Statistical-mechanical models as tools for seismic data analysis and inversion

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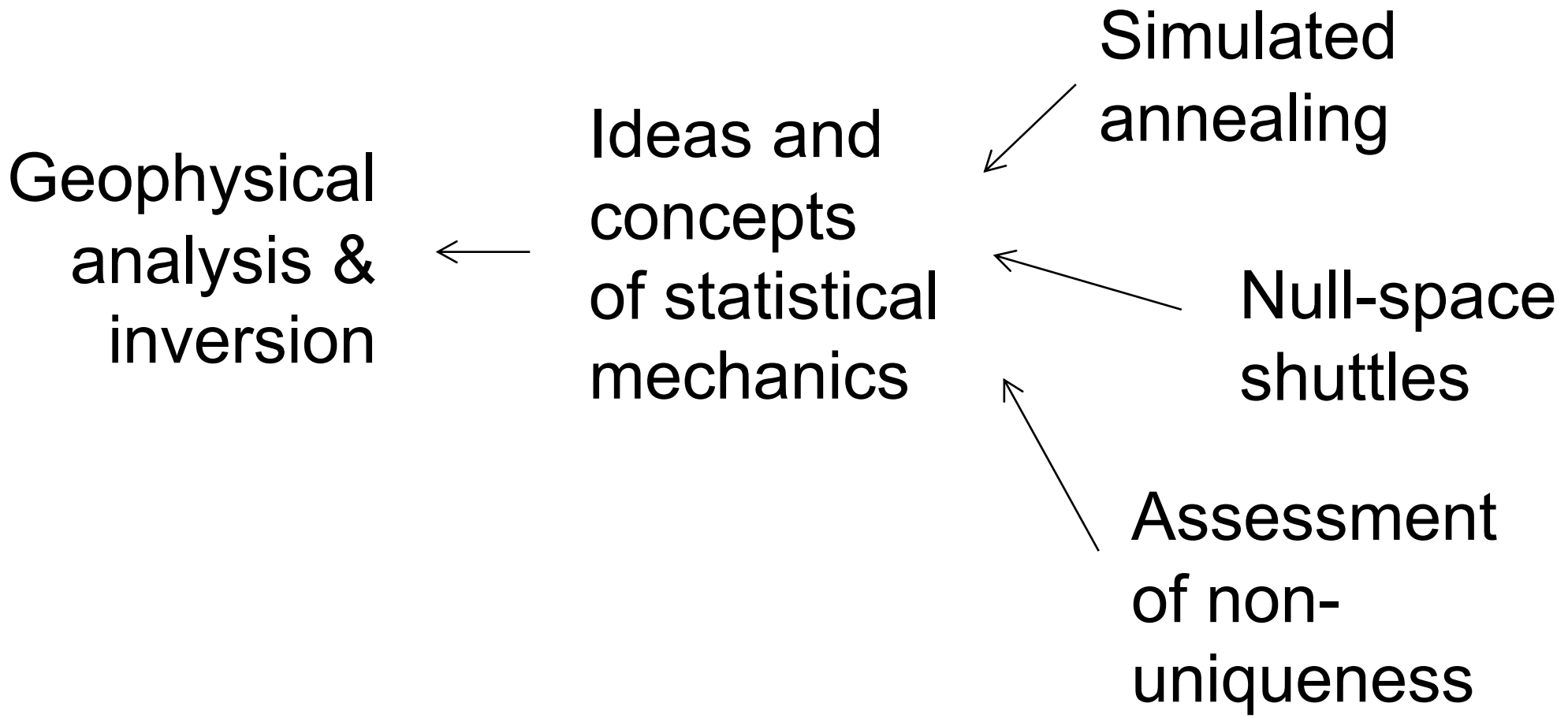
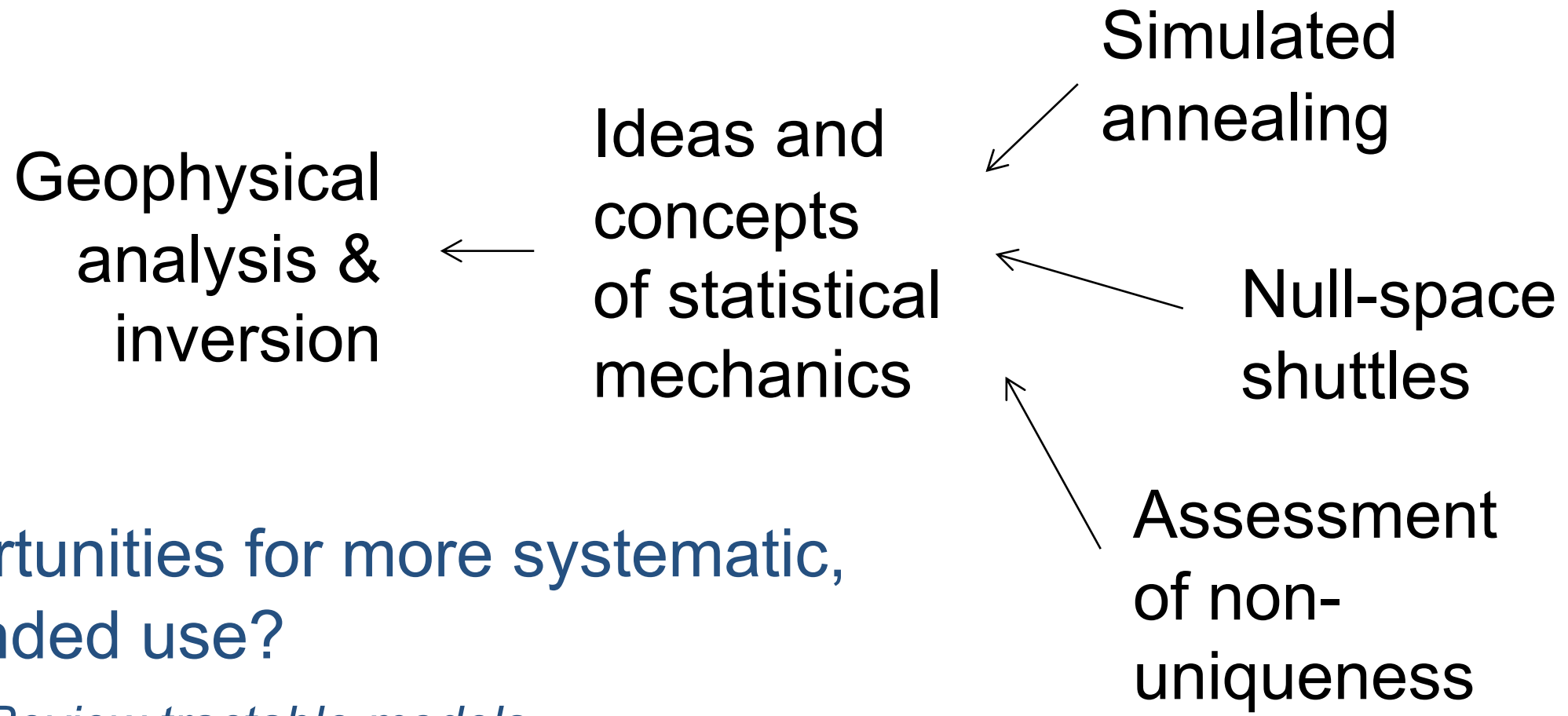
FACULTY OF SCIENCE
Department of Geoscience

Statistical mechanics and geophysical inversion



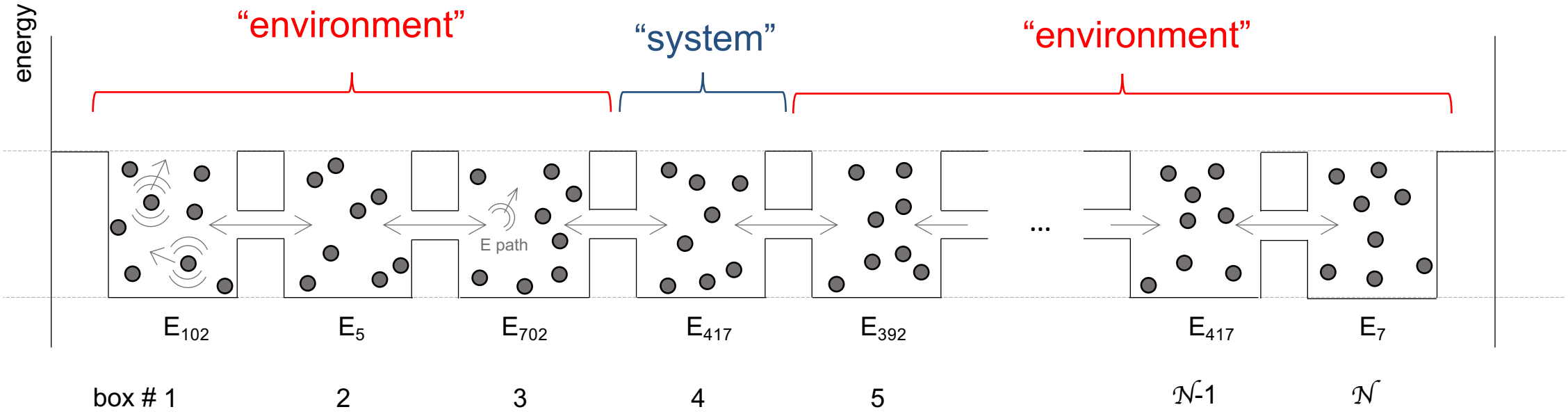
Statistical mechanics and geophysical inversion



Opportunities for more systematic, expanded use?

