Targeted null-space shuttles for hypothesis testing in time-lapse FWI

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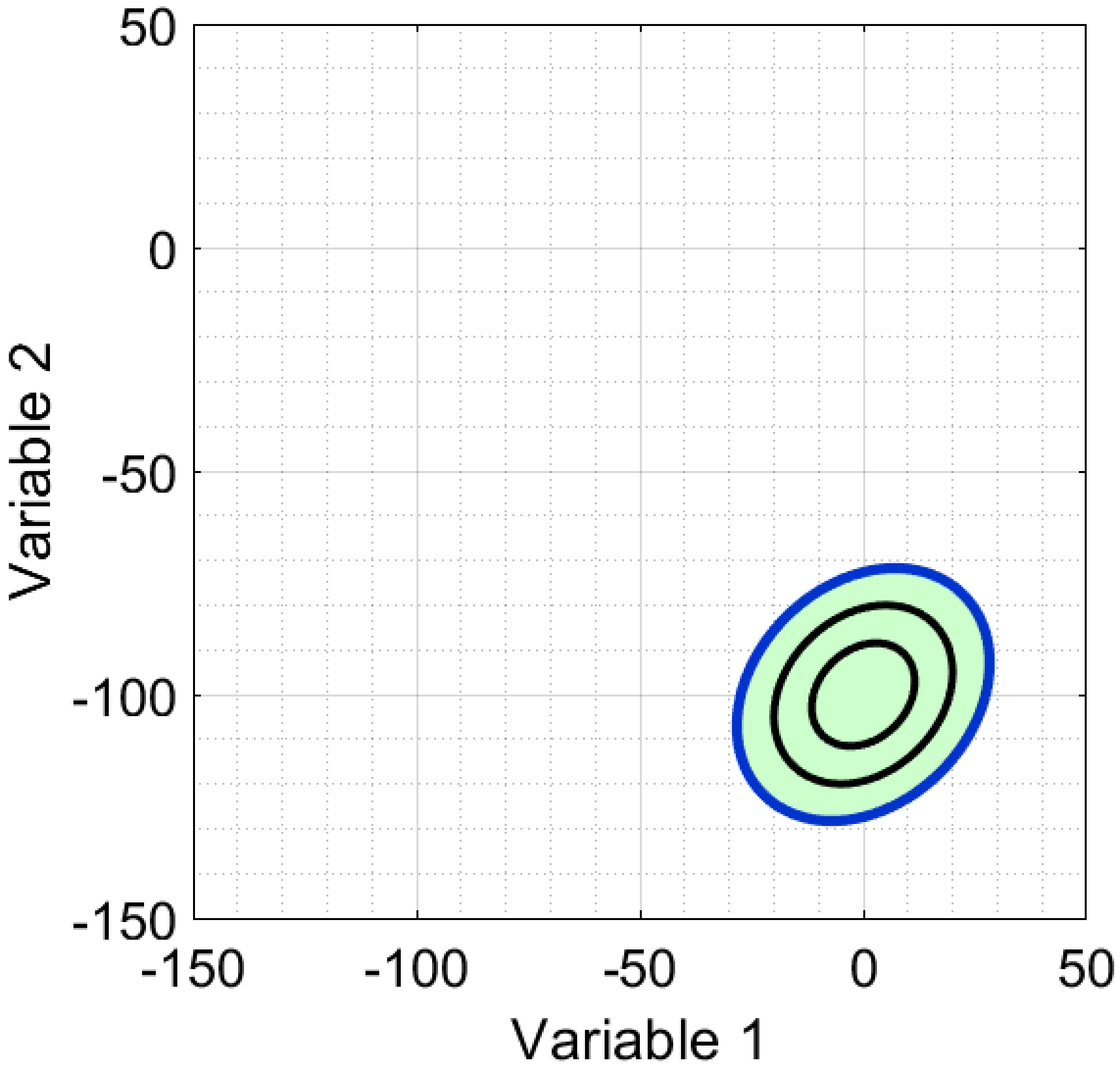
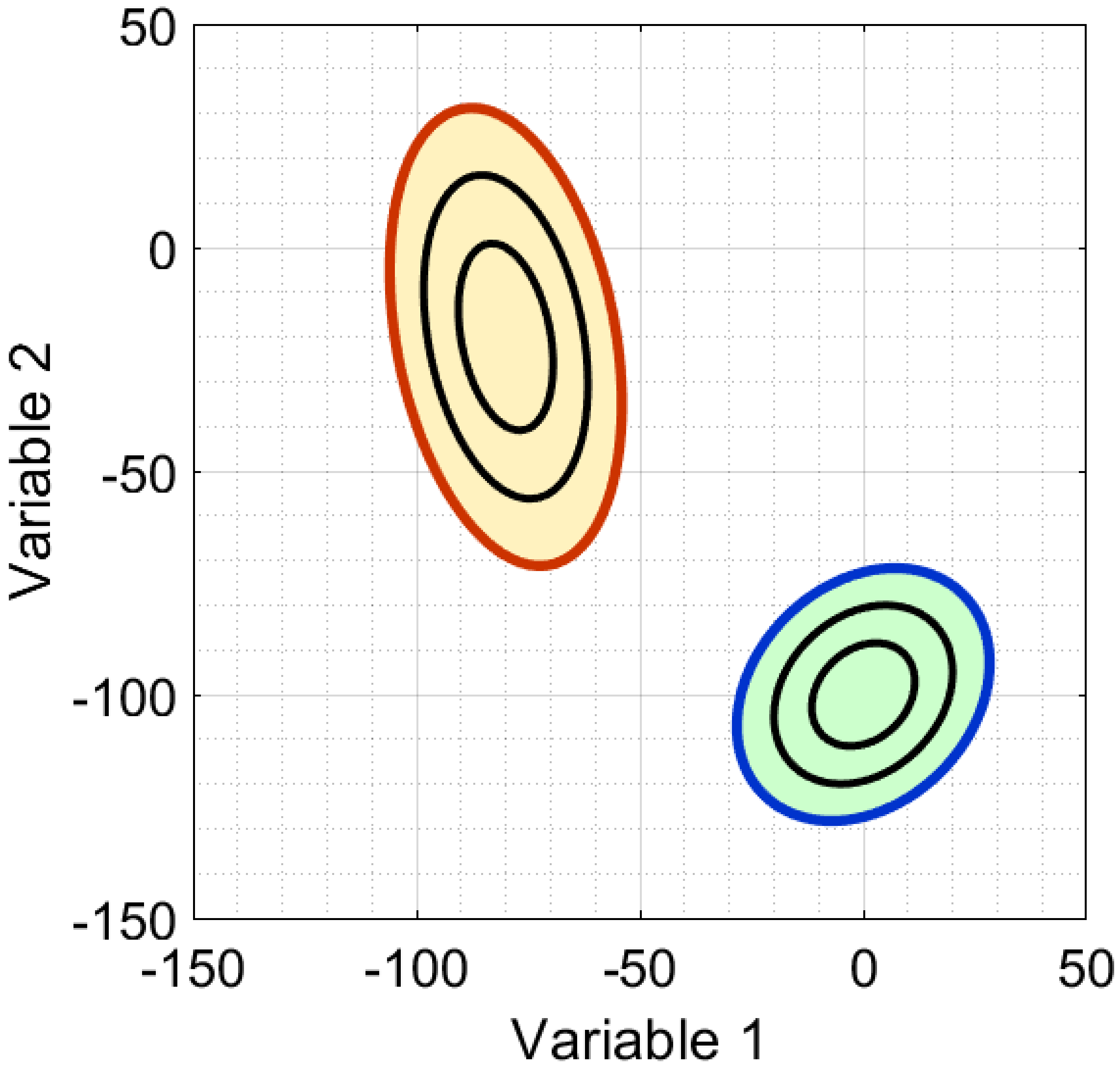
Inversion null-space



Inversion null-space: the set of models that produce good data and prior fit

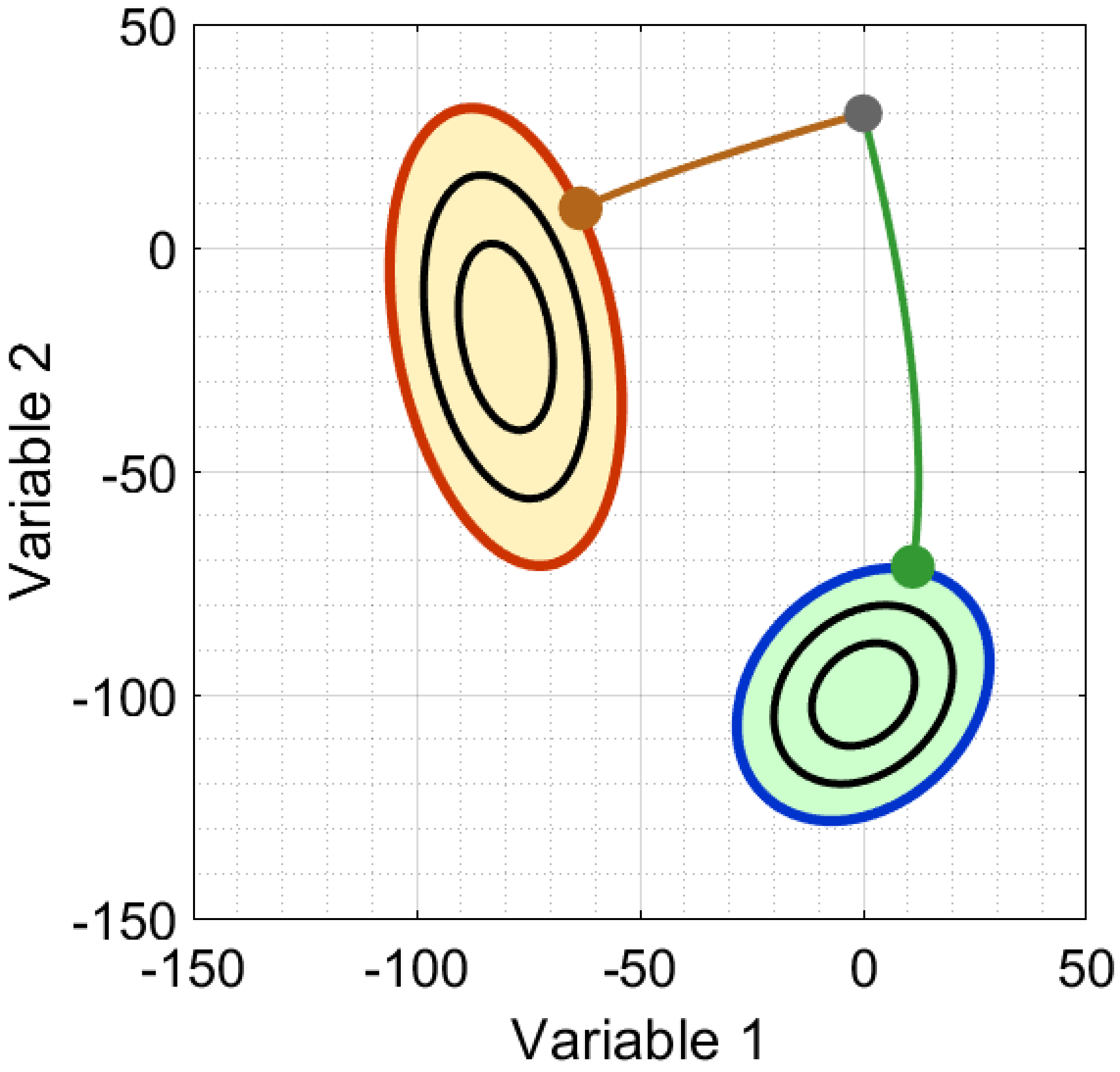
Noise, modeling errors, etc. cause nonzero nullspace in FWI

Time-lapse null-space



In time-lapse FWI we have **two null-spaces**: one for the monitor and one for the baseline

