Towards improving crosstalk suppression in multiparameter FWI by decorrelating parameter classes

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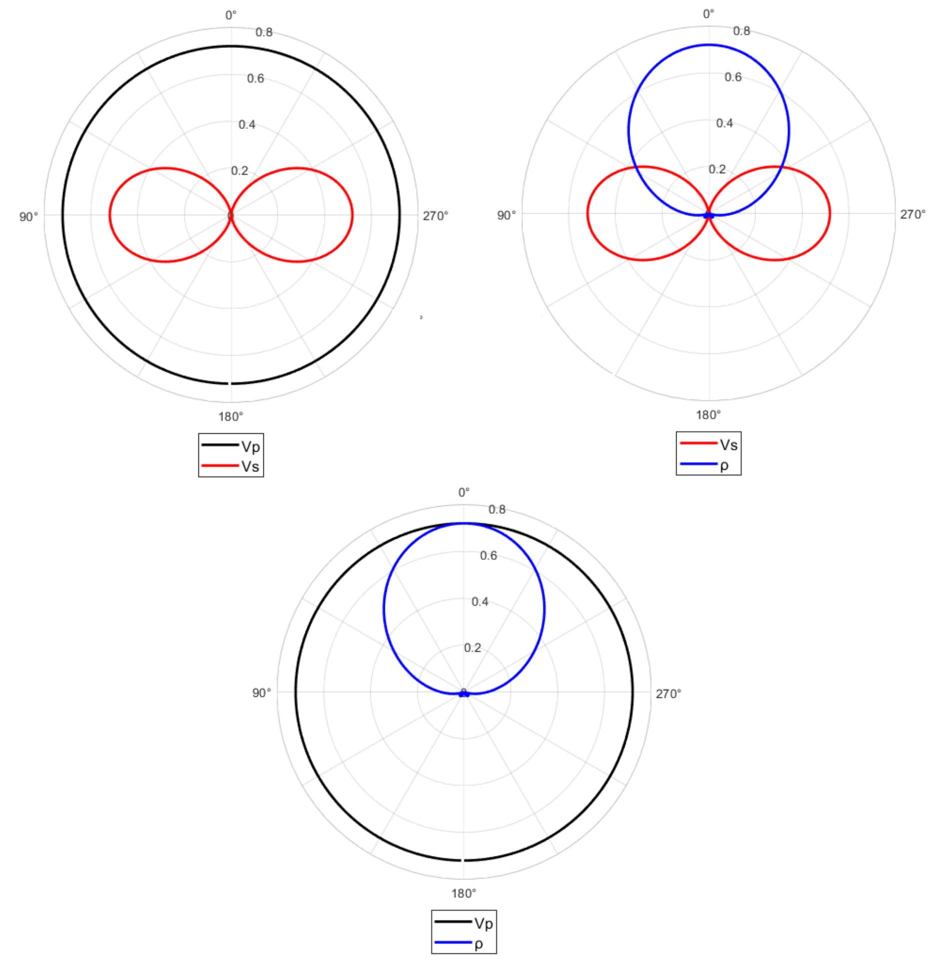
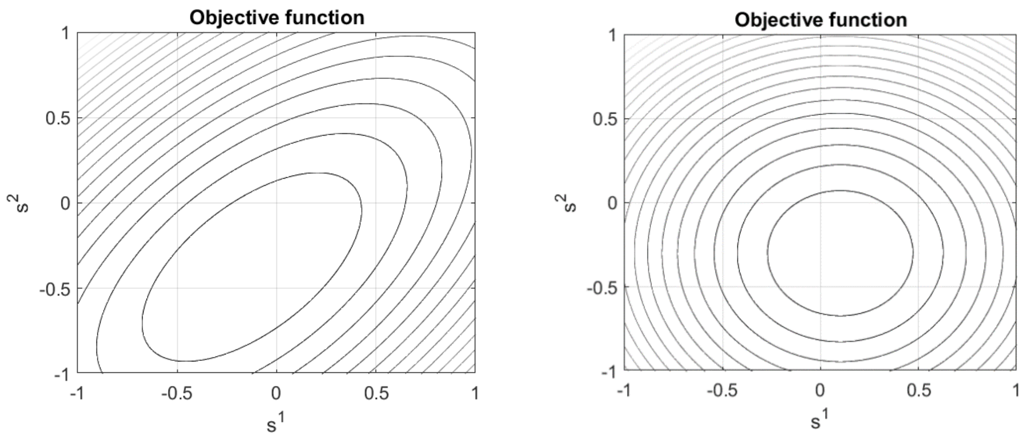
Introduction: multiparameter FWI and crosstalk

Design adequate workflows



Hessian

$$\boldsymbol{H} = \begin{bmatrix} 1 & -0.6 \\ -0.6 & 1 \end{bmatrix} \qquad \boldsymbol{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Crosstalk suppression through its manipulation

$$\boldsymbol{H} = \boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

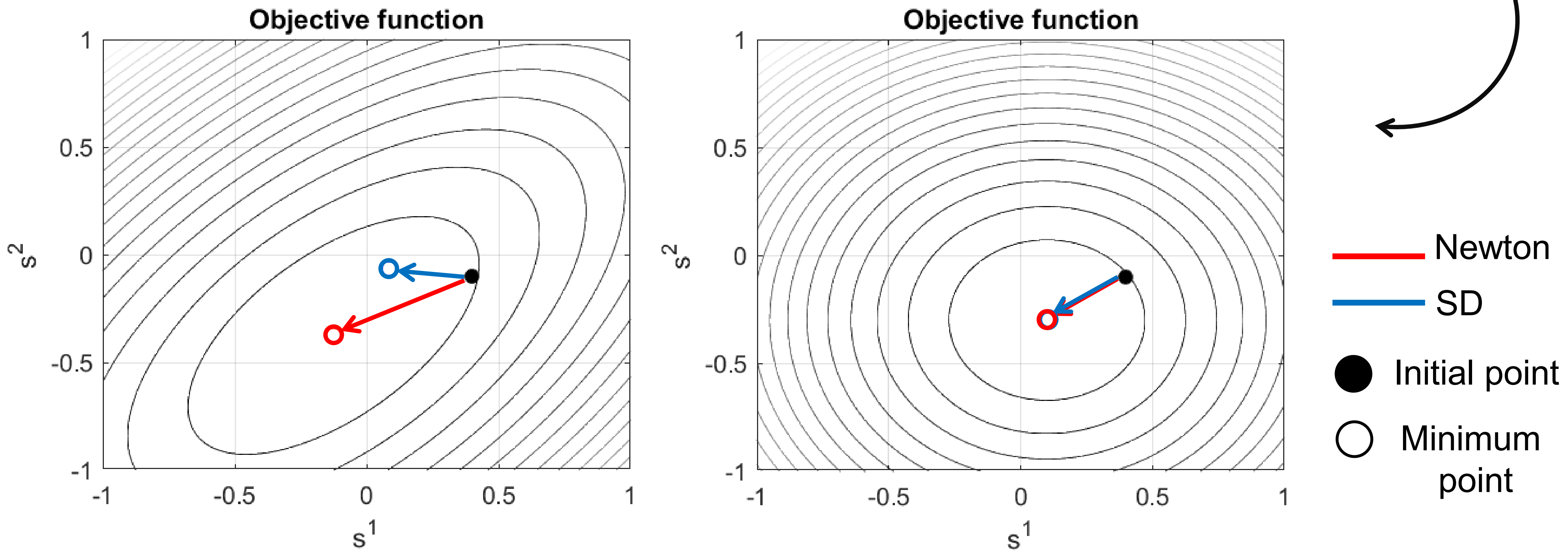
Objective: crosstalk corrected V_P, V_S and ρ using FWI in intermediate model space with no parameter leakage

