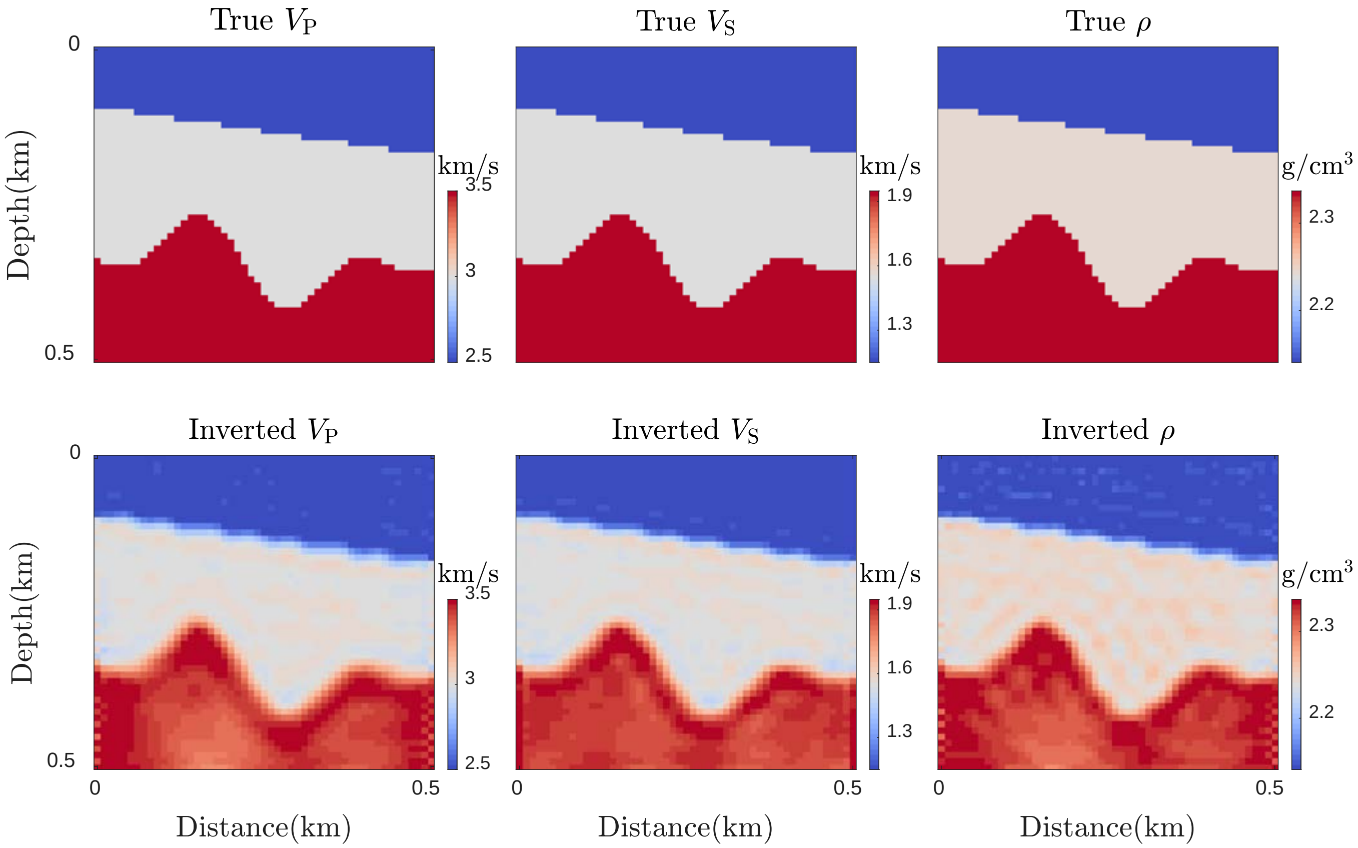
Inversion result: unconstrained

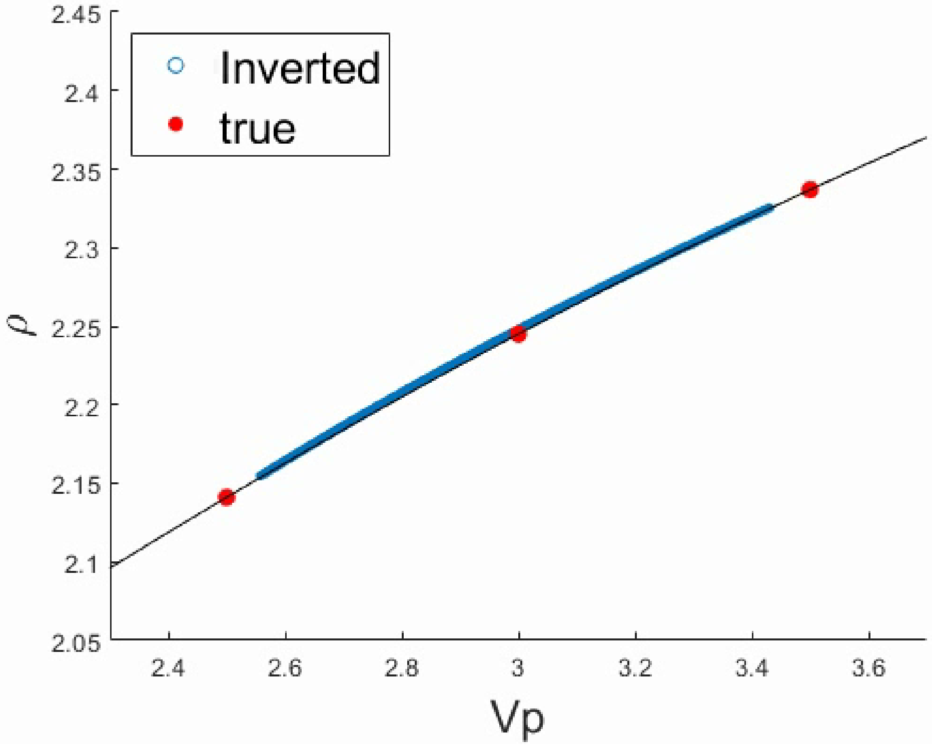




$$\rho = aV_{\mathrm{P}}^2 + bV_{\mathrm{P}} + c$$

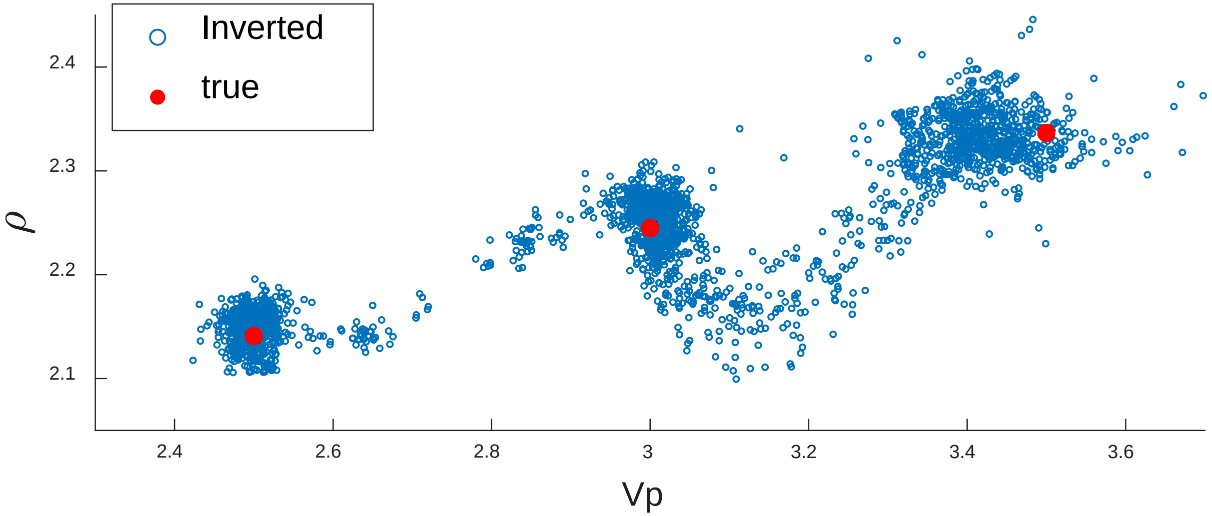
Inversion result: constrained



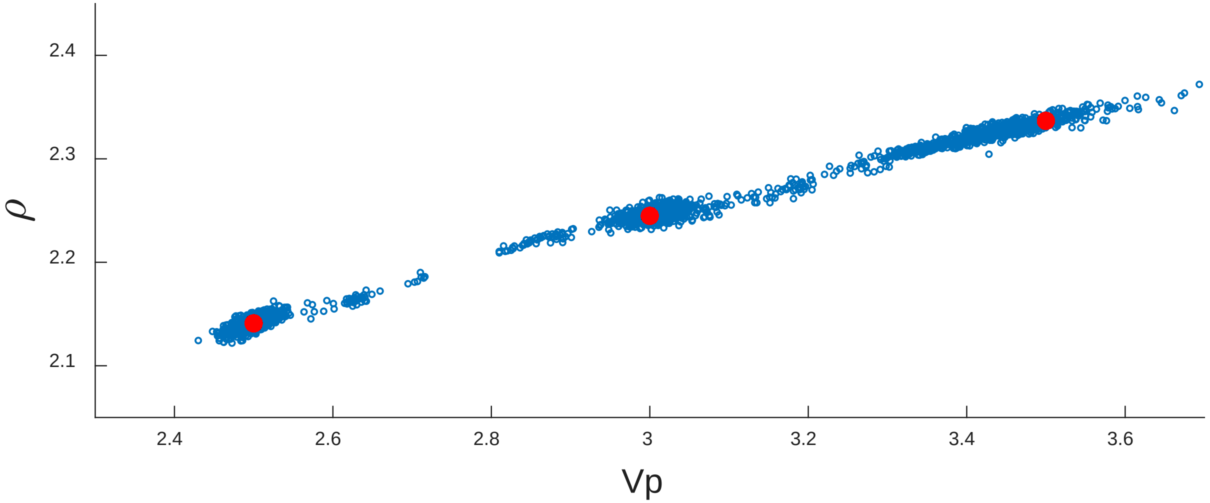


To summarize:

Unconstrained



Constrained



Methods

Extension:

Constraint

