Reducing the influence of remnant noises on FWI with misfit modification

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Department of Geoscience

Objectives

Normal Distribution Probability Density Function

$$L\left(\mathbf{d}|\mathbf{m}\right) = \frac{1}{\sqrt{(2\pi)^N|\mathbf{C_d}|}} \exp\left(-\frac{1}{2}\boxed{\mathbf{r}^T\mathbf{C}_\mathbf{d}^{-1}\mathbf{r}}\right)$$

Misfit function in FWI

$$E = \left[\underbrace{\left(\mathbf{d}_{\text{pre}} - \boxed{\mathbf{d}_{\text{obs}}}\right)}_{\mathbf{r}^T}{}^T \underbrace{\left(\mathbf{d}_{\text{pre}} - \boxed{\mathbf{d}_{\text{obs}}}\right)}_{\mathbf{r}}\right].$$

$\mathbf{d}_{\text{obs}}$ as “true data”, but actually noisy

Modified misfit function

$$E = \frac{1}{2N}\left[\left(\mathbf{d}_{\mathrm{pre}} - \mathbf{d}_{\mathrm{obs}}\right)^{T} \mathbf{C}_{\mathrm{D}}^{-1} \left(\mathbf{d}_{\mathrm{pre}} - \mathbf{d}_{\mathrm{obs}}\right)\right].$$

The gradient of the objective function

$$\frac{\partial \mathrm{E}(\mathbf{m})}{\partial m_i} = \frac{1}{N}\left(\frac{\partial \mathbf{d}_{\mathrm{pre}}}{\partial m_i}\right)^{T} \boxed{\mathbf{C}_{\mathrm{D}}^{-1} \left(\mathbf{d}_{\mathrm{pre}} - \mathbf{d}_{\mathrm{obs}}\right)}.$$

The artifacts generated by matching seismic data with low SNR will be down weighted, improving the quality of imaging.

Methodology

1 Estimation of data covariance matrix

$$c_j = \frac{1}{N} \sum_{k=0}^{N-j-1} \left(\mathbf{d}_{j+k} - \bar{\mathbf{d}}\right) \left(\mathbf{d}_k - \bar{\mathbf{d}}\right) .$$

Inversion results of the 1st FWI ≈ Sample mean $\bar{r}$

2 Dimension of covariance matrix

Time domain:

(Nt*Nr*Ns)*(Nt*Nr*Ns)

Frequency domain:

(Nf*Nr*Ns)*(Nf*Nr*Ns)

Introduction to tests

Research questions:

How noises can influence the FWI results ?

How modified misfit function can improve the inversion result ?

Acoustic FWI in time domain

Elastic FWI in frequency domain

Random noise
Correlated noise

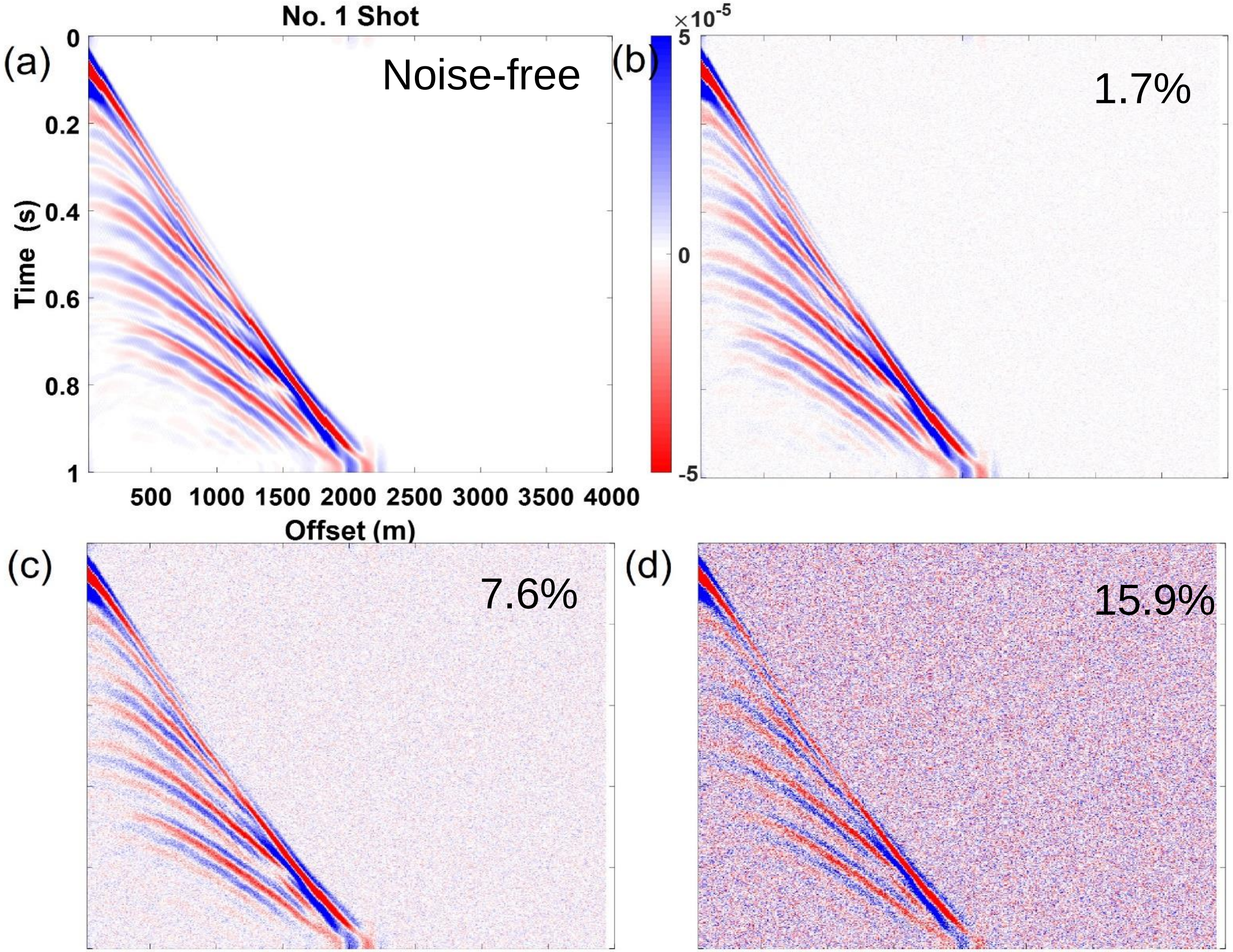
Acoustic FWI in time domain

Random noise

Correlated noise

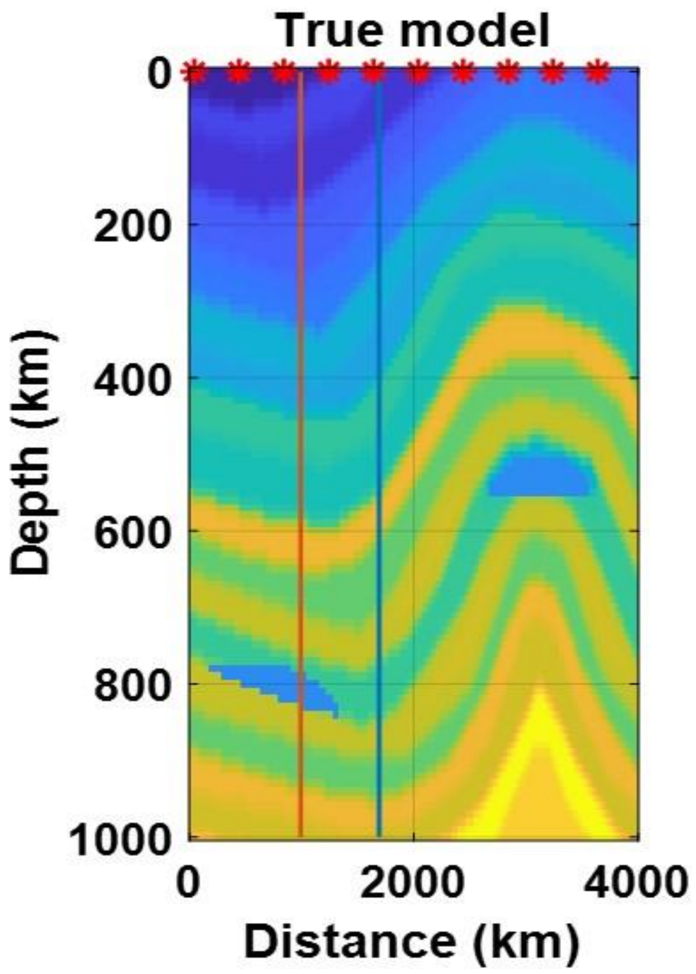
Acoustic FWI in time domain

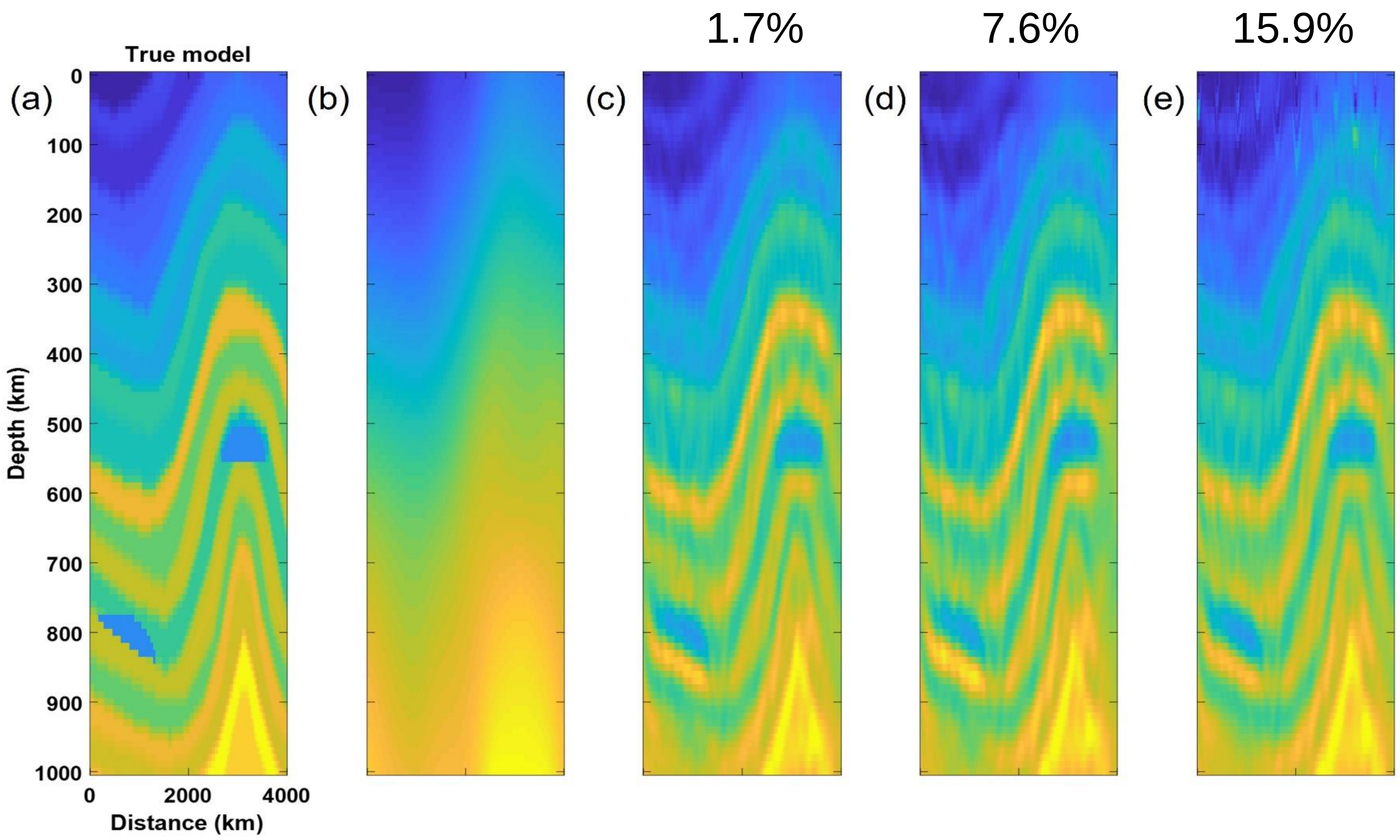
Seismic data with random noises (d_obs)