Background: objective function iso-surfaces

Quadratic Φ : $$\Phi = \mathbf{s}^T\mathbf{H}\mathbf{s} + \mathbf{s}^T\mathbf{p} + C$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & -0.6 \\ -0.6 & 1 \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



2-variable model of descent-based optimization

Background: transformation between model spaces

$[\rho, c_{11}, c_{44}]$ → [Par1, Par2, Par3] → $[\rho, V_P, V_S]$

$c_{11} = {V_P}^2 \rho$

$c_{44} = {V_S}^2 \rho$

$c_{11} = \lambda + 2\mu \qquad c_{44} = \mu$

Innanen (2020 a, b, c, d)

- Re-parameterize= transform between cartesian and oblique systems
- Φ: scalar (invariant)
- Δs: contravariant components (change)

Background: transformation between model spaces

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \quad \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

Transformation rules:

$$\boldsymbol{s} = \boldsymbol{T}\boldsymbol{r}$$

$$\boldsymbol{r} = \boldsymbol{T}^{-1}\boldsymbol{s}$$

T: transformation matrix

↓

can meet constraints

Finding minimizer of Φ in ***r*** implies the minimum point is also found in ***s***

Background: Hessian and crosstalk quantification

$$\mathbf{H}_{(\mathbf{i},\mathbf{j}),(\mathbf{k},\mathbf{l})} = \left(\frac{\partial \mathbf{d_p}}{\partial \mathbf{s_{i,j}}}\right)\left(\frac{\partial \mathbf{d_p}}{\partial \mathbf{s_{k,l}}}\right)^*$$

$\mathbf{d_p}$: predicted data
j, l: parameter class
i, k: position (1: n_z x n_x)