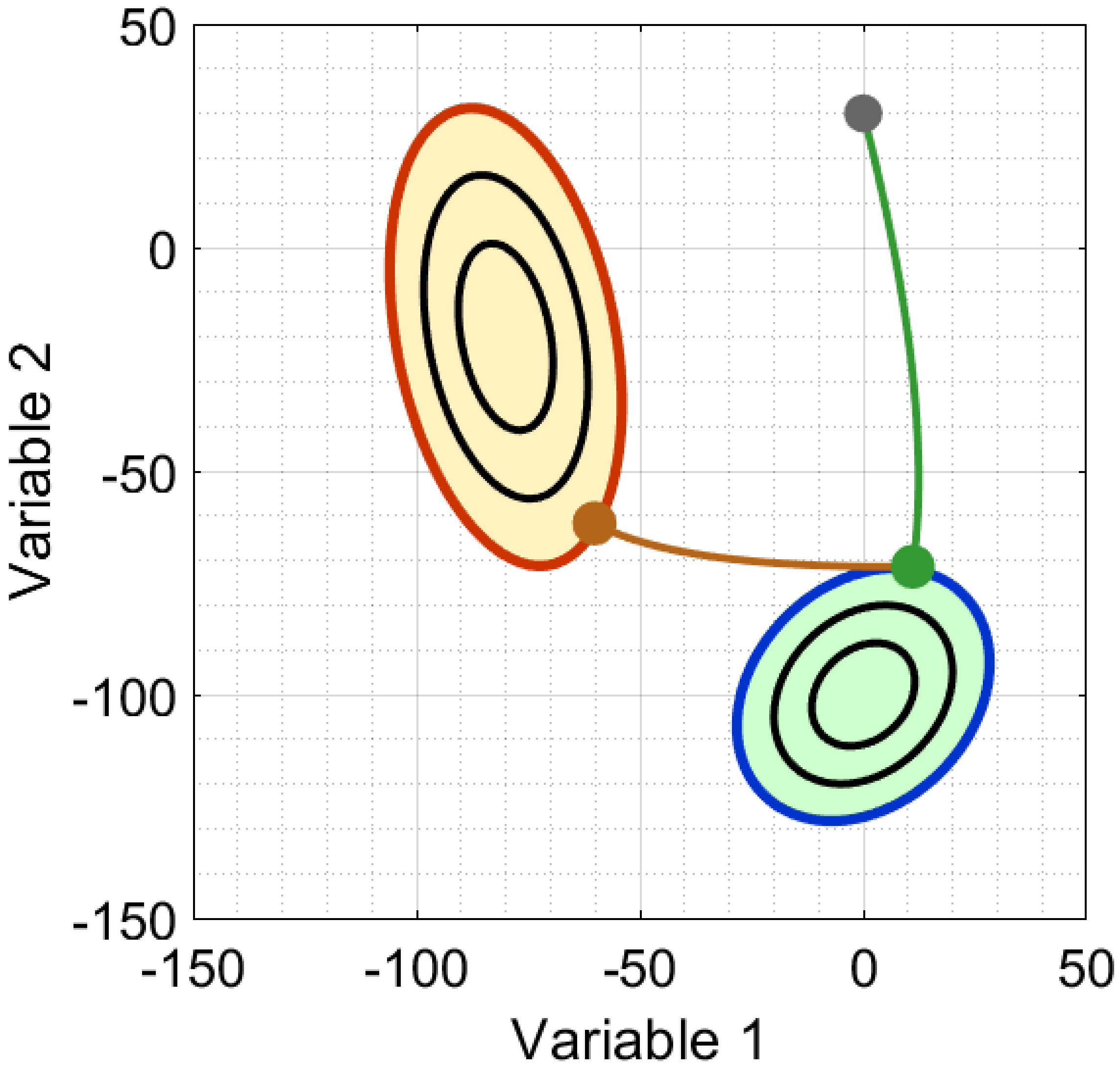
Parallel strategy



Many time-lapse strategies differ in their choice of initial model

This choice changes the null-space location of the inversion result

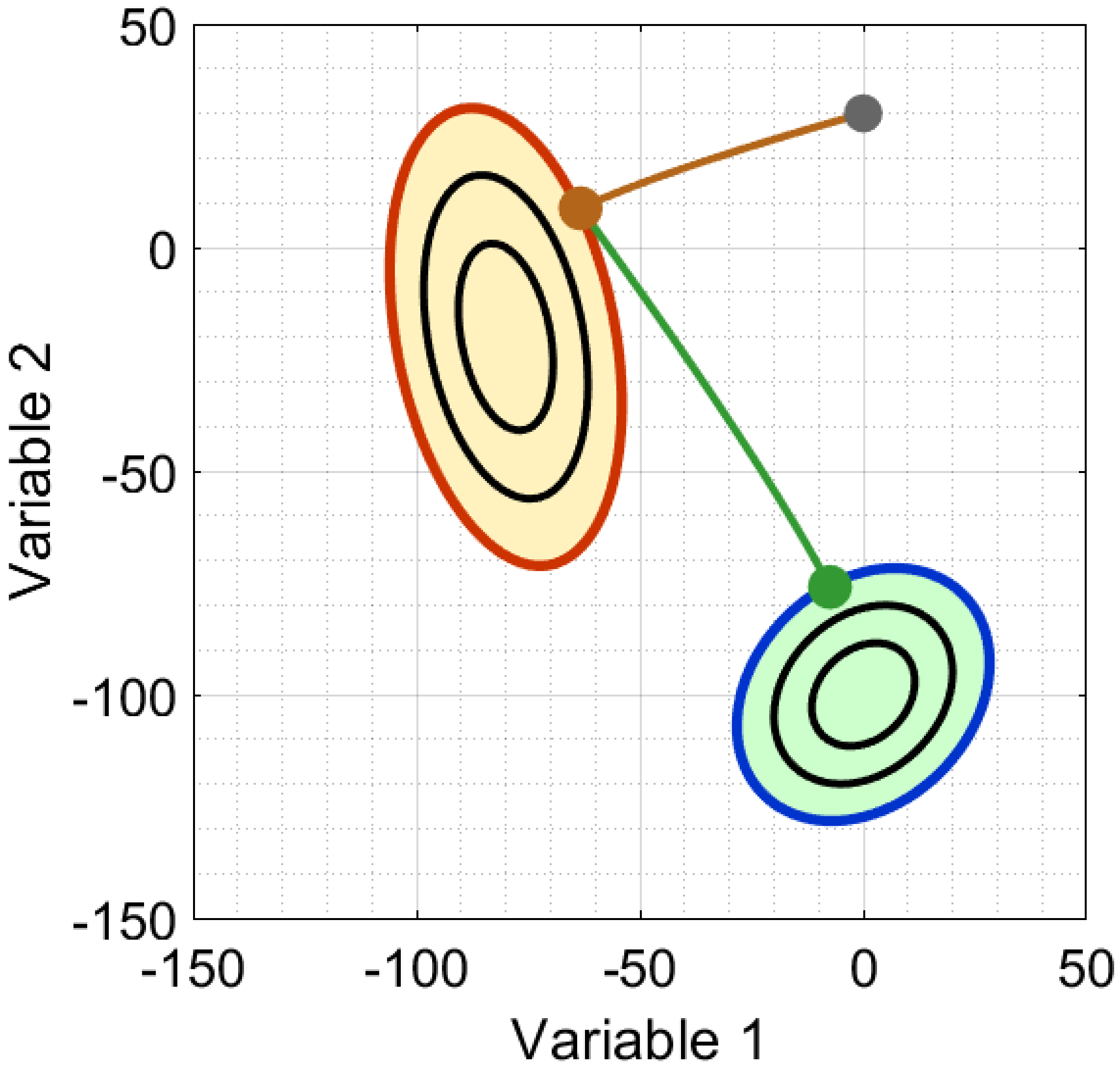
Sequential strategy



Many time-lapse strategies differ in their choice of initial model

This choice changes the null-space location of the inversion result

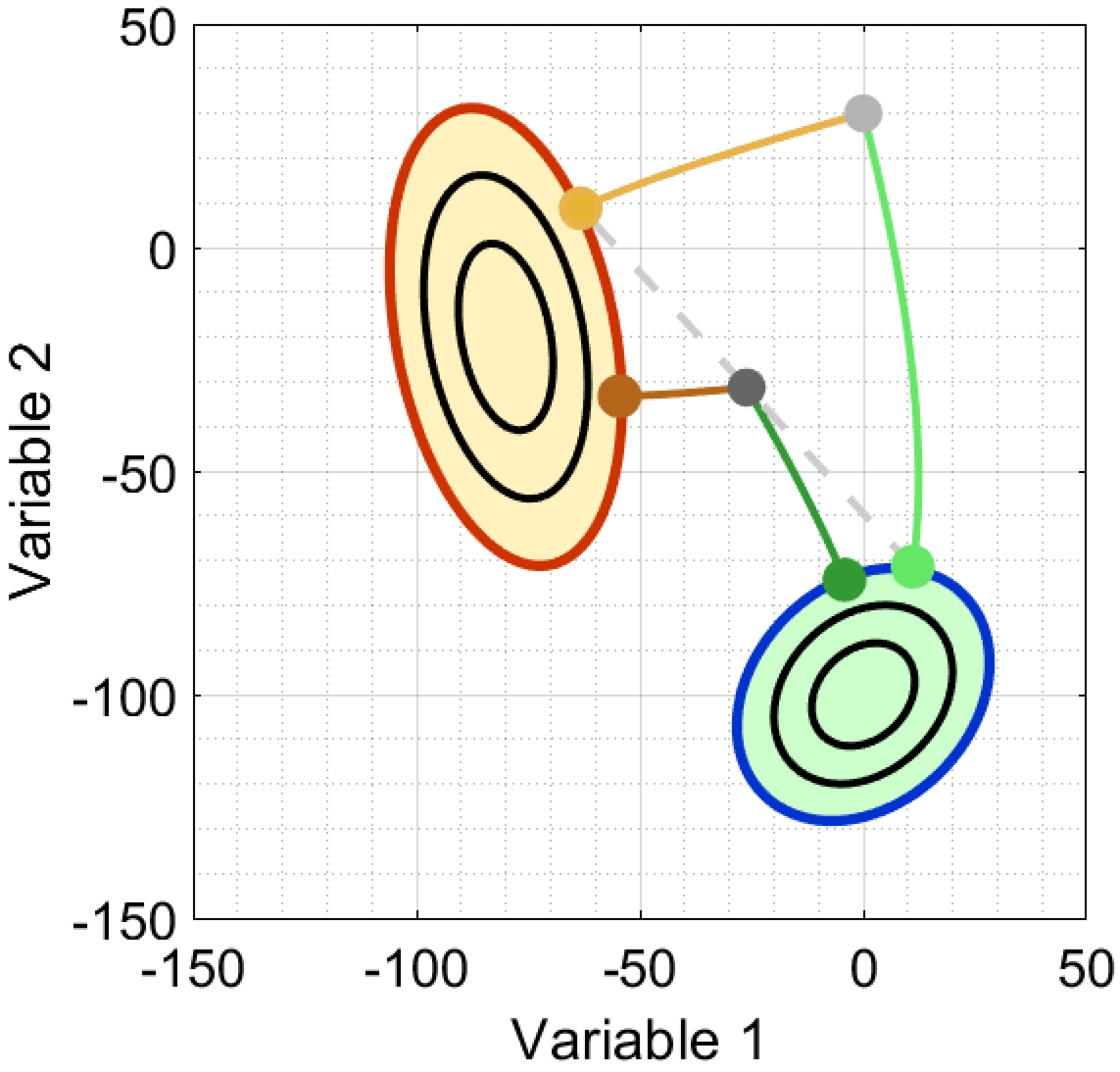
Reverse sequential strategy



Many time-lapse strategies differ in their choice of initial model

This choice changes the null-space location of the inversion result

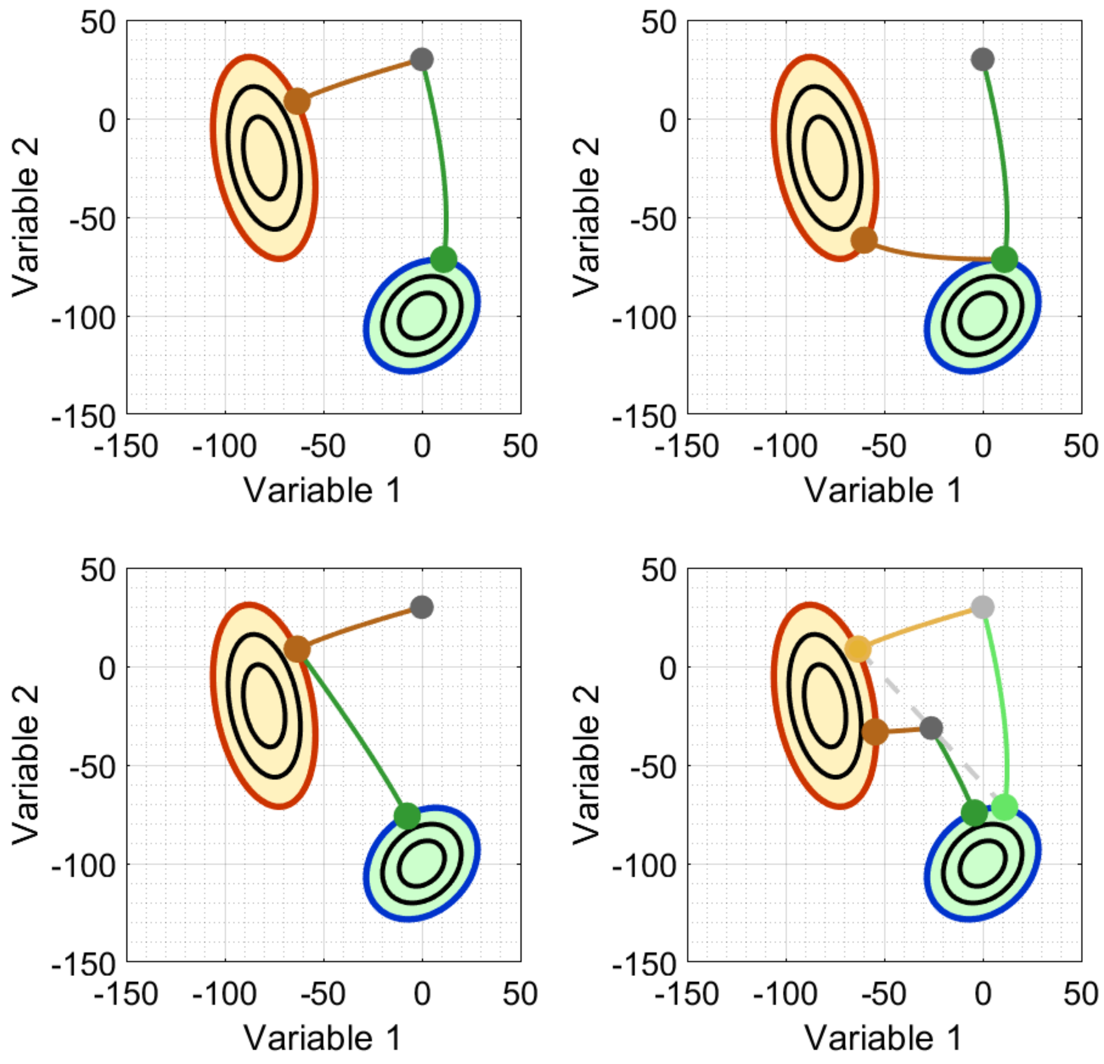
Common model strategy



Many time-lapse strategies differ in their choice of initial model

This choice changes the null-space location of the inversion result

Which starting models give us the best time-lapse differences?



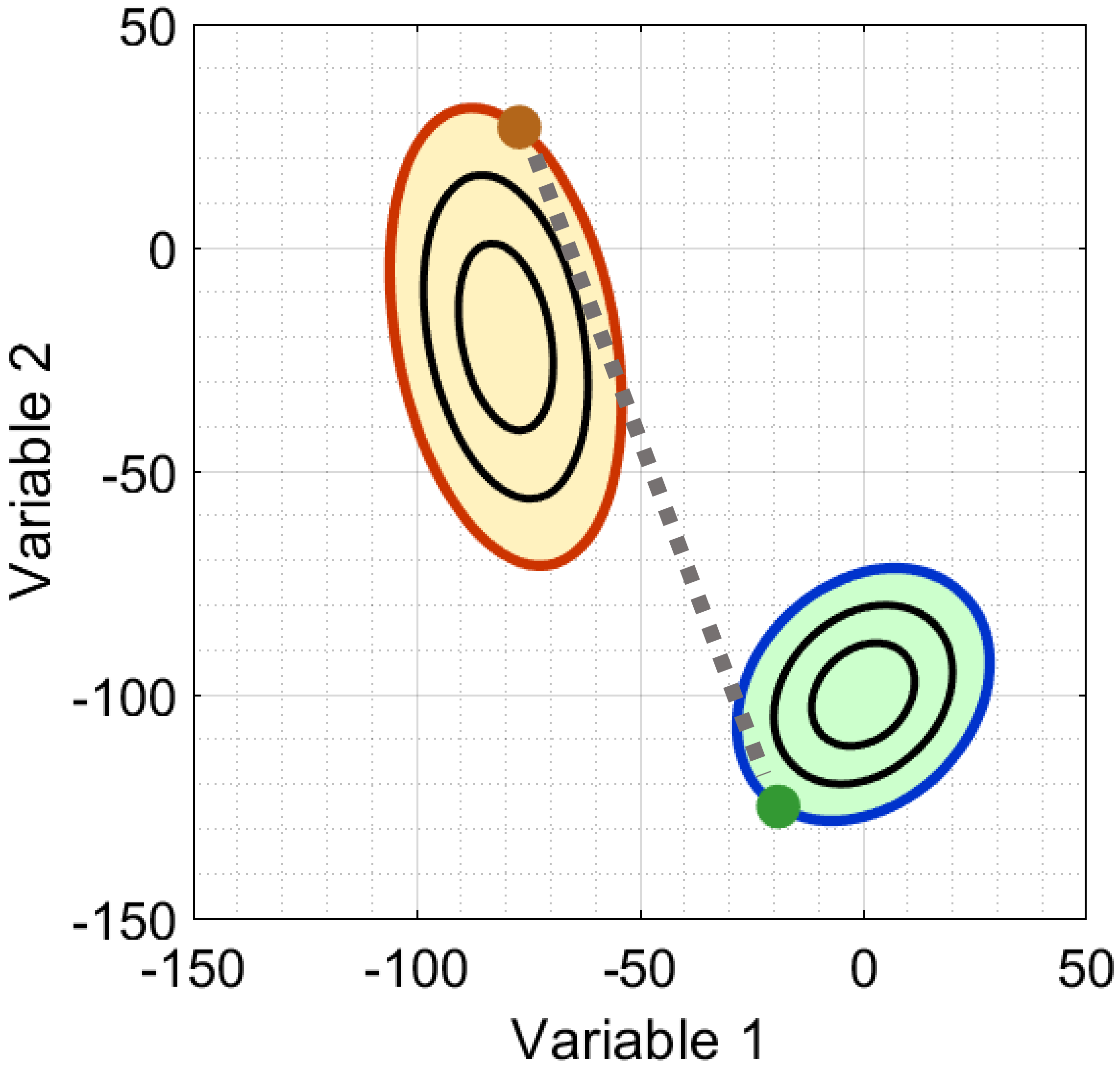
Reframing the problem

Which starting models give us the best time-lapse differences?

→

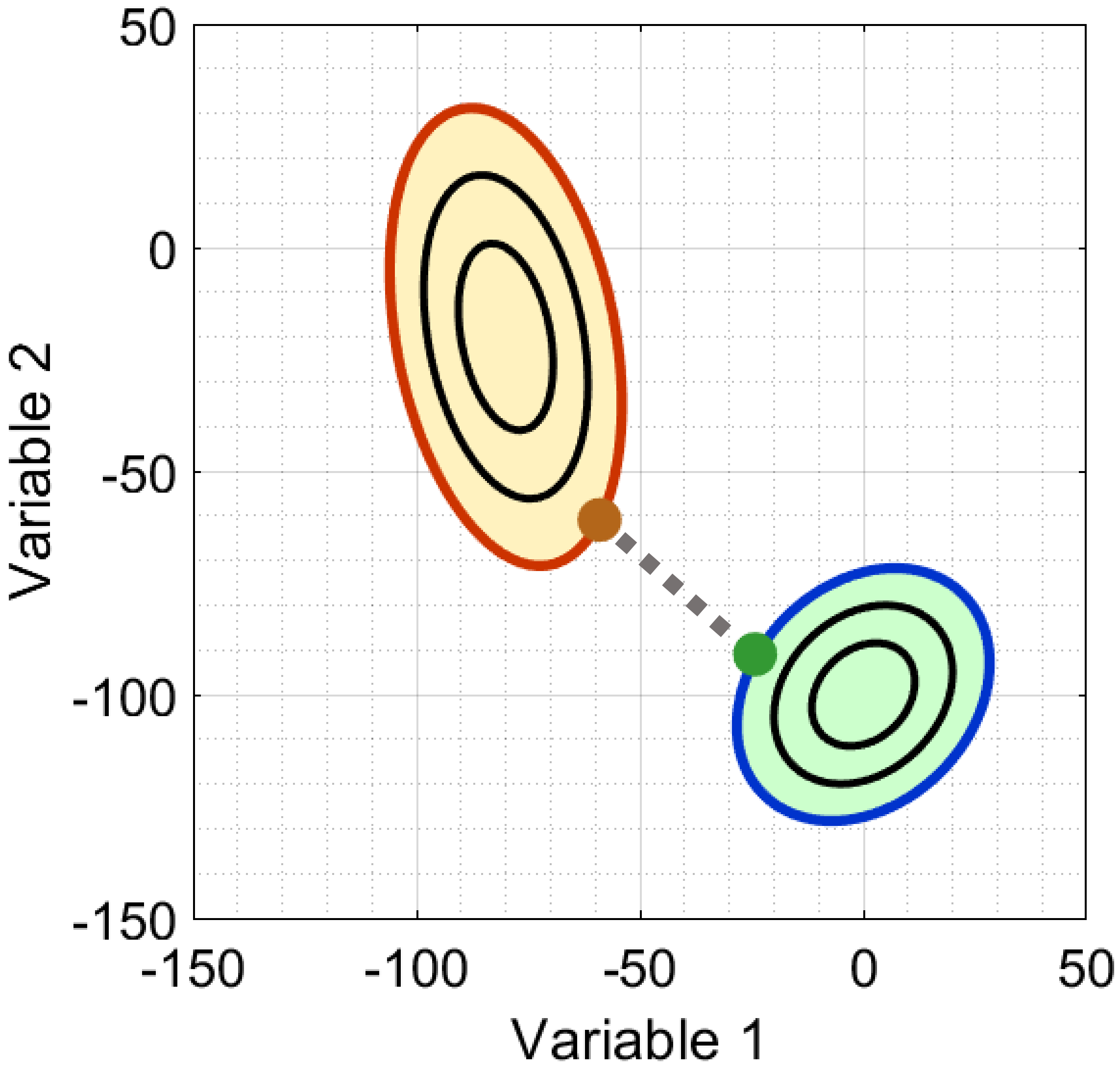
Which points in the null-space give us the best time-lapse differences?

Minimum distance?



Differences in acquisition, noise, etc. can lead to inversion results with unnecessary differences

Minimum distance?



The **minimum difference** includes only changes insisted on by the data

We can find these points with **null-space shuttling**

Null-space shuttles

GEOPHYSICS, VOL. 86, NO. 1 (JANUARY-FEBRUARY 2021); P. R63–R76, 14 FIGS., 1 TABLE.
10.1190/GEO2020-0192.1

Null-space shuttles for targeted uncertainty analysis in full-waveform inversion

Scott D. Keating[1] and Kristopher A. Innanen[1]

ABSTRACT