Acoustic FWI in time domain

Inversion results





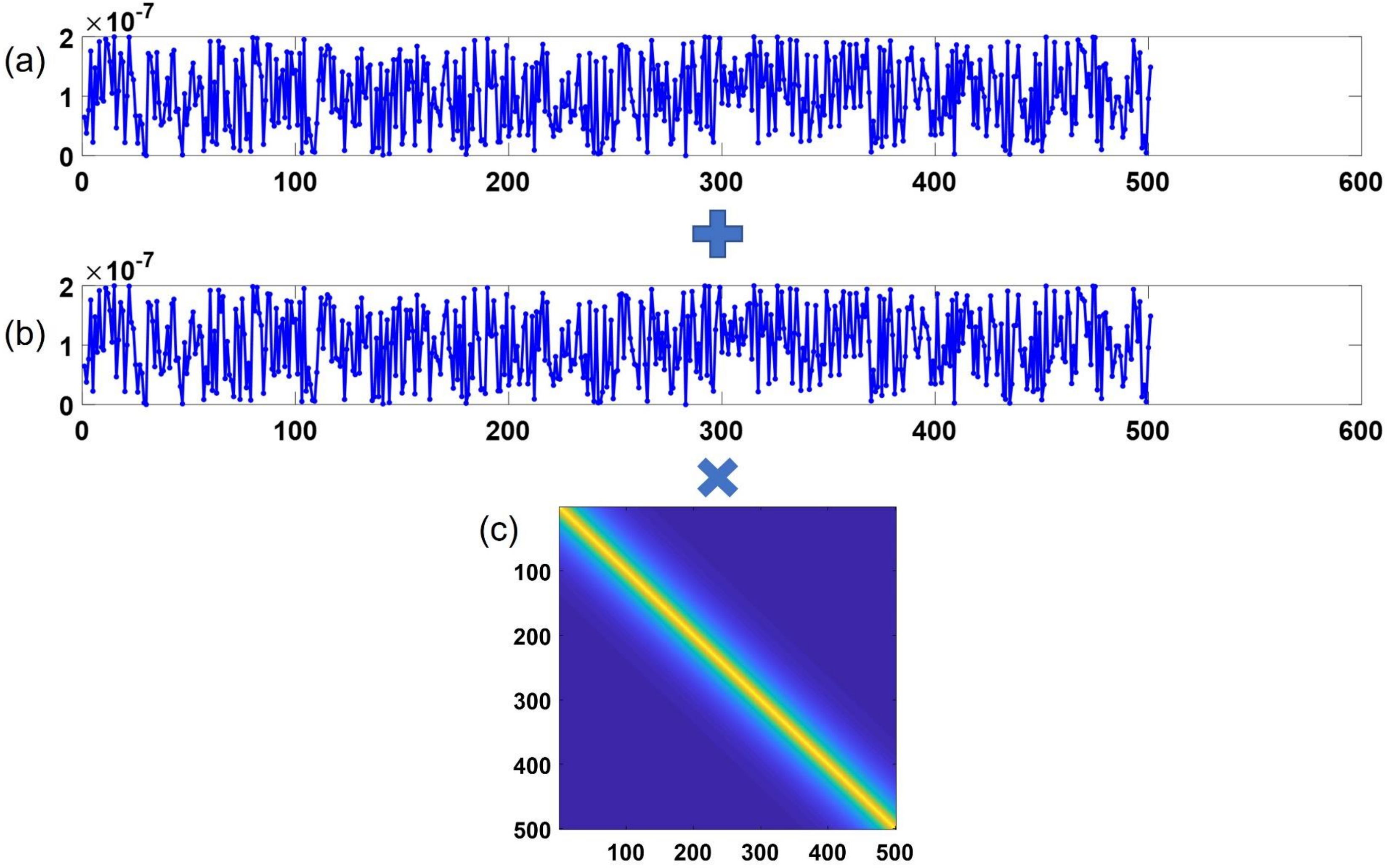
Acoustic FWI in time domain

Random noise

Correlated noise

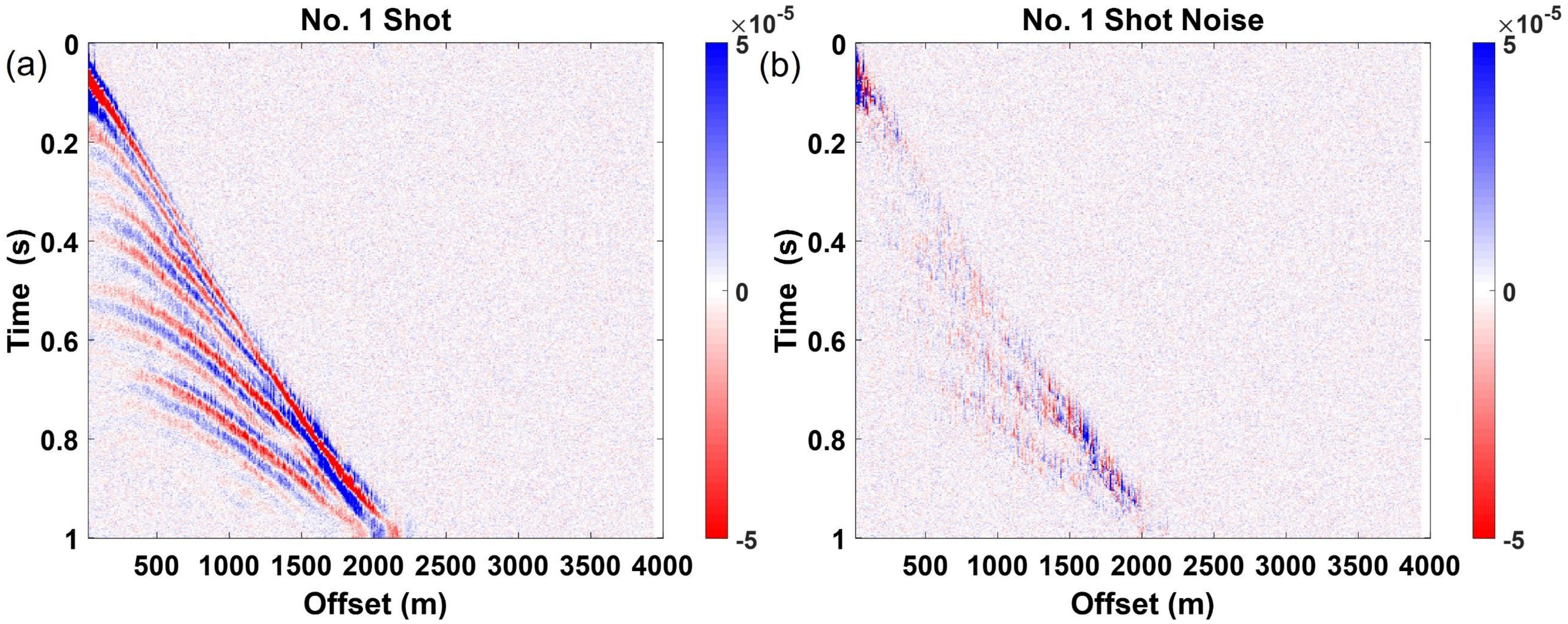
Acoustic FWI in time domain

Generation of correlated noise



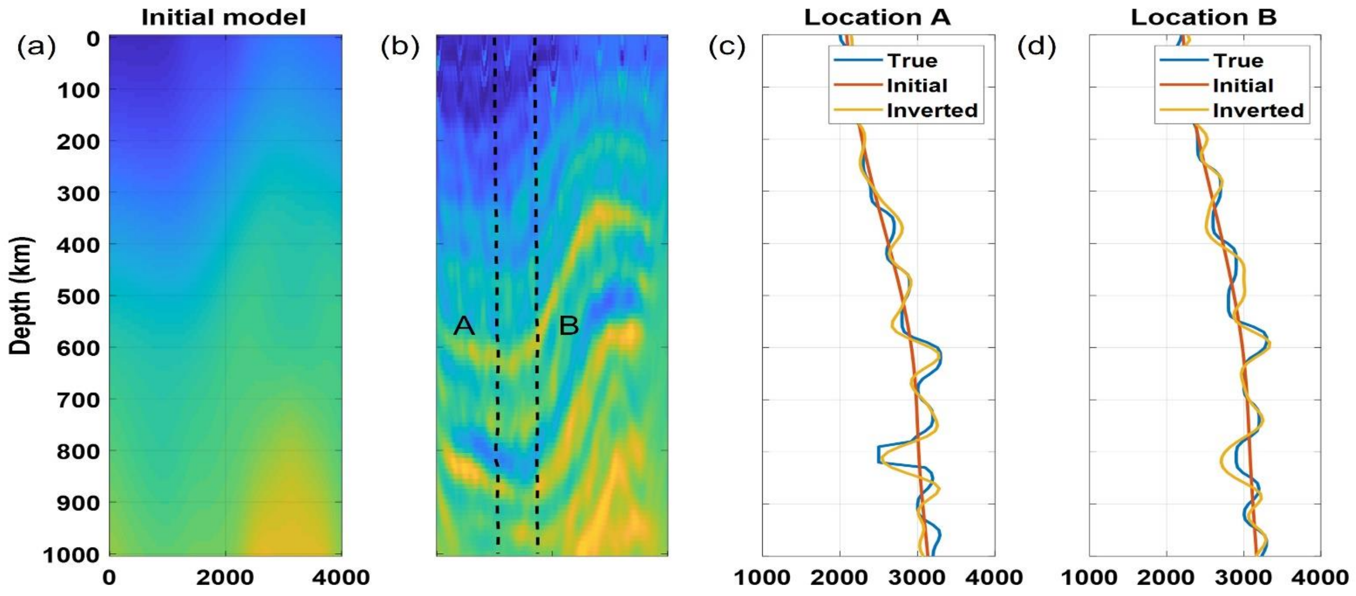
Acoustic FWI in time domain