- *Review tractable models*
- *Port candidates to geophysics*
- *Today: survey of three applications*

Maxwell-Boltzmann statistics



$$P_i = \frac{1}{Z} e^{-\beta E_i}$$

$$\beta = \frac{1}{T}$$

$$Z(\beta) = \sum_{i=1}^{M} e^{-\beta E_i}$$

Geophysical applications

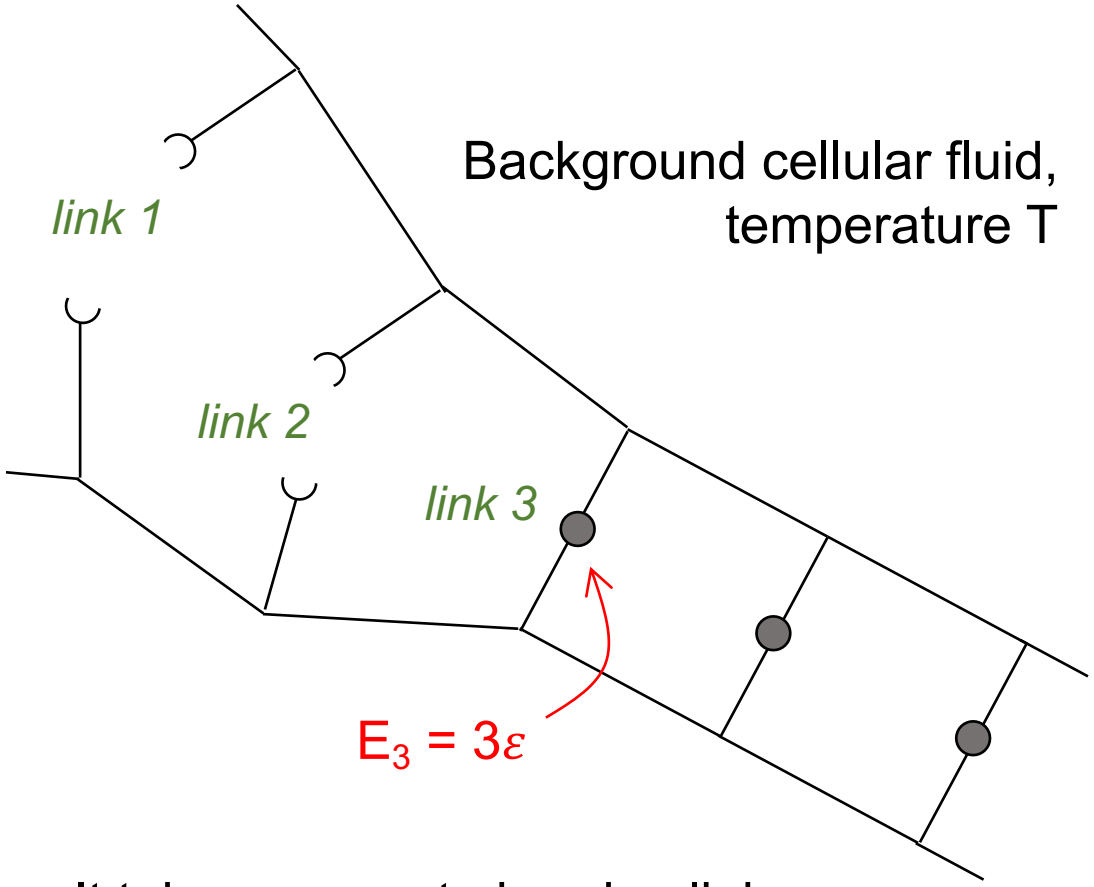
1. Uncertainty analysis in tomographic problems
   - discrete slowness ~ “molecular zipper”
   - $\langle contribution\ of\ grid\ cell \rangle$ as measure of uncertainty
2. The statistics of model-space shuttles
   - extension of FWI uncertainty through null-space shuttles
   - $\langle density \rangle$ in model space akin to a hydrostatic model
3. A phase transition in the O’Doherty-Anstey model
   - stratigraphic filtering as a statistical / combinatorical problem
   - intermediate calculation in OA provides otherwise complex Z
   - two lag regimes separated by a phase transition identified

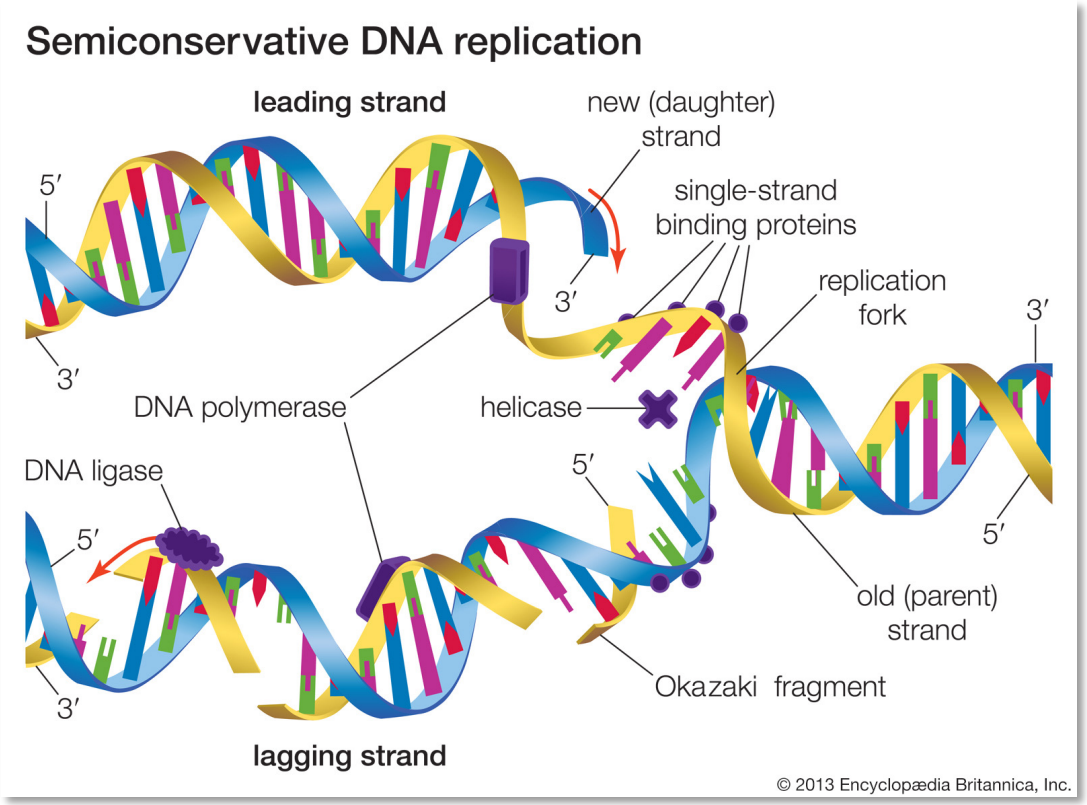
Geophysical applications

1. Uncertainty analysis in tomographic problems
   - discrete slowness ~ “molecular zipper” of Kittel
   - $\langle contribution\ of\ grid\ cell \rangle$ as measure of uncertainty
2. The statistics of null-space shuttles
   - extension of uncertainty char. of Fichtner, Keating, etc.
   - $\langle density \rangle$ in model space akin to a hydrostatic model
3. A phase transition in the O’Doherty-Anstey model
   - stratigraphic filtering as a statistical / combinatorical problem
   - intermediate calculation in OA provides otherwise complex Z
   - two lag regimes separated by a phase transition identified

Application 1: zipper statistics

A DNA molecule is a ladder structure with breakable rungs





source: Encyclopedia Britannica

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- ε if adjacent link is broken, ∞ if it is not
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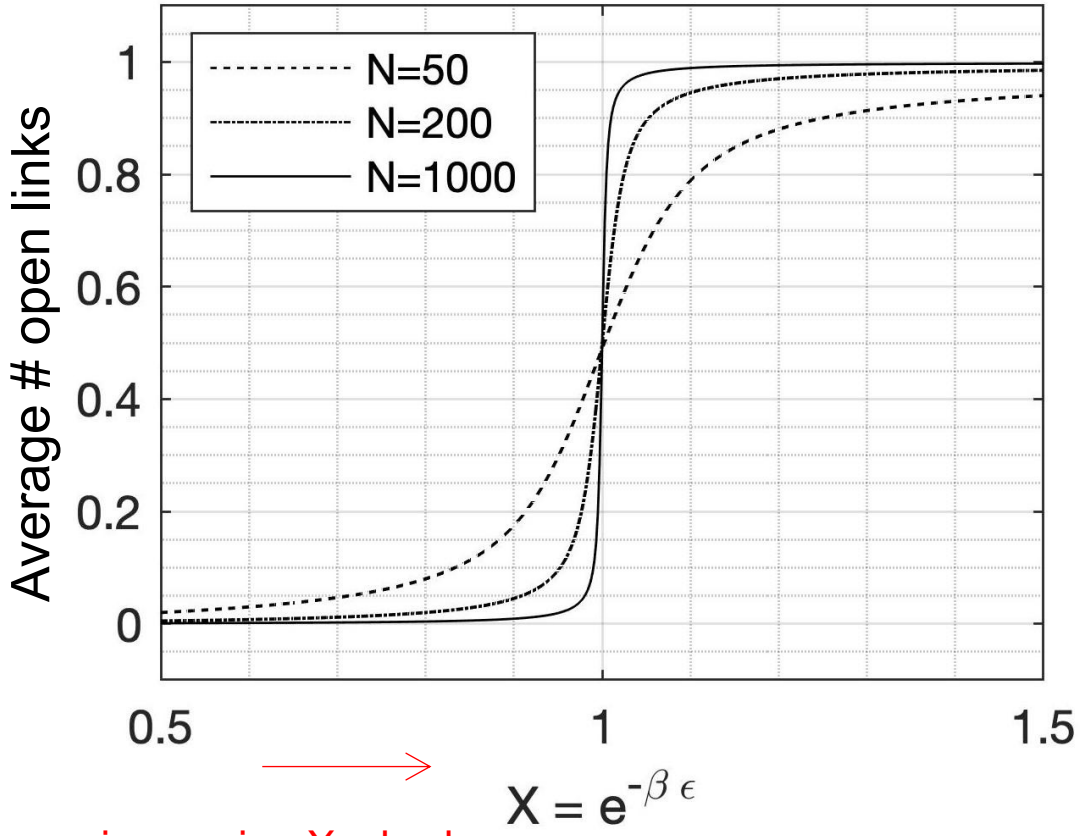
Application 1: zipper statistics