Full-waveform inversion (FWI) is an effective tool for recovering subsurface information, but many factors make this recovery subject to uncertainty. In particular, unwanted noise in measurements can bias results toward models that are not representative of the true subsurface and numerical optimization techniques used in the inversion only allow for approximate minimization of the objective function. Both factors contribute to the nonuniqueness of FWI solutions. Assessing the uncertainty that this nonuniqueness introduces can be difficult, due to the large dimensionality of the inversion problem. Fortunately, complete characterization of inversion uncertainty is seldom necessary for applications using an inversion result, meaning that the entire dimensionality of the problem may not be relevant for practical uncertainty quantification. Typically, it is only the uncertainty in a few specific aspects of the inversion that is important (for instance, confidence in a recovered anomaly). A targeted uncertainty quantification, characterizing only the confidence in a specific feature of the subsurface model, can greatly reduce the dimensionality of the uncertainty characterization problem, potentially making it tractable. We have adopted an approach for quantifying the confidence of the inversion in a chosen hypothesis about the recovered subsurface model. We tested each hypothesis through numerical optimization on the set of equal-objective model-space steps, called null-space shuttles. By approximating the null-space shuttle that maximally violates a given hypothesis about the inversion, this method establishes an effective approximation of the uncertainty in that hypothesis. We tested the use of this technique on several numerical examples for the case of viscoelastic inversion. These examples demonstrate that, at a reasonable computational cost, this method can generate estimates of the lower bound on the maximal uncertainty associated with incomplete numerical optimization. In the viscoelastic examples considered, the velocity variables are much better constrained than the Q and density variables according to this metric.