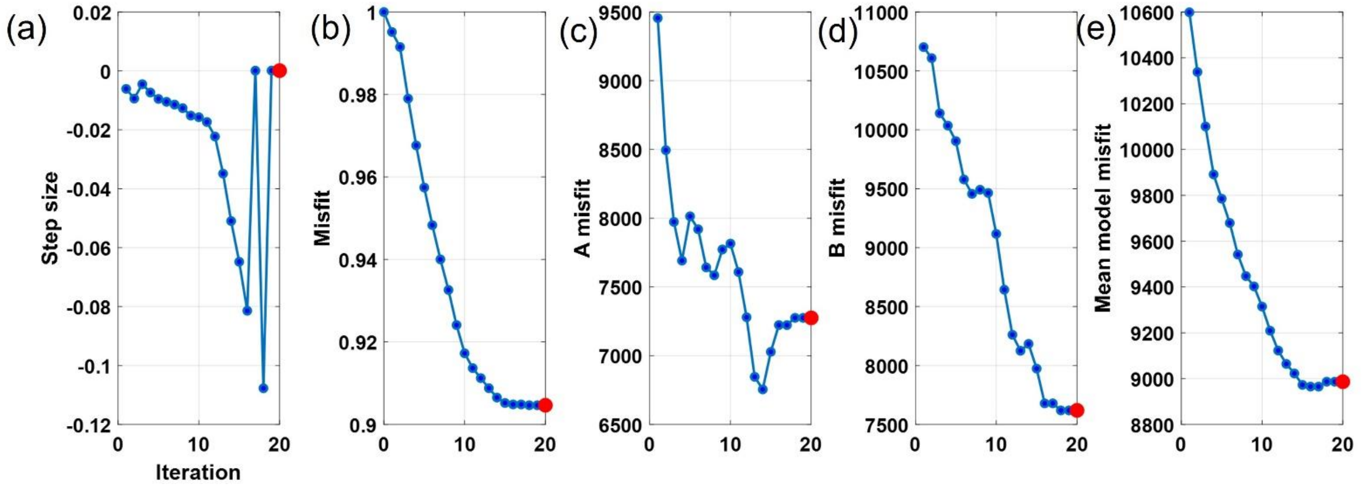
Seismic data with correlated noises



Acoustic FWI in time domain

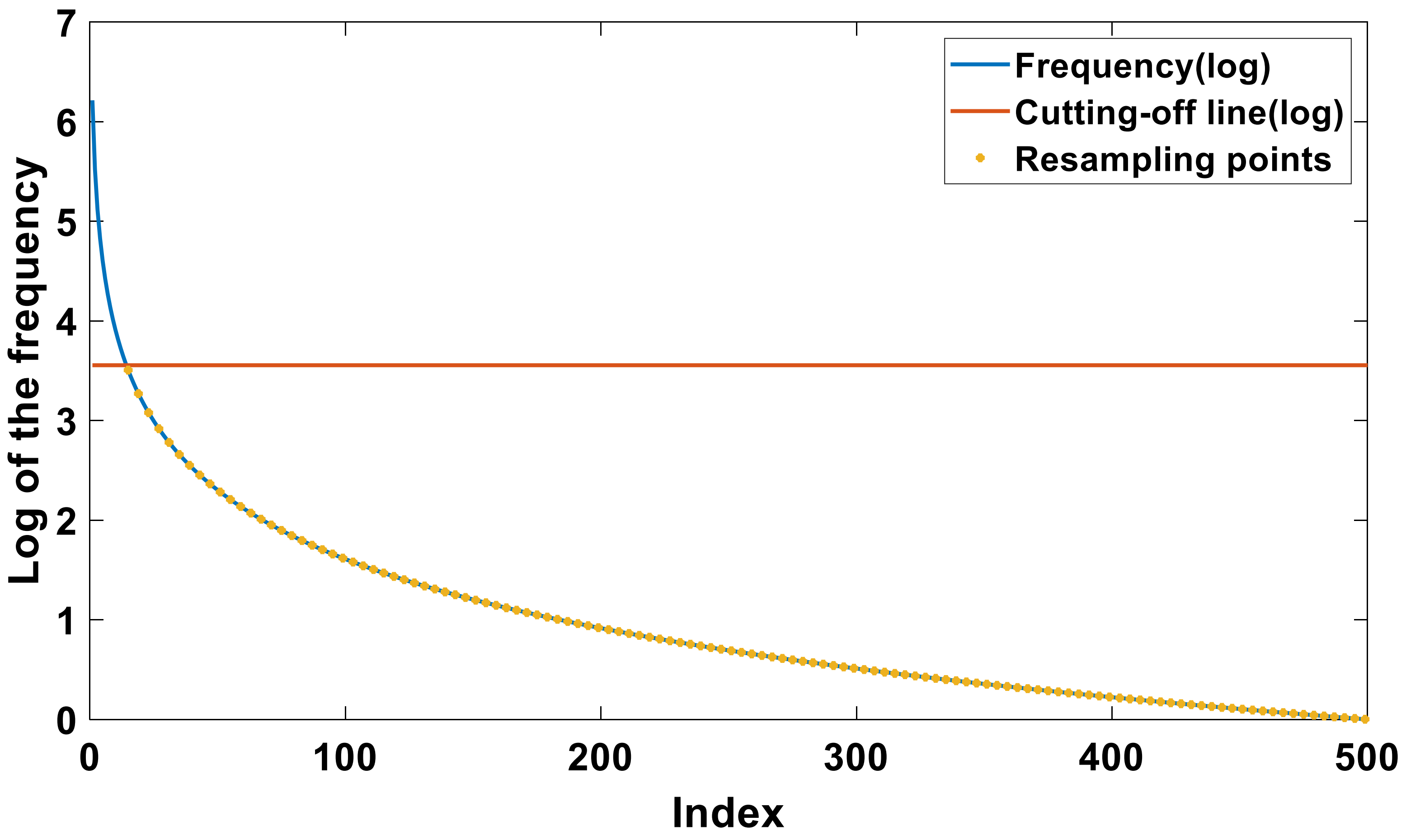
1st FWI inversion result





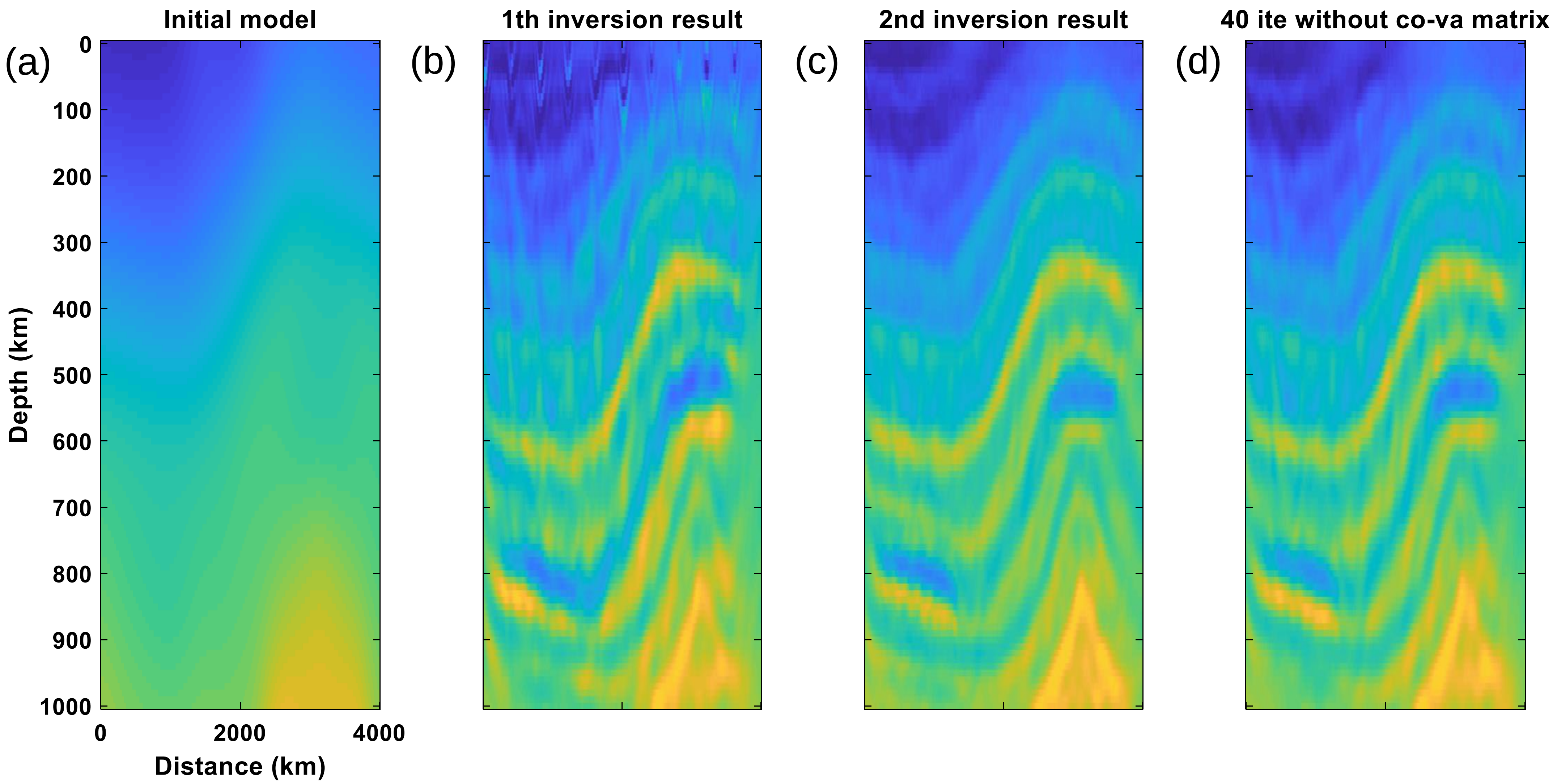
Acoustic FWI in time domain

Resampling in frequency domain



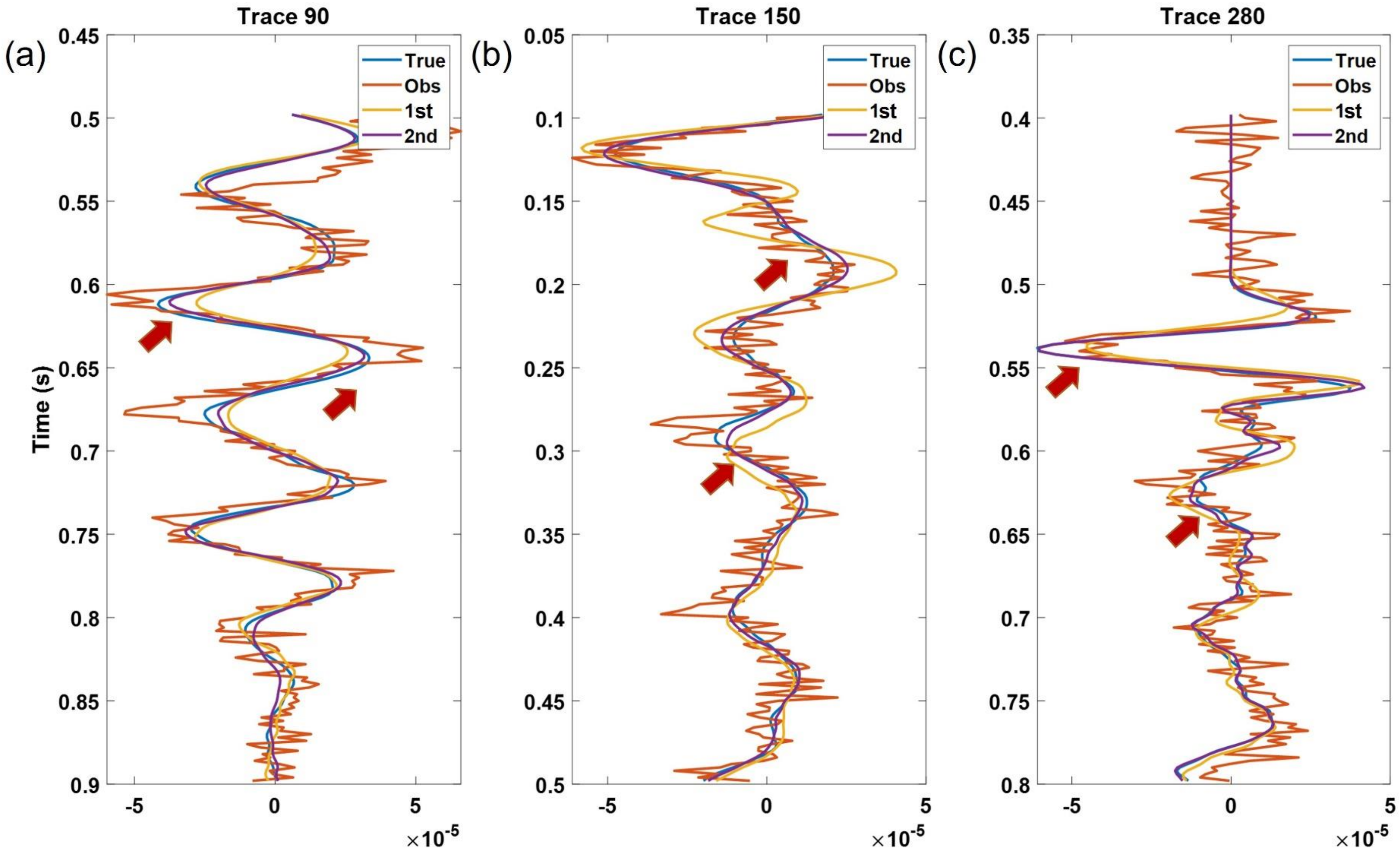