- EFWI parameterized by $V_{\mathrm{P}}, V_{\mathrm{S}}, \rho$ $\longleftarrow$ $\rho = f(V_{\mathrm{P}})$

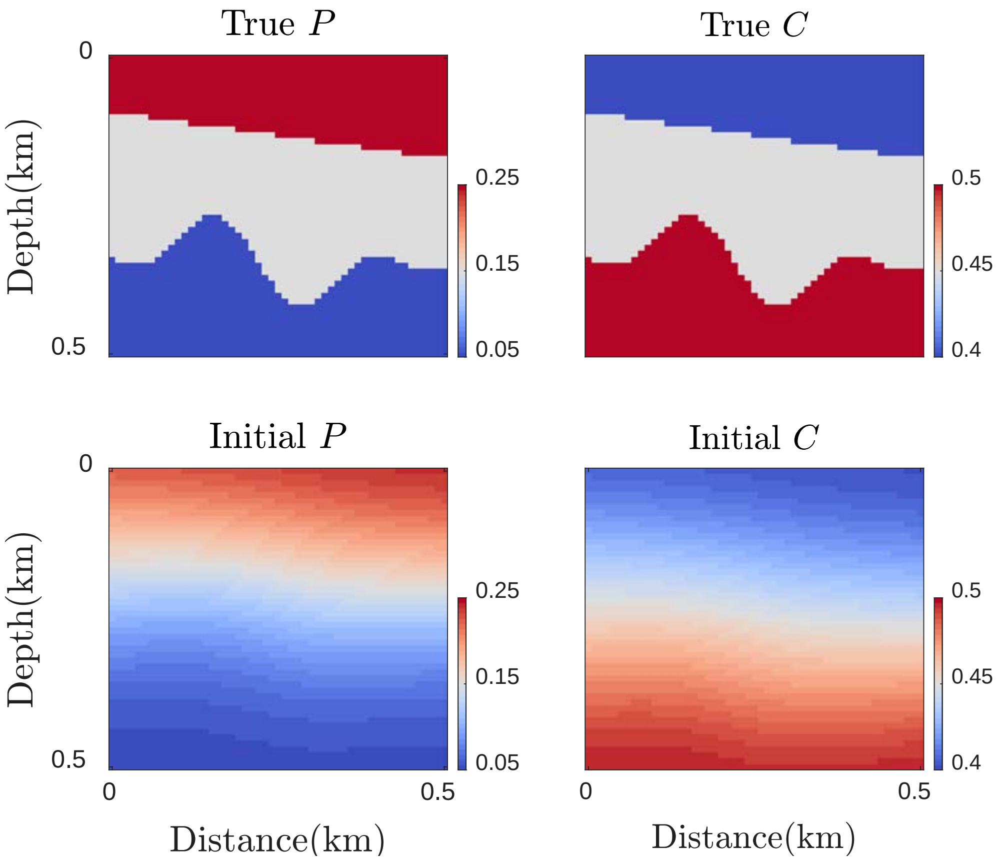
➡ • EFWI parameterized by PCS $\longleftarrow$ $C = f(P)$

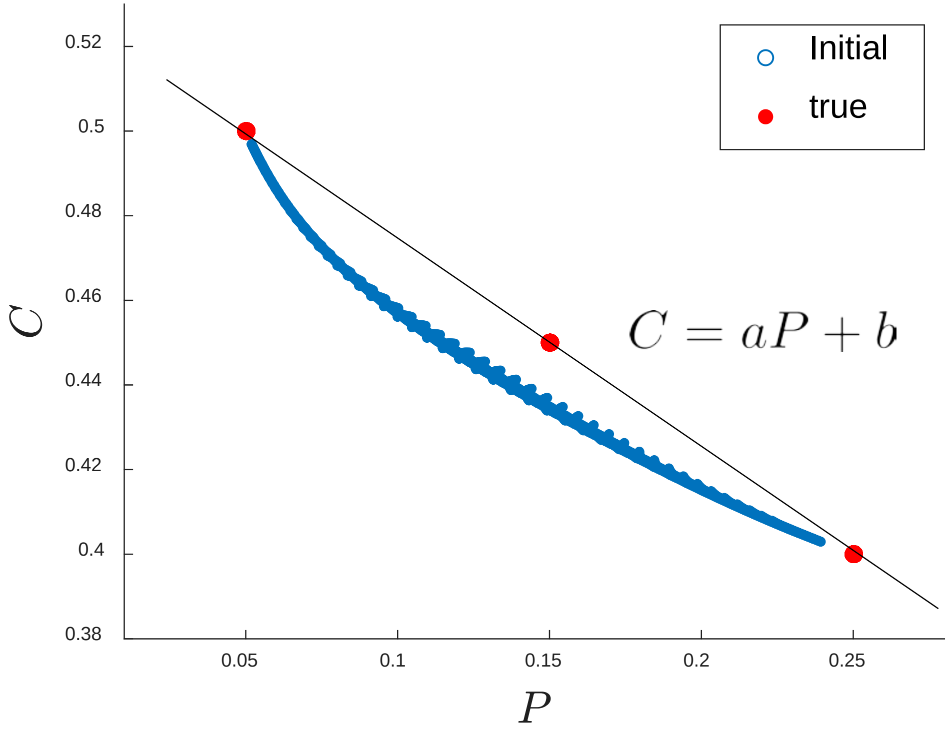
Objective function: $E(\mathbf{m}) = E_d(\mathbf{m}) + \lambda E_c(\mathbf{m})$ $\mathbf{m} : P - C - S$

Model penalty: $E_c(\mathbf{m}) = \|C - f(P)\|^2 \xrightarrow{\text{forcing}} C = f(P)$