increasing X, slowly
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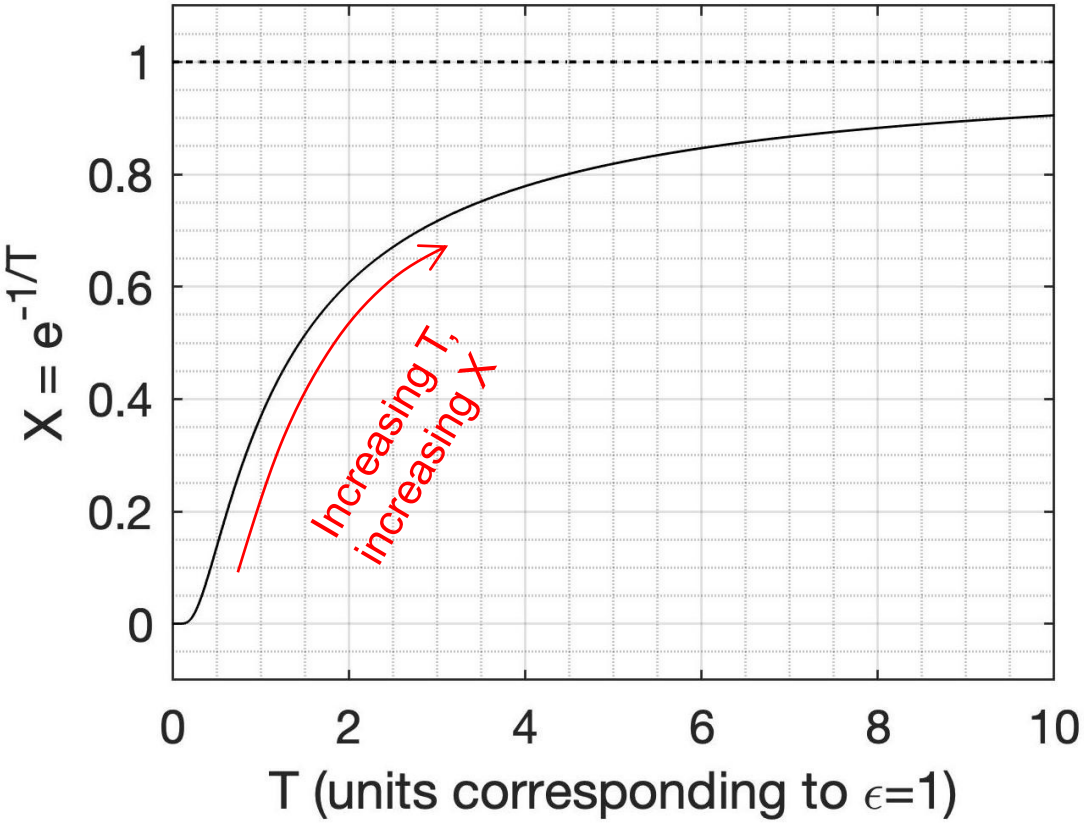
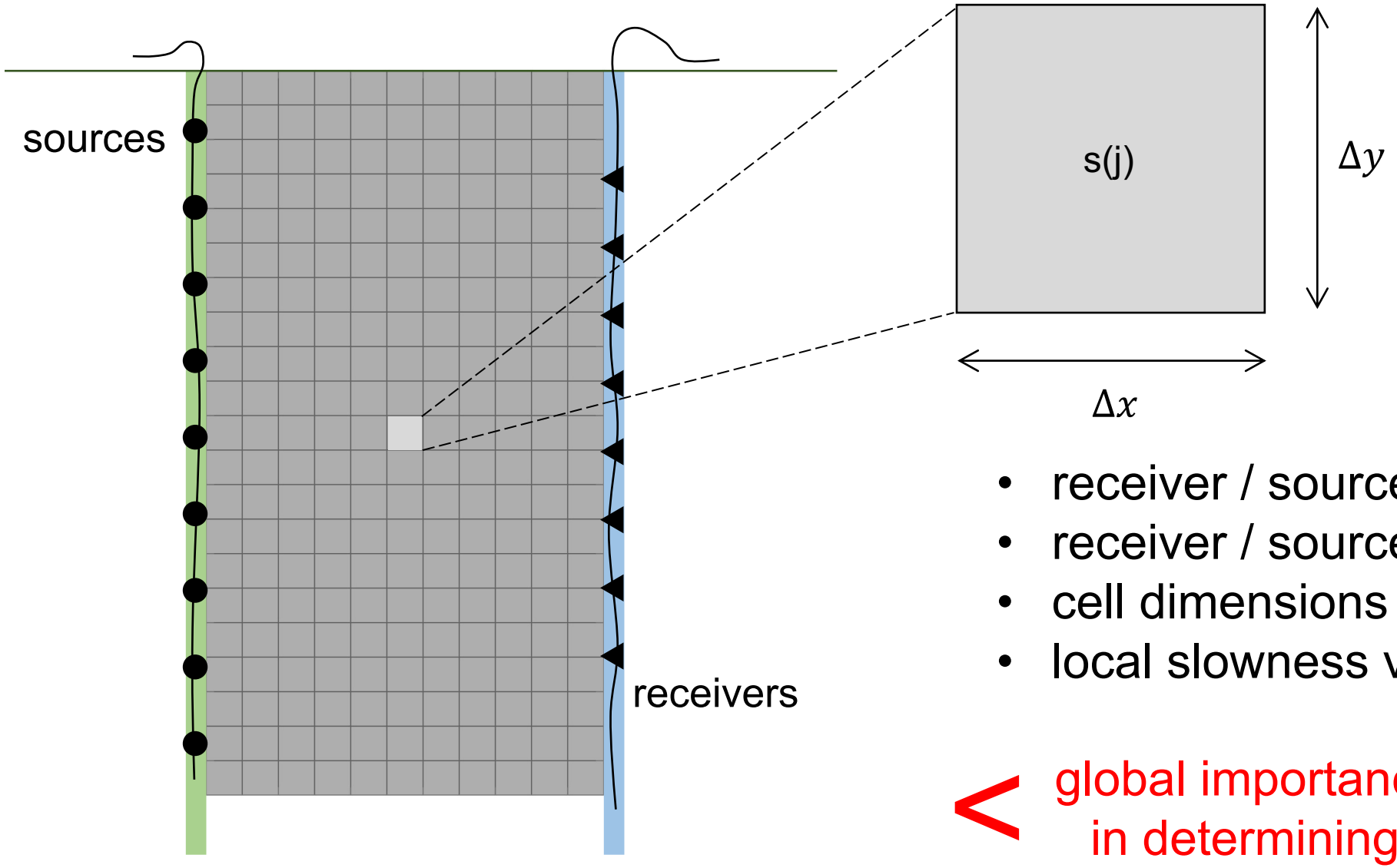


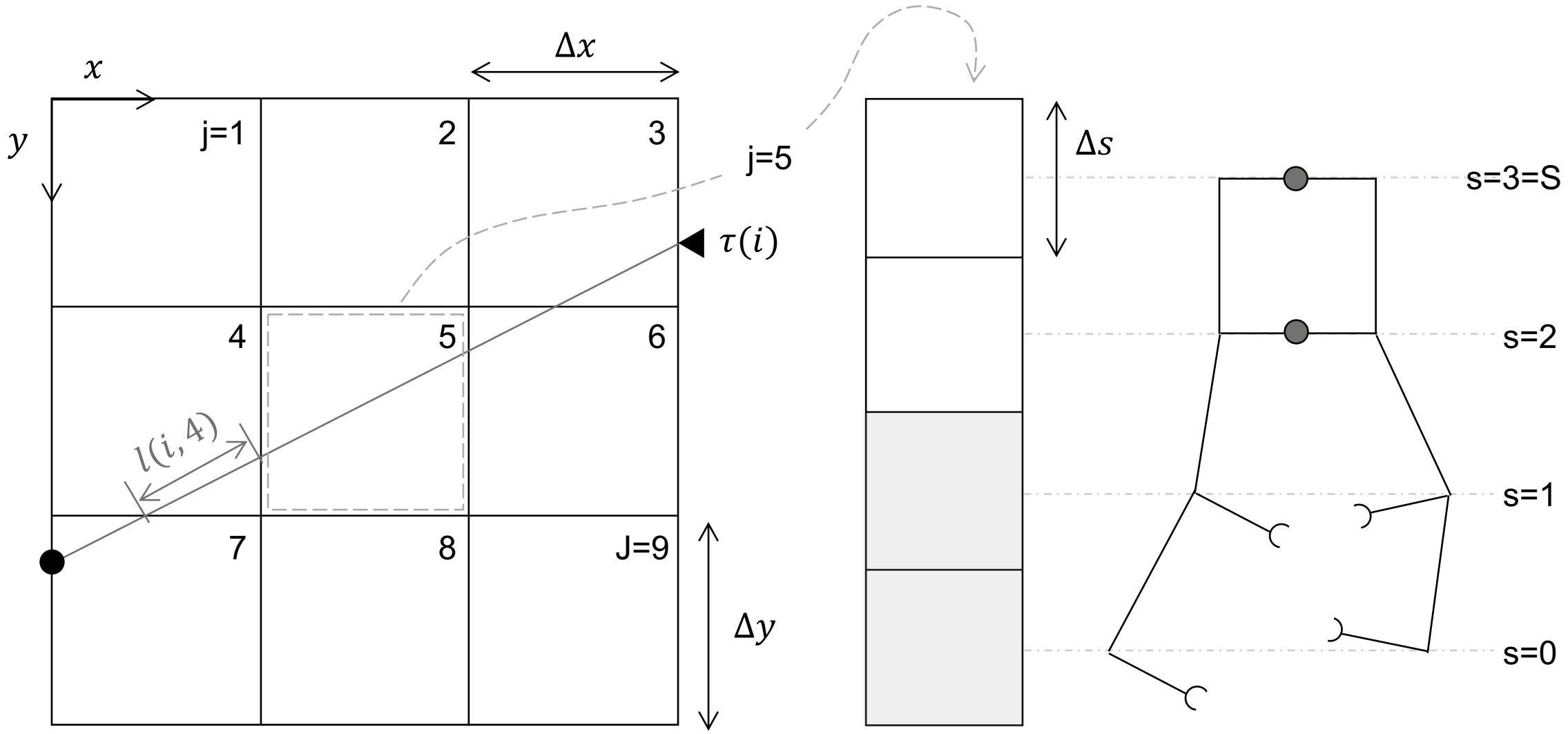
Zipper statistics in a tomography setting



- receiver / source positioning
- receiver / source coverage
- cell dimensions
- local slowness values

< global importance of a cell in determining the data >

Zipper statistics in a tomography setting



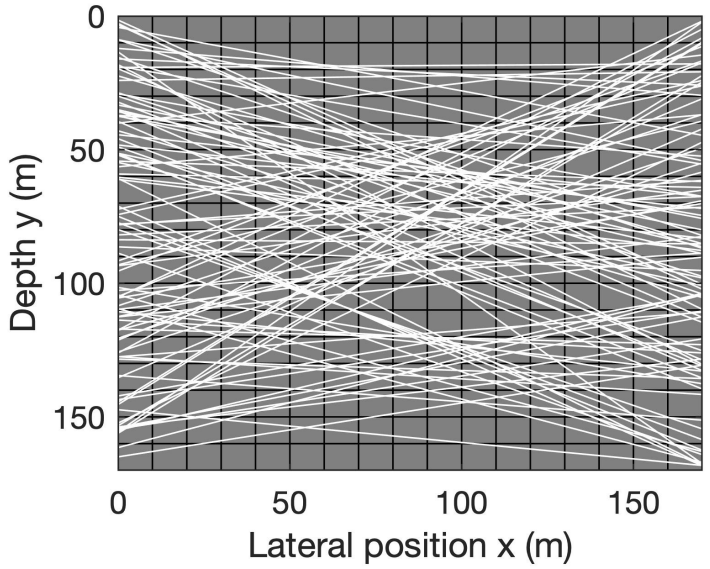
The quantity $\langle \tau \rangle$ as an assessor of uncertainty

