A robust source-independent full-waveform inversion

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University of Calgary
CREWES Sponsors Meeting
Dec 1, 2022

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Department of Geoscience

Outline

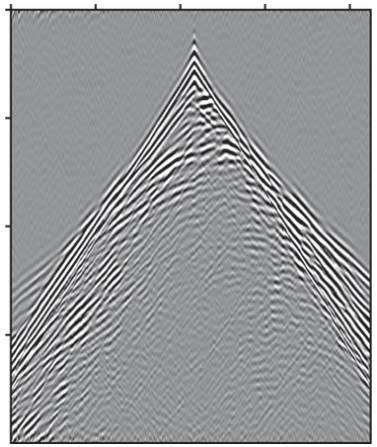
- Introduction
- A robust source-independent FWI
- Numerical example
- Conclusions
- Discussion

FWI

Objective function: $E(\mathbf{m}) = \frac{1}{2}(\mathbf{d}_{syn}(\mathbf{m}) - \mathbf{d}_{obs})^T(\mathbf{d}_{syn}(\mathbf{m}) - \mathbf{d}_{obs})$

$$\frac{1}{v^2(\mathbf{x})}\frac{\partial^2 P(\mathbf{x},t)}{\partial t^2} - \nabla^2 P(\mathbf{x},t) = s(t)\delta(\mathbf{x}-\mathbf{x}_s)$$

Shot gathers



Model update:
(steepest descent)

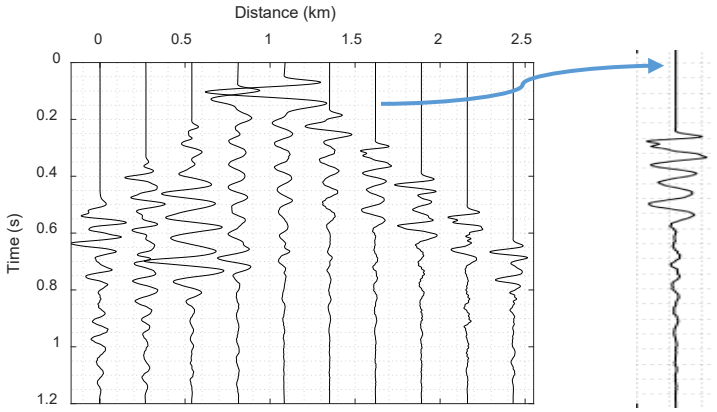
$$\mathbf{m} = \mathbf{m}_0 + \Delta\mathbf{m}$$
$$\Delta\mathbf{m} = -\mu\mathbf{g}$$

Adjoint method + approximation Hessian matrix:

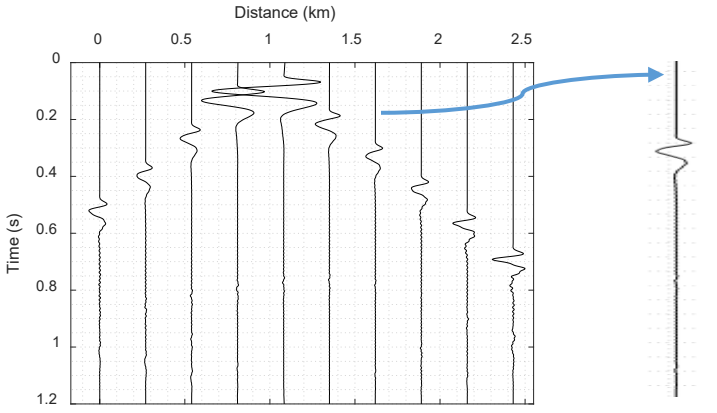
$$\Delta v(\mathbf{x}) = -\mu \sum_{r=1}^{ng}\sum_{i=1}^{ns} \frac{2}{v(\mathbf{x})^3} \frac{\int_0^{t_{\max}} dt[\ddot{P}_f(\mathbf{x},t;\mathbf{x_s})P_b(\mathbf{x},t;\mathbf{x_s})]}{\int_0^{t_{\max}} dt[\ddot{P}_f(\mathbf{x},t;\mathbf{x_s})\ddot{P}_f(\mathbf{x},t;\mathbf{x_s}) + \lambda I_{\max}]}$$

(Tarantola, 1984; Shin et al., 2001)

Introduction – present source-independent FWI



$\mathbf{d}_{obs}$ $\mathbf{d}_{ref}$



$\mathbf{S}_{syn}$ $\mathbf{S}_{ref}$

Some basic problems in FWI:

Inaccurate source wavelet
Low-frequency components lack
High calculation cost
Unbiased starting model required
Cross-talk in elastic FWI
…

- Deconvolution-based method (Lee and Kim, 2003):

$$E(\boldsymbol{m}) = \frac{1}{2}\left\| \frac{\mathbf{d}_{obs}}{\mathbf{d}_{ref}} - \frac{\mathbf{S}_{syn}}{\mathbf{S}_{ref}} \right\|^2 = \frac{1}{2}\left\| \frac{\mathbf{w}_{obs}*\mathbf{G}_{obs}}{\mathbf{w}_{obs}*\mathbf{G}_{ref}} - \frac{\mathbf{w}_{syn}*\mathbf{G}_{syn}}{\mathbf{w}_{syn}*\mathbf{G}'_{ref}} \right\|^2 = \frac{1}{2}\left\| \frac{\mathbf{G}_{obs}}{\mathbf{G}_{ref}} - \frac{\mathbf{G}_{syn}}{\mathbf{G}'_{ref}} \right\|^2$$

- Convolution-based method (Cheong et al., 2004):

$$E(\boldsymbol{m}) = \frac{1}{2}\left\| \mathbf{d}_{obs} * \mathbf{S}_{ref} - \mathbf{S}_{syn} * \mathbf{d}_{ref} \right\|$$

$$= \frac{1}{2}\left\| \mathbf{w}_{obs}*\mathbf{G}_{obs}*\mathbf{w}_{syn}*\mathbf{G}'_{ref} - \mathbf{w}_{syn}*\mathbf{G}_{syn}*\mathbf{w}_{obs}*\mathbf{G}_{ref} \right\|^2$$

$$= \frac{1}{2}\left\| \mathbf{w}_{obs}*\mathbf{w}_{syn}*\mathbf{G}'_{ref}*\mathbf{G}_{obs} - \mathbf{w}_{syn}*\mathbf{w}_{obs}*\mathbf{G}_{ref}*\mathbf{G}_{syn} \right\|^2$$

Drawbacks: More nonlinear, carefully selecting reference traces, inducing noises, sensitive to data noises

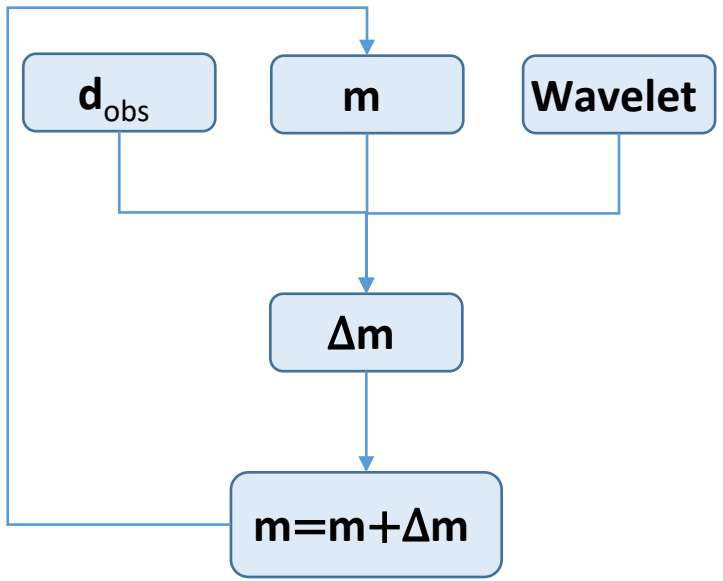
Introduction – present source-independent FWI

- Iterative estimate of source signature method (IES, Song et al., 1995; Pratt, 1999):

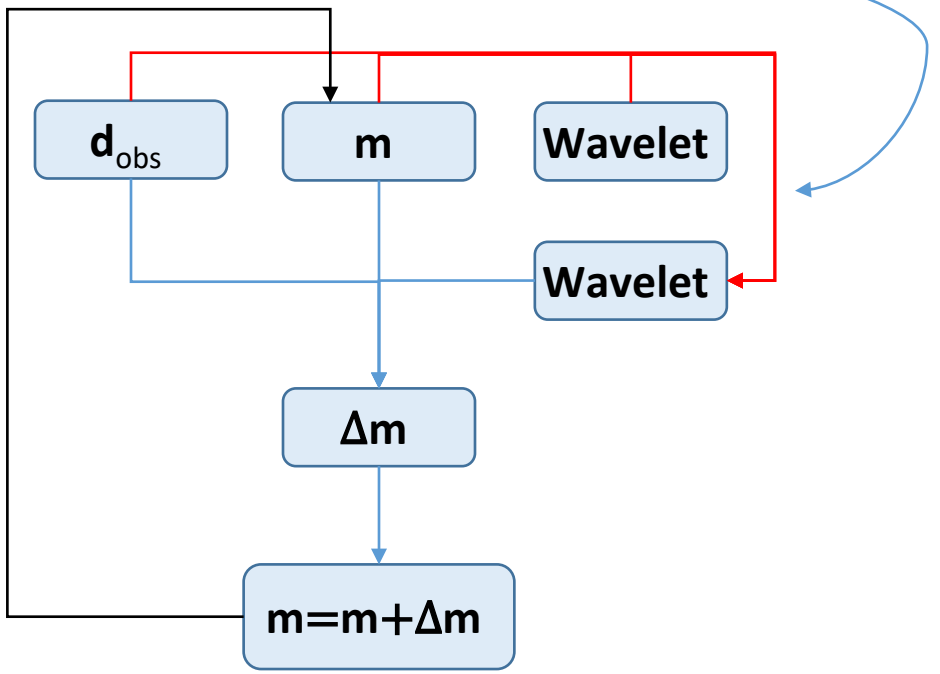
$$E(\boldsymbol{m}) = \frac{1}{2}||\mathbf{d}_{\text{obs}} - \mathbf{S}_{\text{syn}}||^2 = \frac{1}{2}||\mathbf{d}_{\text{obs}} - \mathbf{W}\mathbf{G}_{\text{syn}}||^2$$

(Frequency domain)

$$\Rightarrow \quad \mathrm{W} = \frac{\mathbf{G}_{\text{syn}}{}^{\mathrm{T}}\mathbf{d}_{\text{obs}}{}^{*}}{\mathbf{G}_{\text{syn}}{}^{\mathrm{T}}\mathbf{G}_{\text{syn}}{}^{*}}$$



Known-source-wavelet (KSW) FWI



IES FWI

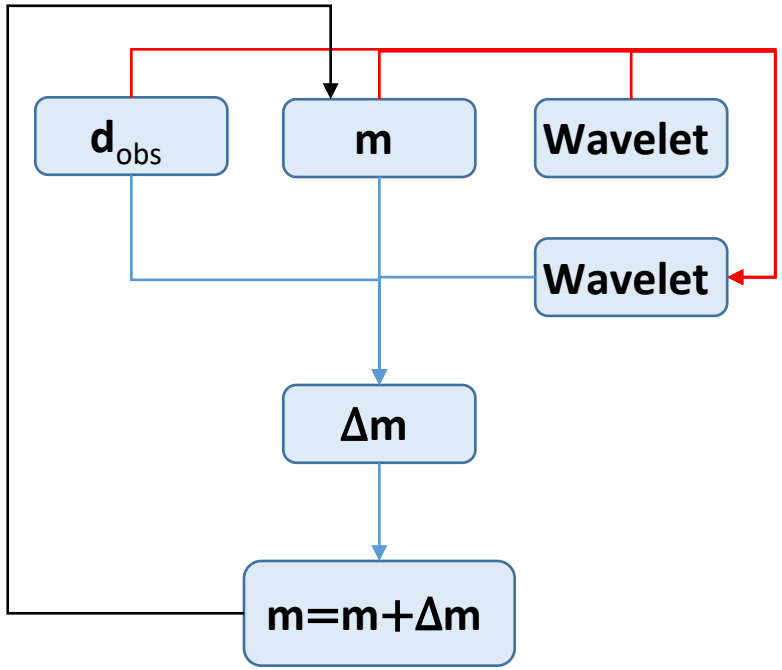
Introduction – A robust source-independent FWI

- Iterative estimate of source signature method (our method):

$$E(\boldsymbol{m}) = \frac{1}{2}||\mathbf{d}_{obs} - \mathbf{S}_{syn}||^2 = \frac{1}{2}||\mathbf{d}_{obs} - \mathbf{W}\mathbf{G}_{syn}||^2 \quad \text{(Frequency domain)}$$

$$W = \frac{1}{n}\sum_{i=1}^{n}\frac{\mathrm{G}_{syn}(\mathrm{i})\mathrm{d}_{obs}(\mathrm{i})^*}{\mathrm{G}_{syn}(\mathrm{i})\mathrm{Gsyn}(\mathrm{i})^*}$$

i is trace number



IES FWI

$$W = \frac{\mathbf{G}_{syn}{}^{\mathrm{T}}\mathbf{d}_{obs}{}^*}{\mathbf{G}_{syn}{}^{\mathrm{T}}\mathbf{G}_{syn}{}^*} = \frac{\sum_{i=1}^{n}\mathrm{G}_{syn}(\mathrm{i})\mathrm{d}_{obs}(\mathrm{i})^*}{\sum_{i=1}^{n}\mathrm{G}_{syn}(\mathrm{i})\mathrm{Gsyn}(\mathrm{i})^*}$$

Old IES FWI

or

$$W = \frac{1}{n}\sum_{i=1}^{n}\frac{\mathrm{G}_{syn}(\mathrm{i})\mathrm{d}_{obs}(\mathrm{i})^*}{\mathrm{G}_{syn}(\mathrm{i})\mathrm{Gsyn}(\mathrm{i})^*}$$

New IES FWI

Suppress noises from data noises, the inaccurate velocity model, and the imperfect forwarding modeling.