Numerical Examples

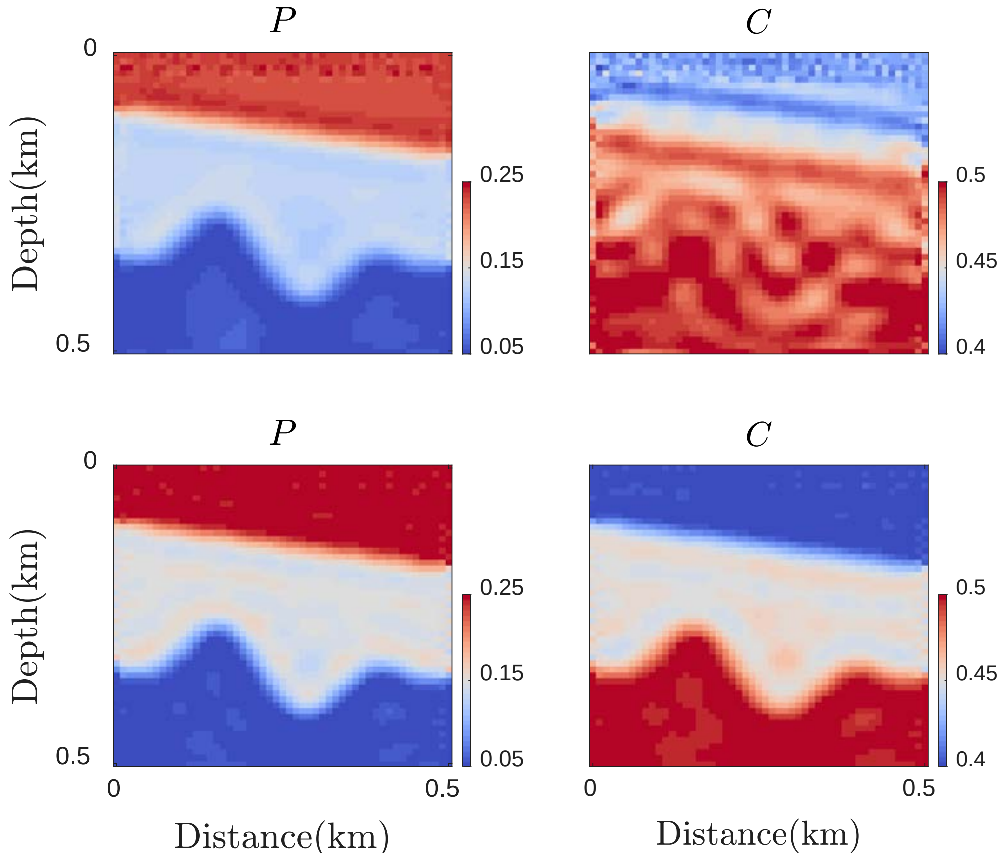
True and Initial models



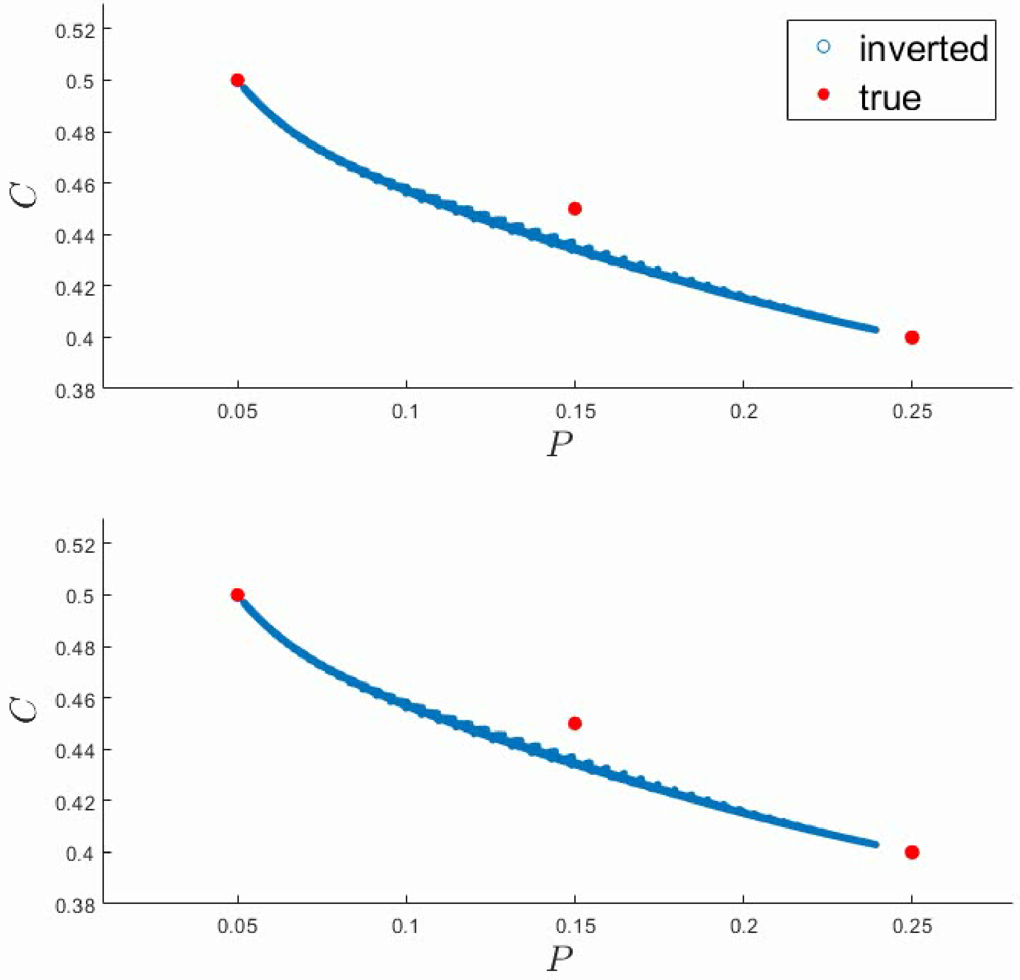


Inversion result

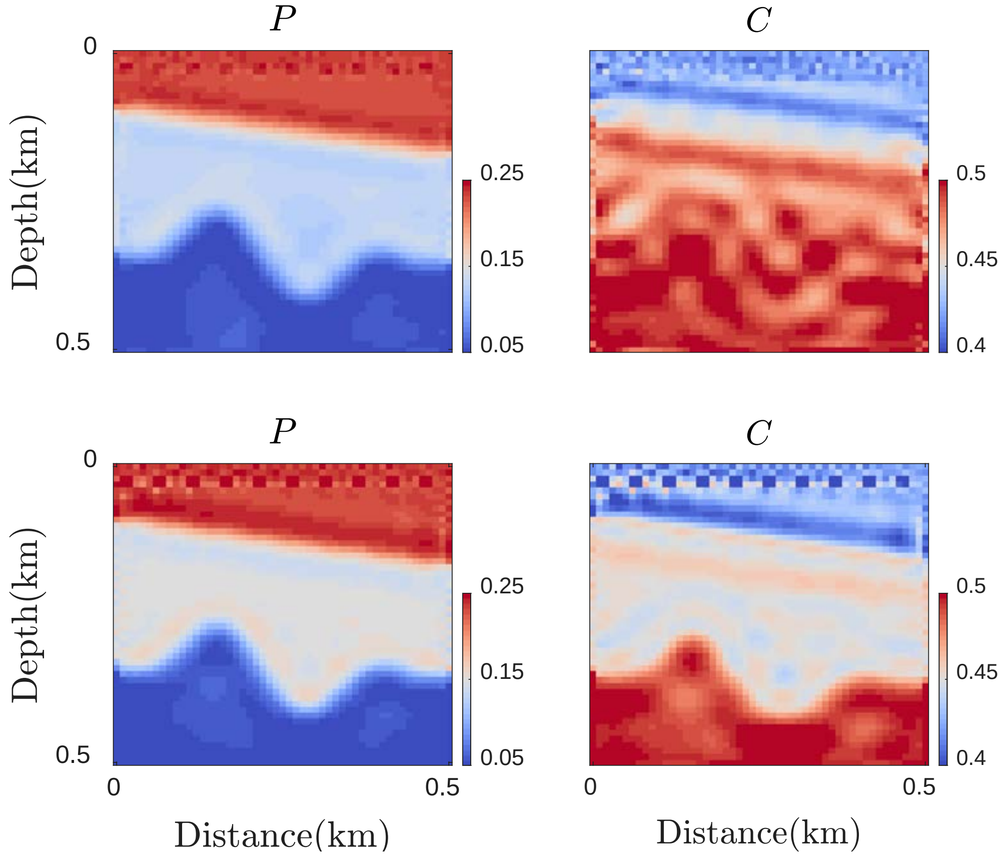
Unconstrained



Constrained

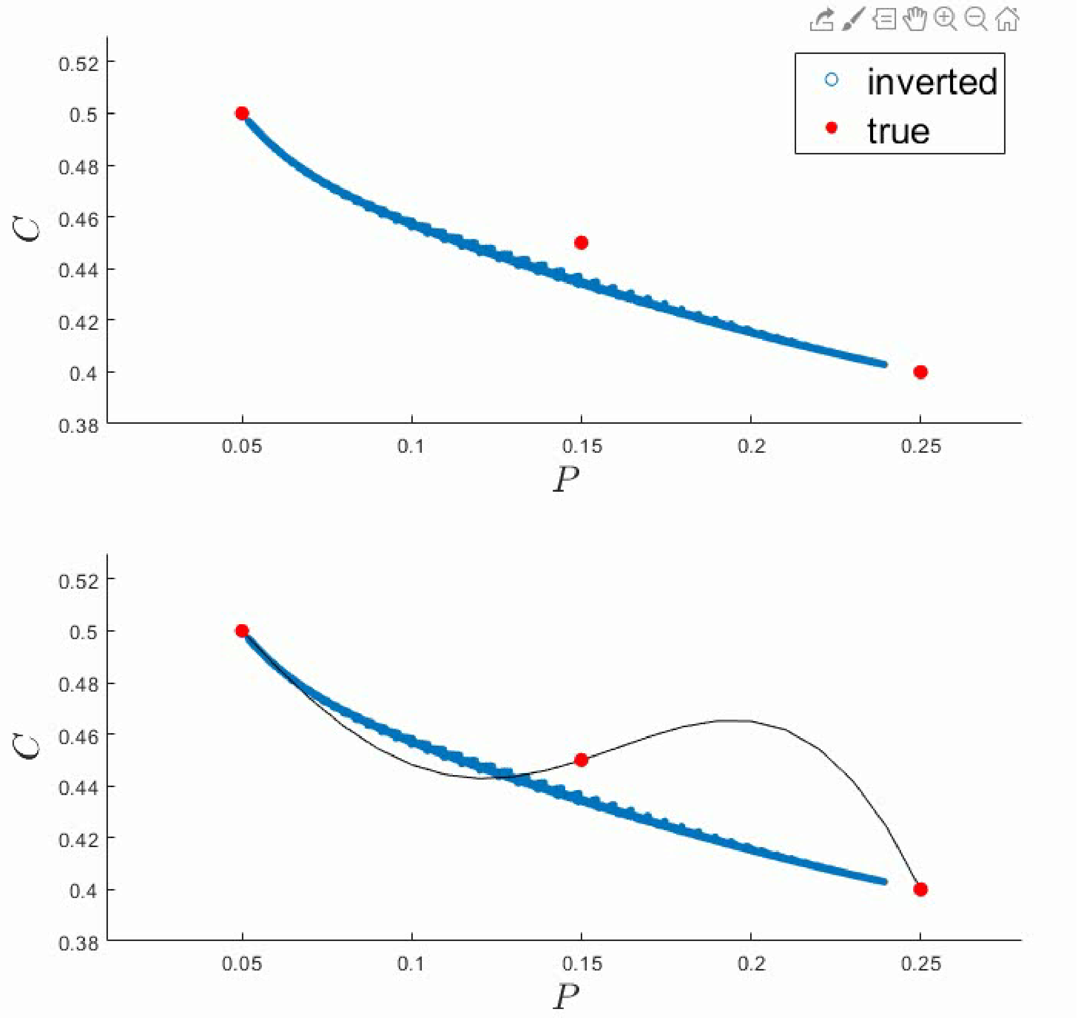


Inversion result



Unconstrained

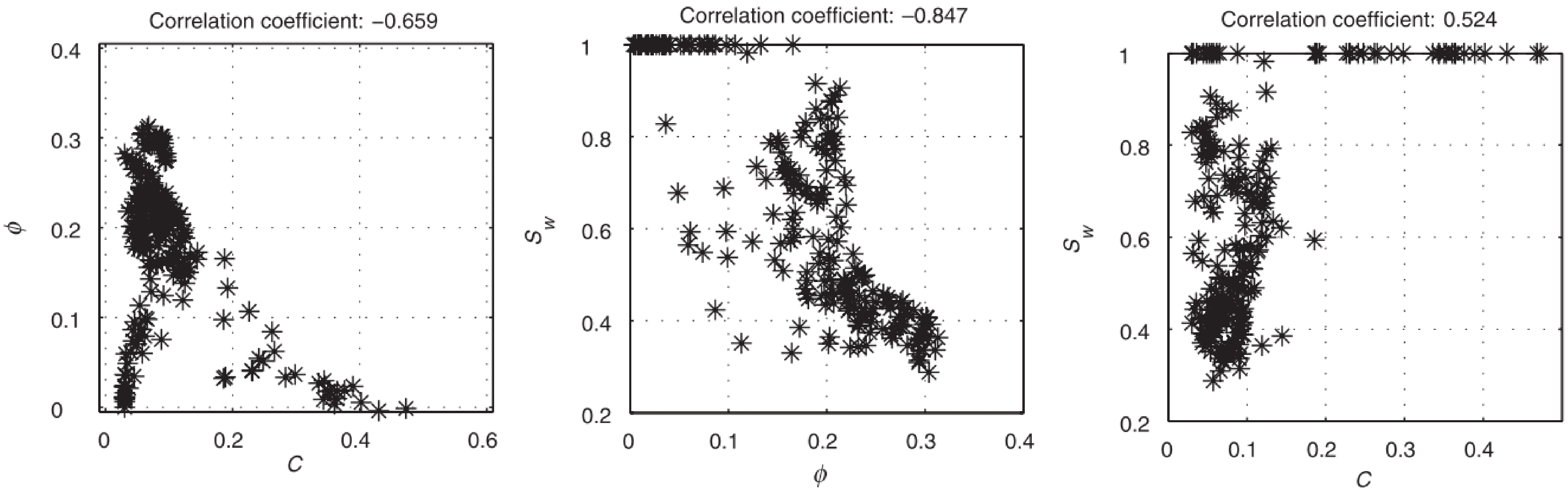
Constrained



Nonlinear Regression: $C = a_1 sin(b_1 P + c_1) + a_2 sin(b_2 P + c_2)$

Limitation of the P-C constraint:

--- It’s greedy to ask for an explicit relation.



Figs from (Spikes et al. 2007)

Methods

Extension:

Constraint

- EFWI parameterized by $V_\mathrm{P}, V_\mathrm{S}, \rho$ ← $\rho = f(V_\mathrm{P})$

