Local optimization approaches for simultaneous AVO inversion based on re-parameterized Zoeppritz equations

Mariana Lume and Kris Innanen

December 3, 2021

NSERC CRSNG
UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
Department of Geoscience

Introduction

Weighted stacking technique:
Smith and Gidlow (1987)
Stewart (1990) and Larsen (1999)

→ Aki-Richards approximations → $\frac{\Delta I}{I}$ $\frac{\Delta J}{J}$ → Rpp, Rps and weights

Assumptions
- Weak contrasts
- Incidence angles < 35° and < critical
- Vp/Vs: 1.5 – 2

Linearized inversions limit the incidence angles → Growing interest in long-offsets scenarios

Objective: Inversion strategy suitable for long-offset acquisitions

Forward Problem: Zoeppritz equations

$$\mathbf{P}\begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = \mathbf{b}$$

$$\mathbf{P} = \begin{bmatrix} -X & -\sqrt{1-B^2X^2} & CX & \sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & -DX \\ 2B^2X\sqrt{1-X^2} & B(1-2B^2X^2) & 2AD^2X\sqrt{1-C^2X^2} & AD(1-2D^2X^2) \\ -(1-2B^2X^2) & 2B^2X\sqrt{1-B^2X^2} & AC(1-2D^2X^2) & -2AD^2X\sqrt{1-D^2X^2} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2B^2X^2 \end{bmatrix}$$

$$\frac{\Delta I}{I} \quad \frac{\Delta J}{J} \quad \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta q}{q} = \frac{\Delta I}{I} - \frac{\Delta J}{J}$$

where:

$$A = \frac{\rho_2}{\rho_1} \qquad B = \frac{\beta_1}{\alpha_1} \qquad C = \frac{I_2}{I_1}\frac{\rho_1}{\rho_2} \qquad D = B\frac{J_2}{J_1}\frac{\rho_1}{\rho_2} \qquad X = sin\theta_1$$

$$\frac{Y_2}{Y_1} = \left(1 + \frac{1}{2}\frac{\Delta Y}{Y}\right) \Big/ \left(1 - \frac{1}{2}\frac{\Delta Y}{Y}\right)$$

Forward Problem: raytracing

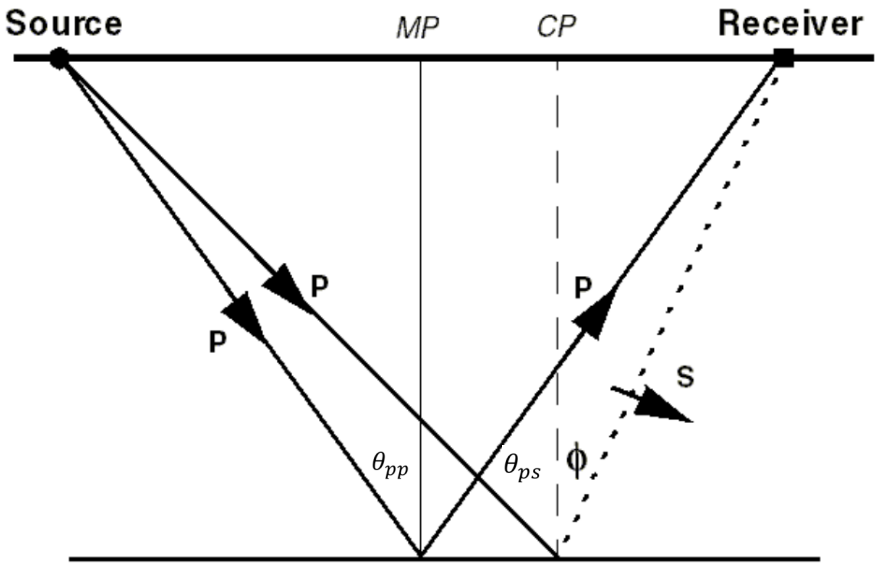


Figure modified from Stewart et al. (1999)

$$\mathbf{P}\begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = \mathbf{b} \begin{cases} X = \sin\theta_{pp}{}_j \\ X = \sin\theta_{ps}{}_j \end{cases} \quad j = 1 \ldots N$$

$$\mathbf{d_{pred}} = \mathbf{Su} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = \begin{bmatrix} R_{pp} \\ R_{ps} \end{bmatrix}$$

Simultaneous inversion: unconstrained optimization

$$\phi(\mathbf{m}) = \frac{1}{2}\sum_{j=1}^{N}\Big(\underbrace{\mathbf{Su}(\mathbf{m},\theta_j)}_{\text{predicted}} - \mathbf{d_{obs}}(\theta_j)\Big)^{\intercal}\mathbf{W}^{\intercal}\mathbf{W}\Big(\underbrace{\mathbf{Su}(\mathbf{m},\theta_j)}_{\text{predicted}} - \mathbf{d_{obs}}(\theta_j)\Big) \longrightarrow \mathbf{W} = \begin{bmatrix} 1/\sigma_{pp} & 0 \\ 0 & 1/\sigma_{ps} \end{bmatrix}$$

Gradient: $$\mathbf{g} = \sum_{j=1}^{N}\mathbf{J}_j^{\intercal}\mathbf{W}^{\intercal}\mathbf{W}\Big(\mathbf{Su}(\mathbf{m},\theta_j) - \mathbf{d_{obs}}(\theta_j)\Big) \qquad J_\mu^i = -S_K^i(P^{-1})_L^K\frac{\partial P_M^L}{\partial m^\mu}u^M$$

$$i = 1,2$$
$$j = 1,2,3$$
$$K = M = L = 1,2,3,4$$

Hessian: $$\mathbf{H} = \sum_{j=1}^{N}\mathbf{J}_j^{\intercal}\mathbf{W}^{\intercal}\mathbf{W}\mathbf{J}_j$$