Full Hessian →

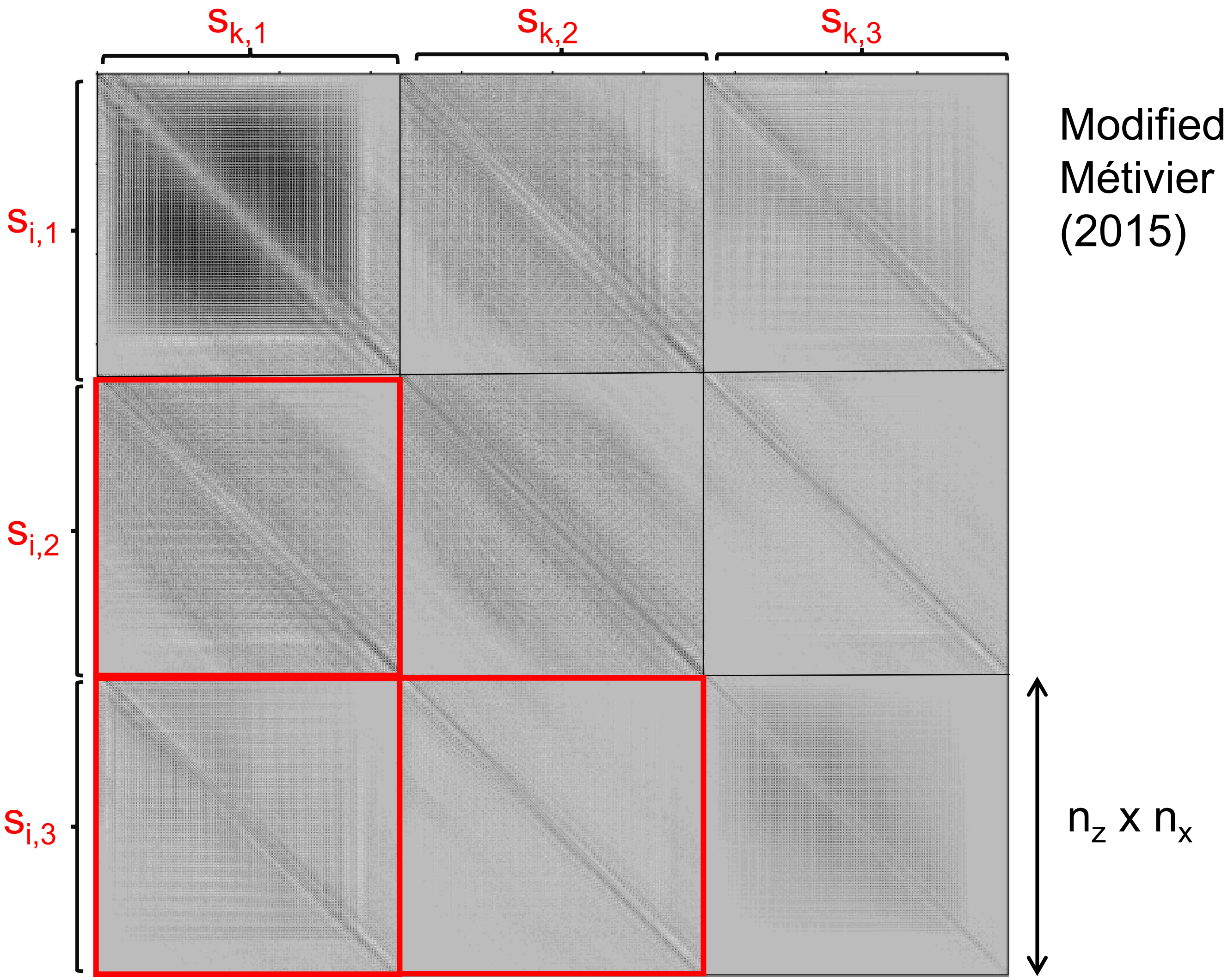
$j \neq l$

↓

Existing trade-off

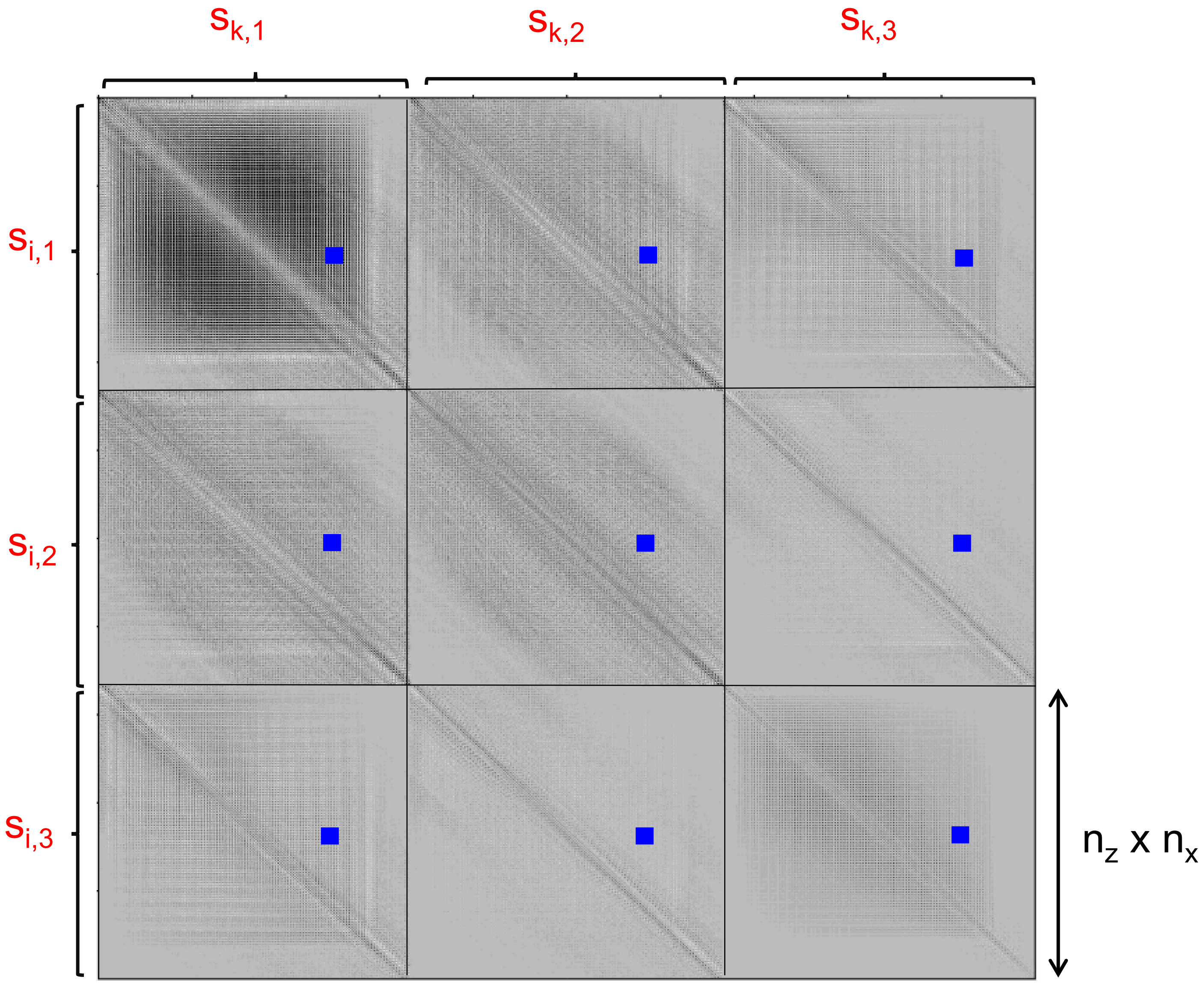
↓

No crosstalk would exist if off-diagonal blocks were zero