➡ • EFWI parameterized by PCS ← $\boxed{C = f(P)}$

Objective function: $E(\mathbf{m}) = E_d(\mathbf{m}) + \lambda E_c(\mathbf{m})$ $\mathbf{m} : P - C - S$

Model penalty: $\boxed{E_c(\mathbf{m}) = \|C - f(P)\|^2 \xrightarrow{\text{forcing}} C = f(P)}$

Methods

Extension:

Constraint

- EFWI parameterized by $V_{\mathrm{P}}, V_{\mathrm{S}}, \rho$ $\longleftarrow$ $\rho = f(V_{\mathrm{P}})$

➡
- EFWI parameterized by PCS $\longleftarrow$ $\boxed{\rho = f(V_{\mathrm{P}})}$

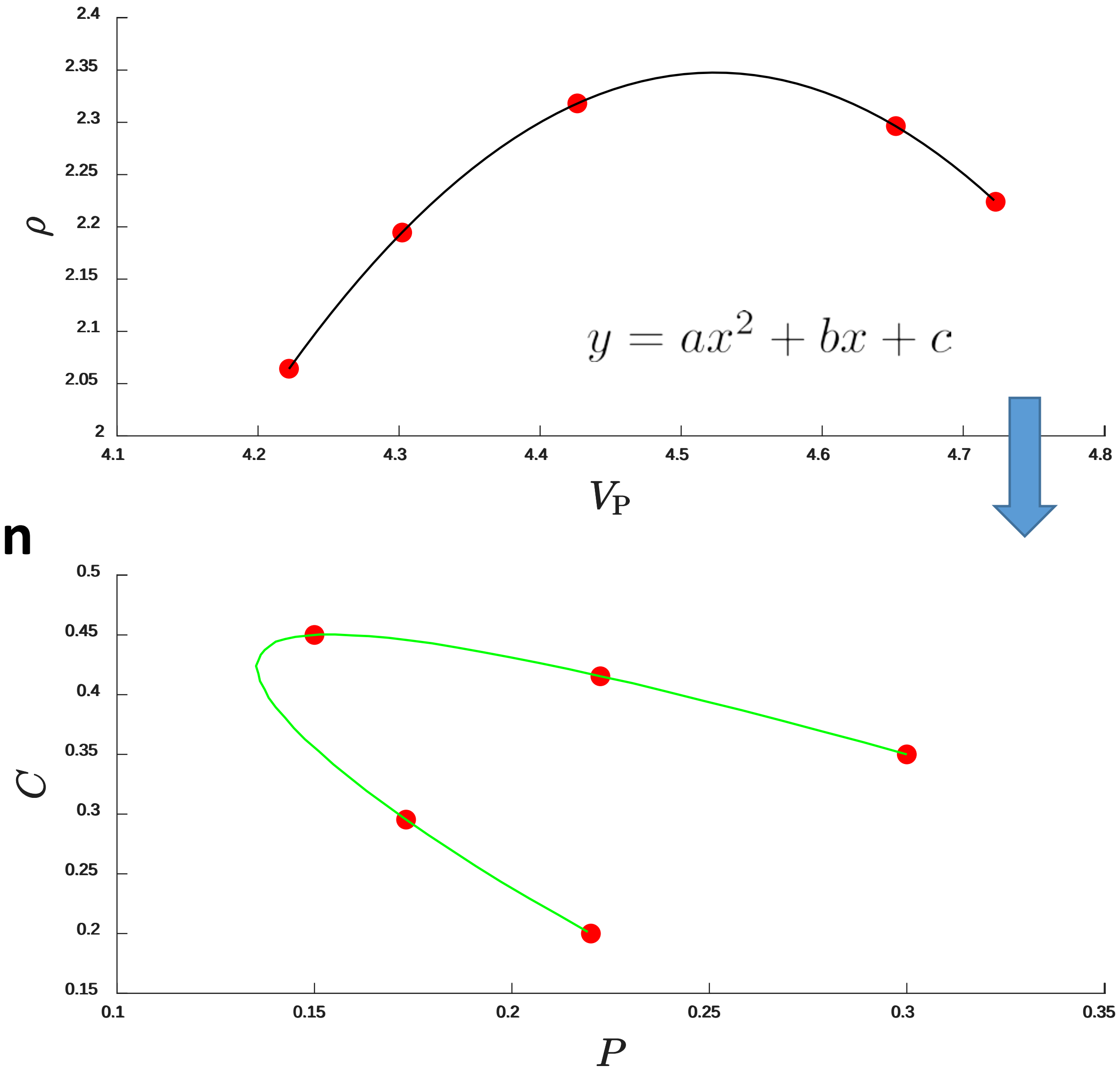
Objective function: $E(\mathbf{m}) = E_d(\mathbf{m}) + \lambda E_c(\mathbf{m})$ $\quad$ $\mathbf{m} : P - C - S$