Acoustic FWI in time domain

2nd FWI inversion result



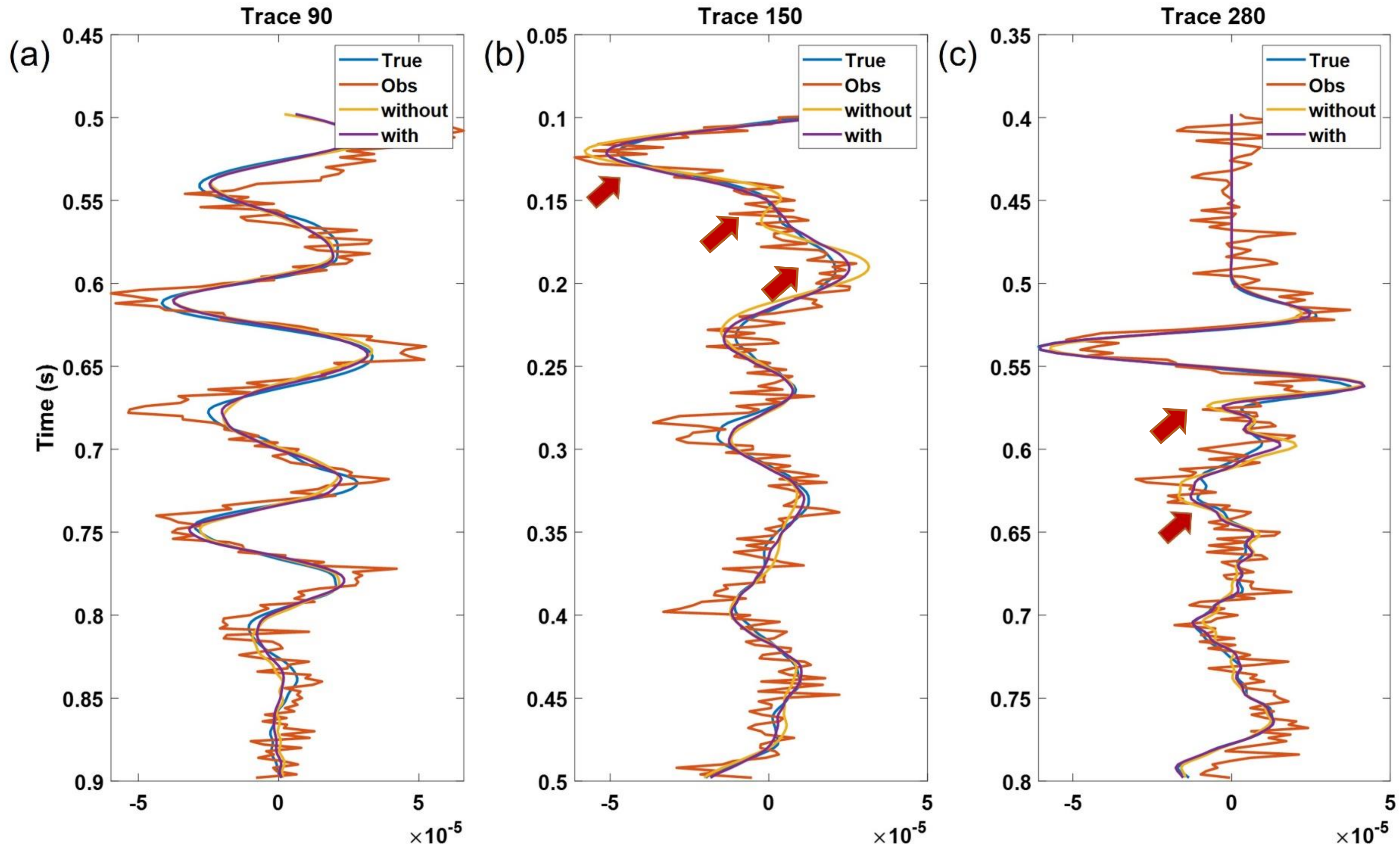
Acoustic FWI in time domain

Trace comparison for the 1st inversion



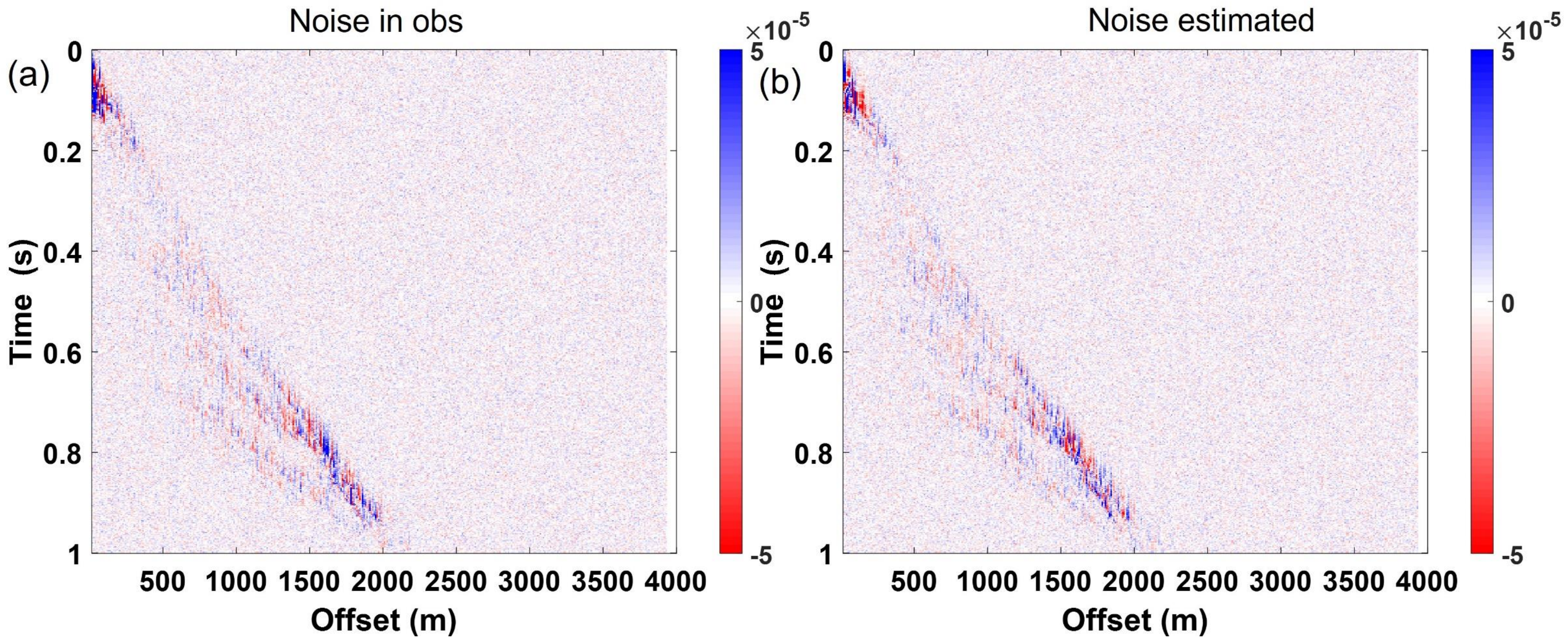
Acoustic FWI in time domain

Trace comparison for the 2nd inversion



Acoustic FWI in time domain

True noise and estimated noise



