Simultaneous prediction of velocity and angle-dependent reflectivity in time domain FWI

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2023-12-8

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Outline

1. Common image gathers (CIGs)
2. Angle-dependent CIGs and reflectivity
3. CIGs applied in FWI

CIGs

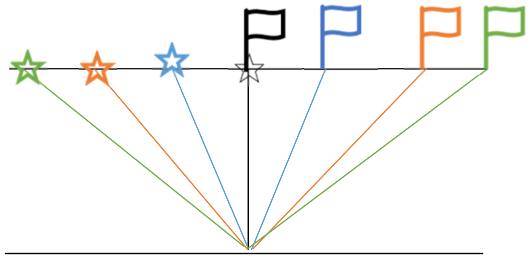
Why CIGs?

CIGs: a series of prestack migrated trace for subsurface attribute interpretation

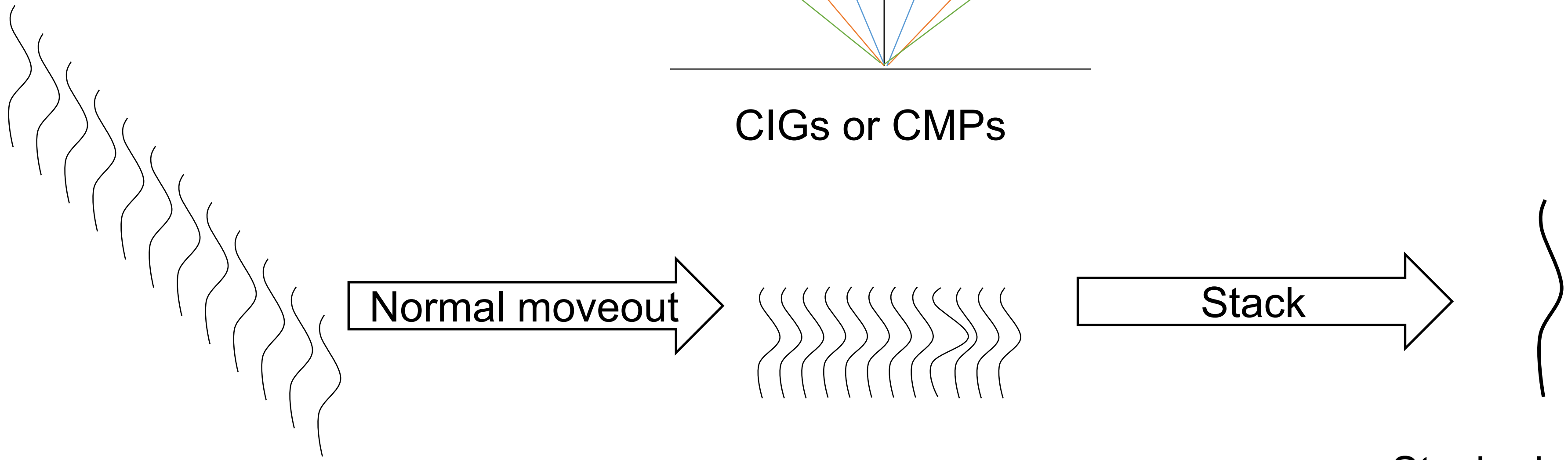
Common midpoint gathers/CMPs are to stack data, CIGs are to migration.

When we stack CIGs, we lose some amplitude information.

If we want reflectivity as a function of offset/angle, we need to look into CIGs before stacking them.



CIGs or CMPs



Stacked seismic trace

Seismic trace sorted by offset/angle

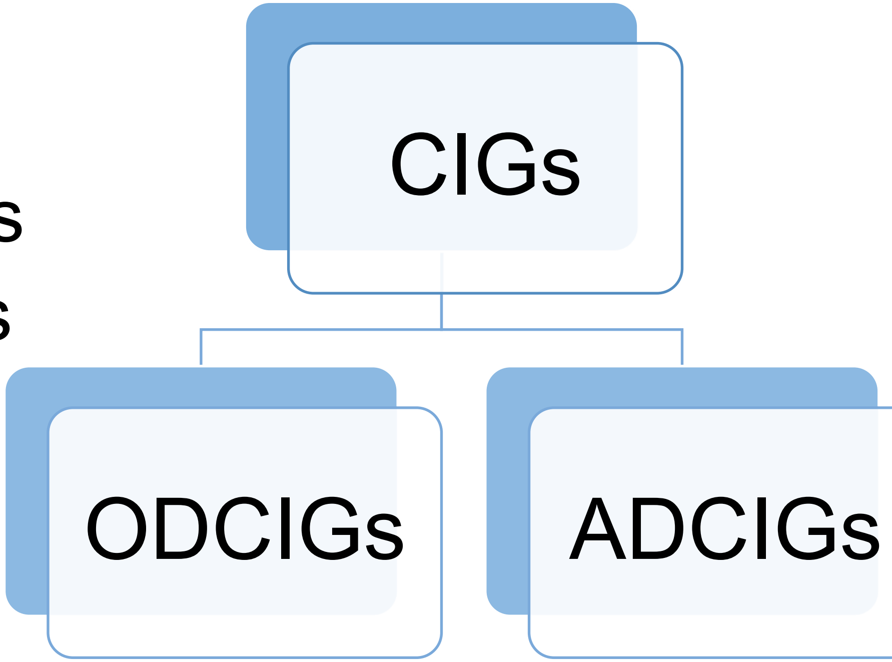
Why CIGs?

CIGs are primary data for:

amplitude variation with offset (AVO) analysis

amplitude variation with angle (AVA) analysis

migration velocity analysis



Types of CIGs:

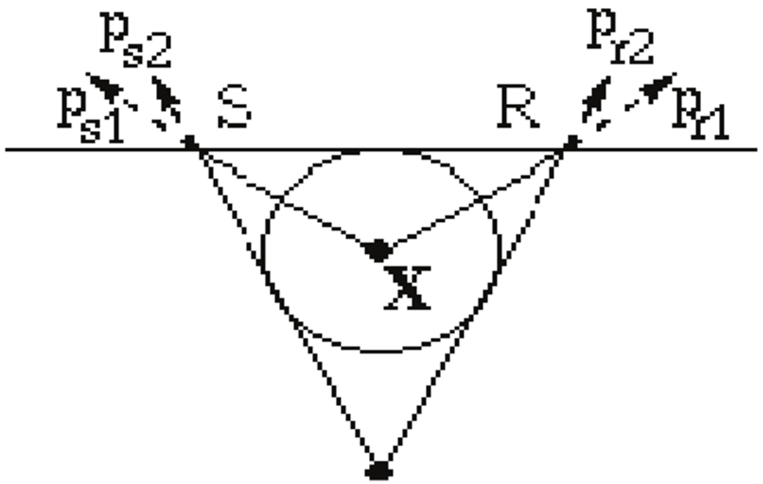
offset domain common image gathers (ODCIGs) contain amplitude variation with offset /AVO information.

angle domain common image gathers (ADCIGs) contain amplitude variation with angle /AVA information.

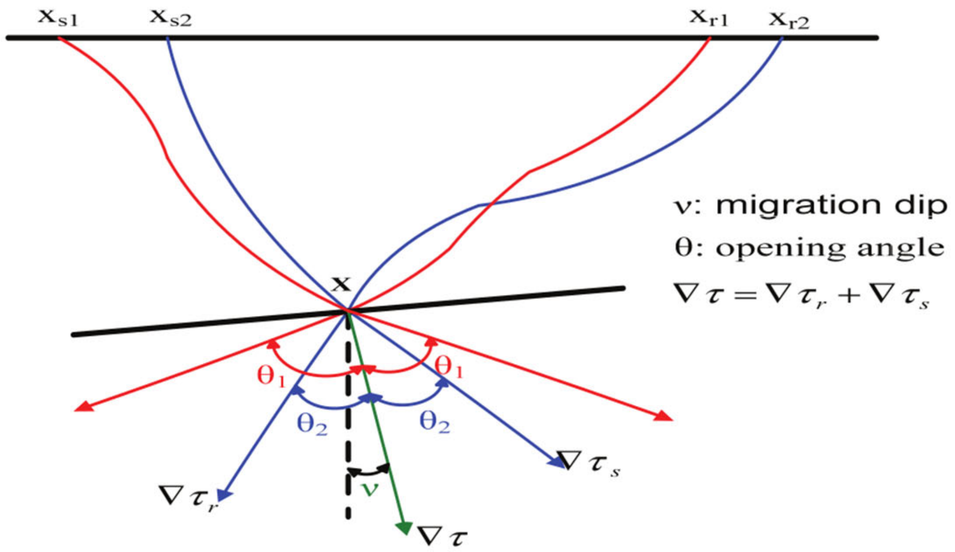
Why not ODCIGs? multipathing

ODCIGs can't handle multipathing for complex velocity model.

Multipath:



Same source-receiver wavefield location and traveltime but different image point.

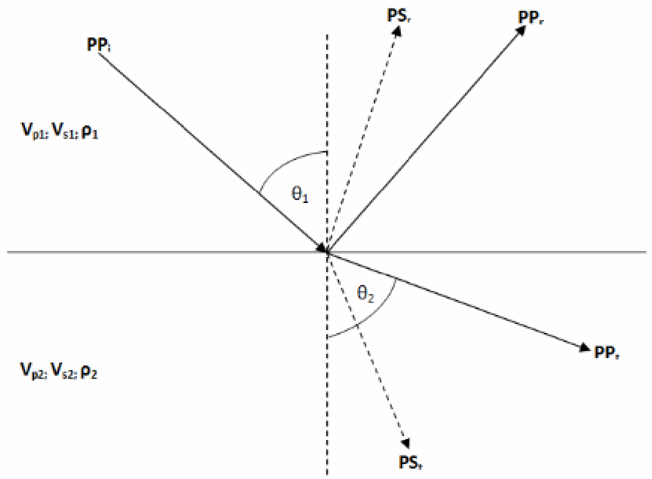


Same subsurface offset and image point but different source-receiver wavefield location .
They have different raypath and reflection angle.