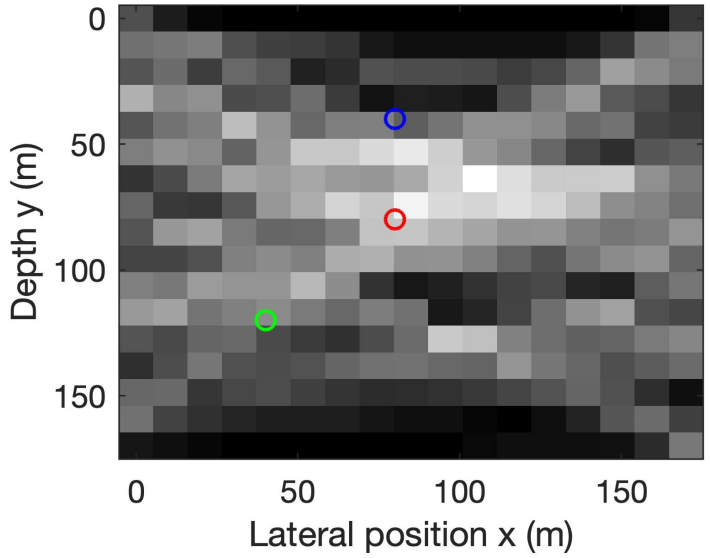
$$X(j) = e^{-\beta s L(j) \Delta s}$$

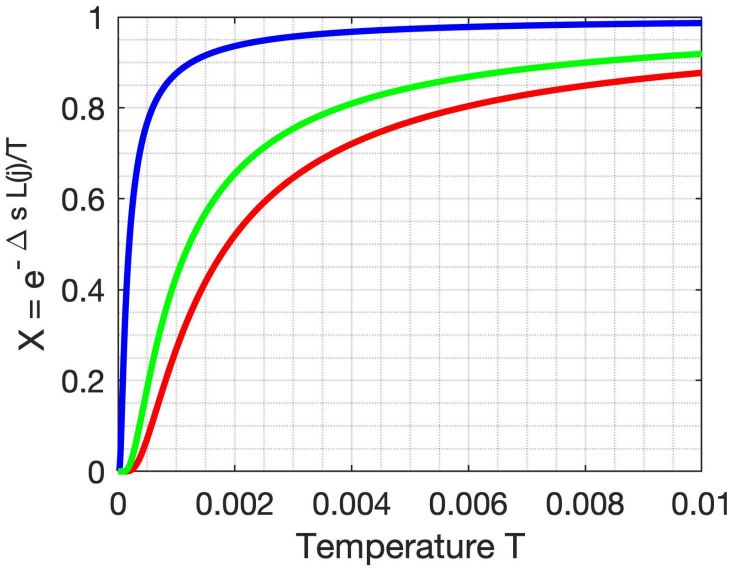
$$\langle\ \tau(j)\ \rangle = \Delta s L(j) \left[\frac{X(j)}{1 - X(j)} - S \frac{X(j)^S}{1 - X(j)^S} \right]$$

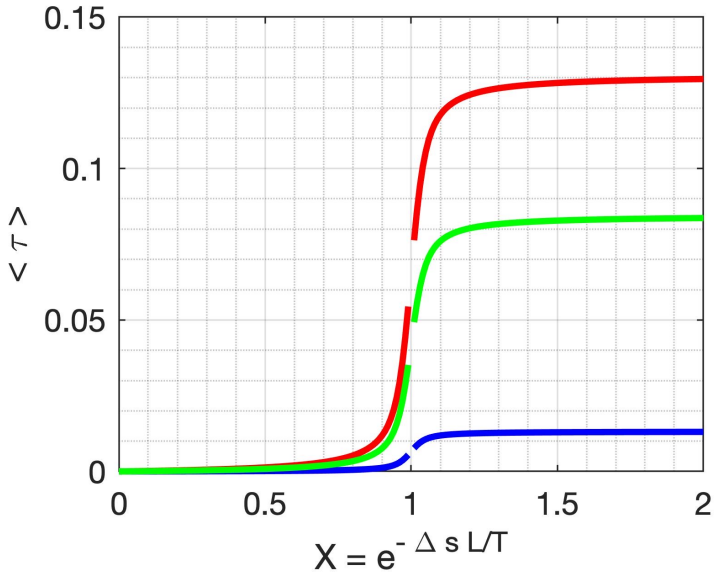
Average traveltime burden to the full dataset of the jth grid cell – how important the cell is in explaining the data

The quantity $\langle \tau \rangle$ as an assessor of uncertainty









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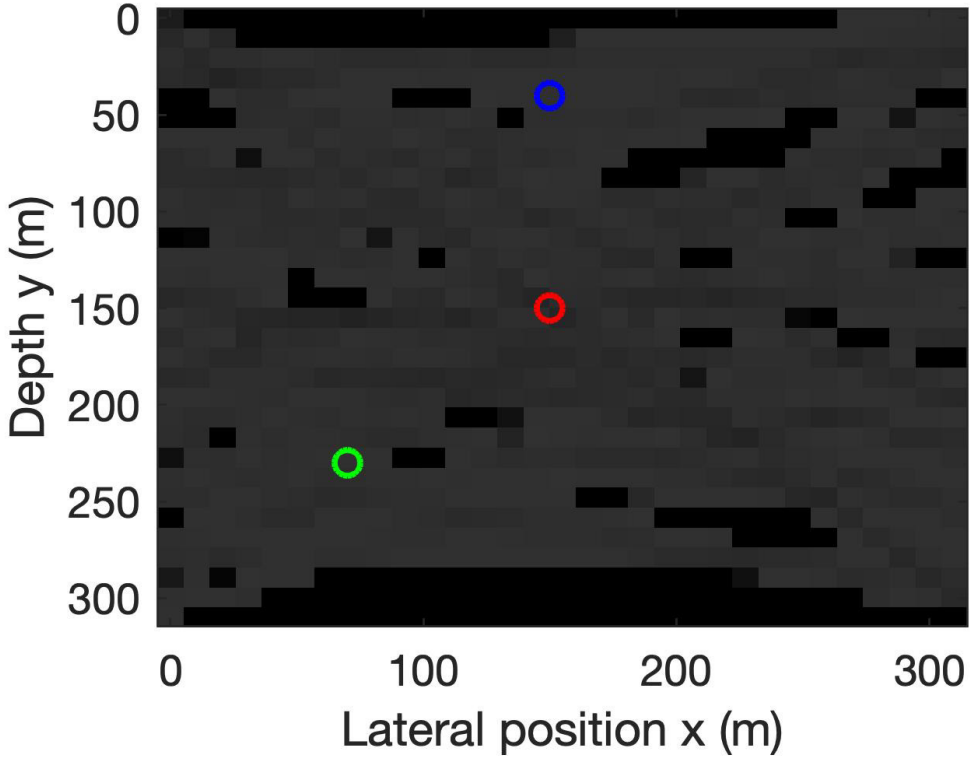

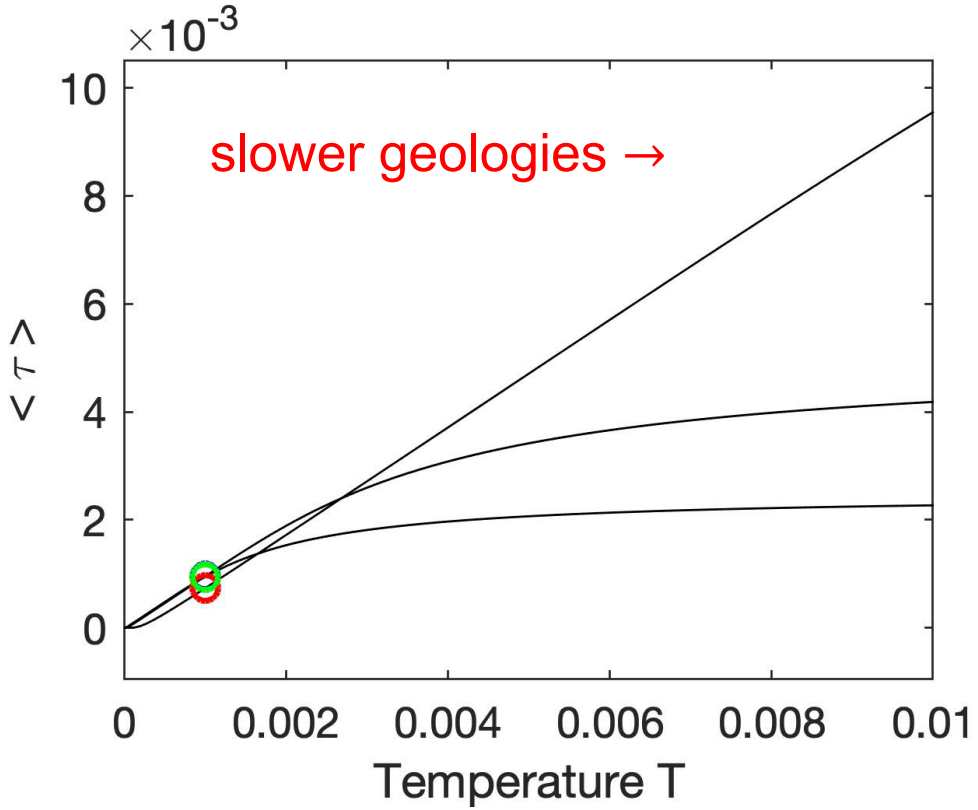

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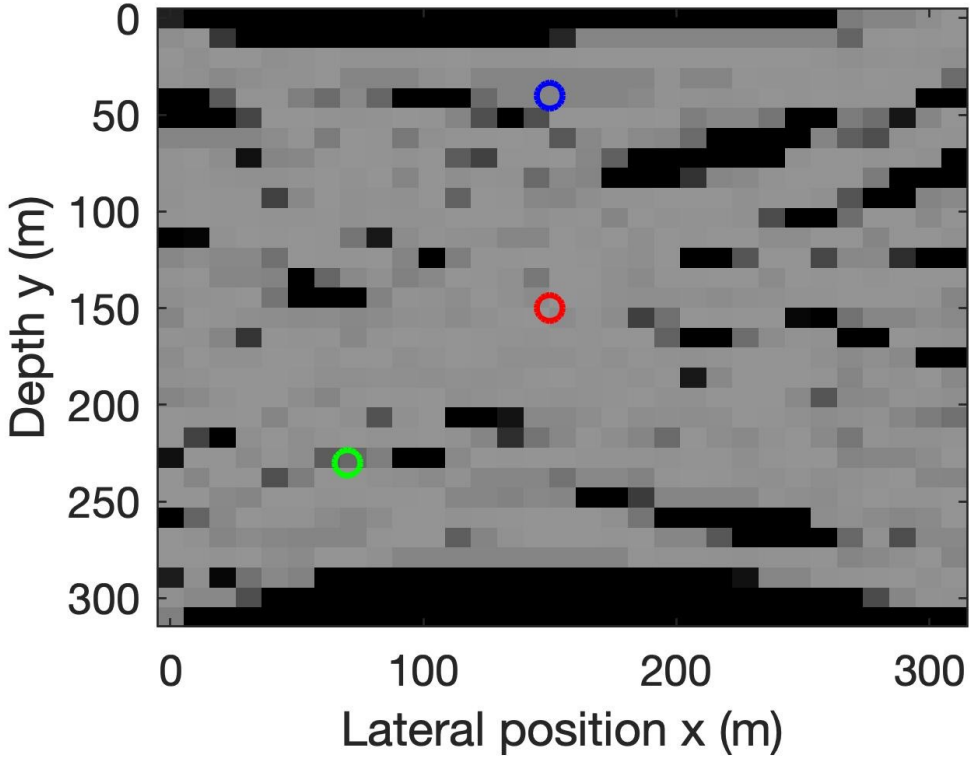