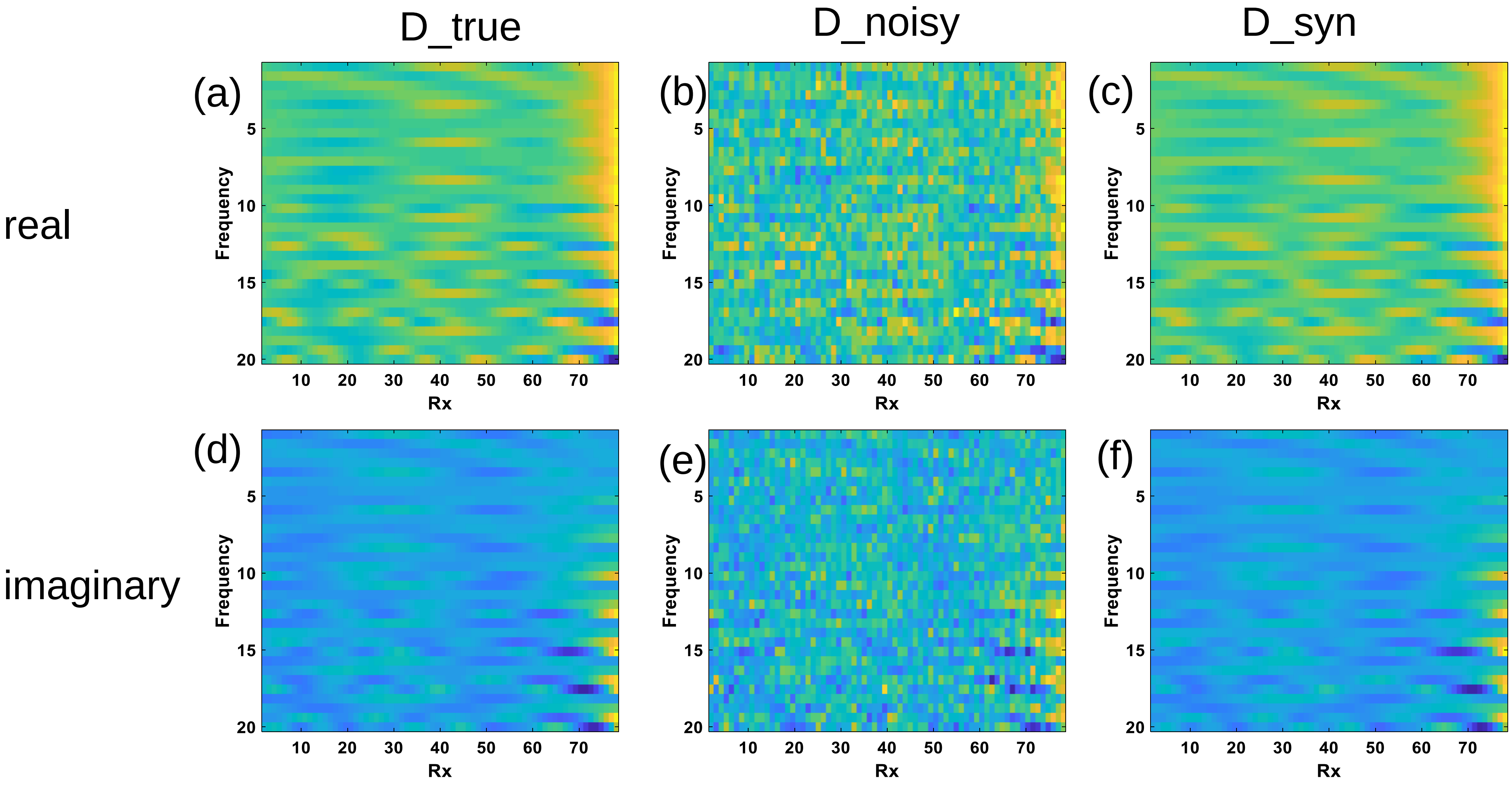
Elastic FWI in frequency domain

Random noise

Correlated noise

Elastic FWI in frequency domain

Seismic data with random noises (d_obs)



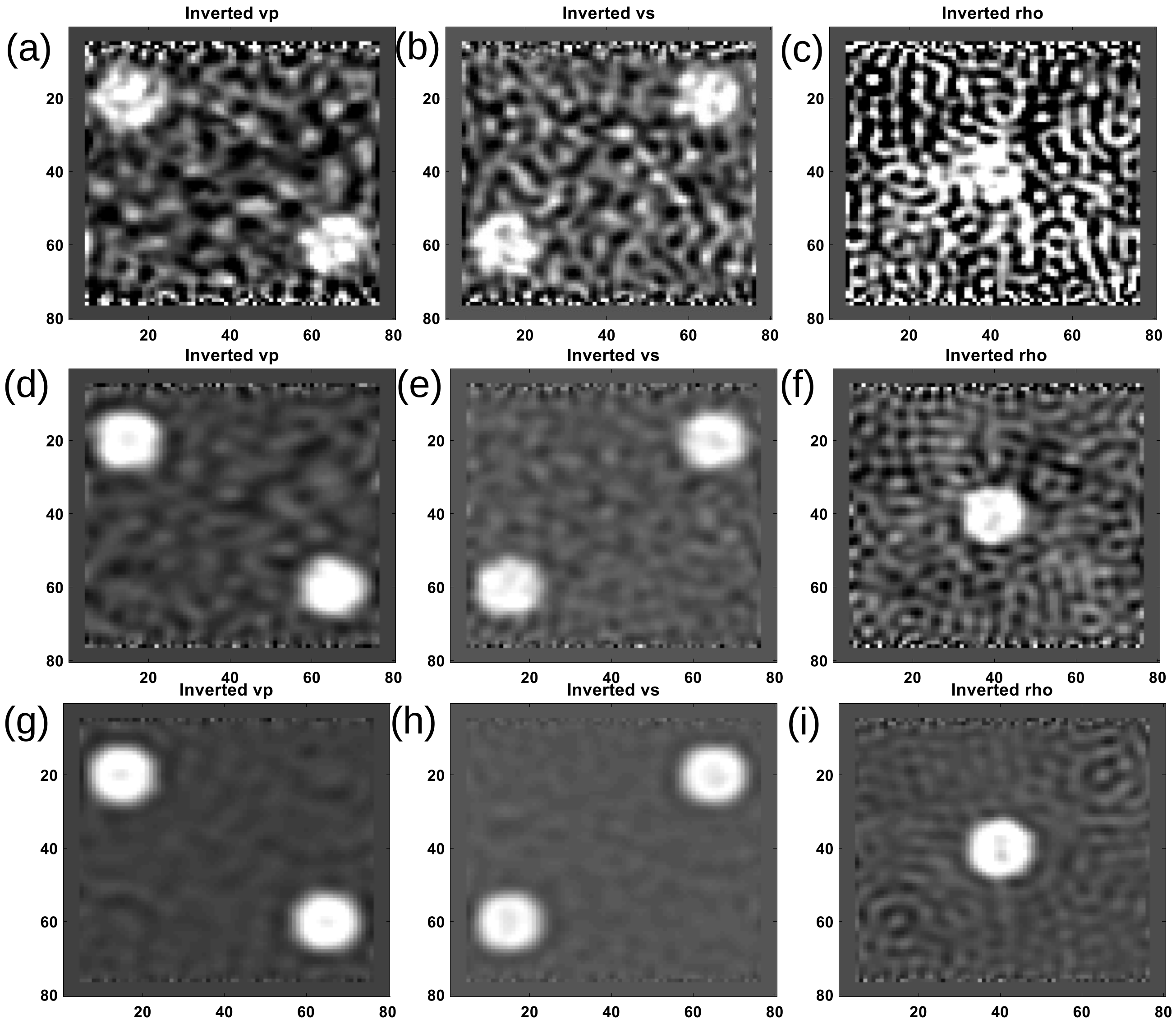
Elastic FWI in frequency domain

Inversion result of seismic data with different SNR (random noises)