Bleistein and Gray (2002), Faranak Mahmoudian and Gary F. Margrave(2009)

Zoeppritz equations represent AVA reflectivity in isotropic elastic media.

$$\begin{bmatrix} R_P \\ R_S \\ T_P \\ T_S \end{bmatrix} = \begin{bmatrix} -\sin\theta_1 & -\cos\phi_1 & \sin\theta_2 & \cos\phi_2 \\ \cos\theta_1 & -\sin\phi_1 & \cos\theta_2 & -\sin\phi_2 \\ \sin 2\theta_1 & \frac{V_{P1}}{V_{S1}}\cos 2\phi_1 & \frac{\rho_2 V_{S2}^2 V_{P1}}{\rho_1 V_{S1}^2 V_{P2}}\sin 2\theta_2 & \frac{\rho_2 V_{S2} V_{P1}}{\rho_1 V_{S1}^2}\cos 2\phi_2 \\ -\cos 2\phi_1 & \frac{V_{S1}}{V_{P1}}\sin 2\phi_1 & \frac{\rho_2 V_{P2}}{\rho_1 V_{P1}}\cos 2\phi_2 & -\frac{\rho_2 V_{S2}}{\rho_1 V_{P1}}\sin 2\phi_2 \end{bmatrix}^{-1} \begin{bmatrix} \sin\theta_1 \\ \cos\theta_1 \\ \sin 2\theta_1 \\ \cos 2\phi_1 \end{bmatrix}$$

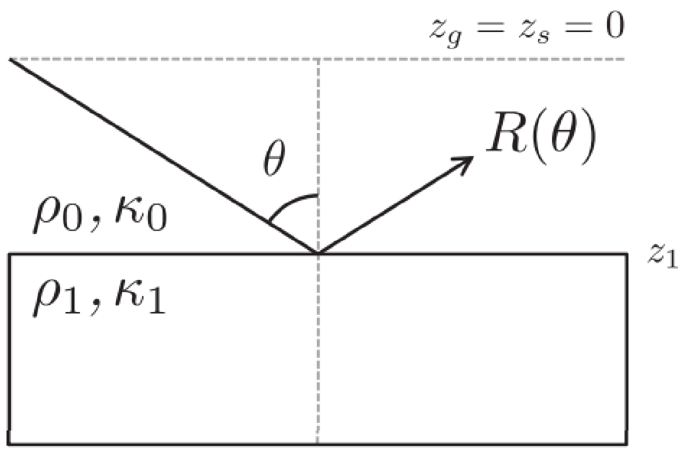


AVA response in isotropic acoustic media is :

$$R(\theta) = \frac{1 - \Omega(\theta)}{1 + \Omega(\theta)},$$

where

$$\Omega(\theta) = \left(\frac{\rho_0}{\rho_1}\right)\sqrt{\frac{\kappa_0}{\kappa_1}\frac{\rho_1}{\rho_0}}\left(\frac{1}{\cos\theta}\right)\sqrt{1 - \frac{\kappa_1}{\kappa_0}\frac{\rho_0}{\rho_1}\sin^2\theta}.$$



Innanen, Kristopher A. "Seismic AVO and the inverse Hessian in precritical reflection full waveform inversion." *Geophysical Journal International* 199.2 (2014): 717-734.

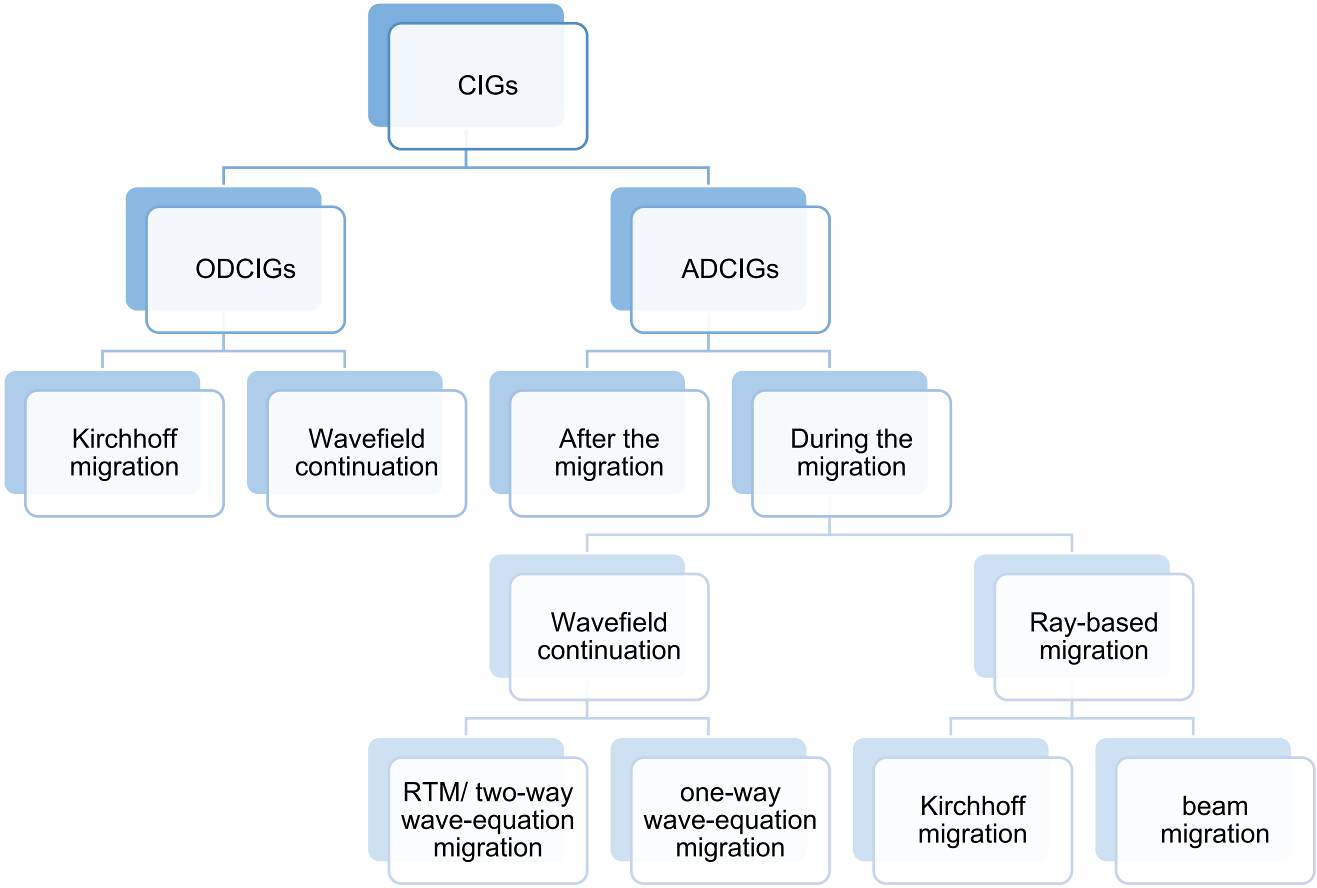
How to get CIGs and AVA information?

assumption & approximation

Kirchhoff depth migration: high frequency approximation

beam migration: Green's function

one-way wave-equation migration: one way approximation assumption

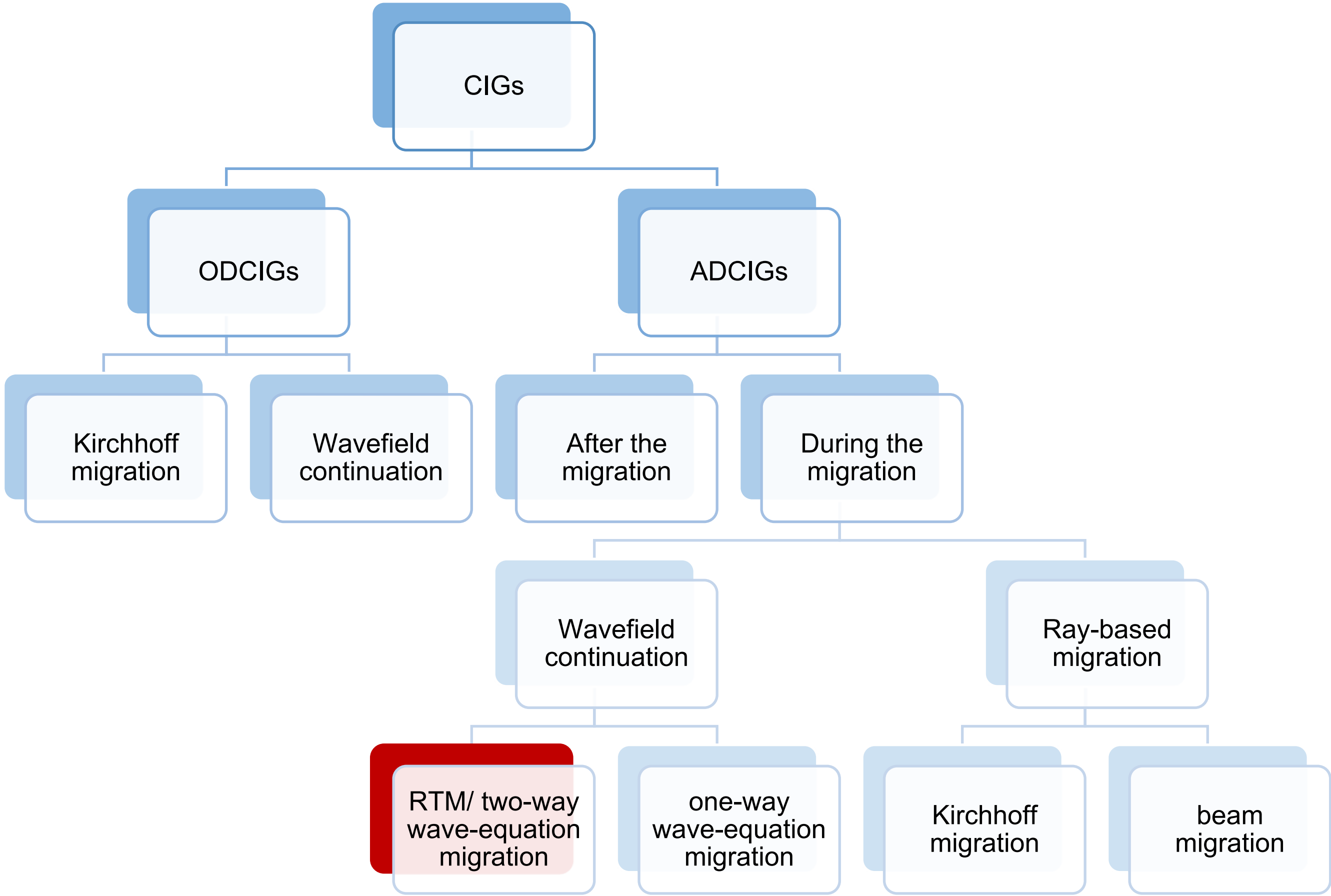


How to get CIGs and AVA information?

