Elastic FWI uncertainty analysis via conventional and machine learning methods

Tianze Zhang, Scott Keating, Kris Innanen and Daniel Trad

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FACULTY OF SCIENCE
Department of Geoscience

Motivations

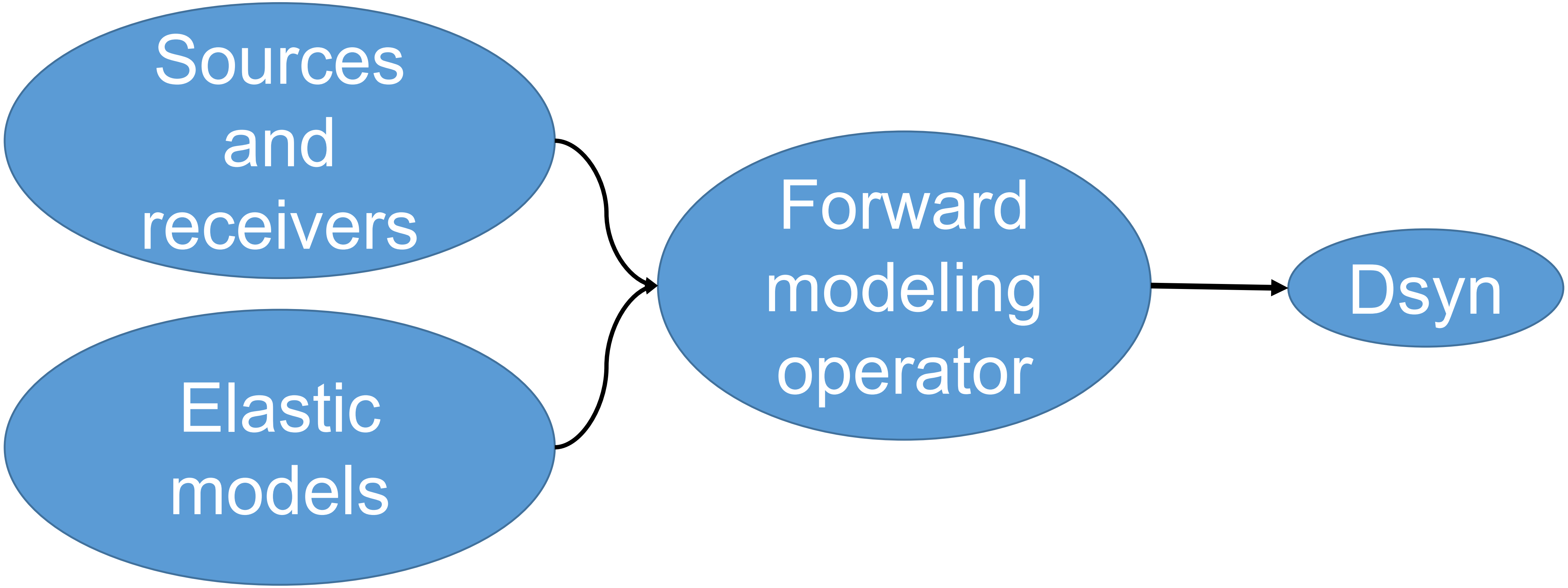
- Uncertainty quantification (UQ) is as important as finding inverse solutions.
- Creating feasible methods for UQ analysis is necessary.

Outline

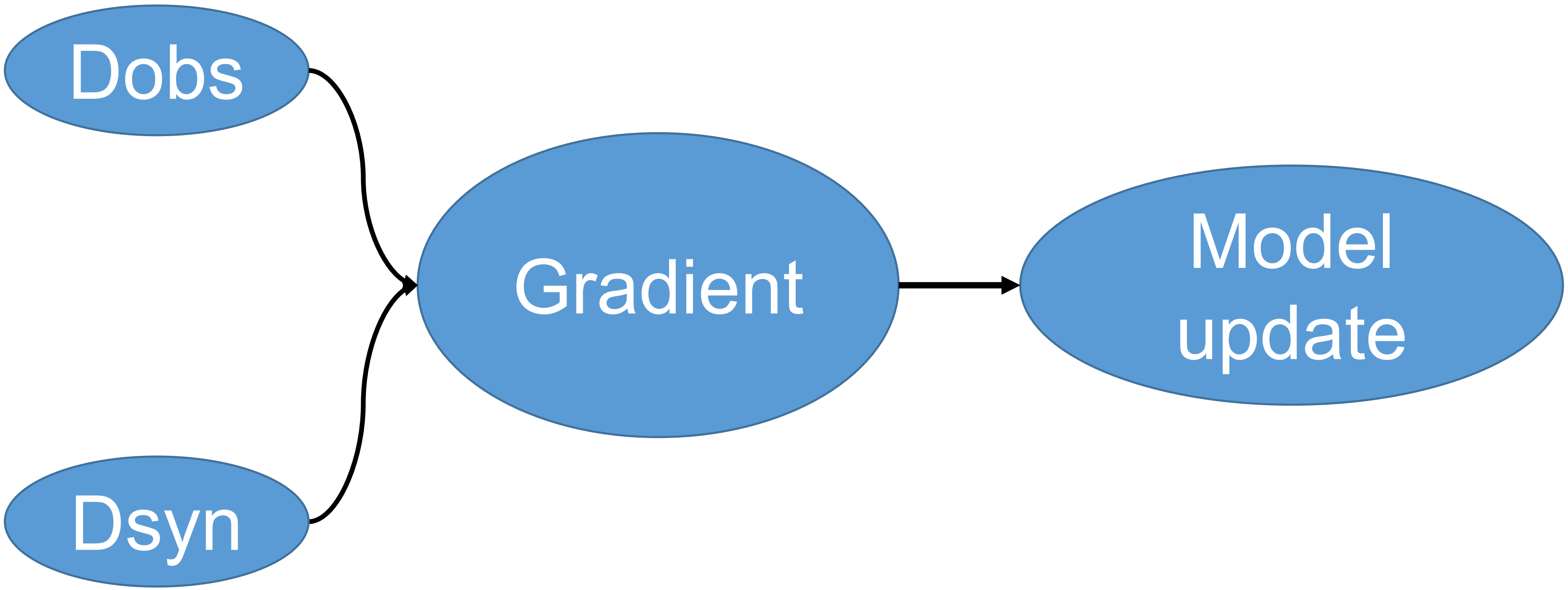
- Introduction to FWI and UQ.
- UQ using inverse Hessian matrix.
- UQ using Bayesian neural network.
- Conclusions and future study.

Part one: Introduction to FWI and UQ

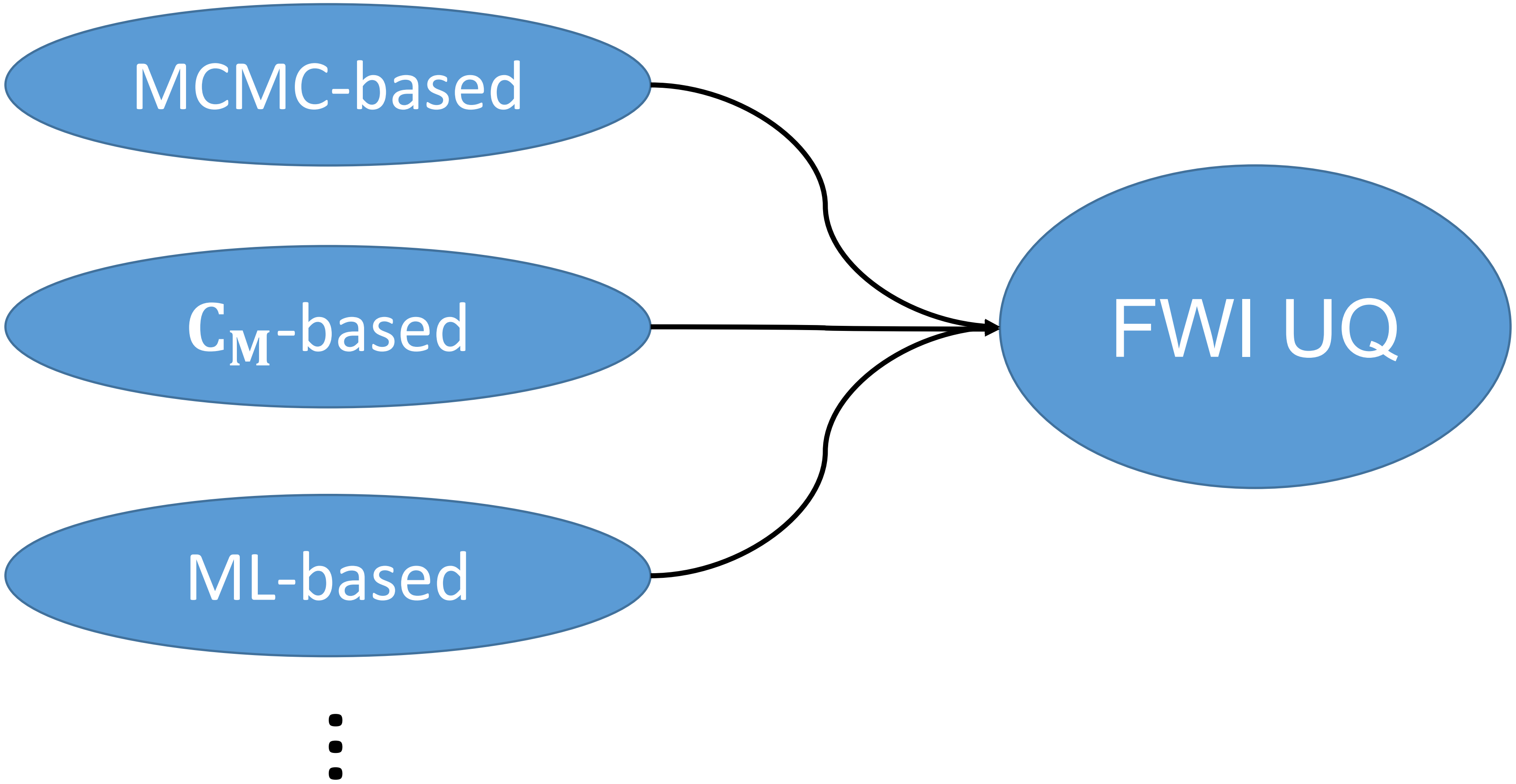
1. Forward modeling



2. Model update



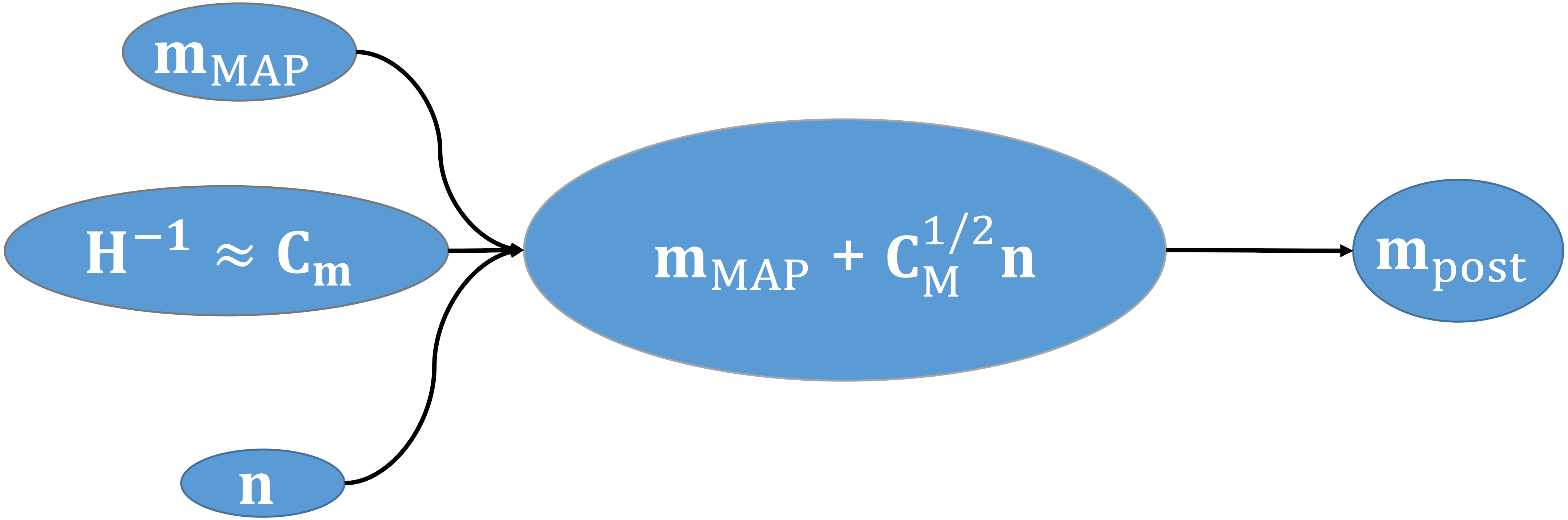
3. Uncertainty quantification methods in FWI