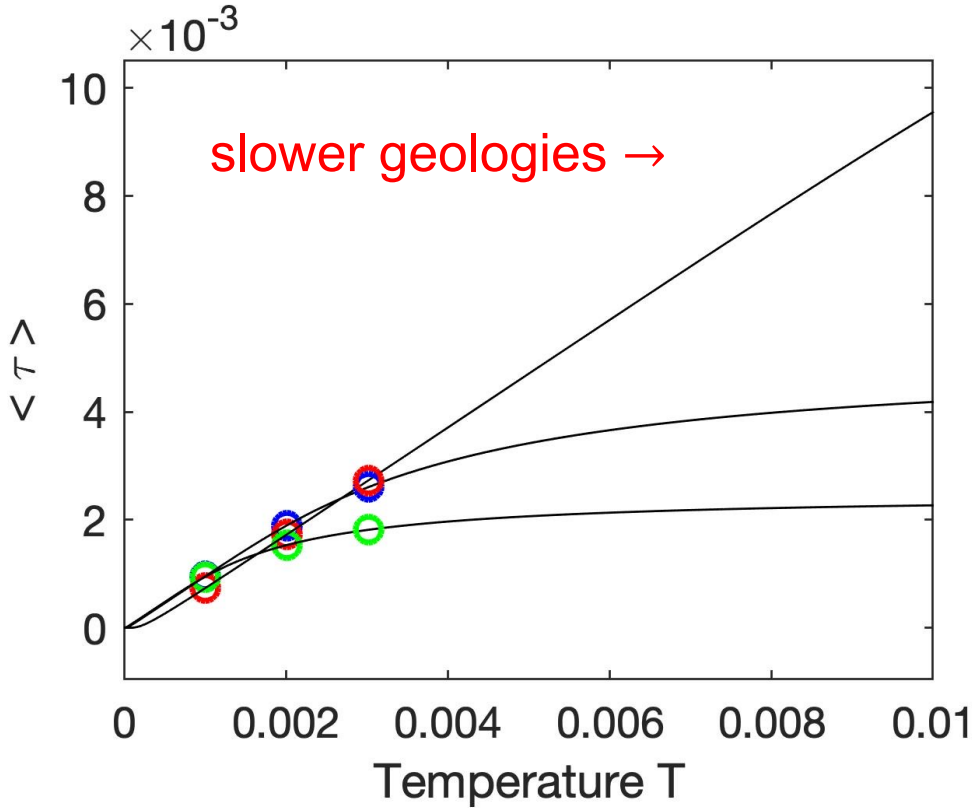

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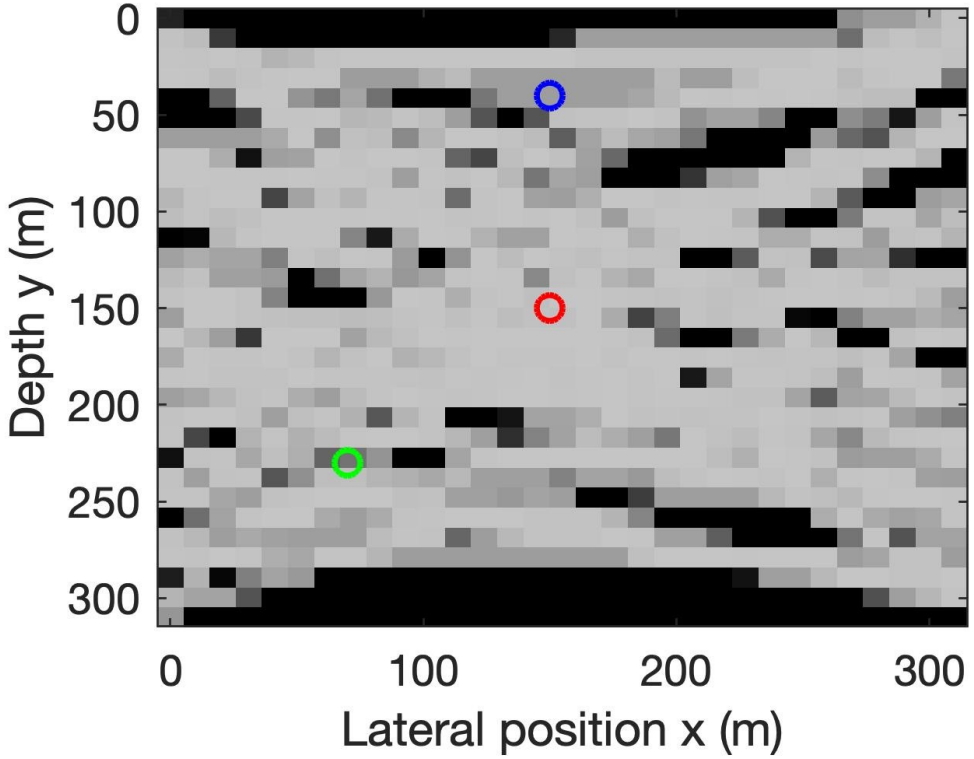

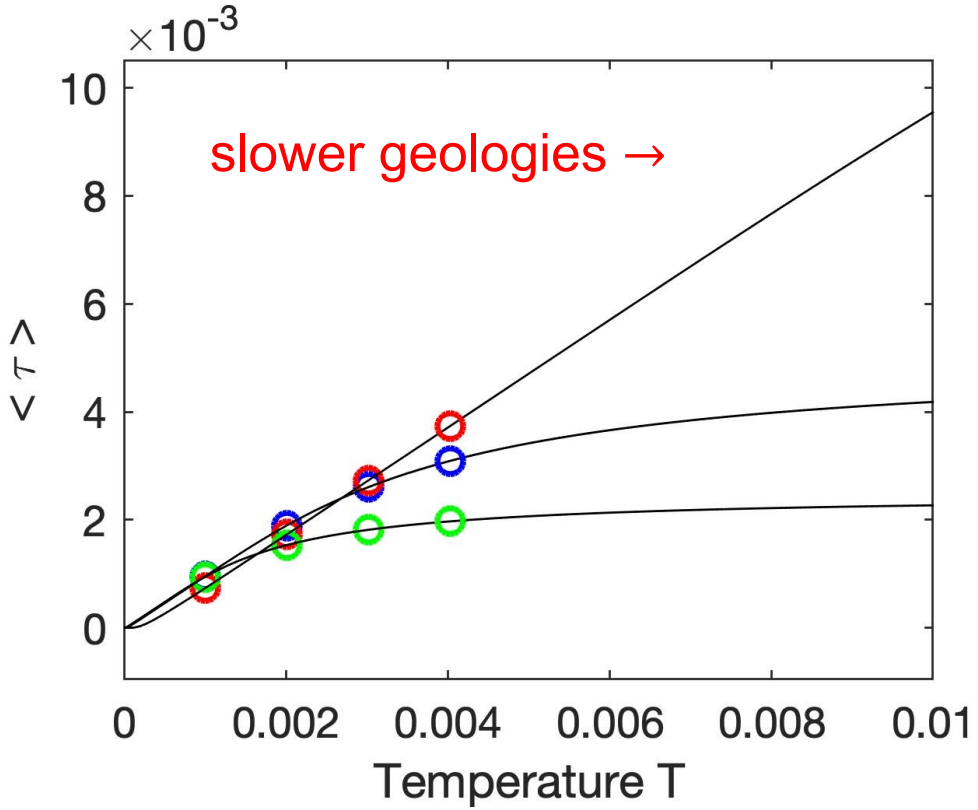

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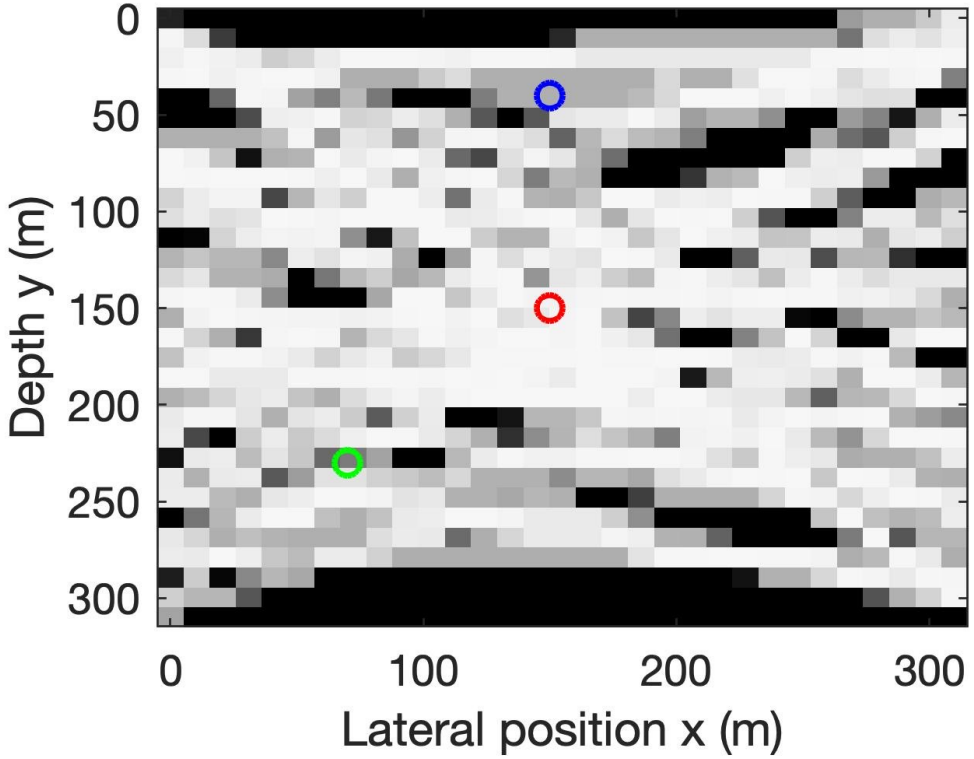