Because the Reverse time migration/RTM / two-way wave-equation migration

is based on the direct solutions of the wave equation, it requires fewer assumptions and approximations than others.

Itself naturally carries the correct propagation amplitude.



Outline

- Common image gathers (CIGs)
- Angle-dependent CIGs and reflectivity
- CIGs applied in FWI

Migration with velocity and reflectivity simultaneously

Media type	Acoustic media	Angle-dependent reflectivity ?	Elastic media
Wavefield	P		Stress and velocity
Model property	Vp		Vp, Vs, Density

$$\nabla^2 P - \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = f(t)$$

$$\frac{\partial \sigma_{xx}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z},$$
$$\frac{\partial \sigma_{zz}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x},$$
$$\frac{\partial \sigma_{xz}}{\partial t} = \mu(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}),$$

$$\frac{\partial v_x}{\partial t} = \frac{1}{\rho}(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}),$$
$$\frac{\partial v_z}{\partial t} = \frac{1}{\rho}(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x}),$$

Migration with velocity and reflectivity simultaneously

Media type	Acoustic media	Angle-dependent reflectivity ? Impedance= Velocity x density	Elastic media
Wavefield	P		Stress and velocity
Model property	Vp		Vp, Vs, Density

$$\nabla^2 P - \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = f(t)$$

$$\nabla\left(\frac{1}{\rho}\nabla P'\right) - \frac{1}{c^2\rho}\frac{\partial}{\partial t}\frac{\partial P'}{\partial t} = 0$$

$$\frac{\partial^2 P}{\partial t^2} - V^2\rho\nabla\cdot\left(\frac{1}{\rho}\nabla P\right) = S$$

$$\frac{\partial \sigma_{xx}}{\partial t} = (\lambda+2\mu)\frac{\partial v_x}{\partial x} + \lambda\frac{\partial v_z}{\partial z},$$

$$\frac{\partial \sigma_{zz}}{\partial t} = (\lambda+2\mu)\frac{\partial v_z}{\partial z} + \lambda\frac{\partial v_x}{\partial x},$$

$$\frac{\partial \sigma_{xz}}{\partial t} = \mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right),$$

$$\frac{\partial v_x}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}\right),$$

$$\frac{\partial v_z}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x}\right),$$

Claerbout, J. F., 1985, Imaging the earth's interior: Blackwell Scientific Publications, Inc.

Migration with velocity and reflectivity simultaneously

Media type	Acoustic media	Acoustic with density media	Elastic media
Wavefield	P	P, Vx, Vz	Stress and velocity
Model property	Vp	Vp, density	Vp, Vs, Density

$$\nabla^2 P - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}P = 0$$

$$\nabla\left(\frac{1}{\rho}\nabla P'\right) - \frac{1}{c^2\rho}\frac{\partial}{\partial t}\frac{\partial P'}{\partial t} = 0$$

$$\begin{cases} P = \frac{\partial P'}{\partial t} \\ \vec{v} = \frac{1}{\rho}\nabla P' \end{cases}$$

$$\begin{cases} \frac{\partial P}{\partial t} = -\rho c^2 \nabla \cdot \vec{v} \\ \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} \end{cases}$$

$Z = \rho V$; $R = \frac{1}{2}\frac{\nabla Z}{Z} = -\frac{1}{2}Z\nabla\left(\frac{1}{Z}\right)$

Migration with velocity and reflectivity simultaneously

Media type	Acoustic media	Acoustic with reflectivity media	Elastic media
Wavefield	P	P, Vx, Vz	Stress and velocity
Model property	Vp	Vp, density	Vp, Vs, Density

$$Z = \rho V;$$

$$\nabla^2 P - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} P = 0$$

$$\frac{\partial^2 P}{\partial t^2} - V^2 \rho \nabla \cdot \left(\frac{1}{\rho}\nabla P\right) = S$$

$$\frac{\partial^2 P}{\partial t^2} - VZ\nabla \cdot \left(\frac{V}{Z}\nabla P\right) = S$$

$$\frac{\partial^2 P}{\partial t^2} - \left[V^2\nabla^2 P + V\nabla V \cdot \nabla P + V^2 Z \nabla\left(\frac{1}{Z}\right) \cdot \nabla P\right] = S$$

$$\boldsymbol{R} = \frac{1}{2}\frac{\nabla Z}{Z} = -\frac{1}{2} Z\nabla\left(\frac{1}{Z}\right)$$

$$\frac{\partial^2 P}{\partial t^2} - \{V^2\nabla^2 P + V\nabla V \cdot \nabla P - 2V^2(R \cdot \nabla P)\} = S$$

Whitmore, N. D., et al. "Seismic modeling with vector reflectivity." *SEG International Exposition and Annual Meeting*. SEG, 2020.

Angle-dependent CIGs

How to get ADCIGs from RTM / two-way wave-equation migration

The ADCIGs IC adds the angle dimension to the output.

Angle calculated from wave propagation direction.

$$R(\vec{x}) = \int p_B(\vec{x};t)p_F^{-1}(\vec{x};t)dt$$

$$\Downarrow$$

$$R(\vec{x},\theta) = \int p_B(\vec{x},\theta;t)p_F^{-1}(\vec{x},\theta;t)dt$$

$$cos2\theta = \frac{\mathbf{S}_{source}\mathbf{S}_{receivers}}{|\mathbf{S}_{source}||\mathbf{S}_{receivers}|}$$

$$\theta = \frac{1}{2}arccos\frac{\mathbf{S}_{source}\mathbf{S}_{receivers}}{|\mathbf{S}_{source}||\mathbf{S}_{receivers}|}$$

Angle-dependent CIGs

How to get wave propagation direction?

Media type	Acoustic media	Acoustic with reflectivity media	Elastic media
Wavefield	P	P, Vx, Vz	Stress and velocity
Model property	Vp	Vp, density	Vp, Vs, Density
Polarization direction	∇P		Vx, Vz
Poynting vector	$-\nabla P \frac{dP}{dt} P$		$S = \tau_{jk} v_k$

Angle-dependent CIGs

How to get wave propagation direction?

Media type	Acoustic media	Acoustic with reflectivity media	Elastic media
Wavefield	P	P, Vx, Vz	Stress and velocity
Model property	Vp	Vp, density	Vp, Vs, Density
Polarization direction	∇P	Direction: Vx, Vz	Vx, Vz
Poynting vector	$-\nabla P \frac{dP}{dt} P$	Amplitude: P	$S = \tau_{jk} v_k$

Angle-dependent CIGs

How to get wave propagation direction?

Media type	Acoustic media	Acoustic with reflectivity media	Elastic media
Wavefield	P	P, Vx, Vz	Stress and velocity
Model property	Vp	Vp, density	Vp, Vs, Density
Polarization direction	∇P	Direction: Vx, Vz	Vx, Vz
Poynting vector	$-\nabla P \frac{dP}{dt} P$	Amplitude: P	$S = \tau_{jk} v_k$