Null-space shuttle – model-space step within the inversion null-space

Targeted null-space shuttles can navigate inversion null-space to points of interest

This can be used to find the minimal time-lapse difference

Targeted shuttling requirements

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Code requirements

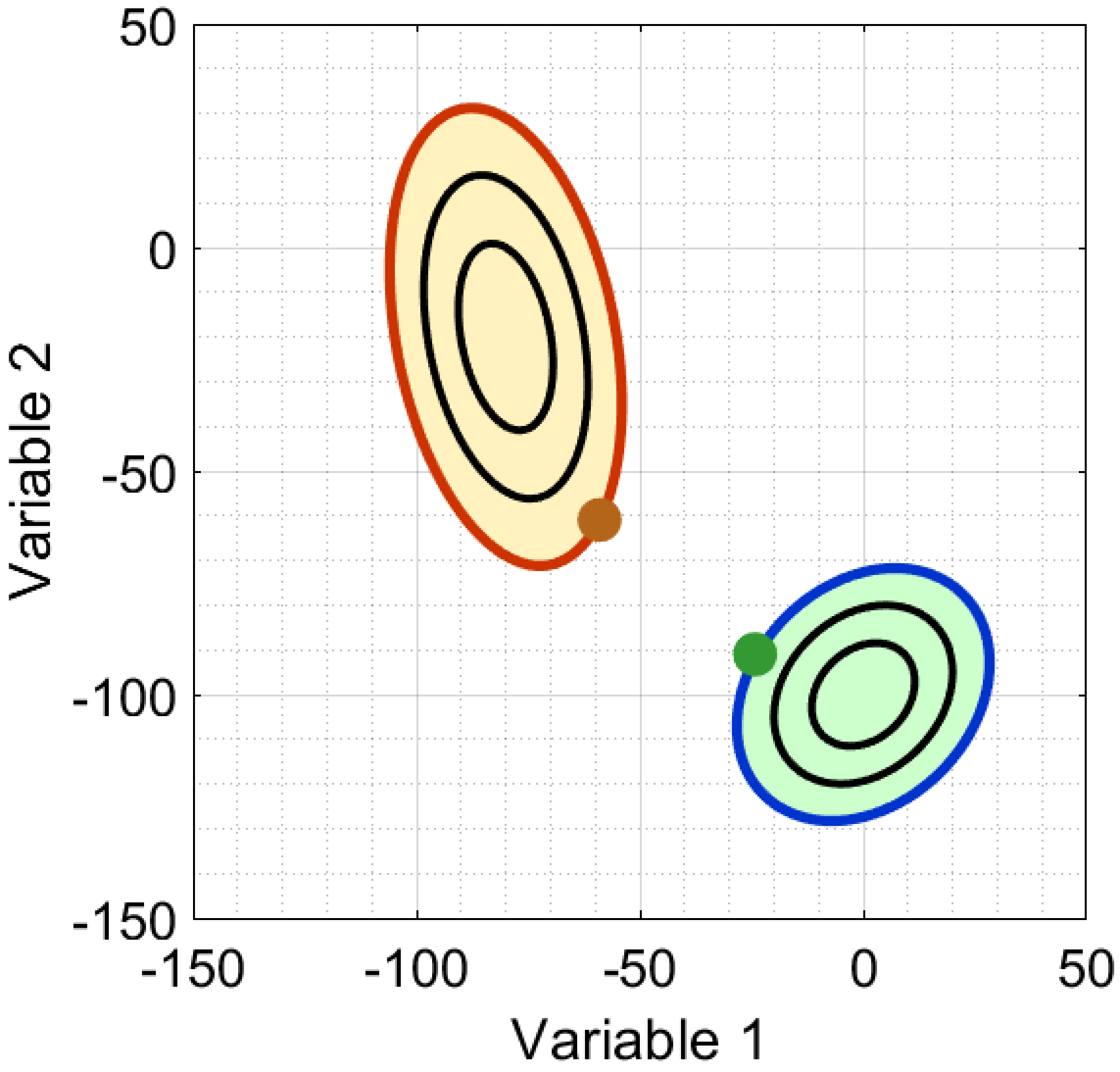
FWI objective function and gradient calculator

Computation requirements

~5-20 FWI gradient evaluations per shuttling iteration

~1-10 shuttling iterations

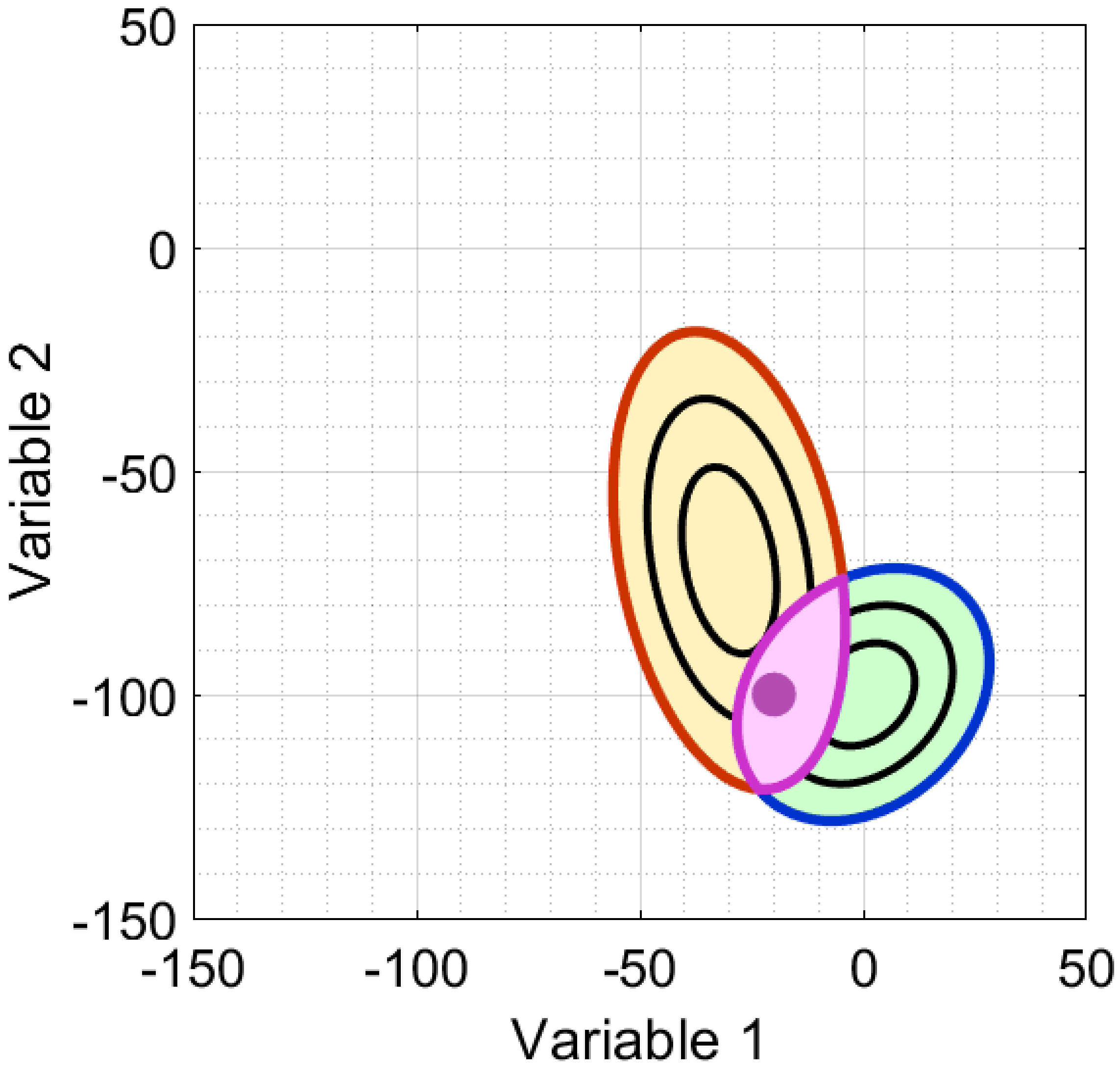
High confidence changes



This approach estimates the **minimum** time-lapse change consistent with data

It may be close to zero if our data don't constrain the changes well

Low confidence changes



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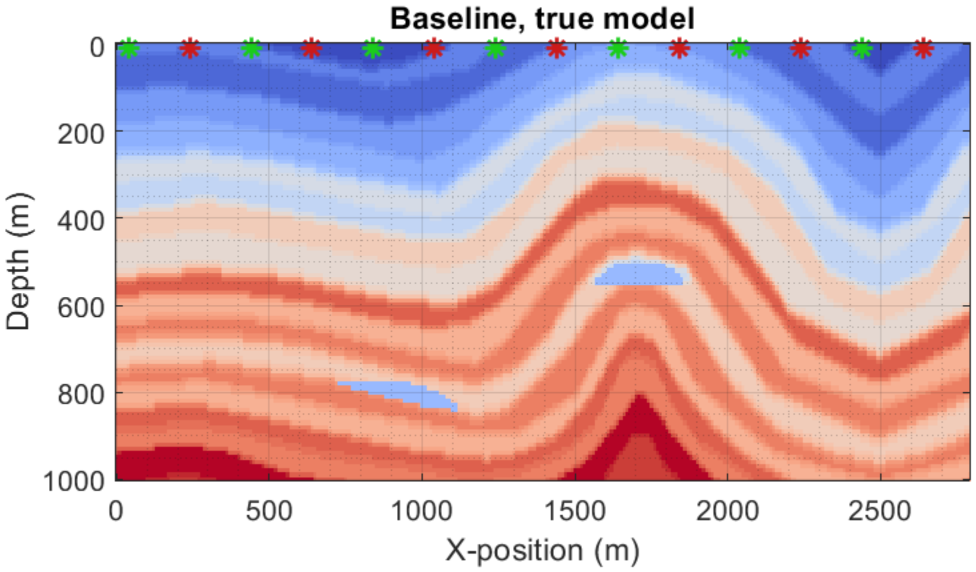
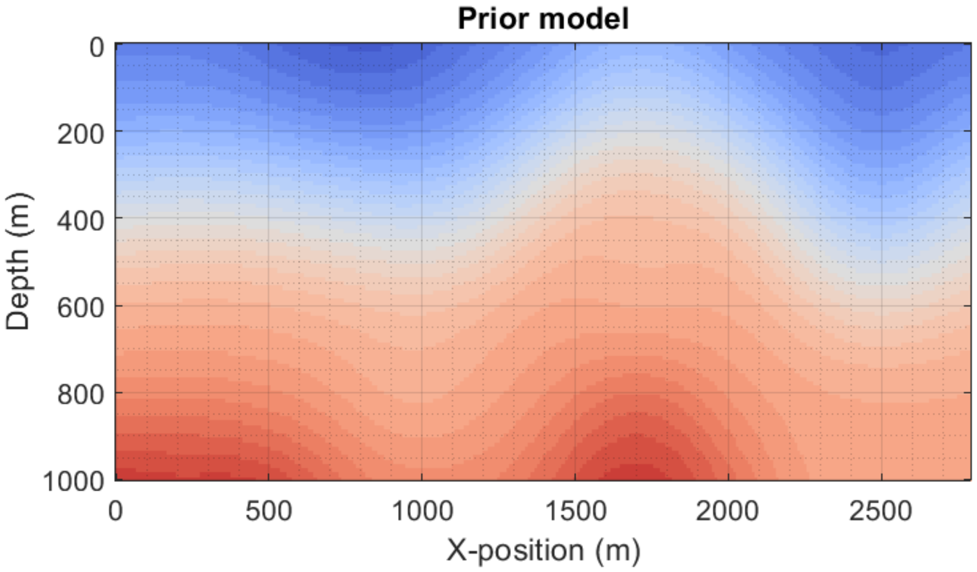
True model