SNR=10dB

SNR=20dB

SNR=30dB



Elastic FWI in frequency domain

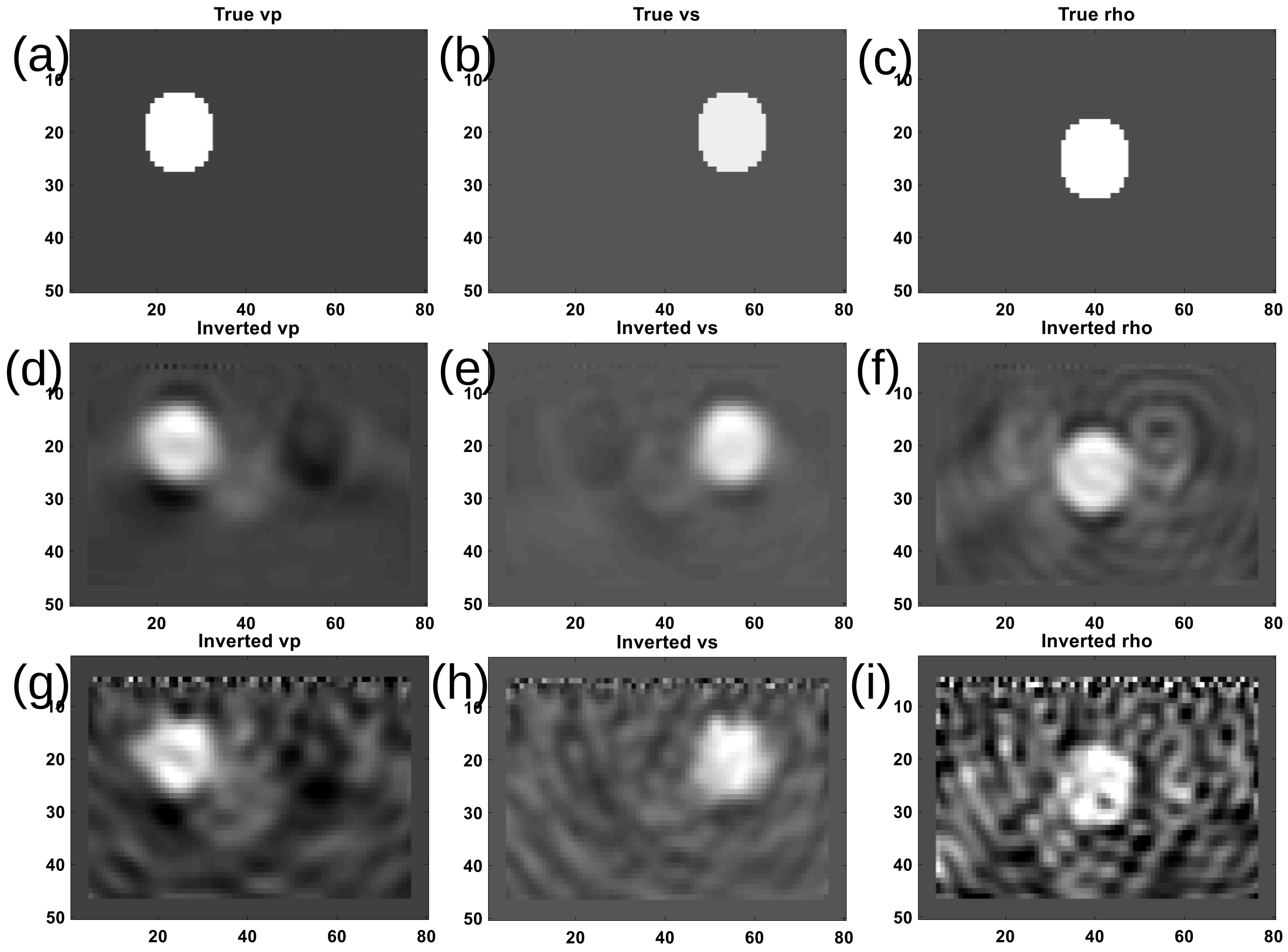
1st FWI inversion result

Vp | Vs | Density

True model

Inverted model of noise-free data

Inverted model of data with noisy data SNR=16dB

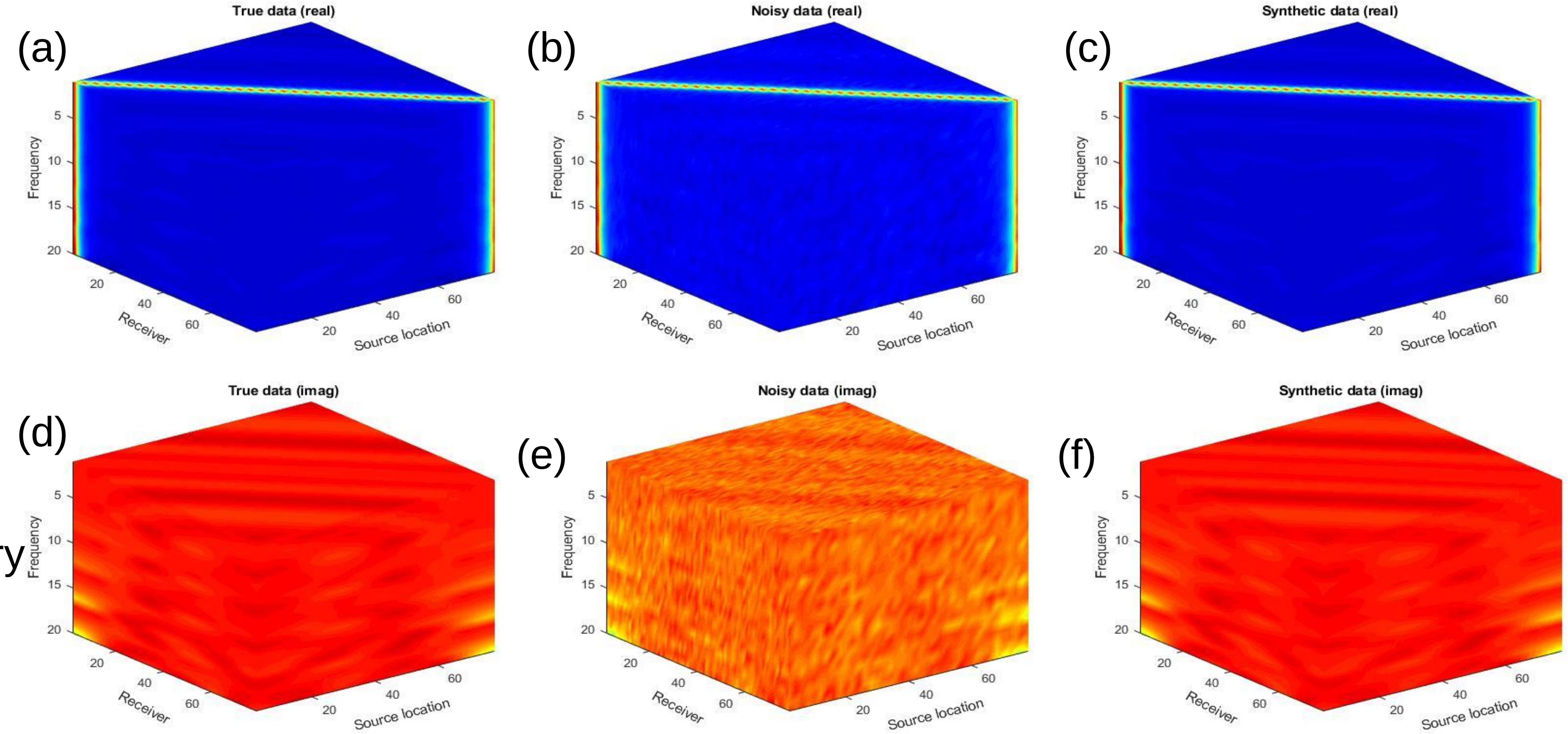


Elastic FWI in frequency domain

Seismic data in frequency domain

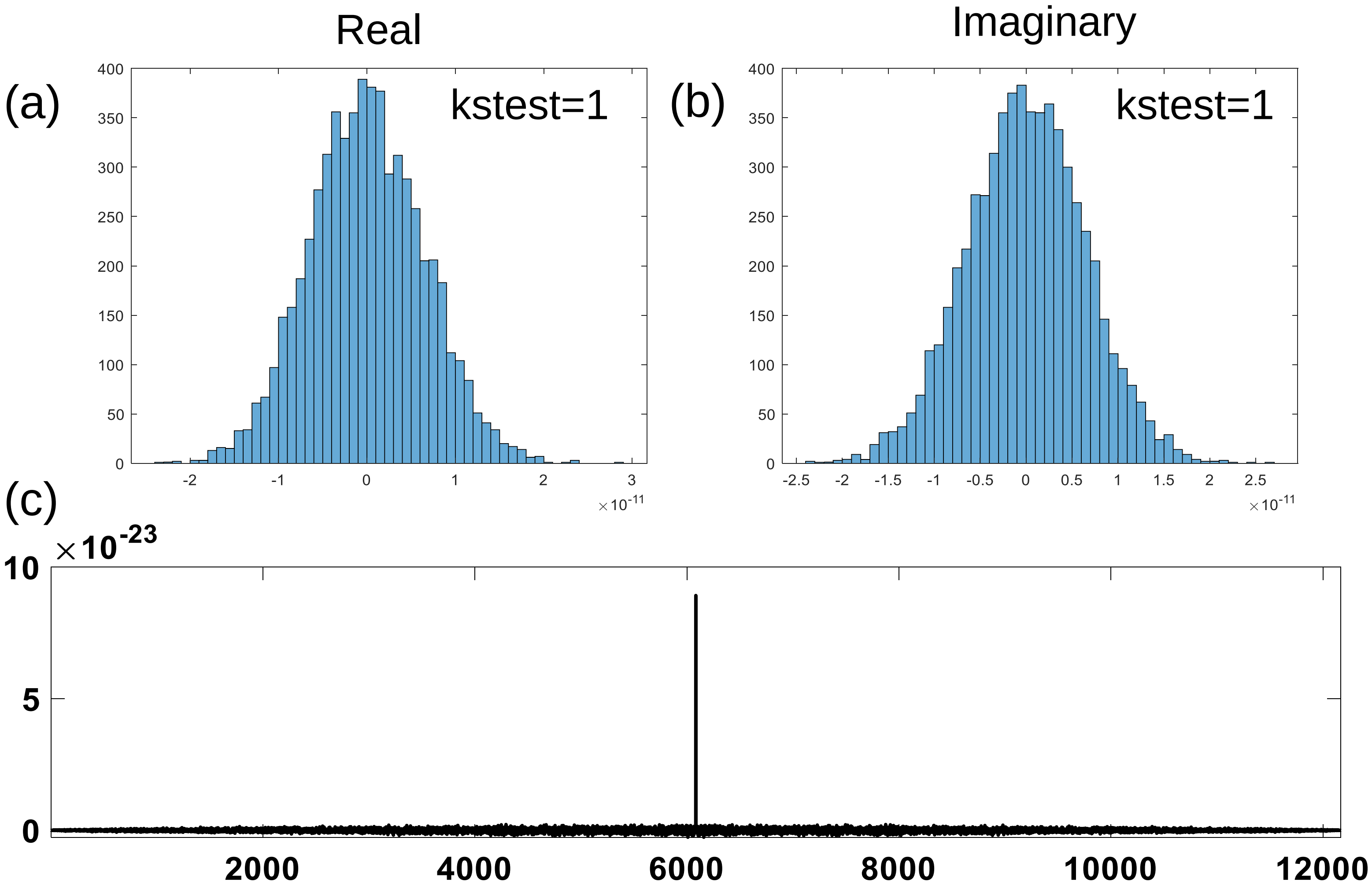
Real

Imaginary



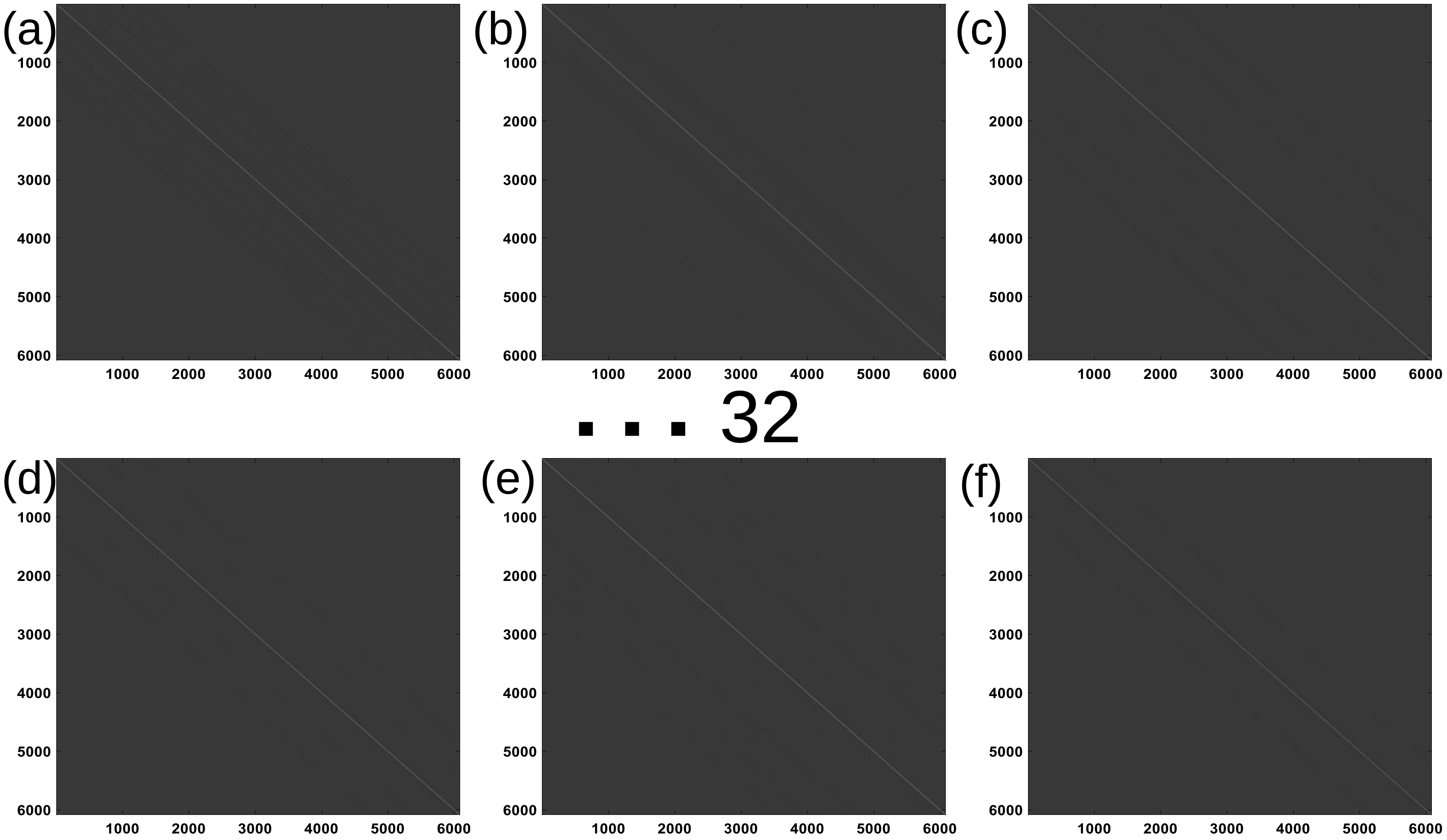