Model penalty: $\boxed{E_c(\mathbf{m}) = \|\rho(\mathbf{m}) - f(V_{\mathrm{P}}(\mathbf{m}))\|^2 \xrightarrow{\text{forcing}} \rho = f(V_{\mathrm{P}})}$

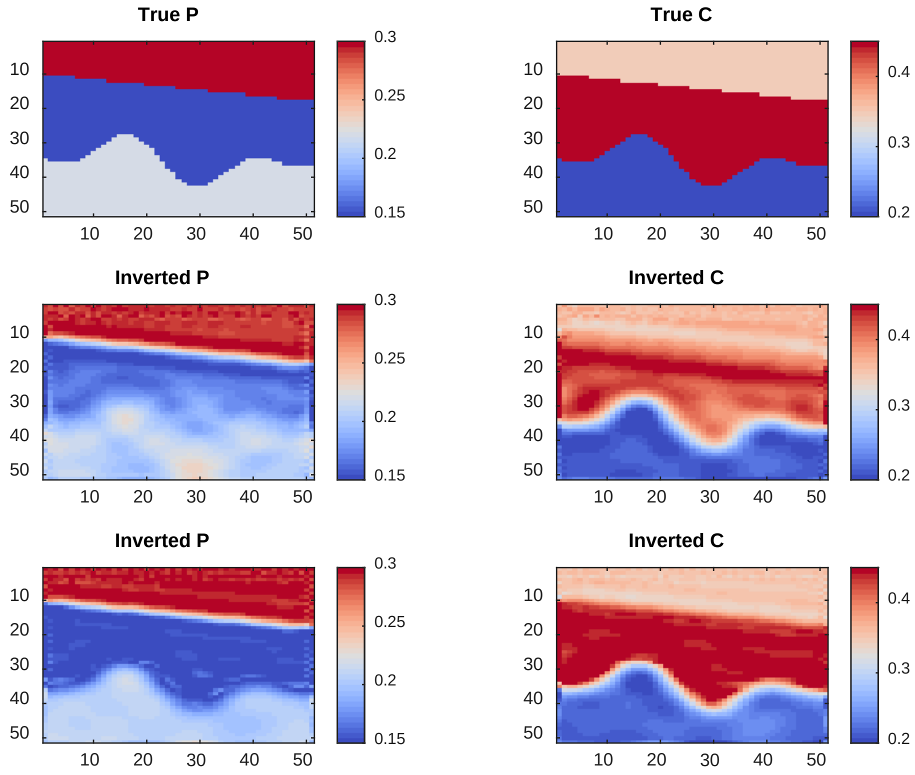
Methods

Fitting a Vp-Den relation

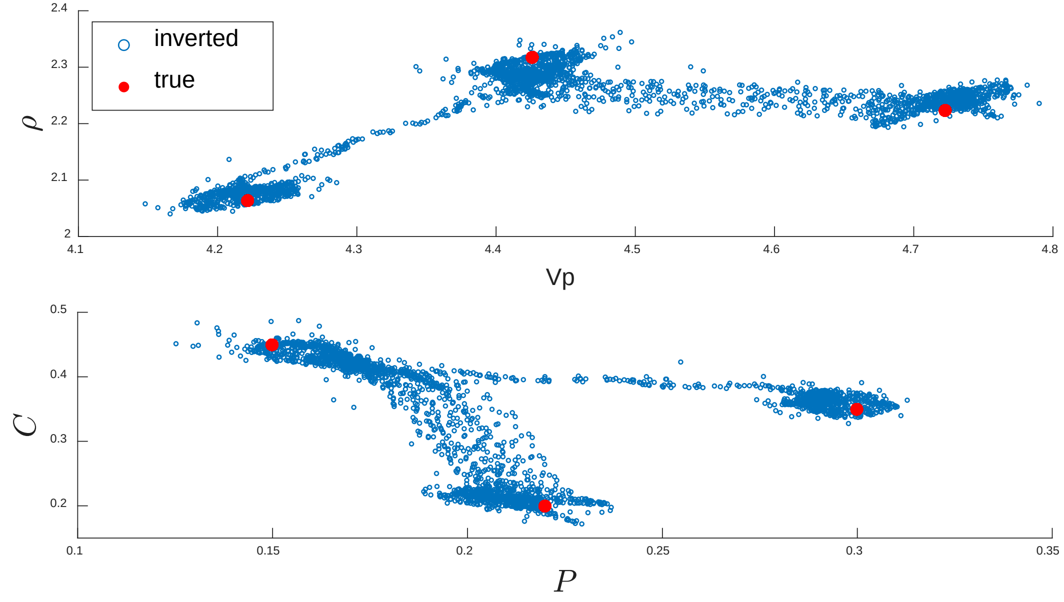
to impose an implicit P-C relation



Numerical Examples