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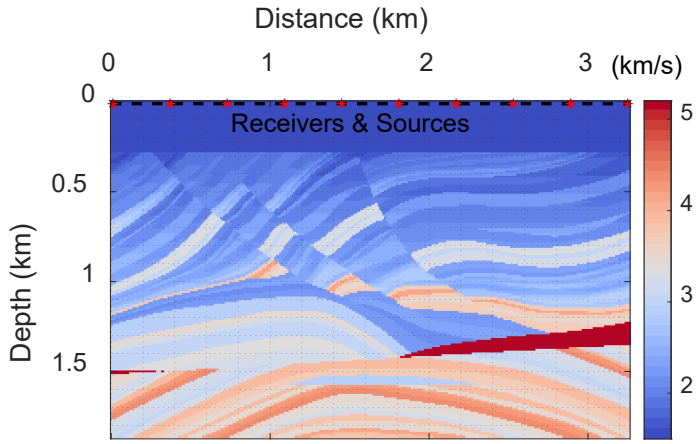
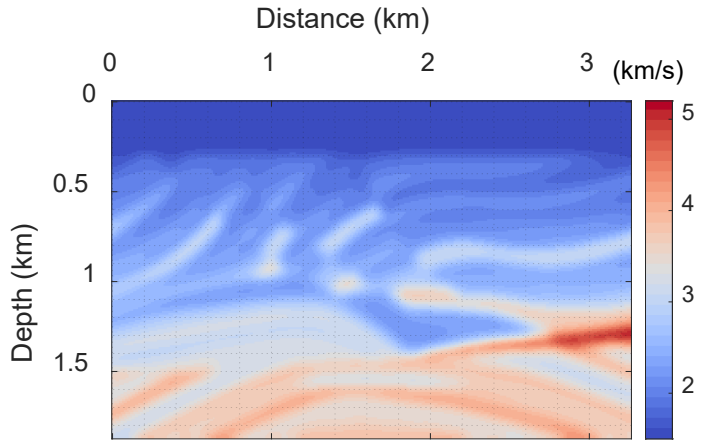
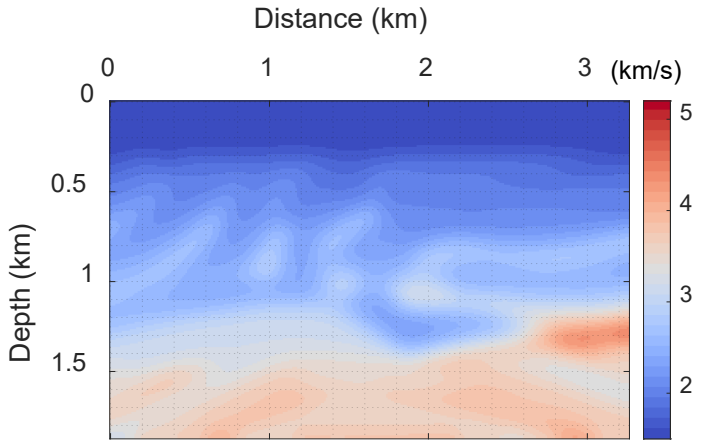
Numerical example