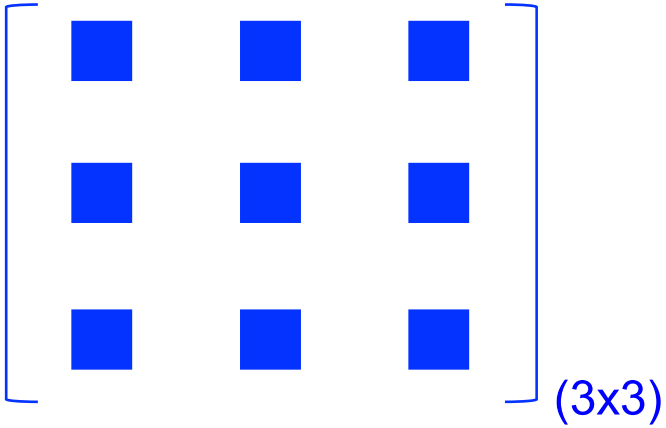
Modified from Métivier et al. (2015)

Background: Hessian and crosstalk quantification



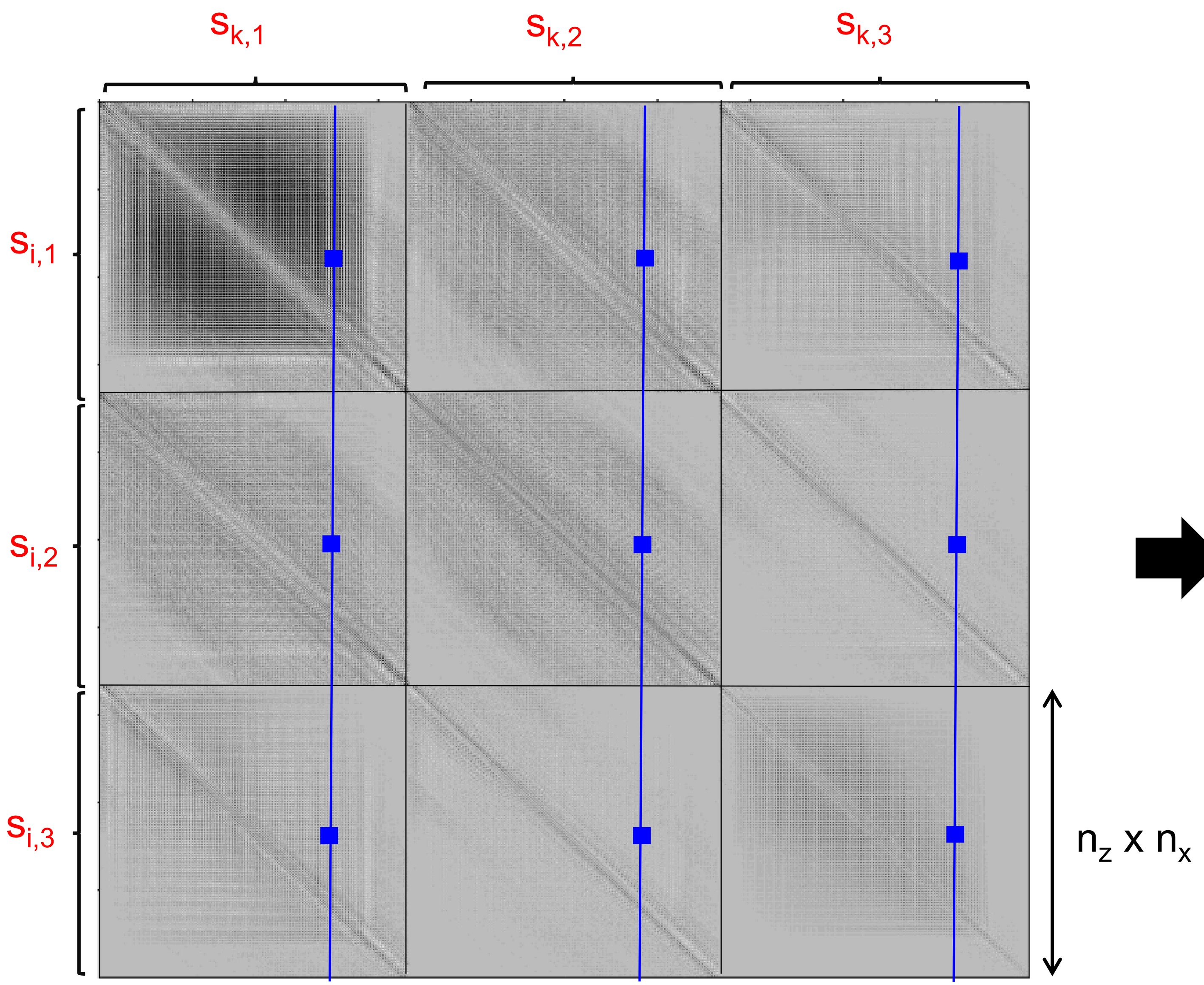
Modified from Métivier et al. (2015)