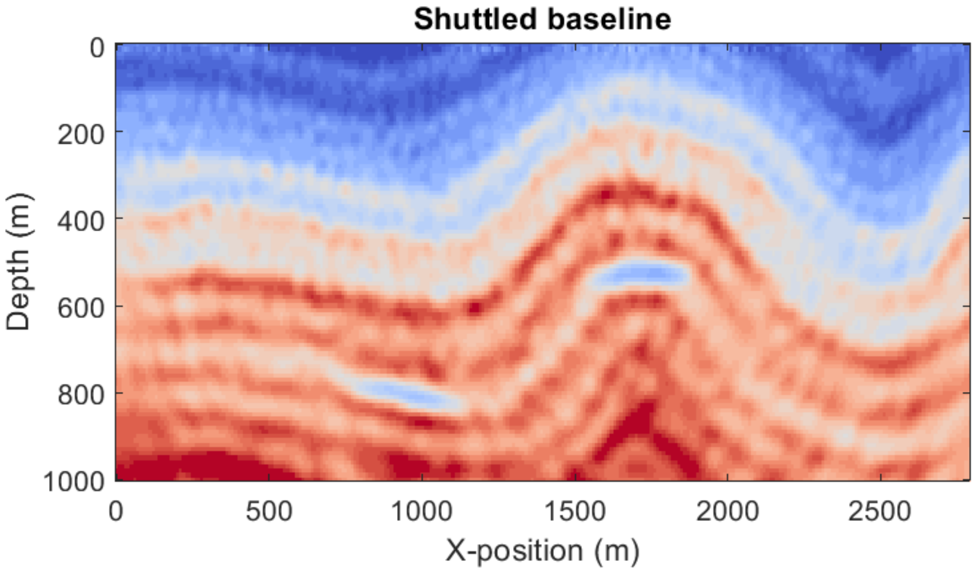


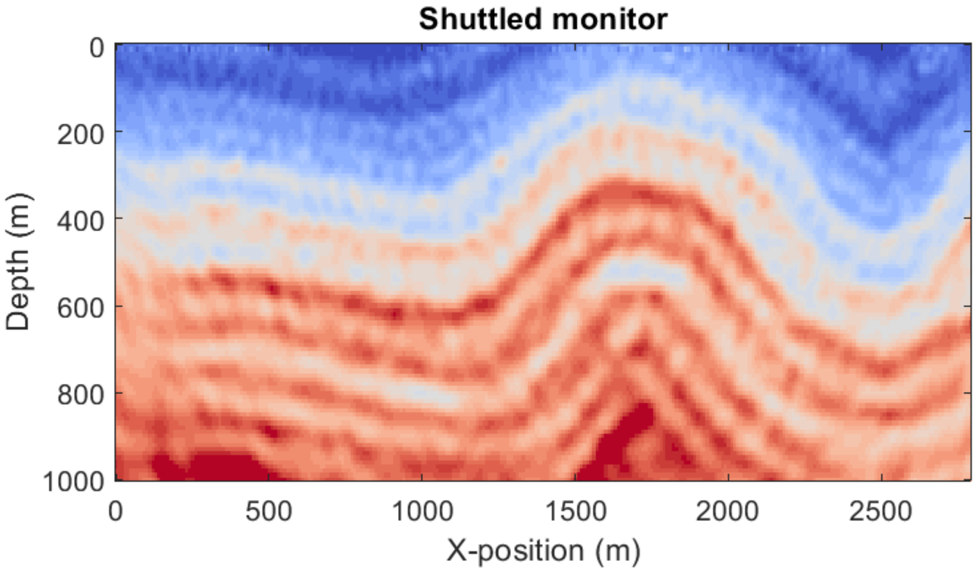
Baseline, true model
Depth (m)
0
200
400
600
800
1000
0
500
1000
1500
2000
2500
X-position (m)

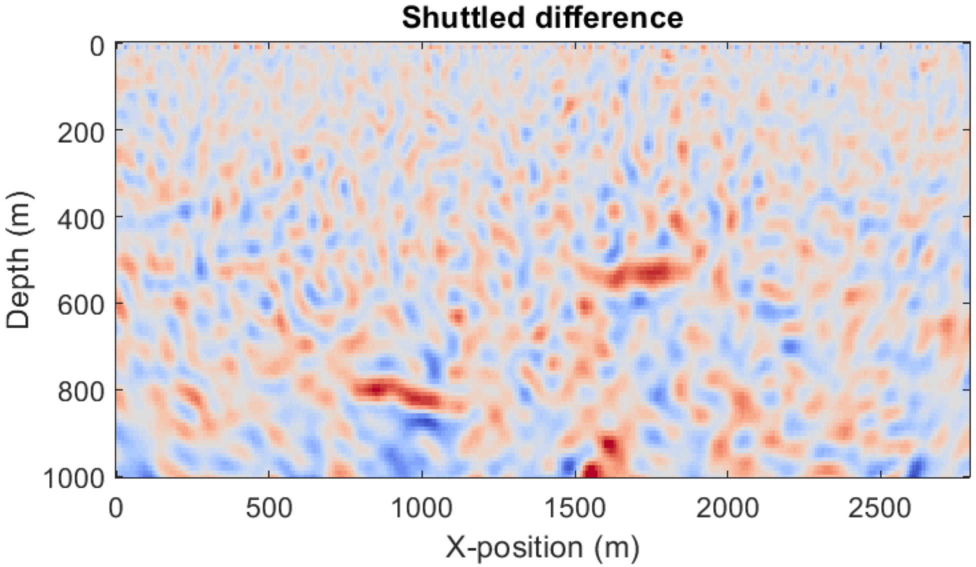


Prior model
Depth (m)
0
200
400
600
800
1000
0
500
1000
1500
2000
2500
X-position (m)