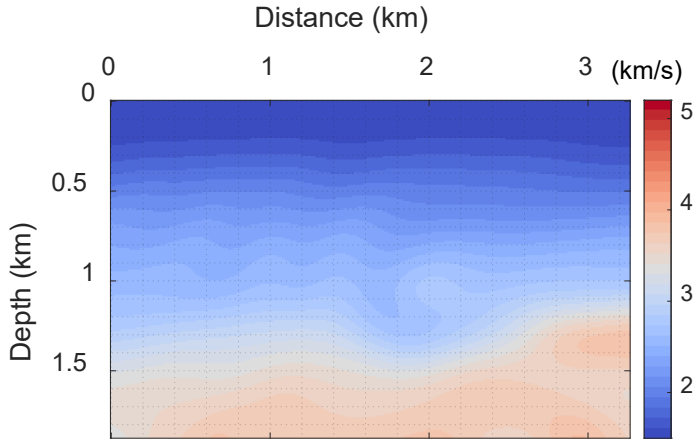
True model



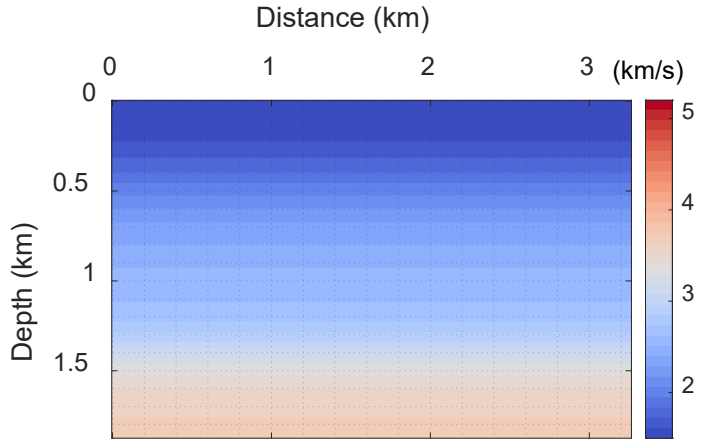
Starting model 1



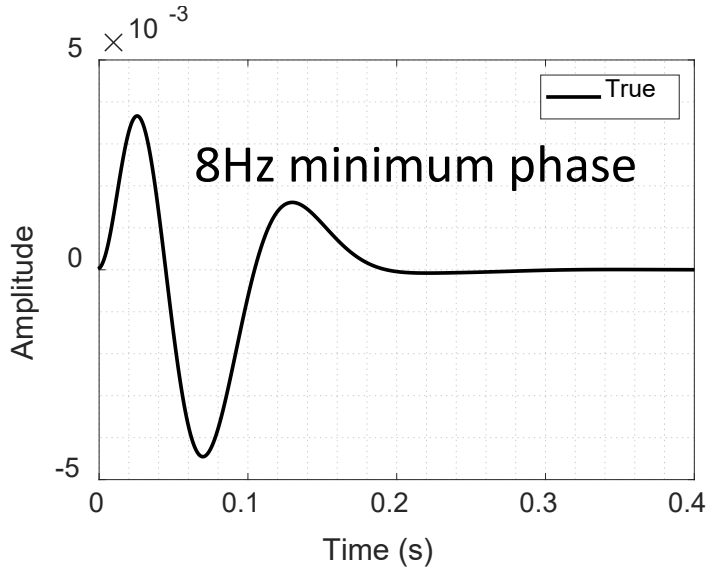
Starting model 2



Starting model 3



Starting model 4

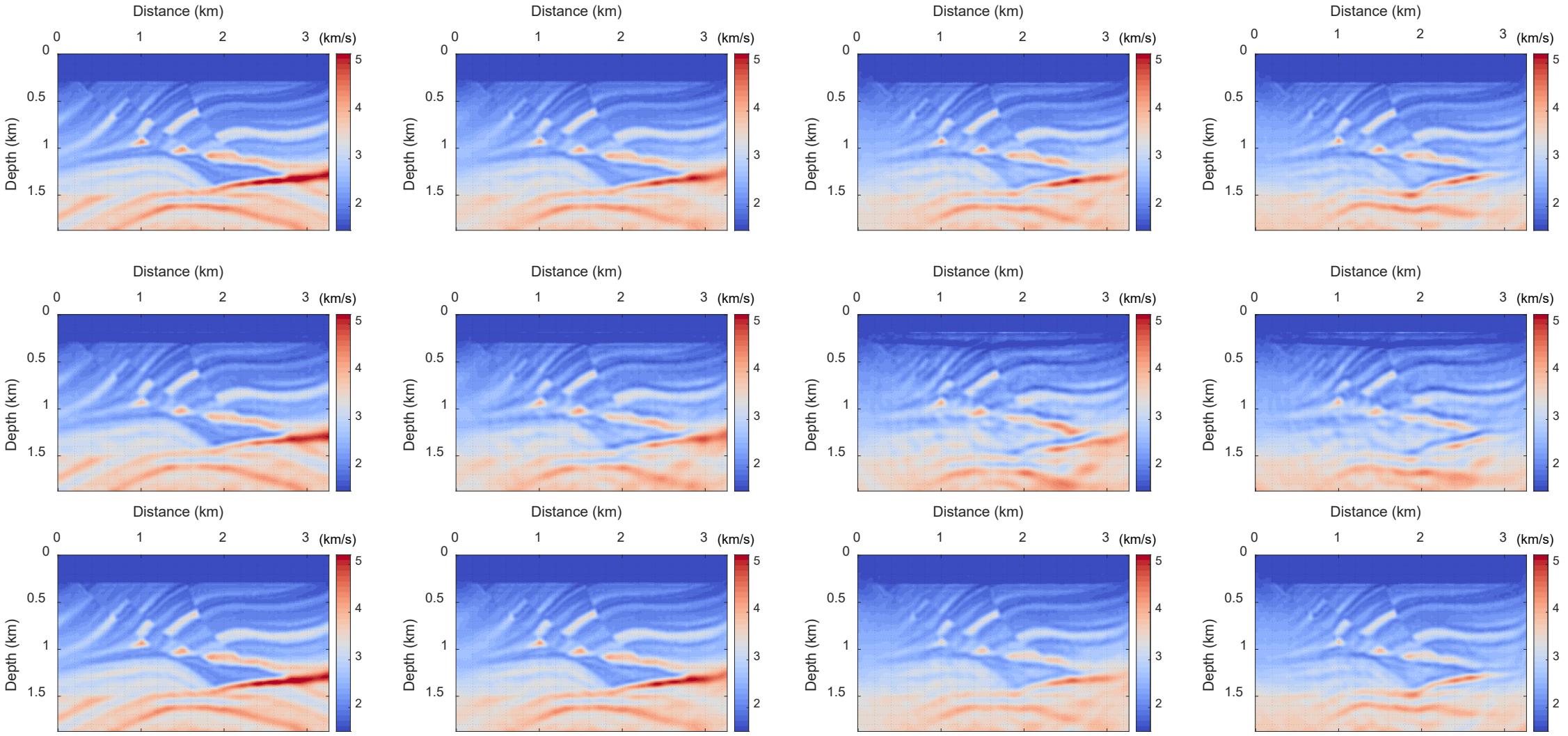


Results of KSW, Old IES, New IES

KSW

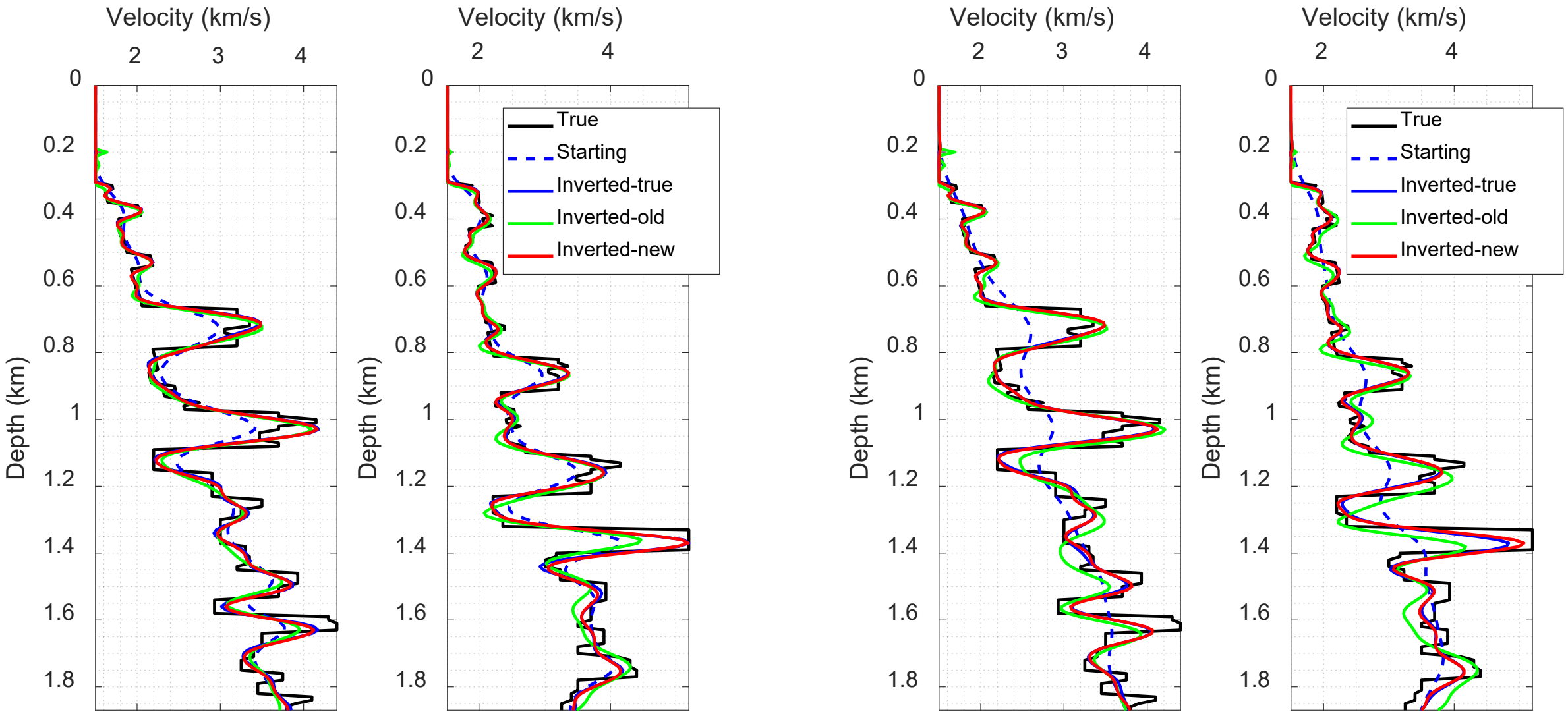
Old IES

New IES



Model 1 Model 2 Model 3 Model 4

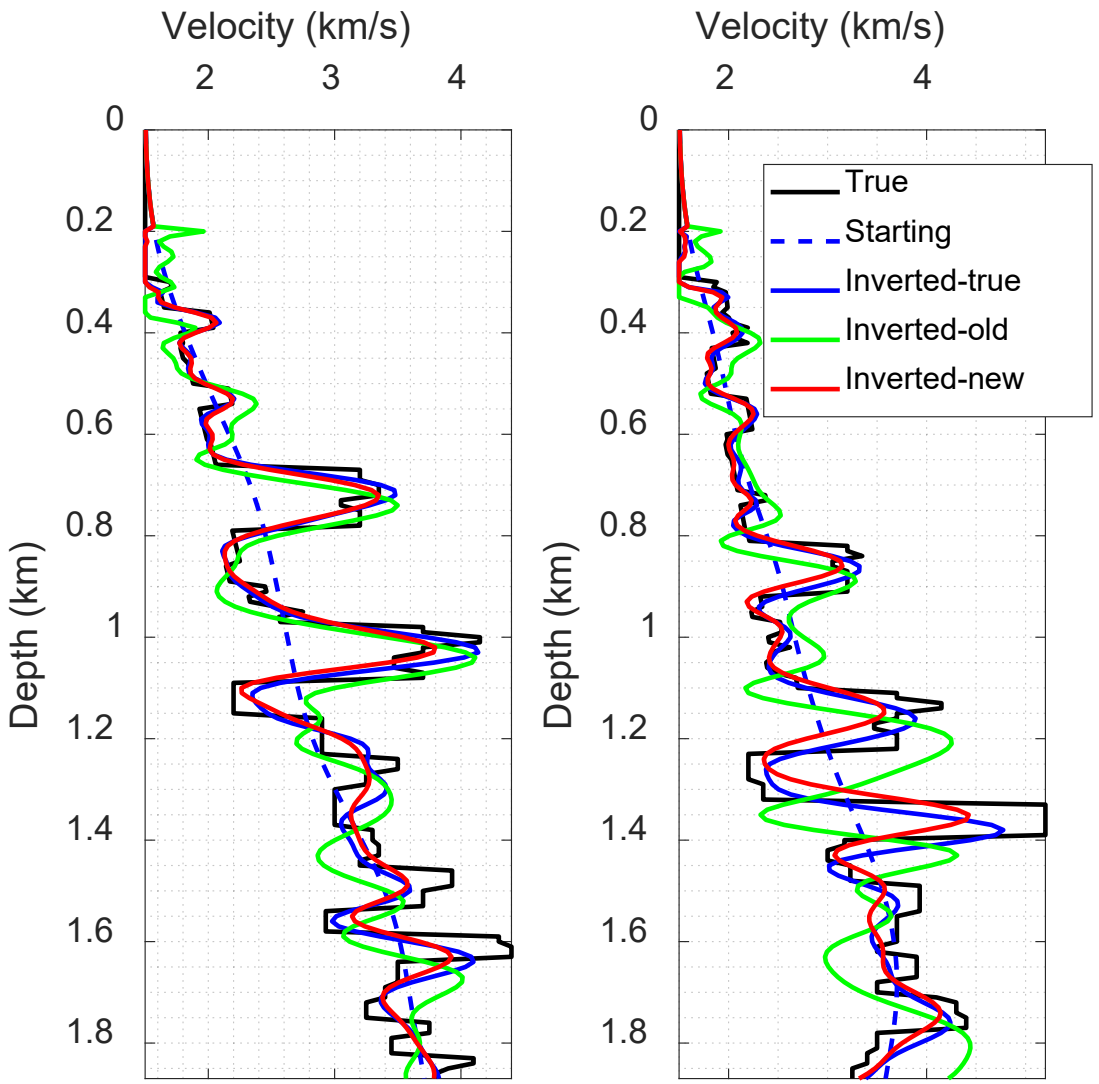
Extract traces at distance 1.5km and 2.5km



