A quantum algorithm for traveltime tomography

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FACULTY OF SCIENCE
Department of Geoscience
Carbon Management Canada

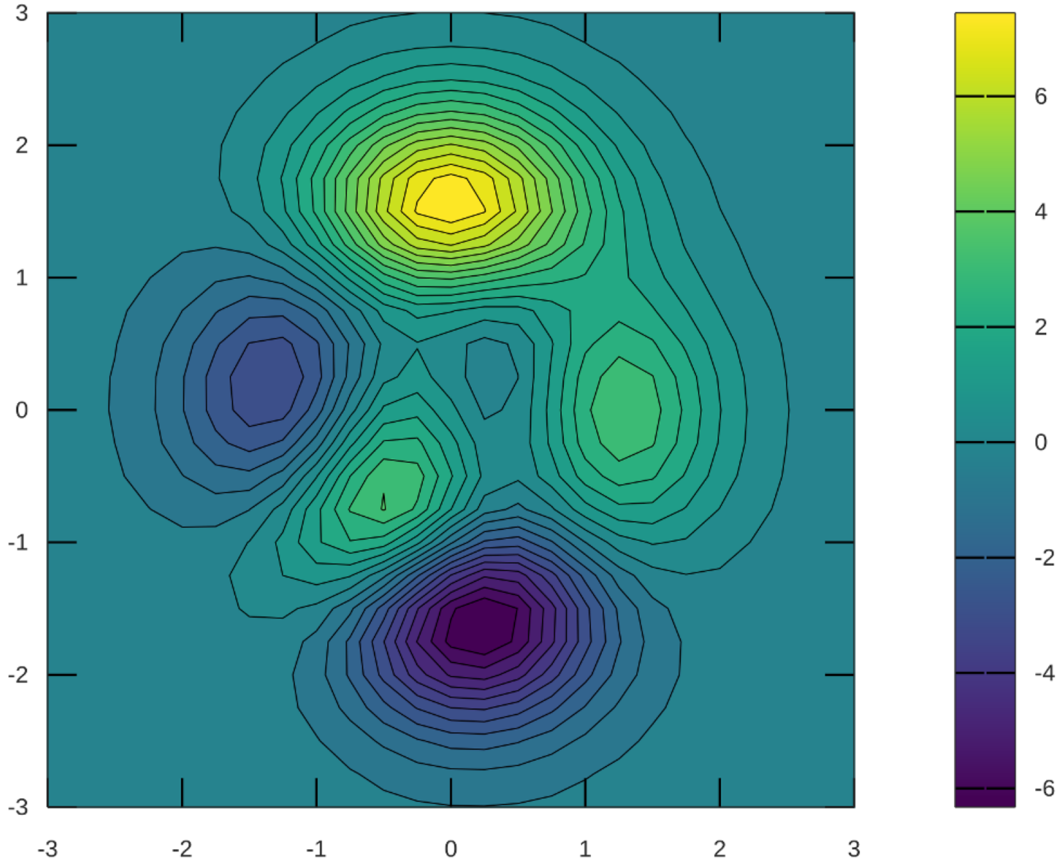
Introduction

Objectives

- Propose a general purpose quantum algorithm to solve a geophysical problem.
- Show an instance of the quantum algorithm running step by step that helps geophysicists to understand quantum techniques.

Gradient methods vs Quantum inversion

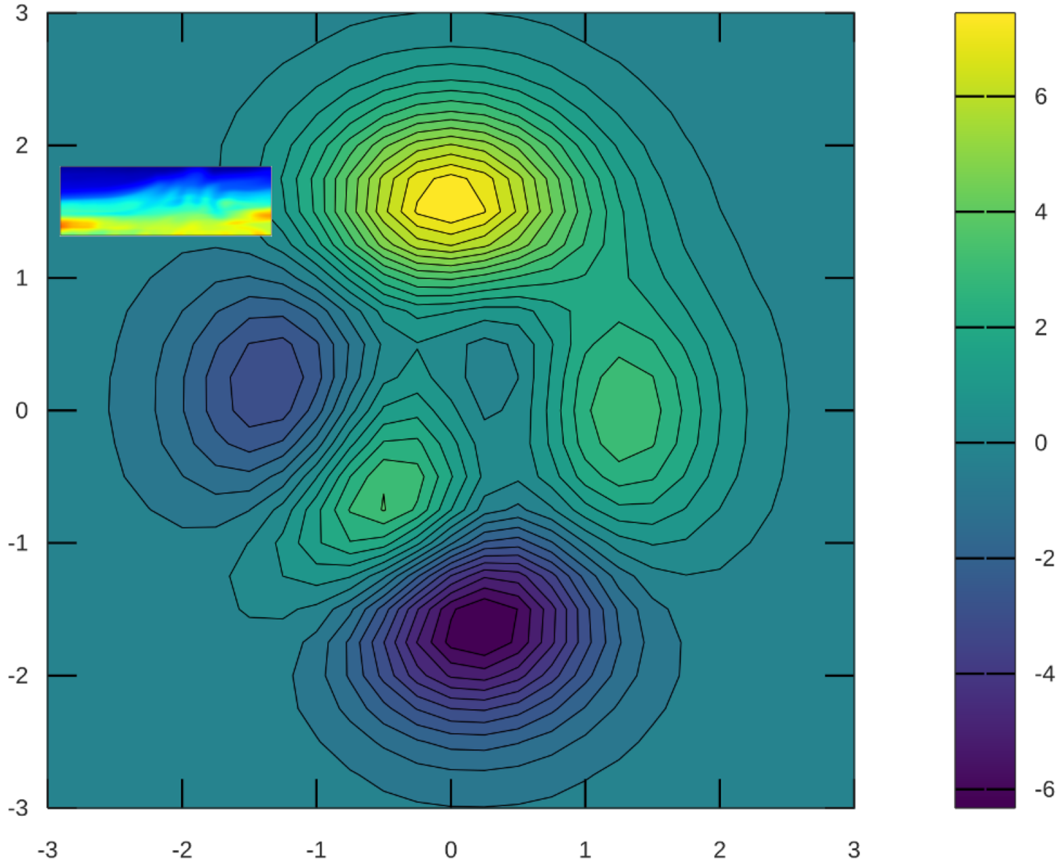
Gradient methods:



Quantum inversion:

Gradient methods vs Quantum inversion

Gradient methods:



Quantum inversion:

Gradient methods vs Quantum inversion

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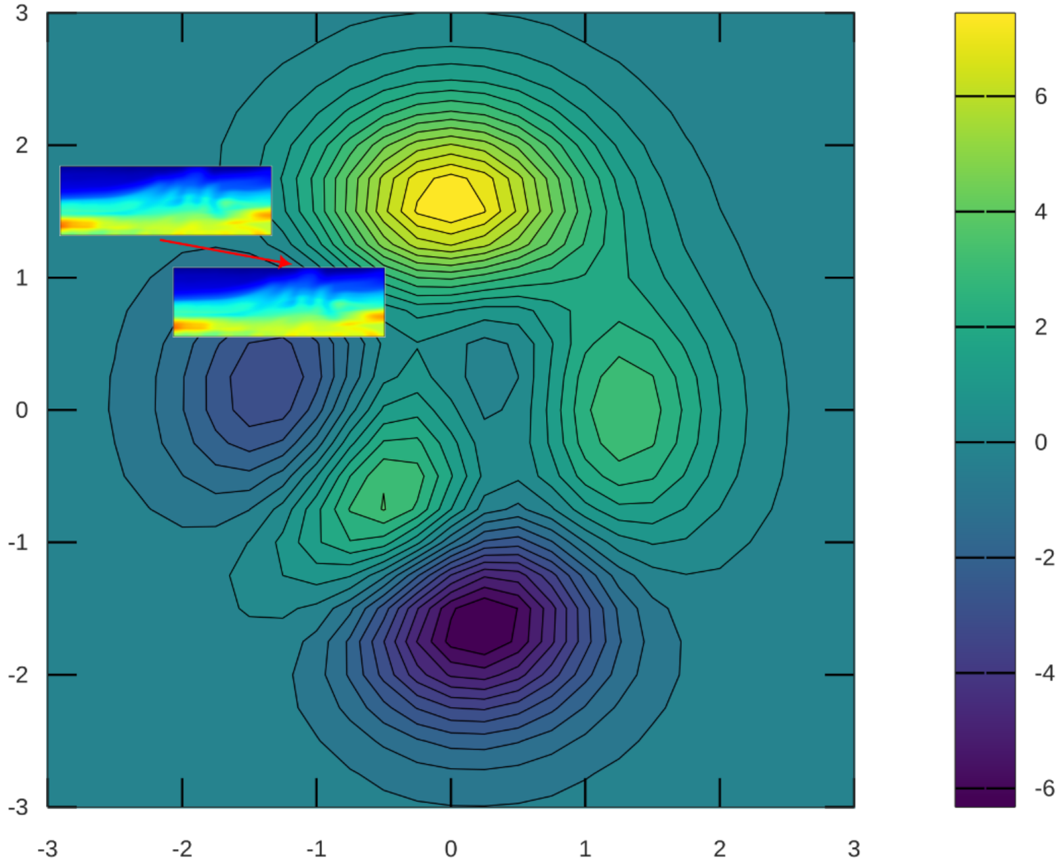
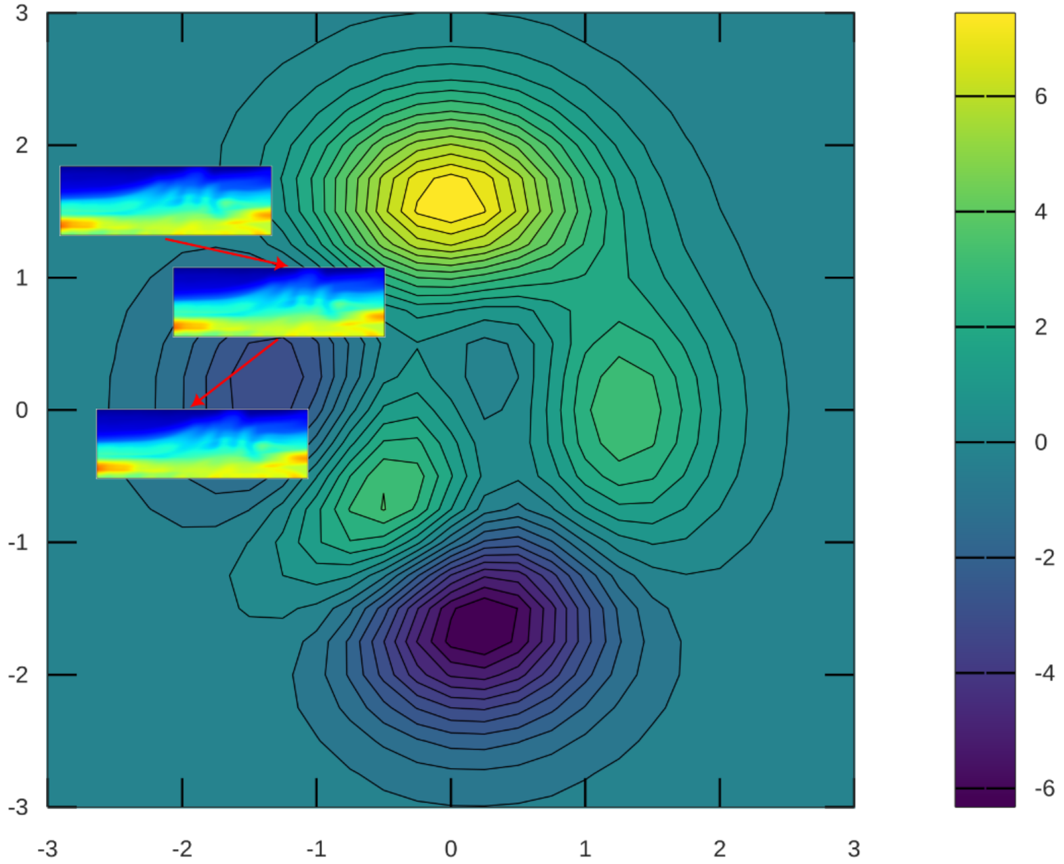


Quantum inversion:

Gradient methods vs Quantum inversion

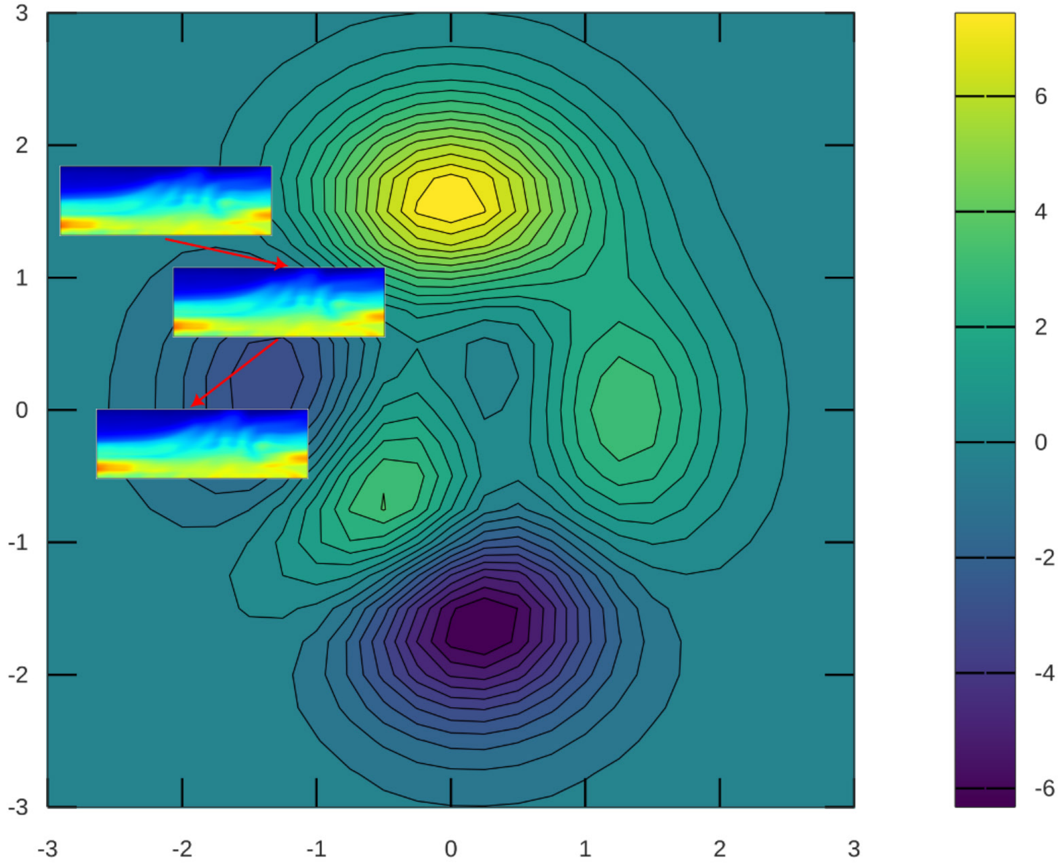
Gradient methods:



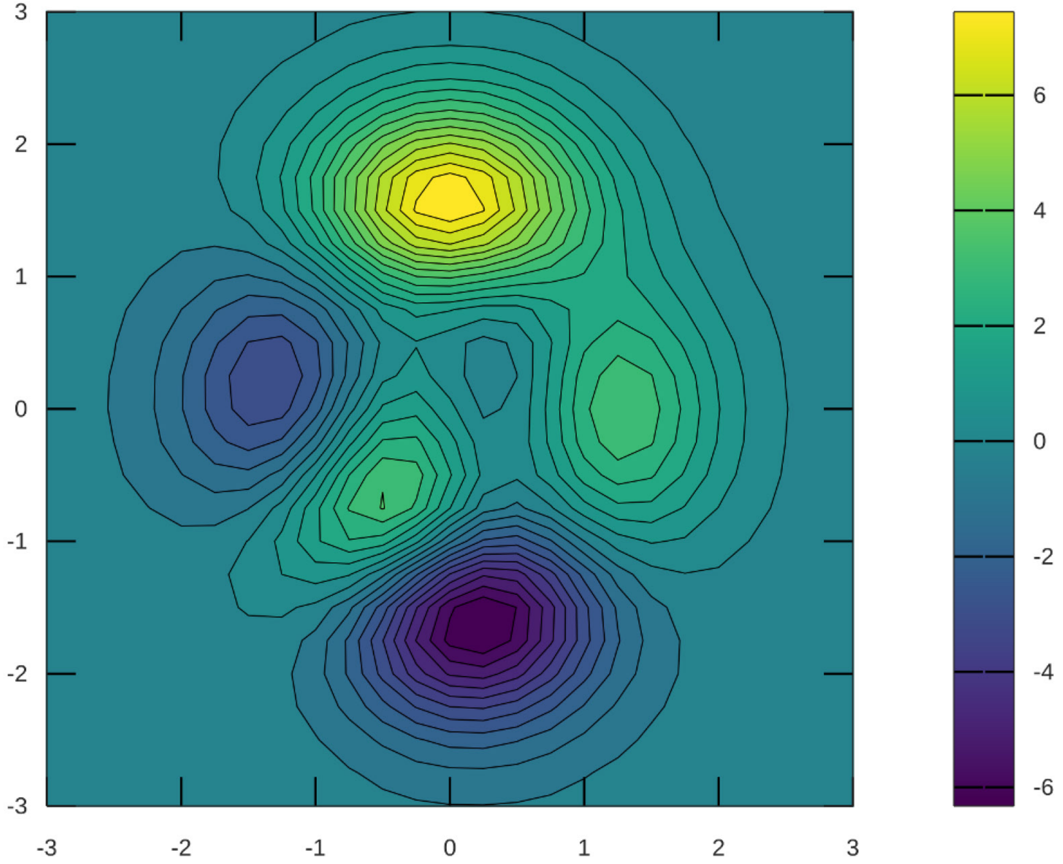
Quantum inversion:

Gradient methods vs Quantum inversion

Gradient methods:



Quantum inversion:



Gradient methods vs Quantum inversion

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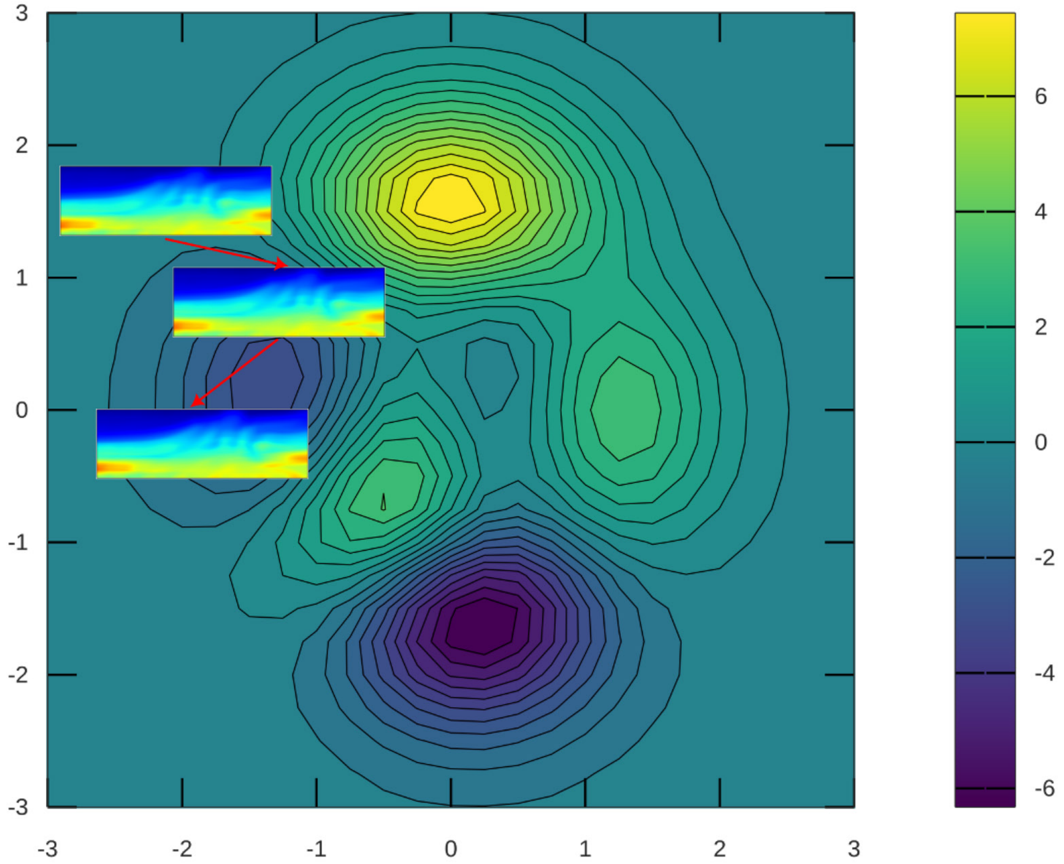
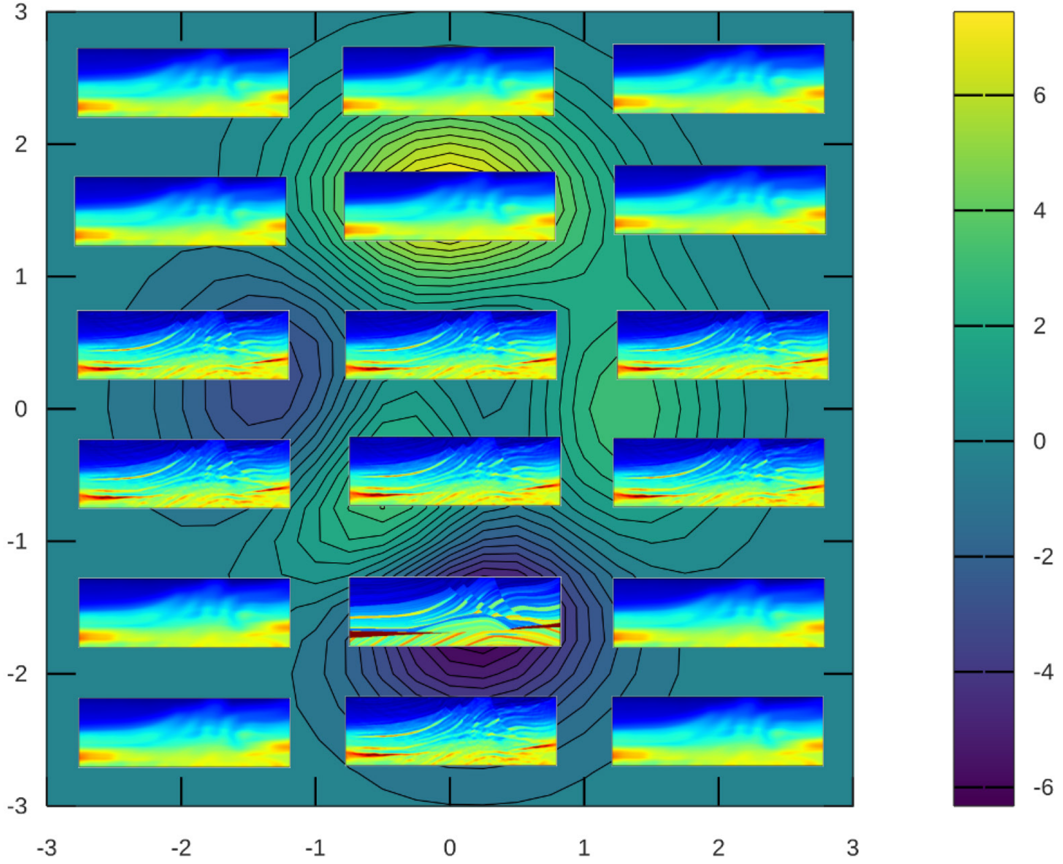


Quantum inversion:



Gradient methods vs Quantum inversion

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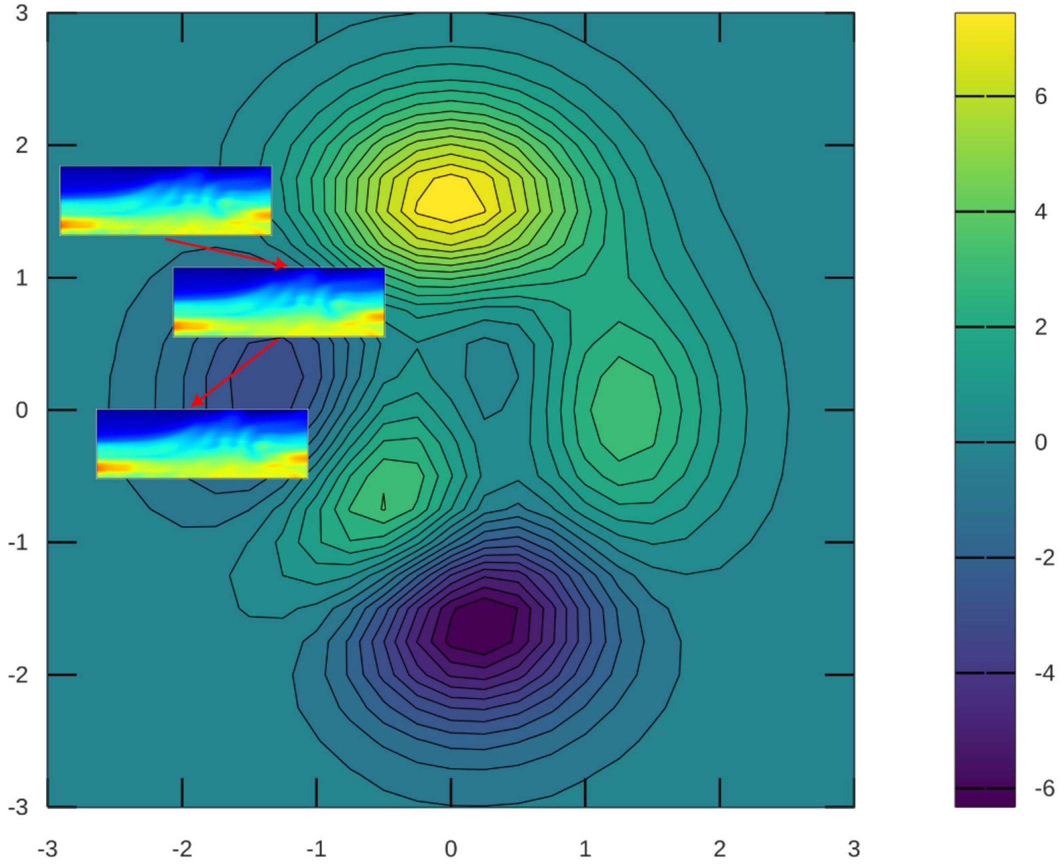
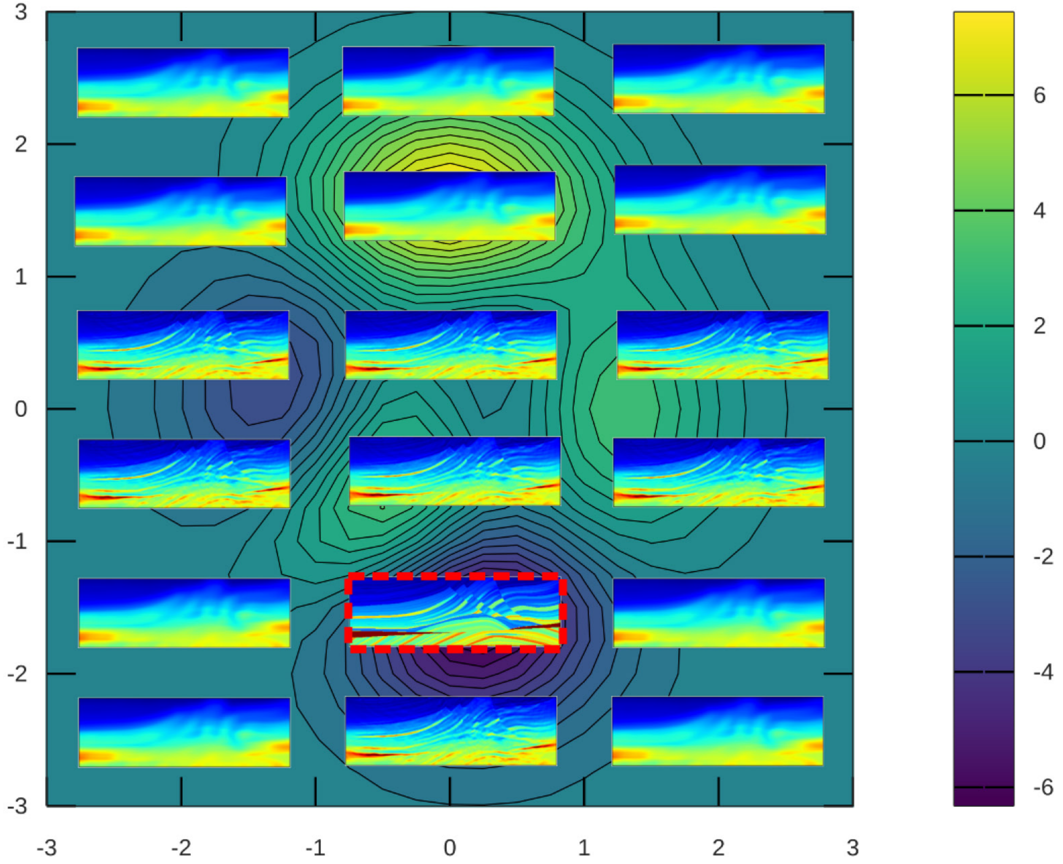
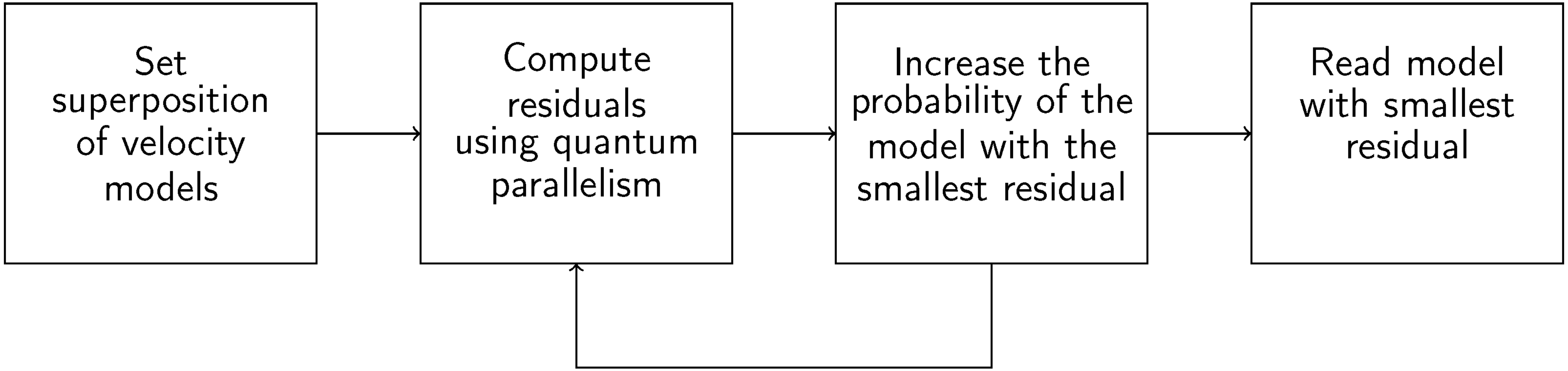


Quantum inversion:



Theory

General quantum inversion algorithm



Set superposition of velocity models

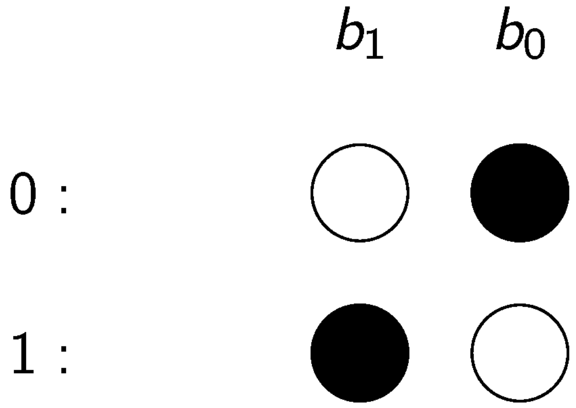
Compute residuals using quantum parallelism

Increase the probability of the model with the smallest residual

Read model with smallest residual

Qubits and superposition

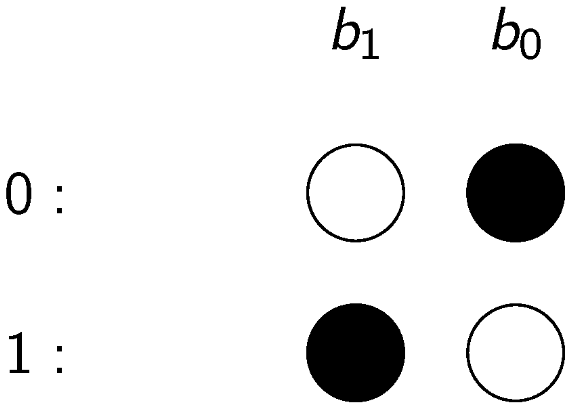
Classical bits:



- $b_1 = 1$
- $b_0 = 0$

Qubits and superposition

Classical bits:



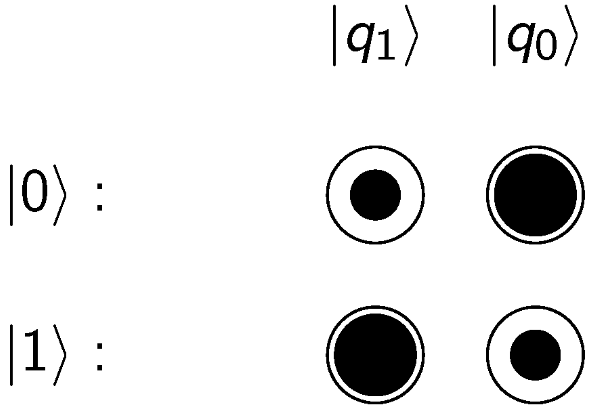
b_1 b_0

$0:$

$1:$

- $b_1 = 1$
- $b_0 = 0$

Quantum bits:



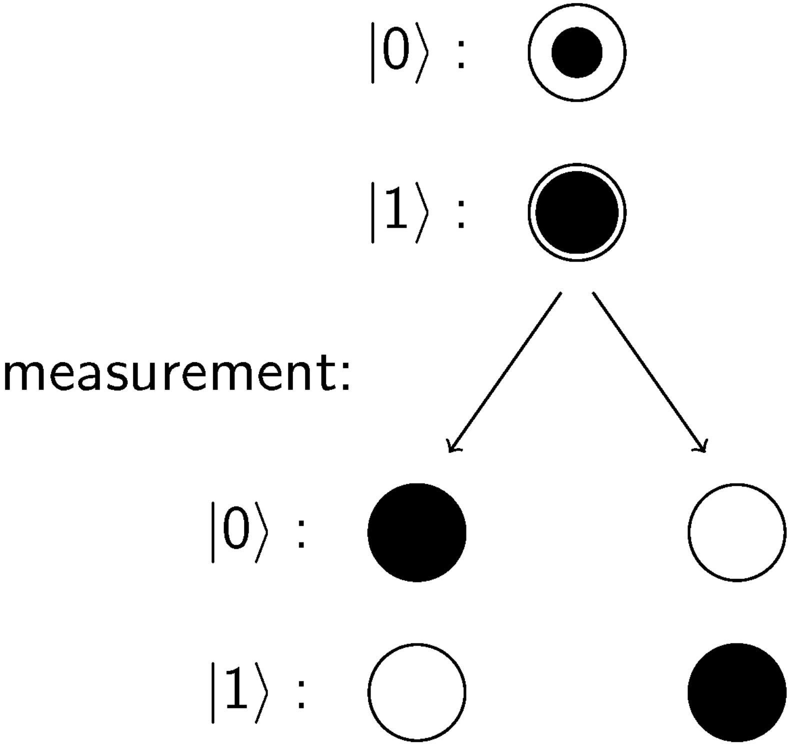
$|q_1\rangle$ $|q_0\rangle$

$|0\rangle :$

$|1\rangle :$

- $|q_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$
- $|q_0\rangle = \alpha_0 |0\rangle + \beta_0 |1\rangle$

Some qubit rules



$|0\rangle$:

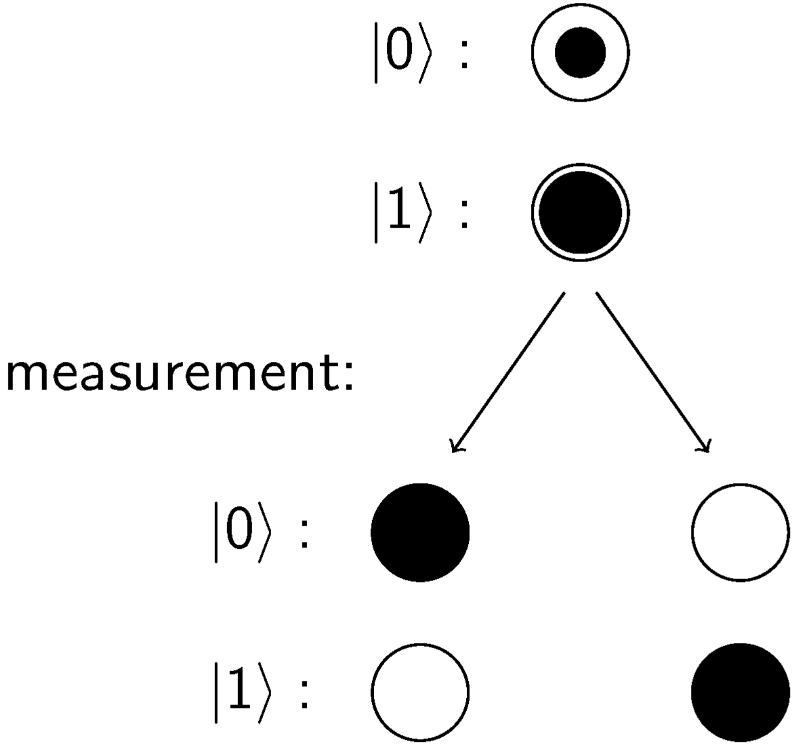
$|1\rangle$:

measurement:

$|0\rangle$:

$|1\rangle$:

Some qubit rules



$|0\rangle$:

$|1\rangle$:

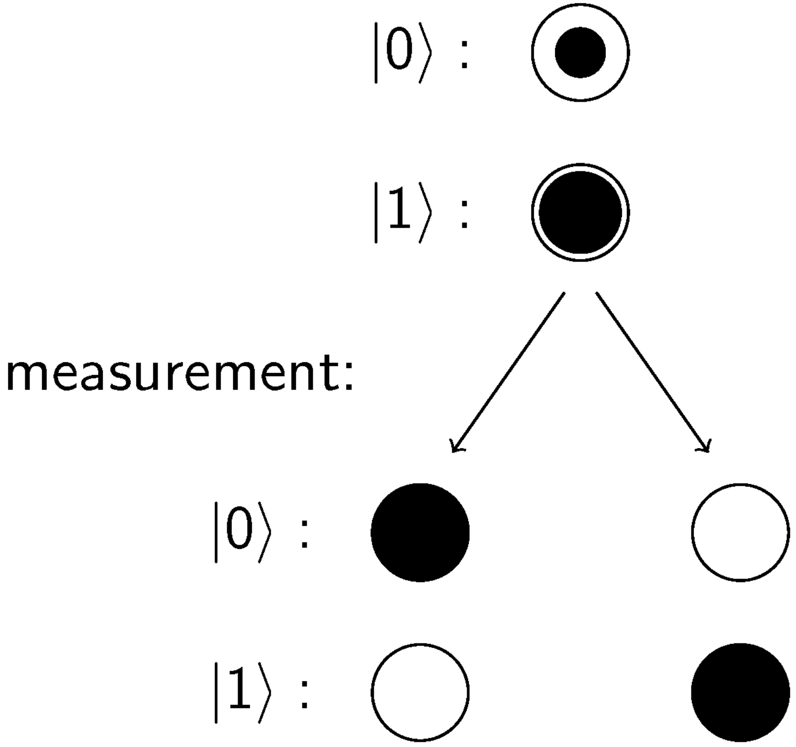
measurement:

$|0\rangle$:

$|1\rangle$:

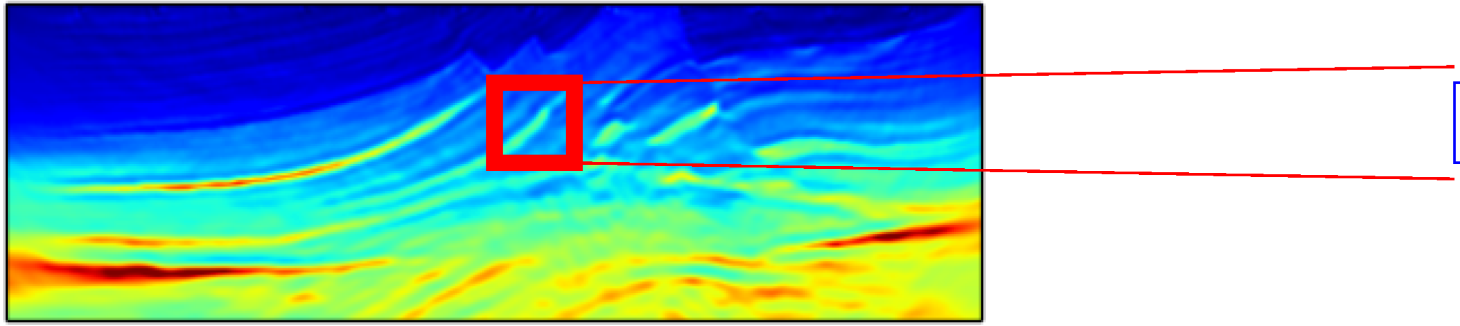
- $|\phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, where $\alpha, \beta \in \mathbb{C}$
- $|\alpha|^2 + |\beta|^2 = 1$

Some qubit rules



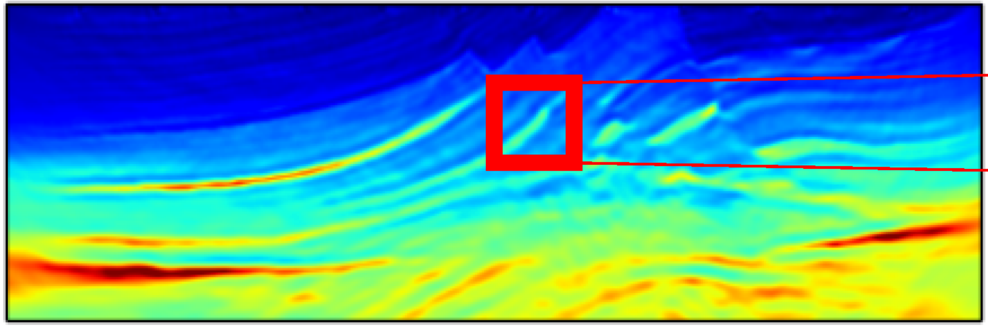
- $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $\alpha, \beta \in \mathbb{C}$
- $|\alpha|^2 + |\beta|^2 = 1$
  - $|\alpha|^2$ is the probability of measuring $|\phi\rangle$ and getting a $|0\rangle$.
  - $|\beta|^2$ is the probability of measuring $|\phi\rangle$ and getting a $|1\rangle$.

Models superposition

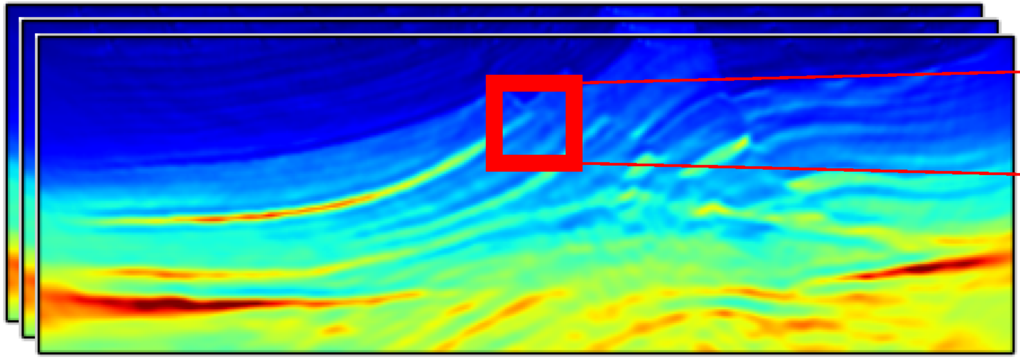


$$\boxed{\underbrace{0100010100101110011}_{r}}$$

Models superposition



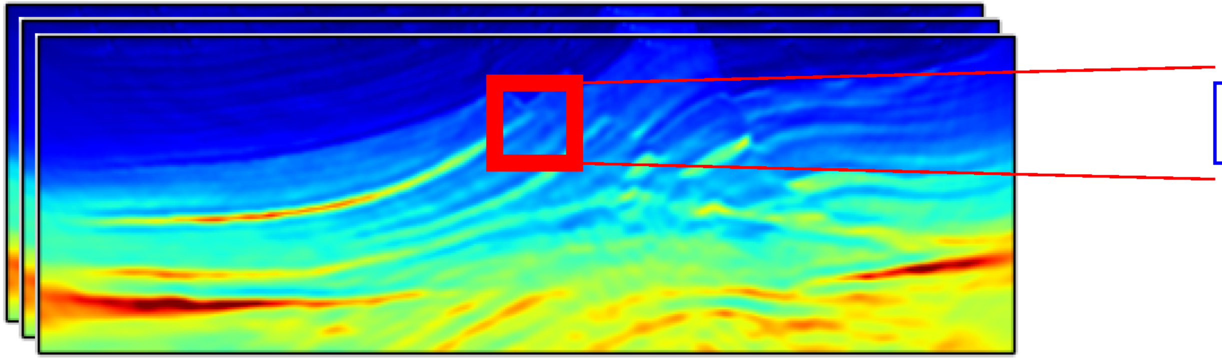
$$\underbrace{\boxed{0100010100101110011}}_{r}$$



$$\underbrace{\boxed{(\alpha_0 \left|0\right\rangle + \beta_0 \left|1\right\rangle) \dots (\alpha_n \left|0\right\rangle + \beta_n \left|1\right\rangle)}}_{\gamma_0 \left|r_0\right\rangle + \cdots + \gamma_{2^n-1} \left|r_{2^n-1}\right\rangle}$$

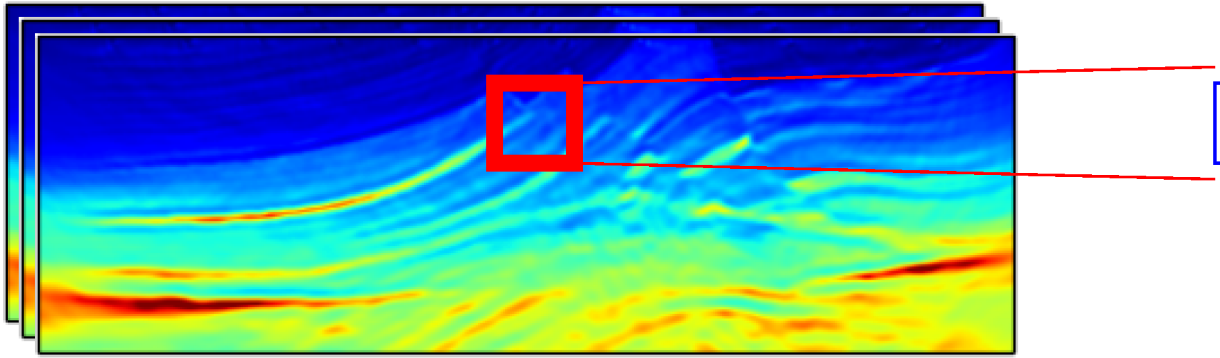
(It's like a polynomial multiplication)

Models superposition

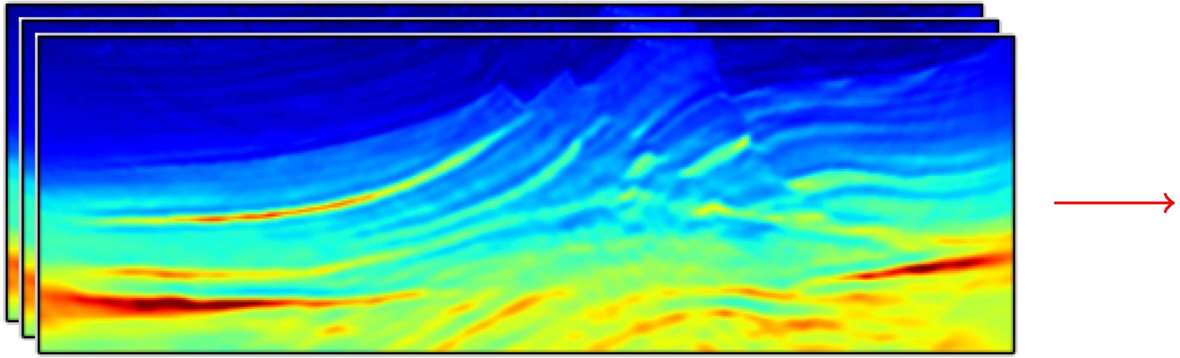


$$\underbrace{\boxed{\left(\alpha_0 |0\rangle + \beta_0 |1\rangle\right) \dots \left(\alpha_n |0\rangle + \beta_n |1\rangle\right)}}_{|\phi_{ij}\rangle}$$

Models superposition



$$\underbrace{\boxed{(\alpha_0 |0\rangle + \beta_0 |1\rangle) \dots (\alpha_n |0\rangle + \beta_n |1\rangle)}}_{|\phi_{ij}\rangle}$$



$\lvert\phi_{1,1}\rangle$	$\lvert\phi_{1,2}\rangle$	$\lvert\phi_{1,3}\rangle$	$\lvert\phi_{1,4}\rangle$	$\lvert\phi_{1,5}\rangle$	$\lvert\phi_{1,6}\rangle$	$\lvert\phi_{1,7}\rangle$	$\lvert\phi_{1,8}\rangle$	$\lvert\phi_{1,9}\rangle$
$\lvert\phi_{2,1}\rangle$	$\lvert\phi_{2,2}\rangle$	$\lvert\phi_{2,3}\rangle$	$\lvert\phi_{2,4}\rangle$	$\lvert\phi_{2,5}\rangle$	$\lvert\phi_{2,6}\rangle$	$\lvert\phi_{2,7}\rangle$	$\lvert\phi_{2,8}\rangle$	$\lvert\phi_{2,9}\rangle$
$\lvert\phi_{3,1}\rangle$	$\lvert\phi_{3,2}\rangle$	$\lvert\phi_{3,3}\rangle$	$\lvert\phi_{3,4}\rangle$	$\lvert\phi_{3,5}\rangle$	$\lvert\phi_{3,6}\rangle$	$\lvert\phi_{3,7}\rangle$	$\lvert\phi_{3,8}\rangle$	$\lvert\phi_{3,9}\rangle$
$\lvert\phi_{4,1}\rangle$	$\lvert\phi_{4,2}\rangle$	$\lvert\phi_{4,3}\rangle$	$\lvert\phi_{4,4}\rangle$	$\lvert\phi_{4,5}\rangle$	$\lvert\phi_{4,6}\rangle$	$\lvert\phi_{4,7}\rangle$	$\lvert\phi_{4,8}\rangle$	$\lvert\phi_{4,9}\rangle$

Some quantum gates

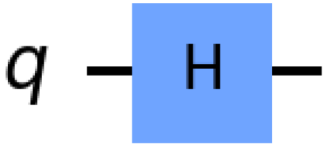
Not gate:

input	$\|0\rangle$	$\|1\rangle$
output	$\|1\rangle$	$\|0\rangle$

$$\alpha |0\rangle + \beta |1\rangle \xmapsto{not} \alpha |1\rangle + \beta |0\rangle$$

Some quantum gates

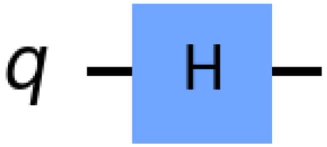
Hadamard gate:



input	$\|0\rangle$	$\|1\rangle$
output	$\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$	$\frac{1}{\sqrt{2}}\|0\rangle - \frac{1}{\sqrt{2}}\|1\rangle$

Some quantum gates

Hadamard gate:

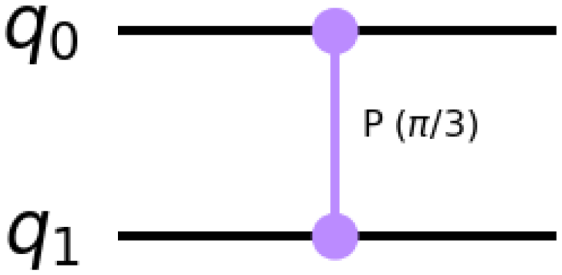


input	$\|0\rangle$	$\|1\rangle$
output	$\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$	$\frac{1}{\sqrt{2}}\|0\rangle - \frac{1}{\sqrt{2}}\|1\rangle$

$$
\begin{aligned}
|00\rangle \quad \xmapsto{HH} \quad & \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \\
= \quad & \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle \\
= \quad & \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle
\end{aligned}
$$

Some quantum gates

Controled phase multiqubit gate:



input	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
output	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$e^{i\phi}\|11\rangle$

$$\alpha|01\rangle + \beta|11\rangle \xmapsto{cpha(\phi)} \alpha|01\rangle + e^{i\phi}\beta|11\rangle$$

$$\alpha|01\rangle + \beta|11\rangle \xmapsto{cpha(\pi)} \alpha|01\rangle - \beta|11\rangle$$

Arithmetic in superposition

Addition:

$$(\alpha_0 \left| r_0 \right\rangle + \cdots + \alpha_n \left| r_n \right\rangle) + \left| s \right\rangle \quad = \quad \alpha_0 \left| r_0 + s \right\rangle + \cdots + \alpha_n \left| r_n + s \right\rangle$$

Arithmetic in superposition

Addition:

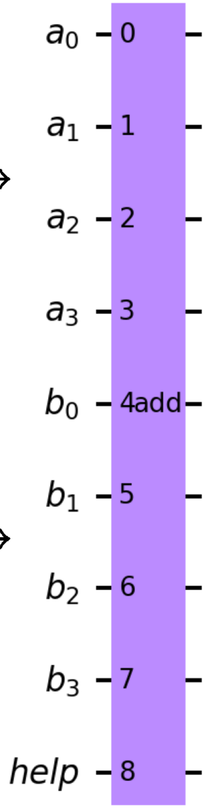
$$(\alpha_0 |r_0\rangle + \cdots + \alpha_n |r_n\rangle) + |s\rangle \quad = \quad \alpha_0 |r_0 + s\rangle + \cdots + \alpha_n |r_n + s\rangle$$

Inputs:

Outputs:

$$\alpha_0 |r_0\rangle + \cdots + \alpha_n |r_n\rangle \longrightarrow$$

$$|s\rangle \longrightarrow$$



$$\longrightarrow \alpha_0 |r_0 + s\rangle + \cdots + \alpha_n |r_n + s\rangle$$

$$\longrightarrow |s\rangle$$

Arithmetic in superposition

Multiplication

$$(\alpha_0 |r_0\rangle + \cdots + \alpha_n |r_n\rangle) \times |s\rangle \quad = \quad \alpha_0 |r_0 \times s\rangle + \cdots + \alpha_n |r_n \times s\rangle$$

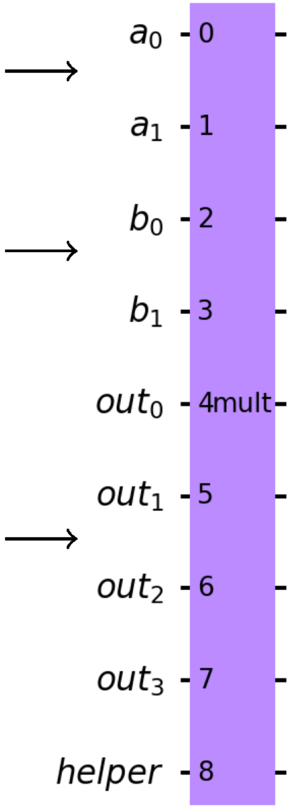
Arithmetic in superposition

Multiplication

$$(\alpha_0 \left|r_0\right\rangle + \cdots + \alpha_n \left|r_n\right\rangle) \times \left|s\right\rangle \quad = \quad \alpha_0 \left|r_0 \times s\right\rangle + \cdots + \alpha_n \left|r_n \times s\right\rangle$$

Inputs:

Outputs:



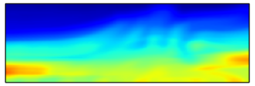
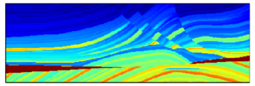
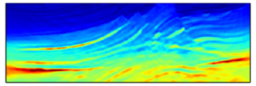
$\alpha_0 \left|r_0\right\rangle + \cdots + \alpha_n \left|r_n\right\rangle \longrightarrow$ $\longrightarrow \alpha_0 \left|r_0\right\rangle + \cdots + \alpha_n \left|r_n\right\rangle$

$\left|s\right\rangle \longrightarrow$ $\longrightarrow \left|s\right\rangle$

$\left|t\right\rangle \longrightarrow$ $\longrightarrow \left|t\right\rangle + (\alpha_0 \left|r_0 \times s\right\rangle + \cdots + \alpha_n \left|r_n \times s\right\rangle)$

Compute in superposition

1. Initial equal superposition.

$\alpha_0 \left|0\right\rangle$  $+ \cdots + \alpha_k \left|0\right\rangle$  $+ \cdots + \alpha_n \left|0\right\rangle$ 