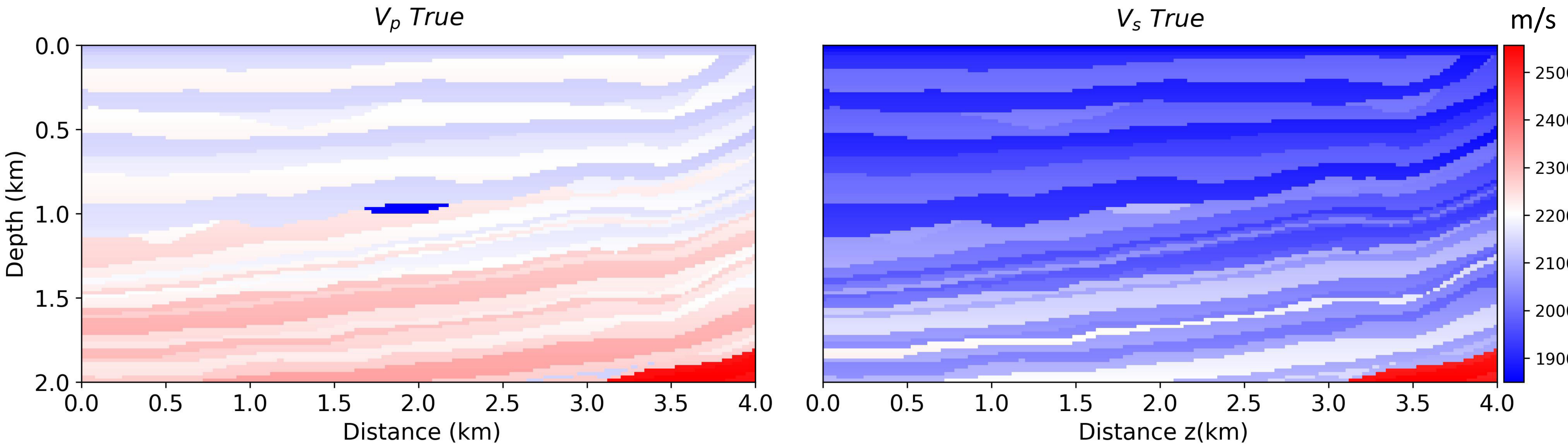
Part two: UQ of EFWI using inverse Hessian matrix

Part two: UQ using inverse Hessian

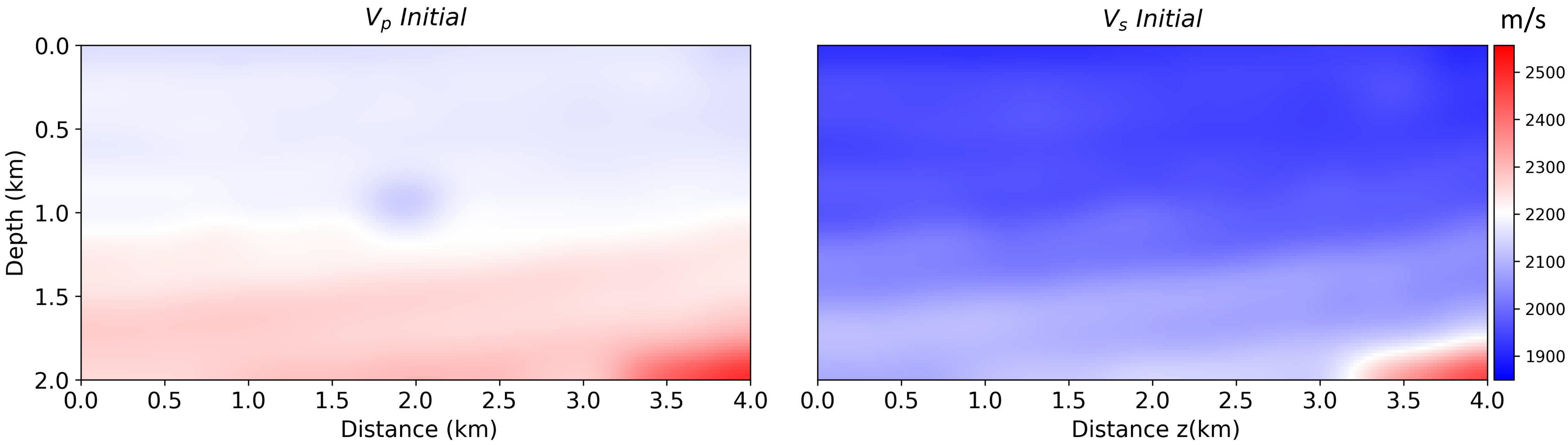
Uncertainty quantification using posterior model covariance matrix



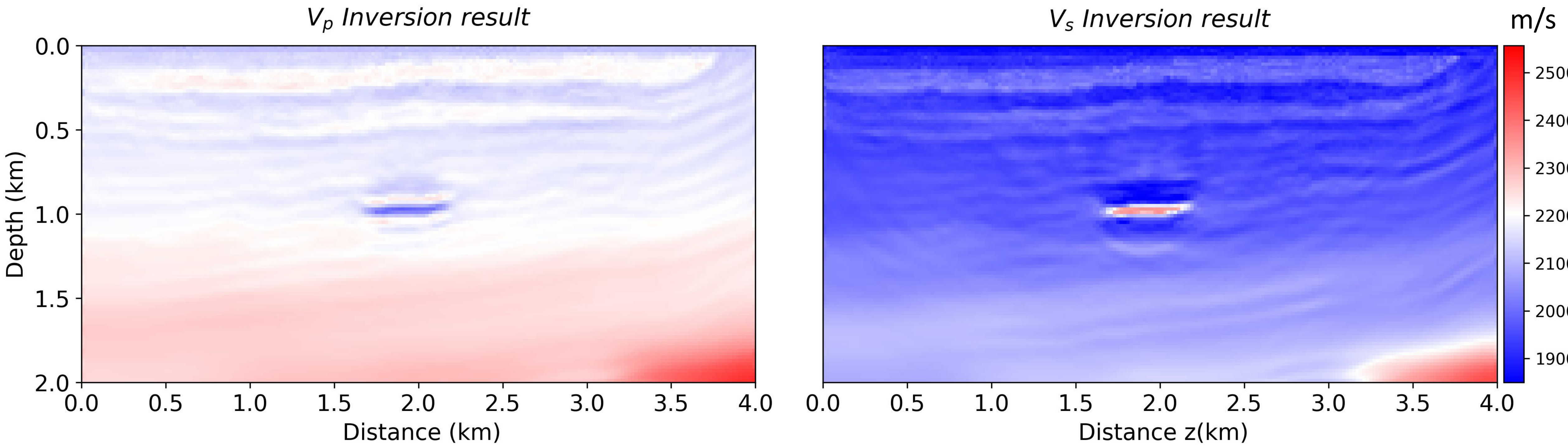
True models


V_p True
V_s True
m/s
Depth (km)
0.0
0.5
1.0
1.5
2.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
Distance (km)
Distance z(km)
2500
2400
2300
2200
2100
2000
1900

Initial models

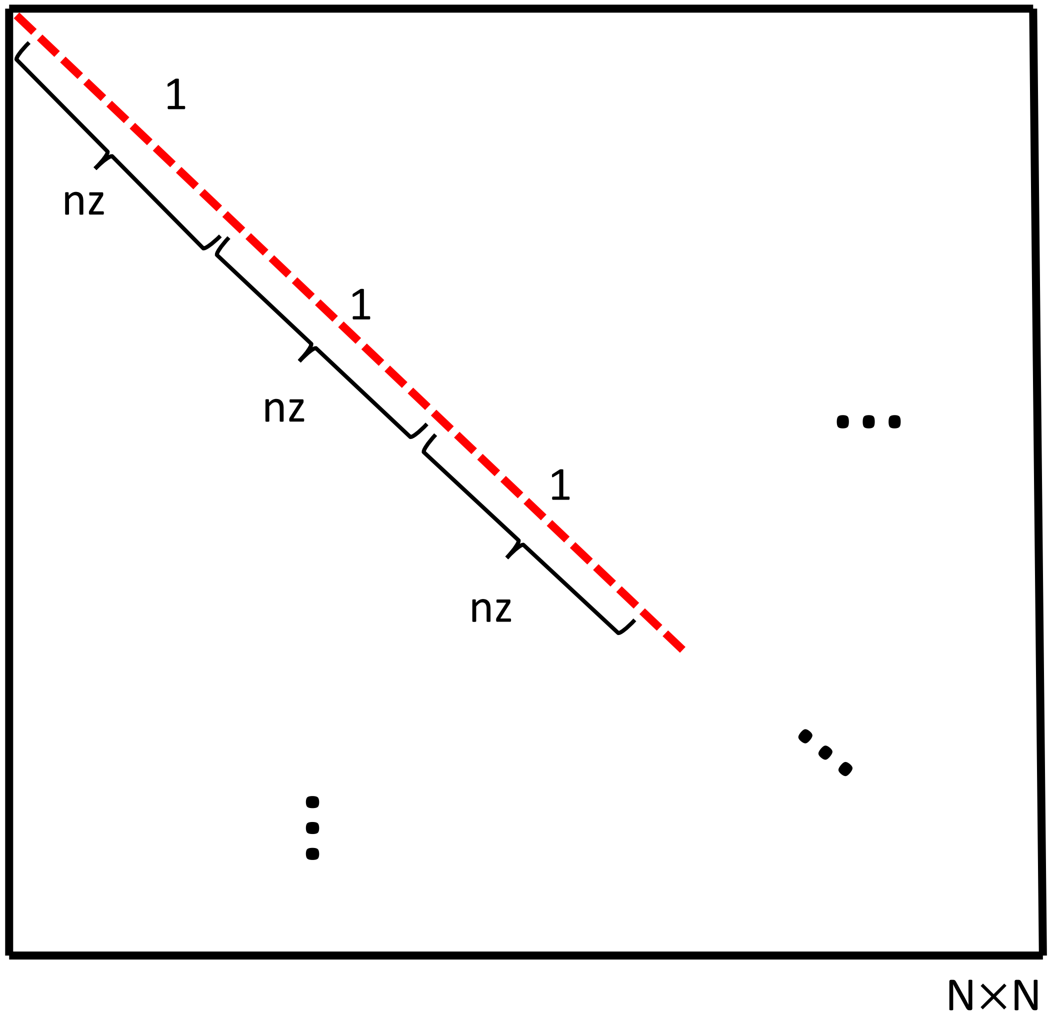


Inversion results ($B_0 = I$)

