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Learning the elastic wave equation with Fourier Neural Operators

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Outline

- Introduction to the Fourier Neural Operator
- Learning the elastic wave equation with Fourier Neural Operator
- Computational performance comparison
- Conclusions and future study

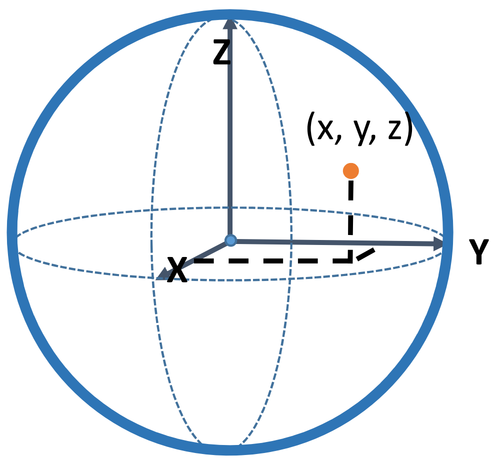
Part one: Introduction to the Fourier Neural Operator

Manifold

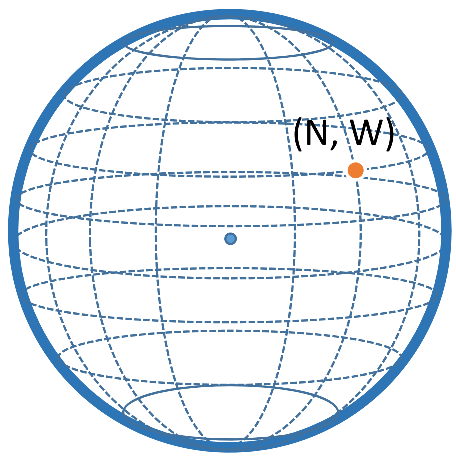
Topological space that locally resembles Euclidean space near each point.

On the Manifold, we use the local manifold coordinate to represent data.
(Dimension is reduced)

The 2D surface of earth is a manifold of 3D space



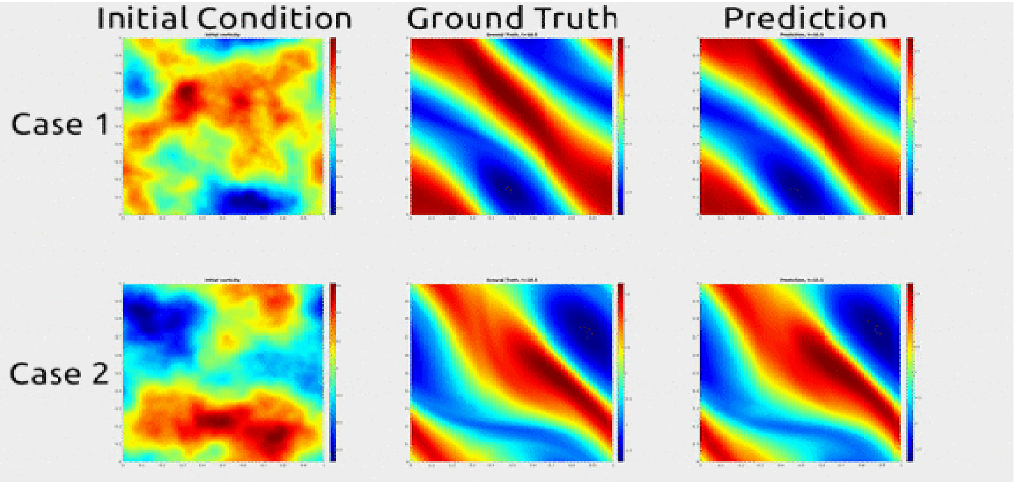
3D Cartesian Coordinate to represent a city on earth ($\mathbb{R}^3$)



Using Longitude and Latitude to represent a the same city $\mathbb{R}^2$

Part one: Introduction to the Fourier Neural Operator

The main reference article : **FOURIER NEURAL OPERATOR FOR PARAMETRIC PARTIAL DIFFERENTIAL EQUATIONS (Zongyi Li, et. al.)**