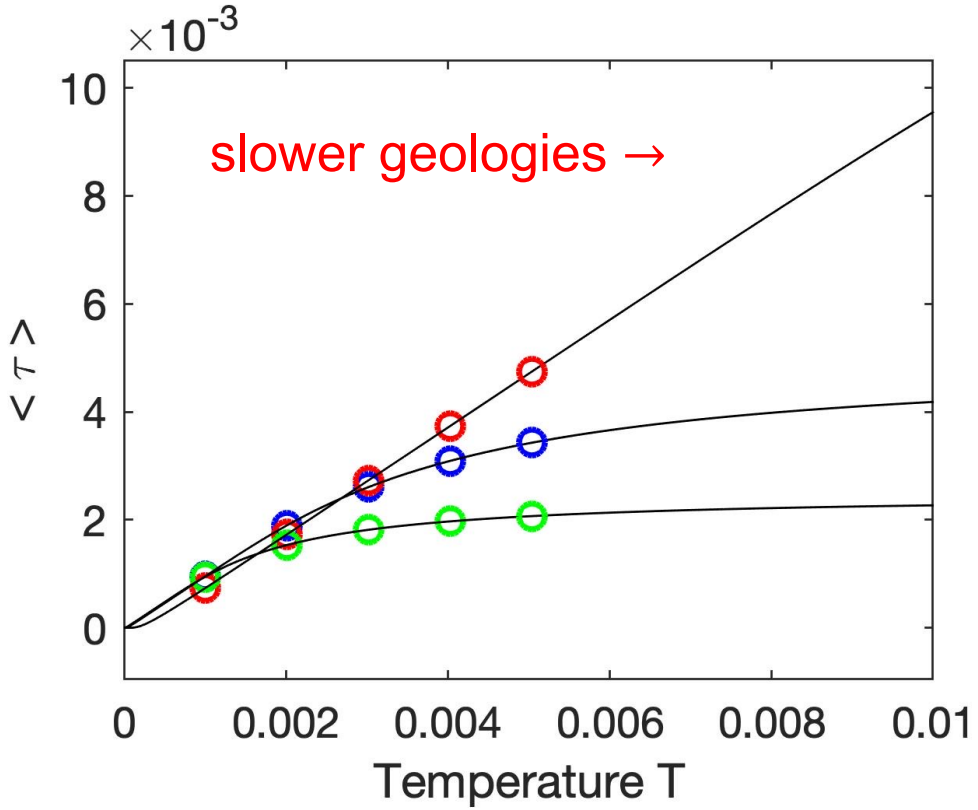

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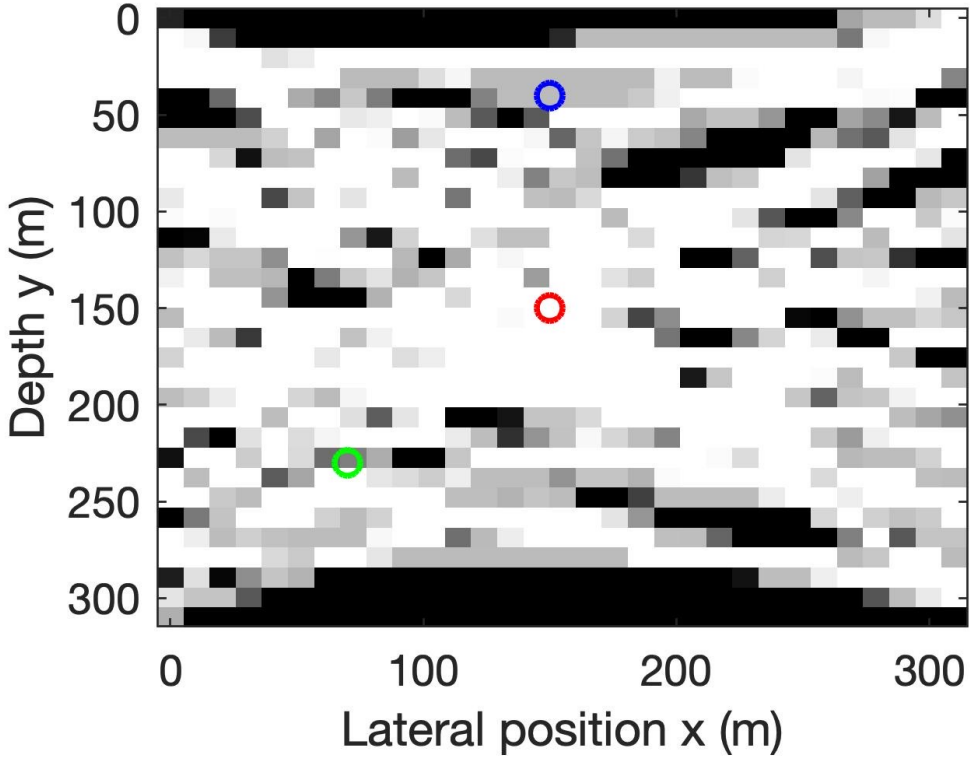

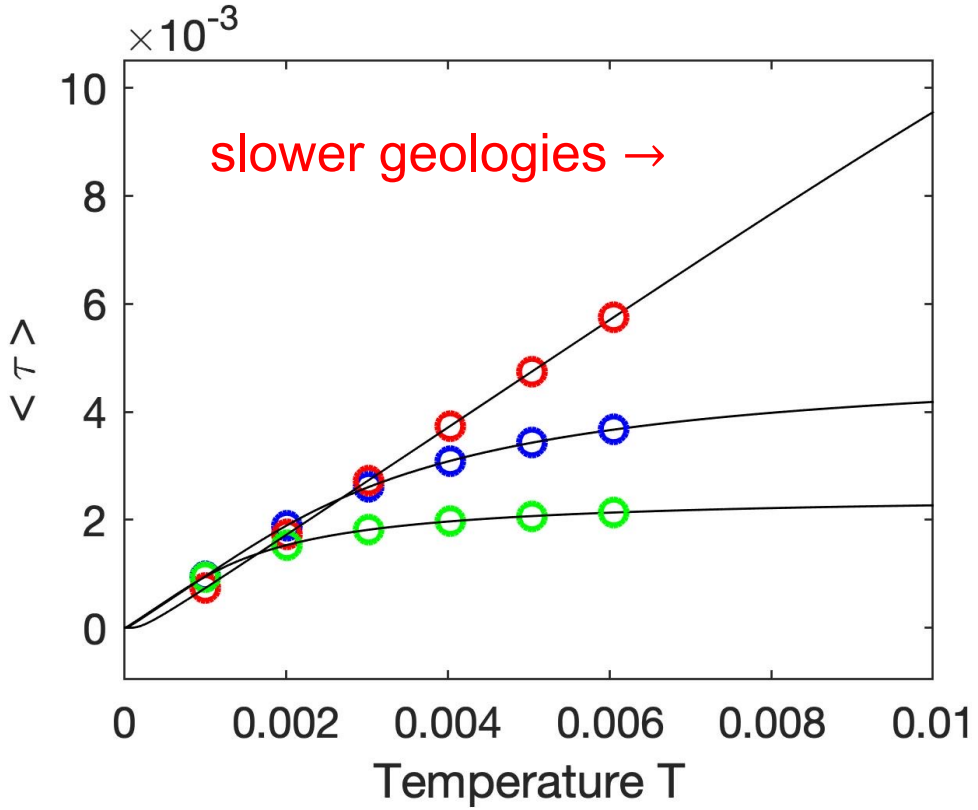

Application 2: a phase transition in the O'Doherty-Anstey model

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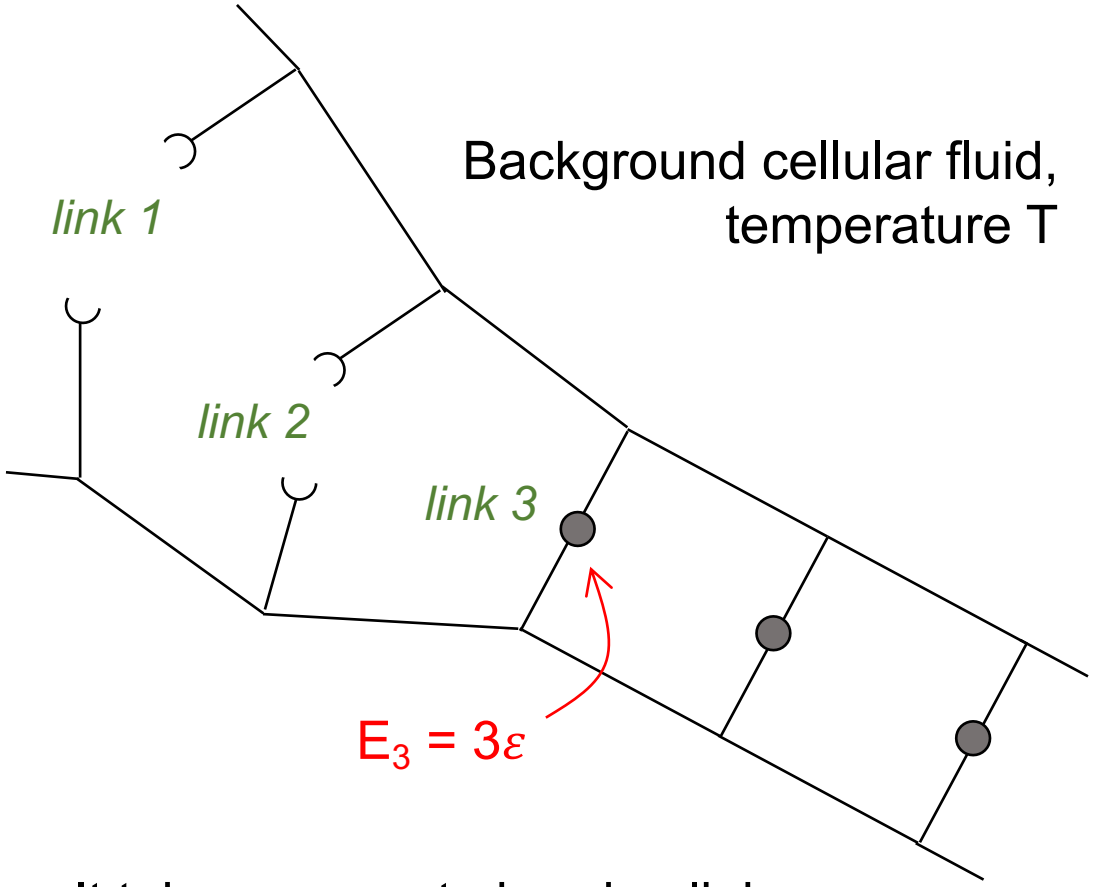