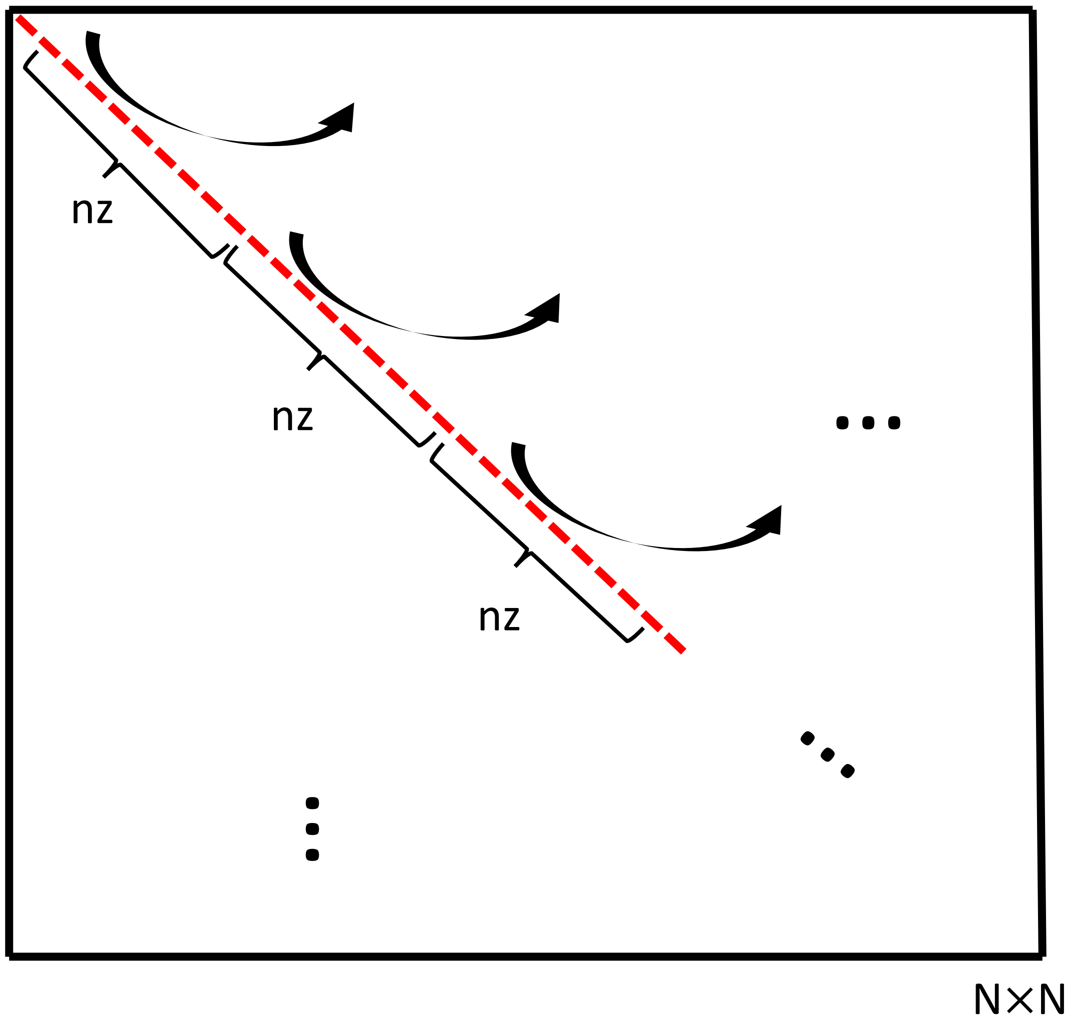


Part two: UQ using inverse Hessian

$B_0 = I$

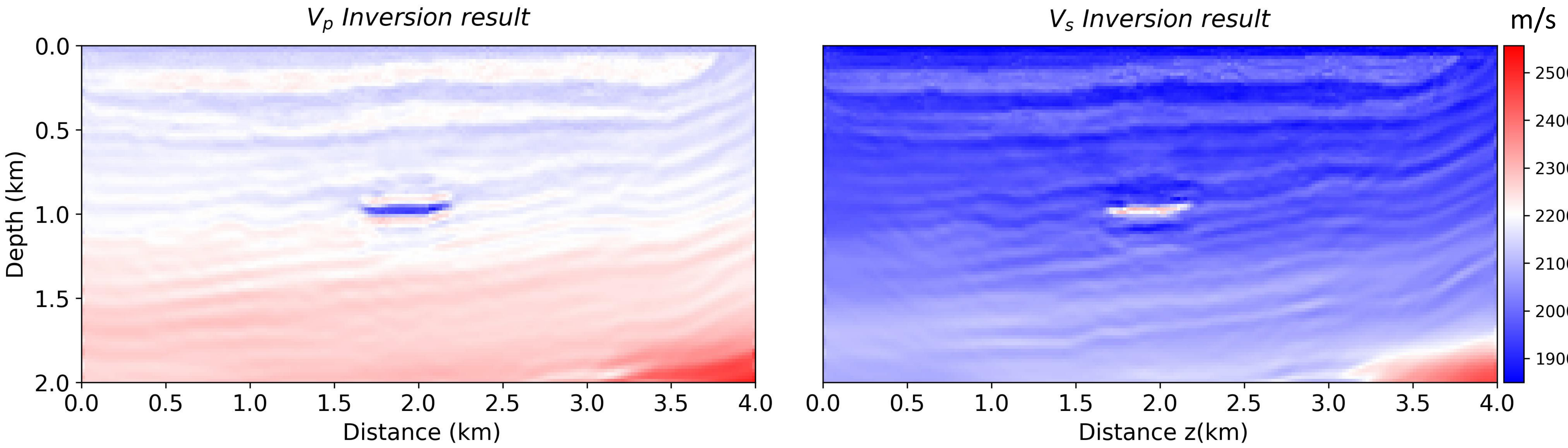


No identity $B_0(a)$

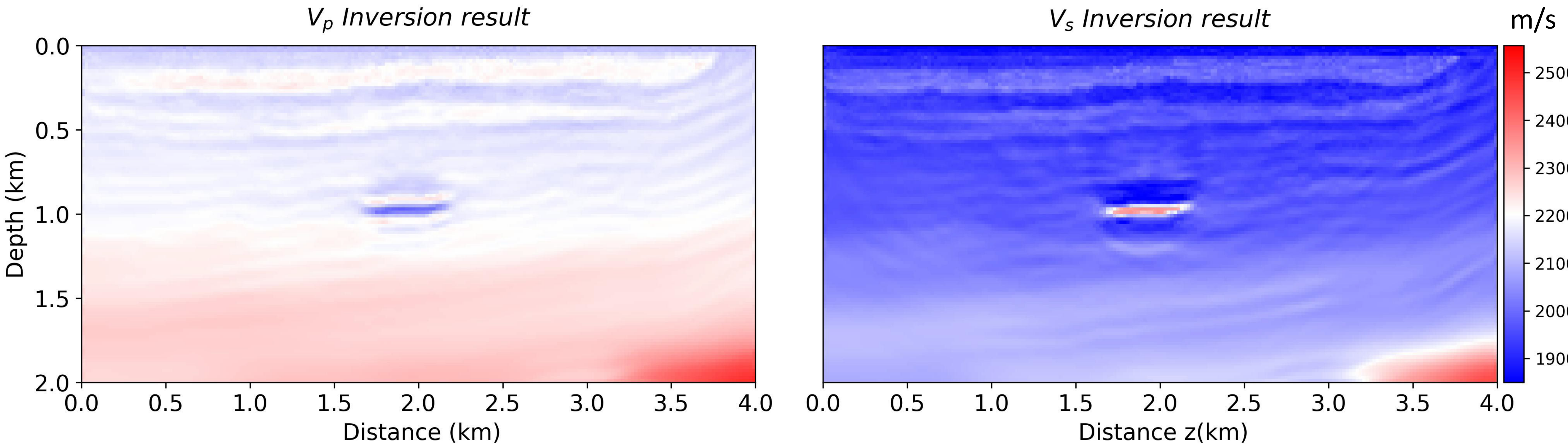


N=nz×nx

Inversion results ($a = 12$)