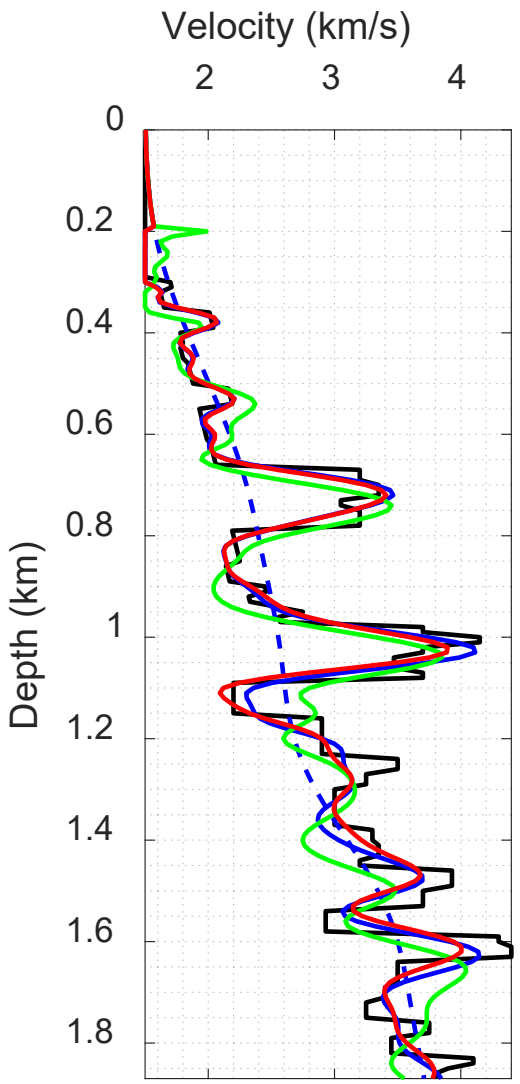
Model 1

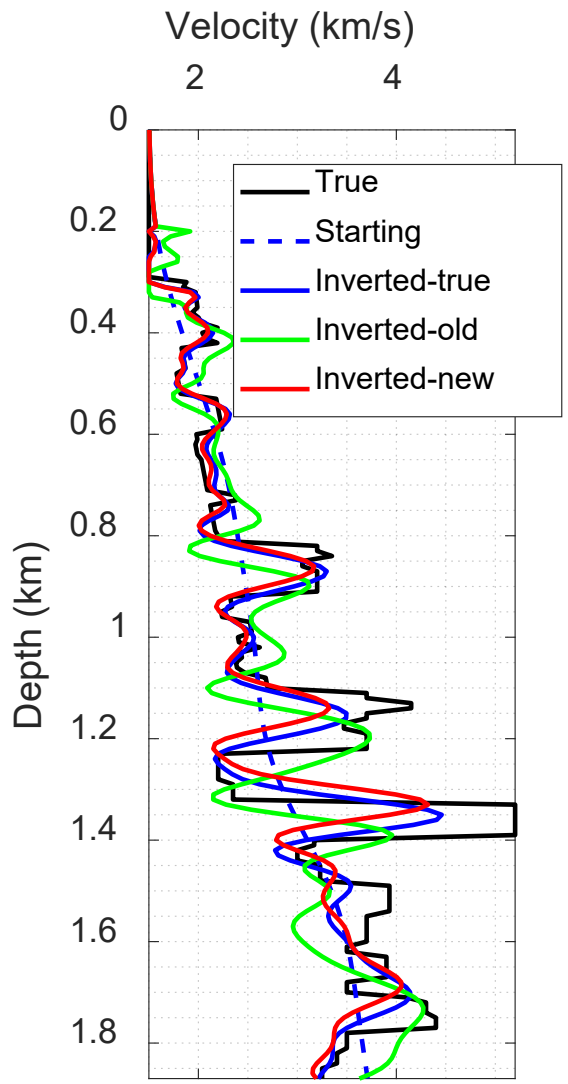
Model 2

Extract traces at distance 1.5km and 2.5km



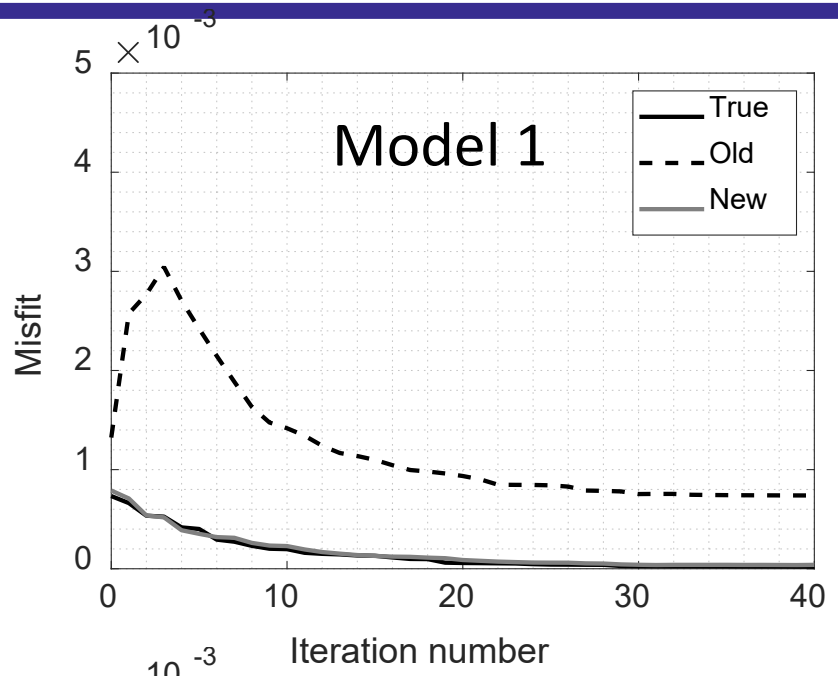
Model 3

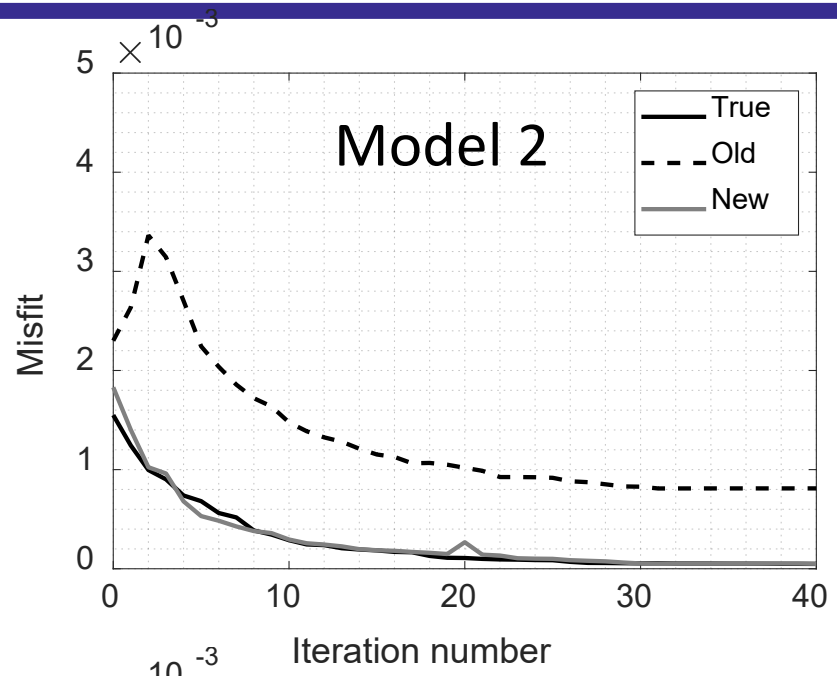


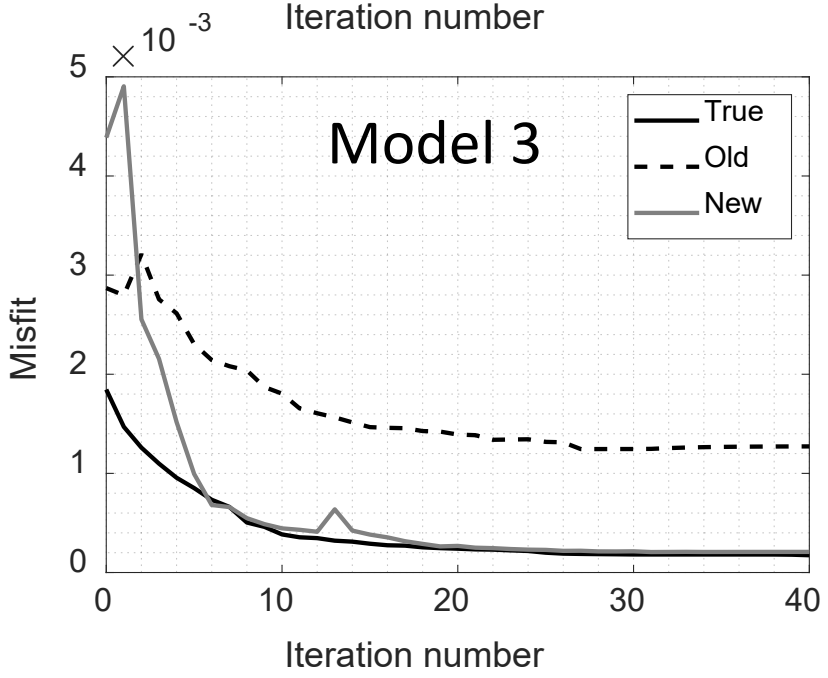


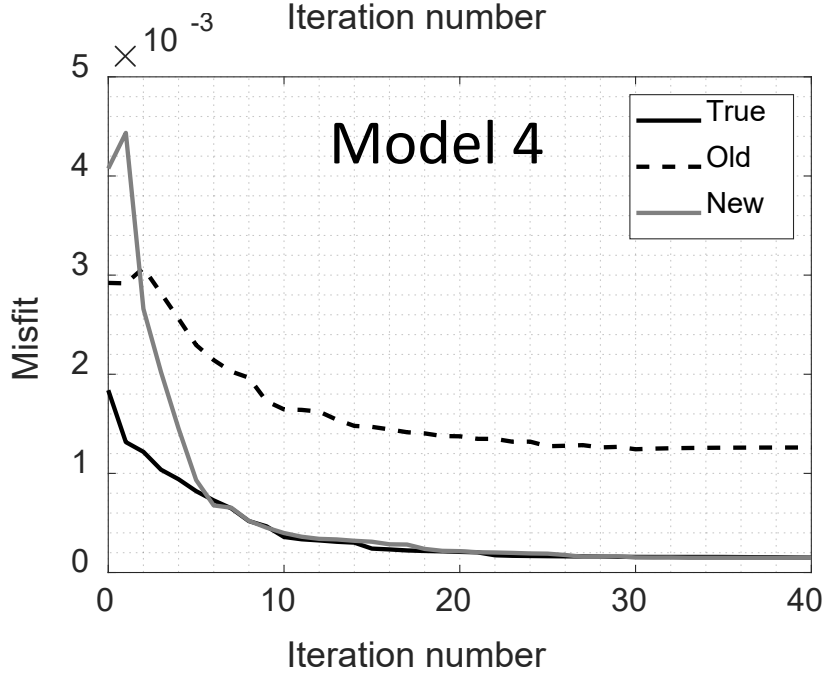
Model 4

Misfit vs Iteration number