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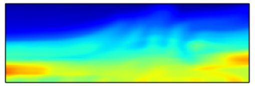
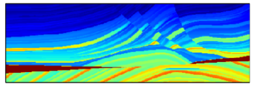
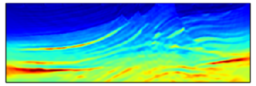
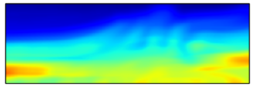
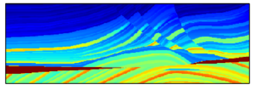
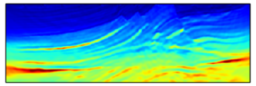
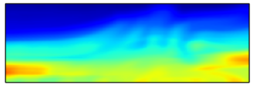
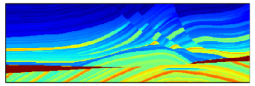
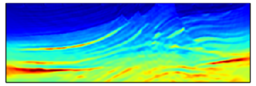
$\alpha_0 \left|0\right\rangle$  $+ \cdots + \alpha_k \left|0\right\rangle$  $+ \cdots + \alpha_n \left|0\right\rangle$ 

2. Perform forward modelling and compute residuals in superposition.

$\alpha_0 \left|r_0\right\rangle$  $+ \cdots + \alpha_k \left|r_k\right\rangle$  $+ \cdots + \alpha_n \left|r_n\right\rangle$ 

Compute in superposition

1. Initial equal superposition.

$\alpha_0 \left|0\right\rangle$  $+ \cdots + \alpha_k \left|0\right\rangle$  $+ \cdots + \alpha_n \left|0\right\rangle$ 

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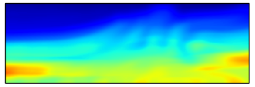
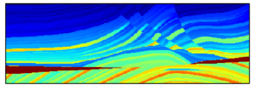
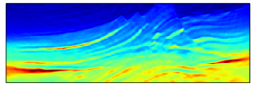
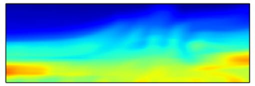
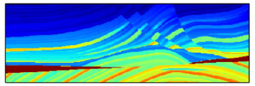
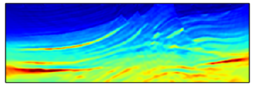
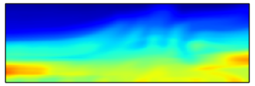
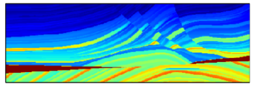
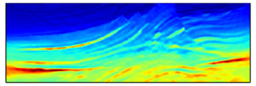
$\alpha_0 \left|r_0\right\rangle$  $+ \cdots + \alpha_k \left|r_k\right\rangle$  $+ \cdots + \alpha_n \left|r_n\right\rangle$ 

3. Flip the sign of residuals minimum.

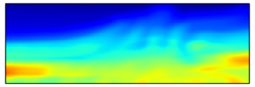
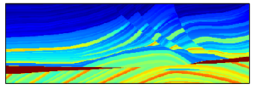
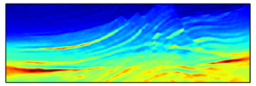
$\alpha_0 \left|r_0\right\rangle$  $+ \cdots - \alpha_k \left|r_k\right\rangle$  $+ \cdots + \alpha_n \left|r_n\right\rangle$ 

Compute in superposition

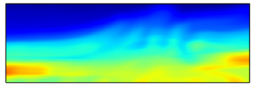
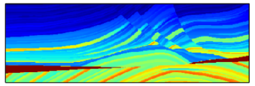
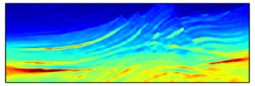
1. Initial equal superposition.

$\alpha_0 \left|0\right\rangle$  $+ \cdots + \alpha_k \left|0\right\rangle$  $+ \cdots + \alpha_n \left|0\right\rangle$ 

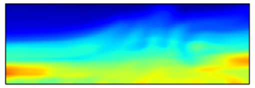
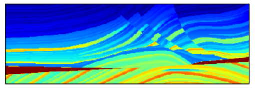
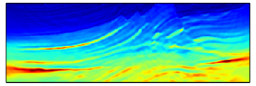
2. Perform forward modelling and compute residuals in superposition.

$\alpha_0 \left|r_0\right\rangle$  $+ \cdots + \alpha_k \left|r_k\right\rangle$  $+ \cdots + \alpha_n \left|r_n\right\rangle$ 

3. Flip the sign of residuals minimum.

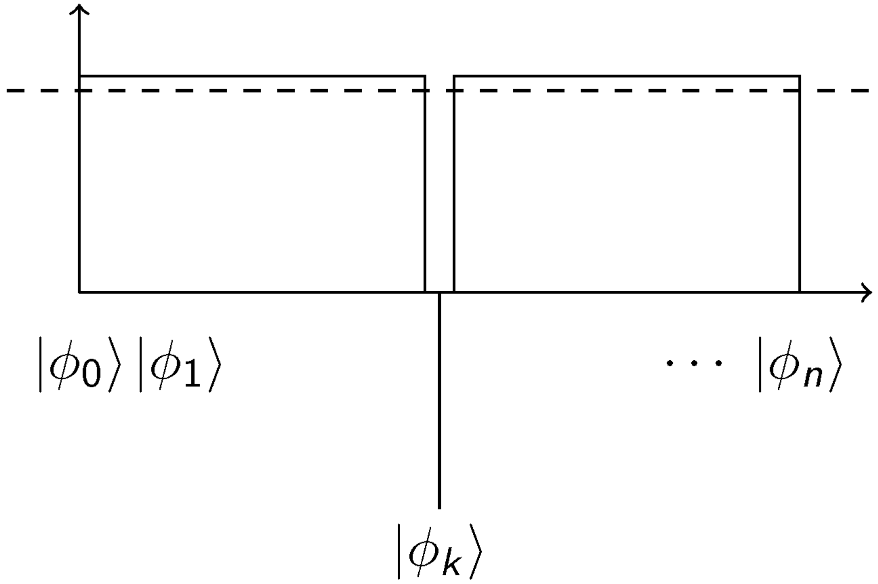
$\alpha_0 \left|r_0\right\rangle$  $+ \cdots - \alpha_k \left|r_k\right\rangle$  $+ \cdots + \alpha_n \left|r_n\right\rangle$ 

4. Increase the probability of the flipped term with *Mirror* circuit.

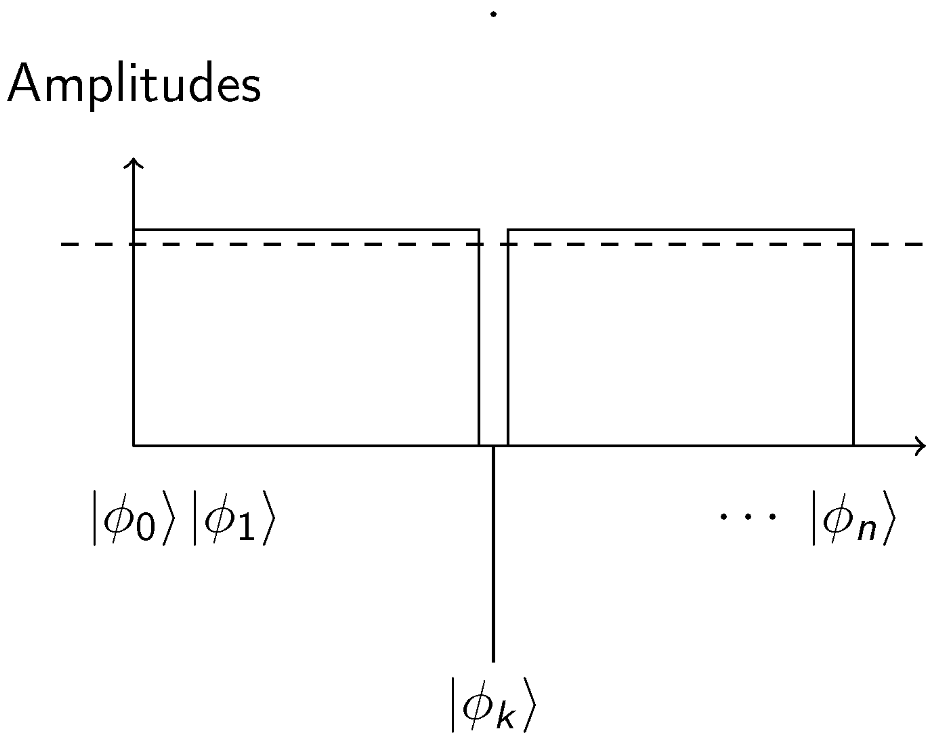
$\alpha'_0 \left|r_0\right\rangle$  $+ \cdots + \alpha'_k \left|r_k\right\rangle$  $+ \cdots + \alpha'_n \left|r_n\right\rangle$ 

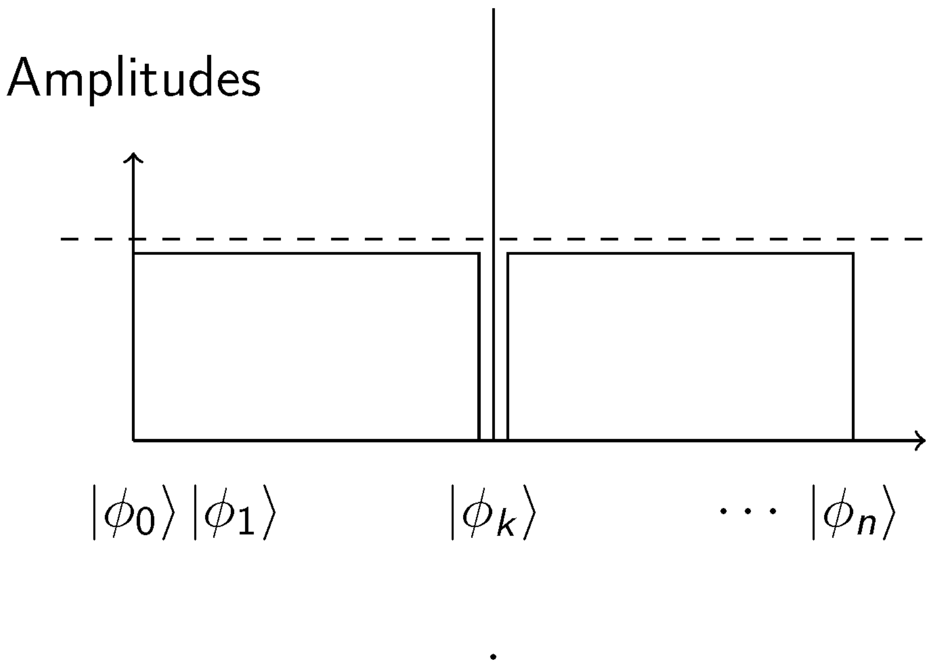
Phase amplification or Grover's iteration or Mirror

Amplitudes

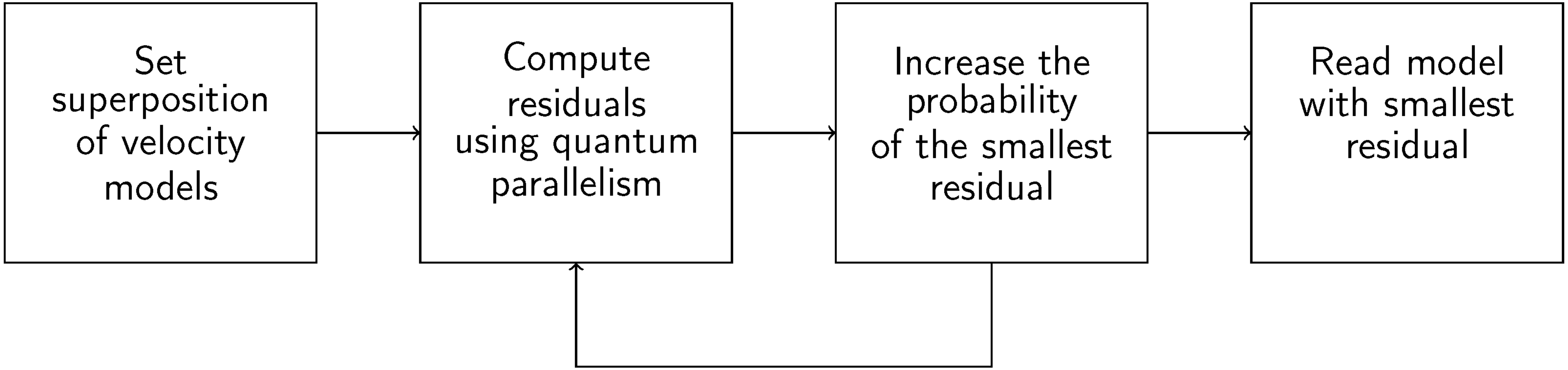


Phase amplification or Grover's iteration or Mirror





General quantum inversion algorithm

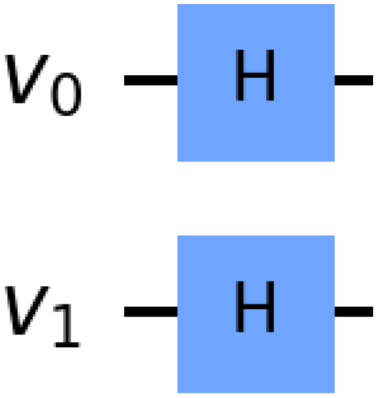


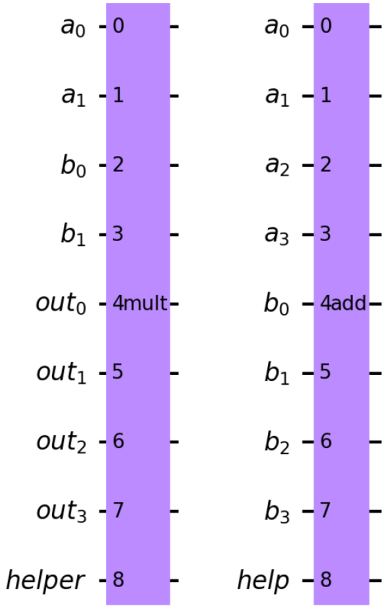
Set superposition of velocity models

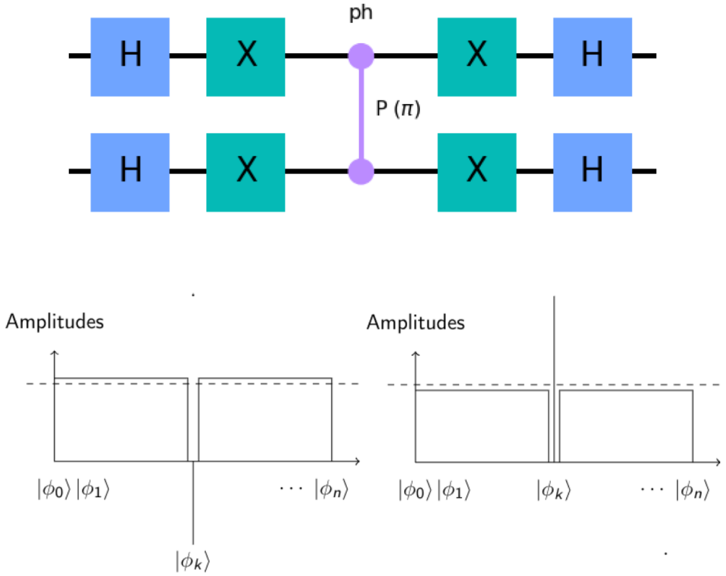
Compute residuals using quantum parallelism

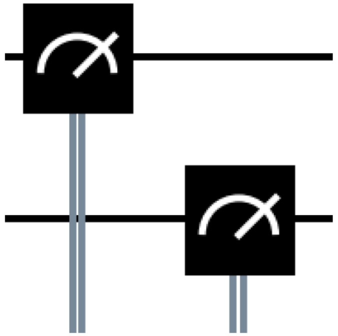
Increase the probability of the smallest residual

Read model with smallest residual





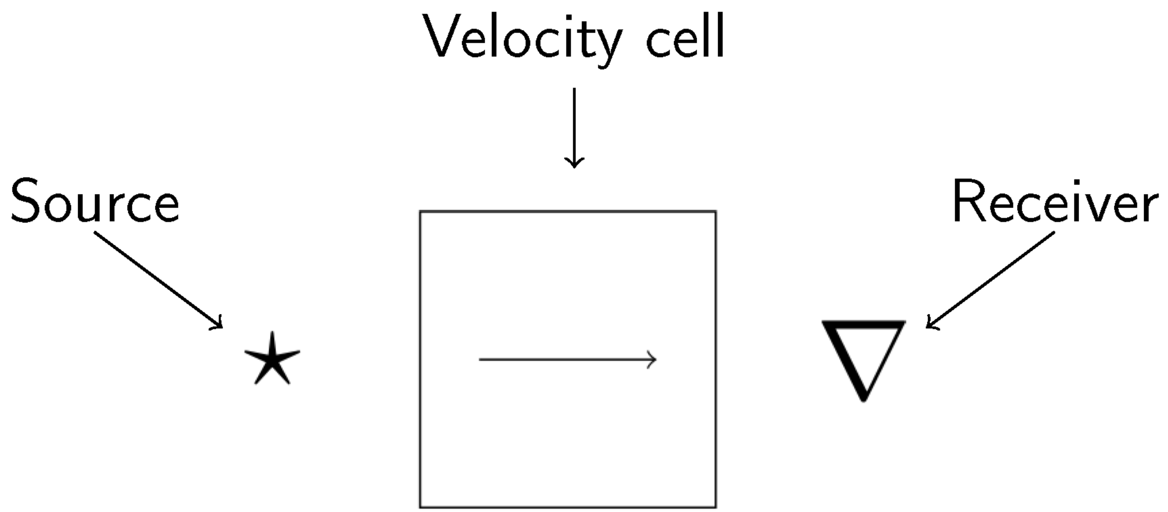




Proof of concept

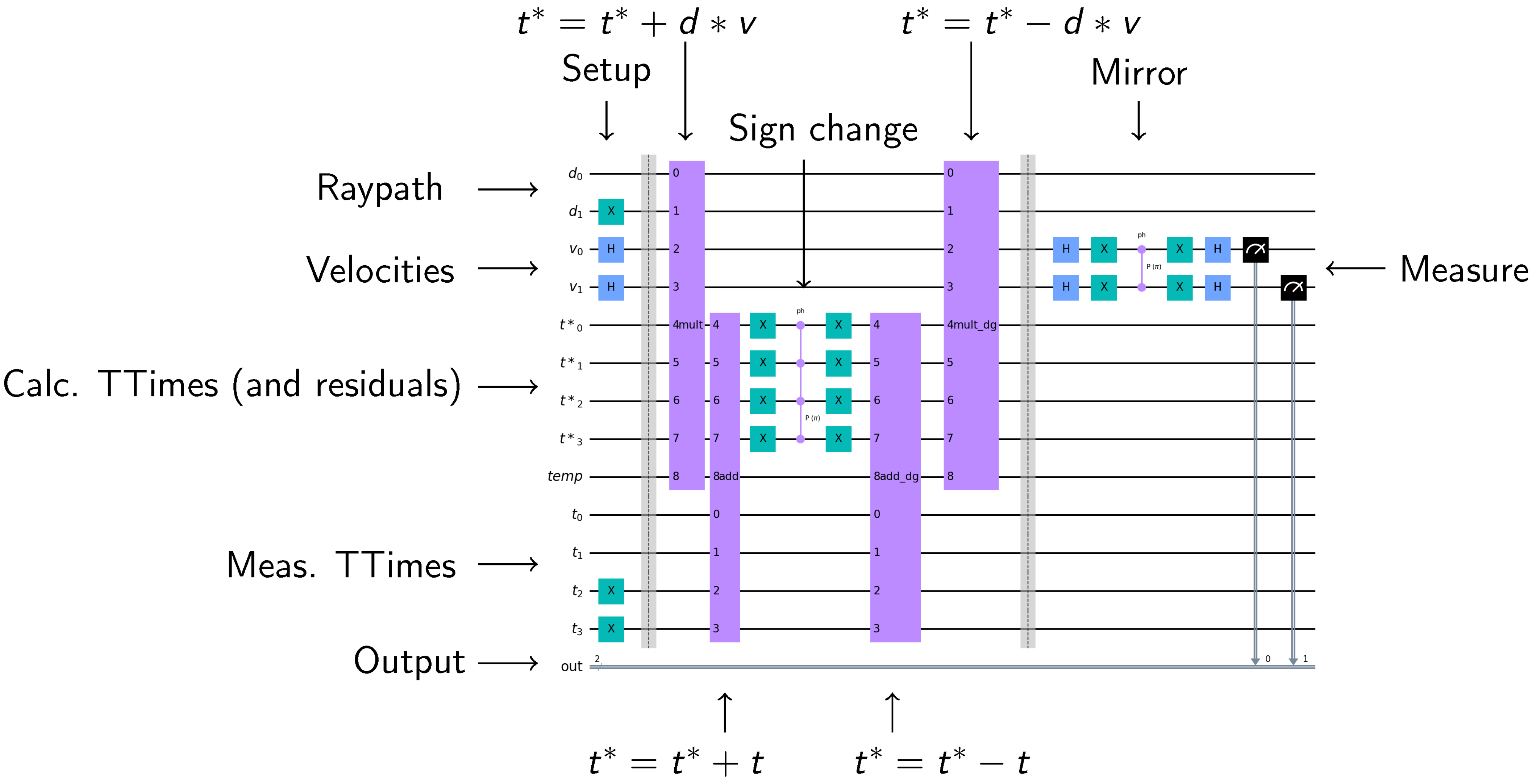
An algorithm must be seen to be believed
-Donald Knuth

Proof of concept problem, quantum circuit and simulation



- Raypath length is 2 distance units.
- Traveltime is 4 time units.
- Modelling is $t = dv$.
- Answer should be 2 slowness units.