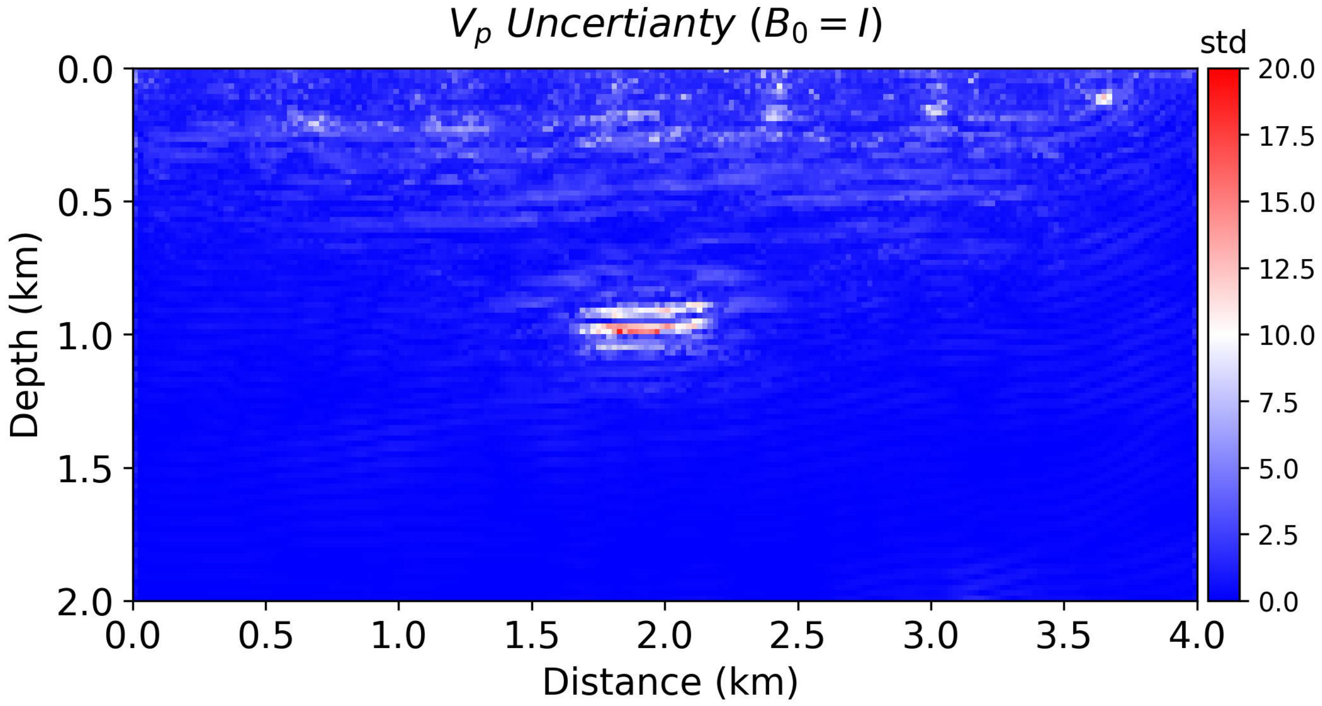


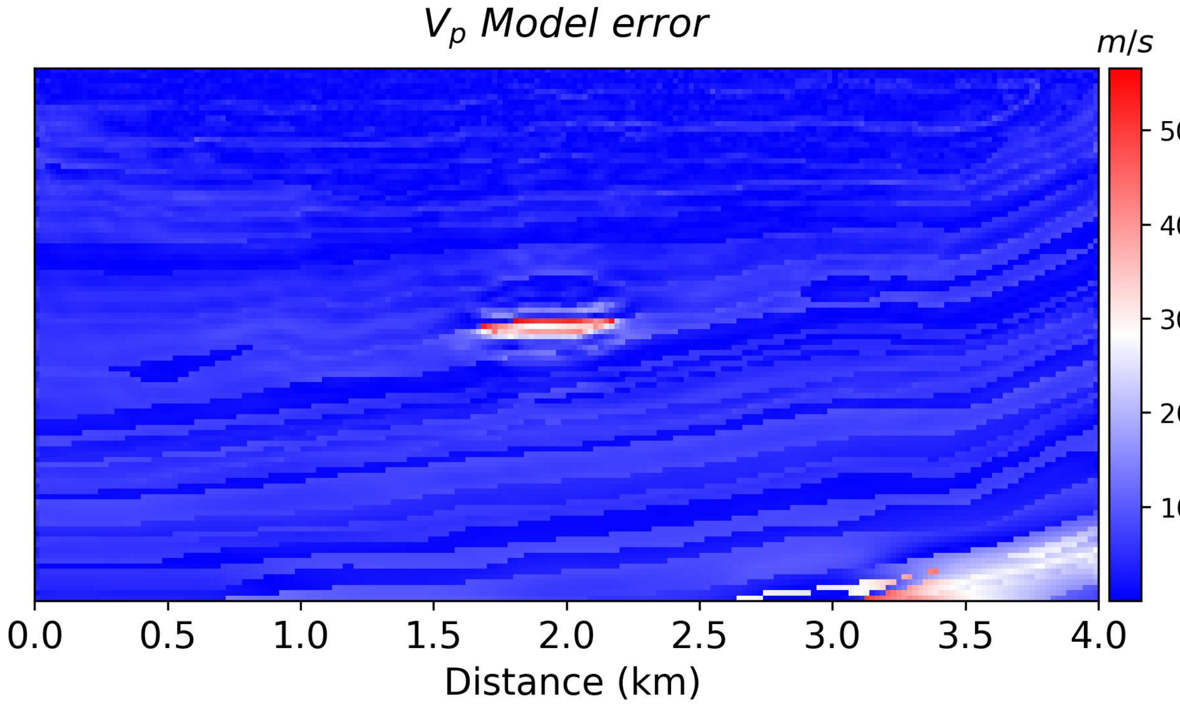
Inversion results ($B_0 = I$)



Part two: UQ using inverse Hessian

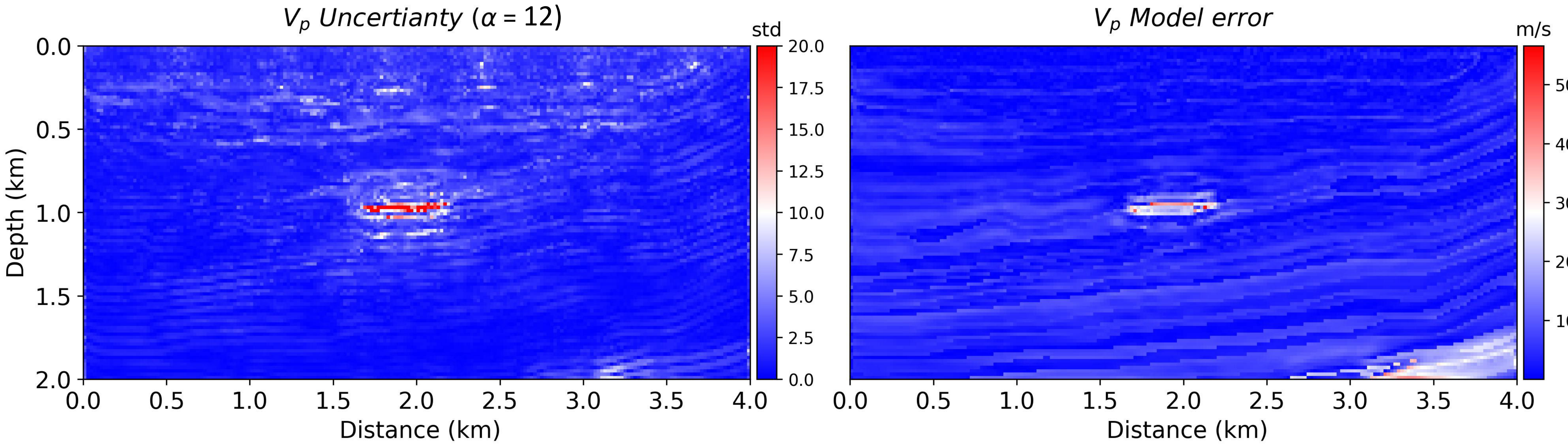
$$\mathbf{m}_{\text{post}} = \mathbf{m}_{\text{MAP}} + \mathbf{C}_{\text{M}}^{1/2}\mathbf{n}\ (\boldsymbol{B_0} = \boldsymbol{I})$$





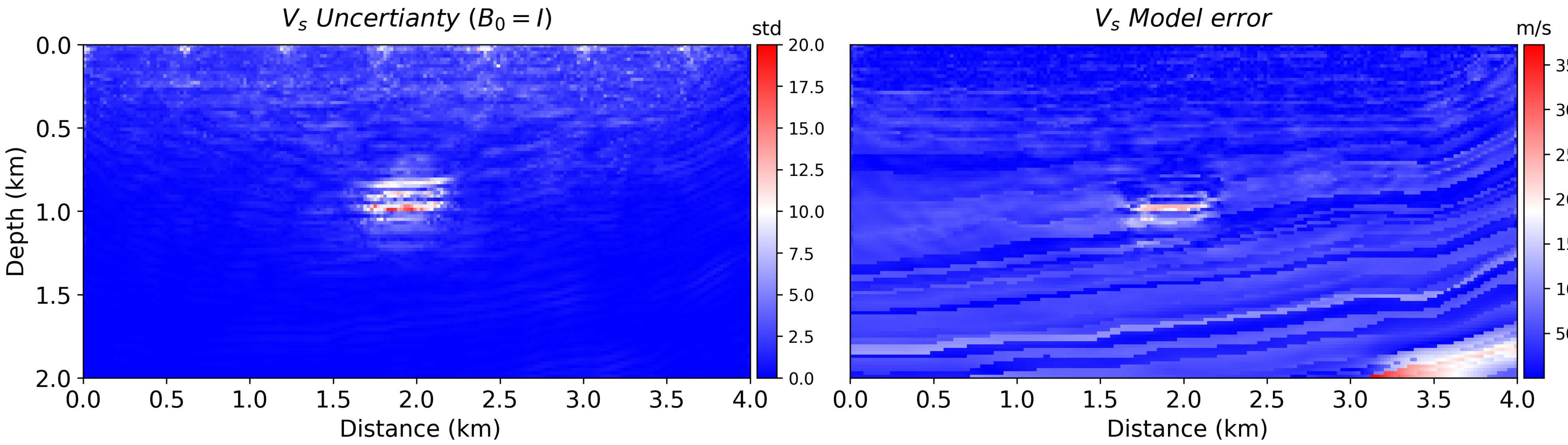
Part two: UQ using inverse Hessian

$$\mathbf{m}_{\mathrm{post}} = \mathbf{m}_{\mathrm{MAP}} + \mathbf{C}_{\mathrm{M}}^{1/2}\mathbf{n}\ (\alpha = 12)$$