2-d Navier-Stokes equation

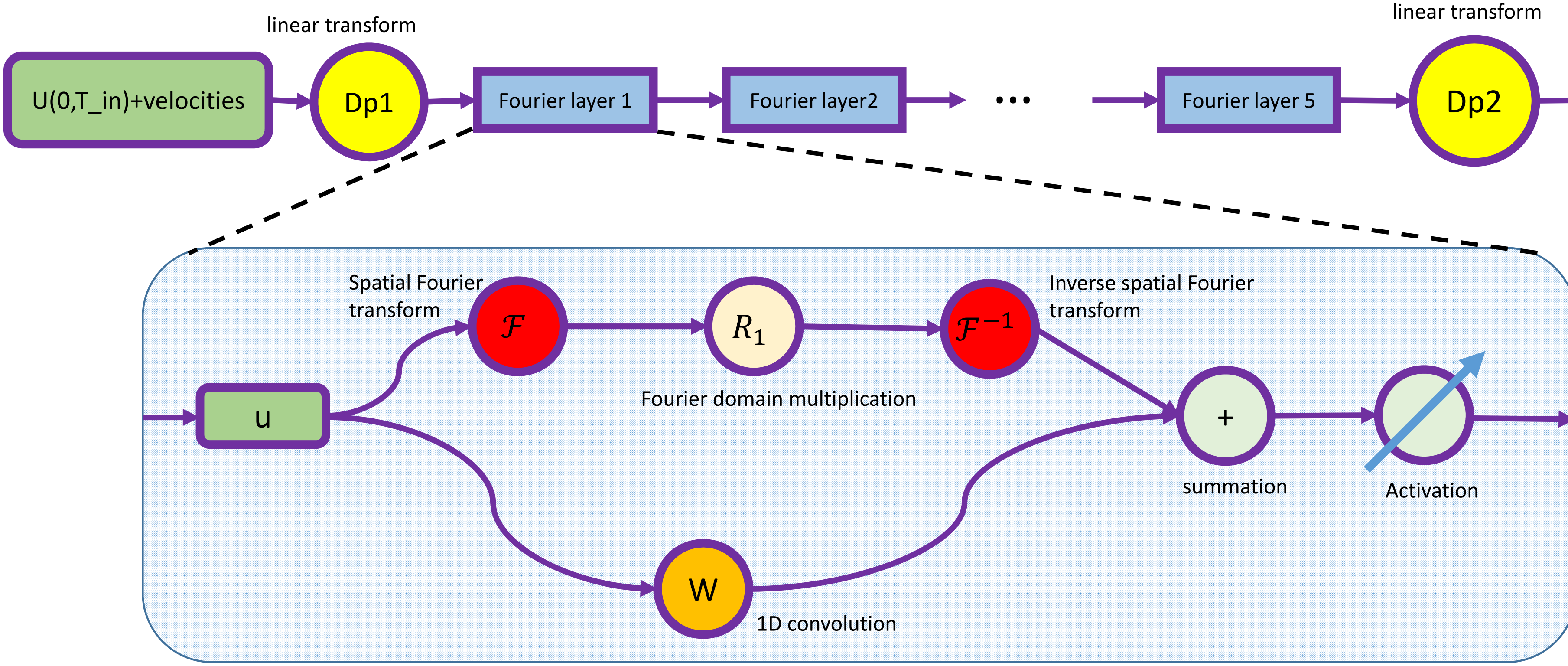
$$\begin{aligned} \partial_t w(x,t) + u(x,t)\cdot\nabla w(x,t) &= \nu\Delta w(x,t) + f(x), \\ \nabla\cdot u(x,t) &= 0, \\ w(x,0) &= w_0(x), \end{aligned}$$

By Li, Zongyi, et al.

Part one: Introduction to the Fourier Neural Operator

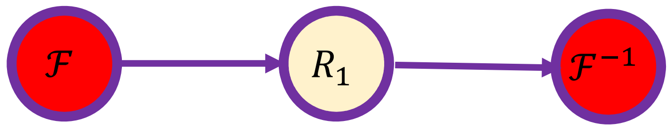
The main mathematical operations includes:
(1) Linear transform (dimension projection).
(2) Spatial Fourier transformation.
(3) Nonlinear activation.

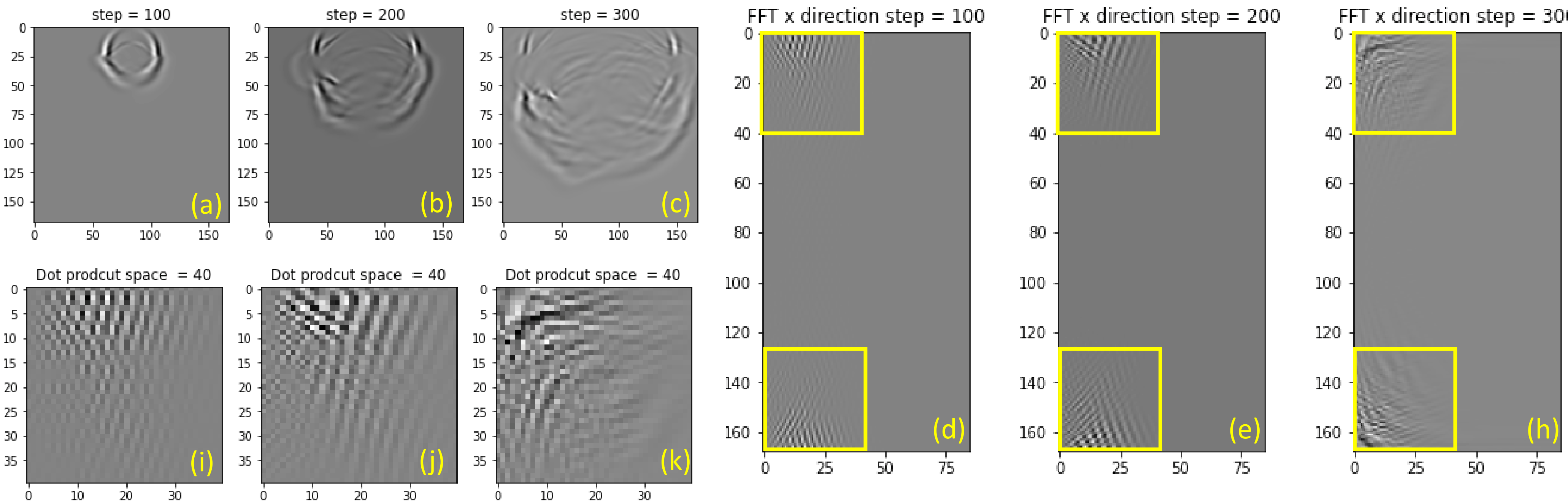
The input and output of the network:
(1) Input are the ***0->t1*** steps of the fields and velocity models.
(2) Output is the ***t1+1->tmax*** steps of the fields