Point-wise Hessian



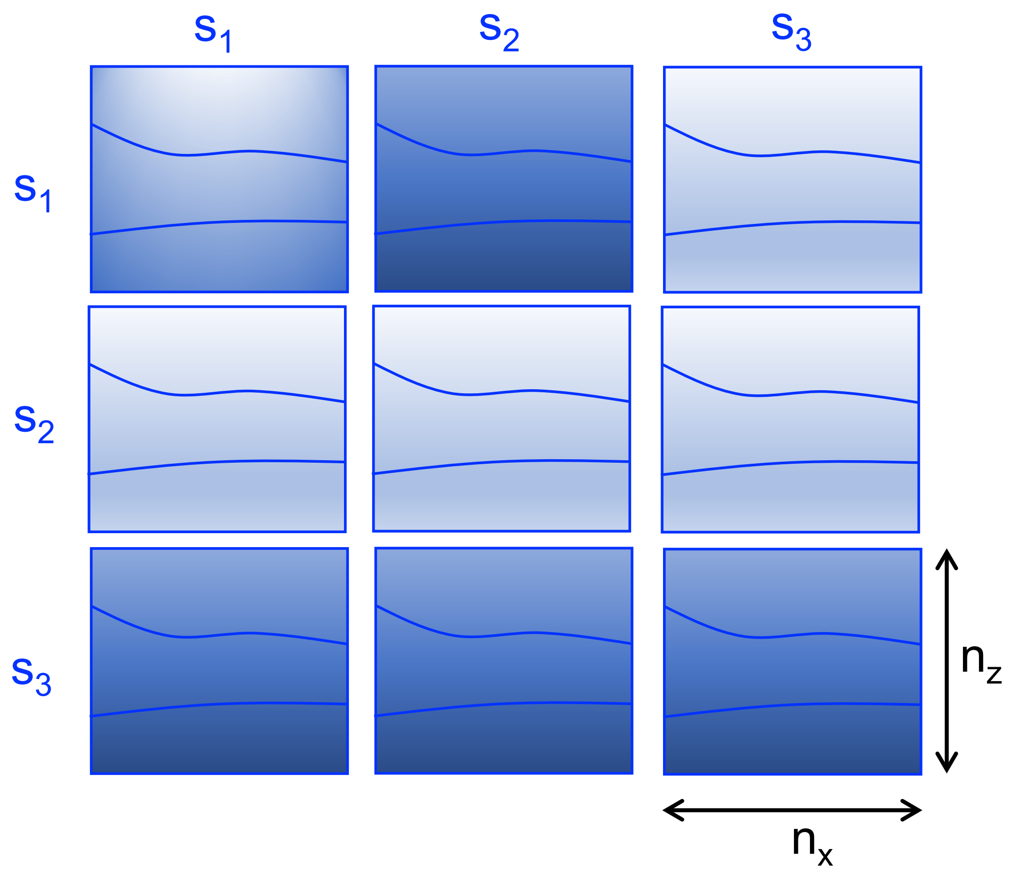
Crosstalk between
s_1, s_2 and s_3 at a fixed point

Background: Hessian and crosstalk quantification



Modified from Métivier et al. (2015)

Point probes Hessian



Crosstalk at one point with all the other unkowns.

Background: design of transformation matrices

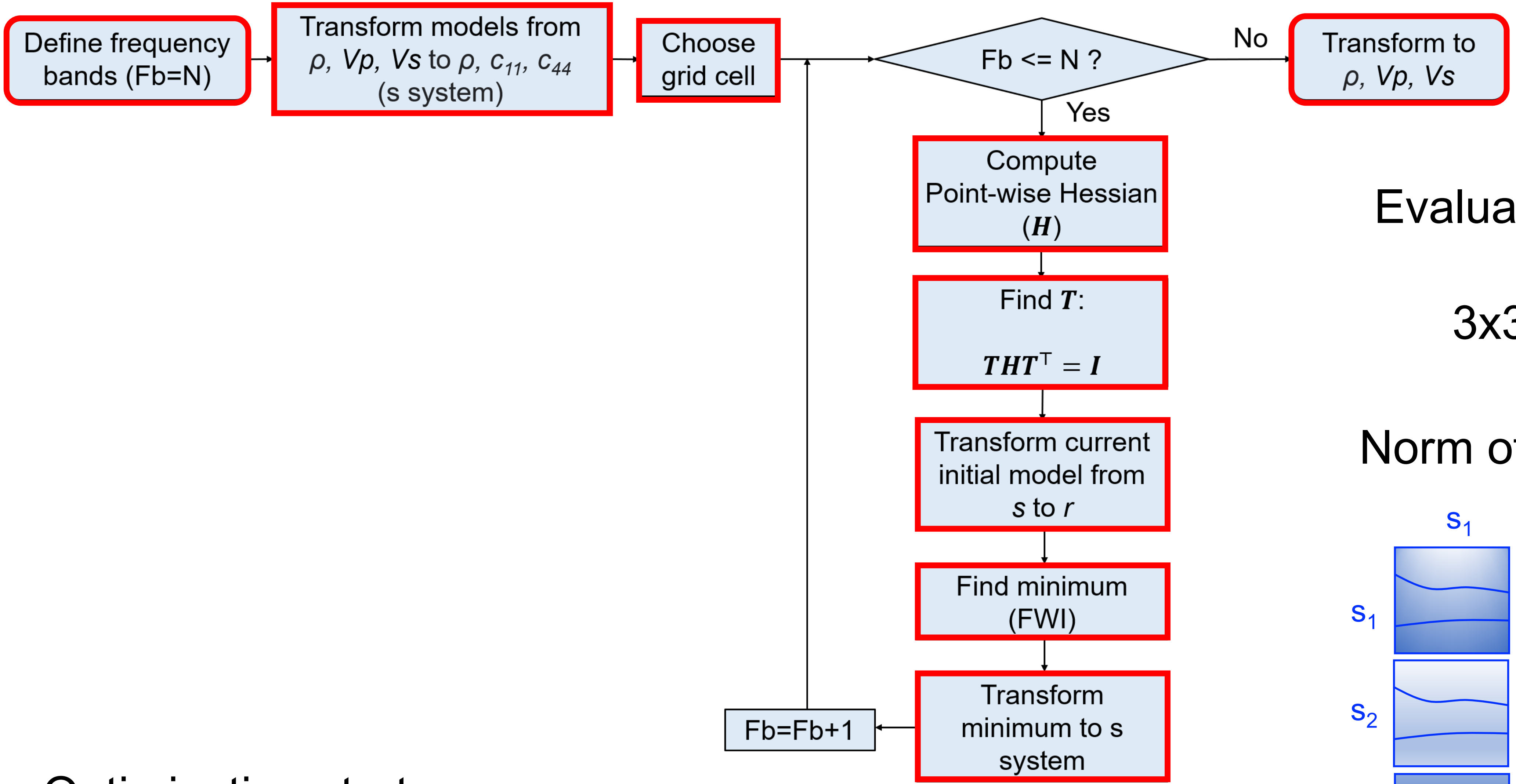
Transformation rule for the Hessian:

$$\boldsymbol{T}\boldsymbol{H}(s)\boldsymbol{T}^{\mathsf{T}} = \boldsymbol{H}(r) = \boldsymbol{I}$$

↑ constraint

$$\mathbf{T} \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{bmatrix}_{(3x3)} \mathbf{T}^{\mathsf{T}} = \mathbf{I}$$

Workflow and evaluation



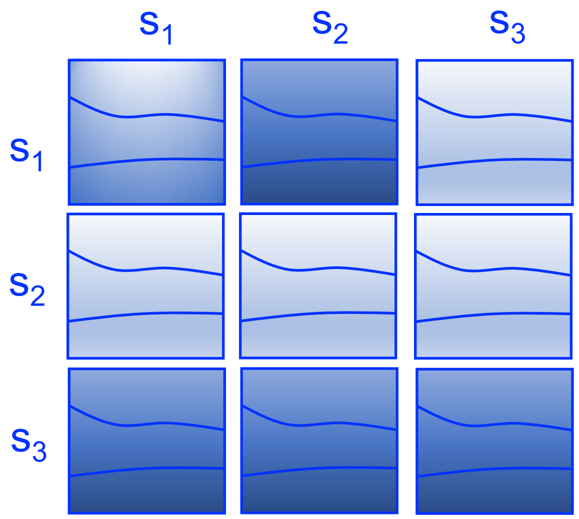
Evaluation metric

↓

3x3 matrix

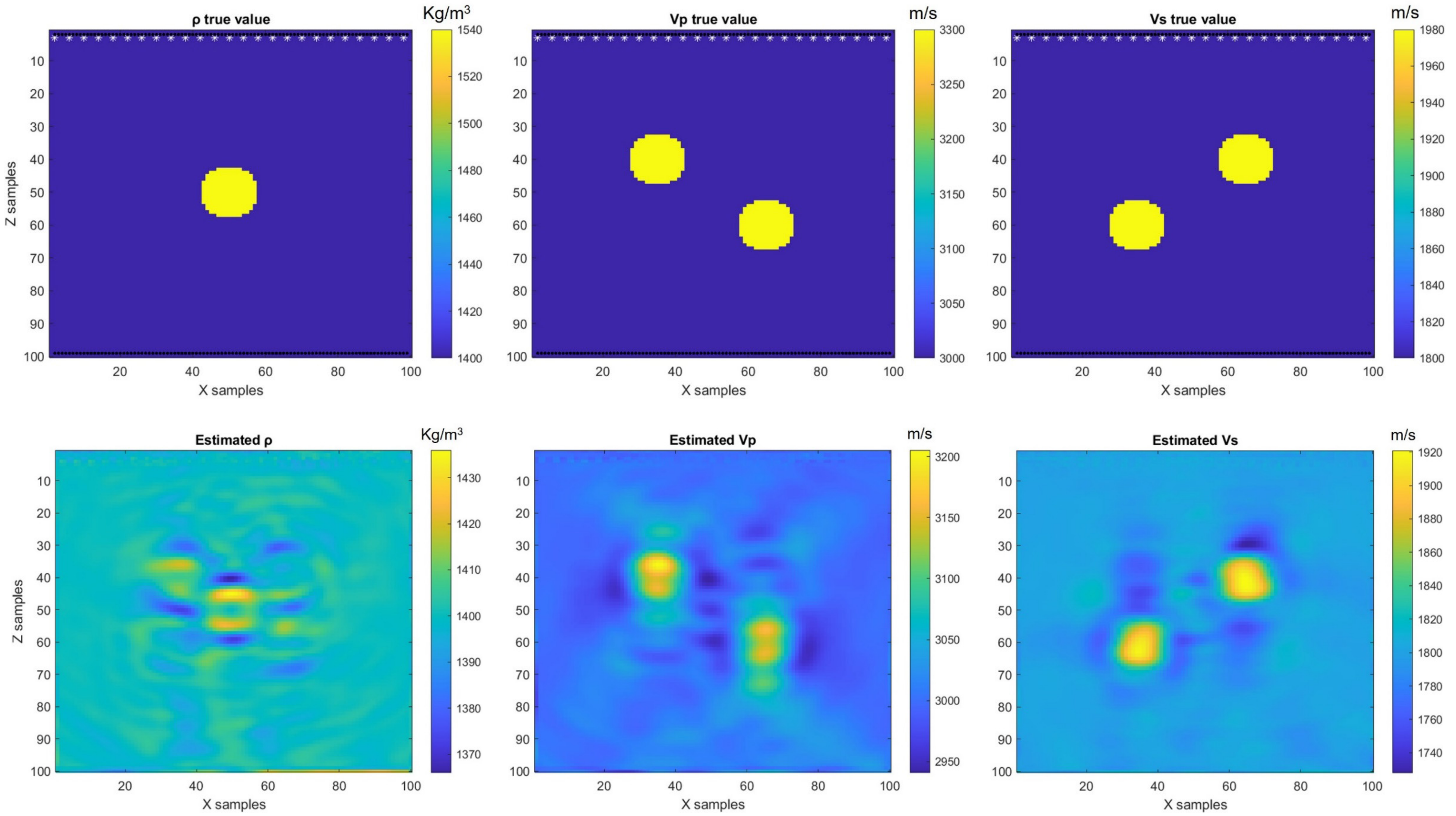
↓

Norm of each block

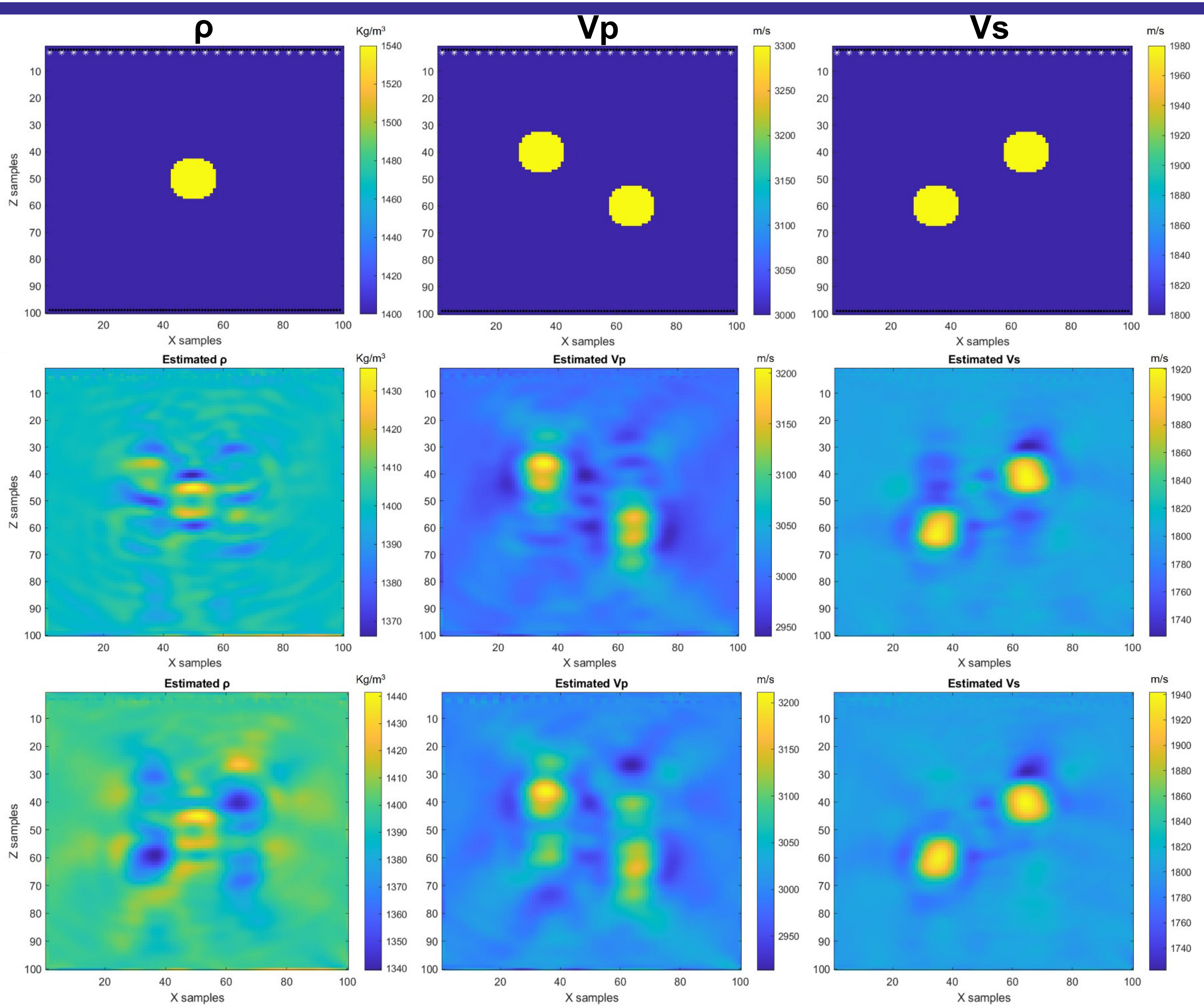