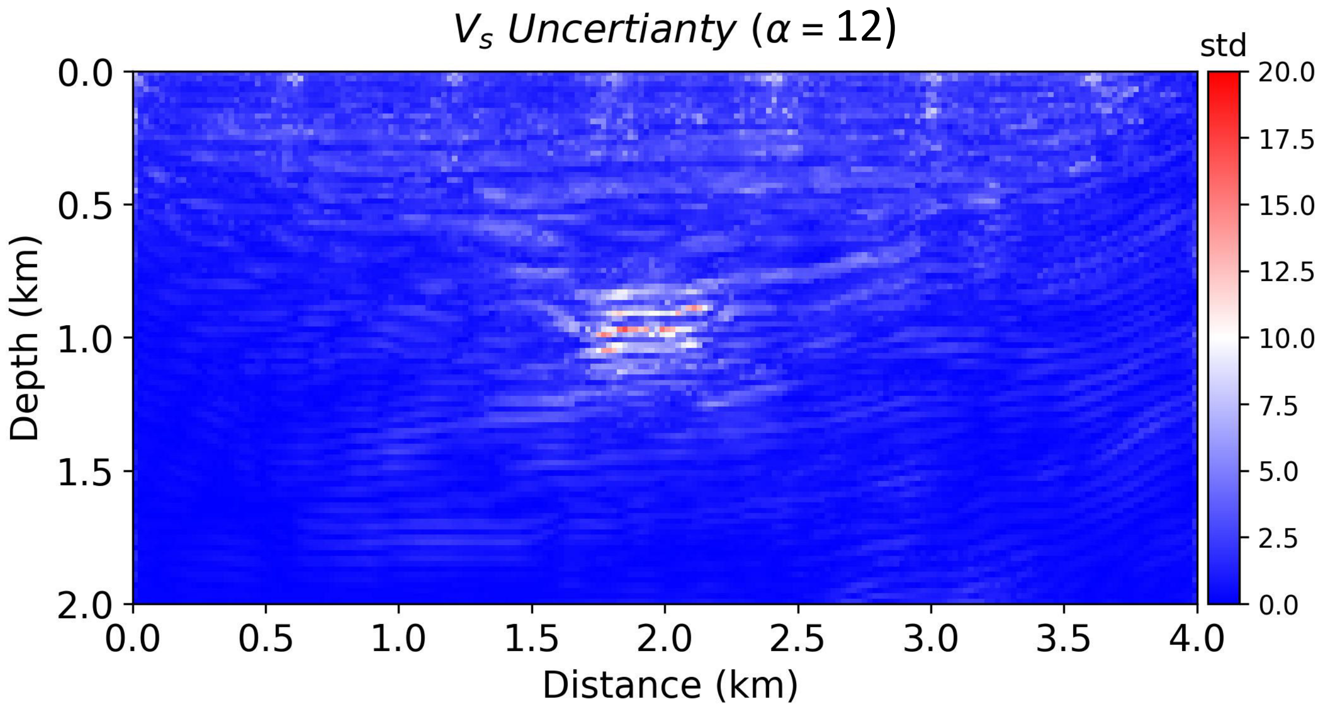
Part two: UQ using inverse Hessian

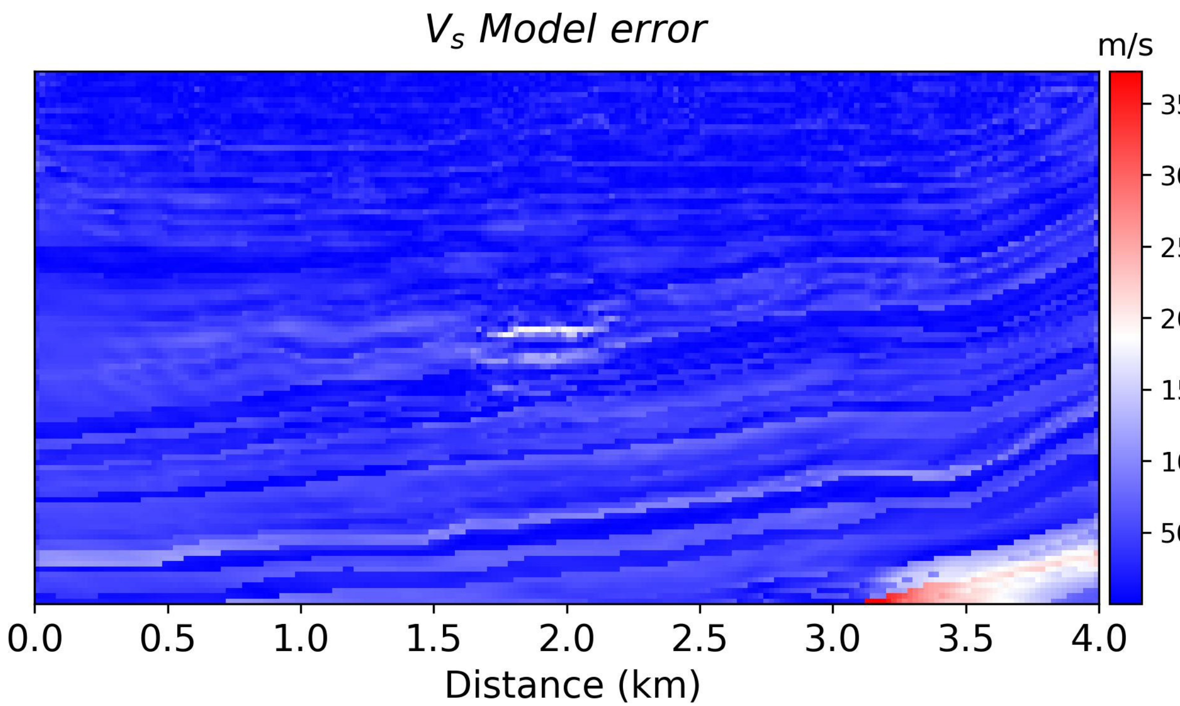
$$\mathbf{m}_{\mathrm{post}} = \mathbf{m}_{\mathrm{MAP}} + \mathbf{C}_{\mathrm{M}}^{1/2}\mathbf{n}\;(\boldsymbol{B_0} = \boldsymbol{I})$$



Part two: UQ using inverse Hessian

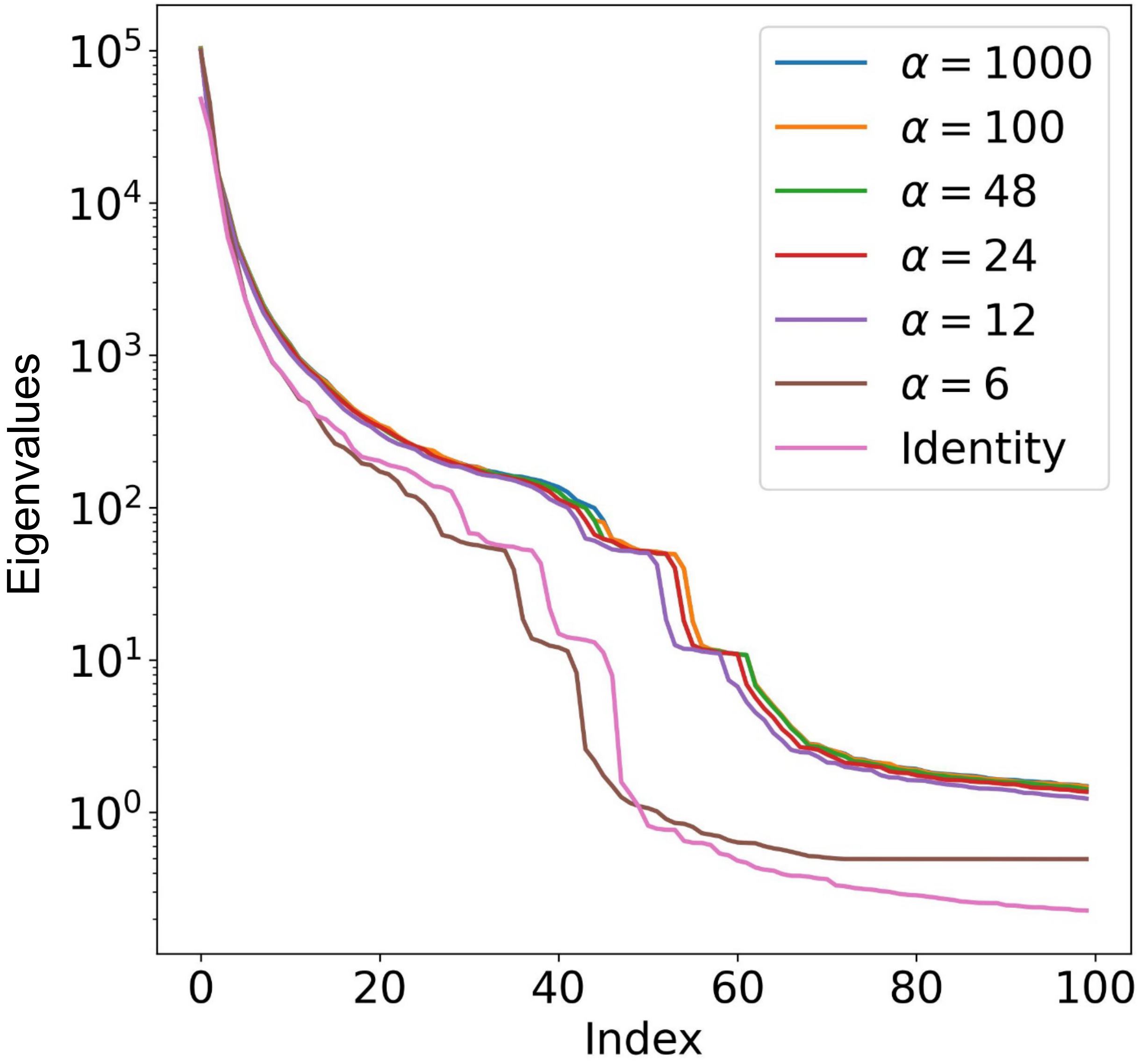
$$\mathbf{m}_{\mathrm{post}} = \mathbf{m}_{\mathrm{MAP}} + \mathbf{C}_{\mathrm{M}}^{1/2}\mathbf{n}\ (\alpha = 12)$$





Random singular value decomposition of $\mathbf{C_m}$

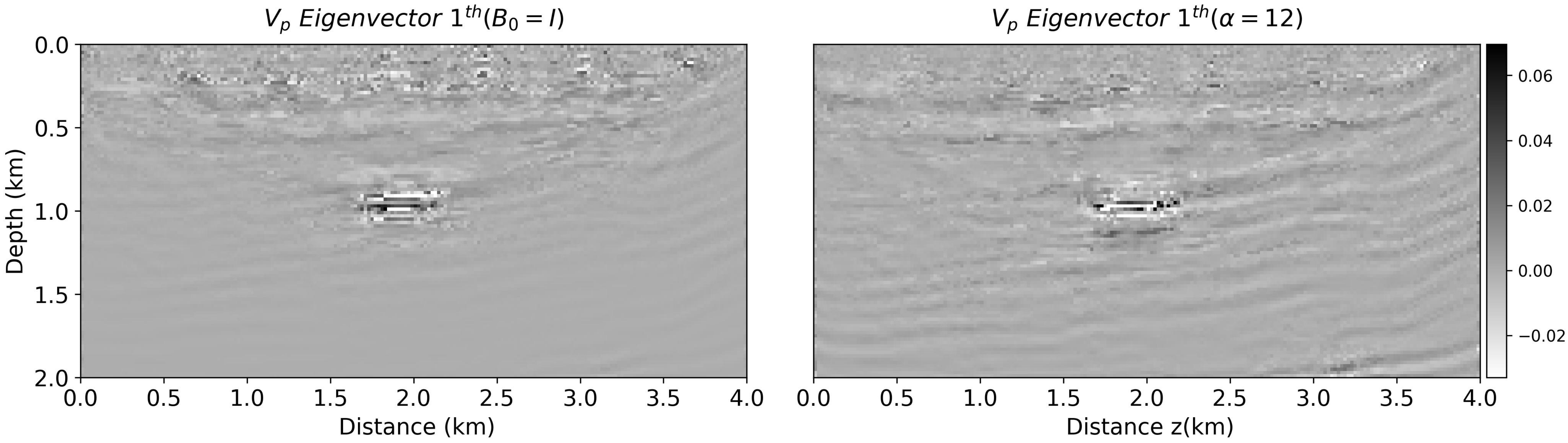
$$\mathbf{C_m} \approx \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{\Gamma}^{\dagger}$$



Part two: UQ using inverse Hessian

$\mathbf{C_m} \approx \boldsymbol{\Gamma\Lambda\Gamma}^{\dagger}$

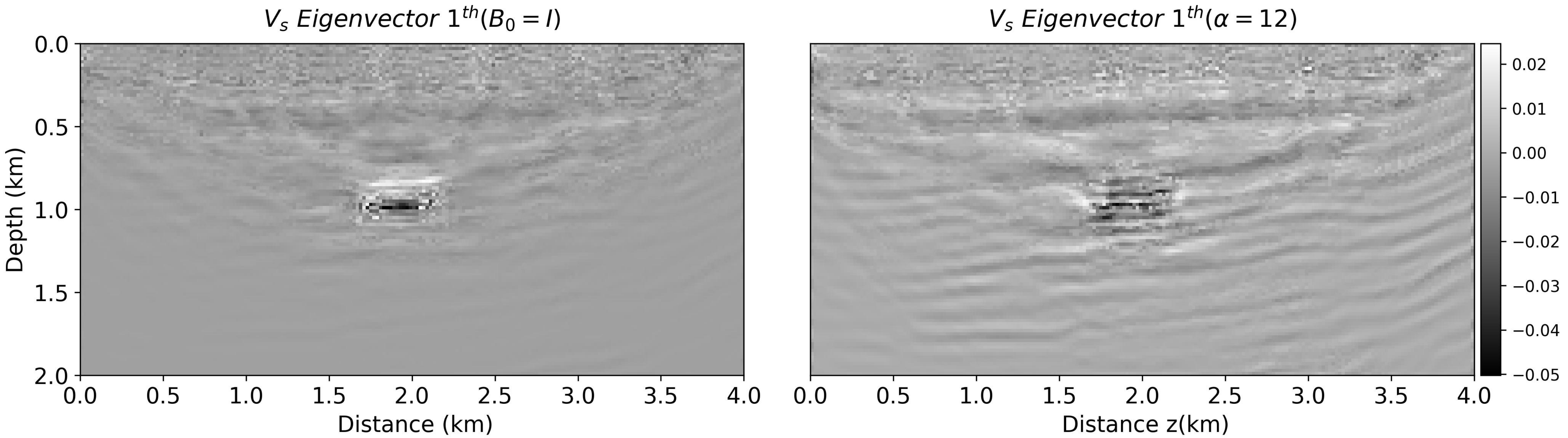
$\mathbf{C_m}$ Eigenvector comparisons for V_p