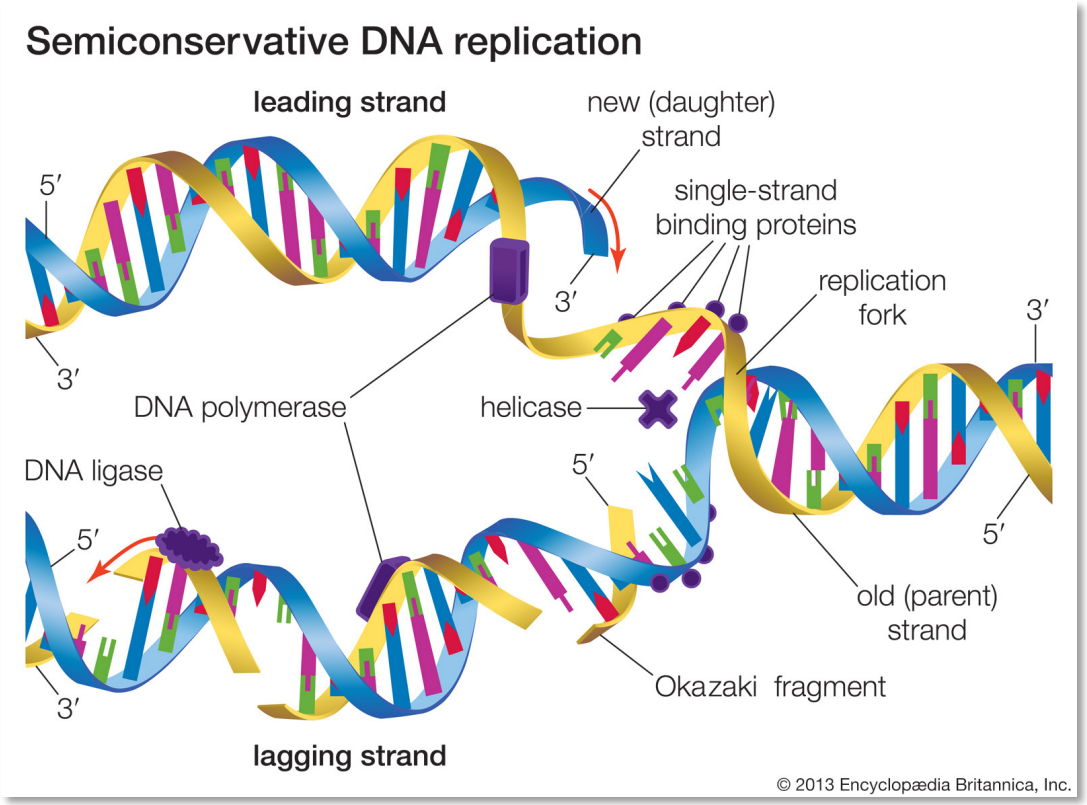

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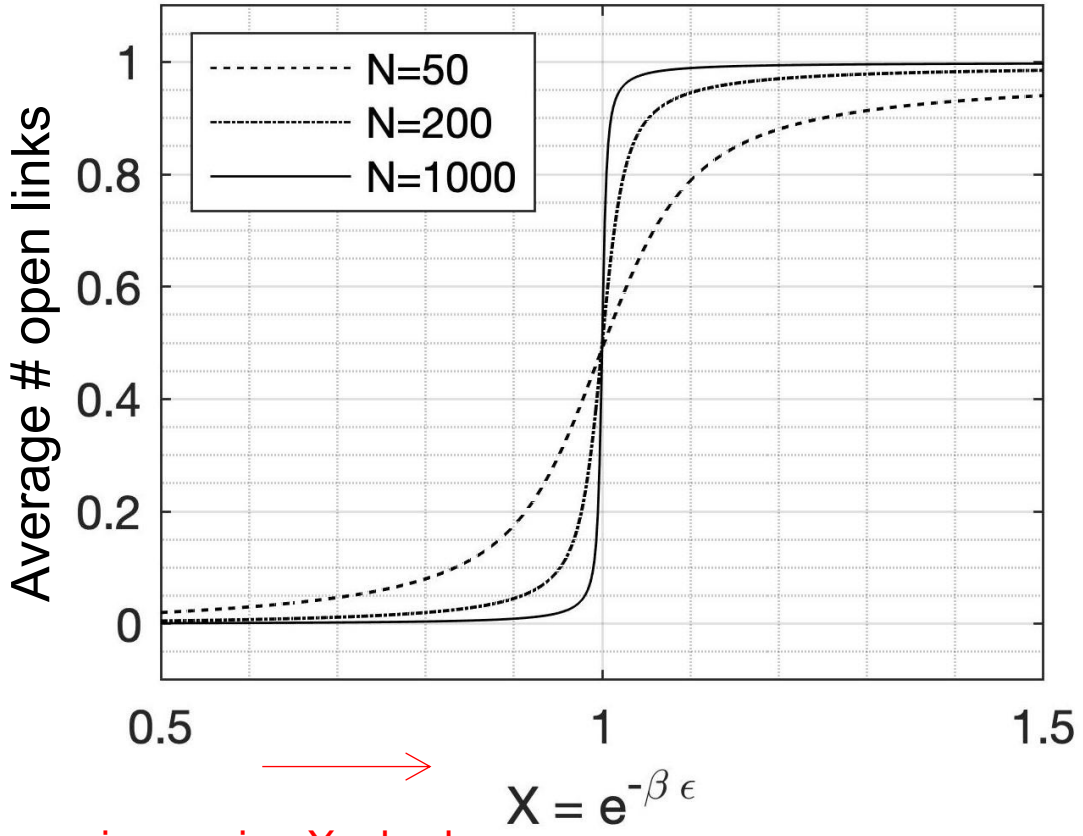

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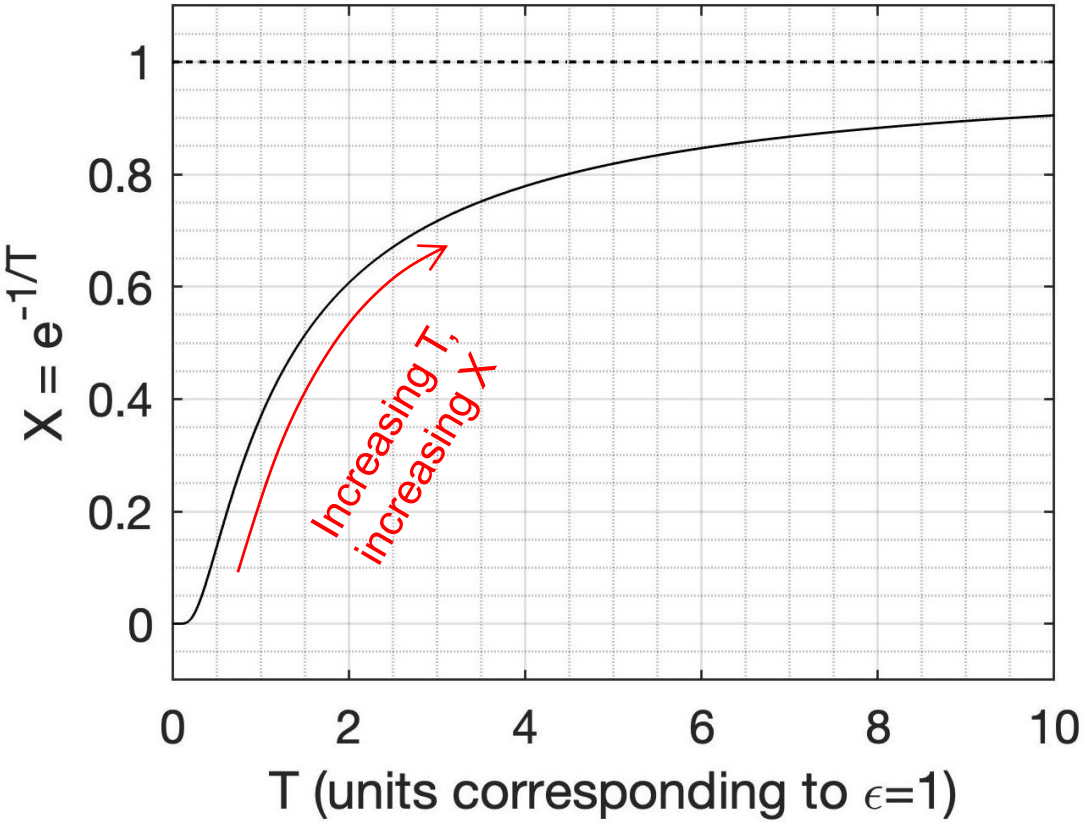