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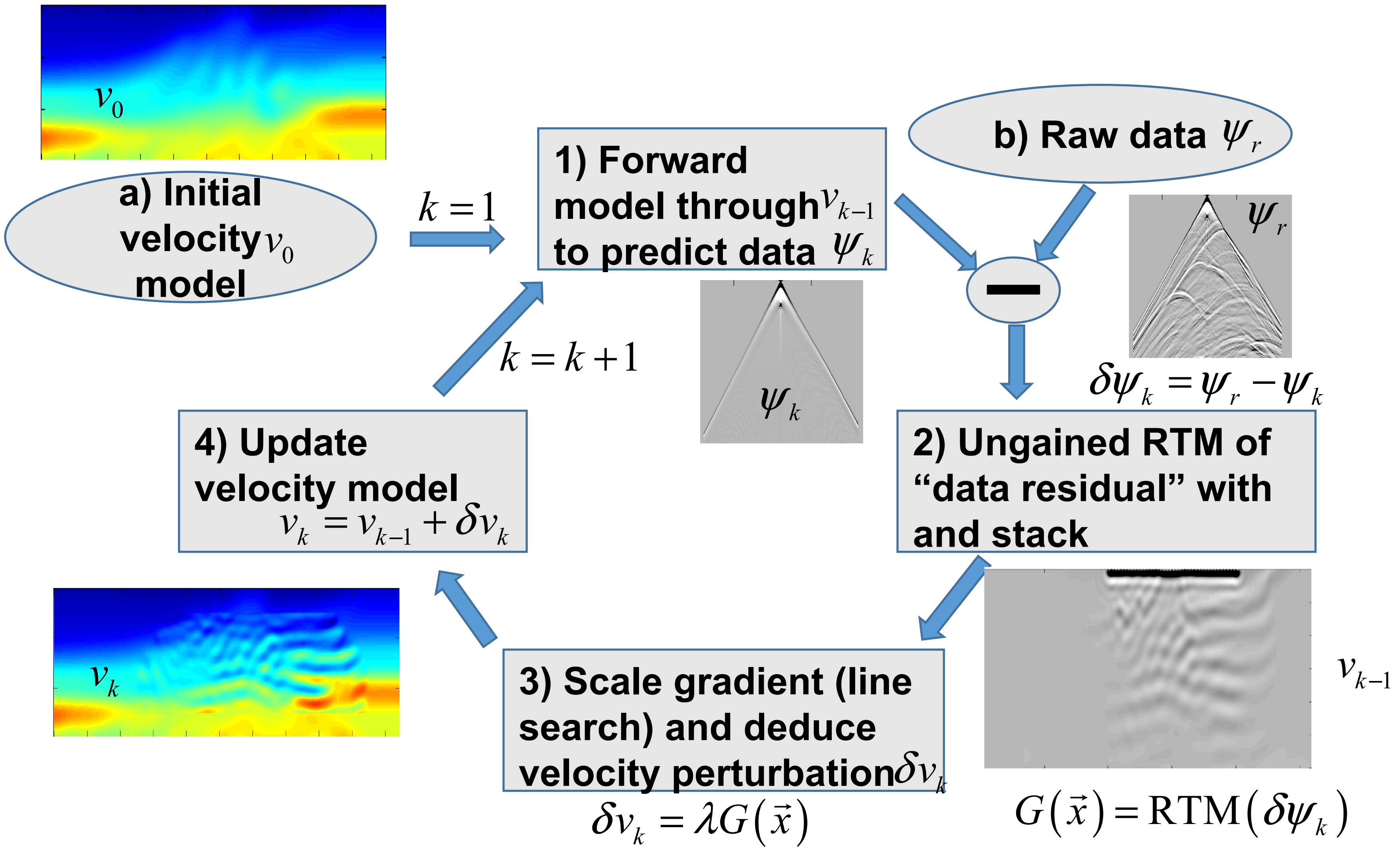
Full Waveform Inversion Basics (Inversion)



$$J = \|d_{predicted} - d_{acquired}\|^2$$

$$\mathbf{d}_{\text{predicted}} = \mathbf{L}(\mathbf{v}_{\text{iter}})$$

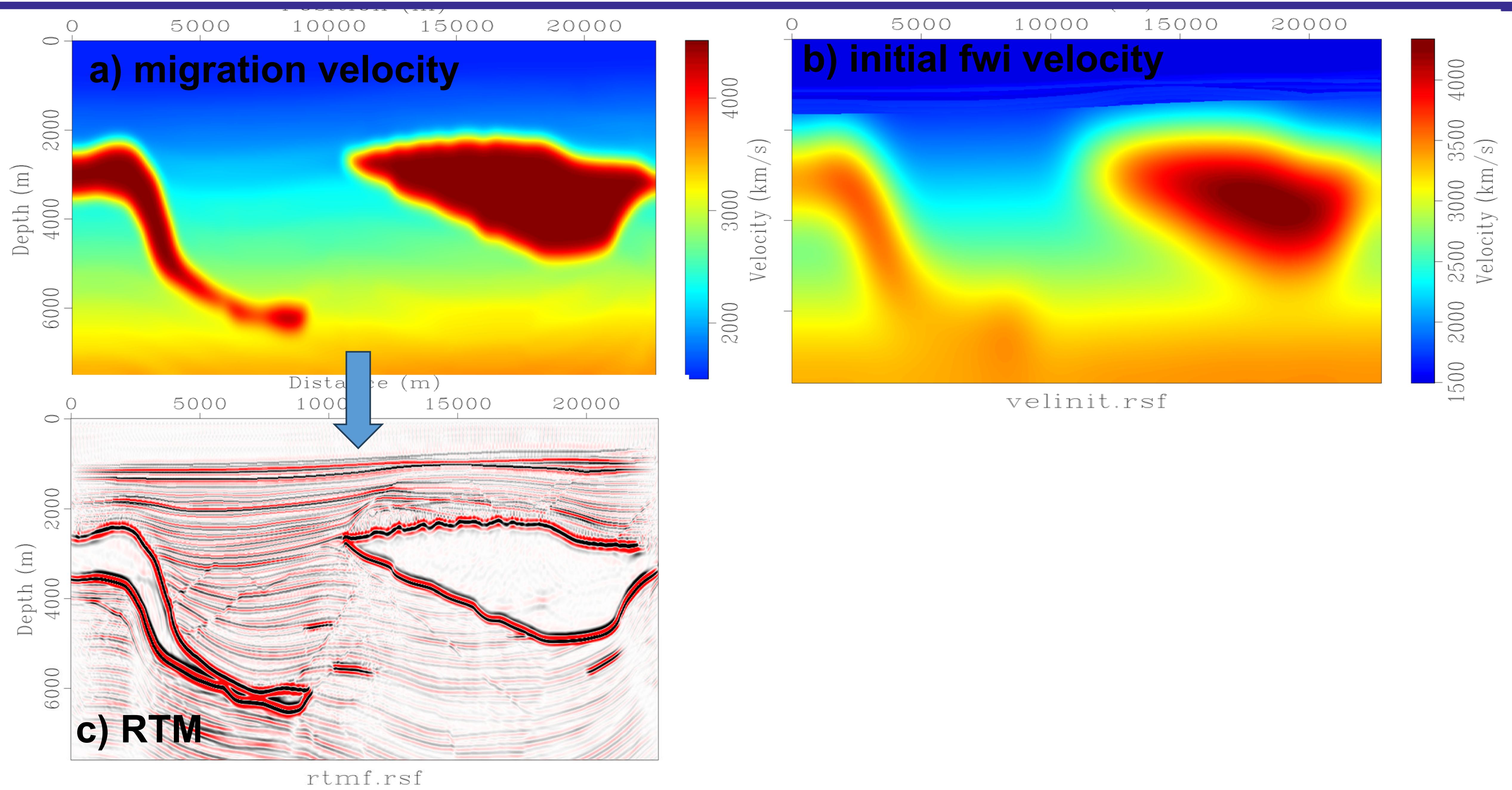
$$\mathbf{v}_{\text{iter}} = \mathbf{v}_{\text{iter}-1} + \alpha\mathbf{\Delta v}$$

$$\Delta v = RTM(Residuals)$$

Pan, W., 2017. Waveform Inversion for Estimating Subsurface Properties: Phase-Encoding Strategies, Optimization Methods, Interparameter Tradeoffs Quantification and Reduction. Ph.D. thesis. University of Calgary.

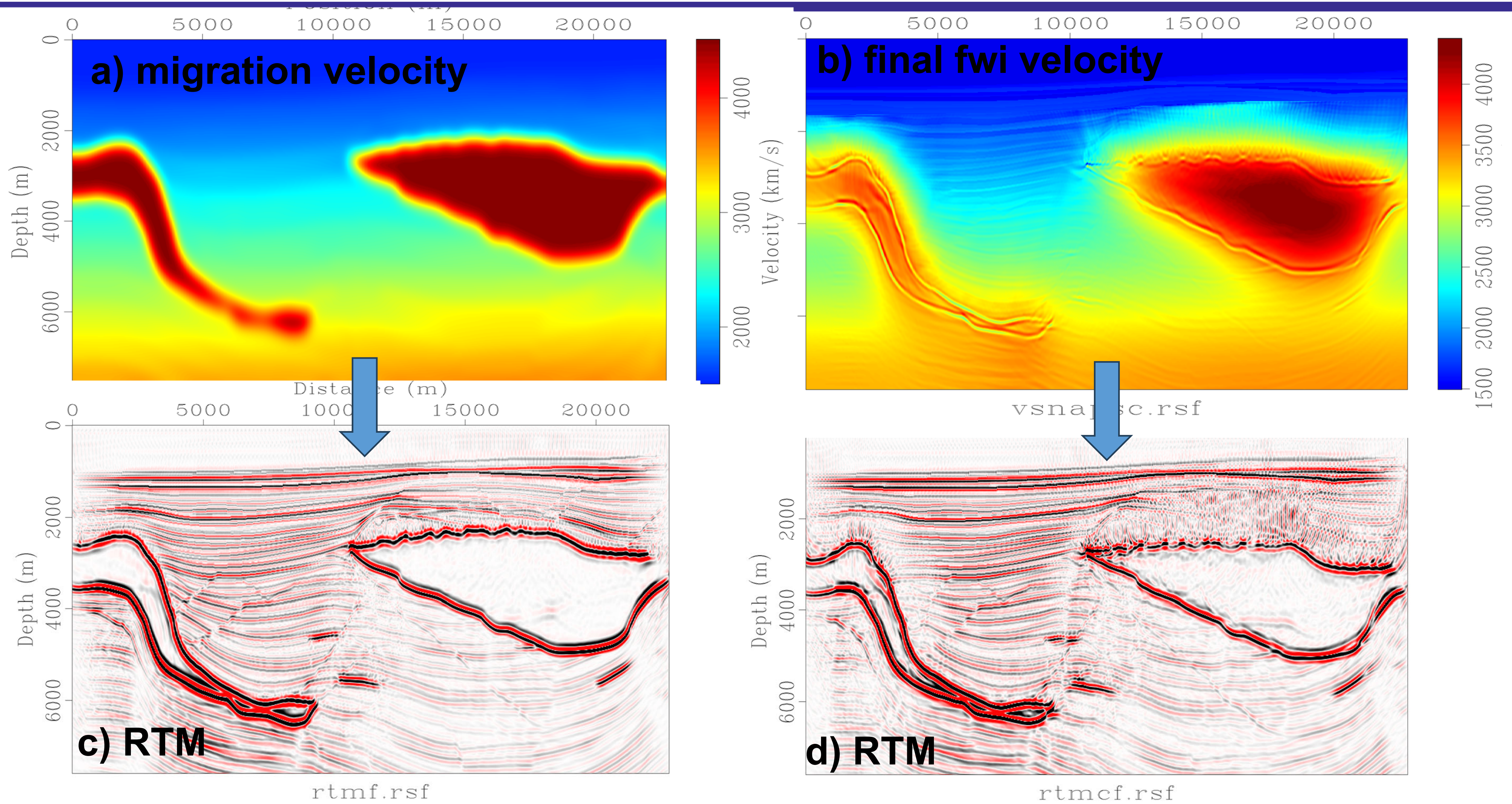
rtm vs fwi

(Trad and Sanchez, 2023,Crewes)