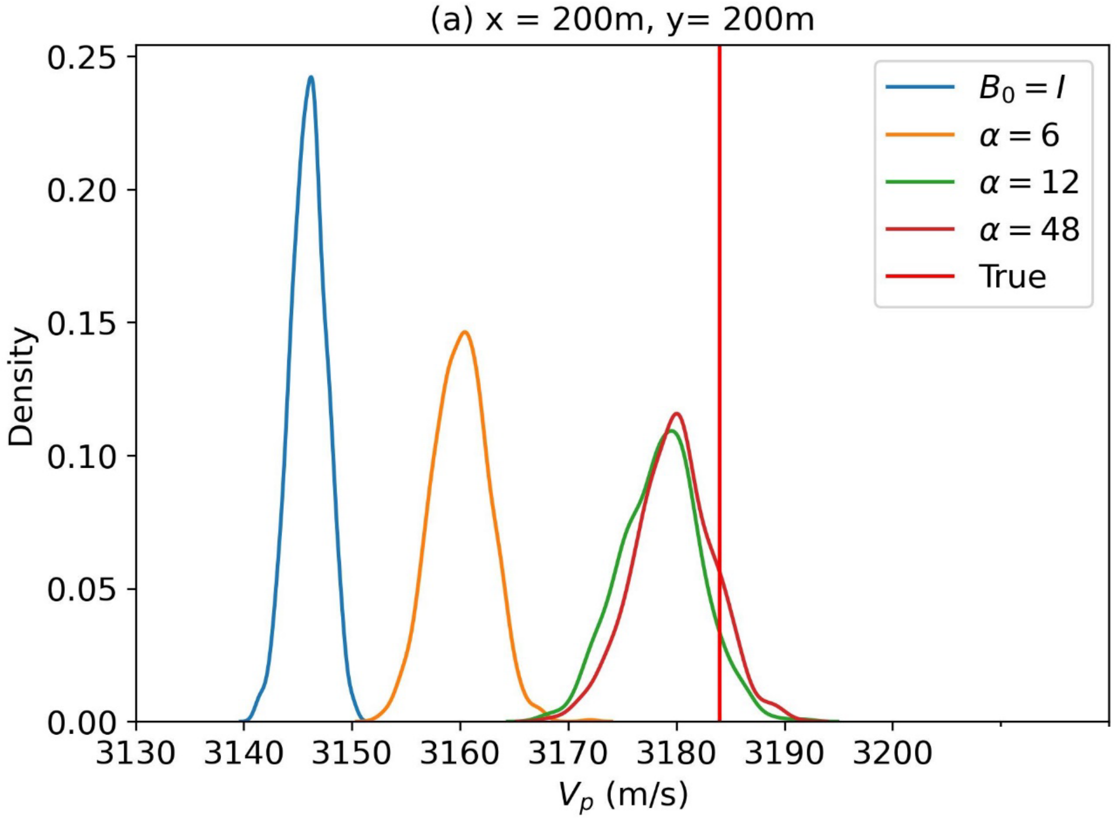
Part two: UQ using inverse Hessian

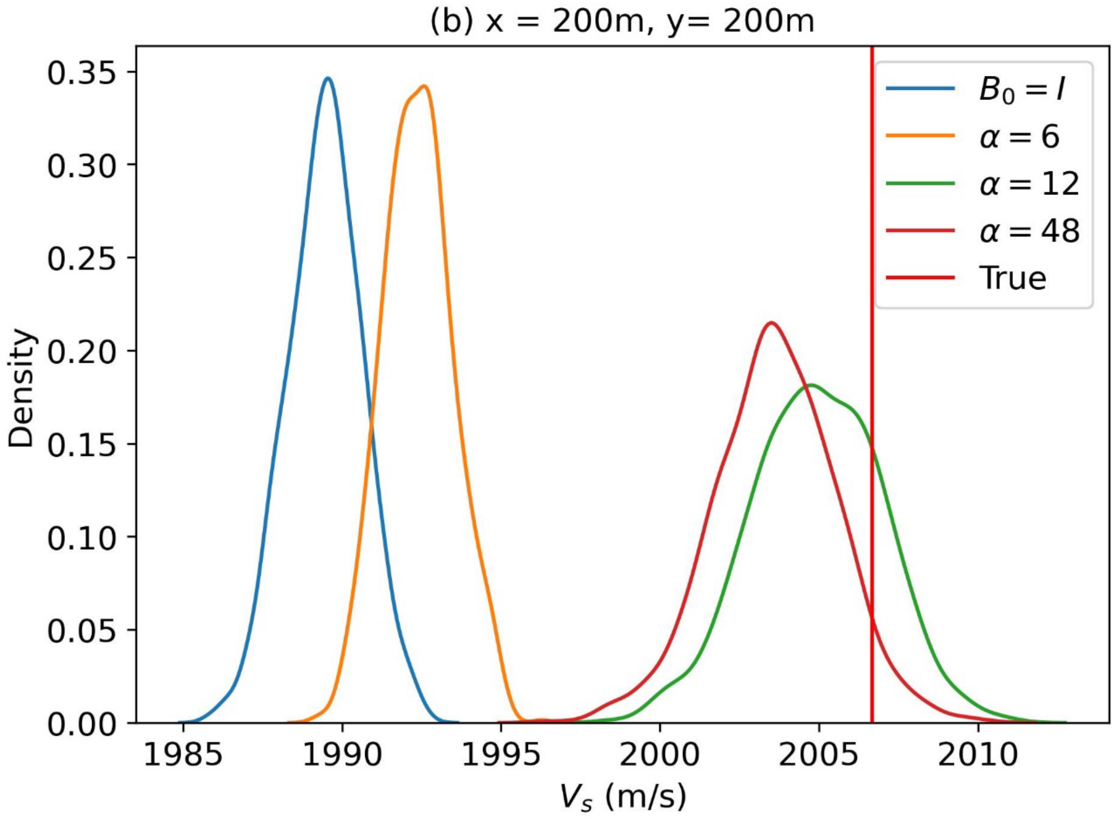
$\mathbf{C_m} \approx \boldsymbol{\Gamma\Lambda\Gamma}^{\dagger}$