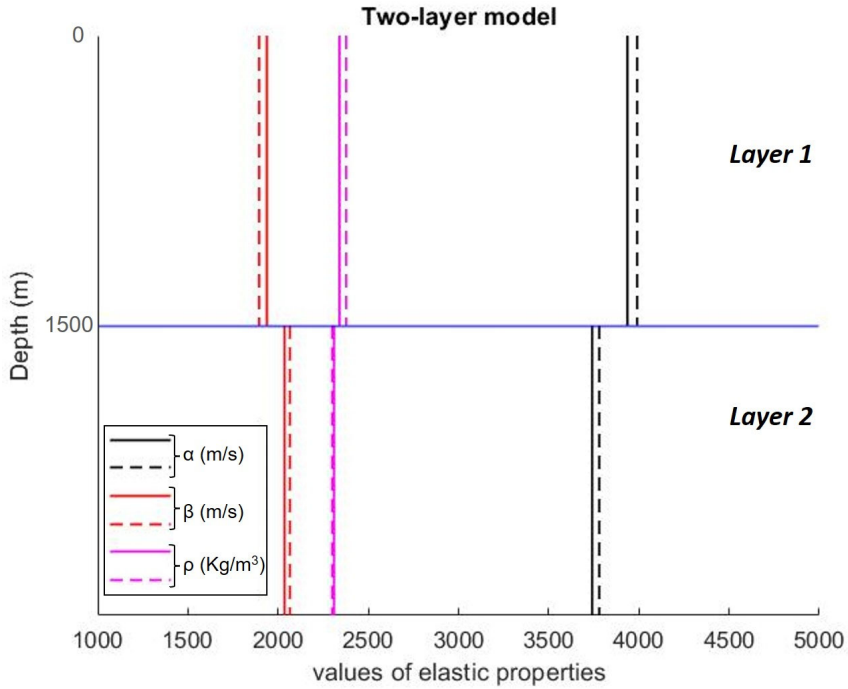
Updates:

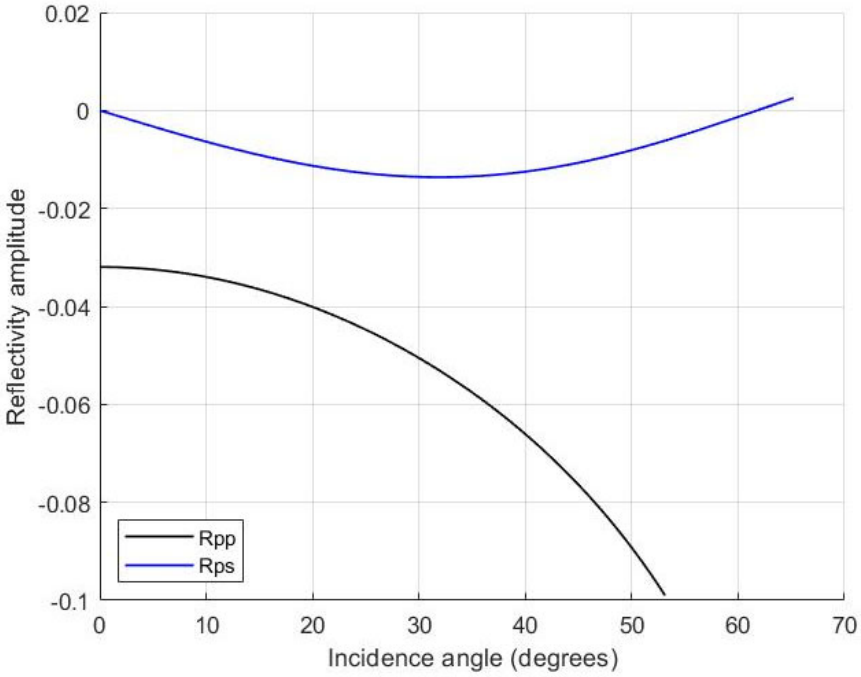
$$\Delta\mathbf{m} = -\mathbf{B}^{-1}\mathbf{g}$$

- $\mathbf{B} = \mathbf{I} \rightarrow$ Steepest Descent
- $\mathbf{B} = \mathbf{H} \rightarrow$ Gauss-Newton
- $\mathbf{B} = \mathbf{H} + \lambda diag(\mathbf{H}) \rightarrow$ Levenberg-Marquardt