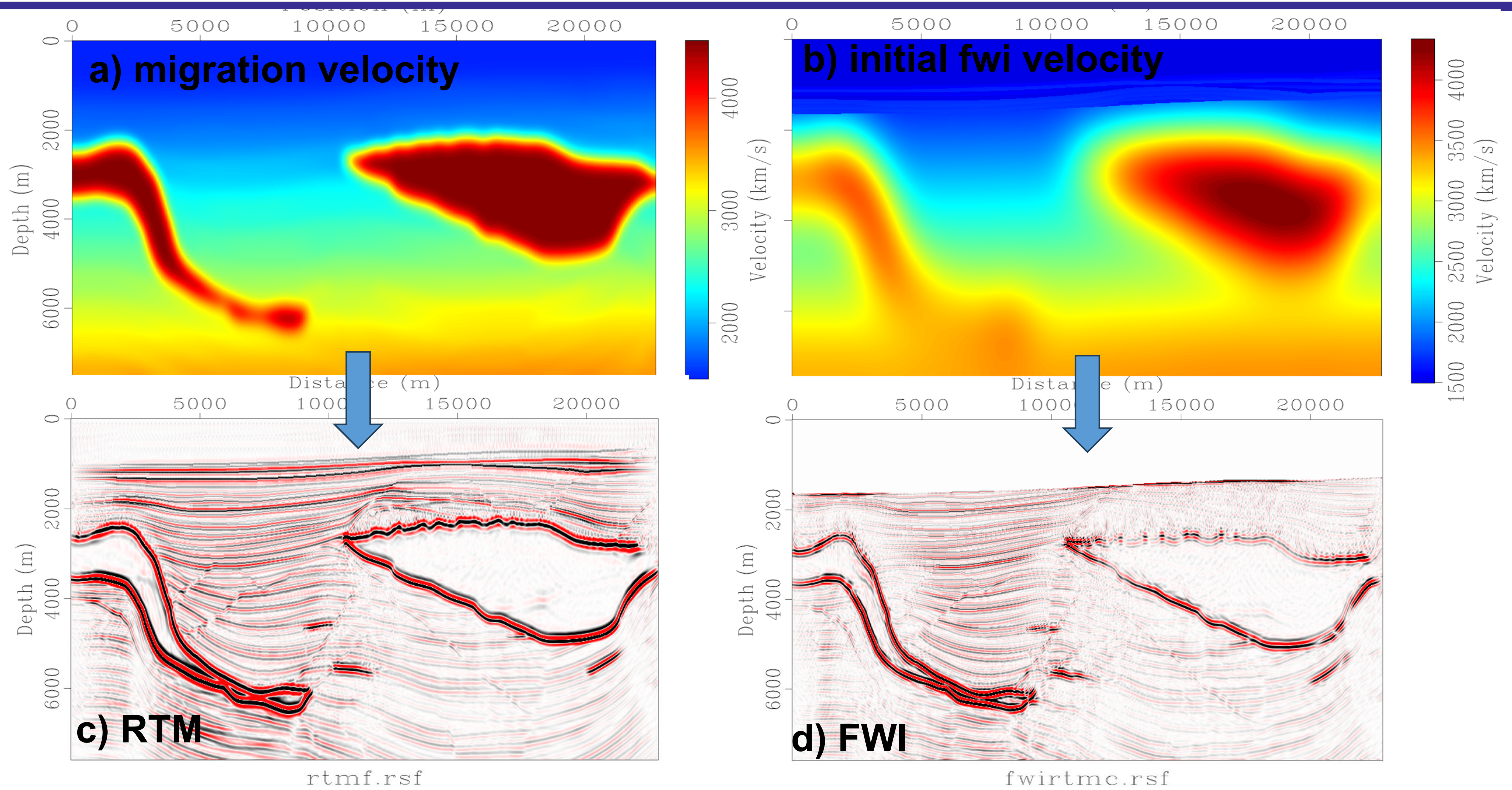
rtm vs fwi

(Trad and Sanchez, 2023,Crewes)



rtm vs fwi

(Trad and Sanchez, 2023,Crewes)



Pseudo code for time domain FWI

```
For iter=1:niter
    For is=1:ns
        For it=1:nt
        D_cal=stepforward(S,v, wavelet)
        Δd= D_cal -D_obs
        end
        For it=nt:1
        G+= stepforward(G,v, Δd)
        Δm= ∇²S × G
        end
    End
    grad= Δm
    cg=-grad+β*cg_old
    α=line search(v)
    v=v+ α*cg
end
```

D_{obs} : Observed data from v_{true}

S: shot wavefield

G: receiver wavefield from Δd

Δm : gradient update, reflectivity

$\Delta m = \nabla^2 S \times G$

$\text{Velocity}(x,z) = \text{FWI}(D_{obs})$

Yang, Pengliang, Jinghuai Gao, and Baoli Wang. "A graphics processing unit implementation of time-domain full-waveform inversion." *Geophysics* 80.3 (2015): F31-F39.

FWI and Angle gather

FWI:

Velocity(x,z)=FWI(D_{obs})

$\Delta m = \nabla^2 S \times G$

Angle domain FWI:

[Velocity(x,z),Reflectivity(x,z,ϴ)]=FWI(D_{obs})

$$R(\overrightarrow{x}) = \int p_B(\overrightarrow{x};t)p_F^{-1}(\overrightarrow{x};t)dt$$

↓

$$R(\overrightarrow{x},\theta) = \int p_B(\overrightarrow{x},\theta;t)p_F^{-1}(\overrightarrow{x},\theta;t)dt$$

Jin, Hu, George A. McMechan, and Huimin Guan. "Comparison of methods for extracting ADCIGs from RTM." *Geophysics* 79.3 (2014): S89-S103.

FWI:

$Velocity(x,z) = FWI(D_{obs})$

$\Delta m = \nabla^2 S \times G$

Angle domain FWI:

$[Velocity(x,z), Reflectivity(x,z,\Theta)] = FWI(D_{obs})$

$\Delta m\ (x,z,\Theta)$ = Angle domain image condition$(\nabla^2 S, G)$;

Angle domain image condition$(\nabla^2 Sp, Svx, Svz, Gp, Gvx, Gvz)$;

Acoustic with reflectivity media
P, Vx, Vz
Velocity, density
Direction: Vx, Vz
Amplitude: P

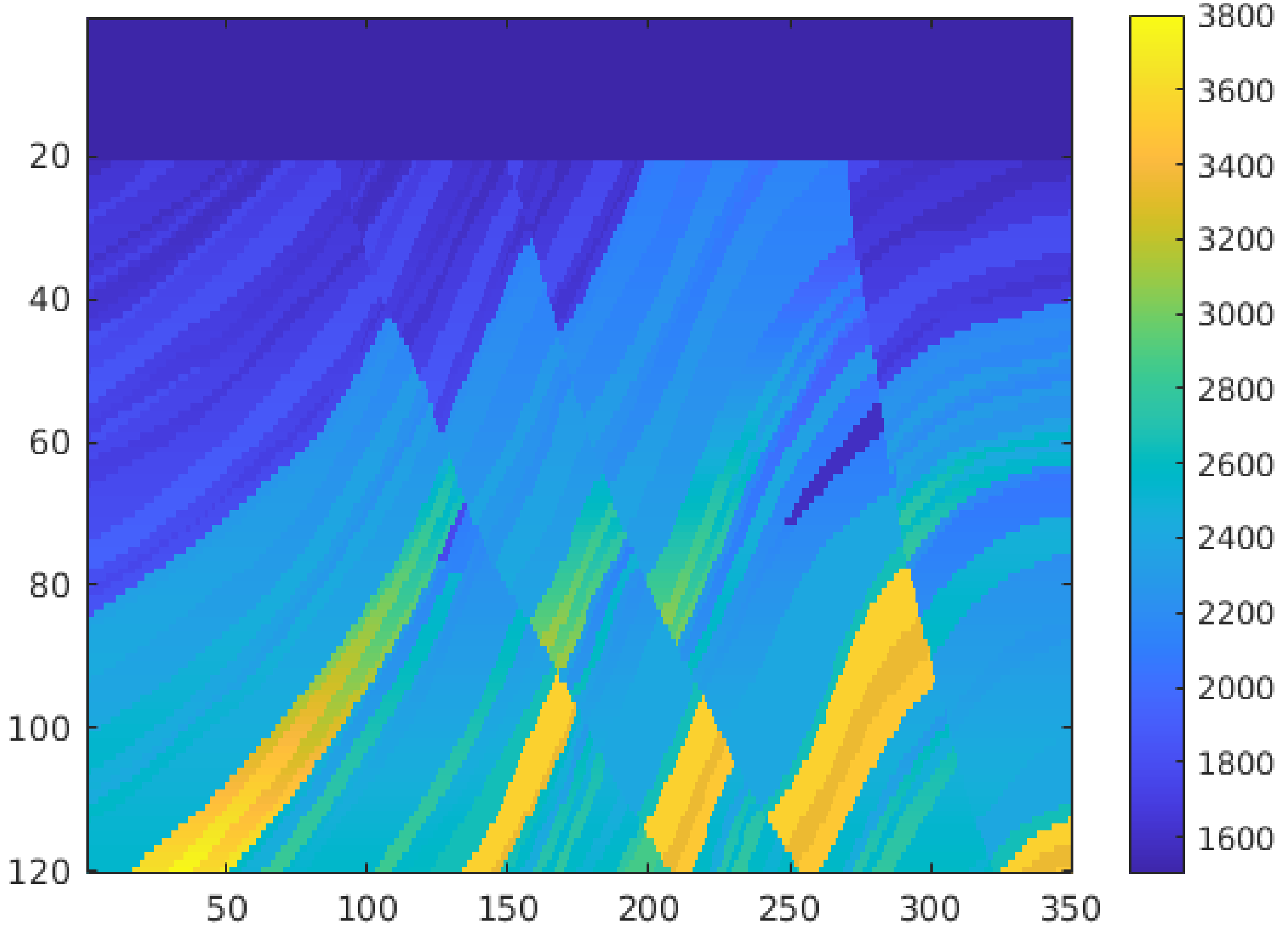
Jin, Hu, George A. McMechan, and Huimin Guan. "Comparison of methods for extracting ADCIGs from RTM." *Geophysics* 79.3 (2014): S89-S103.