$\mathbf{C_m}$ Eigenvector comparisons for V_s



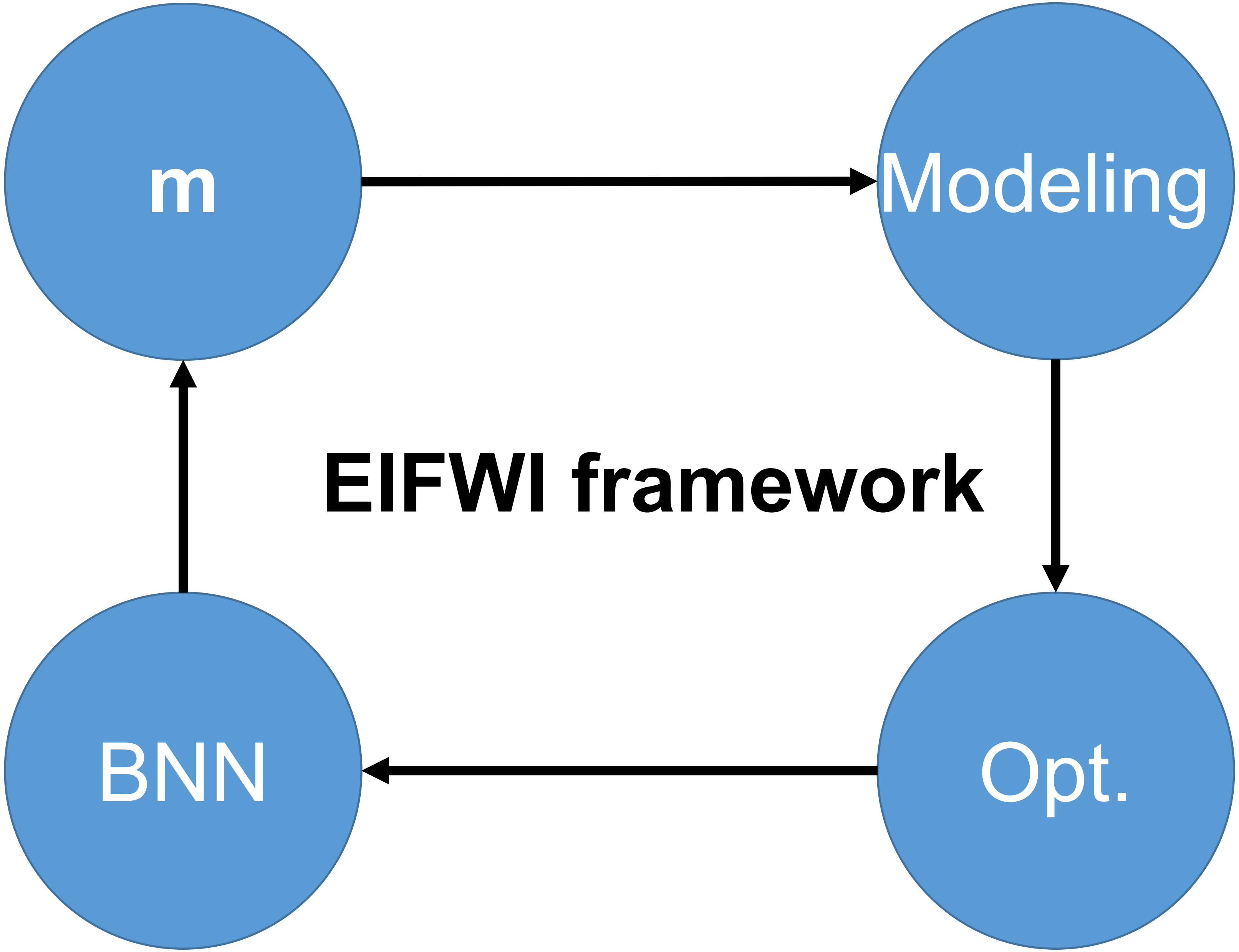
Posterior probability distributions



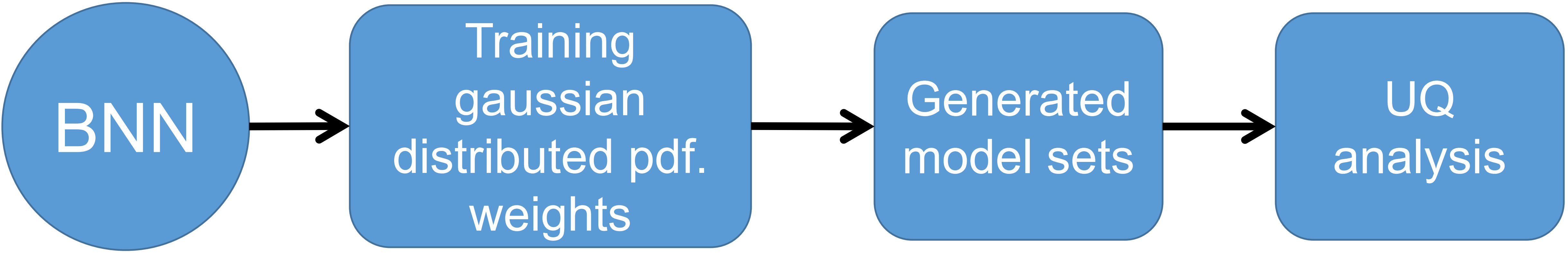


Part three: UQ using Bayesian neural network

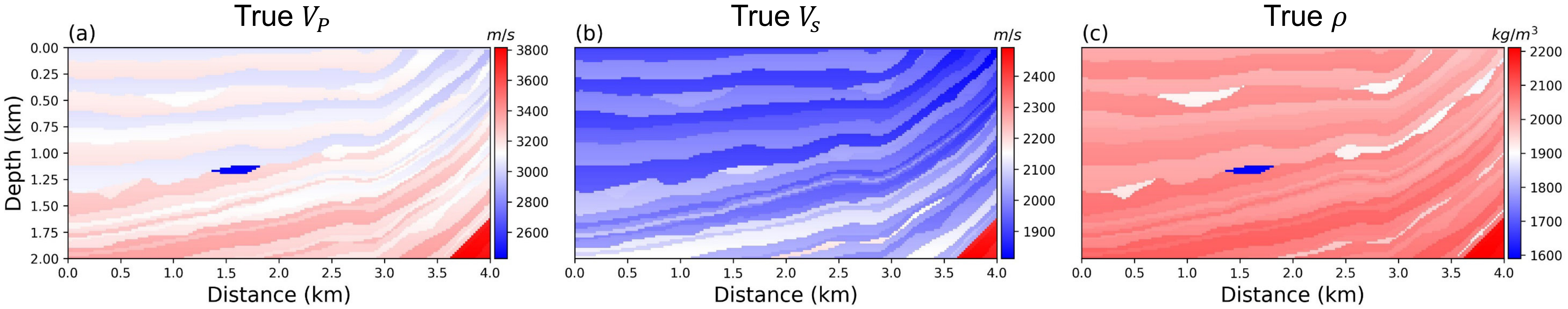
Part three: UQ using BNN



Part three: UQ using BNN

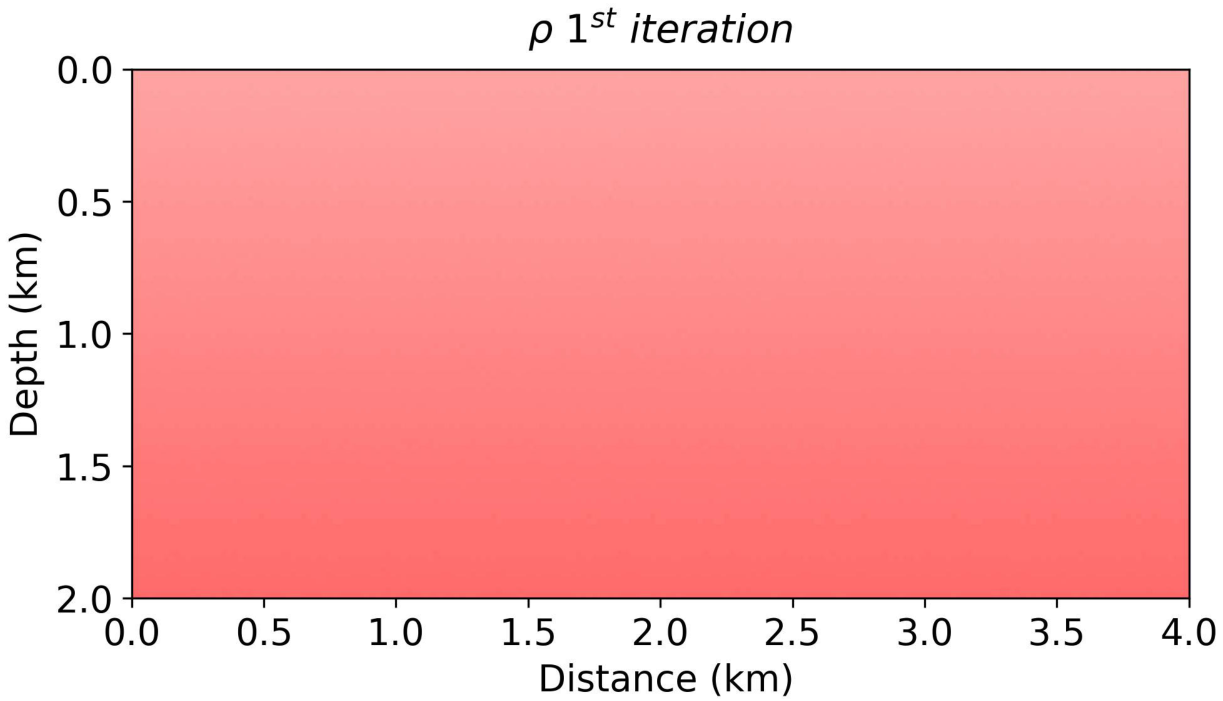


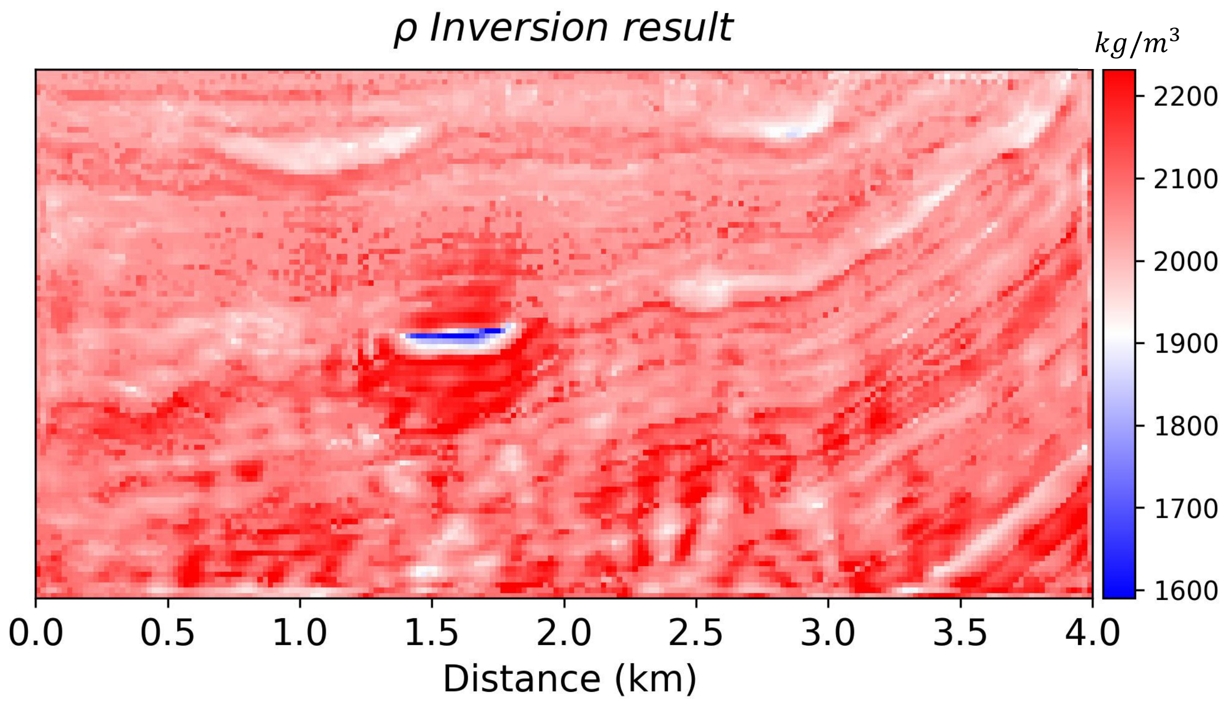
True models



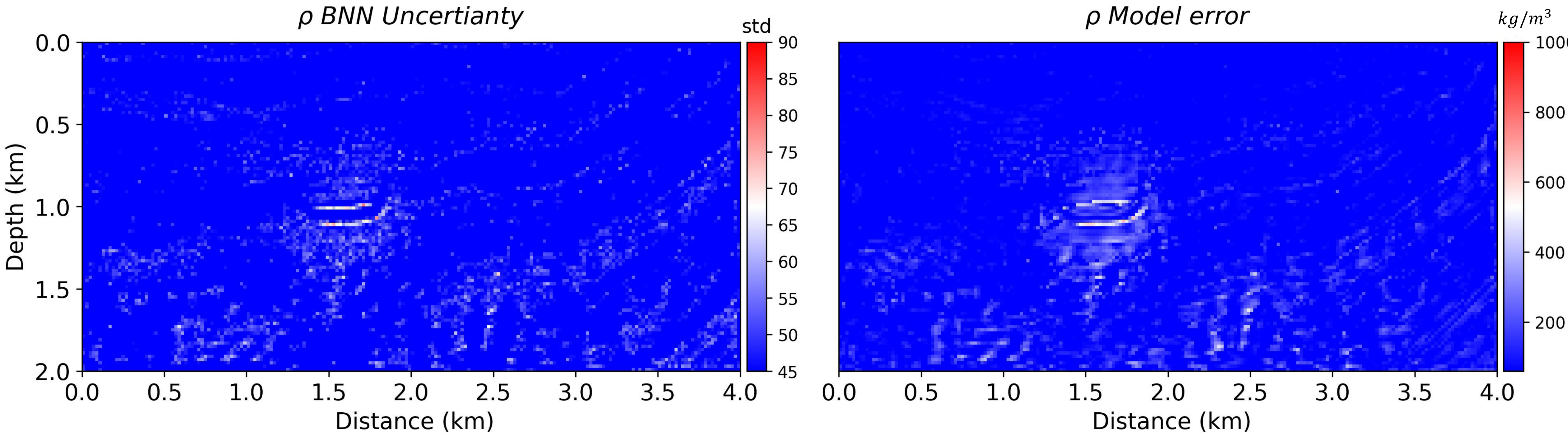
True V_P
(a)
m/s
Depth (km)
Distance (km)
True V_s
(b)
m/s
Distance (km)
True ρ
(c)
kg/m^3
Distance (km)