Optimization strategy:
Steepest Descent

Results: baseline/reference FWI



Results: re-parameterized FWI



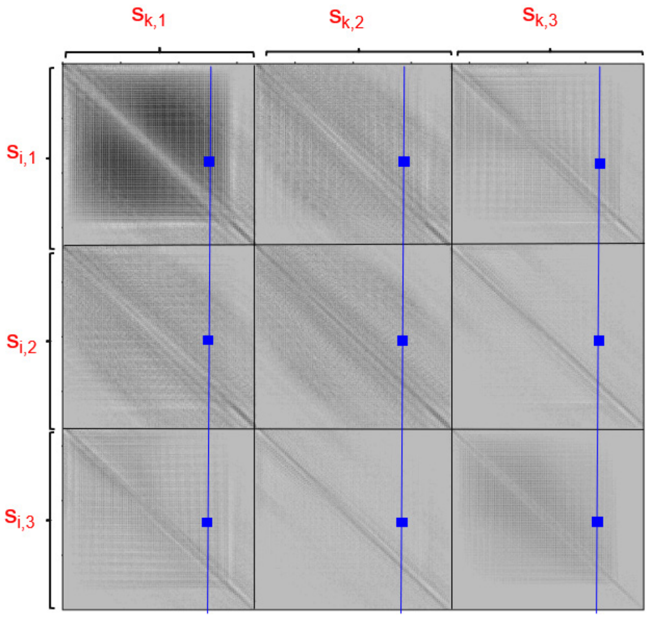
True values

Baseline FWI

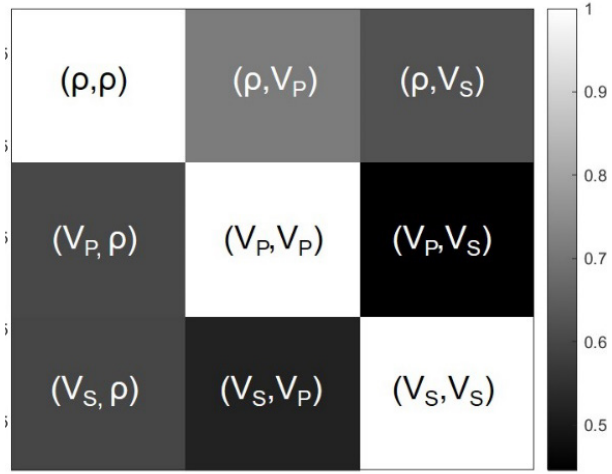
Re-parameterized FWI

T: x=50 z=20

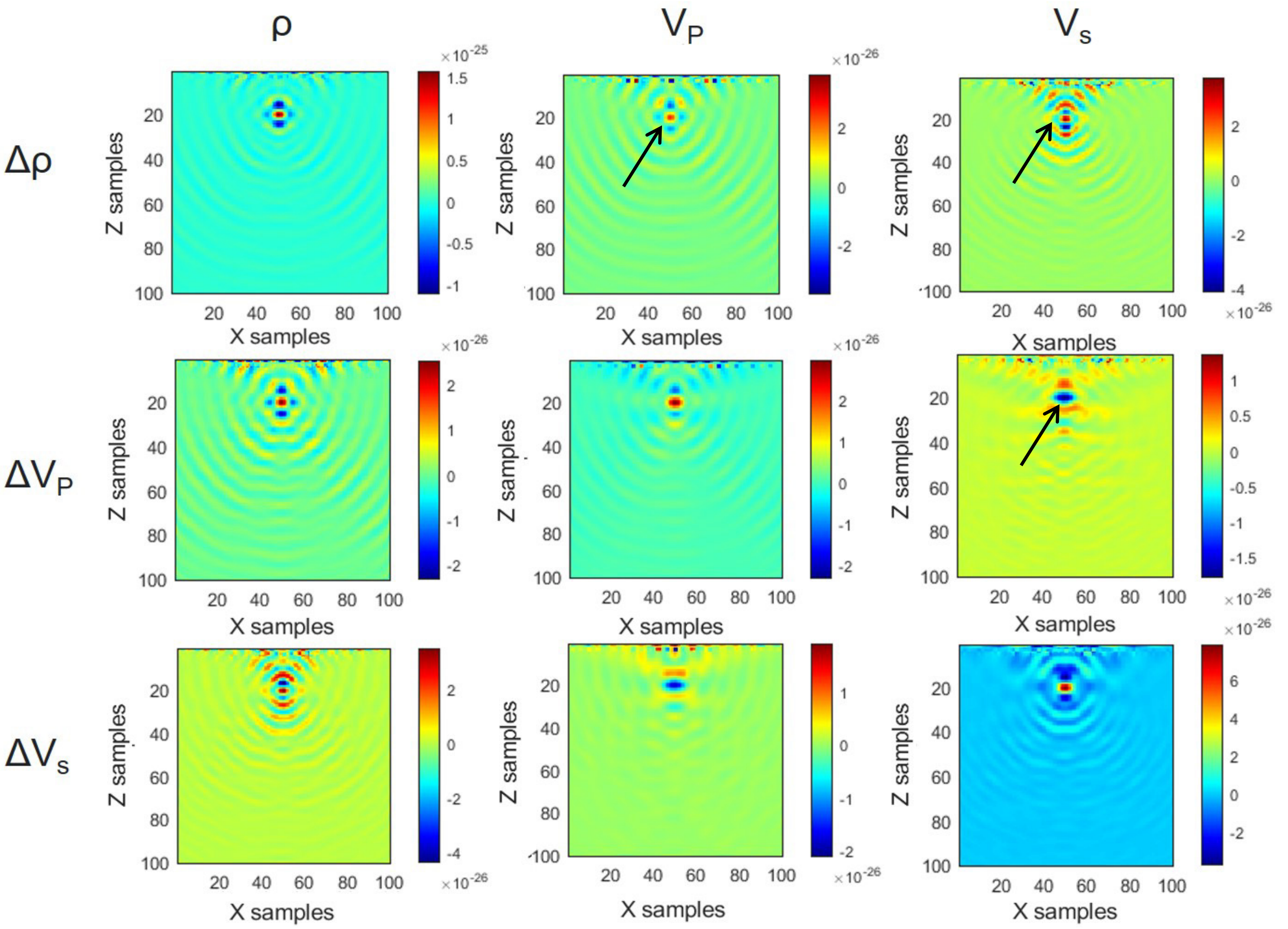
Results: evaluation of estimates (reference FWI)



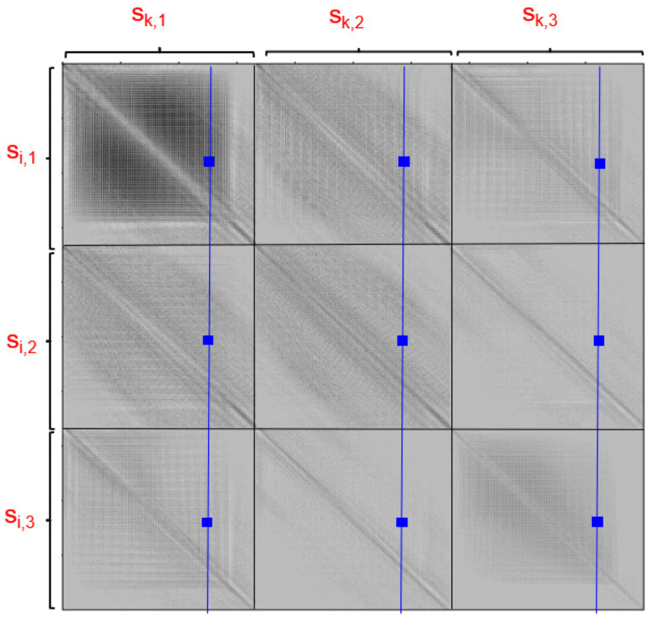
Normalized crosstalk metric with perturbation at x=50 z=20