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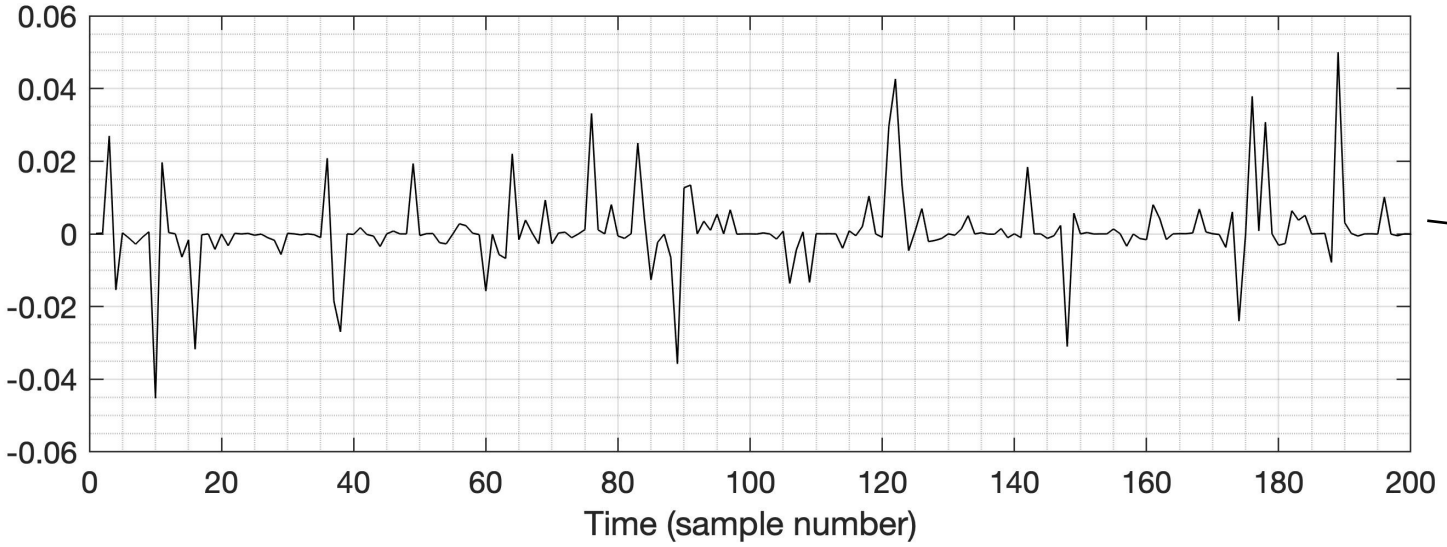
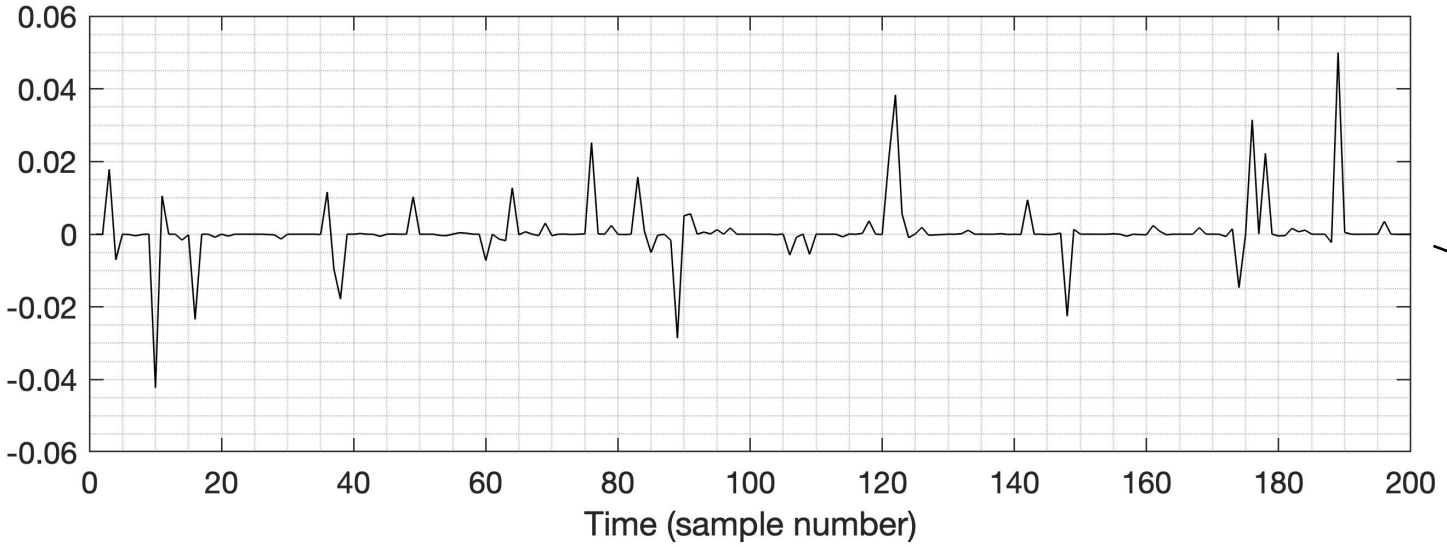
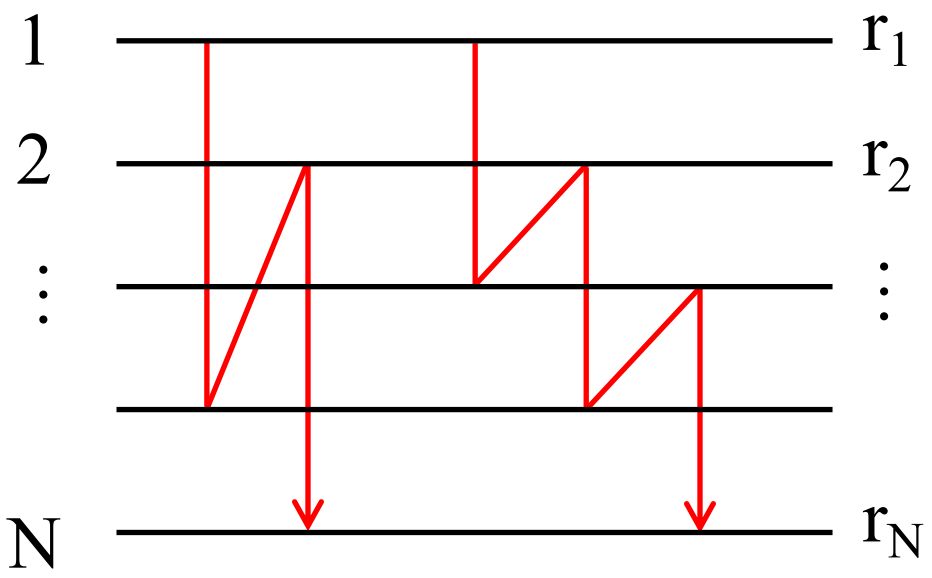




$$|T(\omega)| = e^{-R(\omega)t}$$

Application 2: a phase transition in the O'Doherty-Anstey model



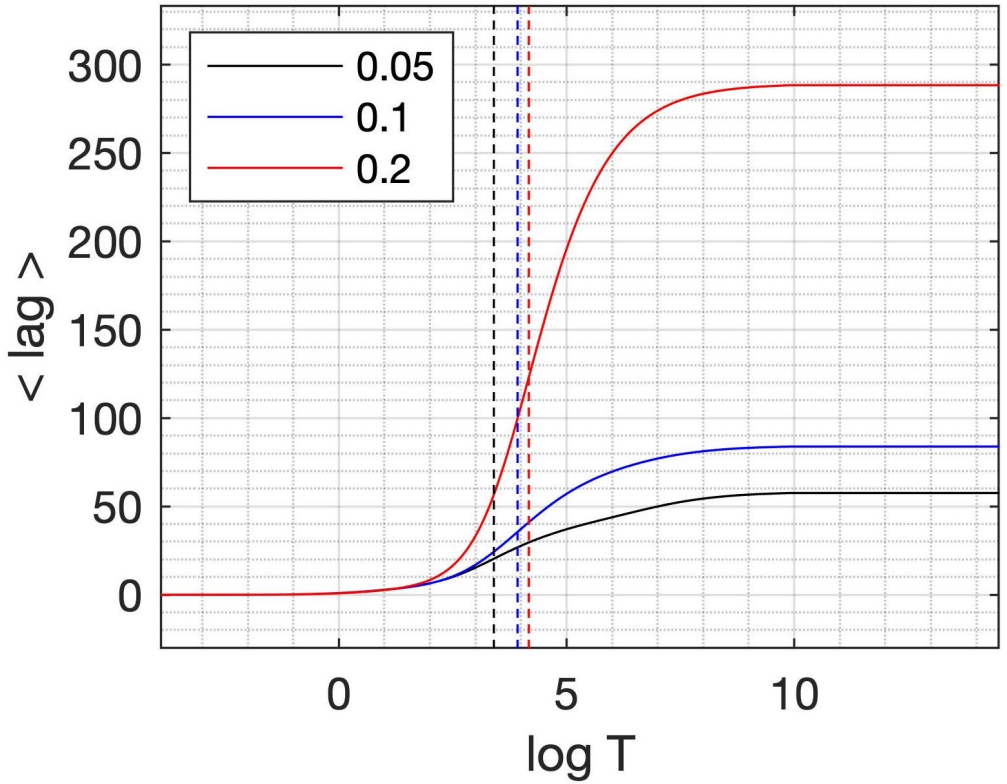
oa(l) = # raypaths contributing at lag l

note… rapid growth with N and |r|

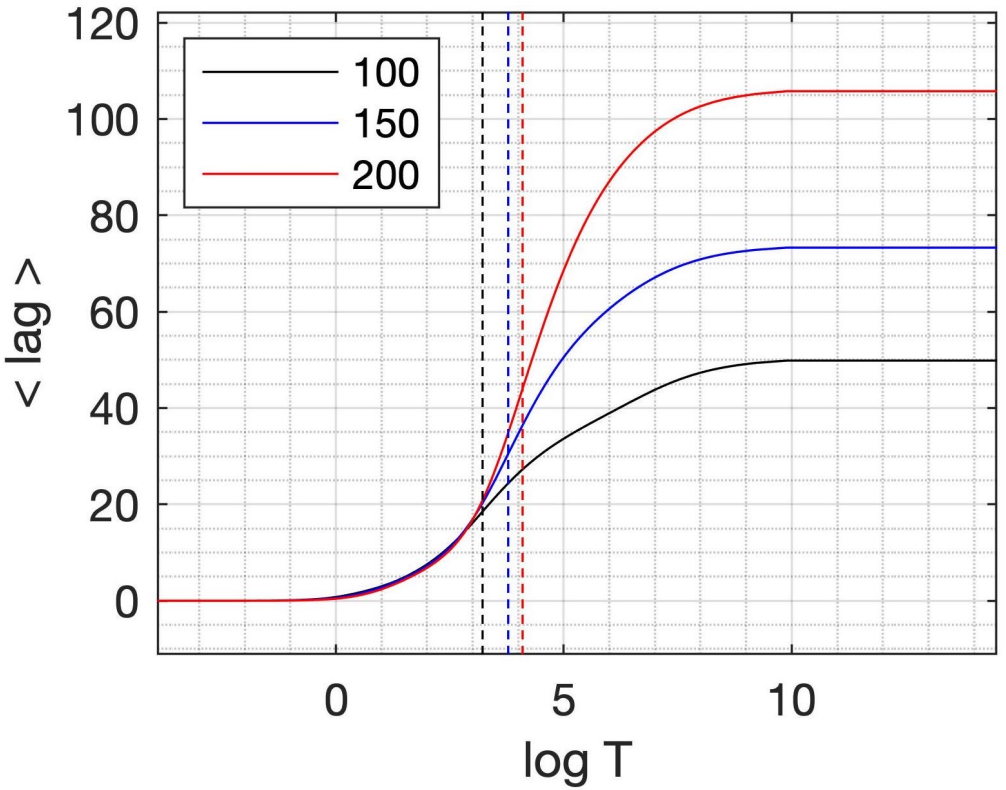
$$Z(\beta) = c \sum_{l=1}^{L} \mathrm{oa}(l) e^{-\beta l}$$

Application 2: a phase transition in the O'Doherty-Anstey model

<lag> versus <r>



<lag> versus # interfaces



T : "lag temperature"

Summary

- Statistical mechanics concepts pervade inverse theory
- Lots of unused “stuff” in the theory – port it over?
- Approach: if possible, use tractable models
- Possibility: assess model uncertainty
  - Zipper model ~ discrete slowness
  - $< \tau >$ ~ relevance of slowness cell
- Possibility: identify new macroscopic behaviours
  - phase transitions implicit in stratigraphic filtering

Acknowledgments and further information

Thanks to:

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CREWES faculty, staff & researchers

More on statistical mechanics:

Kestin, J., Dorfman, J. R., 1971, A course in statistical thermodynamics: Academic Press.

Kittel, C., 1969, Phase transition of a molecular zipper: American Journal of Physics, 37, 917.

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