Elastic FWI in frequency domain

Data residuals



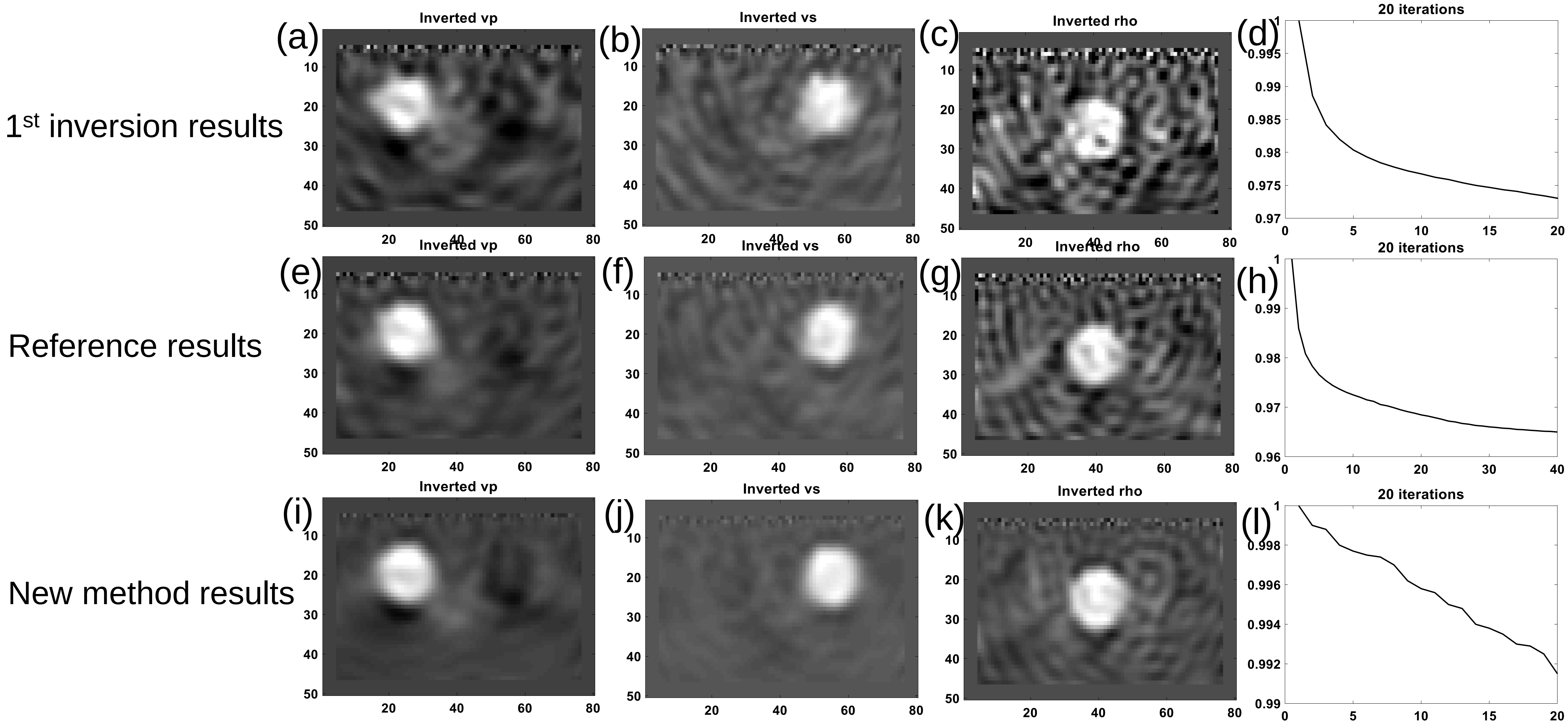
Elastic FWI in frequency domain

Data covariance matrices



Elastic FWI in frequency domain

Inversion results



Elastic FWI in frequency domain

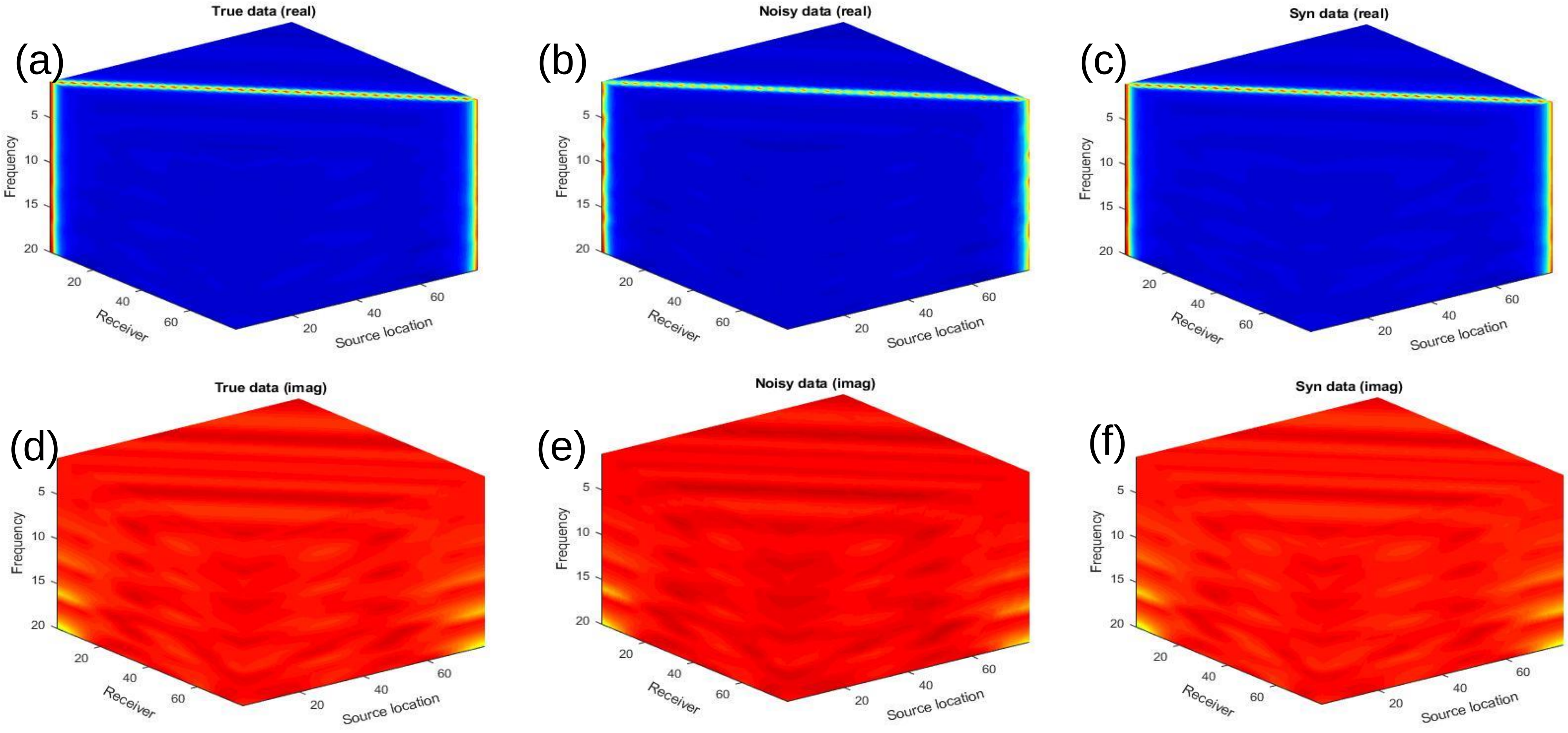
Random noise

Correlated noise

Seismic data in frequency domain

Real

Imaginary



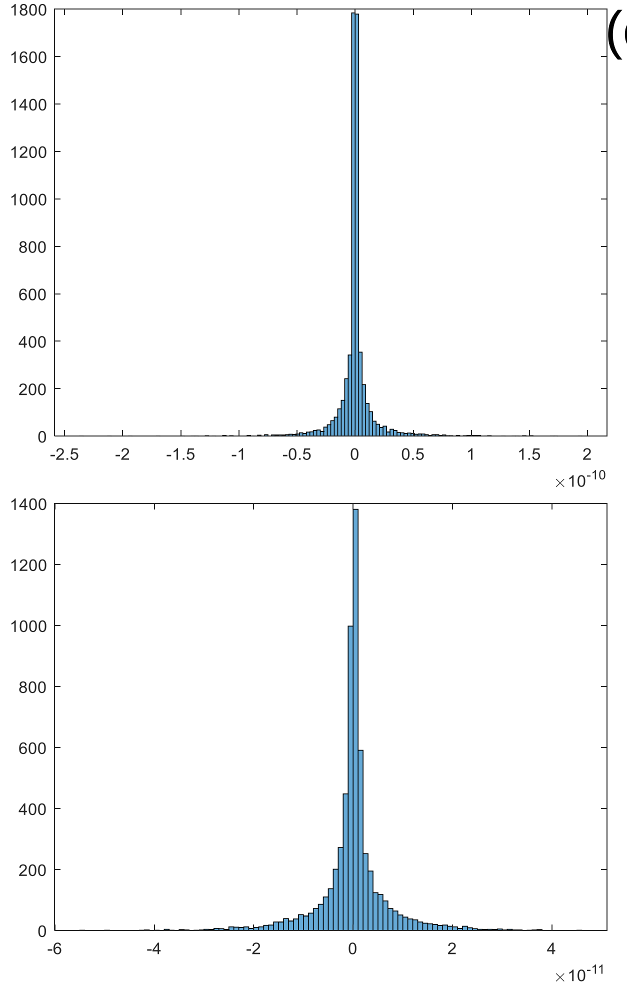
Data residuals and data covariance matrices