Unconstrained



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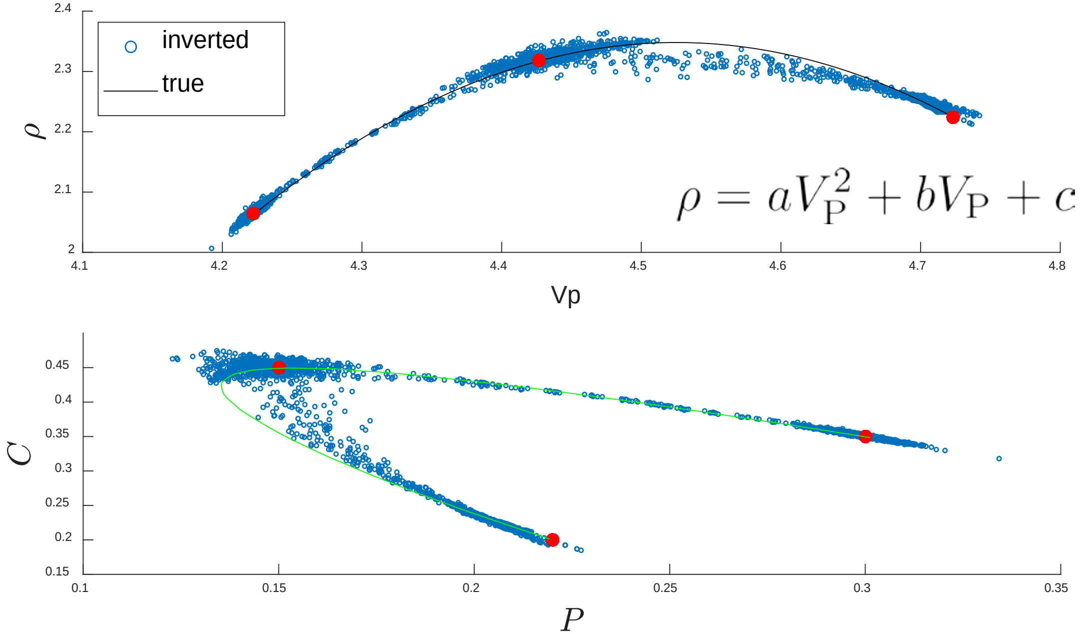
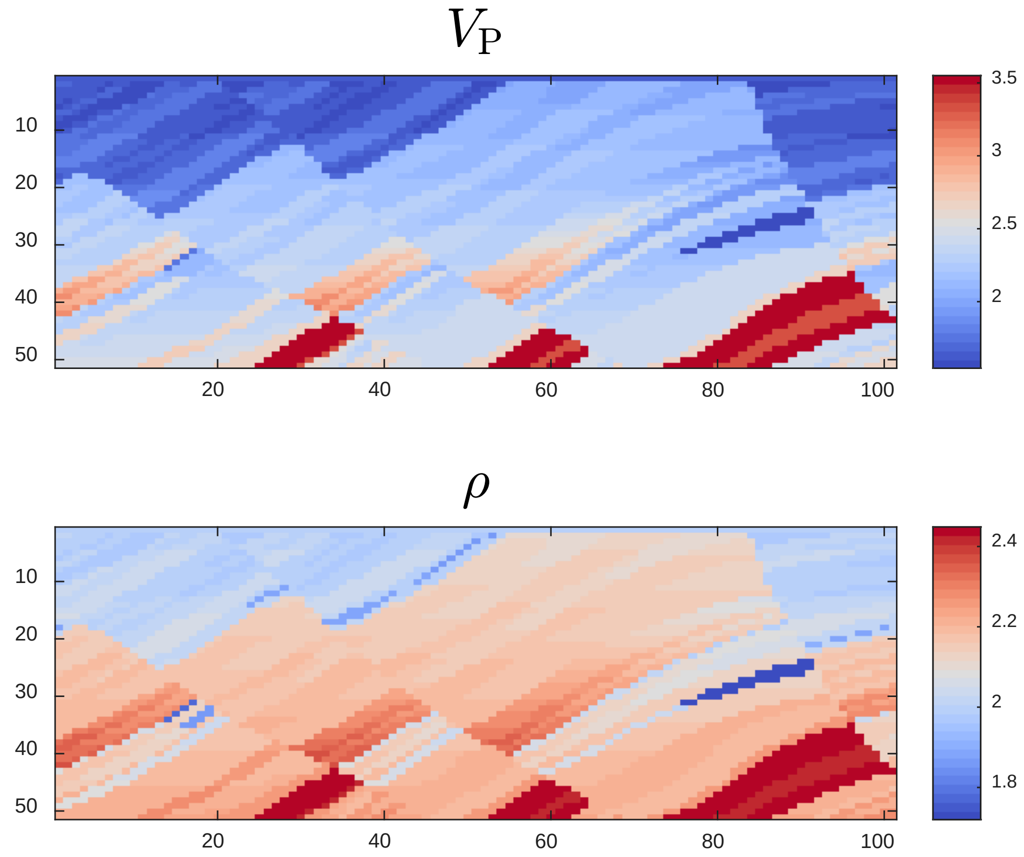
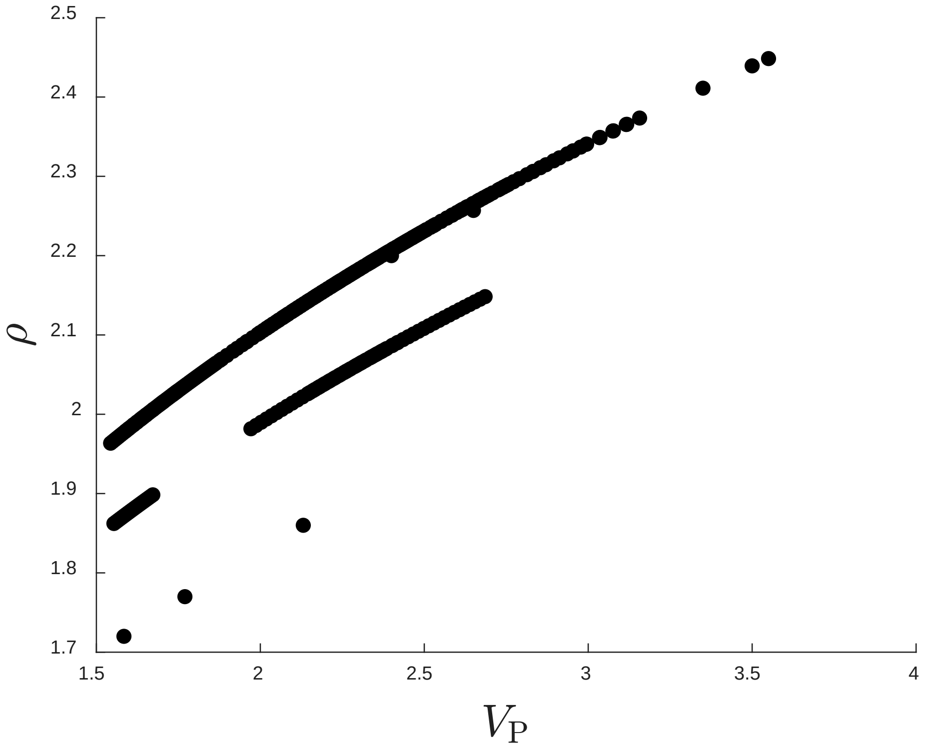


Last:

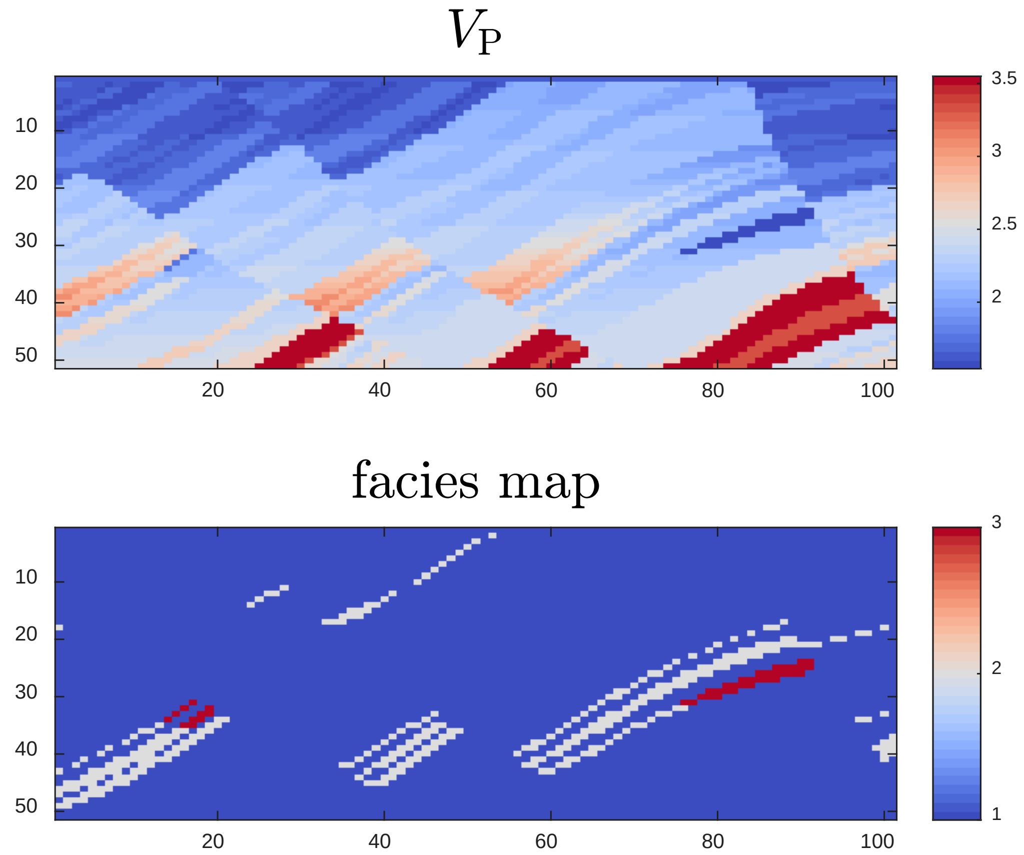
Study with more complex model: Multi-facies (multiple Vp-Den relations)