True model features

Dashed lines: initial model
Continuous lines: true model



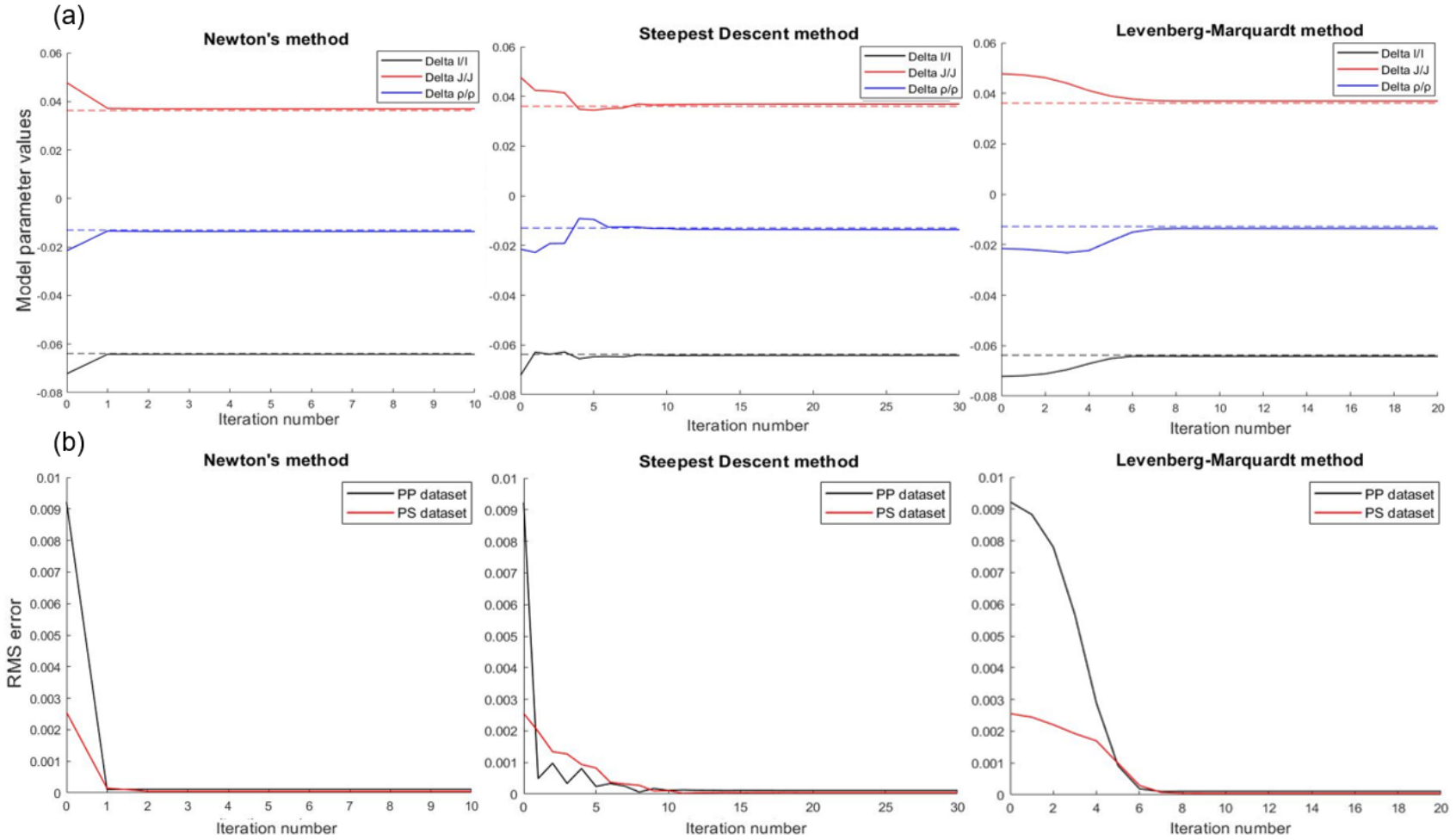


☑ Underlying assumptions of weighted stacking inversion

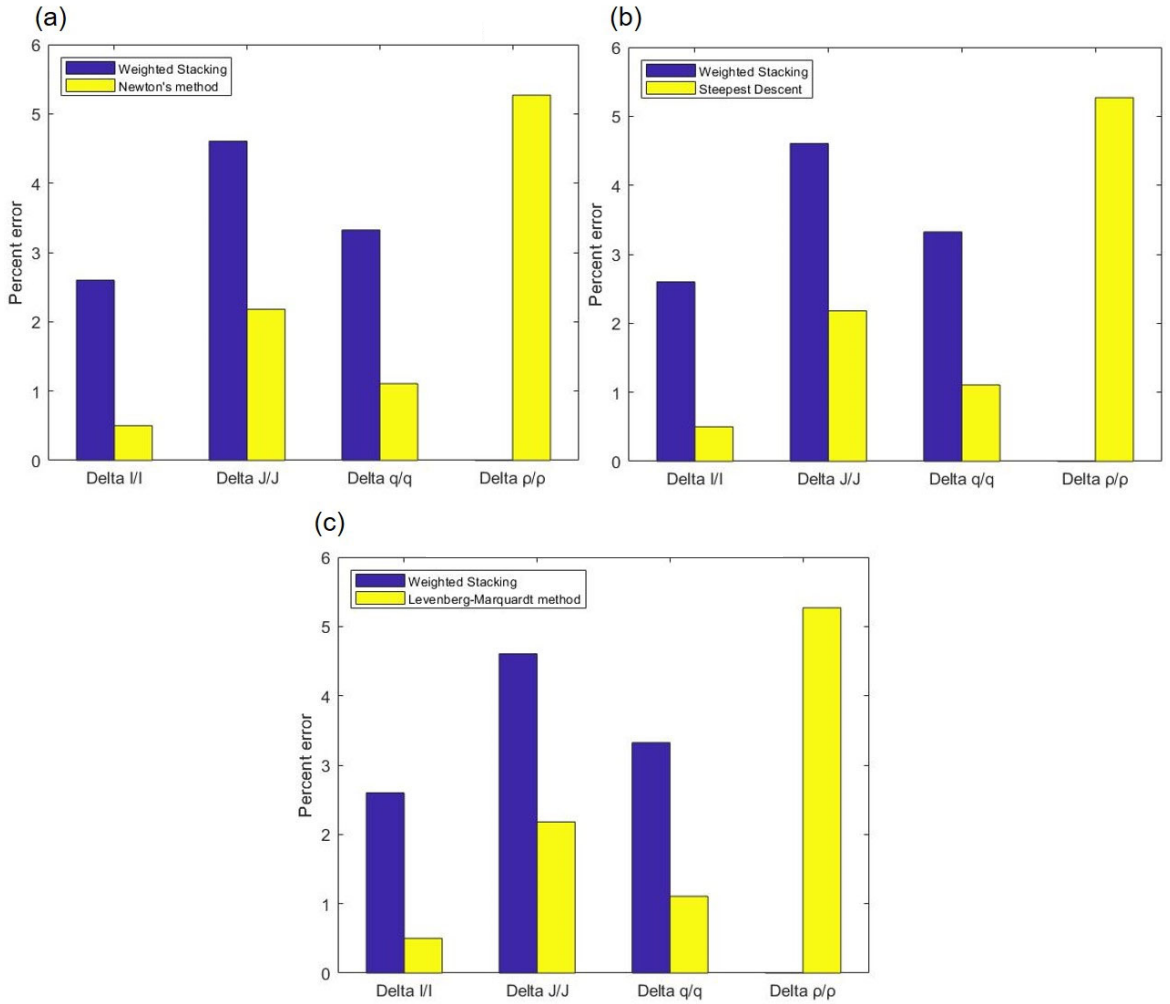