Pseudo code for angle domain FWI

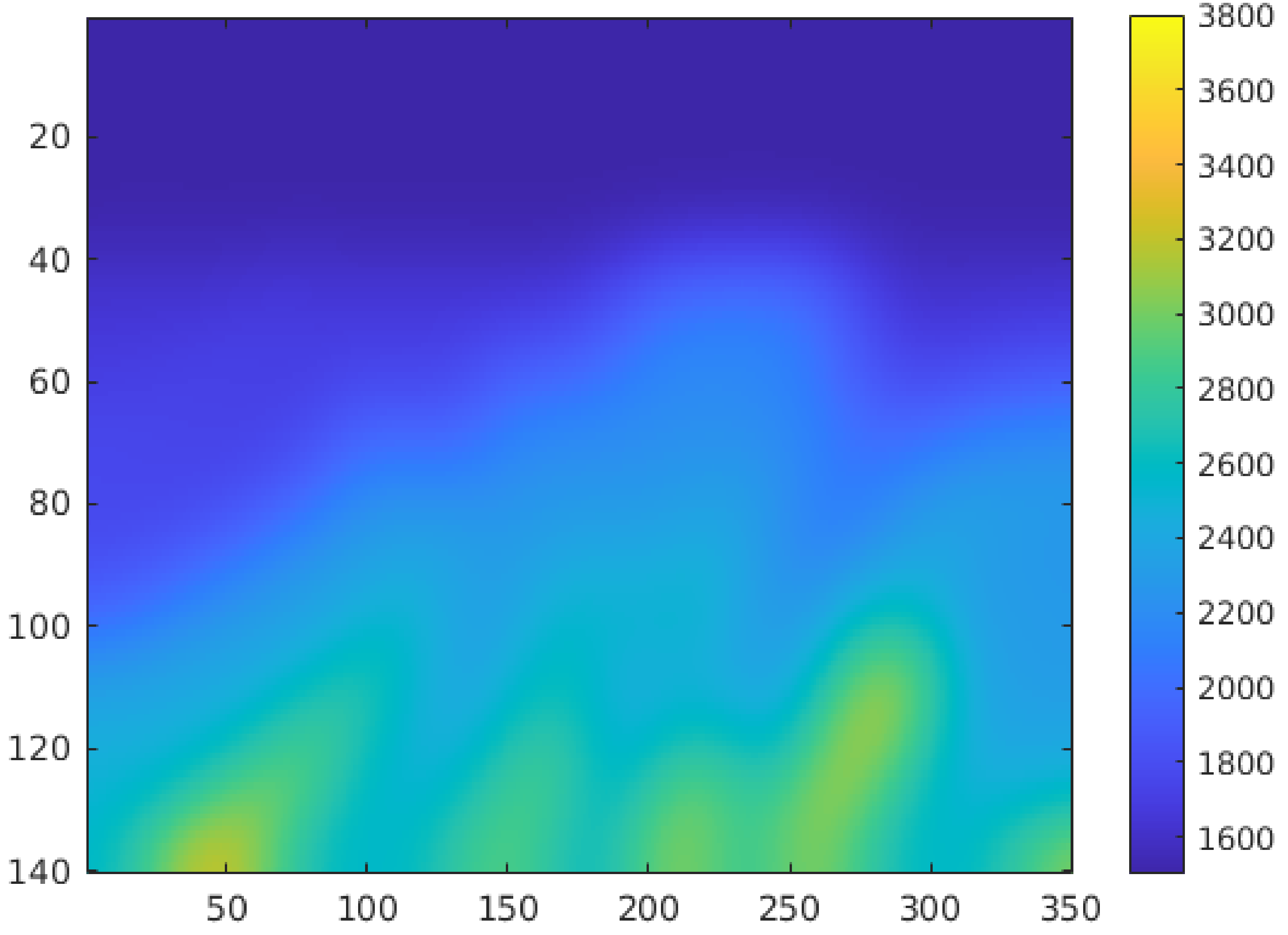
```
For iter=1:niter
    For is=1:ns
        For it=1:nt
            D_cal=stepforward(S,v, wavelet)
            Δd= D_cal -D_obs
        end
        For it=nt:1
            G+= stepforward(G,v, Δd)
            Δm (x,z,Θ) = ADCIG(∇²S,G)
        end
    End
    For i Θ =1:n Θ
        grad= Δm(i Θ) % read from Δm
        cg(i Θ)=-grad+β × cg_old(i Θ)
        α=line search(v)
        v=v+ α × cg
    end
end
```

Sequential FWI, update model with small angle gradient first, use the updated model for large angle.
Different stepsize for each angle