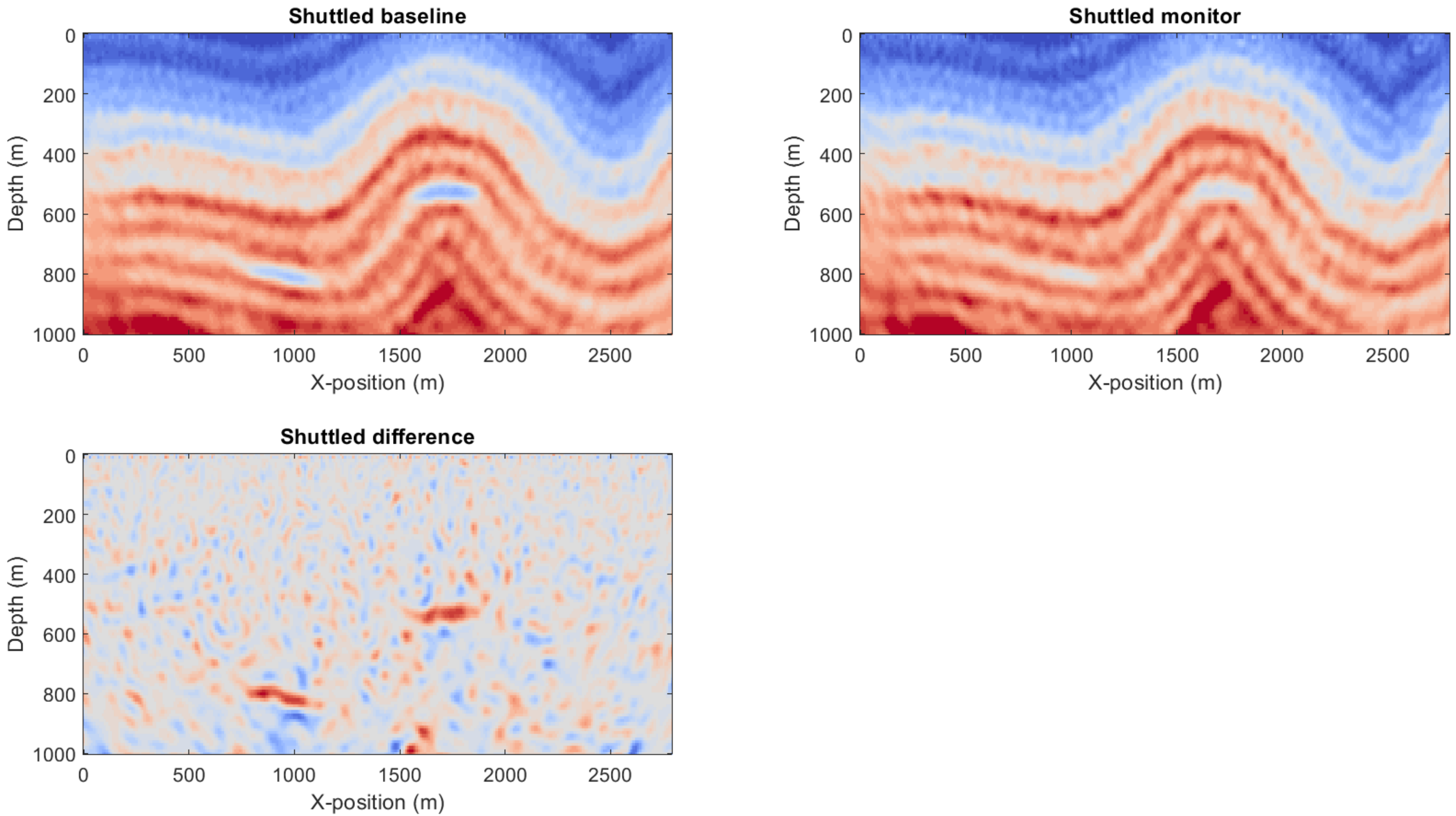
0 iterations shuttling



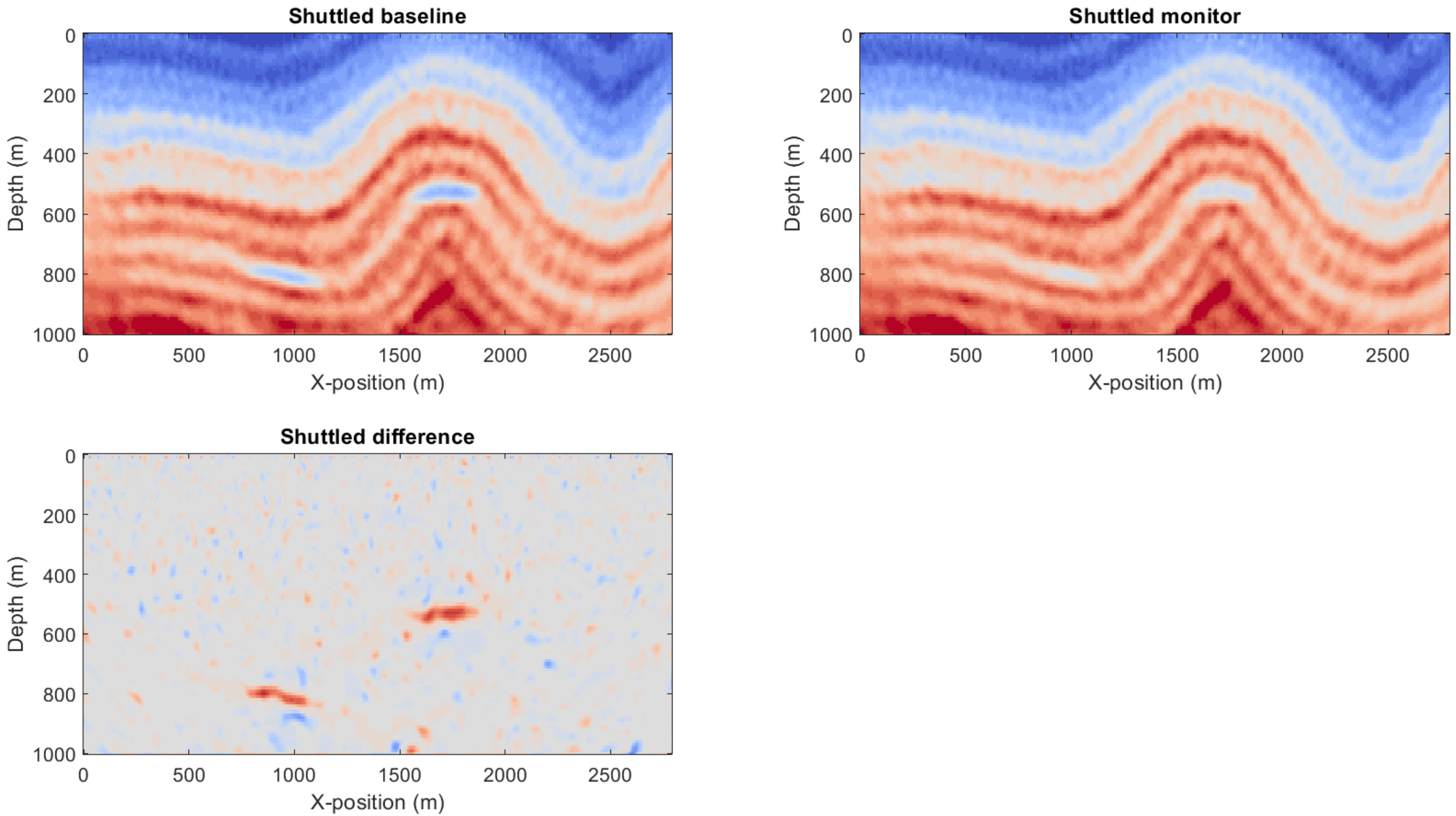




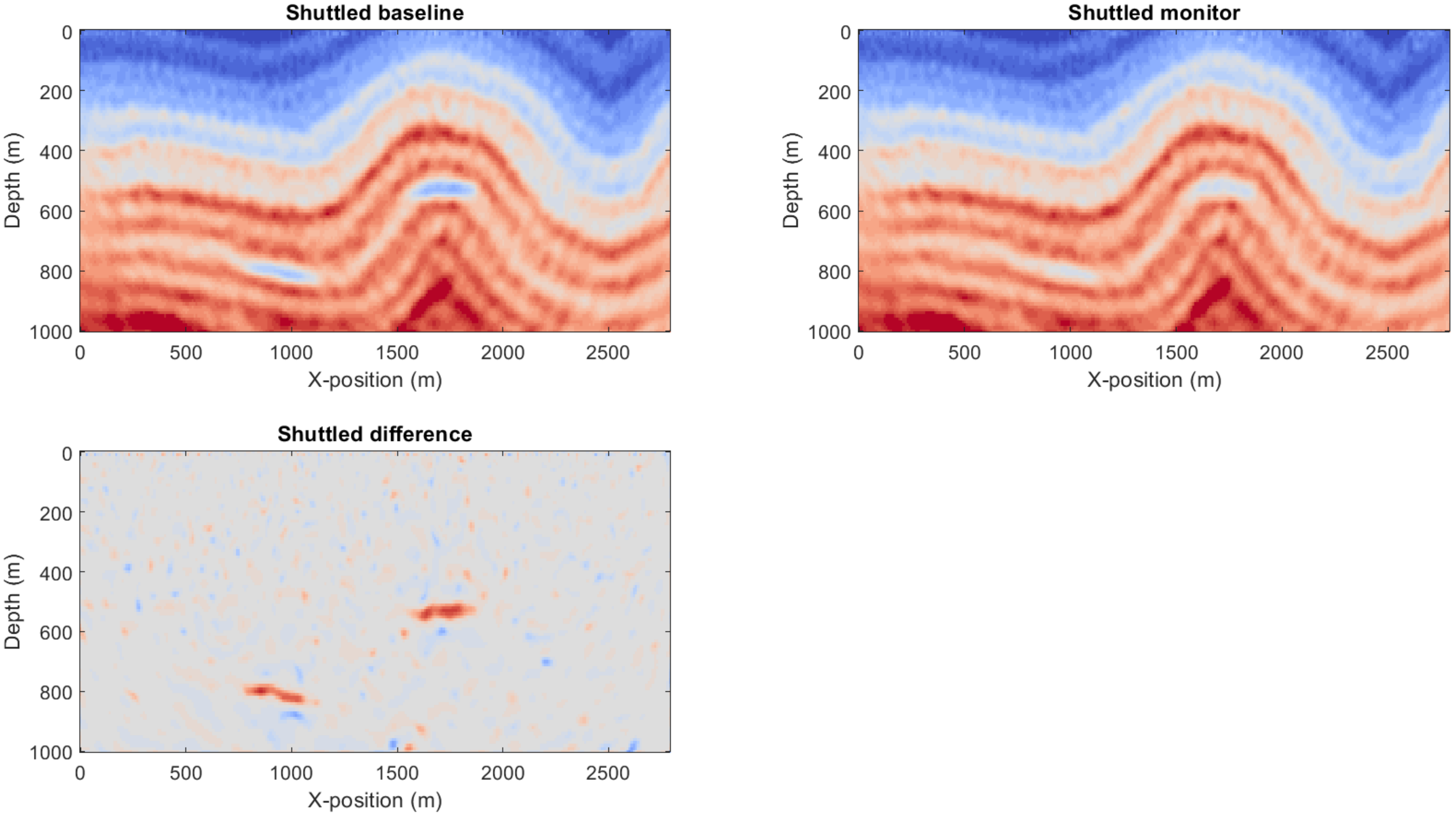
1 iteration shuttling



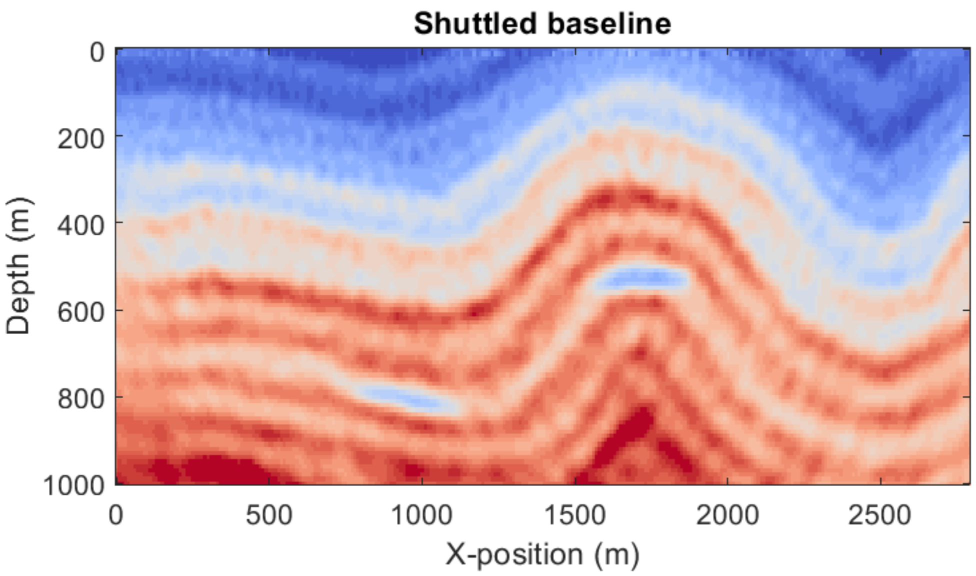
2 iterations shuttling

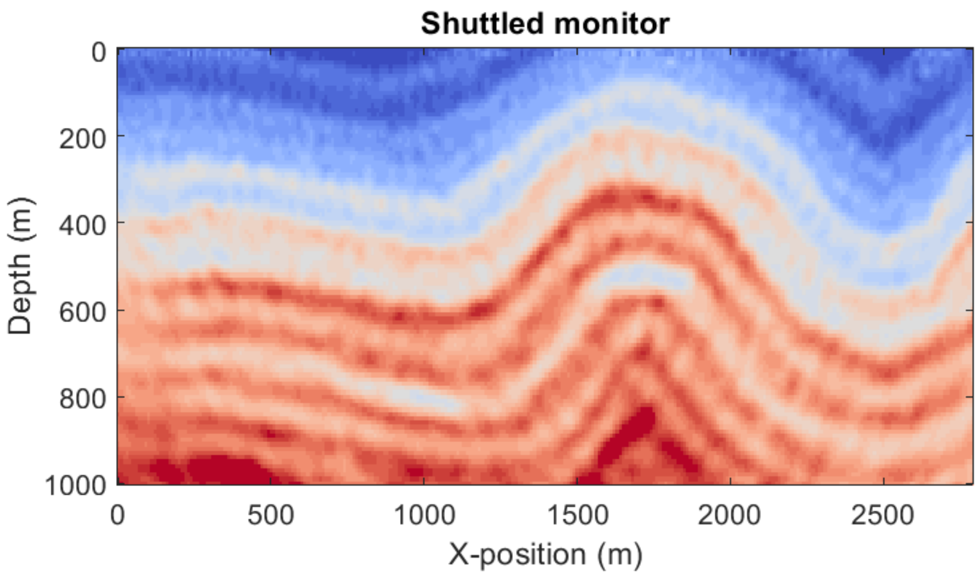


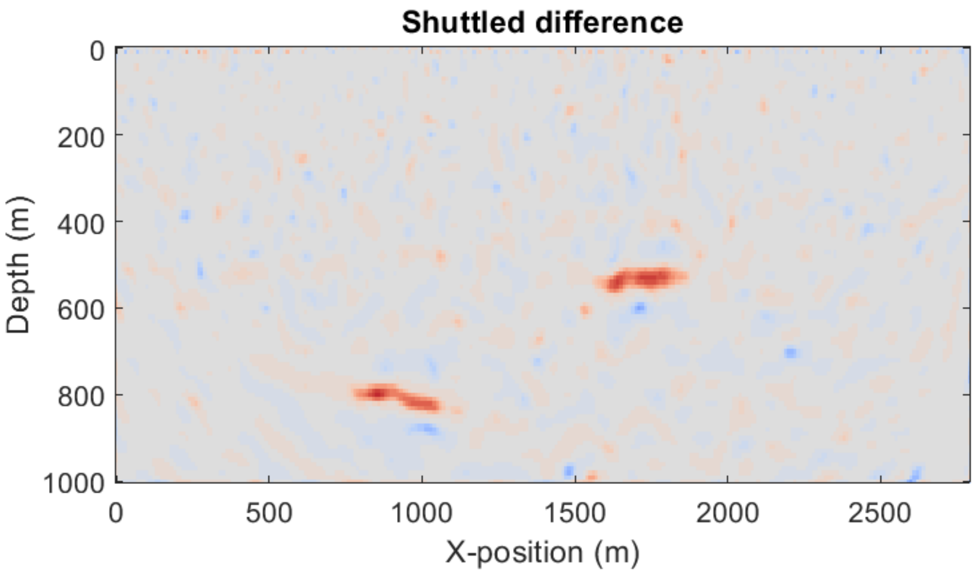
3 iterations shuttling



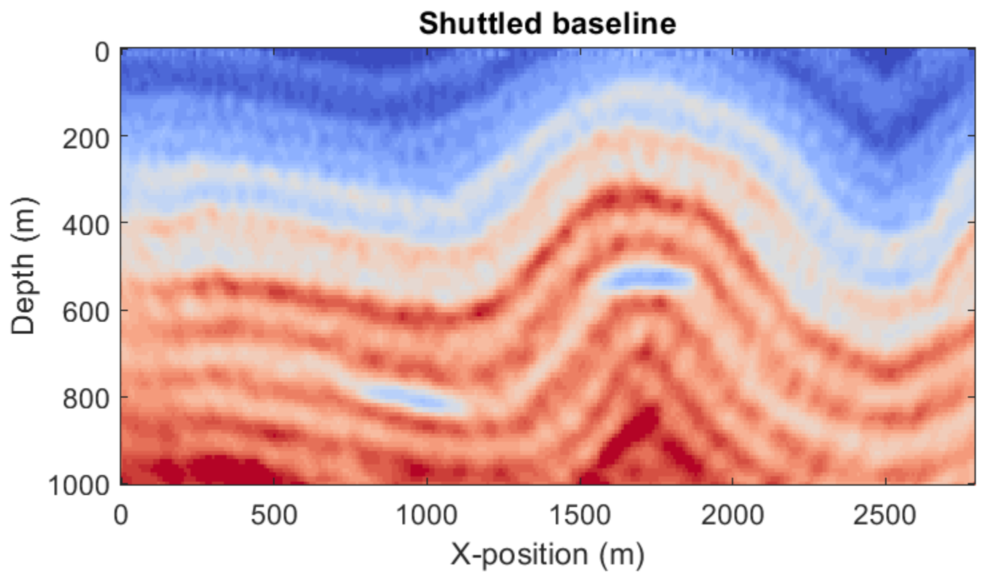
4 iterations shuttling

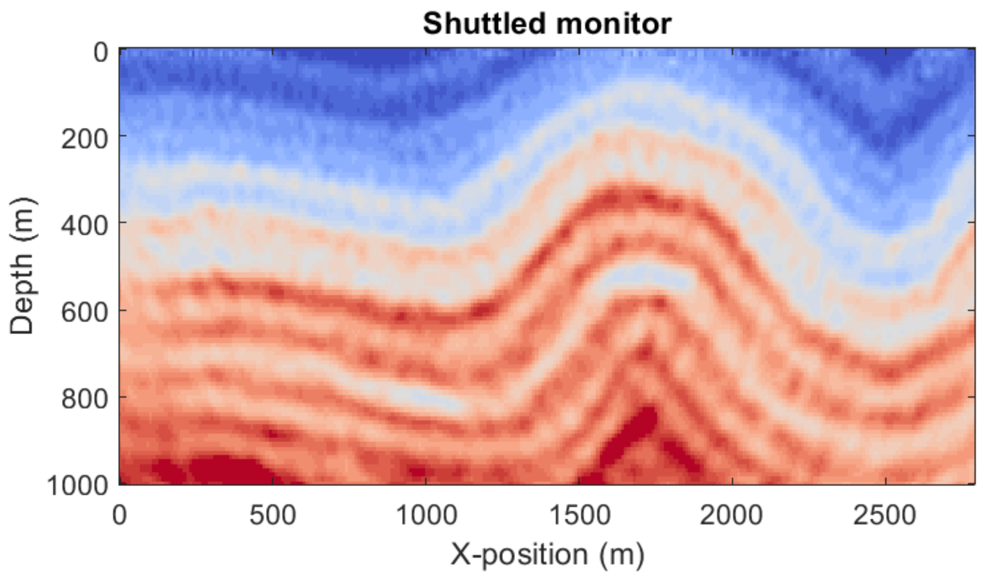