Proof of concept problem, quantum circuit and simulation



$t^* = t^* + d * v$

Setup

Sign change

$t^* = t^* - d * v$

Mirror

Raypath

Velocities

Calc. TTimes (and residuals)

Meas. TTimes

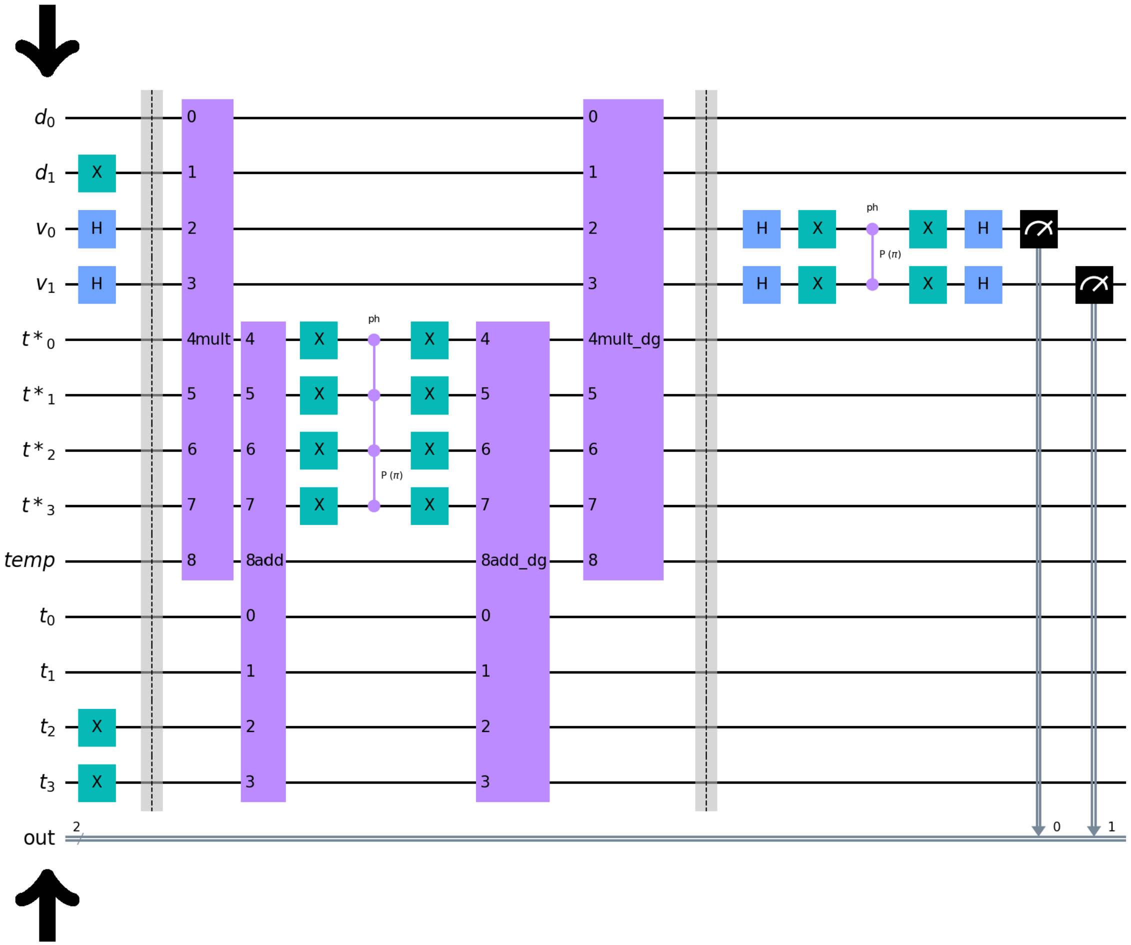
Output

Measure

$t^* = t^* + t$

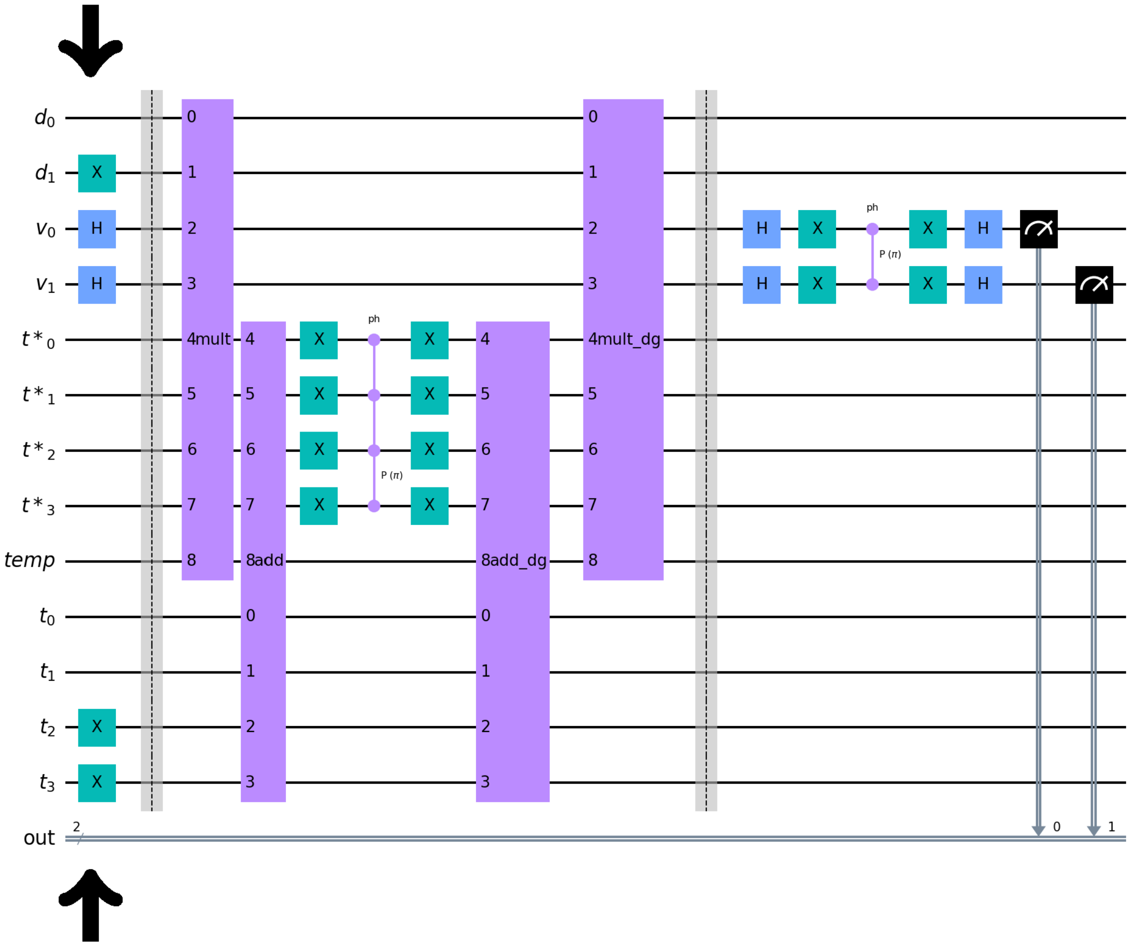
$t^* = t^* - t$

Proof of concept problem, quantum circuit and simulation



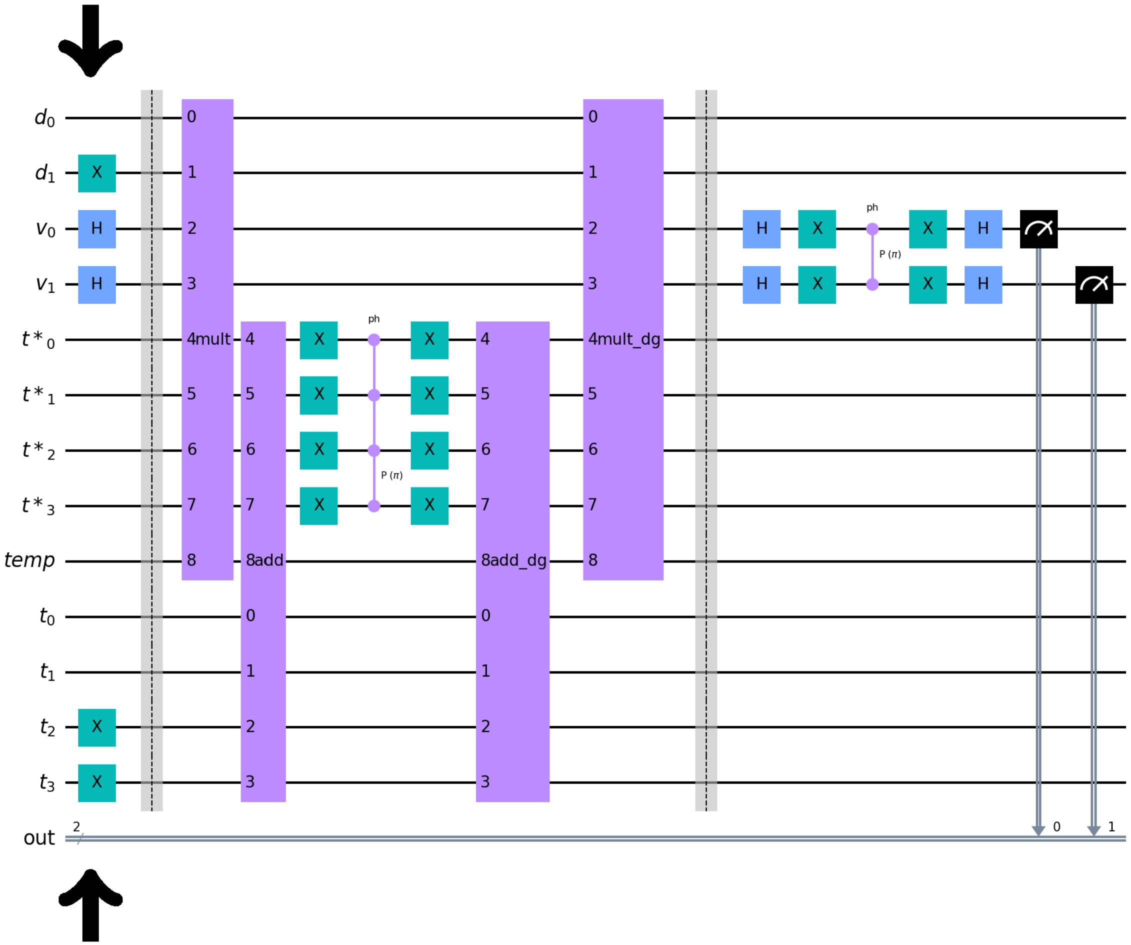
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup				
Mult				
Add				
Flip sign				
Add†				
Mult†				

Proof of concept problem, quantum circuit and simulation



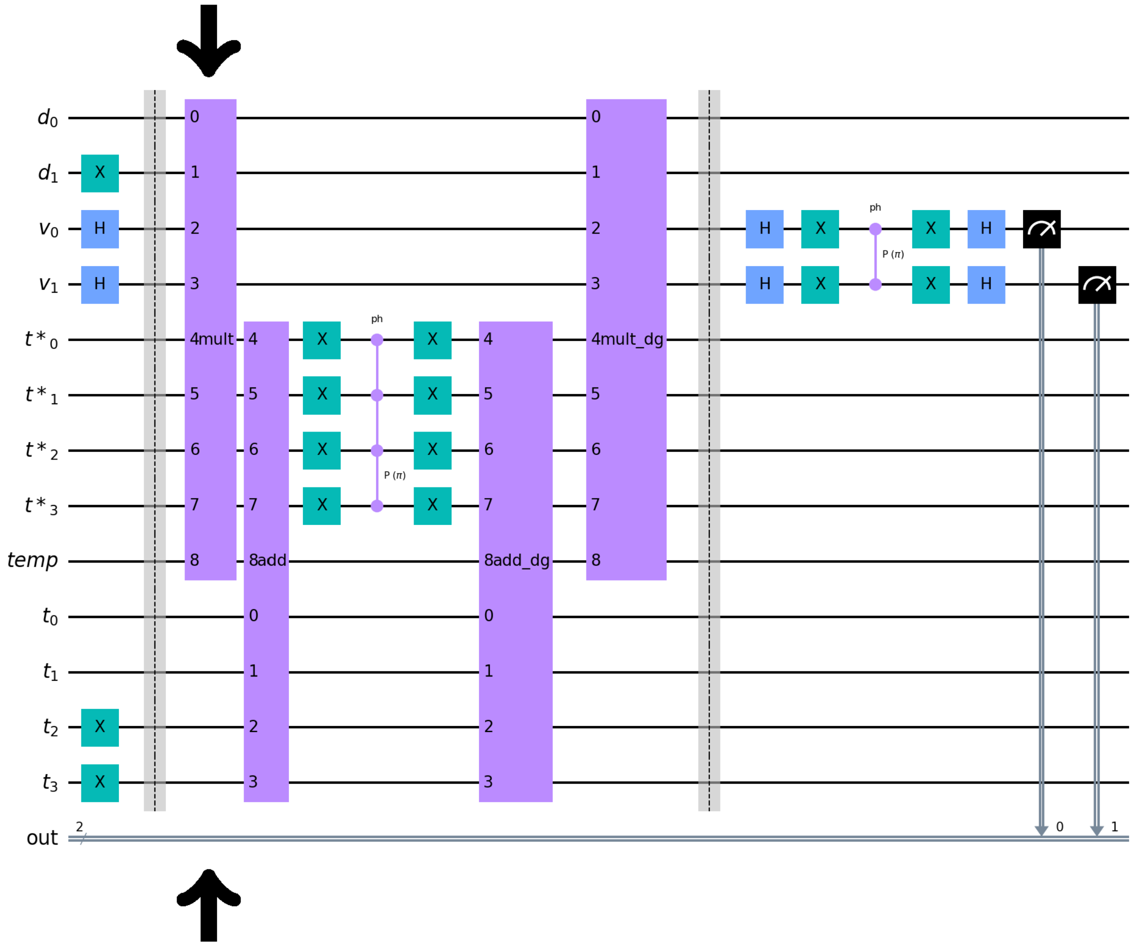
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
Mult				
Add				
Flip sign				
Add†				
Mult†				

Proof of concept problem, quantum circuit and simulation



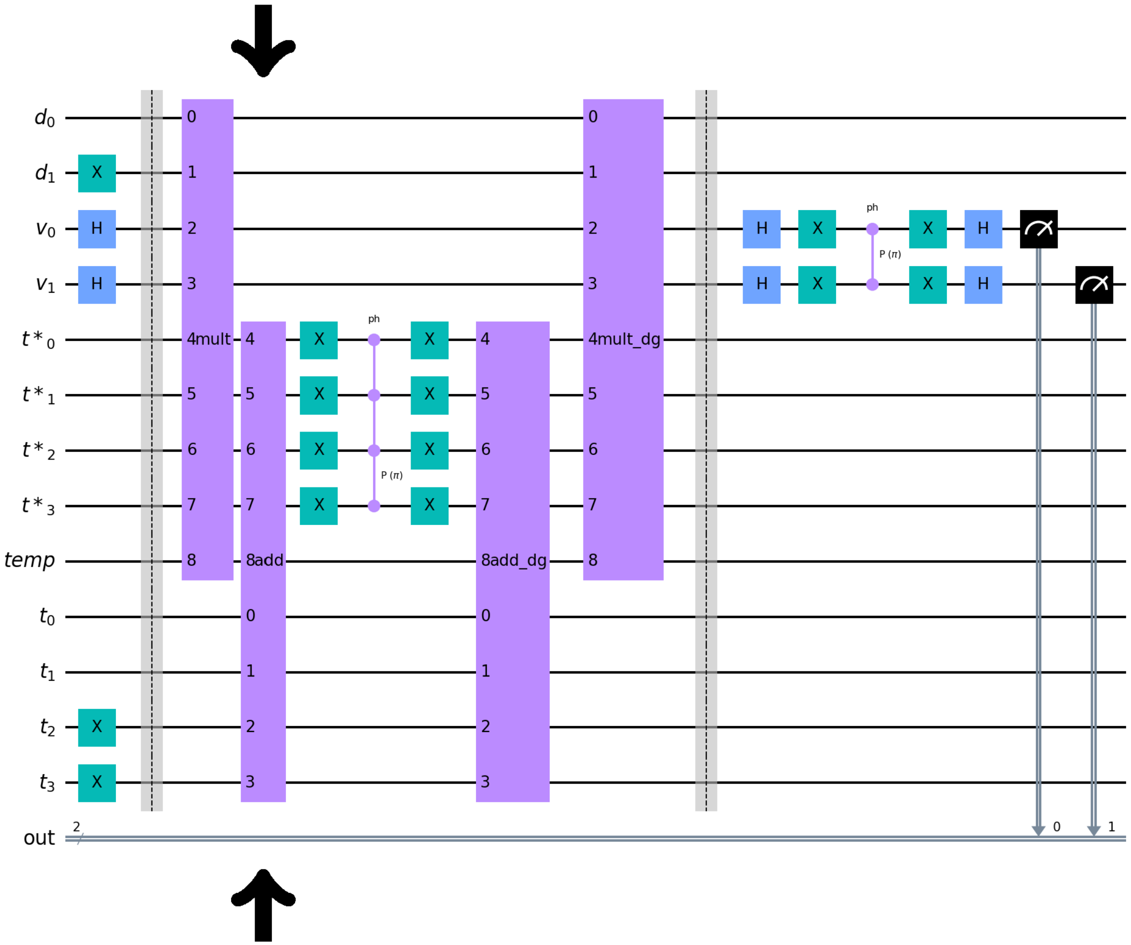
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
Mult				
Add				
Flip sign				
Add†				
Mult†				