BNN ElFWI results for ρ

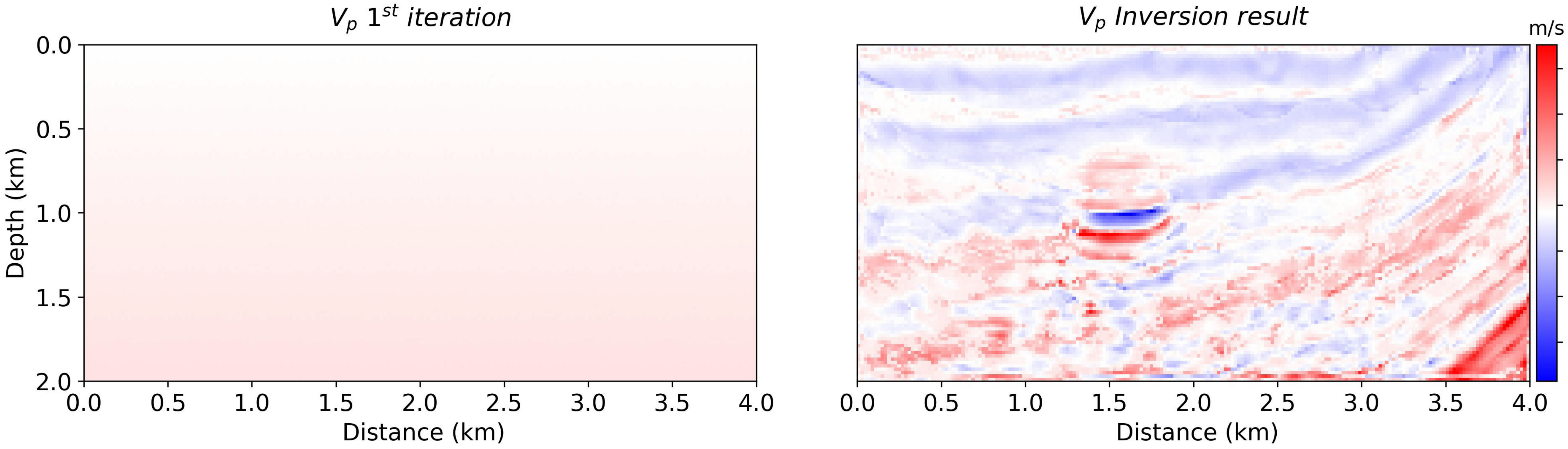




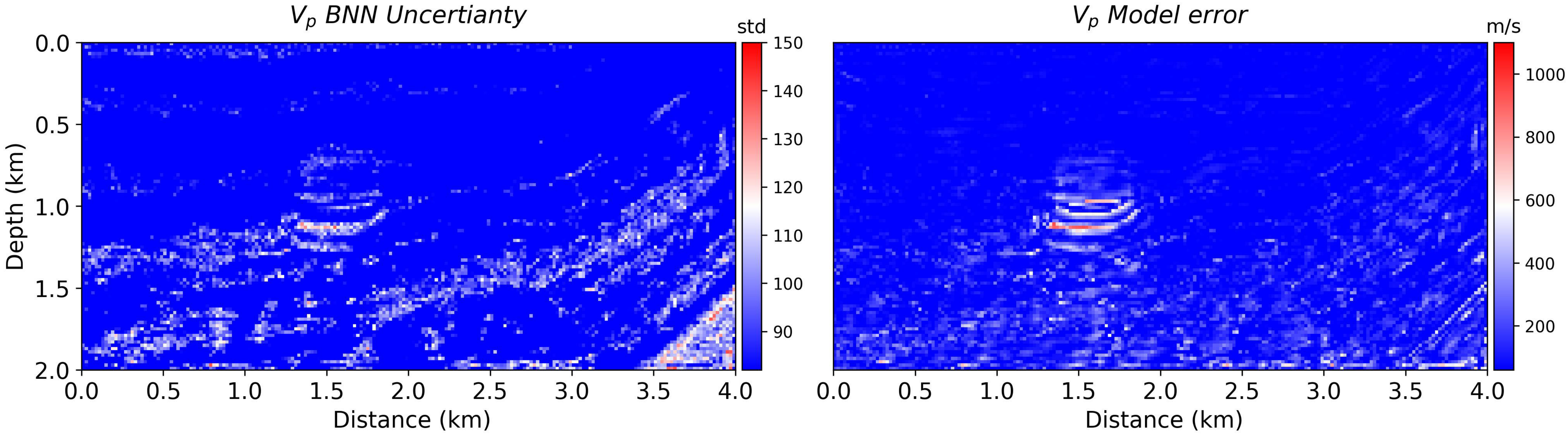
BNN ElFWI UQ for ρ



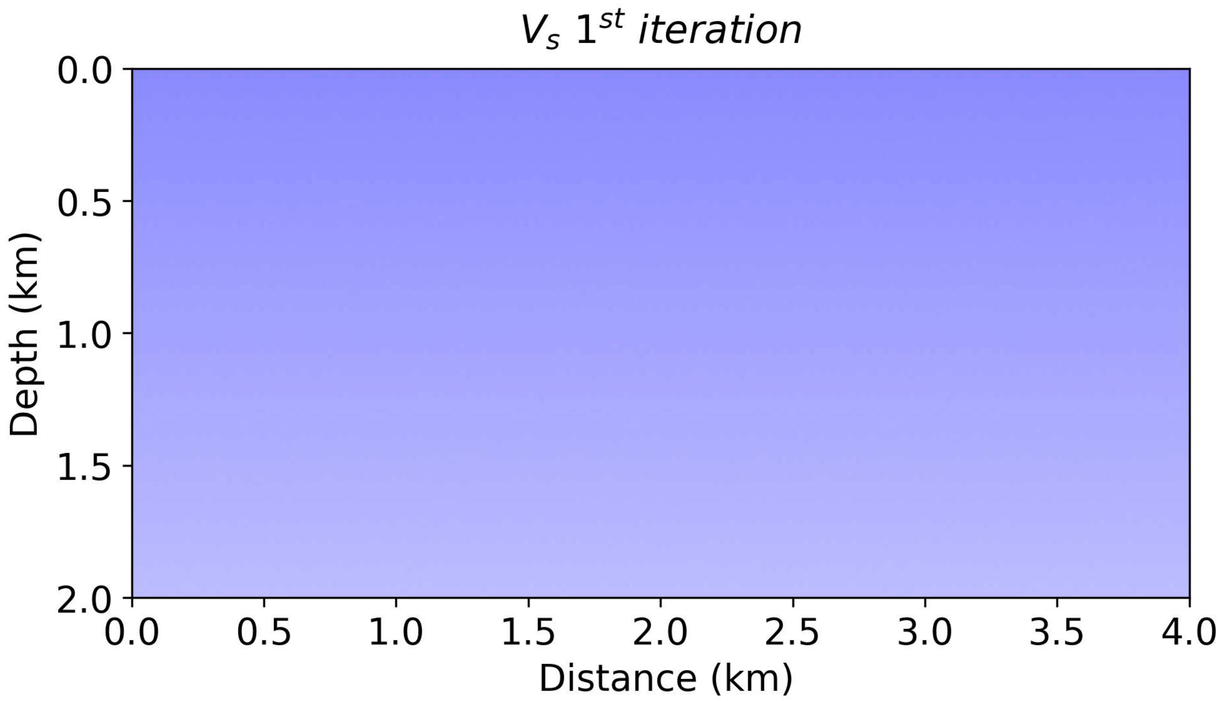
BNN ElFWI result for V_p

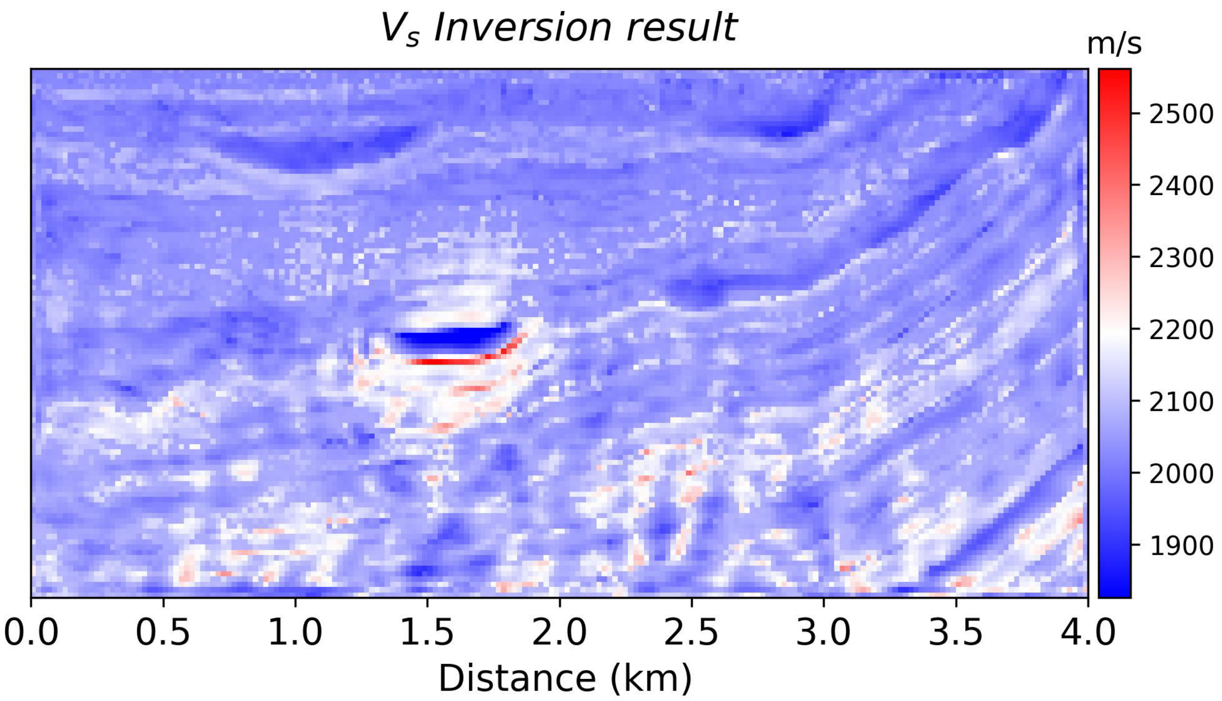


BNN EIFWI UQ for V_p

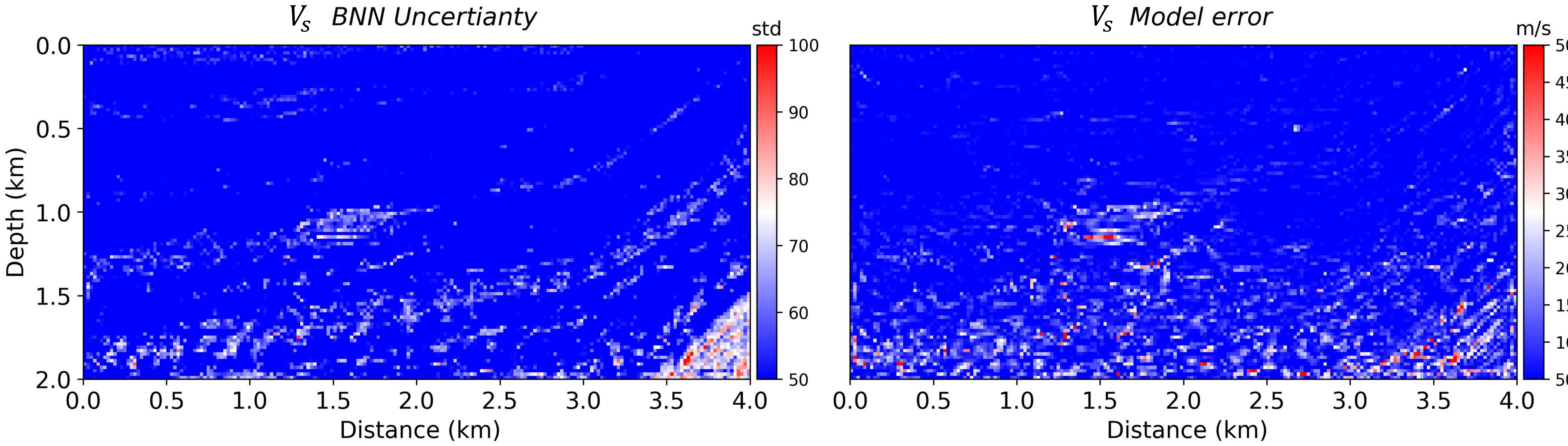


BNN ElFWI results for V_s





BNN EIFWI UQ for V_s



Conclusions

1. FWI uncertainties given by approximate Hessian are reasonable.
2. Initialization of $\boldsymbol{B_0}$ influence the uncertainty quantification.
3. BNN gives reasonable uncertainty quantification for FWI.

Future study

1. Prior model covariance matrix influence on uncertainty.
2. Influence of the regularization terms on uncertainty.

Thanks all CREWES sponsors and students.