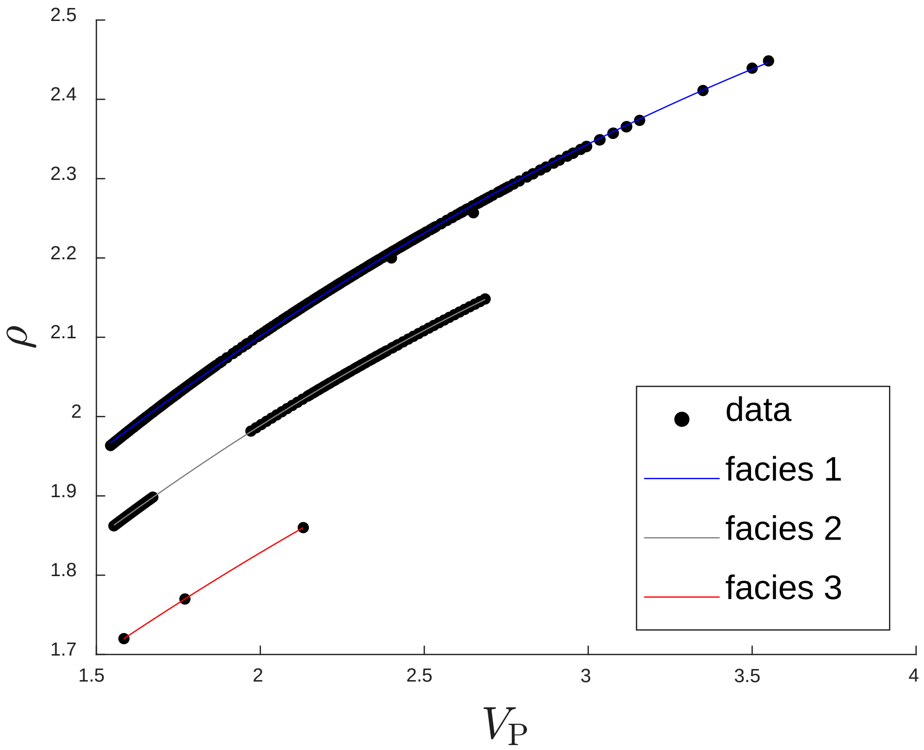
Complex model: Multi-facies (multiple Vp-Den relations)





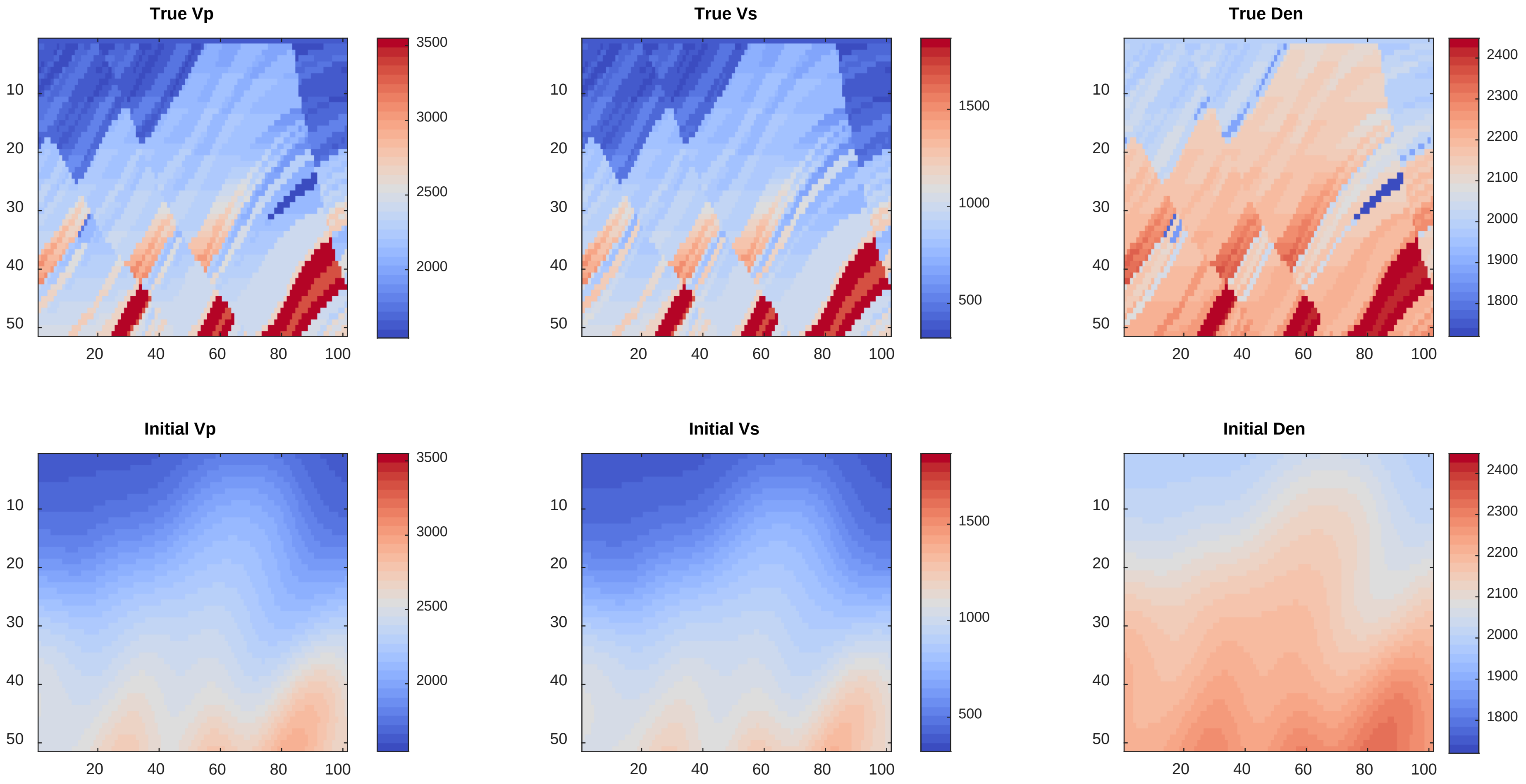
Updating each model cell honoring its corresponding facies