Proof of concept problem, quantum circuit and simulation



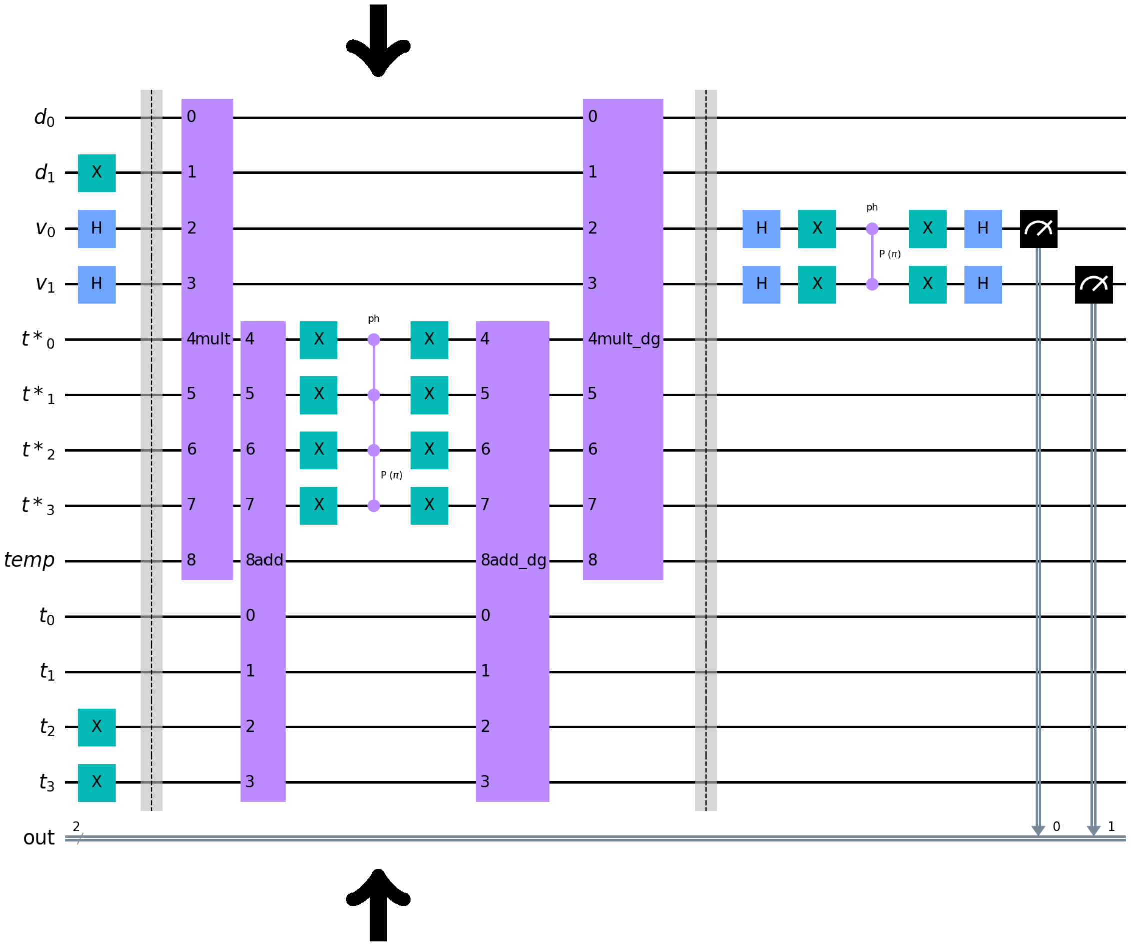
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
Mult	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle + \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
Add				
Flip sign				
Add†				
Mult†				

Proof of concept problem, quantum circuit and simulation



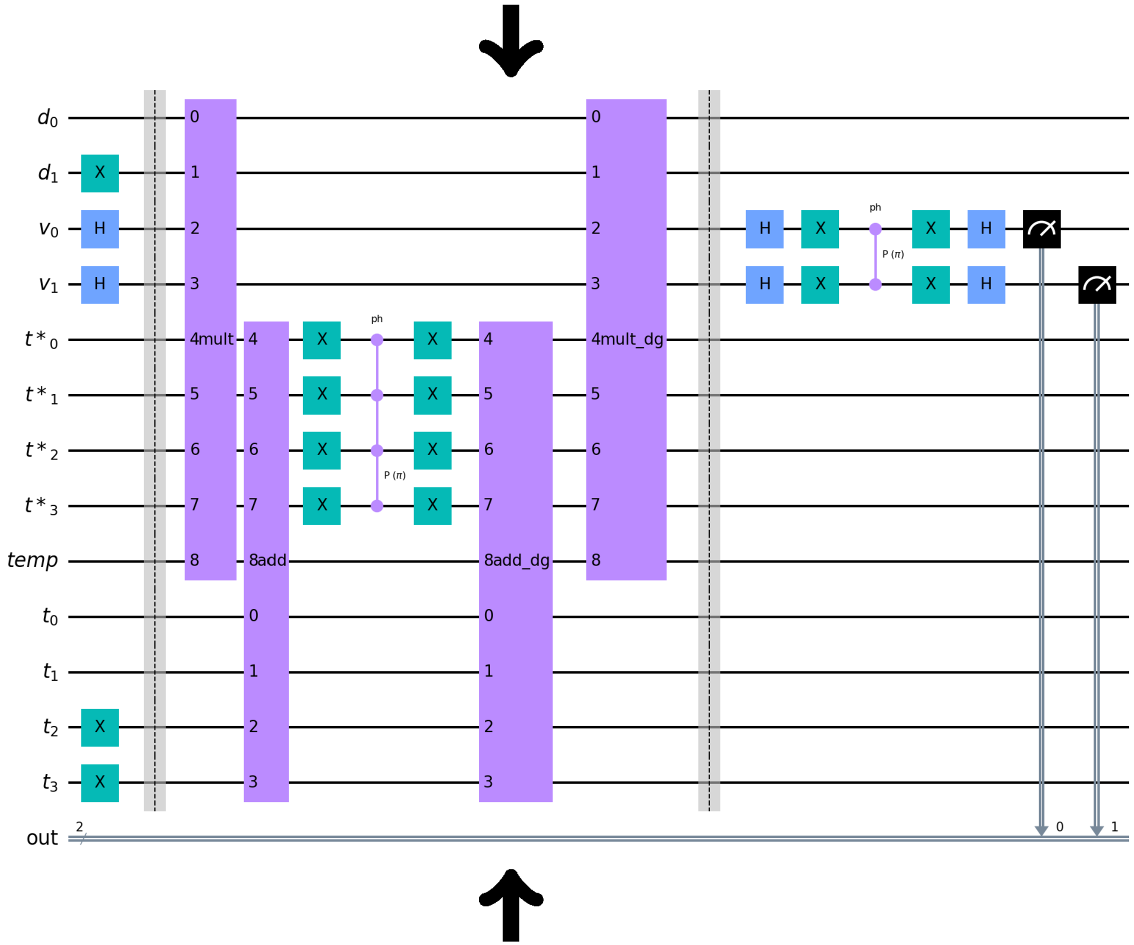
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle+\frac{1}{2}\|1\rangle+\frac{1}{2}\|2\rangle+\frac{1}{2}\|3\rangle$	$\|2\rangle$
	$\|-4\rangle$	$\frac{1}{2}\|00\rangle+\frac{1}{2}\|01\rangle+\frac{1}{2}\|02\rangle+\frac{1}{2}\|03\rangle$		$\|2\rangle$
Mult	$\|-4\rangle$	$\frac{1}{2}\|00\rangle+\frac{1}{2}\|21\rangle+\frac{1}{2}\|42\rangle+\frac{1}{2}\|63\rangle$		$\|2\rangle$
Add	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle+\frac{1}{2}\|-21\rangle+\frac{1}{2}\|02\rangle+\frac{1}{2}\|23\rangle$		$\|2\rangle$
Flip sign				
Add†				
Mult†				

Proof of concept problem, quantum circuit and simulation



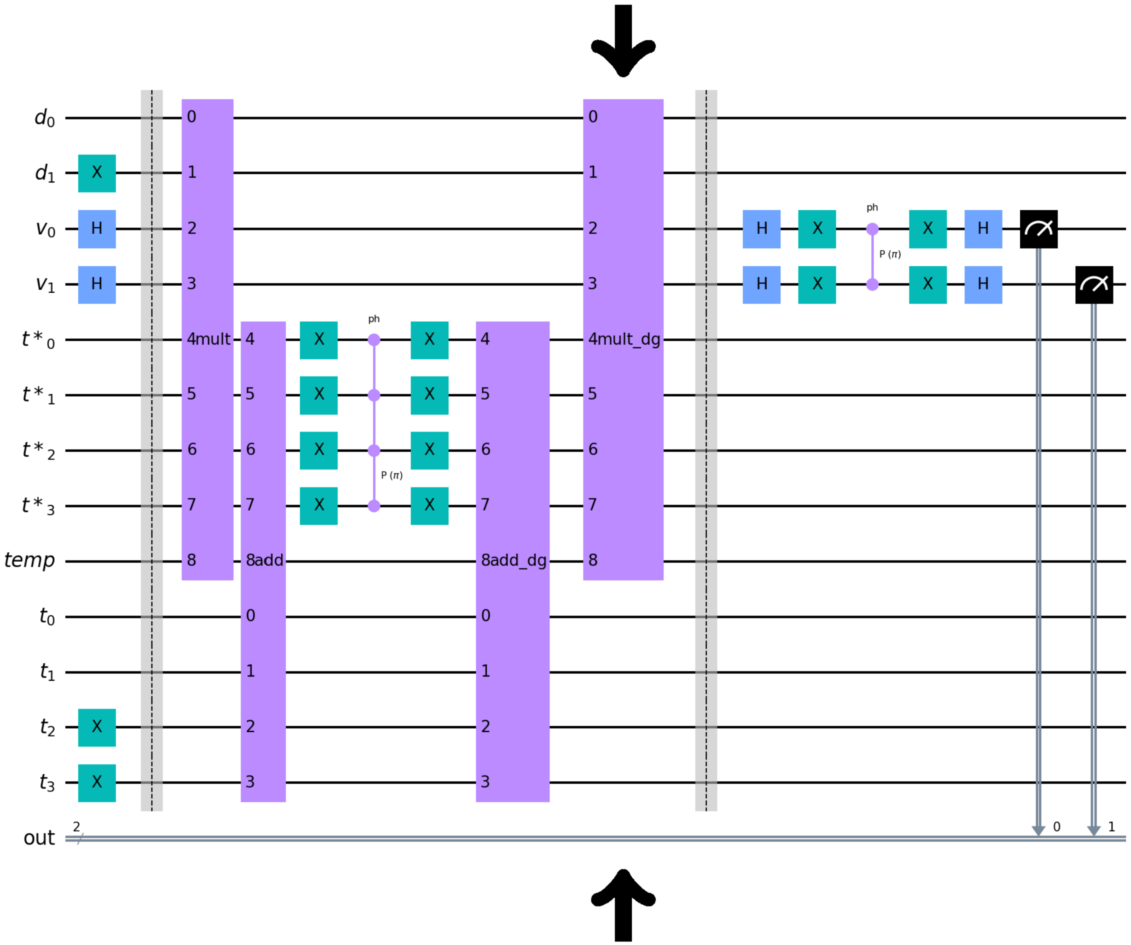
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
Mult	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle + \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
Add	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
Flip sign	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle - \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
Add†				
Mult†				

Proof of concept problem, quantum circuit and simulation



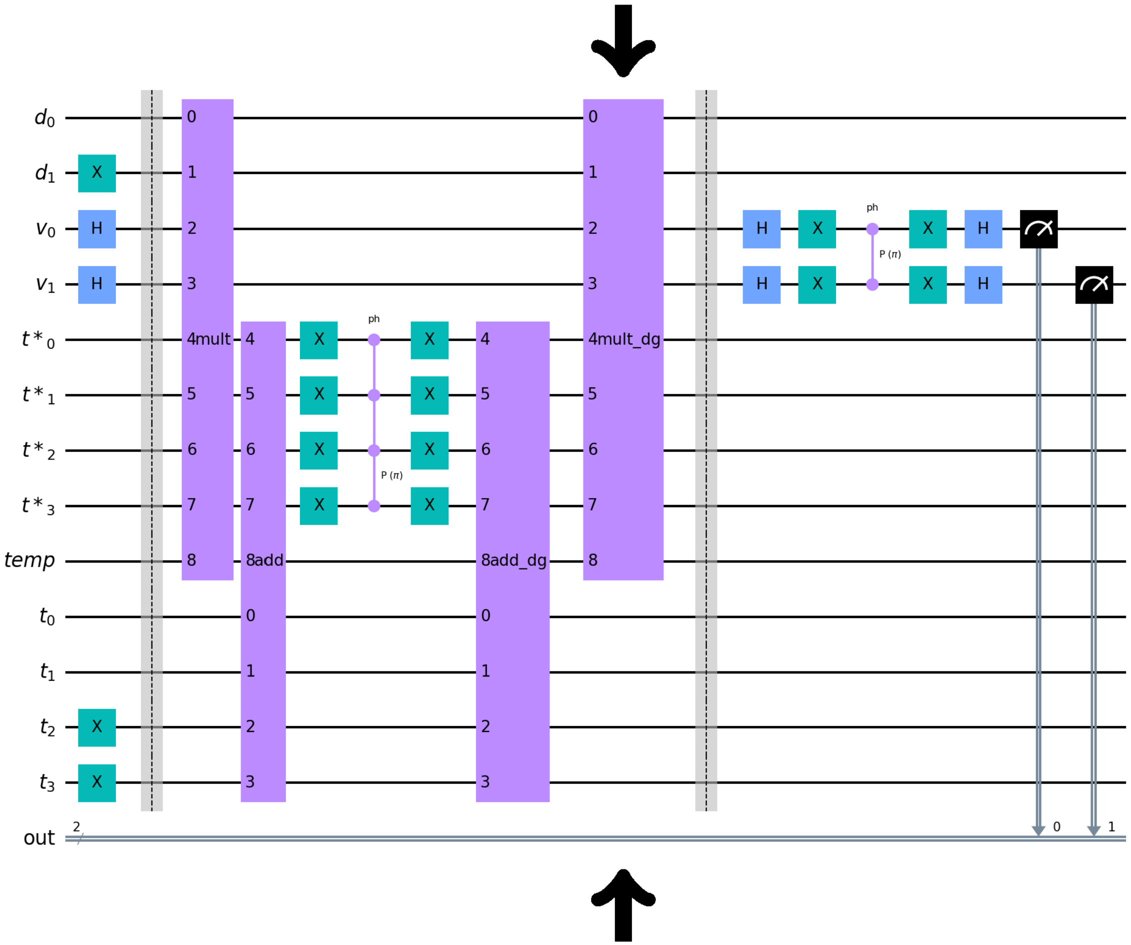
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
Mult	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle + \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
Add	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
Flip sign	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle - \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
Add†	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle - \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
Mult†				

Proof of concept problem, quantum circuit and simulation



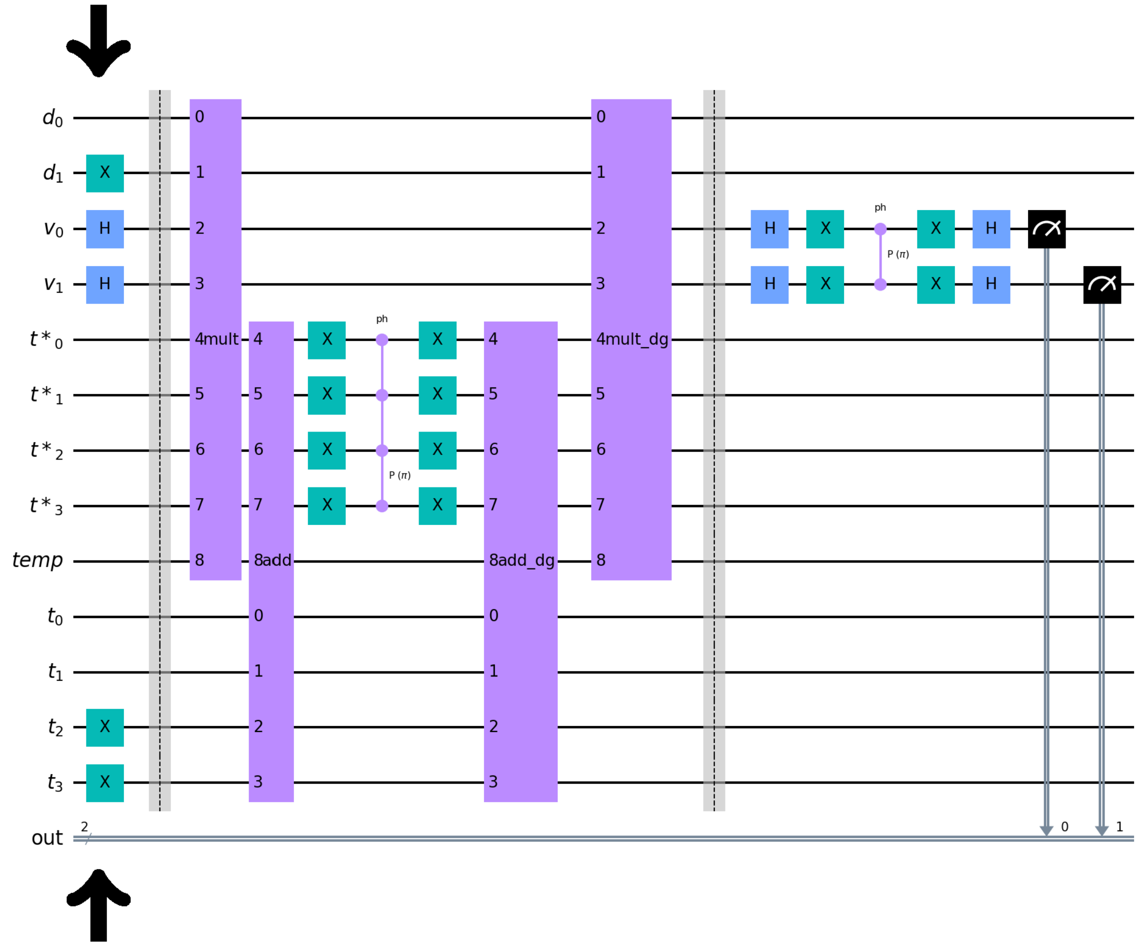
Step	t	t^*	v	d
Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
Mult	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle + \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
Add	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
Flip sign	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle - \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
Add†	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle - \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
Mult†	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle - \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$

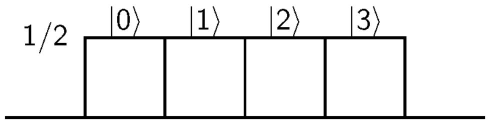
Proof of concept problem, quantum circuit and simulation