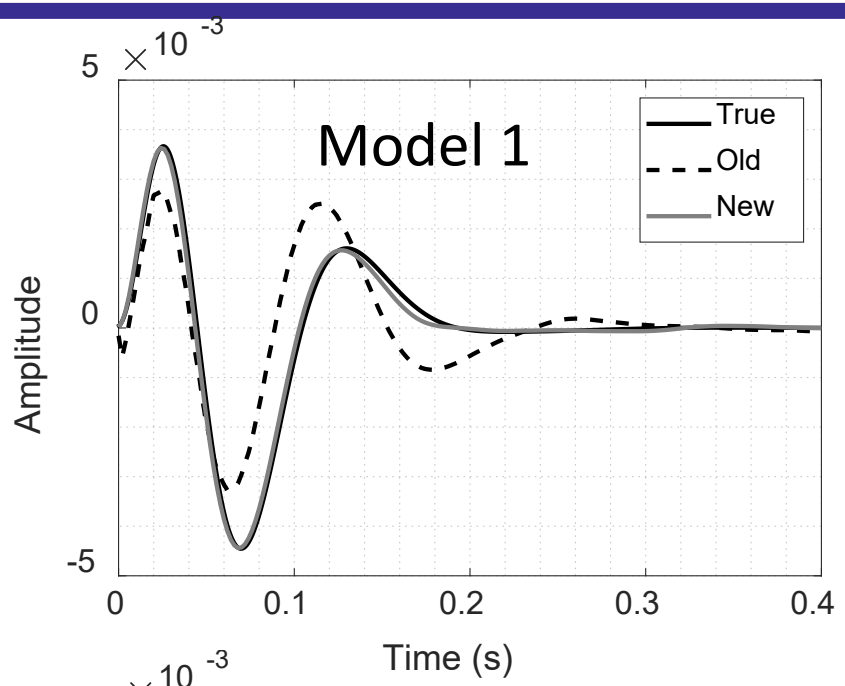


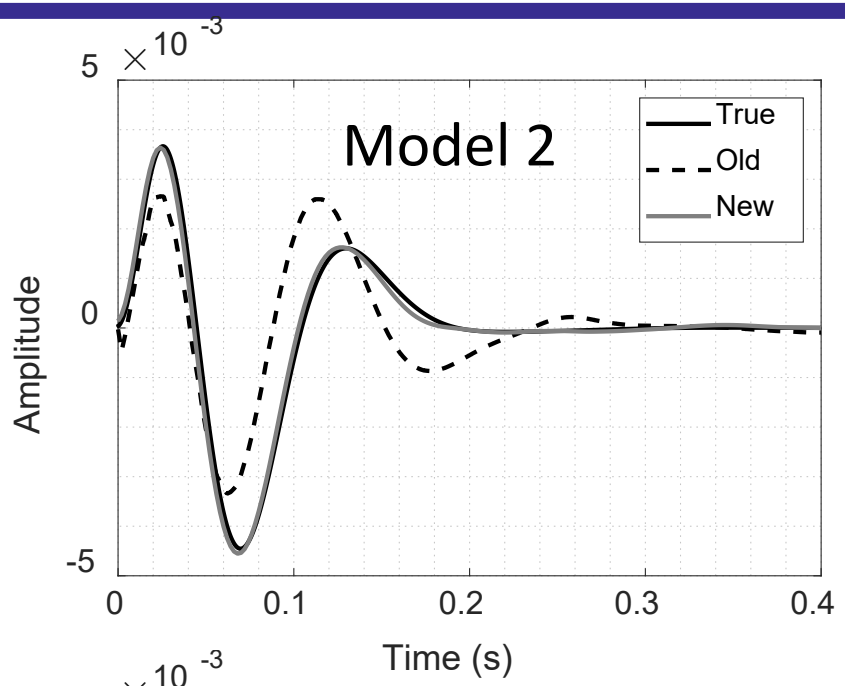


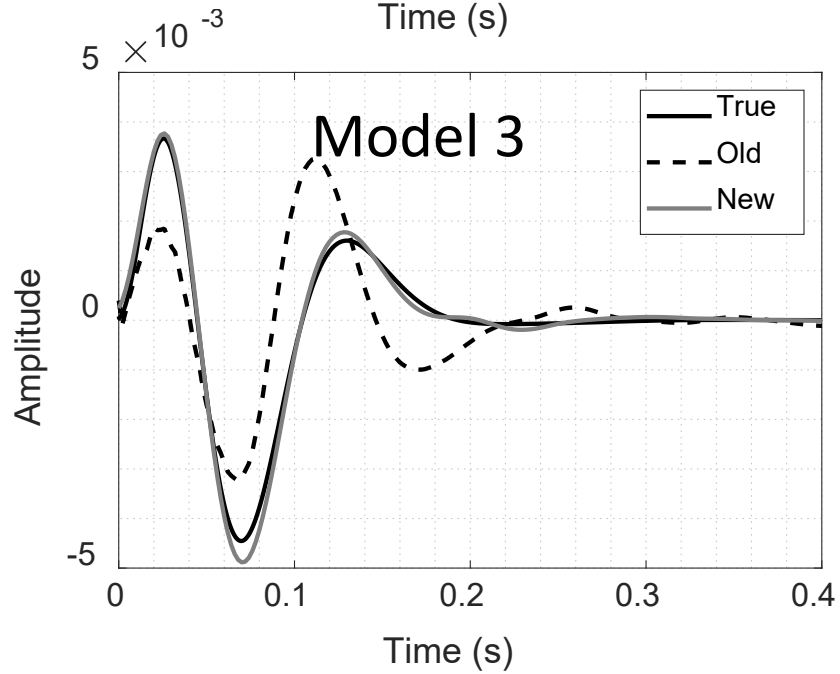


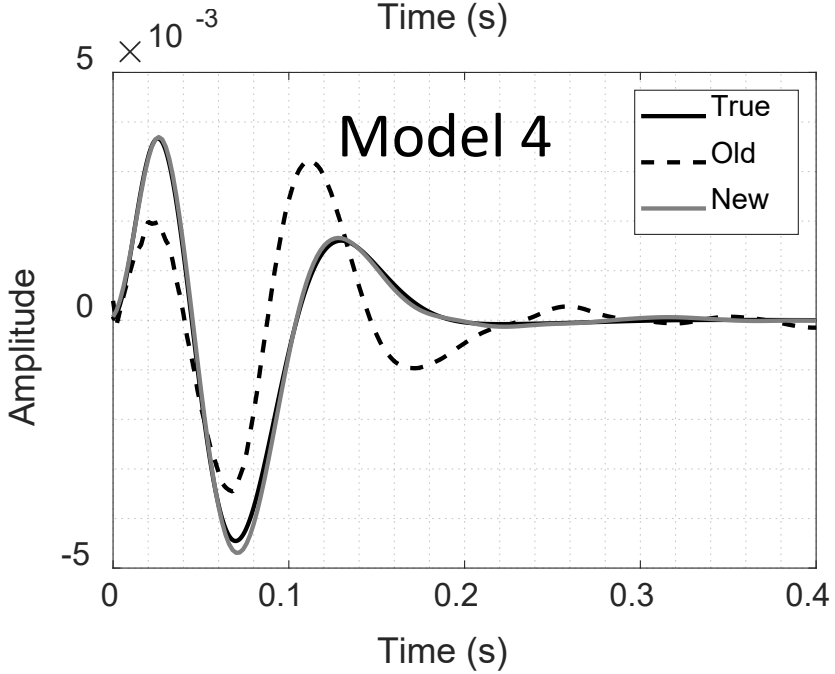


Estimated wavelets

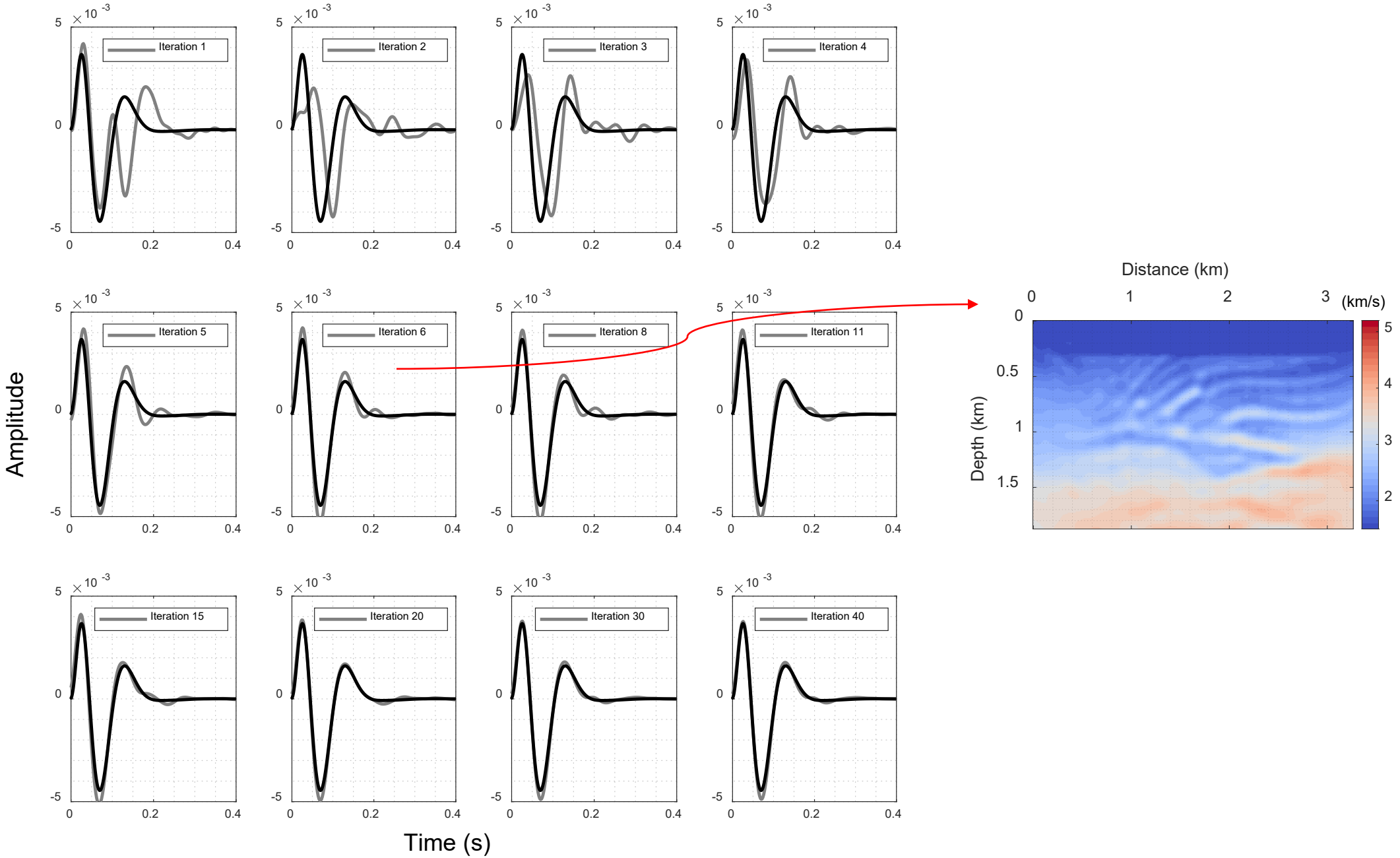




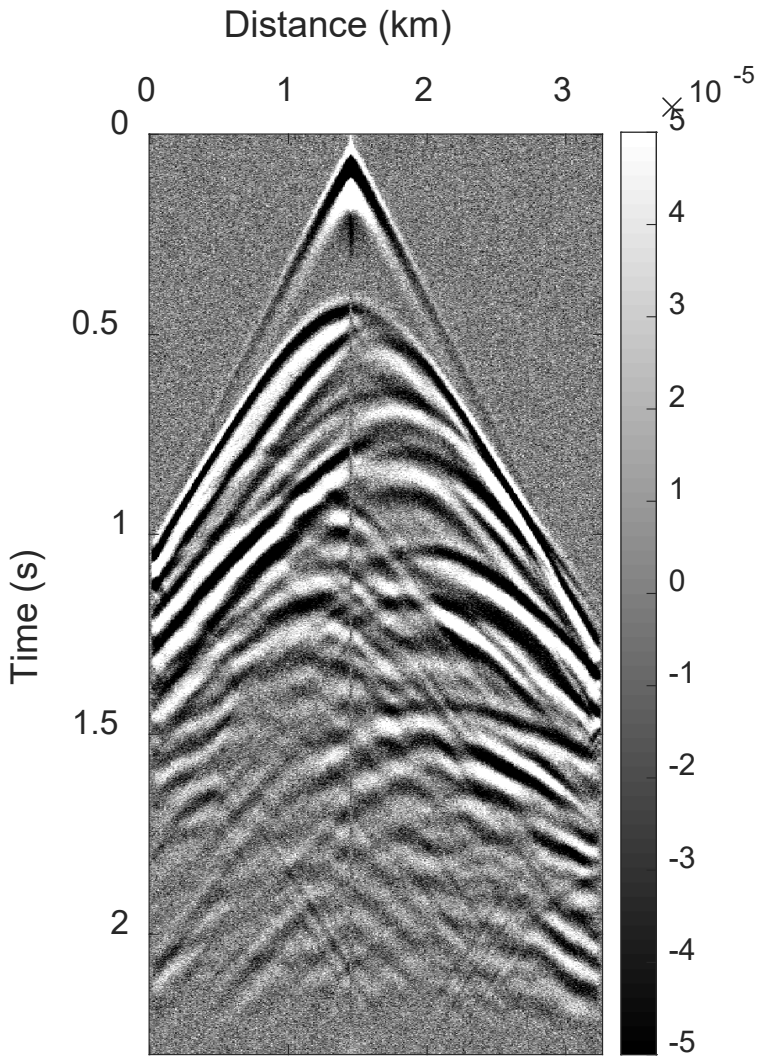




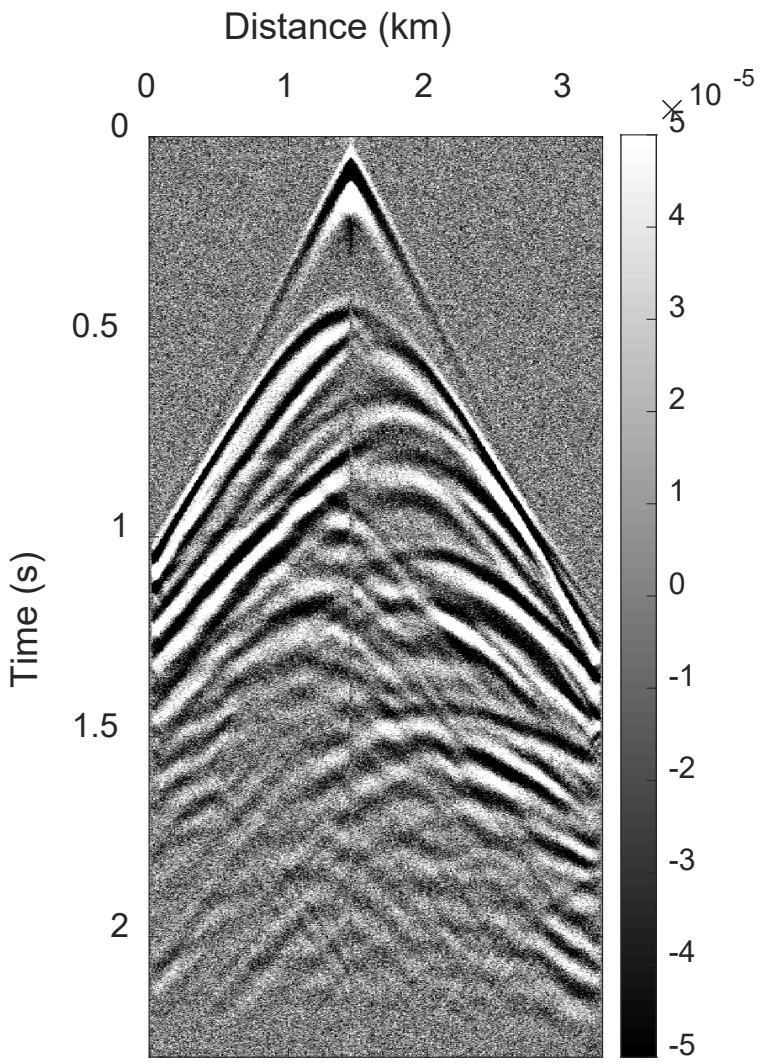
Evolution of estimated wavelet for starting model 3