Maximum P-P angle: 53°

Maximum P-S angle: 65°

Results: broadband and noise free reflectivities

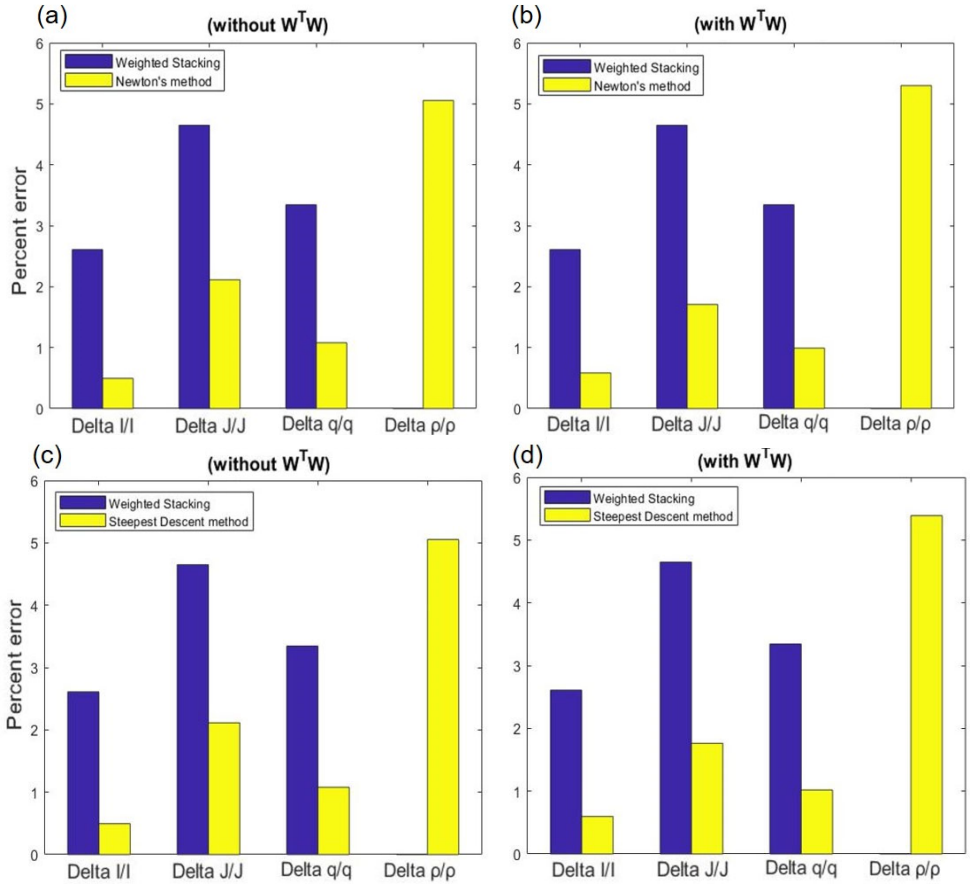


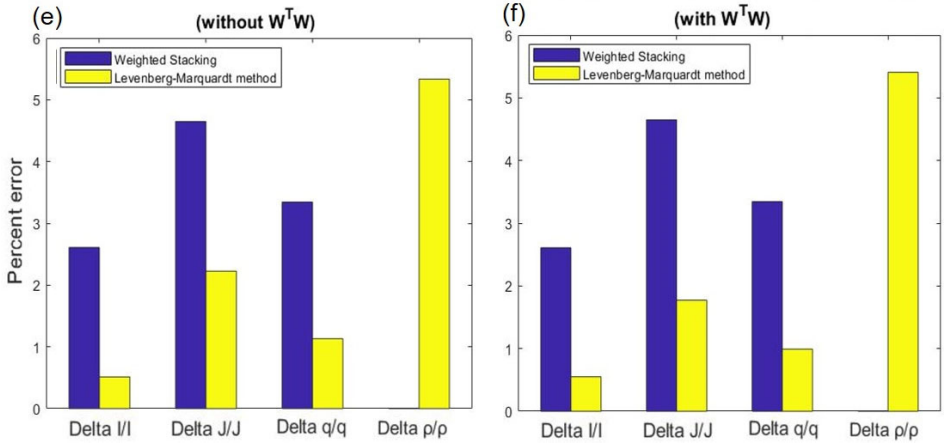
Results: broadband and noise free reflectivities