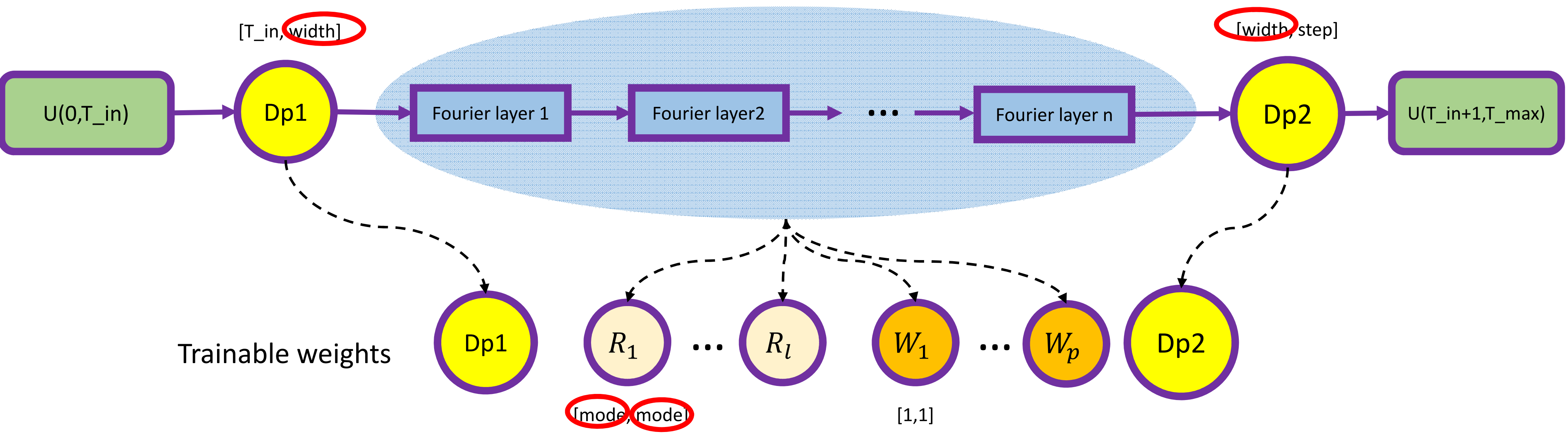
Part one: Introduction to the Fourier Neural Operator

Dot Product in Fourier layer



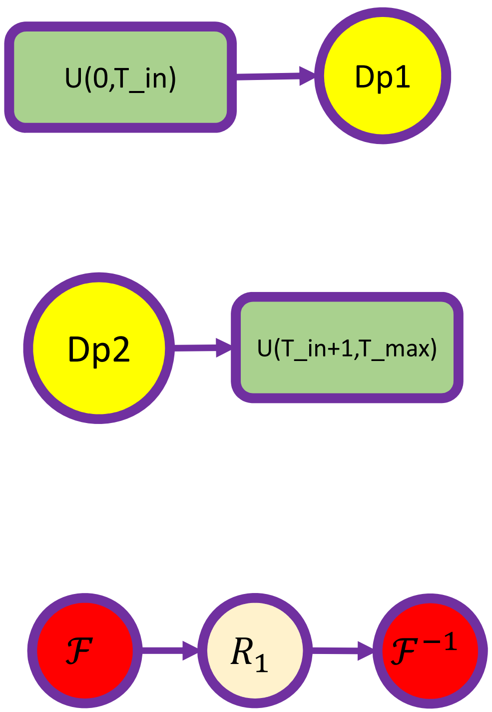


Part one: Introduction to the Fourier Neural Operator



Part one: Introduction to the Fourier Neural Operator

Why are these operations in the network



Operations with respect to time

Operations with respect to time

Spatial Fourier domain dot product.
(e.g. Pseudo spectral forward modeling)

Operations with respect to space

PDEs

BURGERS' EQUATION

$$\partial_t u(x,t) + \partial_x (u^2(x,t)/2) = \nu \partial_{xx} u(x,t),$$
$$u(x,0) = u_0(x),$$

NAVIER-STOKES EQUATION

$$\partial_t w(x,t) + u(x,t) \cdot \nabla w(x,t) = \nu \Delta w(x,t) + f(x),$$
$$\nabla \cdot u(x,t) = 0,$$
$$w(x,0) = w_0(x),$$