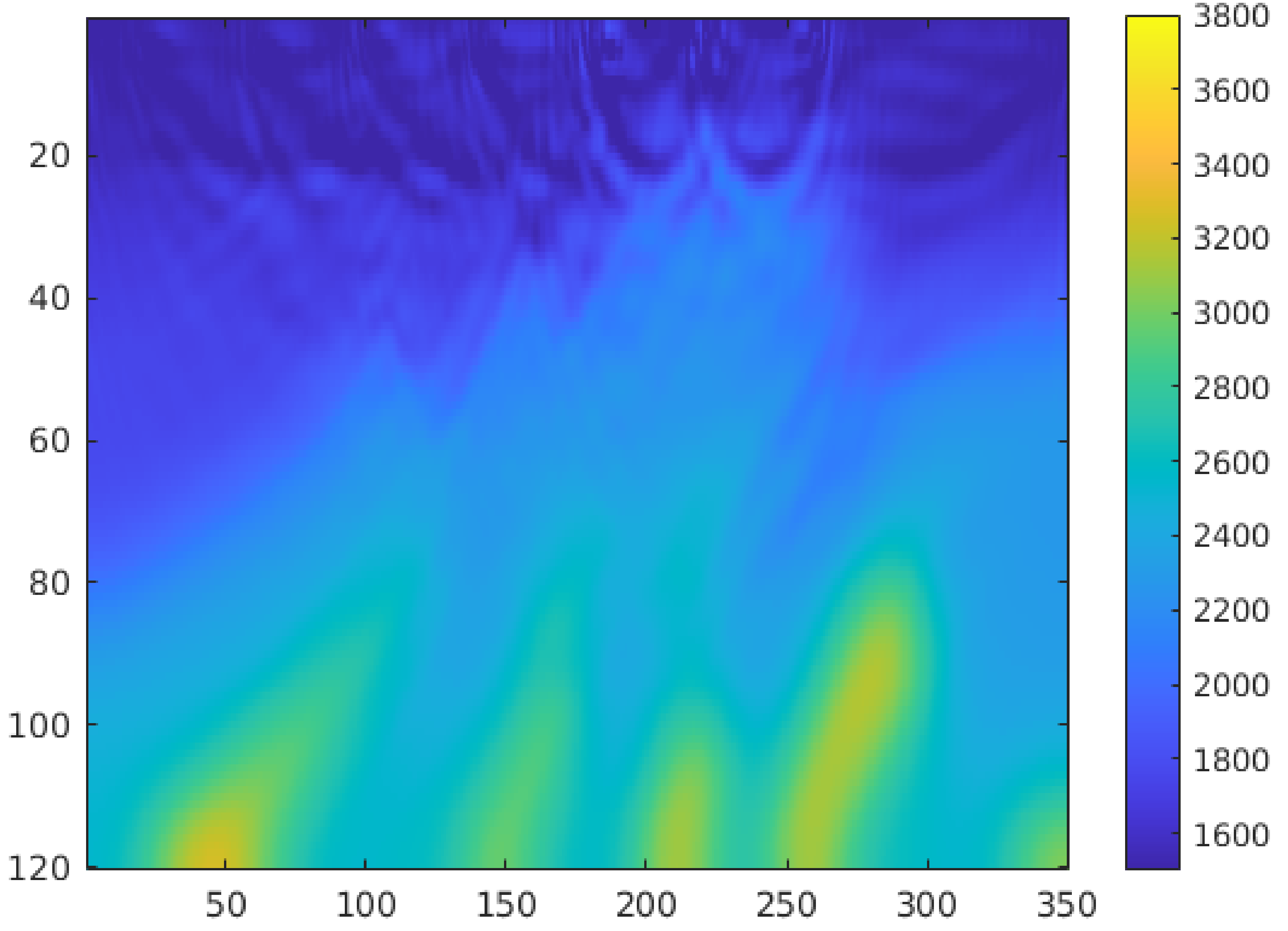
True velocity model



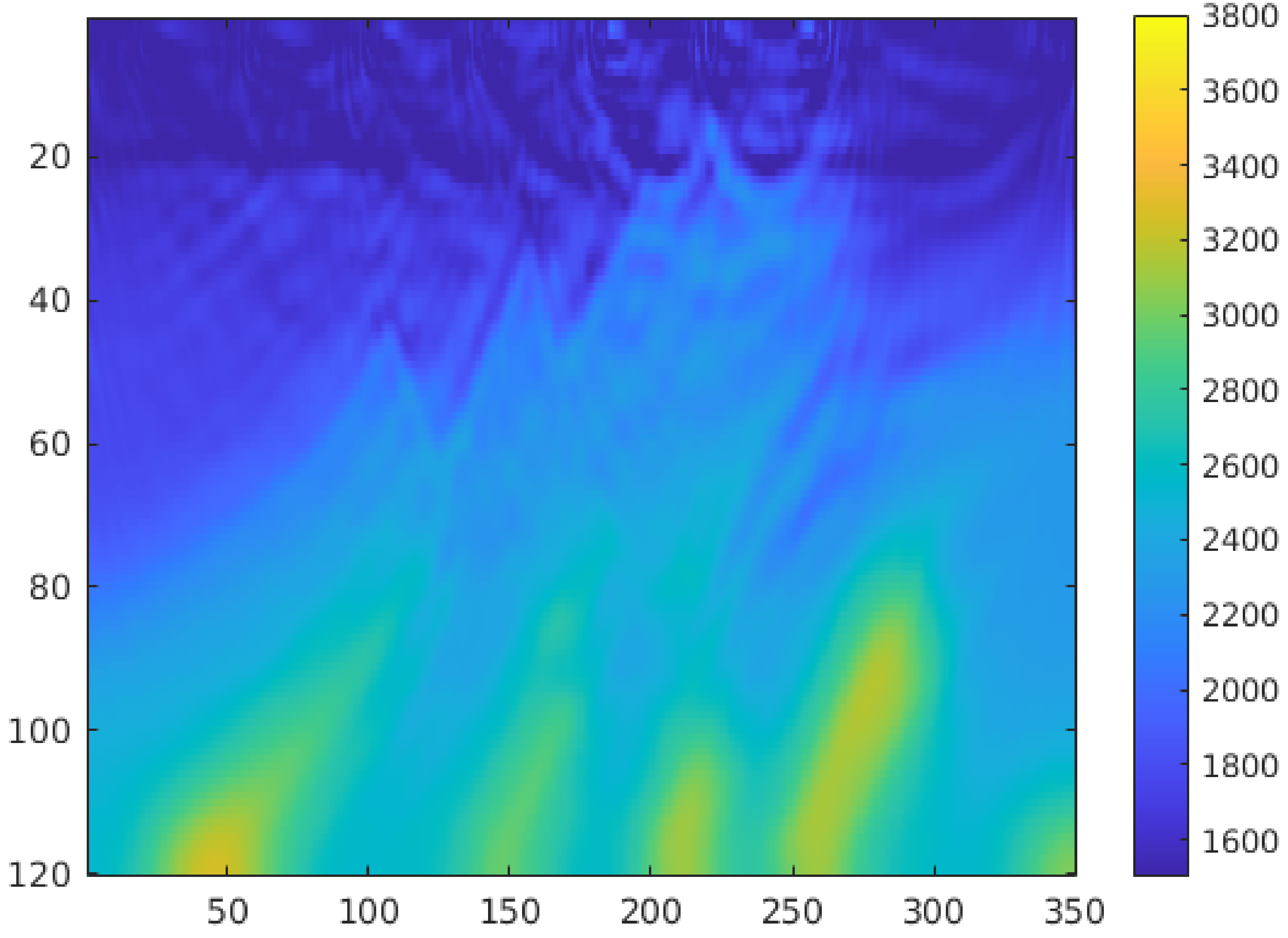
Initial velocity model



Updated velocity model, iteration 10

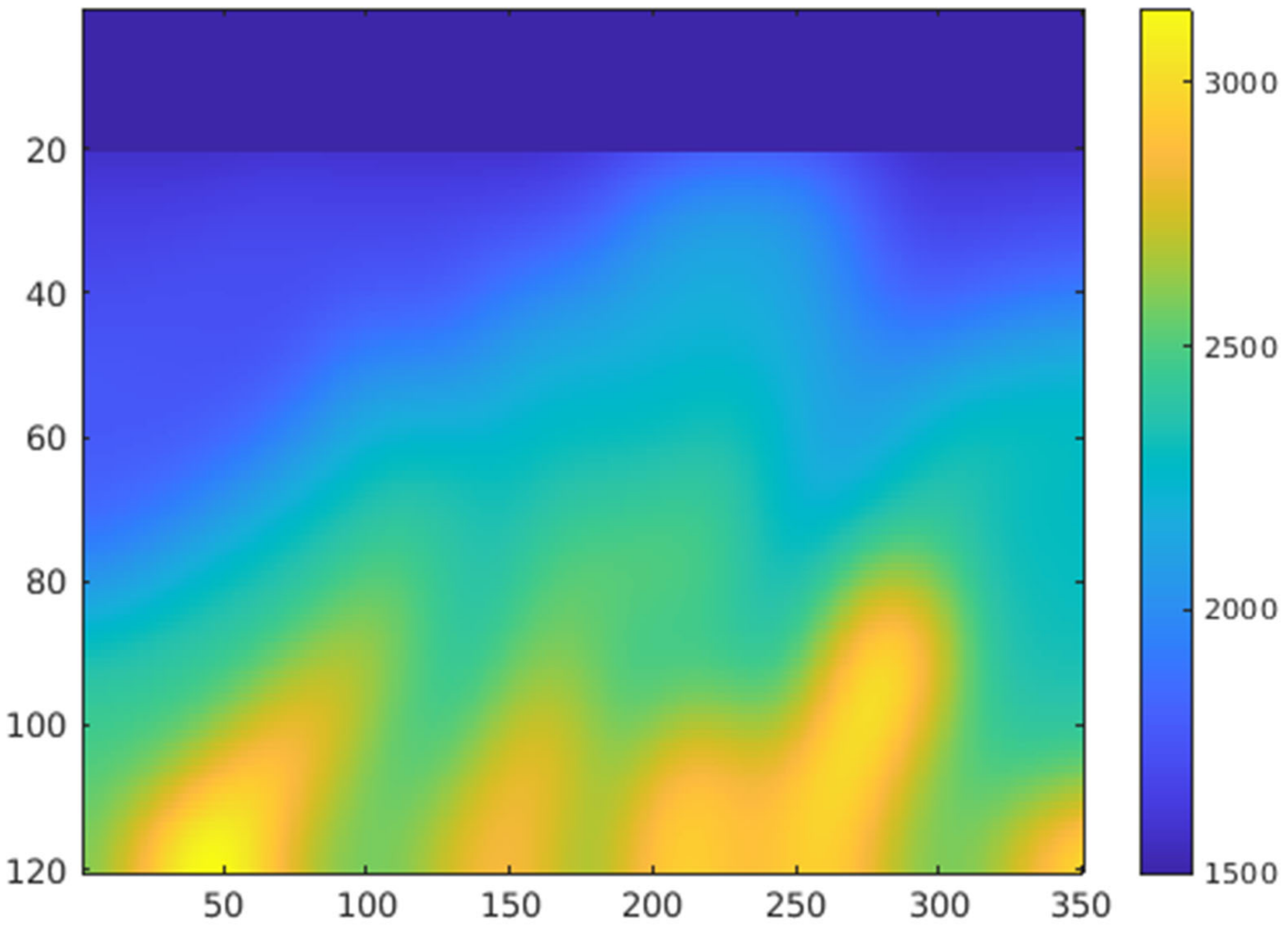


Updated velocity model, iteration 20