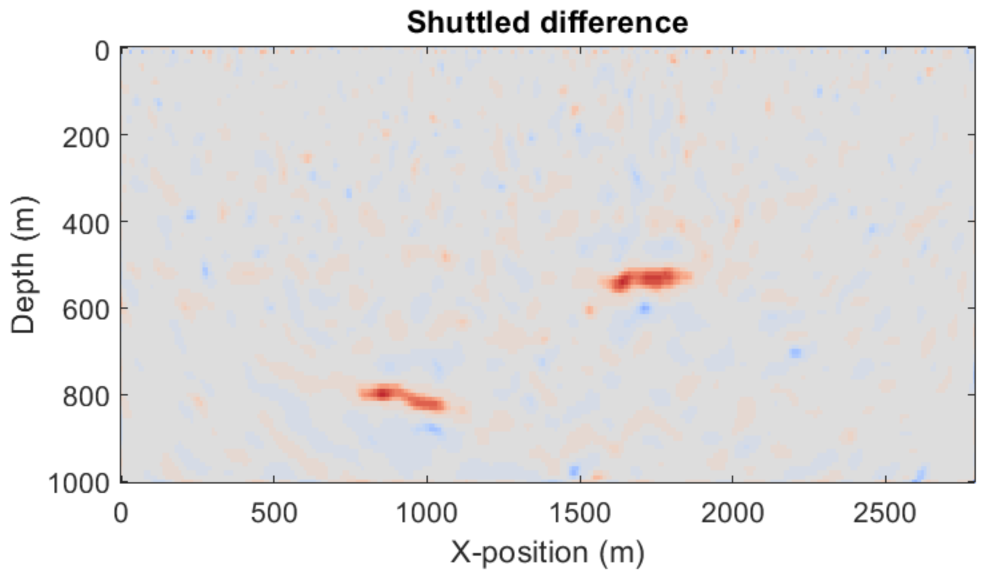




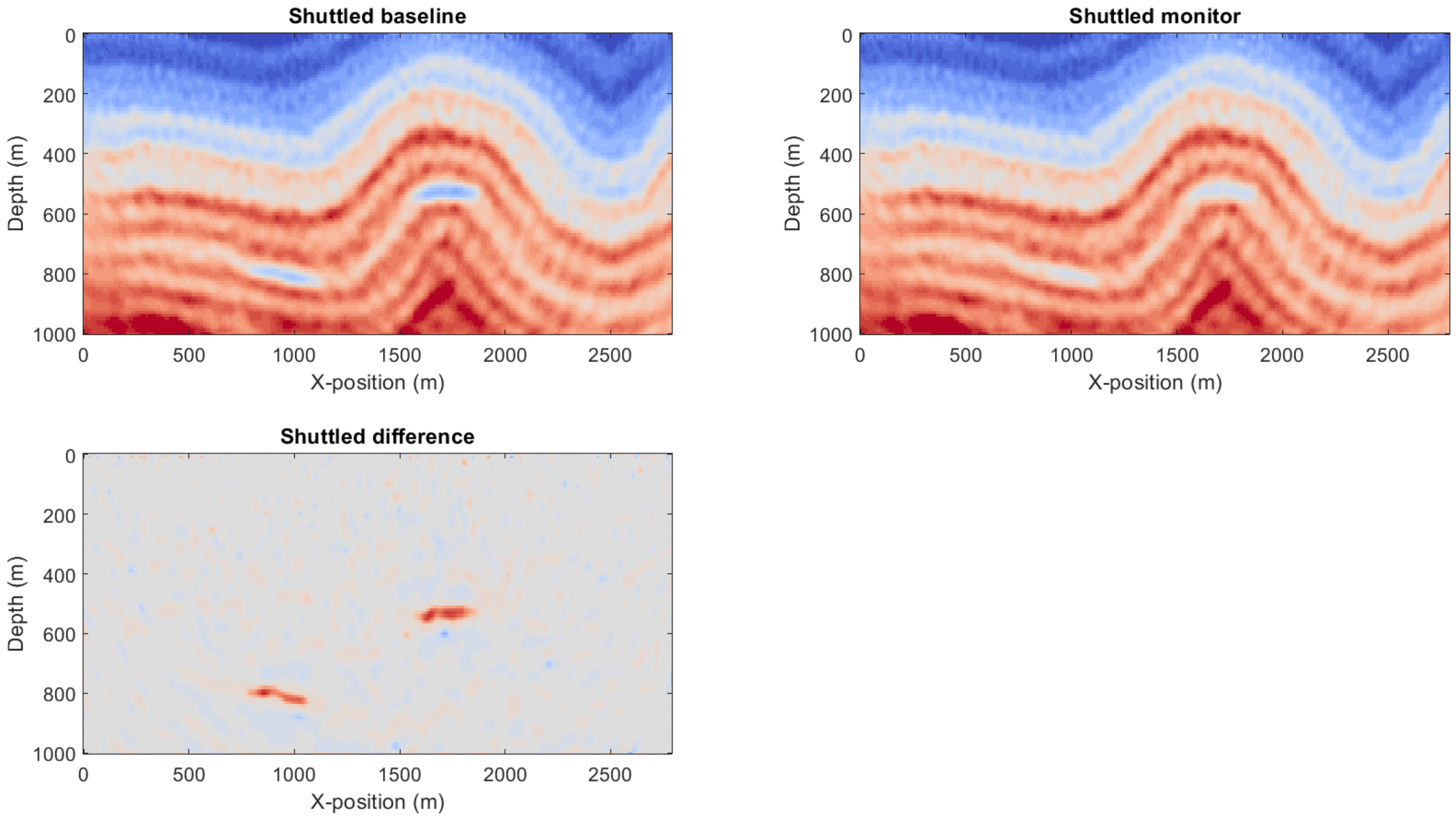
5 iterations shuttling



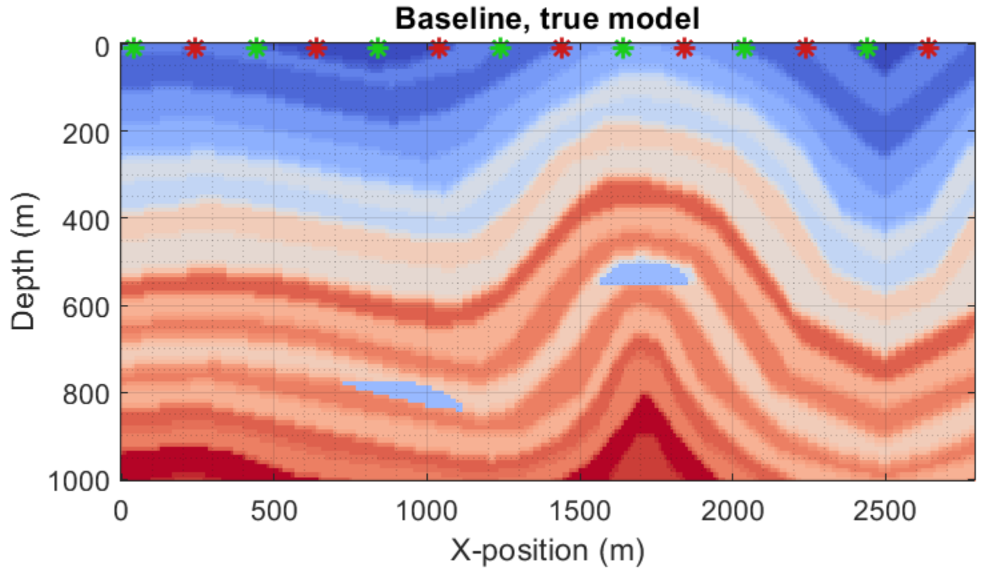




6 iterations shuttling



True model



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Depth (m)
0
200
400
600
800
1000
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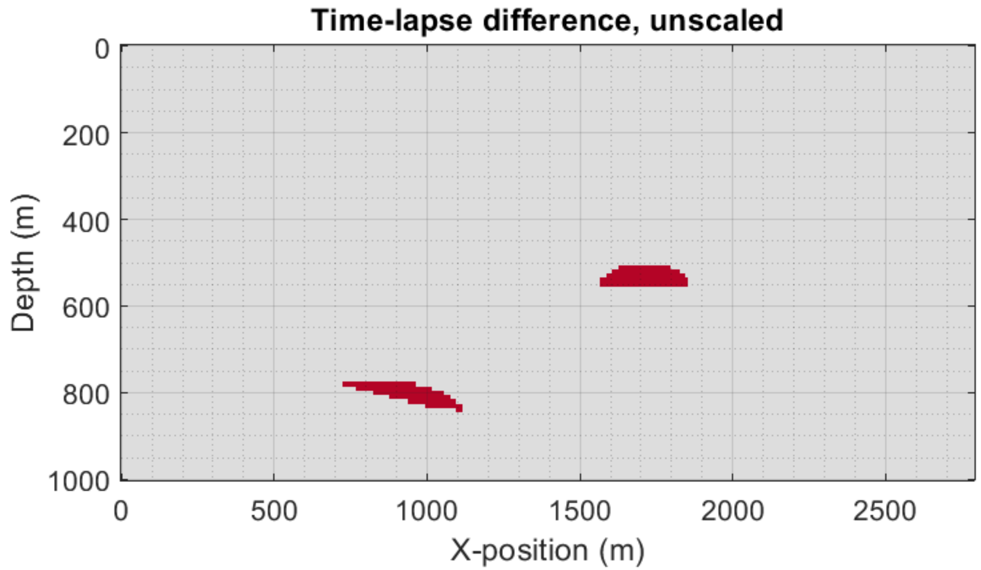
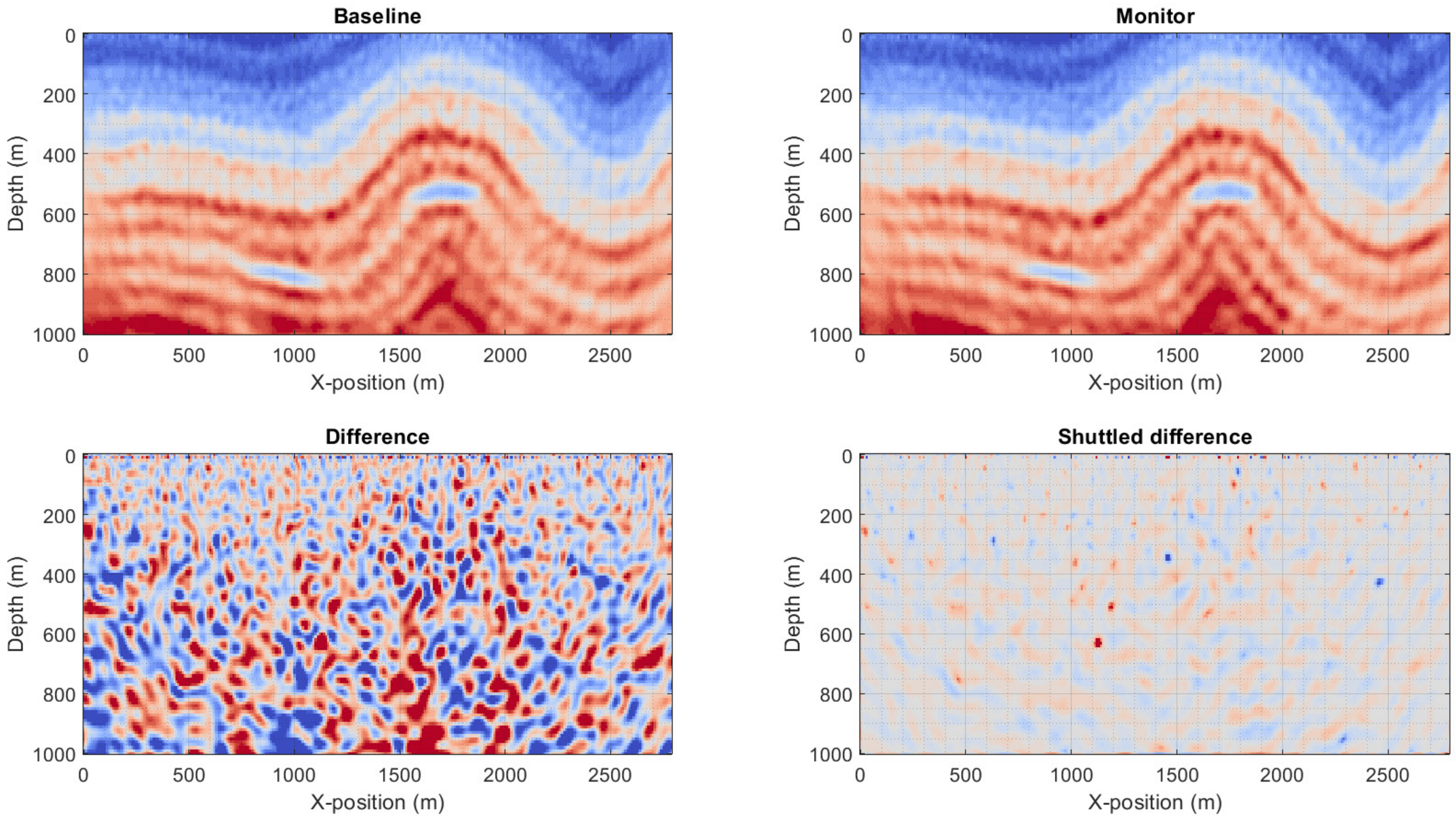
2500
X-position (m)