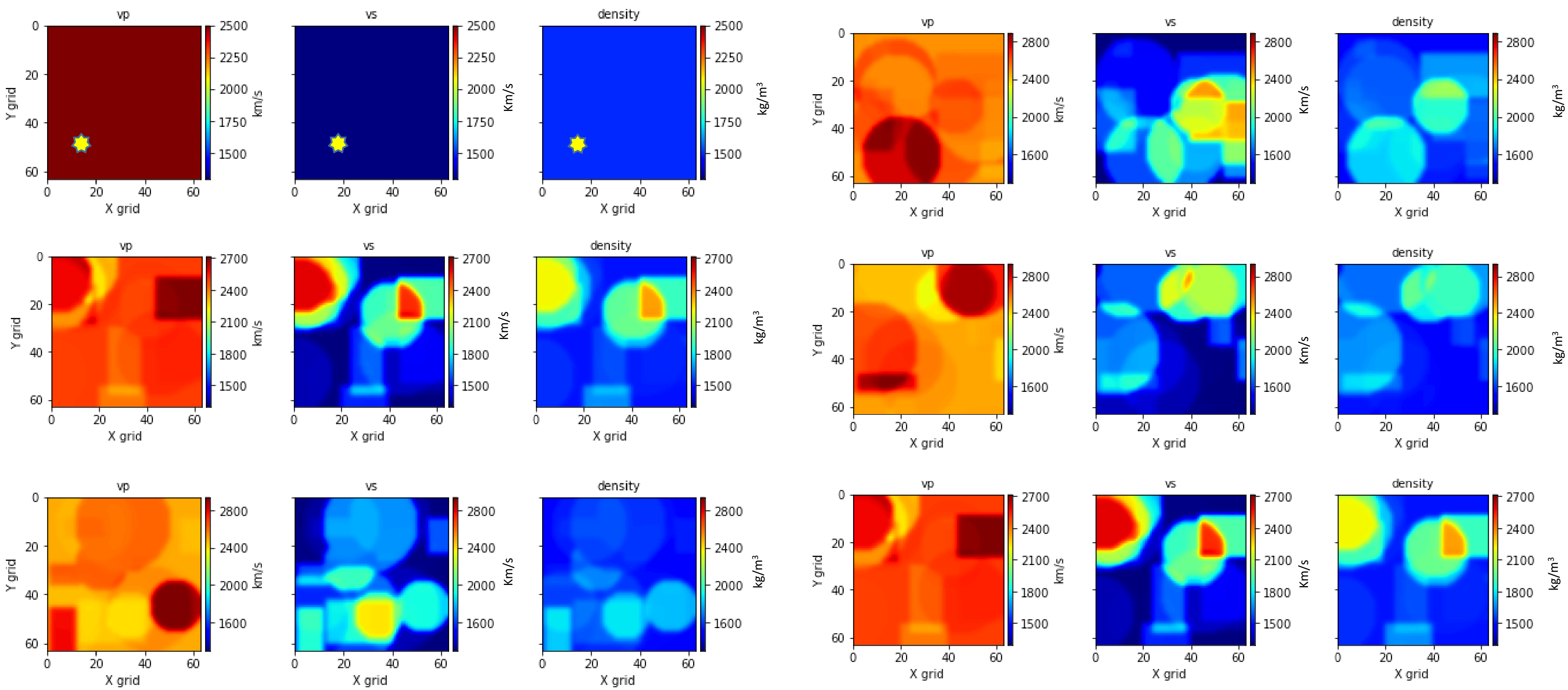
DARCY FLOW equation

$$-\nabla \cdot (a(x) \nabla u(x)) = f(x)$$
$$u(x) = 0$$

Part Two: Learning the elastic wave equation with Fourier Neural Operator

Learning the elastic wave equation with Fourier Neural Operator

Data set preparation



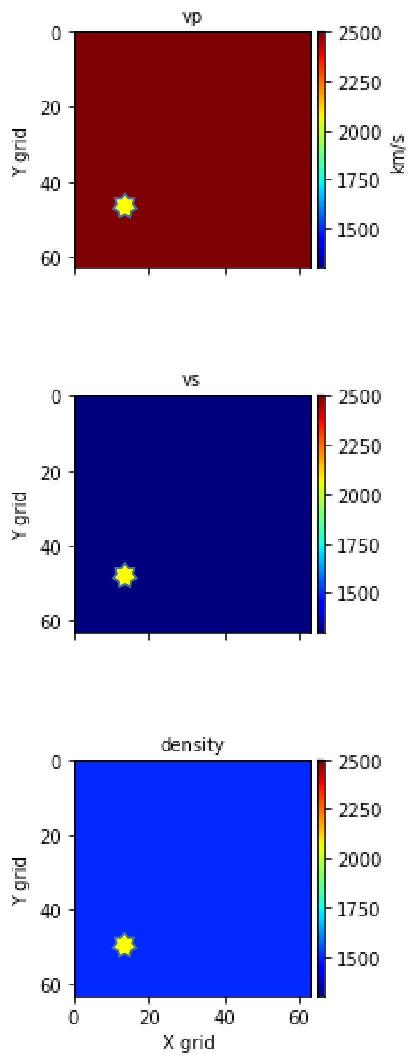
Learning the elastic wave equation with Fourier Neural Operator

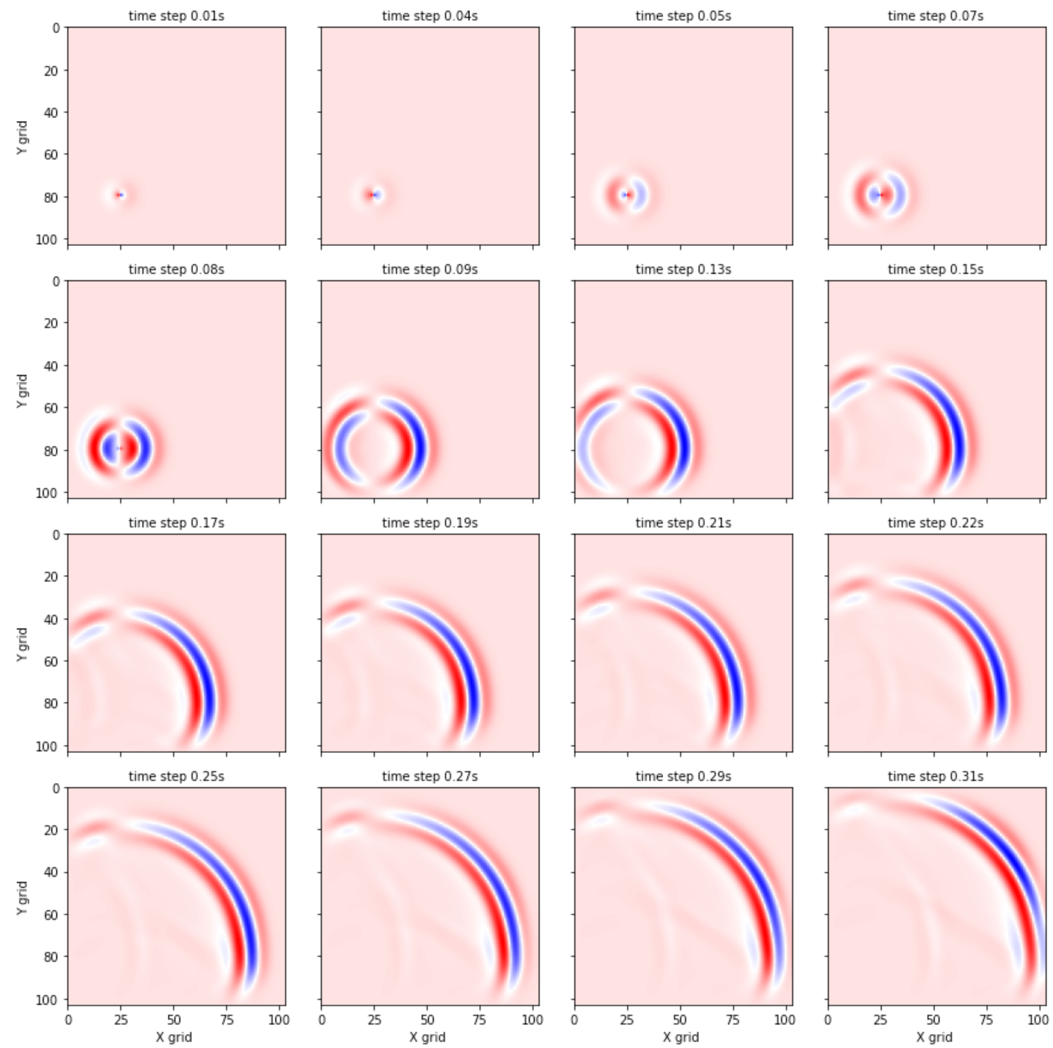
Grid size: 64×64
Dx=Dz=10m
Ricker's wavelet (15Hz)
Dt = 0.001
Step =450
Number of models:100
Number of Training model: 80
Number of Testing model: 20
T_in = 50
Step = 400
Elastic Stress and velocity wave equation.