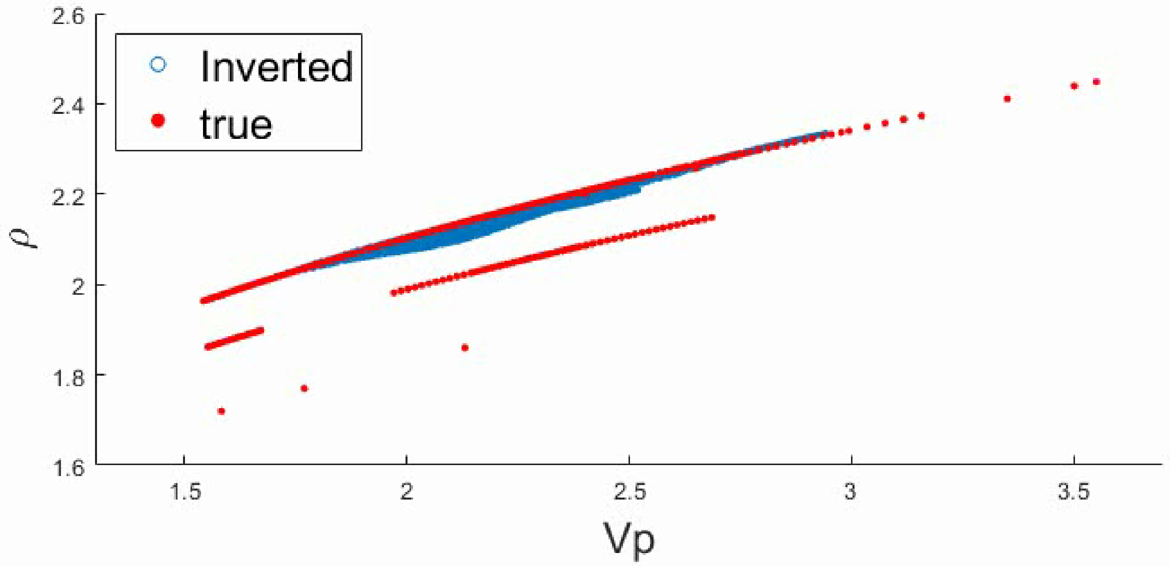


$$E_c(\mathbf{m}) = \|\mathbf{m}_1 - f(\mathbf{m}_2)\|^2 \qquad f = f(x, z)$$

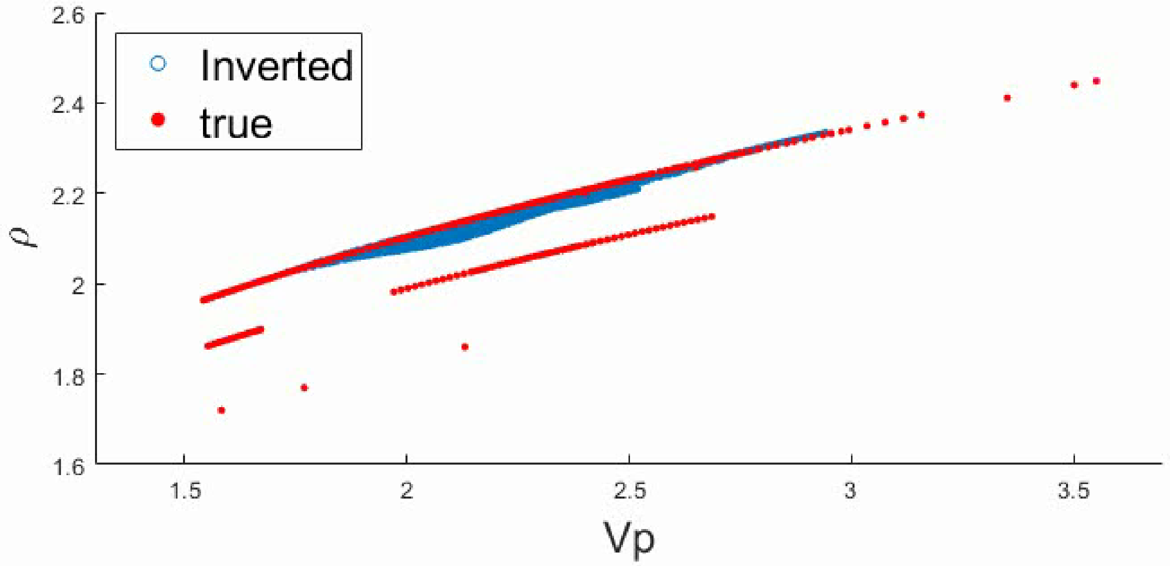
True and Initial models



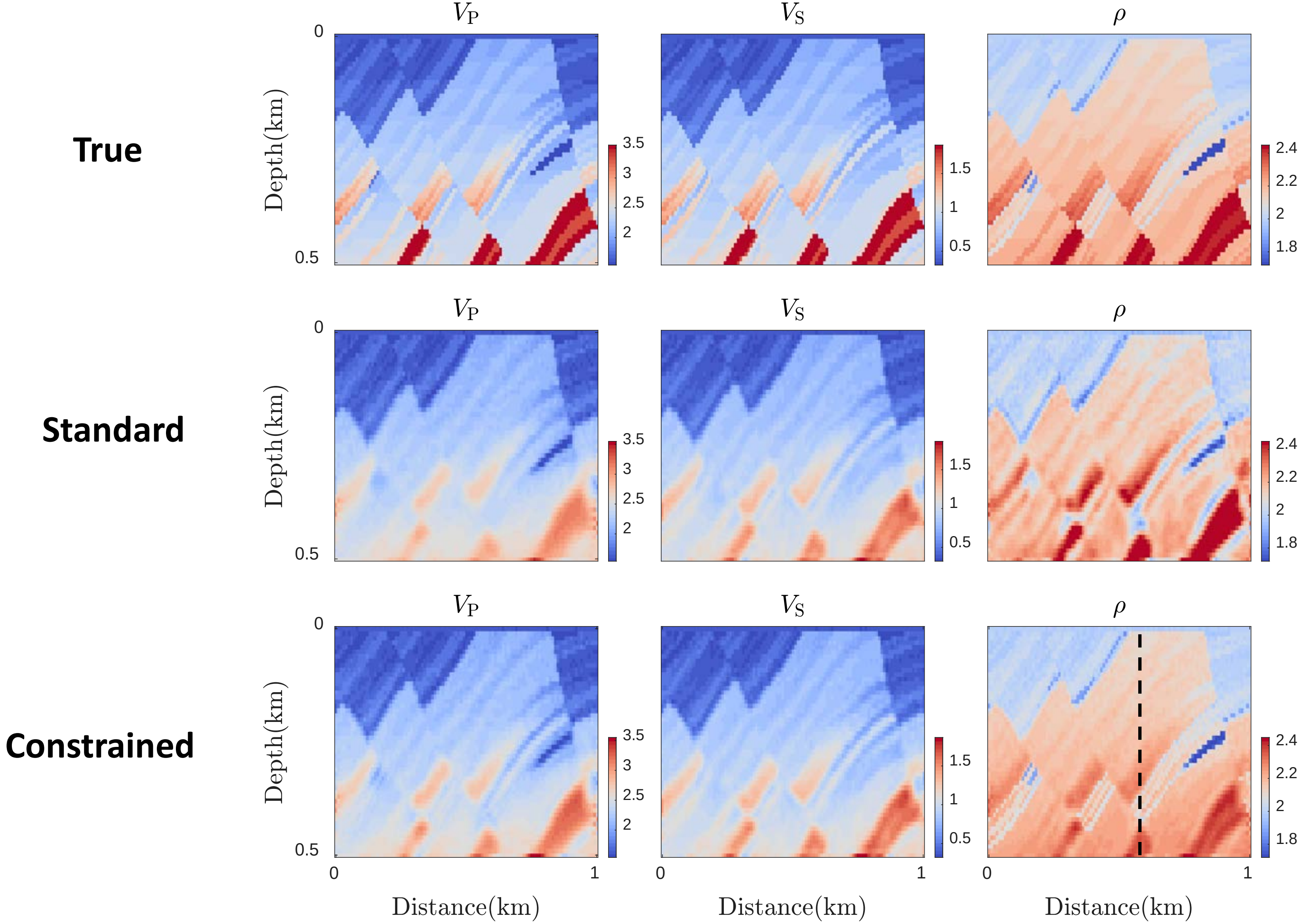
Standard

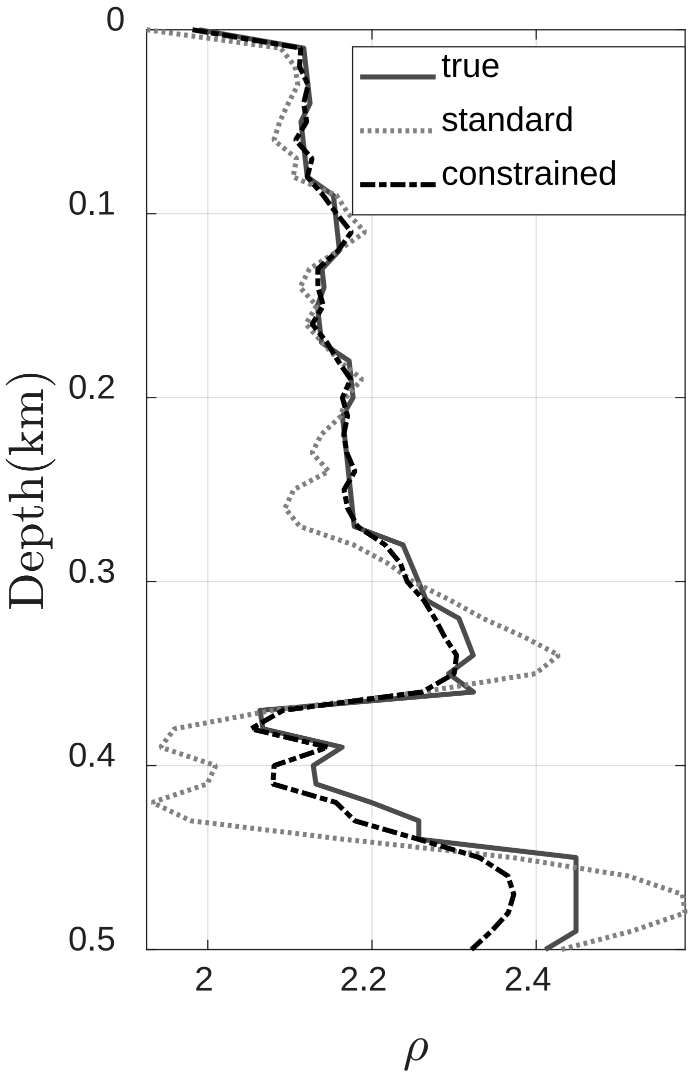


Constrained



Inversion result





Conclusions

- We provide ways to constraining FWI with relations between model parameters;

- The method can improve the estimation of both elastic attributes and rock physics properties;

- A reliable estimation of facies distribution is the key to apply this method to complex models.

Acknowledgments

- CREWES sponsors
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