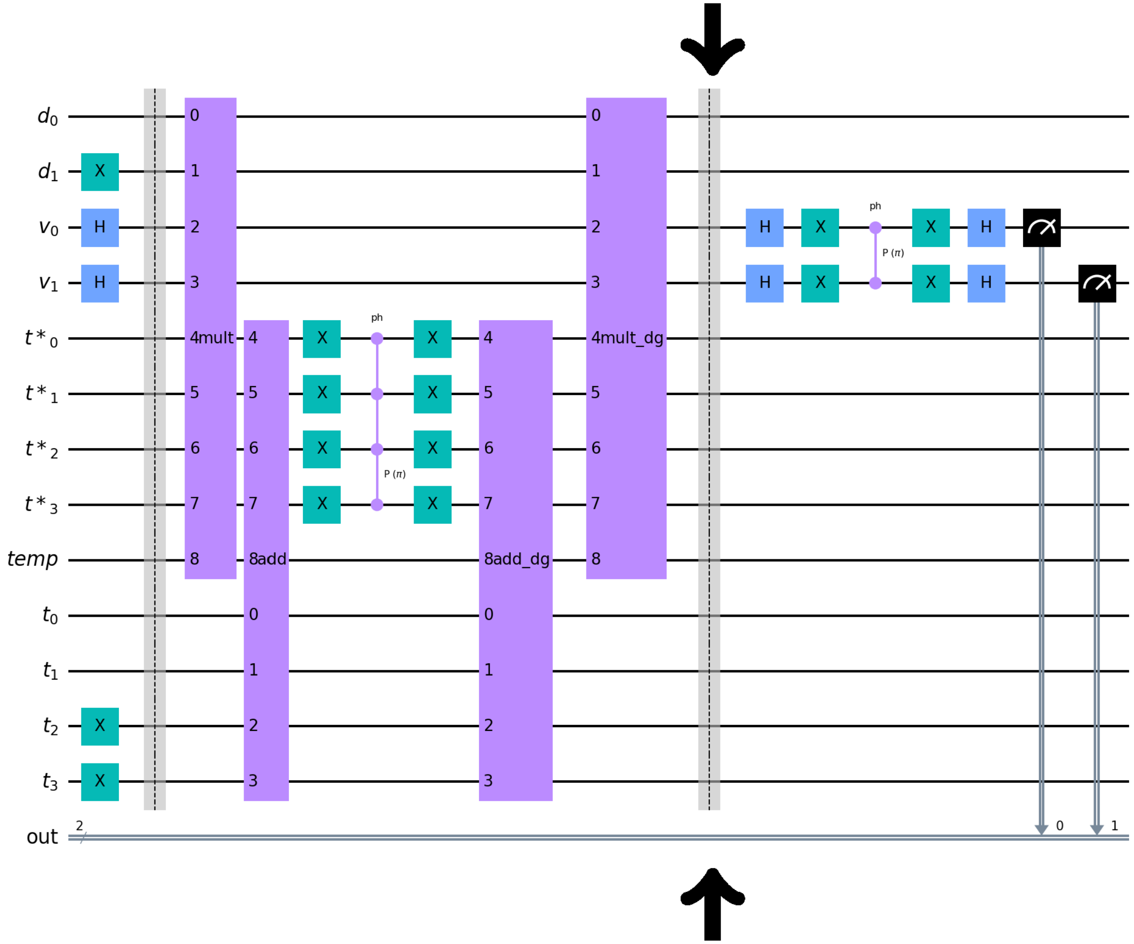
Step	t	t^*	v	d
Initial	$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$
Setup	$\lvert -4\rangle$	$\lvert 0\rangle$	$\frac{1}{2}\lvert 0\rangle + \frac{1}{2}\lvert 1\rangle + \frac{1}{2}\lvert 2\rangle + \frac{1}{2}\lvert 3\rangle$	$\lvert 2\rangle$
	$\lvert -4\rangle$	$\frac{1}{2}\lvert 00\rangle + \frac{1}{2}\lvert 01\rangle + \frac{1}{2}\lvert 02\rangle + \frac{1}{2}\lvert 03\rangle$		$\lvert 2\rangle$
Mult	$\lvert -4\rangle$	$\frac{1}{2}\lvert 00\rangle + \frac{1}{2}\lvert 21\rangle + \frac{1}{2}\lvert 42\rangle + \frac{1}{2}\lvert 63\rangle$		$\lvert 2\rangle$
Add	$\lvert -4\rangle$	$\frac{1}{2}\lvert -40\rangle + \frac{1}{2}\lvert -21\rangle + \frac{1}{2}\lvert 02\rangle + \frac{1}{2}\lvert 23\rangle$		$\lvert 2\rangle$
Flip sign	$\lvert -4\rangle$	$\frac{1}{2}\lvert -40\rangle + \frac{1}{2}\lvert -21\rangle - \frac{1}{2}\lvert 02\rangle + \frac{1}{2}\lvert 23\rangle$		$\lvert 2\rangle$
Add†	$\lvert -4\rangle$	$\frac{1}{2}\lvert 00\rangle + \frac{1}{2}\lvert 21\rangle - \frac{1}{2}\lvert 42\rangle + \frac{1}{2}\lvert 63\rangle$		$\lvert 2\rangle$
Mult†	$\lvert -4\rangle$	$\frac{1}{2}\lvert 00\rangle + \frac{1}{2}\lvert 01\rangle - \frac{1}{2}\lvert 02\rangle + \frac{1}{2}\lvert 03\rangle$		$\lvert 2\rangle$
	$\lvert -4\rangle$	$\lvert 0\rangle$	$\frac{1}{2}\lvert 0\rangle + \frac{1}{2}\lvert 1\rangle - \frac{1}{2}\lvert 2\rangle + \frac{1}{2}\lvert 3\rangle$	$\lvert 2\rangle$

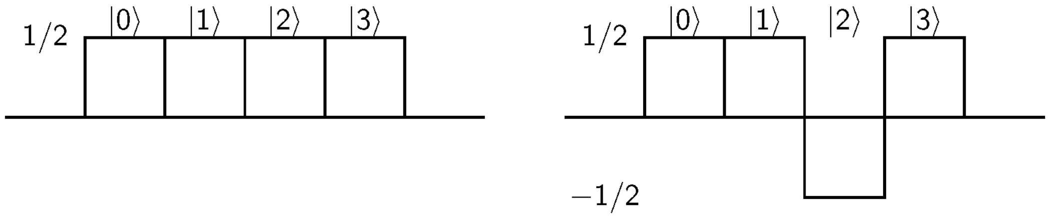
Proof of concept problem, quantum circuit and simulation



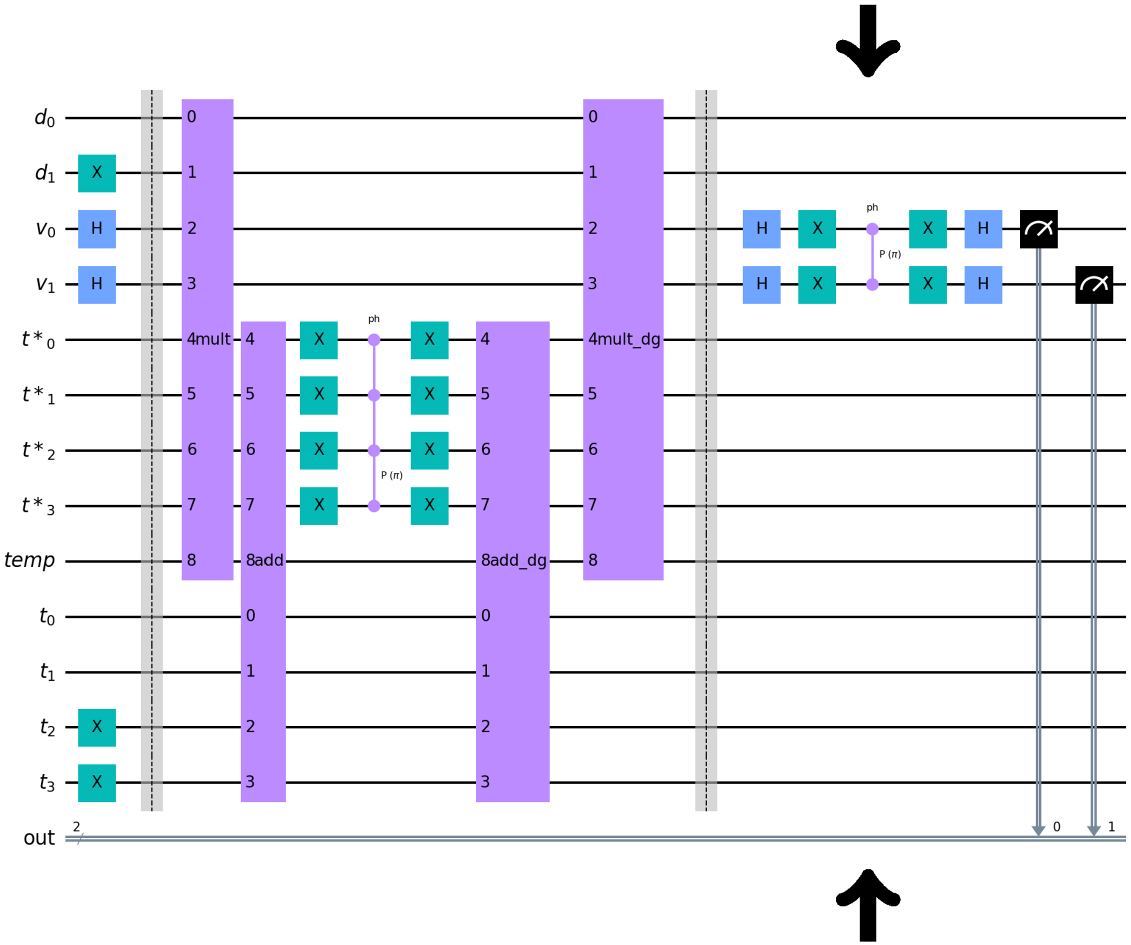


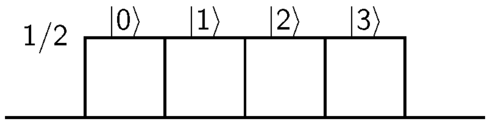
Proof of concept problem, quantum circuit and simulation