Data set dimension for training:

Training data set input:
[80, nz, nx, 50]
Training data set Output:
[80, nz, nx, 400]

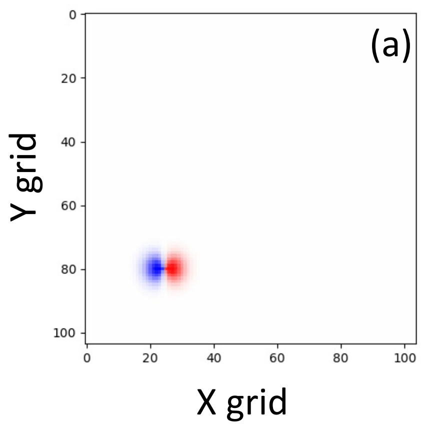
Testing data set input:
[20, nz, nx, 50]
Training data set Output:
[20, nz, nx, 400]



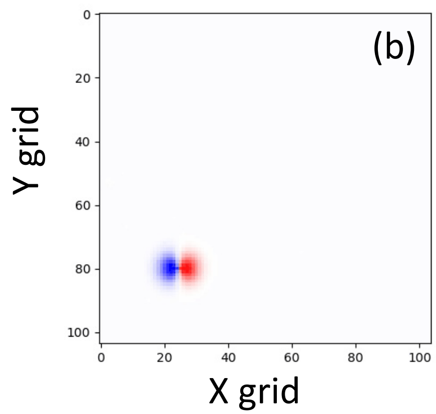


Learning the elastic wave equation with Fourier Neural Operator

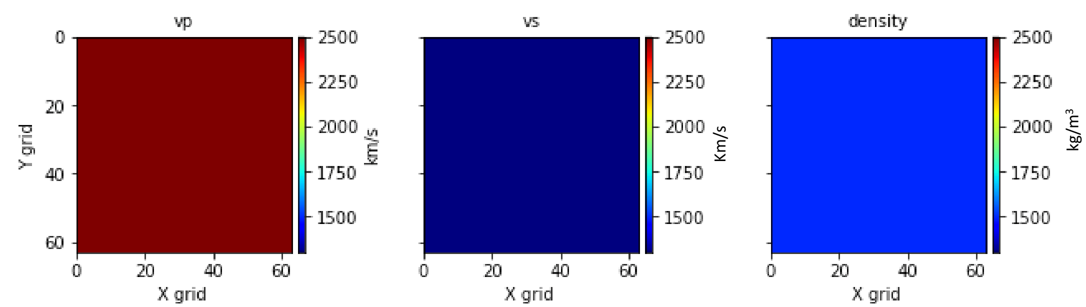
Wave fields Generated with FD



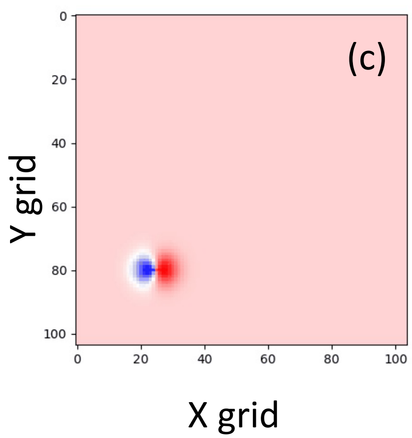
Wave fields Generated with FN



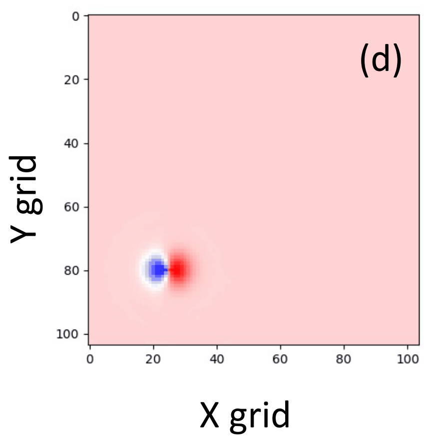
Velocity Model 1



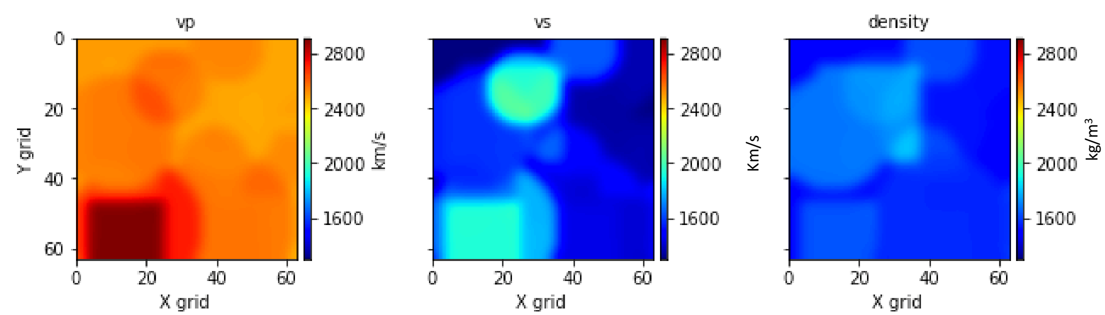
Wave fields Generated with FD



Wave fields Generated with FN

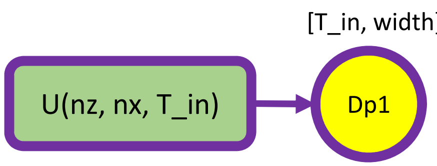


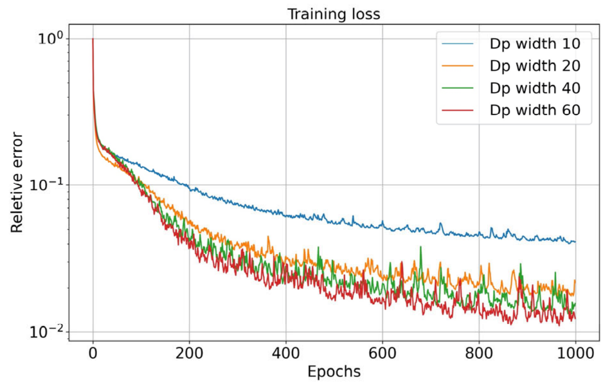
Velocity Model 2

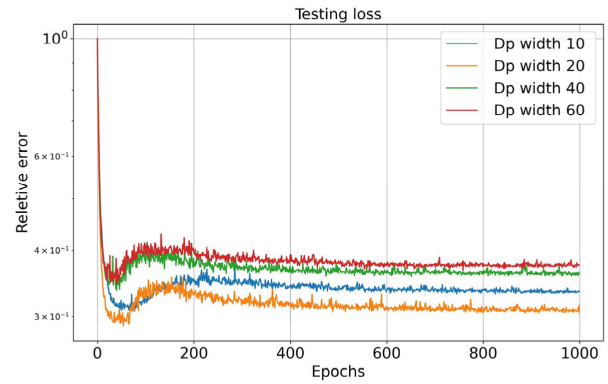