Real

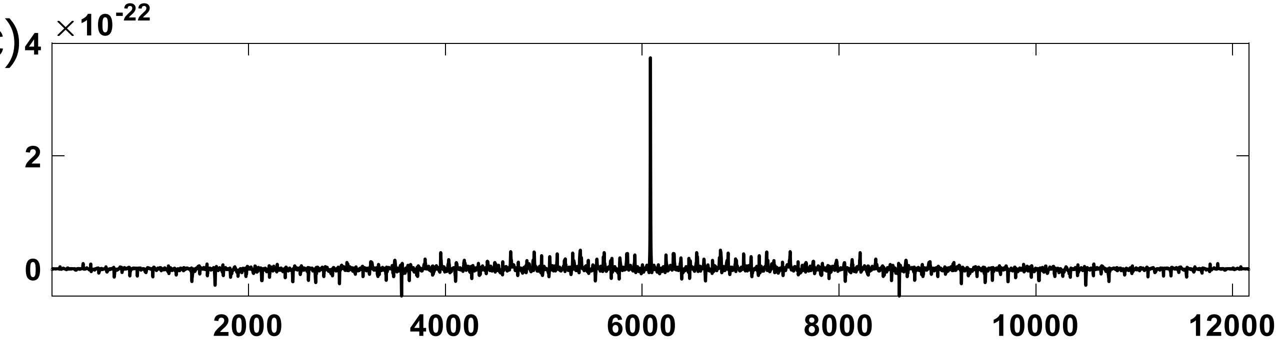
(a)

Imaginary

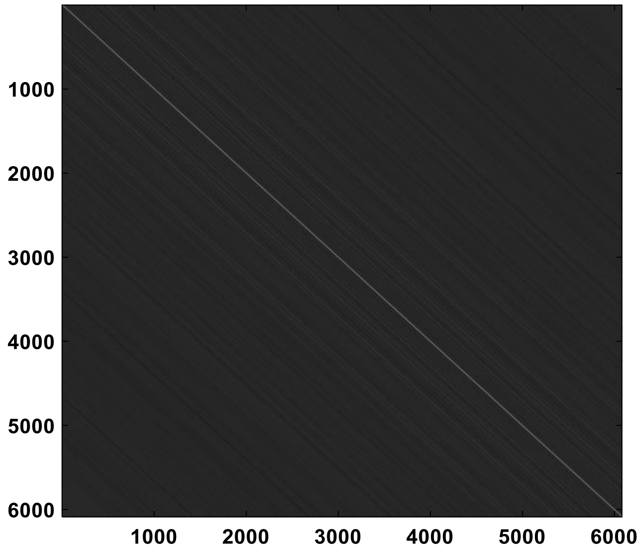
(b)



(c)



(d)



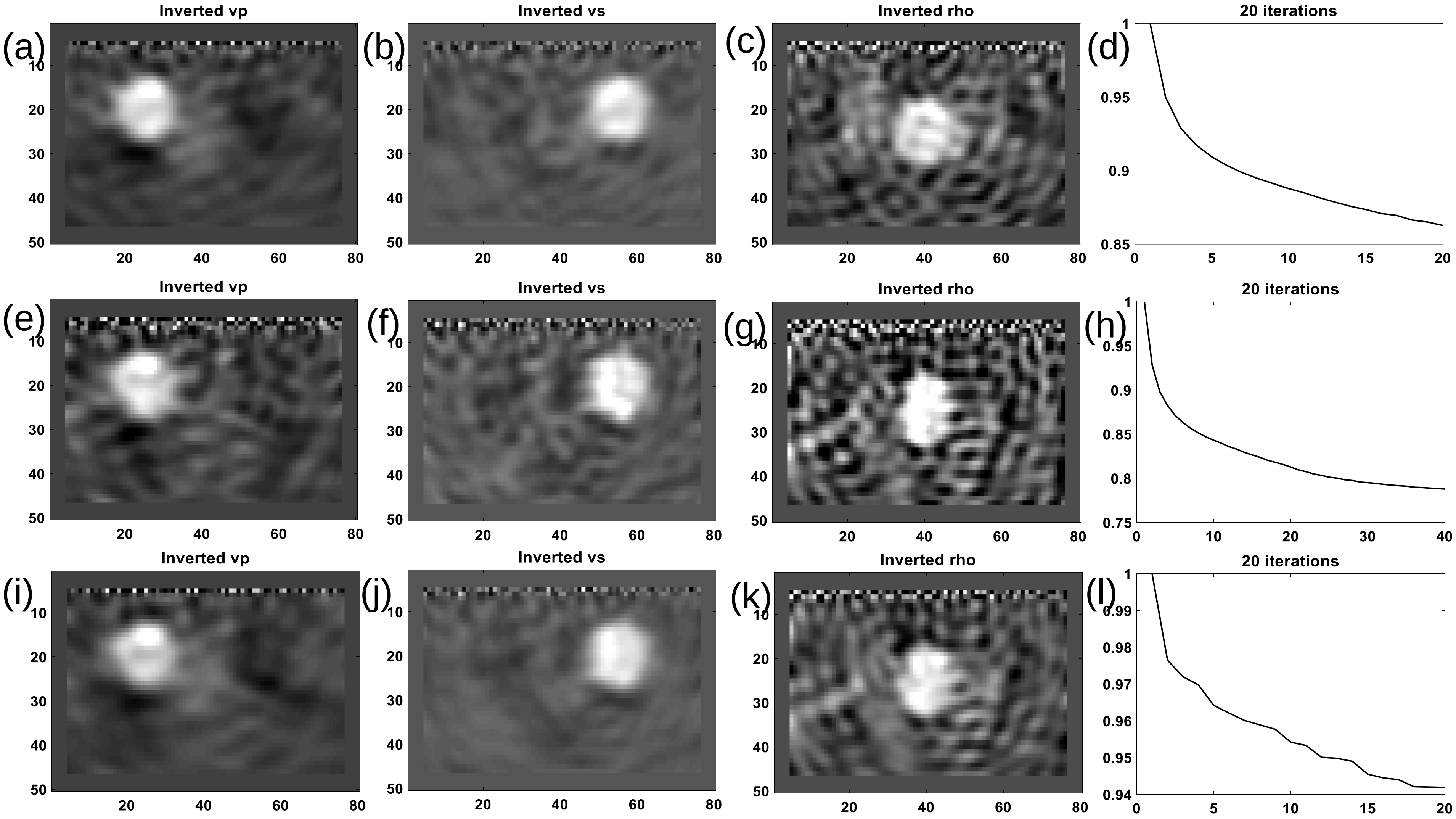
Elastic FWI in frequency domain

Inversion results

1st inversion results

Reference results

New method results



Conclusions

- Inversion algorithm is more resistant to the influence of random noises.
- The influence can be ignored if the random noise magnitude is within 20 percent of the true data.
- FWI with modified misfit can better estimate the random and correlated noises in the observing data by incorporating the data covariance matrix.
- The new method can yield better imaging with less artifacts.

Further study

- Field data tests need to be conducted to examine the effectiveness of the method.
- Local minimum.

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SEG

CREWES faculty, staff and students

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Methodology

Modified misfit function

$$E = \frac{1}{2N}\left[\left(\mathbf{d}_{\text{pre}} - \mathbf{d}_{\text{obs}}\right)^{T} \mathbf{C}_{\mathrm{D}}^{-1}\left(\mathbf{d}_{\text{pre}} - \mathbf{d}_{\text{obs}}\right)\right].$$

The gradient of the objective function

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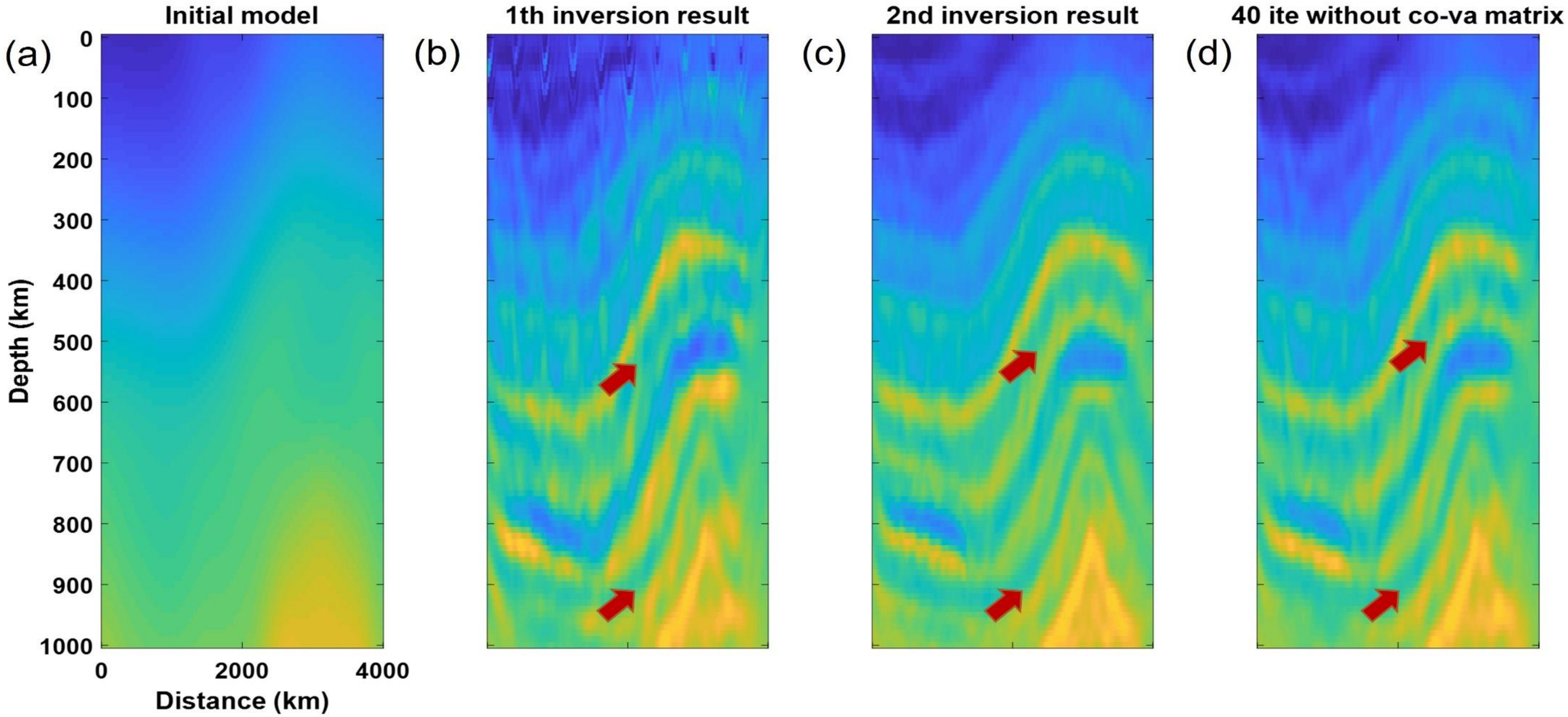
The artifacts generated by matching seismic data with low SNR will be down weighted, improving the quality of imaging.

Acoustic FWI in time domain

2nd FWI inversion result



Methodology

Modified misfit function

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