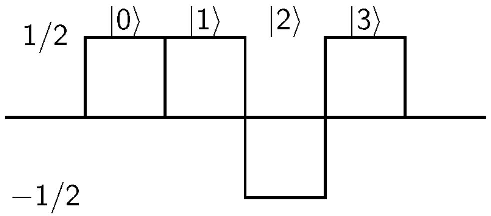


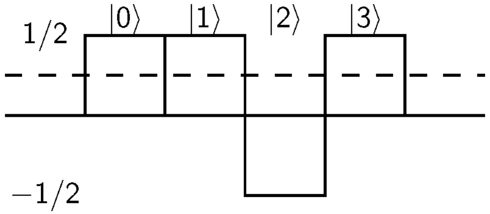


Proof of concept problem, quantum circuit and simulation

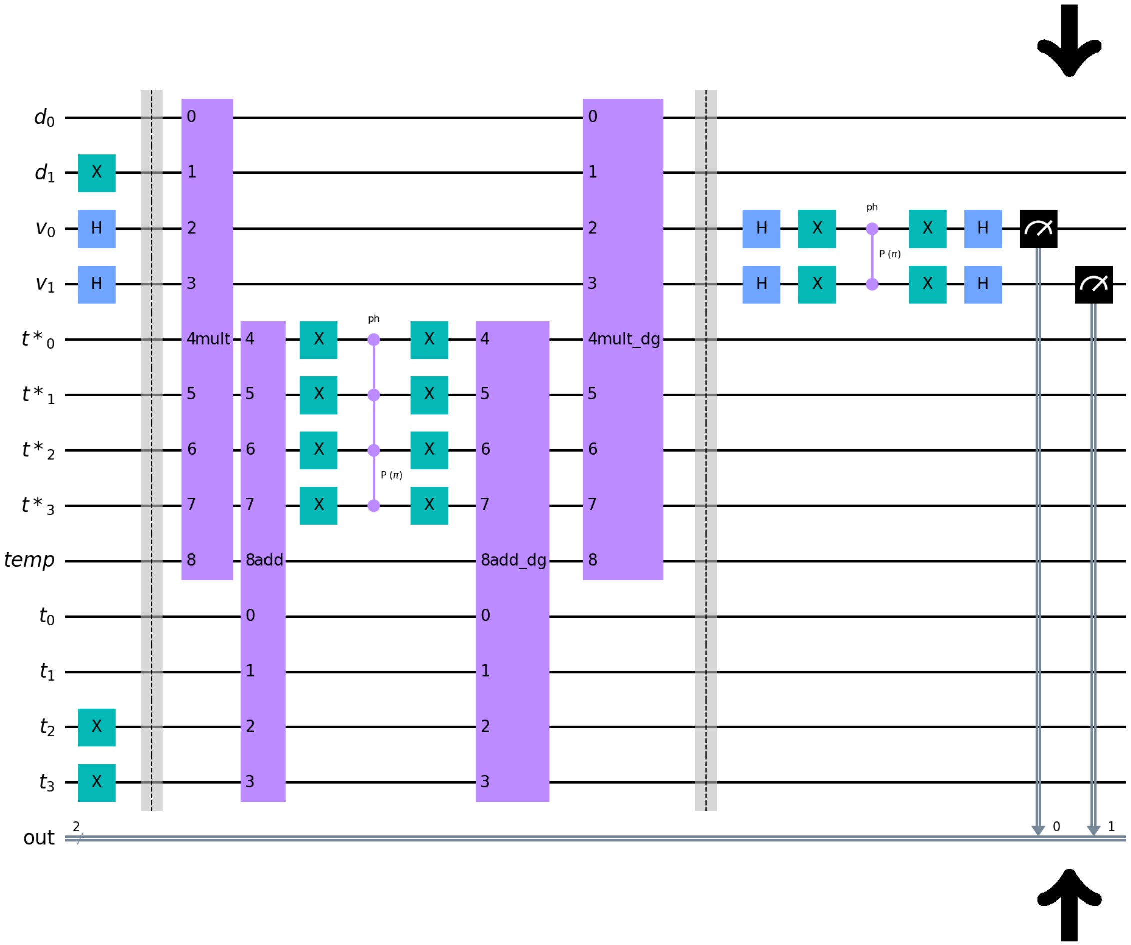


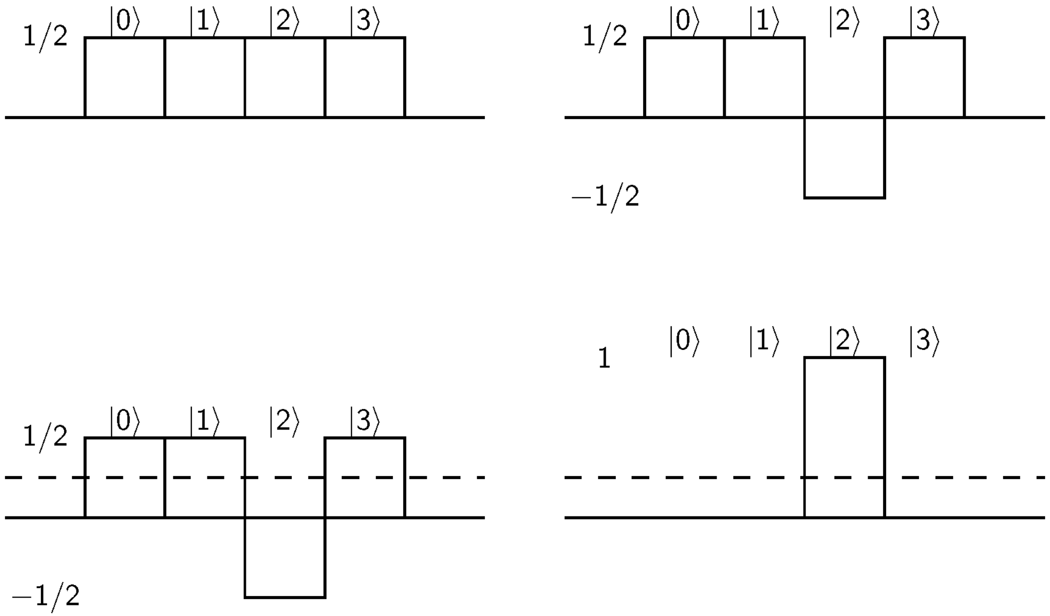




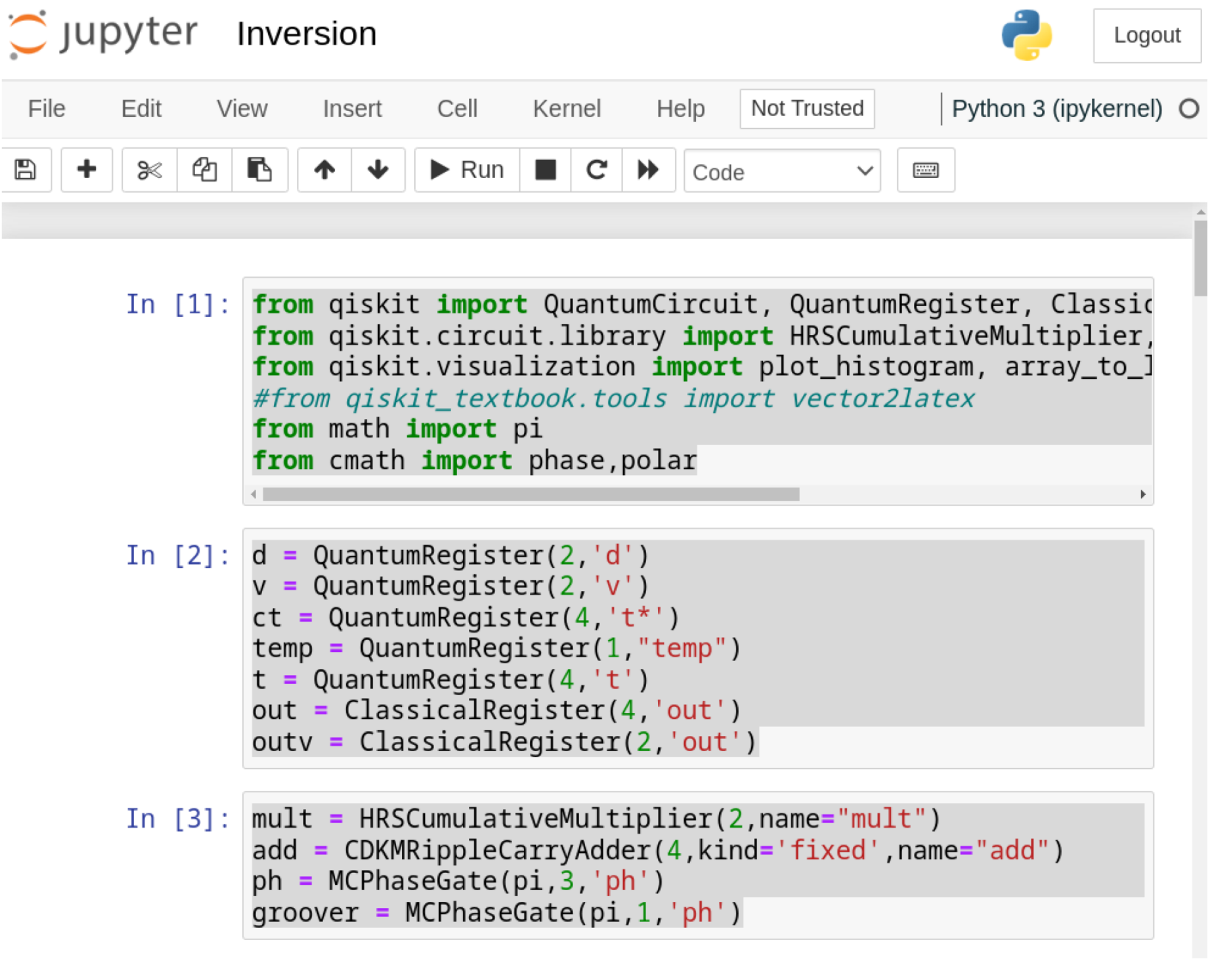


Proof of concept problem, quantum circuit and simulation





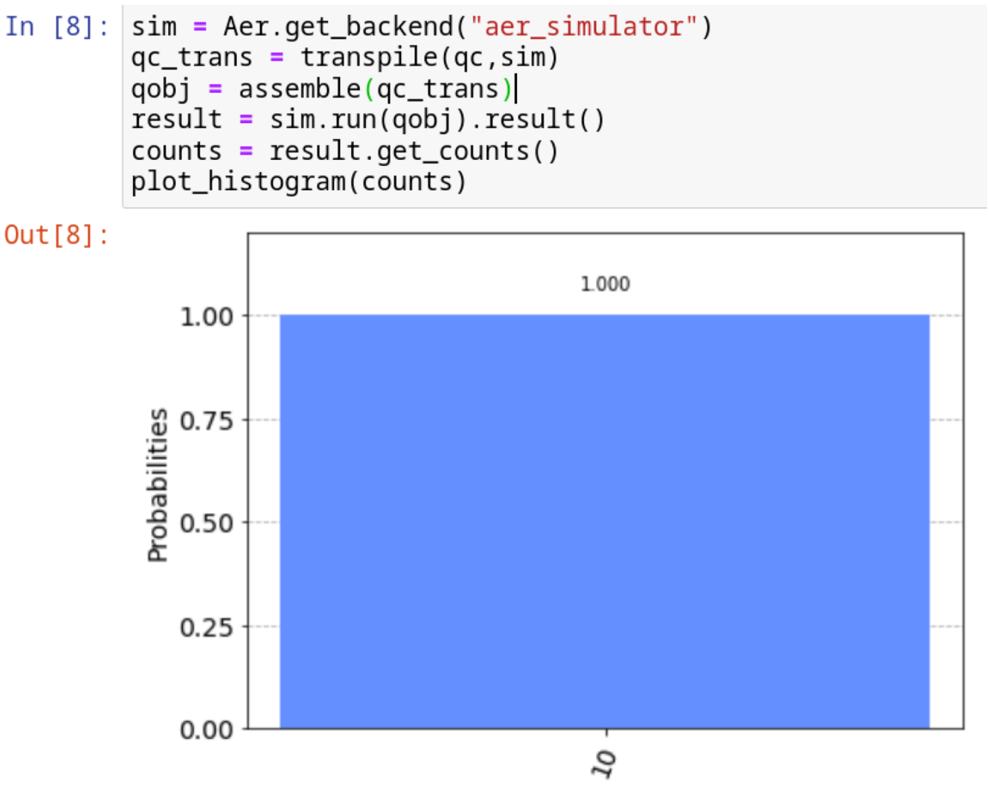
Proof of concept problem, quantum circuit and simulation



```
In [1]: from qiskit import QuantumCircuit, QuantumRegister, Classic
        from qiskit.circuit.library import HRSCumulativeMultiplier,
        from qiskit.visualization import plot_histogram, array_to_
        #from qiskit_textbook.tools import vector2latex
        from math import pi
        from cmath import phase,polar
```

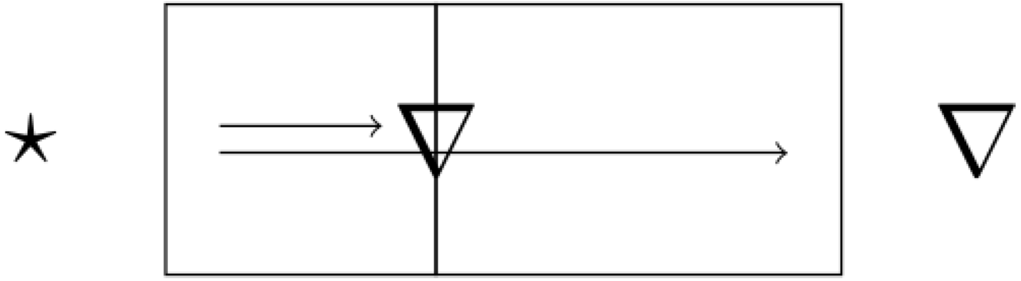
```
In [2]: d = QuantumRegister(2,'d')
        v = QuantumRegister(2,'v')
        ct = QuantumRegister(4,'t*')
        temp = QuantumRegister(1,"temp")
        t = QuantumRegister(4,'t')
        out = ClassicalRegister(4,'out')
        outv = ClassicalRegister(2,'out')
```

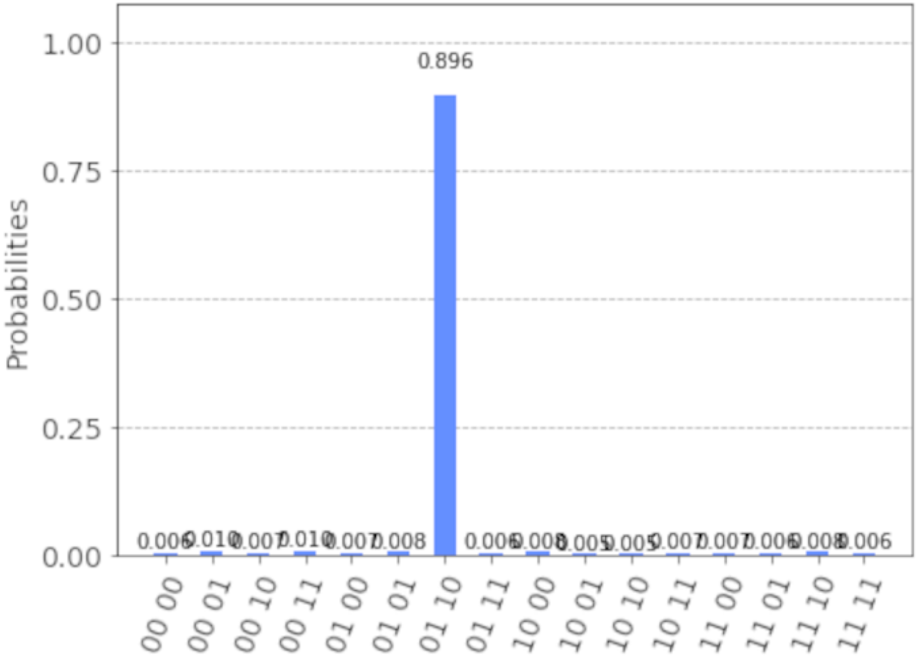
```
In [3]: mult = HRSCumulativeMultiplier(2,name="mult")
        add = CDKMRippleCarryAdder(4,kind='fixed',name="add")
        ph = MCPhaseGate(pi,3,'ph')
        groover = MCPhaseGate(pi,1,'ph')
```

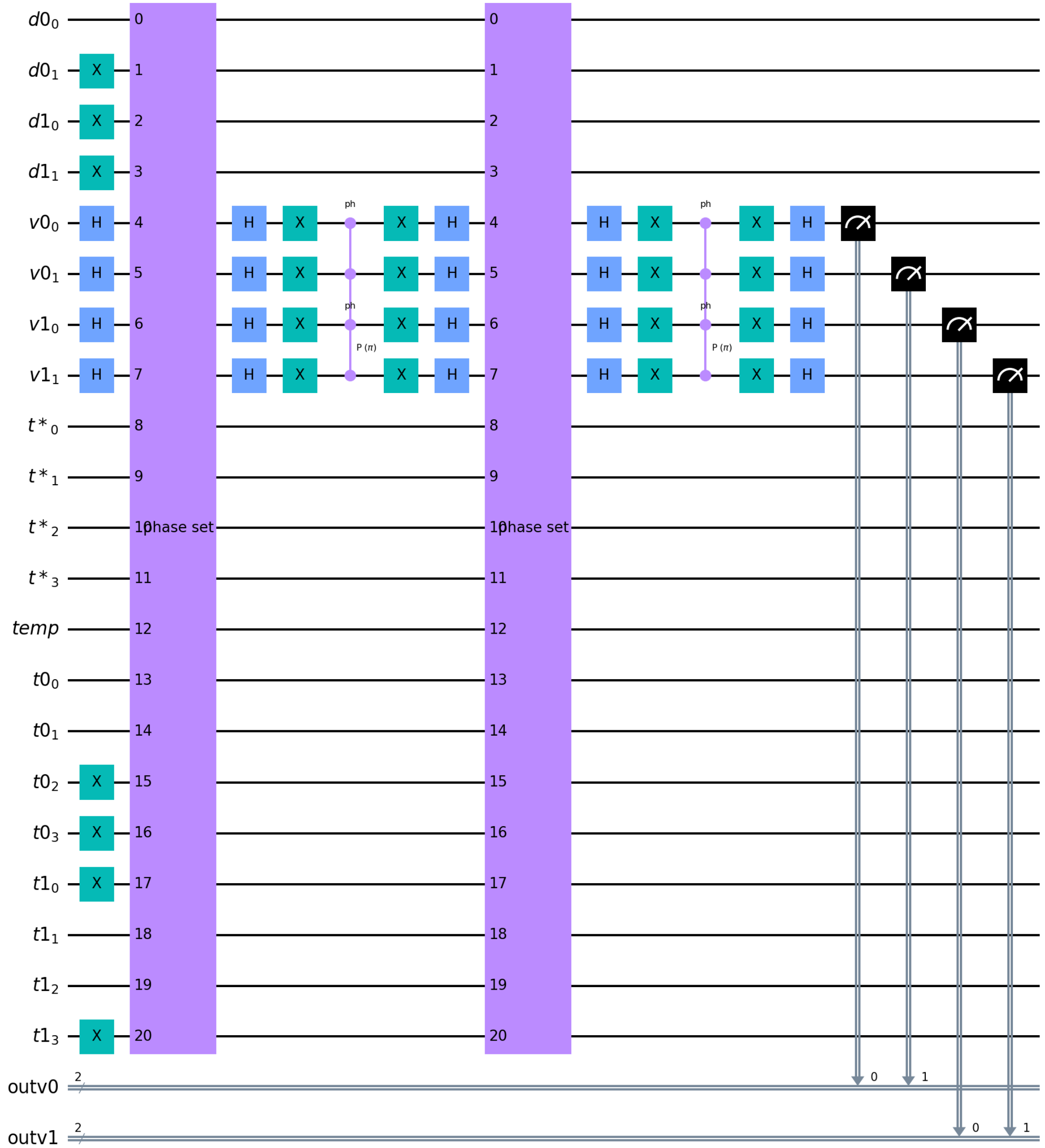


```
In [8]: sim = Aer.get_backend("aer_simulator")
        qc_trans = transpile(qc,sim)
        qobj = assemble(qc_trans)
        result = sim.run(qobj).result()
        counts = result.get_counts()
        plot_histogram(counts)
Out[8]:
```

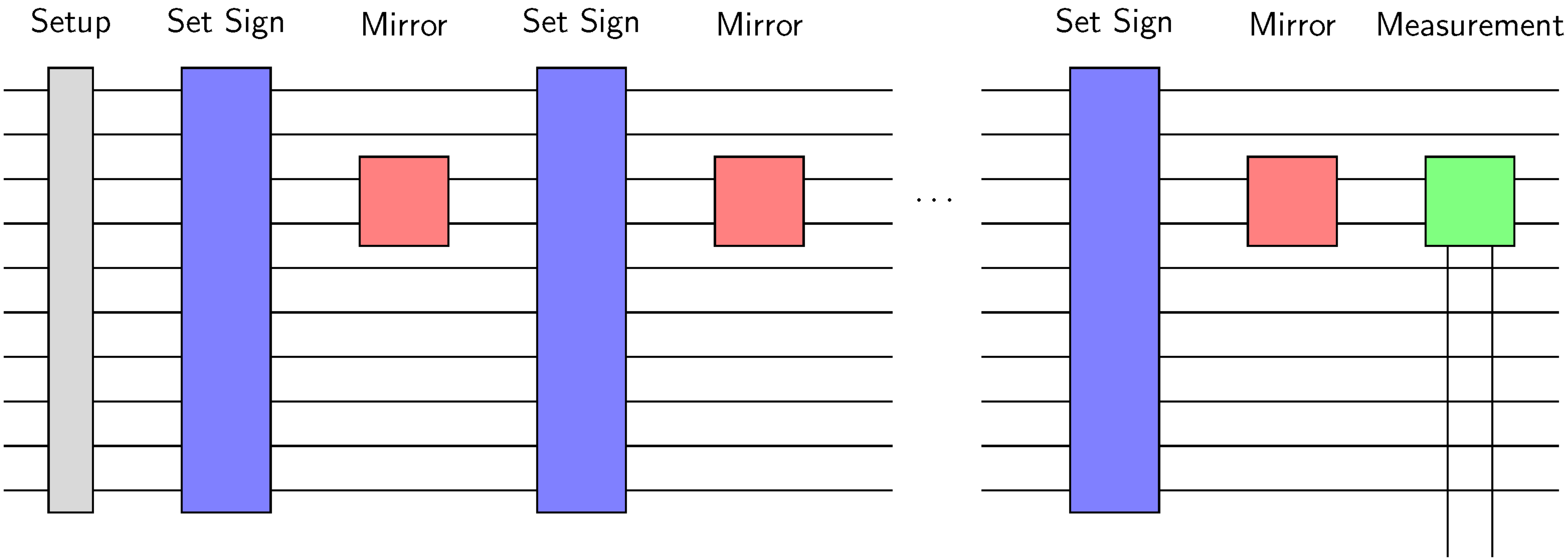
Proof of concept problem, quantum circuit and simulation



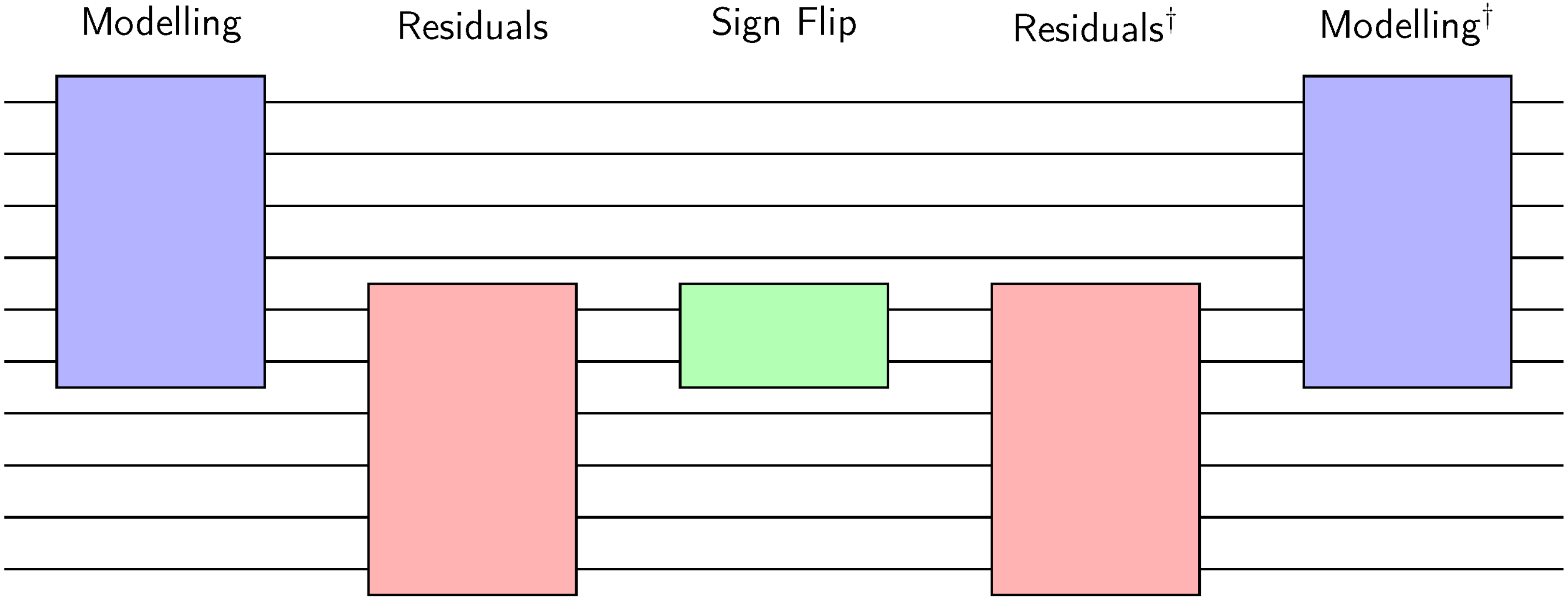




General Form

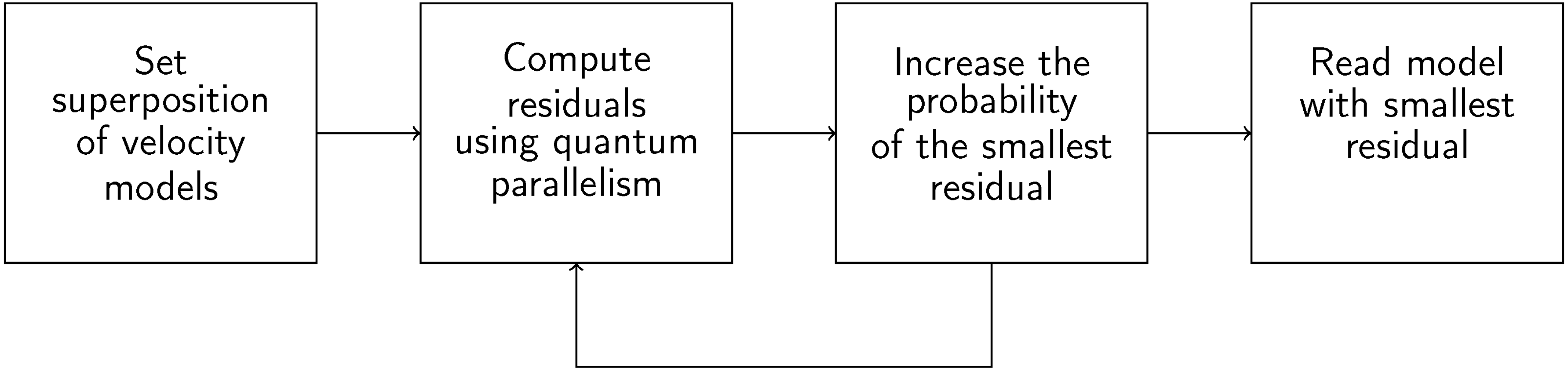


General Form: Set Sign



Discussion

Discussion



Set superposition of velocity models

Compute residuals using quantum parallelism

Increase the probability of the smallest residual

Read model with smallest residual

Time Complexity This algorithm follows the same structure of the *Grover's Quantum Search Algorithm*. The number of applications of mirror has been found to be $O(\sqrt{N})$ where N is the number of models.

Discussion

Computing in superposition The main attractive is using quantum superposition. It can calculate the residuals on a set of velocity models in parallel. It is not possible to get all of them but just one, so other stages are needed.

Only forward modelling and residual calculation There is no need to compute gradients or Hessians because the whole velocity space is searched in parallel.

Encodings Classical computers use floating point formats to encode the optimization values. The number of available qubits constrains the range and precision of the numbers in the quantum calculations. There are more ways to encode numbers using qubits that can be explored.

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