Learning the elastic wave equation with Fourier Neural Operator

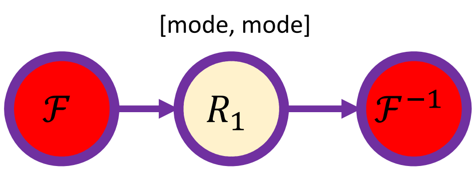
Training with different Dp width

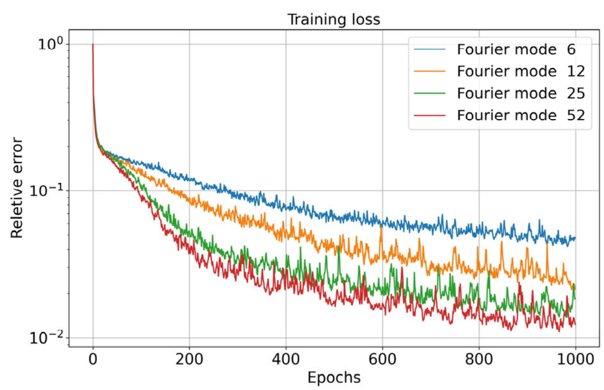


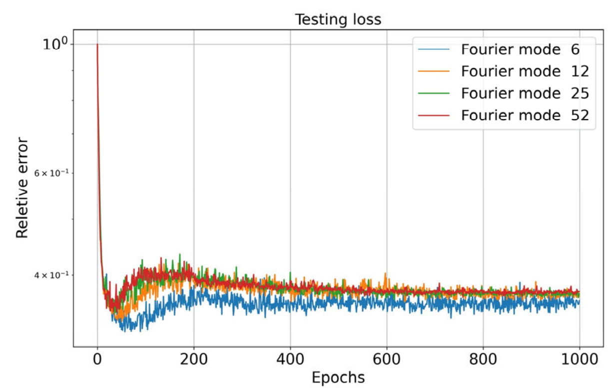




Training with different Fourier model

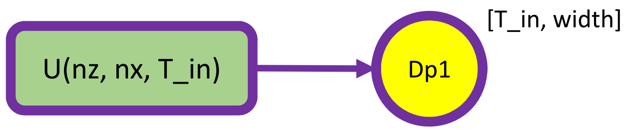


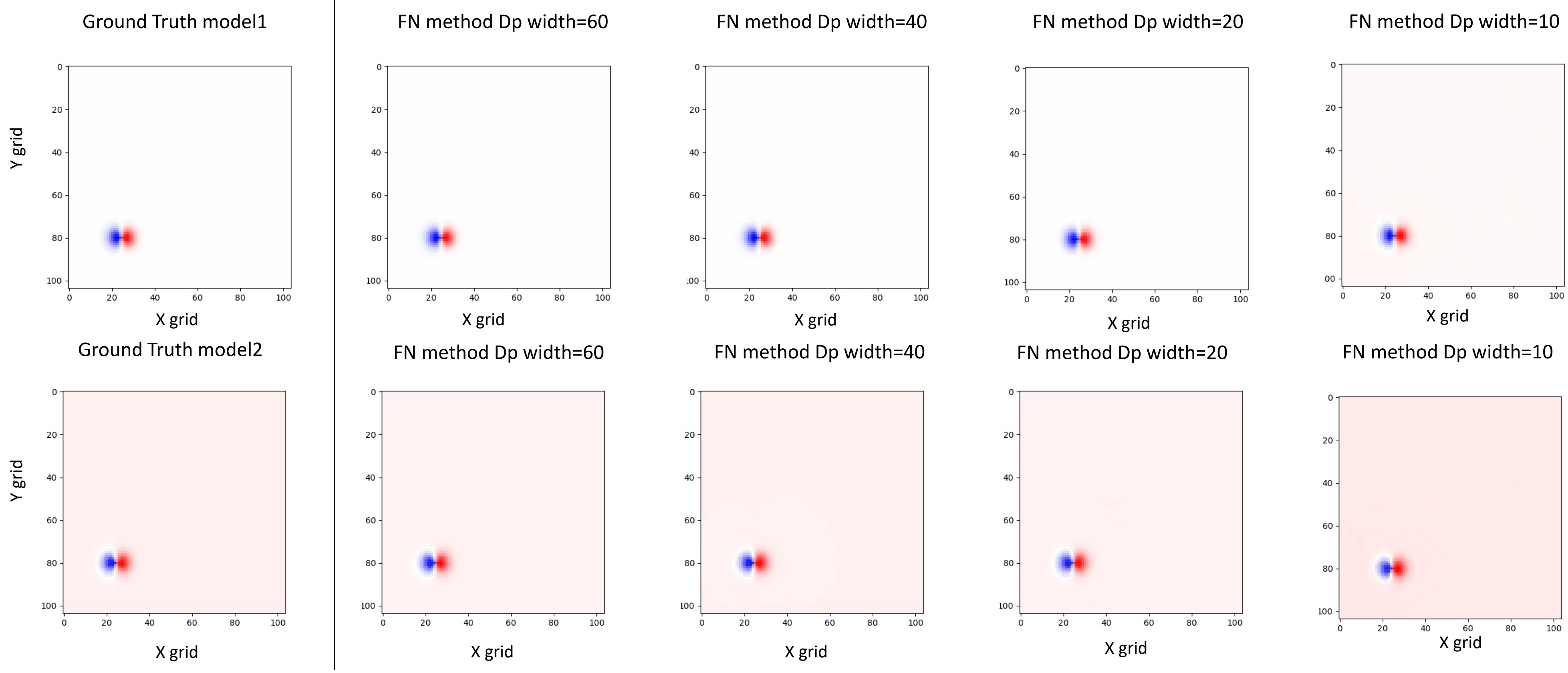




Learning the elastic wave equation with Fourier Neural Operator

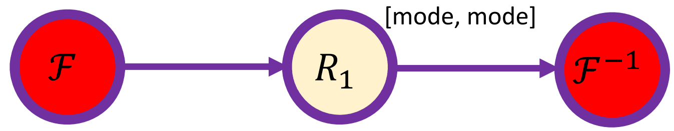
Training with different Dp width

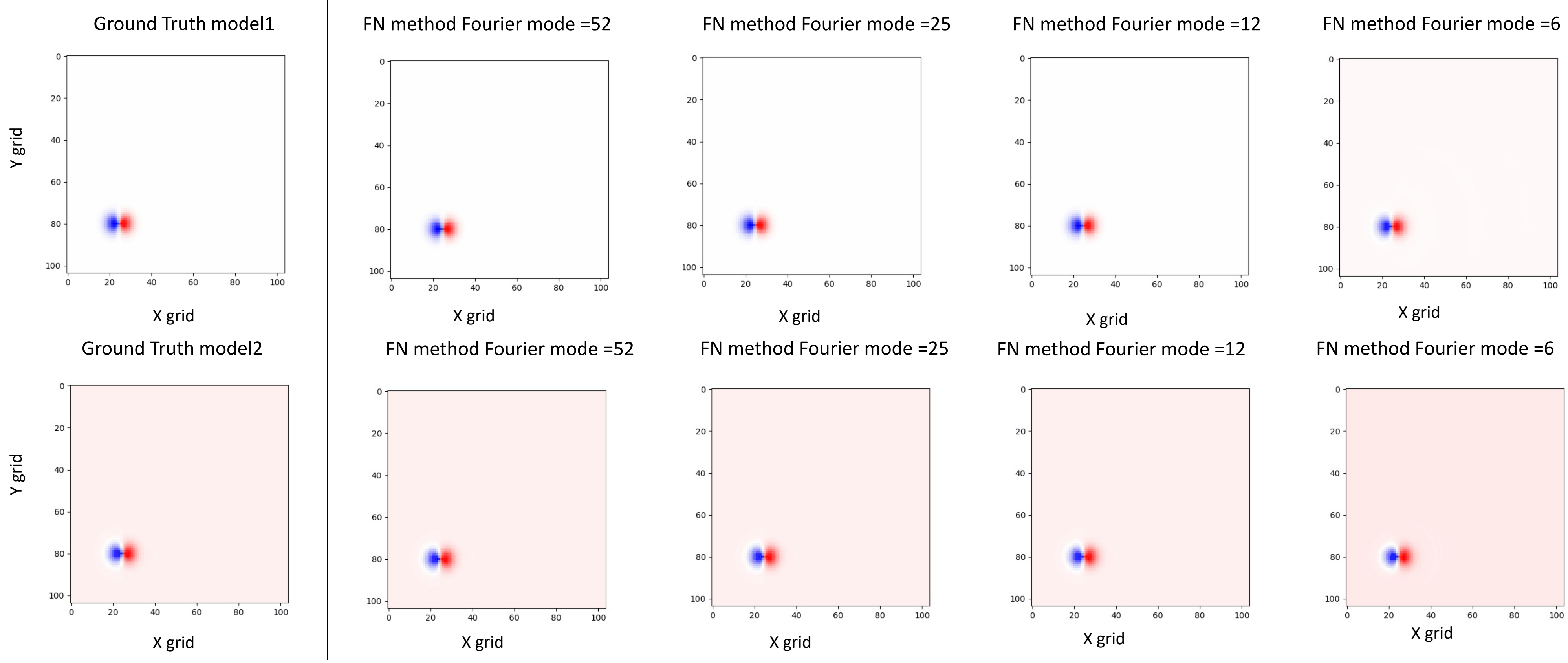




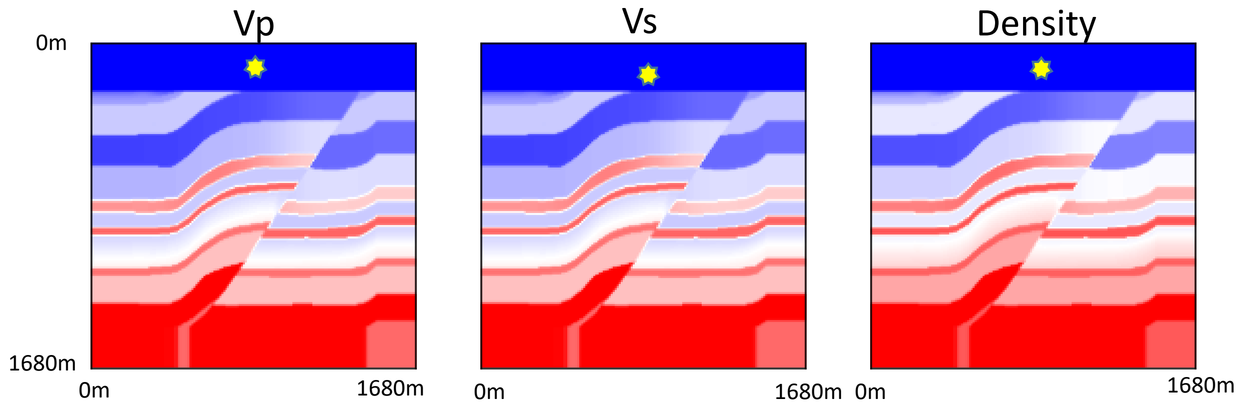
Learning the elastic wave equation with Fourier Neural Operator

Training with different Fourier model

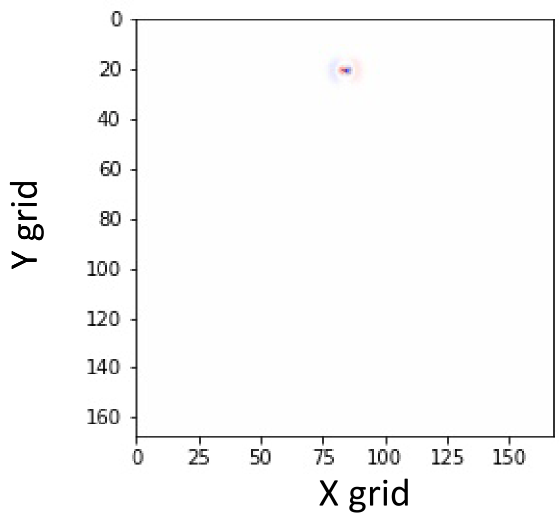




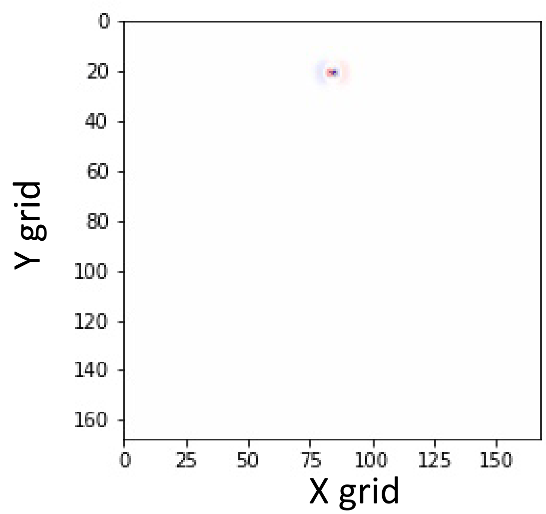
Learning the elastic wave equation with Fourier Neural Operator