Results: broadband and noisy reflectivities

IID noise
- P-P SNR: 8
- P-S SNR: 4



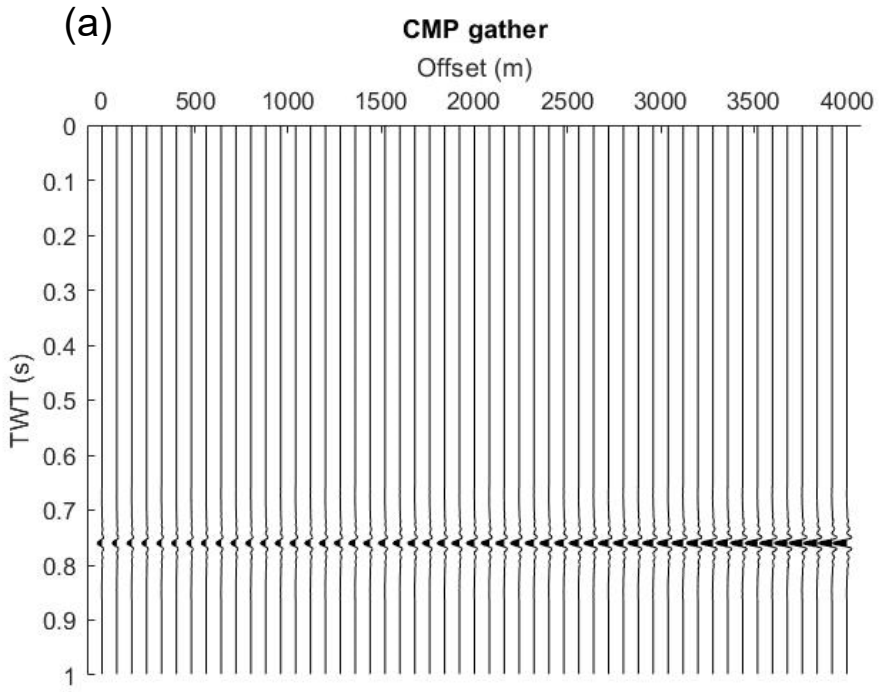


Maximum likelihood solutions

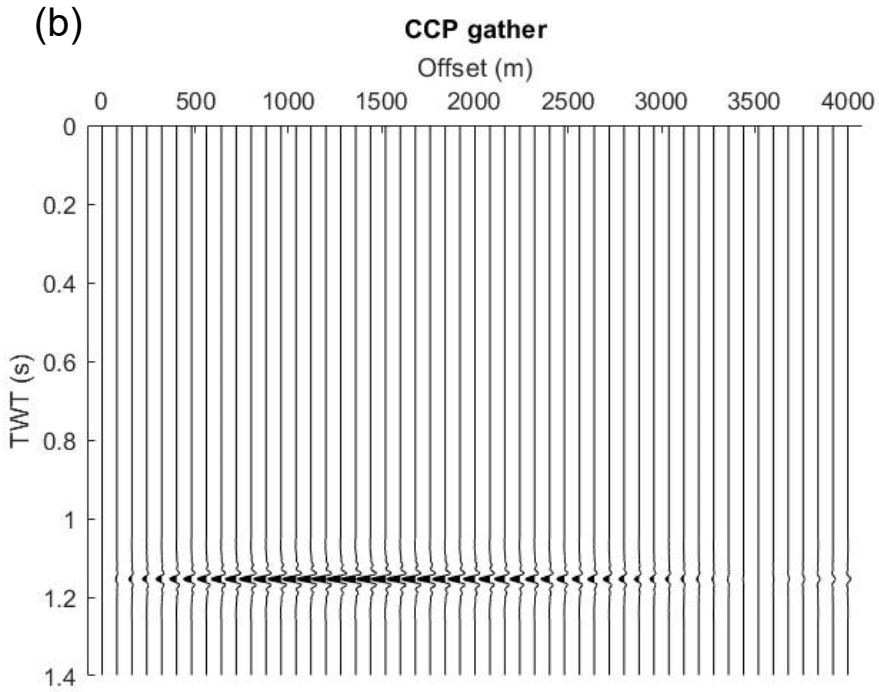
Results: band-limited and noisy reflectivities

Ormsby wavelet
- P-P dataset: → 5-10-60-75Hz
- P-S dataset: → 5-10-40-55Hz

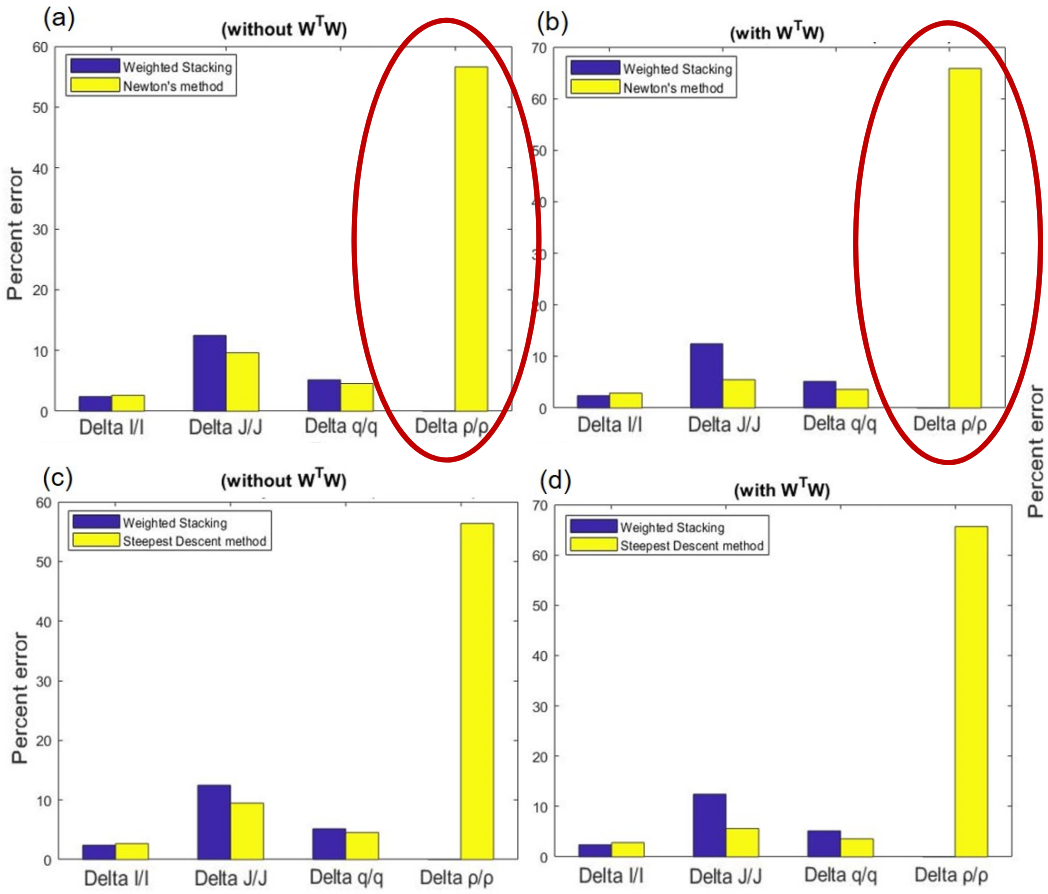
(a)

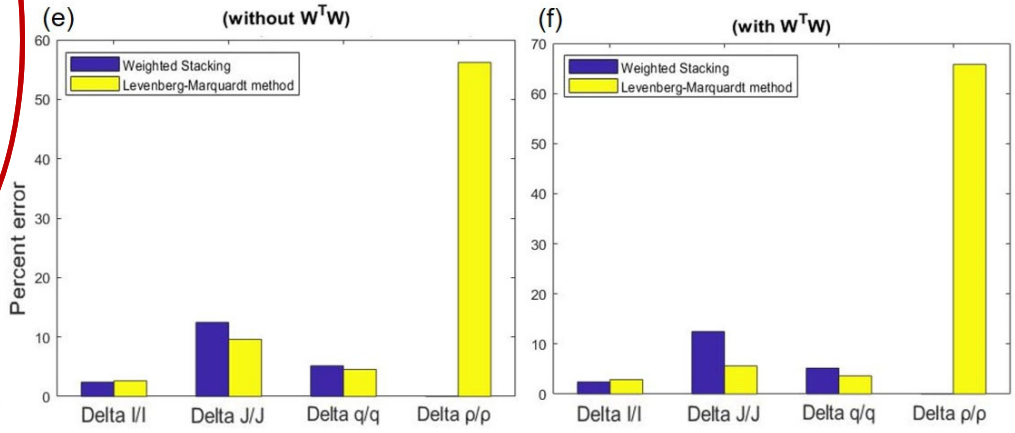


(b)