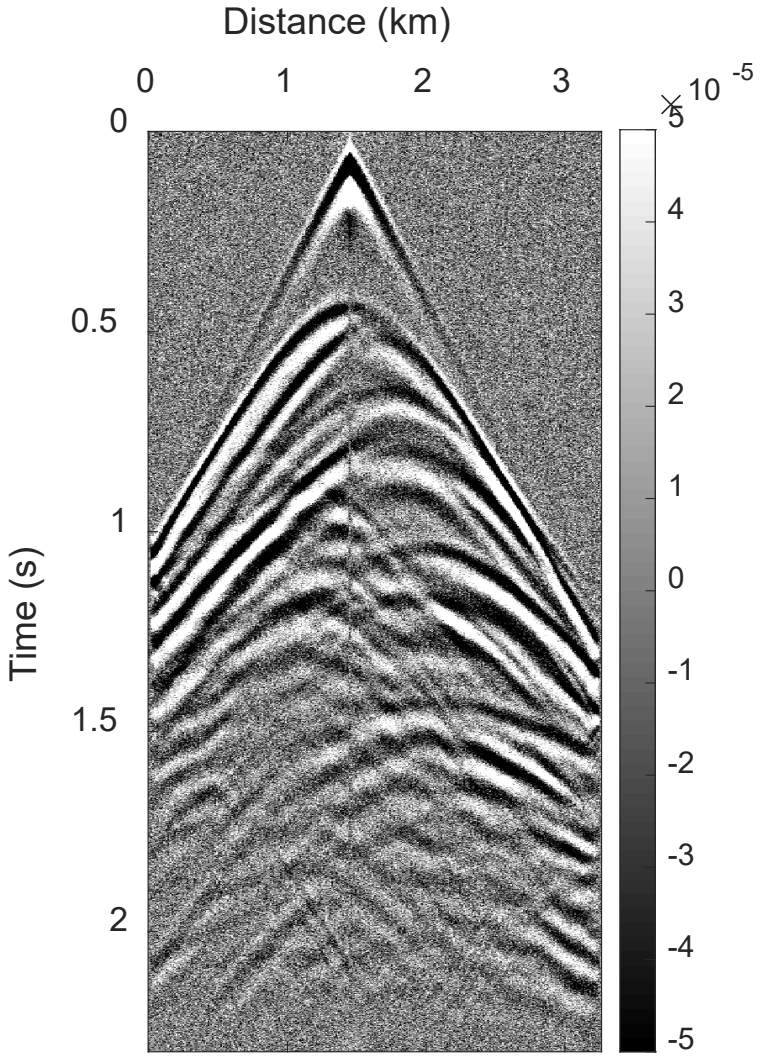
Noisy data tests



SNR = 5dB



SNR = 2dB



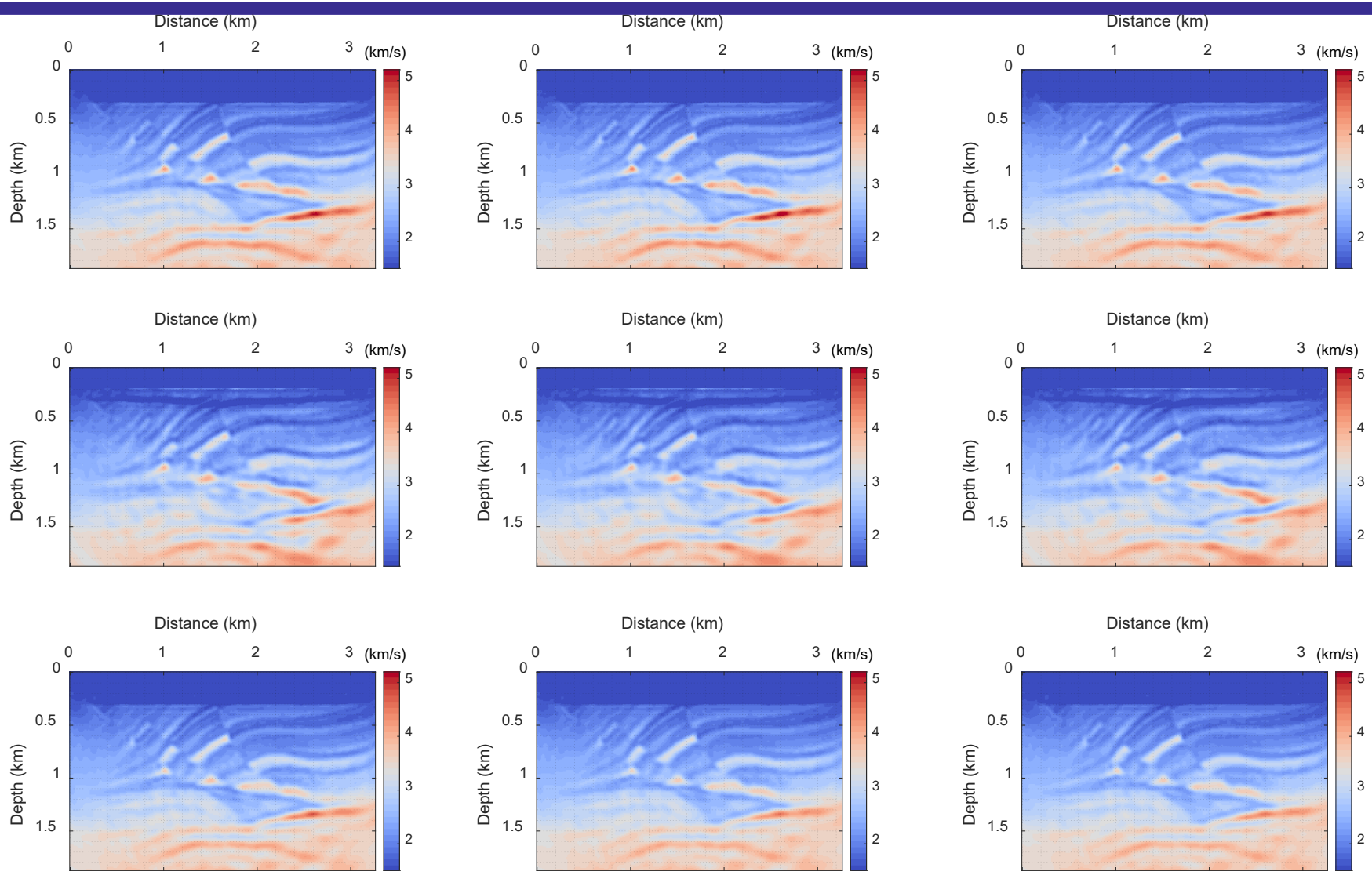
SNR = 1dB

Results of noisy data using starting model 3

KSW

Old IES

New IES

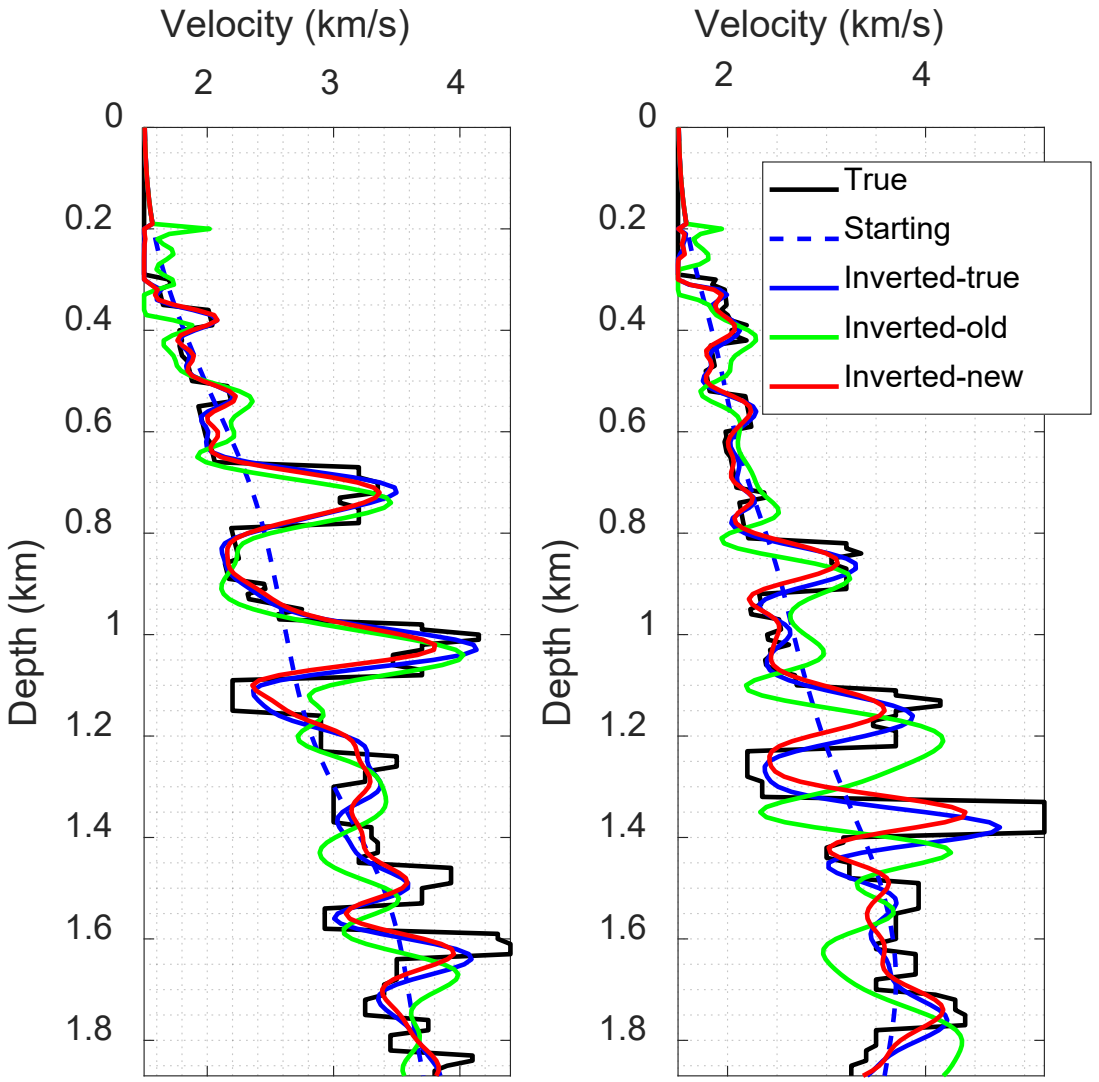


SNR = 5dB

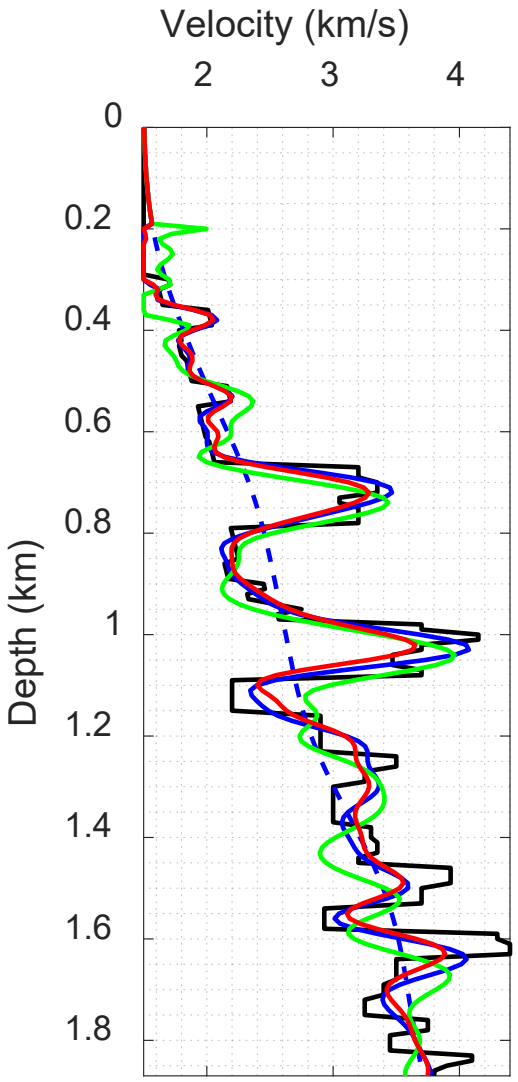
SNR = 2dB

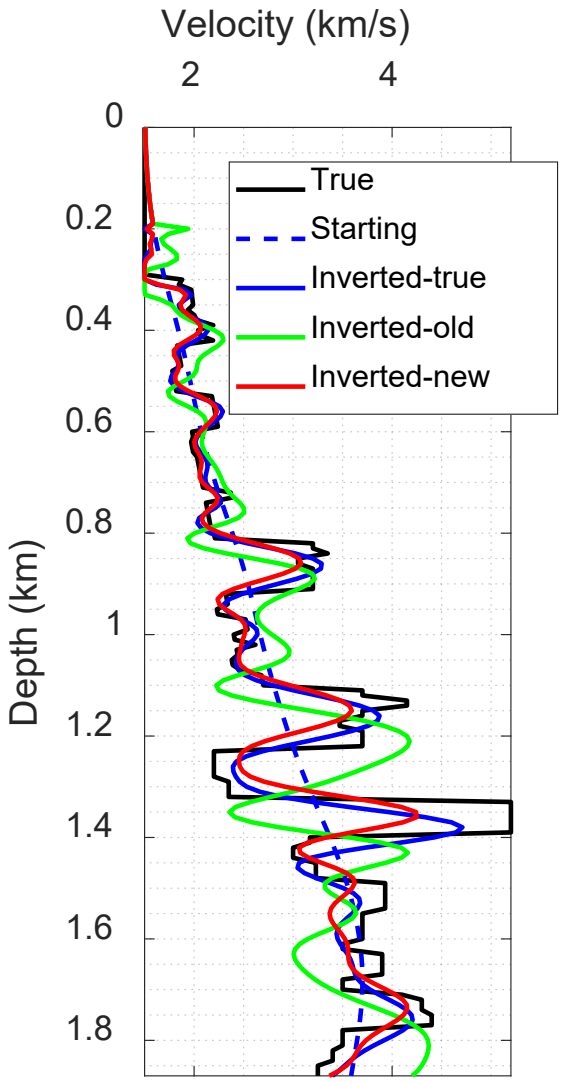
SNR = 1dB

Extract traces at distance 1.5km and 2.5km



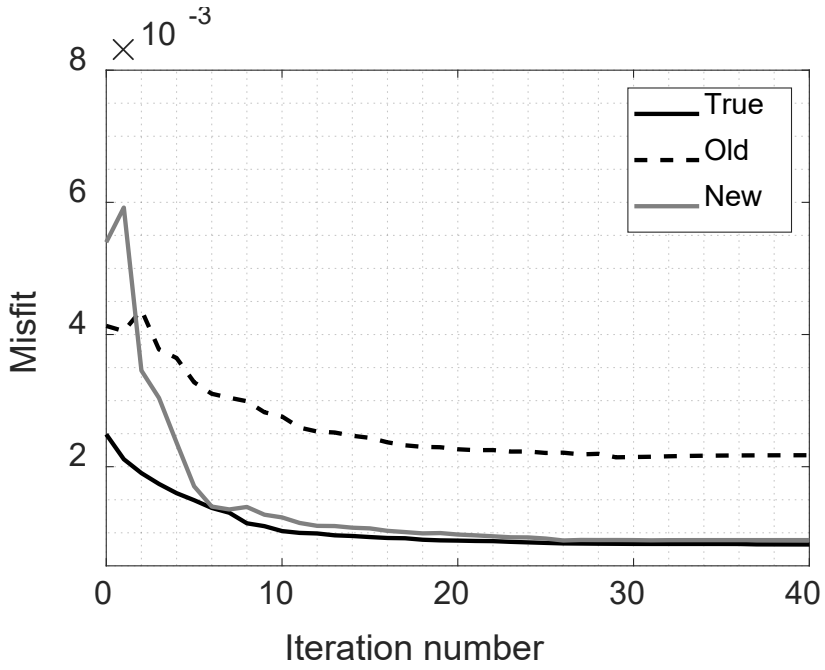
SNR = 5dB



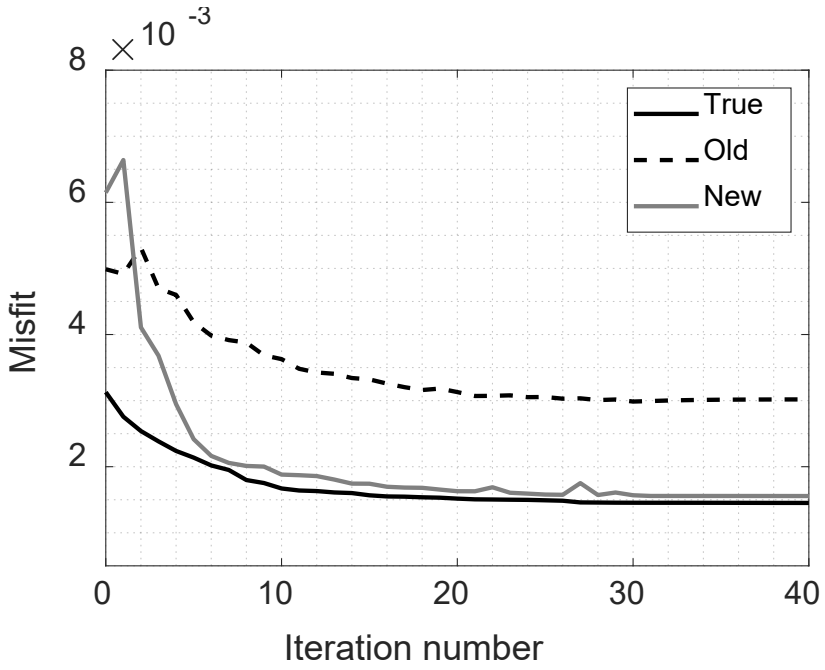


SNR = 1dB

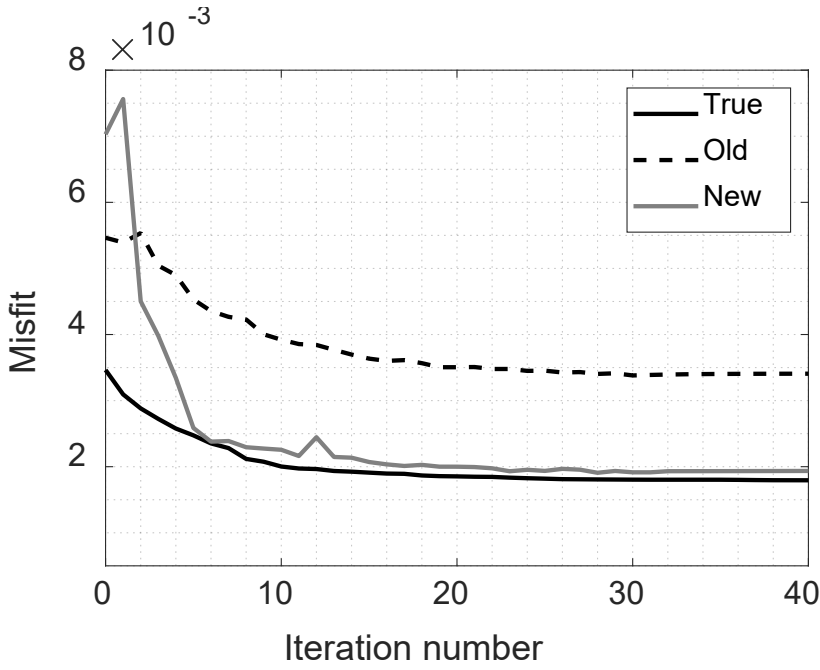
Misfit vs Iteration number



SNR = 5dB

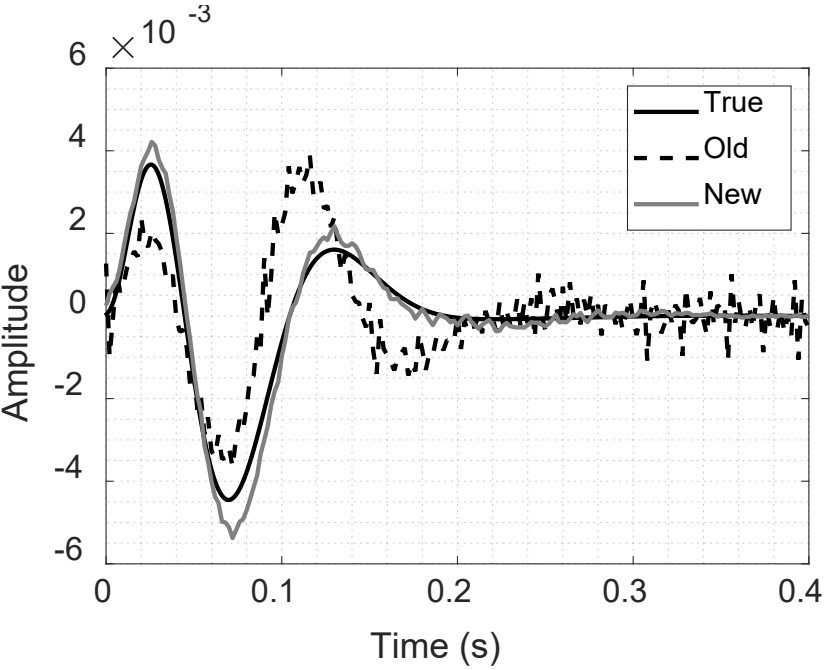


SNR = 2dB

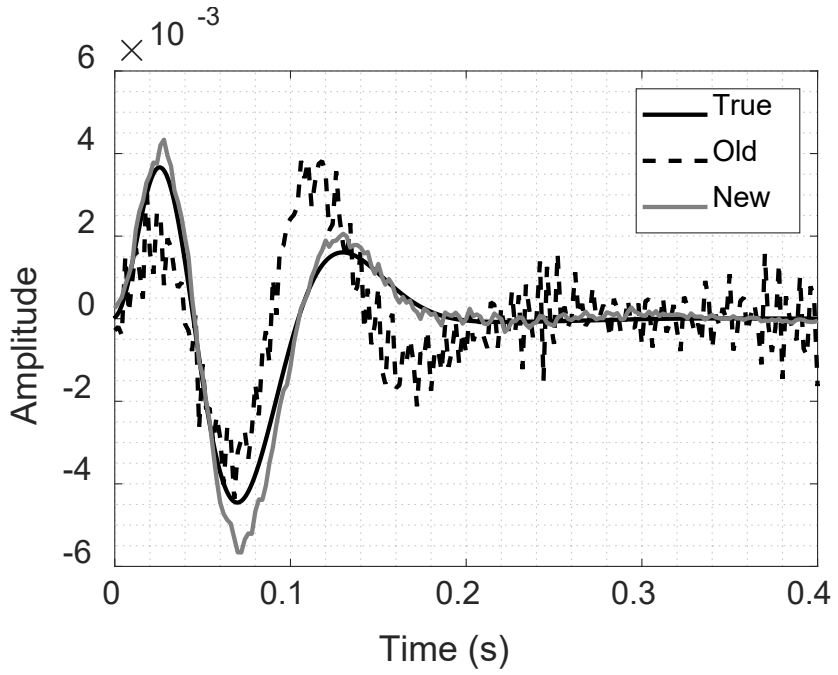


SNR = 1dB

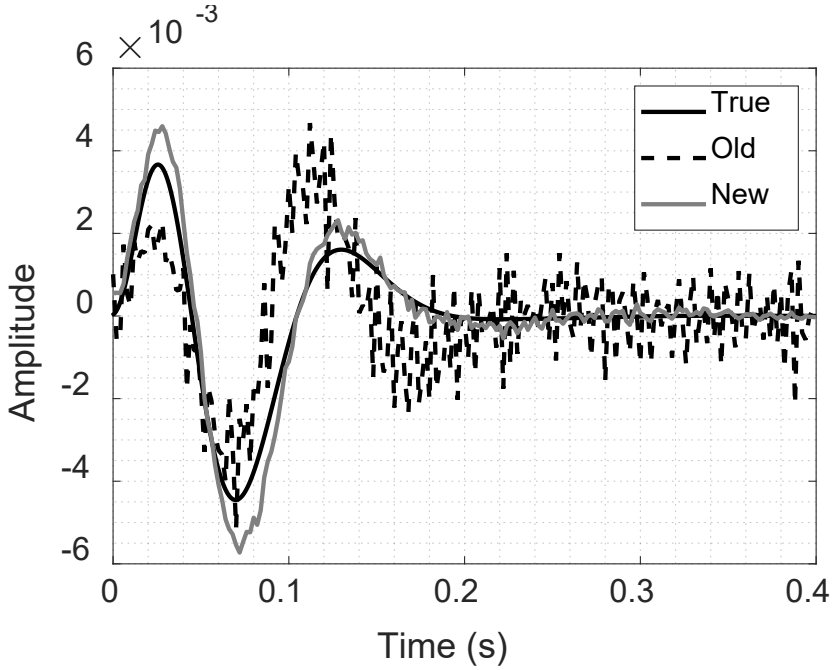
Estimated wavelets



SNR = 5dB

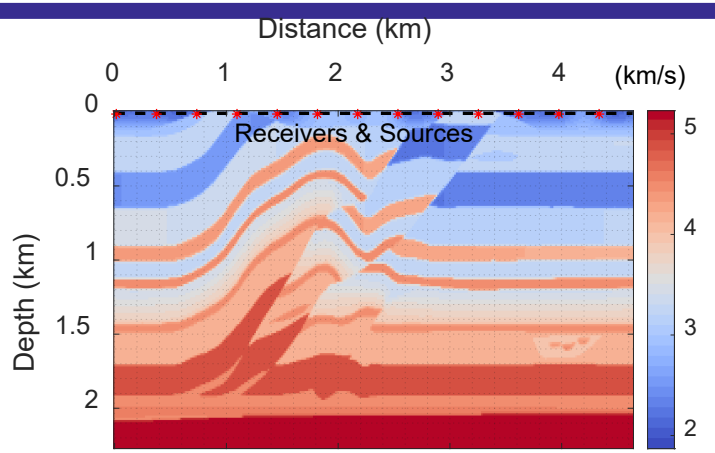


SNR = 2dB

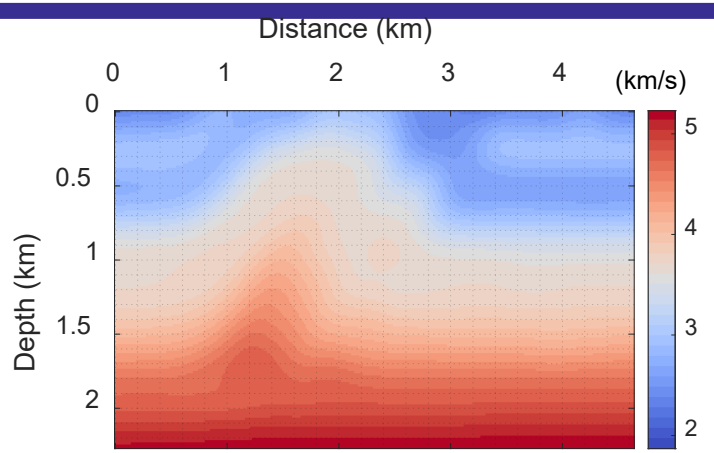


SNR = 1dB

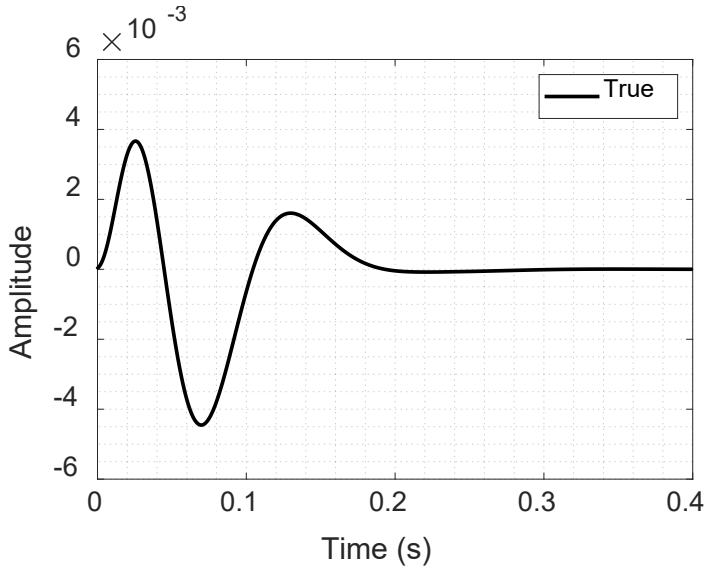
Tests using different wavelets



True model

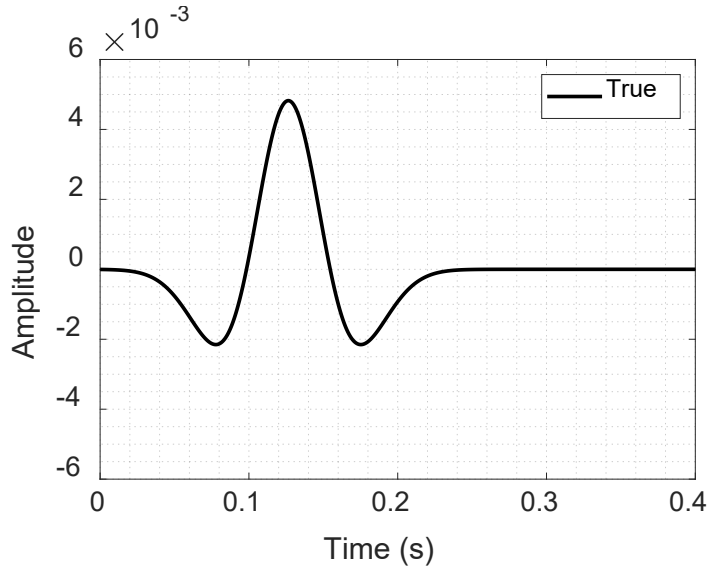