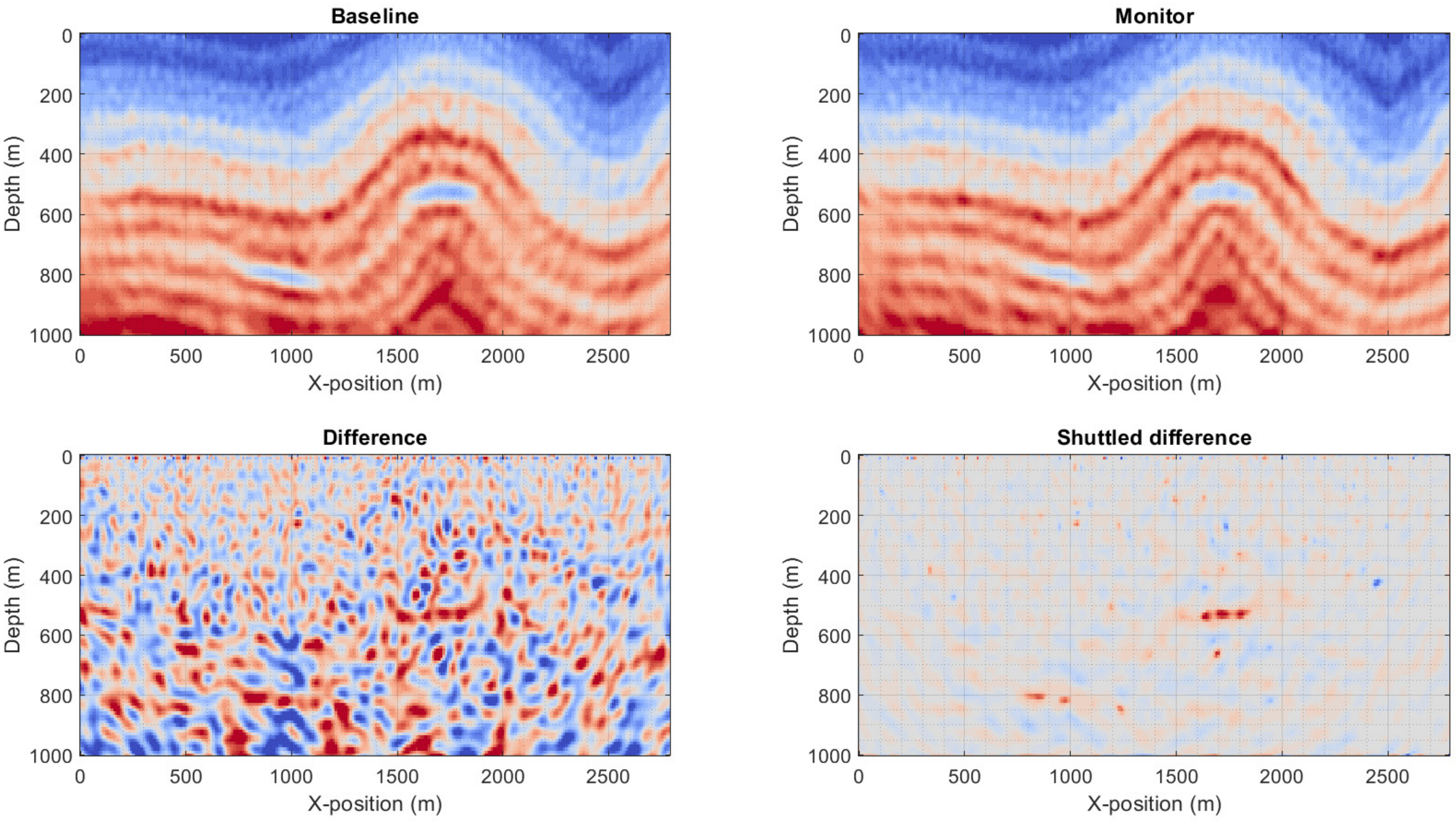


Time-lapse difference, unscaled
Depth (m)
0
200
400
600
800
1000
0
500
1000
1500
2000
2500
X-position (m)

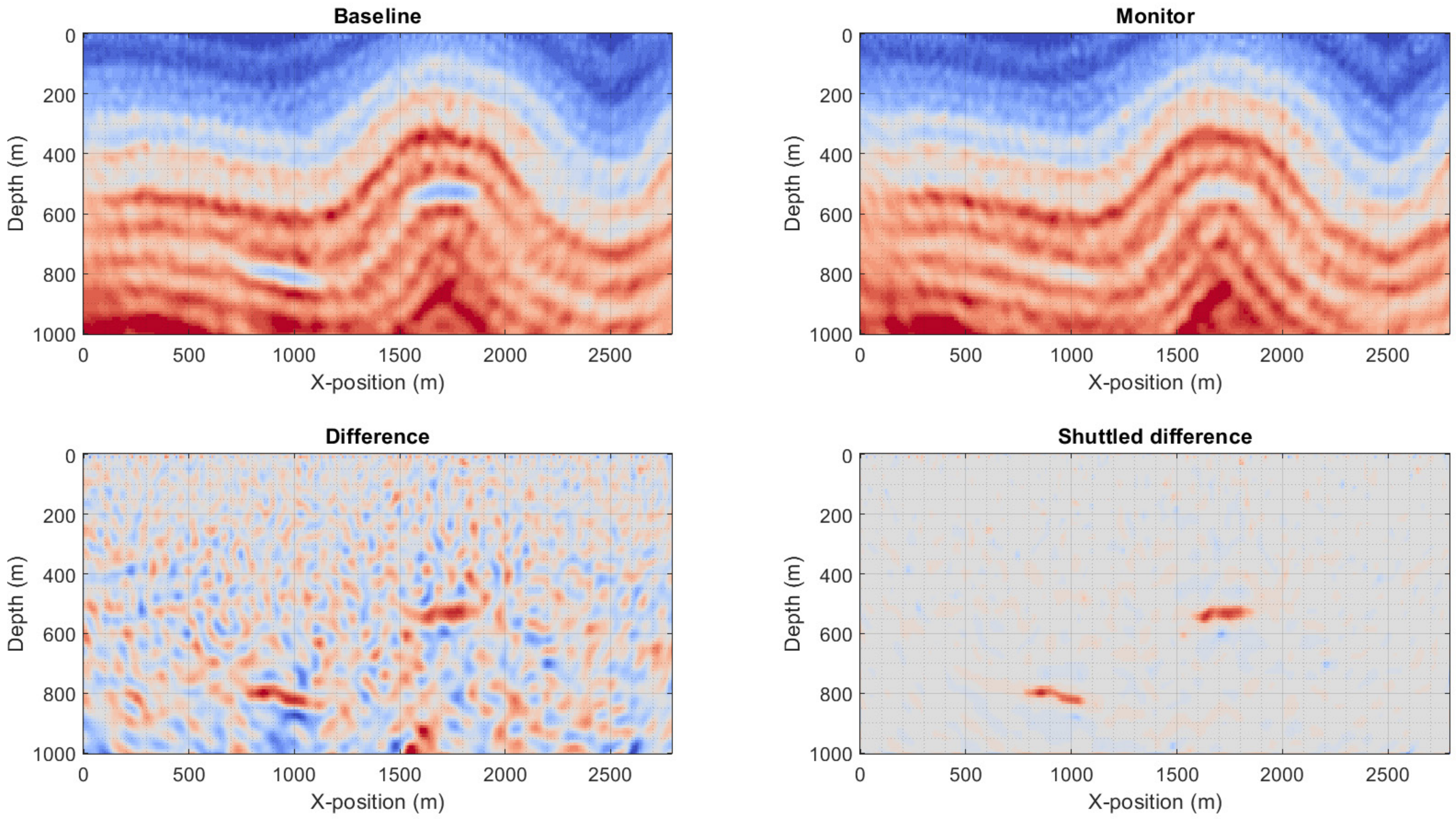
$\Delta v_P = 50\ m/s$



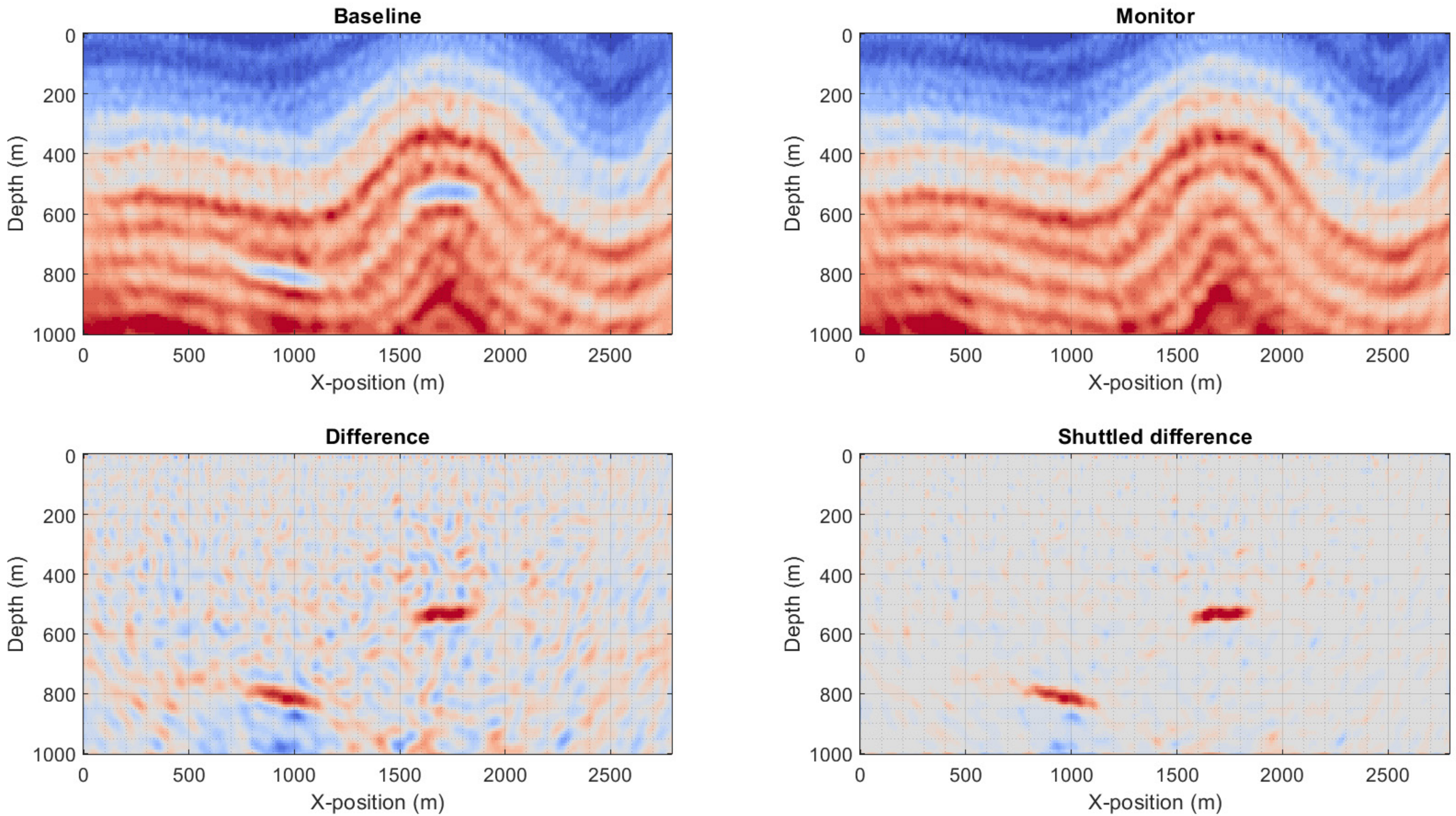
$\Delta v_P = 150\ m/s$



$\Delta v_P = 300\ m/s$



$\Delta v_P = 500\ m/s$



Conclusions

Targeted null-space shuttling may help to find optimal time-lapse differences

This can help to mitigate the effects of survey non-repeatability on time-lapse estimates

Acknowledgements

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- CFREF
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