Results: band-limited and noisy reflectivities

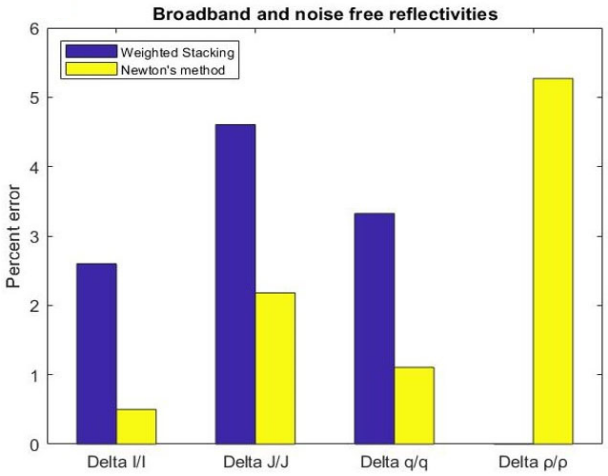


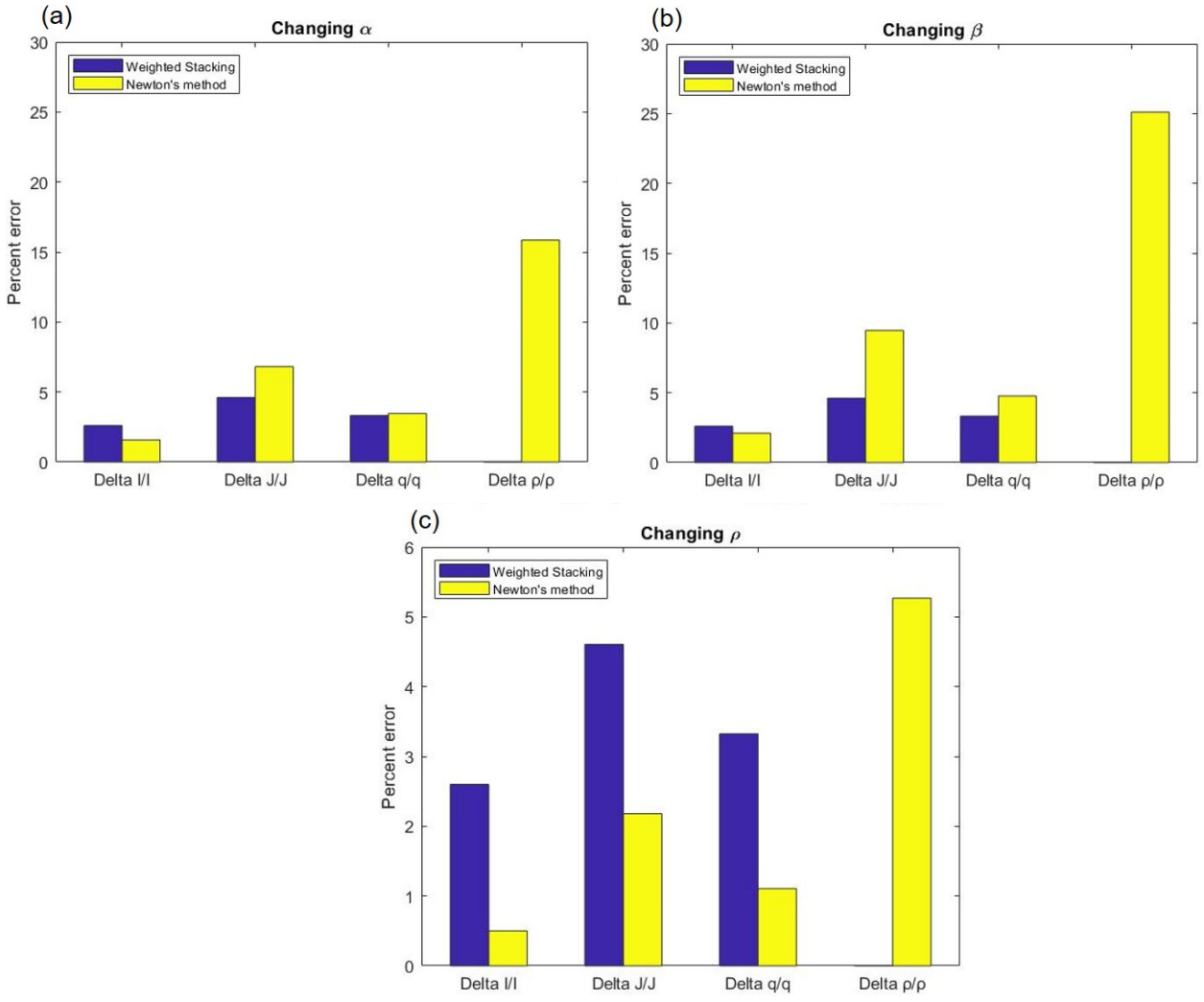


Maximum likelihood solutions

Results: effects of the initial model

Results with original initial model

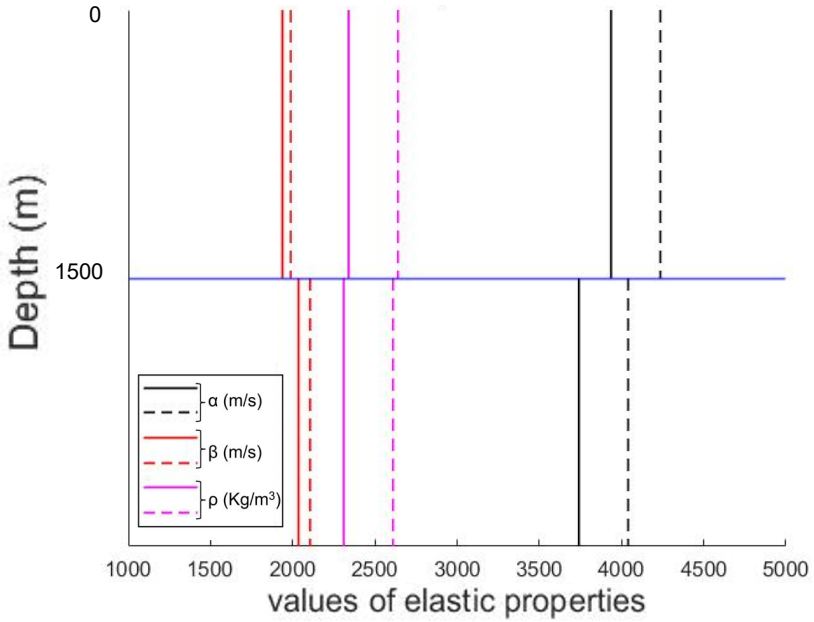


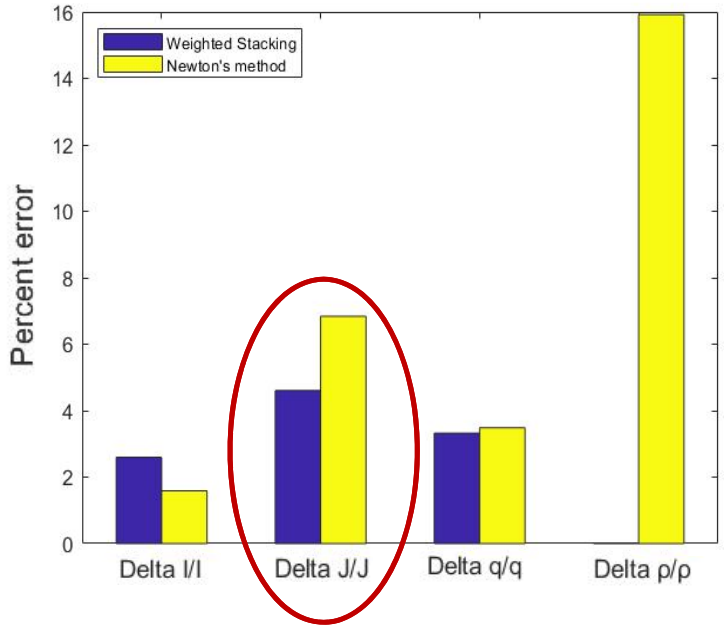


Results: effects of the initial model

Good initial model → Close to true contrasts and relationships between α and β perturbations