Perturbed location: x=50 z=20

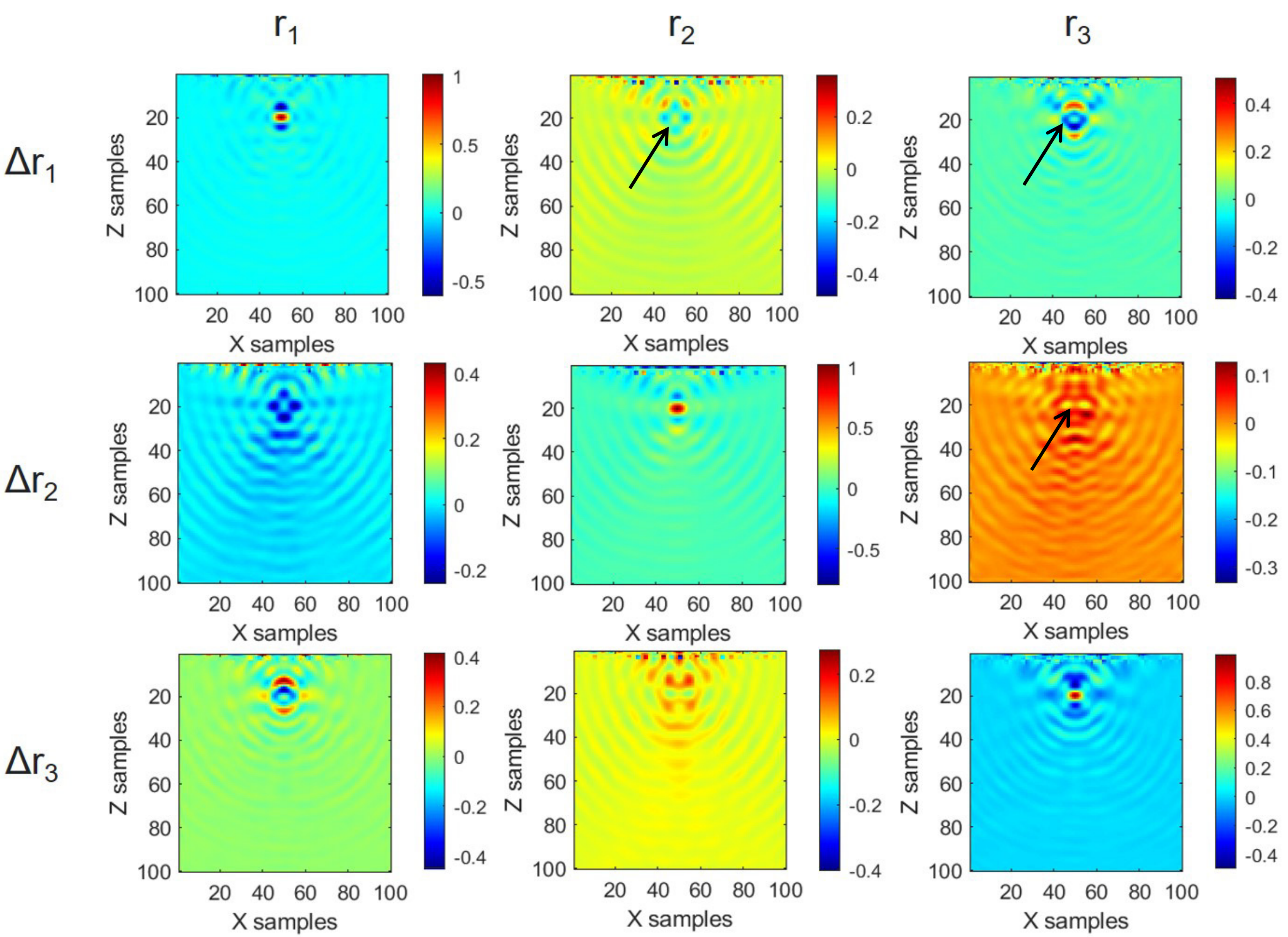


Results: evaluation of estimates (re-parameterized FWI)



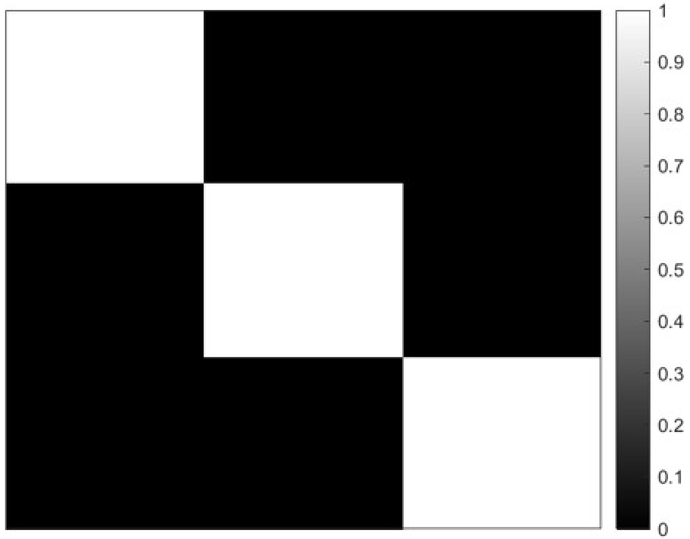
Normalized crosstalk metric
with perturbation at x=50 z=20

Perturbed location: x=50 z=20



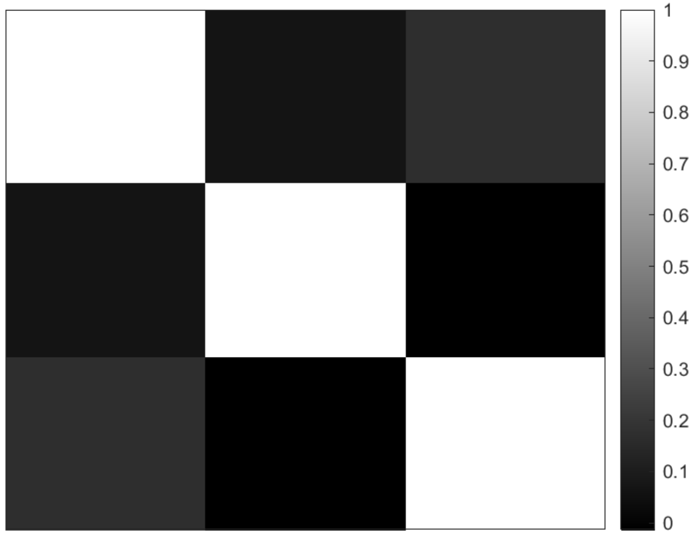
Results: evaluation of estimates from re-parameterized FWI

Normalized Hessian
at x=50 z=20



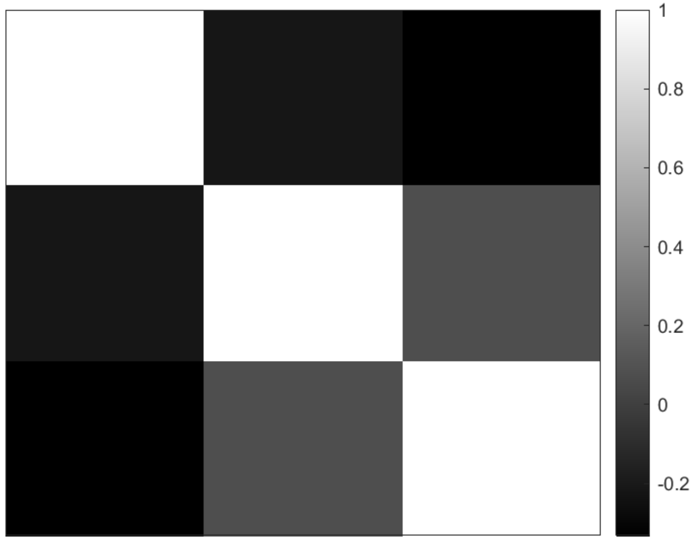
at design position

Normalized Hessian
at x=20 z=20



off design position

Normalized Hessian
at x=80 z=80



off design position

Conclusions

- Treat FWI parameterization as coordinate transform problem
- Seek set of parameters within which **H** = **I**
- Approach: design **T** based on single point (computable)
  - It works! At design point
  - Crosstalk induced at other locations
- Next steps:

  Different procedure to compute **T** with contribution of all locations

Acknowledgments

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References

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Thank you!