Starting model



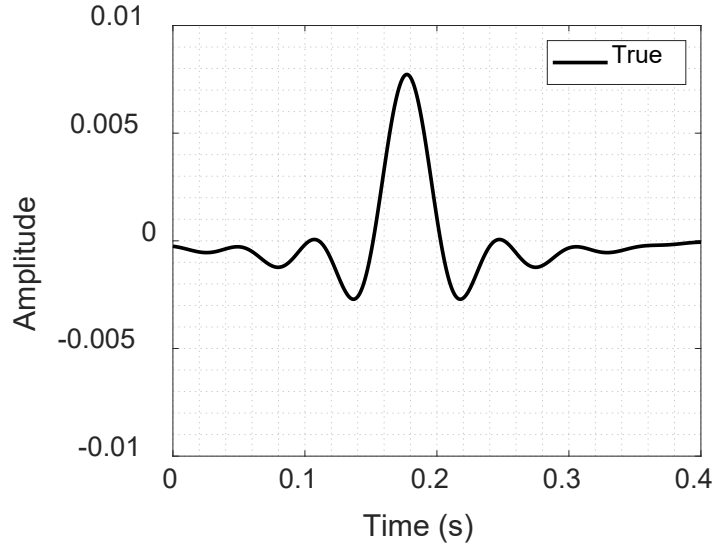
Wavelet 1

8Hz minimum phase wavelet



Wavelet 2

8Hz Ricker wavelet



Wavelet 3

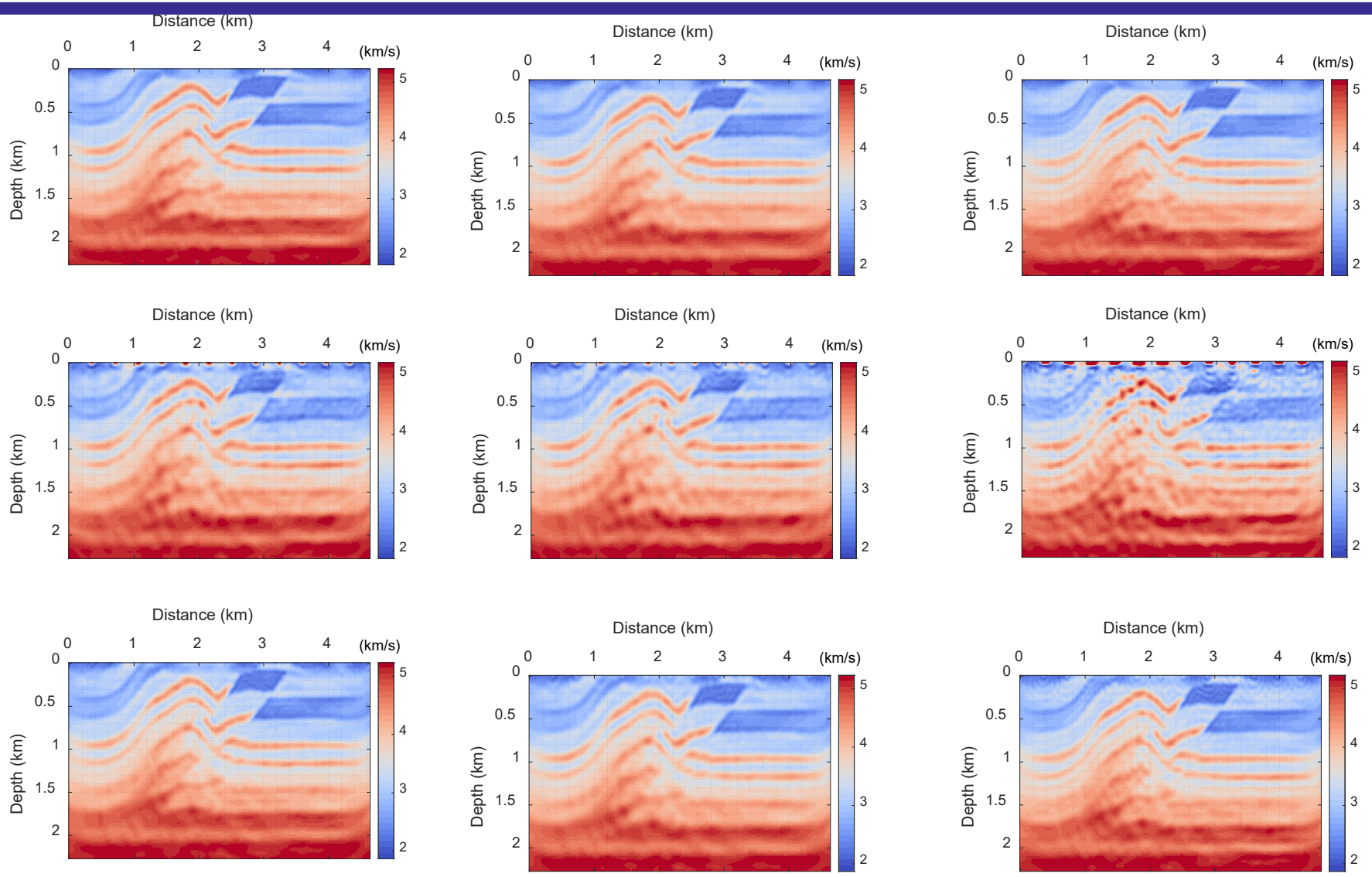
1Hz-3Hz-15-20Hz Ormsby wavelet

Results of noisy data using starting model 3

KSW

Old IES

New IES

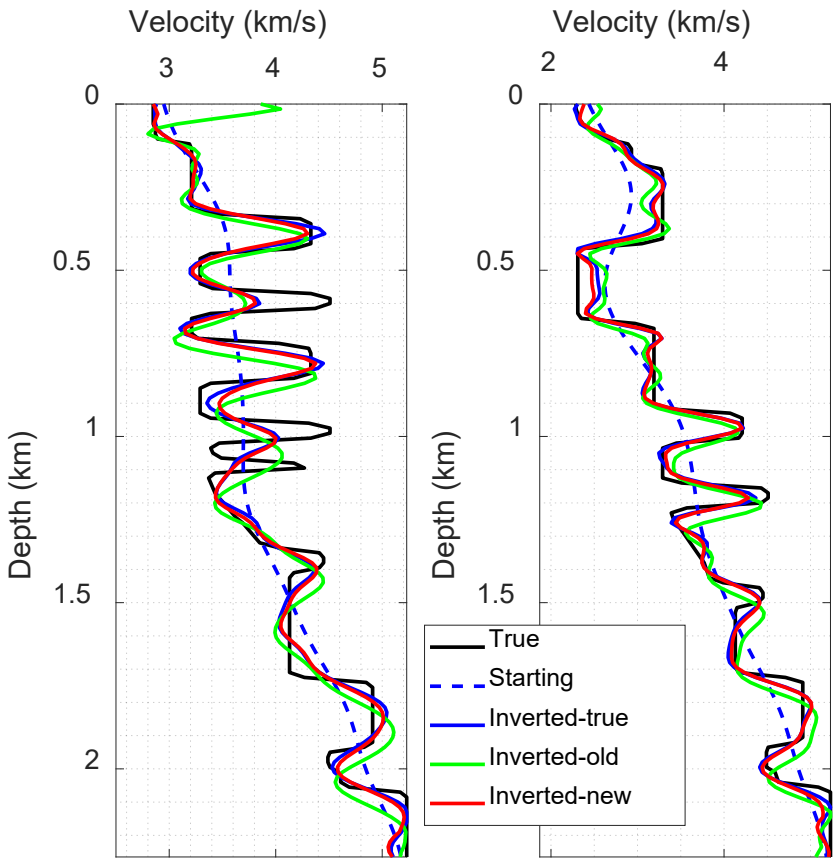


Wavelet 1

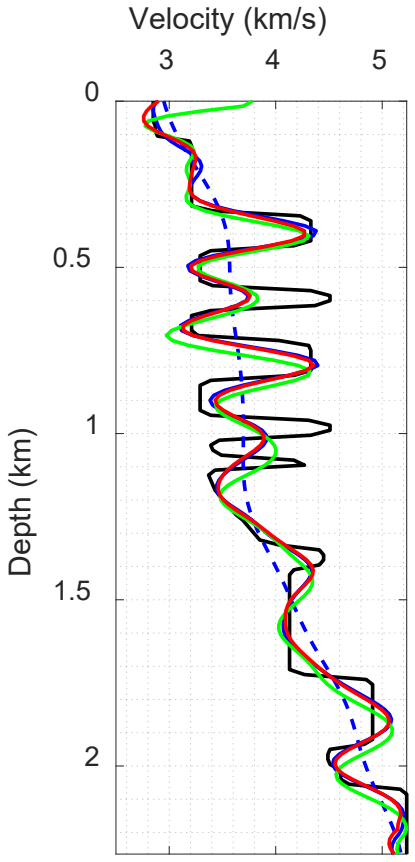
Wavelet 2

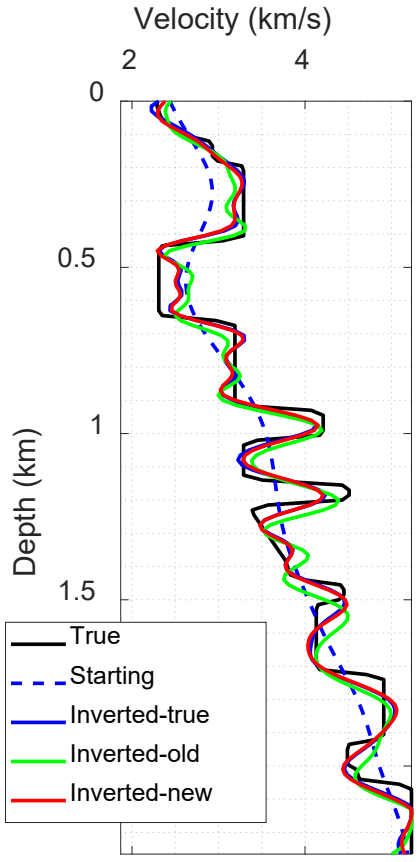
Wavelet 3

Extract traces at distance 1.5km and 2.5km

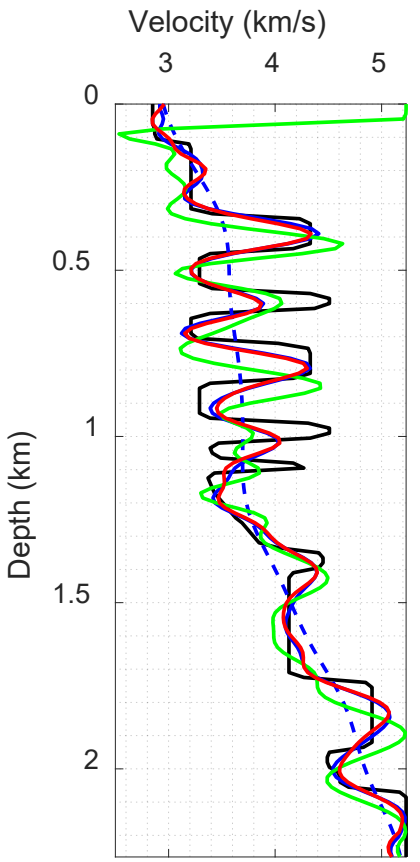


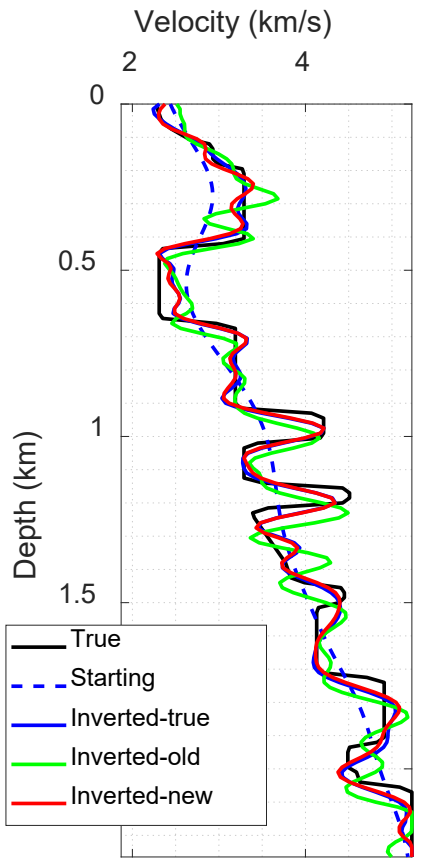
Wavelet 1





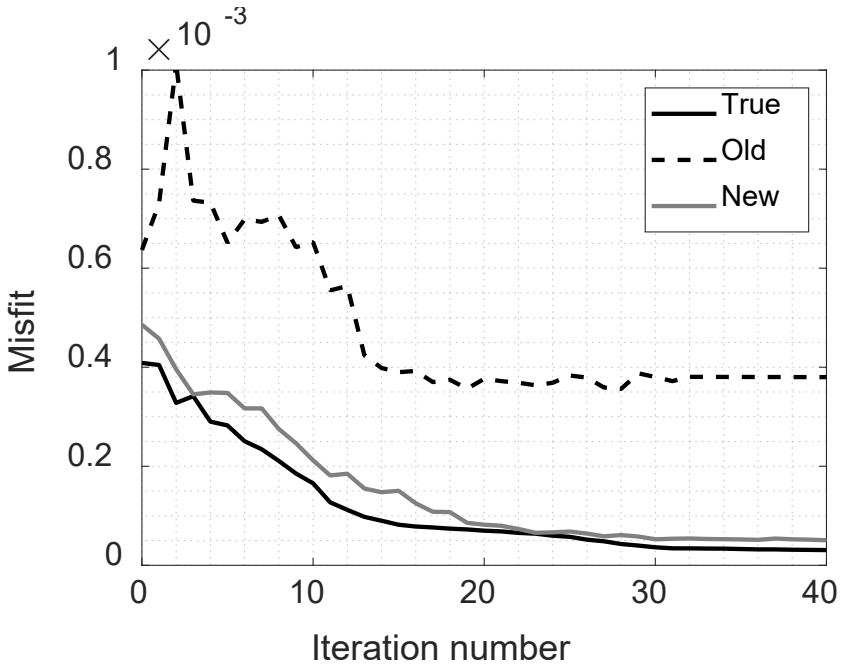
Wavelet 2



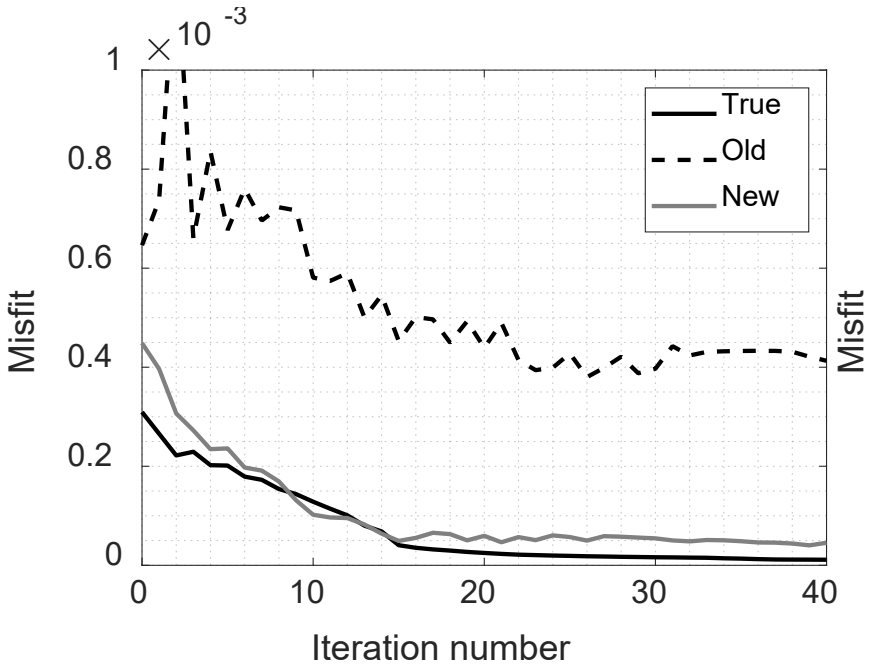


Wavelet 3

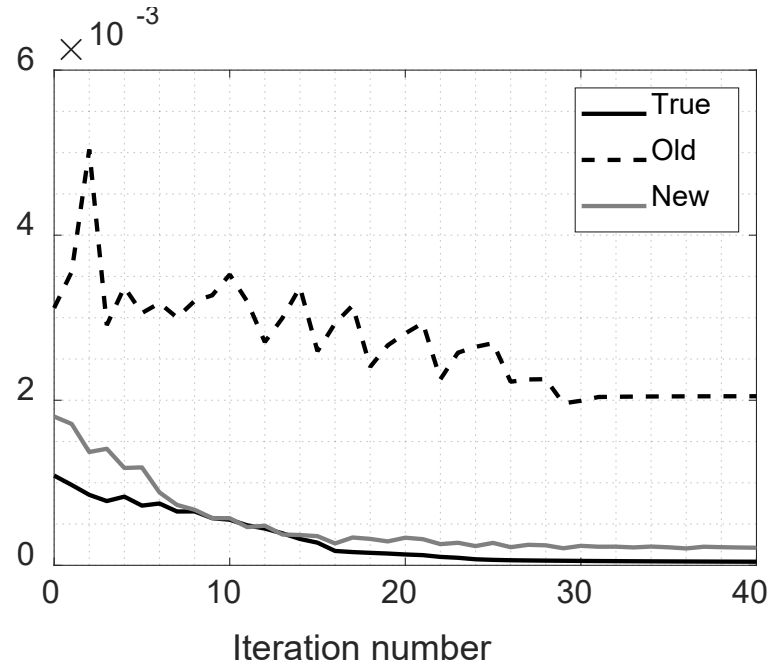
Misfit vs Iteration number



Wavelet 1

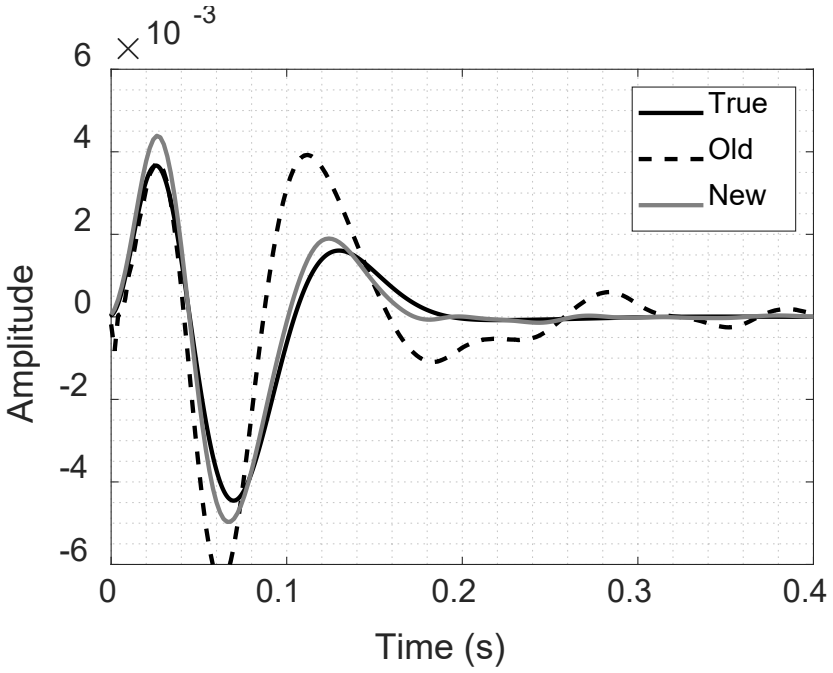


Wavelet 2

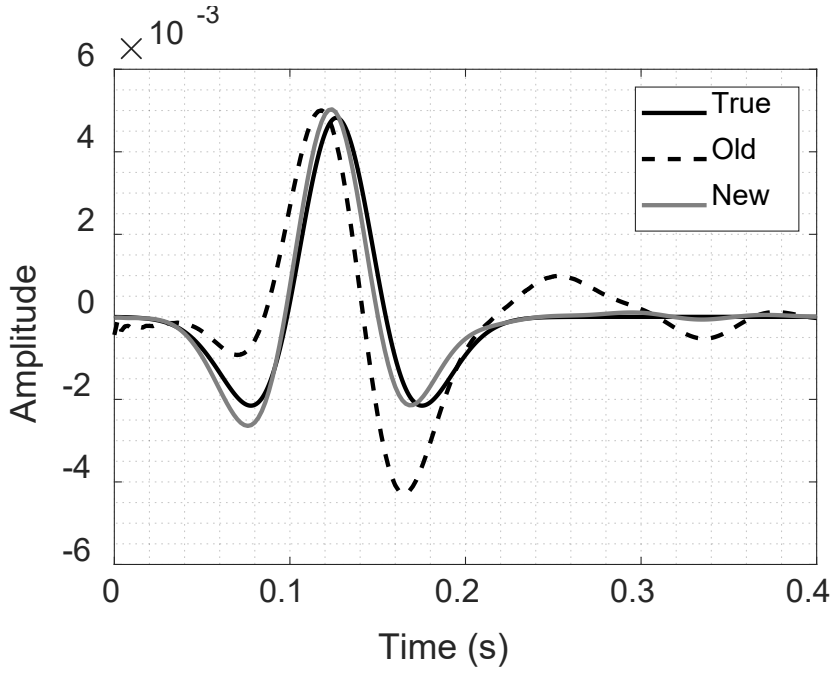


Wavelet 3

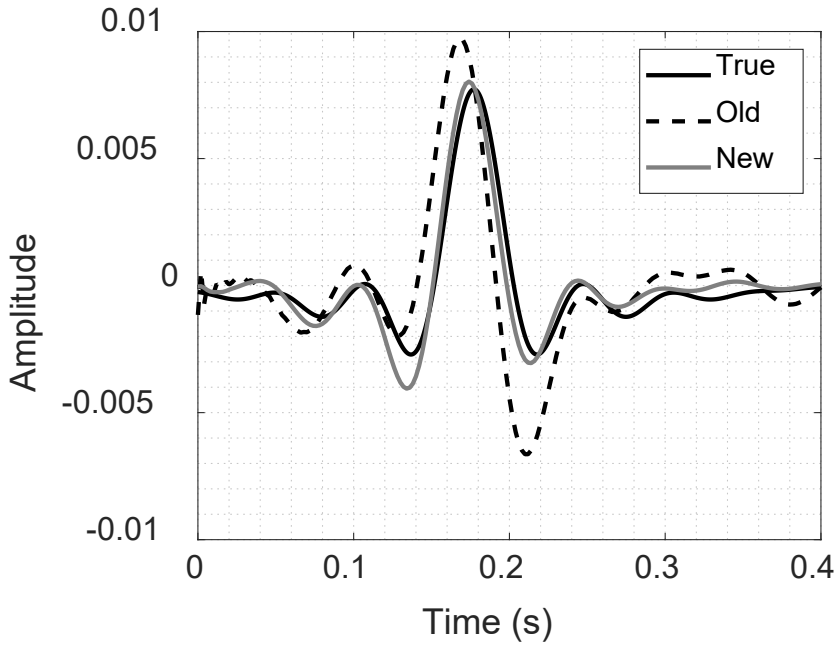
Estimated wavelets



Wavelet 1



Wavelet 2



Wavelet 3

Outline

- ❑ Introduction
- ❑ A robust source-independent FWI
- ❑ Numerical example
- ❑ Conclusions
- ❑ Discussion

Conclusions

- ✓ A robust source-independent FWI method has been developed
- ✓ It does not require any prior wavelet information
- ✓ It does not require an accurate starting model, even an 1D starting model is feasible
- ✓ It is stable for random noises
- ✓ A good estimate of the source wavelet can be obtained from a poorly converged model based on the new proposed wavelet estimation equation
- ✓ It is stable for different types of source wavelets
- ✓ The performance of the new source-independent FWI is similar to that of the known-source-wavelet FWI

Discussion

- Coherent noise tests
- Source-independent elastic FWI
- Source-independent time-lapse FWI
- Solving wavelet non-repeatability for time-lapse seismic
- Field data application
- Other applications

Acknowledges

We thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors, NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 543578-19.

Thank you!