Dashed lines: new initial model
Continuous lines: true model





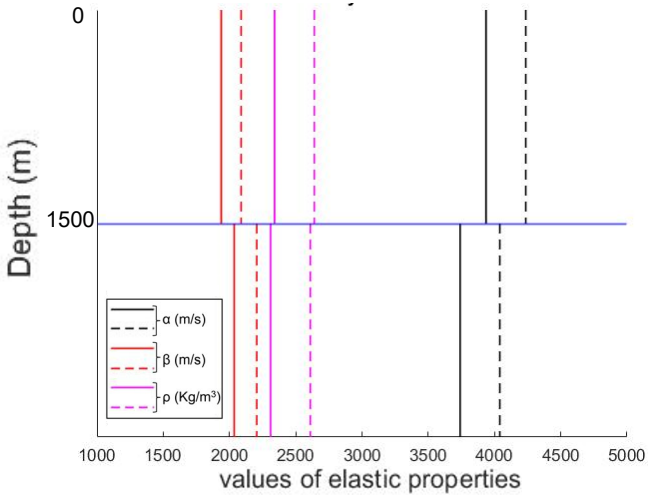
α:β → 6:1

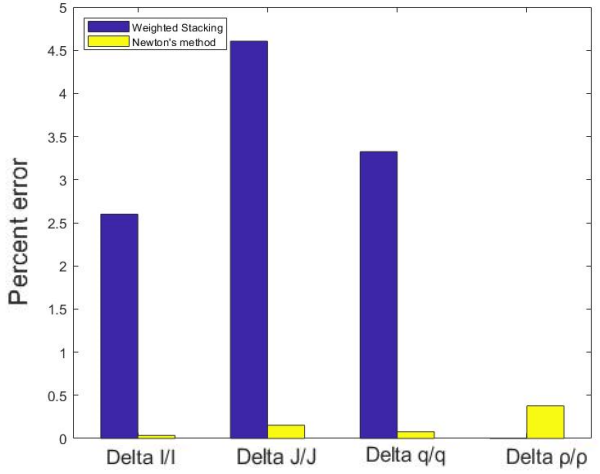
Results: effects of the initial model

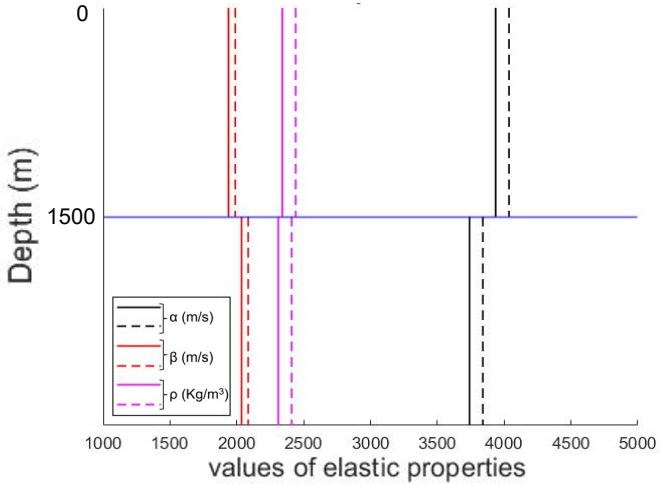
α:β ⟶ 2:1

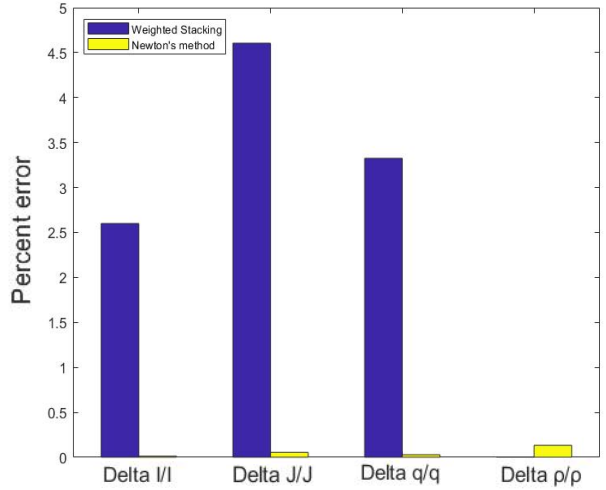
Dashed lines: new initial model

Continuous lines: true model

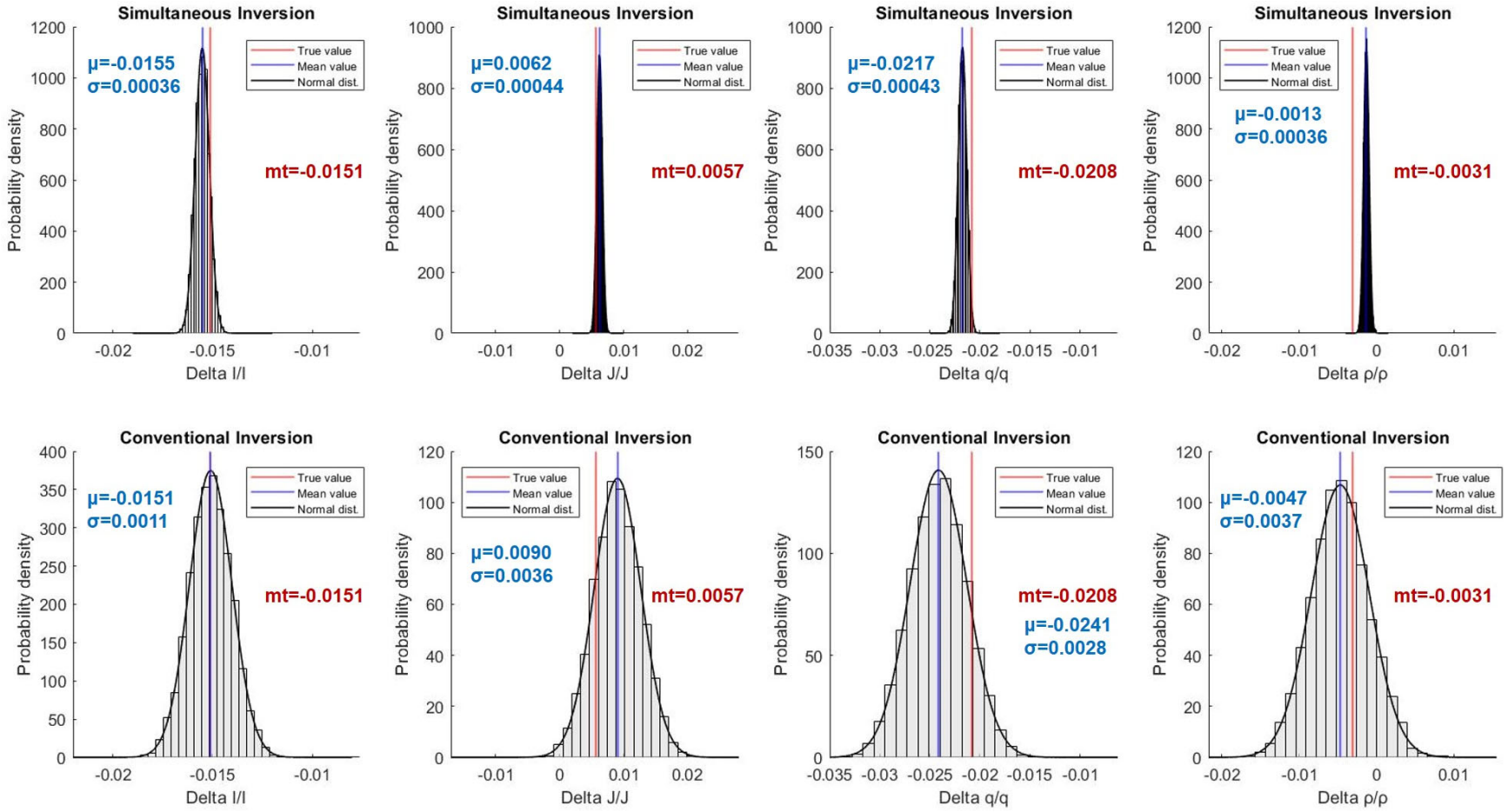








Results: advantages of the simultaneous nonlinear inversion



.

mt:
Band-limited true values

μ:
Maximum-likelihood sol.

σ:
Standard deviation

Conclusions

- Outperformance of the nonlinear inversion over the weighted stacking approach.
- Convergence to the same minimum point with different convergence features.
- Including noise information produced improvements on the fractional shear impedance and fractional Vp/Vs ratio.
- High accuracy and precision of the simultaneous nonlinear inversion results.
- Negative impact on the fractional density.

Acknowledgments

- CREWES sponsors
- CREWES faculty, staff and students
- Natural Science and Engineering Research Council of Canada (CRDPJ 543578-19)

References