Wave fields Generated with FD



Wave fields Generated with FN



Part Three: Computational performance comparison

Part three: Computational performance comparison

Table 1. Forward modeling computational performance comparison

Modeling method	CPU	GPU	CPU/GPU Ratio
Finite Difference method (PY)	1.345670445s	0.95749302s	1.4
FNO(Dp width=10, Model=33)	0.10359570s	0.00122648s	84
FNO(Dp width=20, Model=33)	0.13362584s	0.00125602s	106
FNO(Dp width=30, Model=33)	0.29434162s	0.00130555s	225
FNO(Dp width=60, Model=33)	0.48886882s	0.00244271s	200
FNO(Dp width=40, Model=10)	0.11607934s	0.00357925s	32
FNO(Dp width=40, Model=20)	0.15244401s	0.00177402s	85
FNO(Dp width=40, Model=40)	0.26186507s	0.00196767s	133
FNO(Dp width=40, Model=60)	0.33643302s	0.00242882s	138

Part Four : Conclusions and future study

Conclusions and future study

Conclusions

- The Fourier network can be trained to learn PDEs and can give promising wavefields.
- Different Fourier models and width can lead to different accuracy of the fields
- Experiments shows that the Fourier network can generate fields about 10 times faster than the Finite Difference method on CPU and 1000 times faster on GPU

Future study

- Study the methods generality on different models (i.e., different sizes, types of sources and propagation time). The ultimate goal for this method should be the a fast forward modeling method that can be universally applied on all kinds of velocity models.
- The implementation on the MCMC types of inversion.

Thanks

- Thanks all CREWES sponsors and students
- Thanks China Scholarship Council

Thanks for listening