- Downton, J. E., Russell, B. H., and Lines, L. R., 2000, AVO for managers: pitfalls and solutions: CREWES Research Report, 12, 46.1–46.21.
- Downton, J. E., and Ursenbach, C., 2005, Linearized AVO inversion with supercritical angles: SEG Technical Program Expanded Abstracts 2005.
- Larsen, J. A., 1999, AVO analysis by simultaneous P-P and P-S weighted stacking: M.Sc. thesis, University of Calgary.
- Liu, Y., and Liu, X., 2016, Application and performances of unconstrained optimization methods in seafloor AVO inversion: Arabian Journal of Geosciences, 9, 1–7.
- Nocedal, J., and Wright, S. J., 2006, Numerical Optimization: Springer.
- Skopintseva, L., Ayzenberg, M., Landrø, M., Nefedkina, T., and Aizenberg, A. M., 2011, Long-offset AVO inversion of PP reflections from plane interfaces using effective reflection coefficients: Geophysics, 76, No. 6, C65–C79.
- Smith, G. C., and Gidlow, P. M., 1987, Weighted stacking for rock property estimation and detection of gas: Geophys. Prosp, 35, 993–1014.
- Stewart, R. R., 1990, Joint P and P-SV inversion: CREWES Research Report, 2.
- Stewart, R. R., Gaiser, J. E., Brown, R. J., and Lawton, D. C., 1999, Converted-wave seismic exploration: A tutorial: CREWES Research Report, 11, 3, 42.