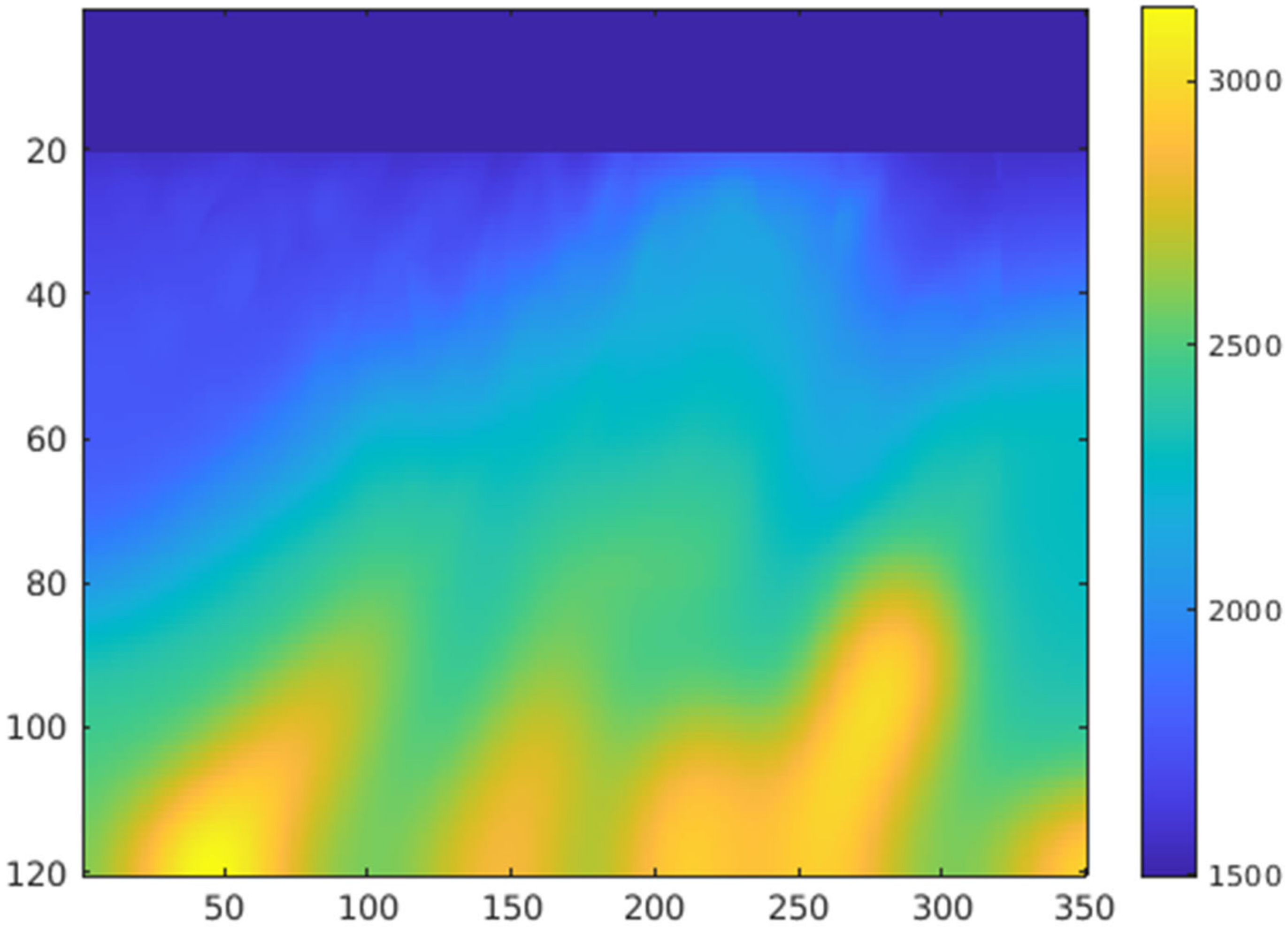


Angle domain FWI

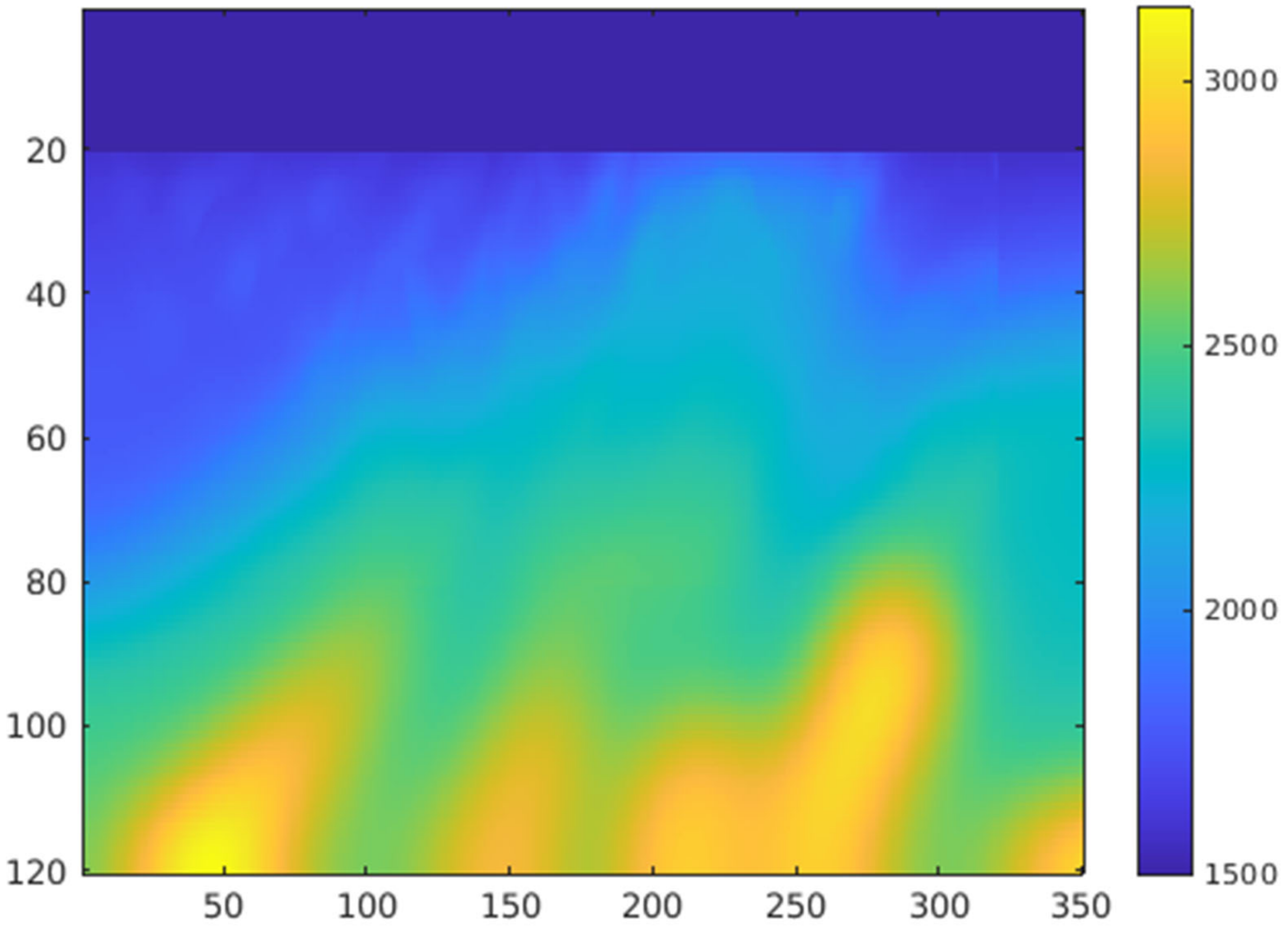
Initial velocity model



Updated vel model, 0-30 degree, iter 1

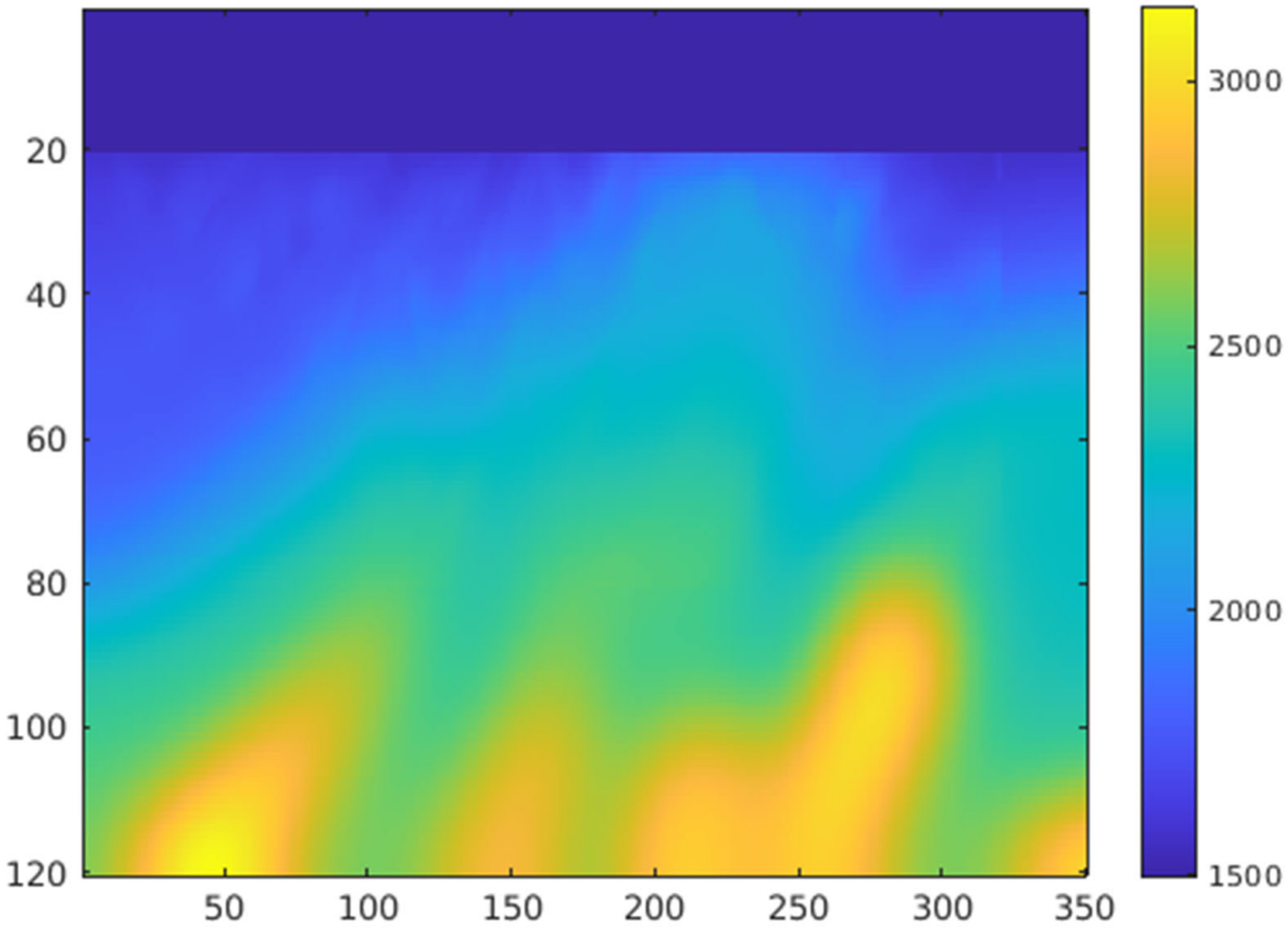


Updated vel model, 30-60 degree, iter 1



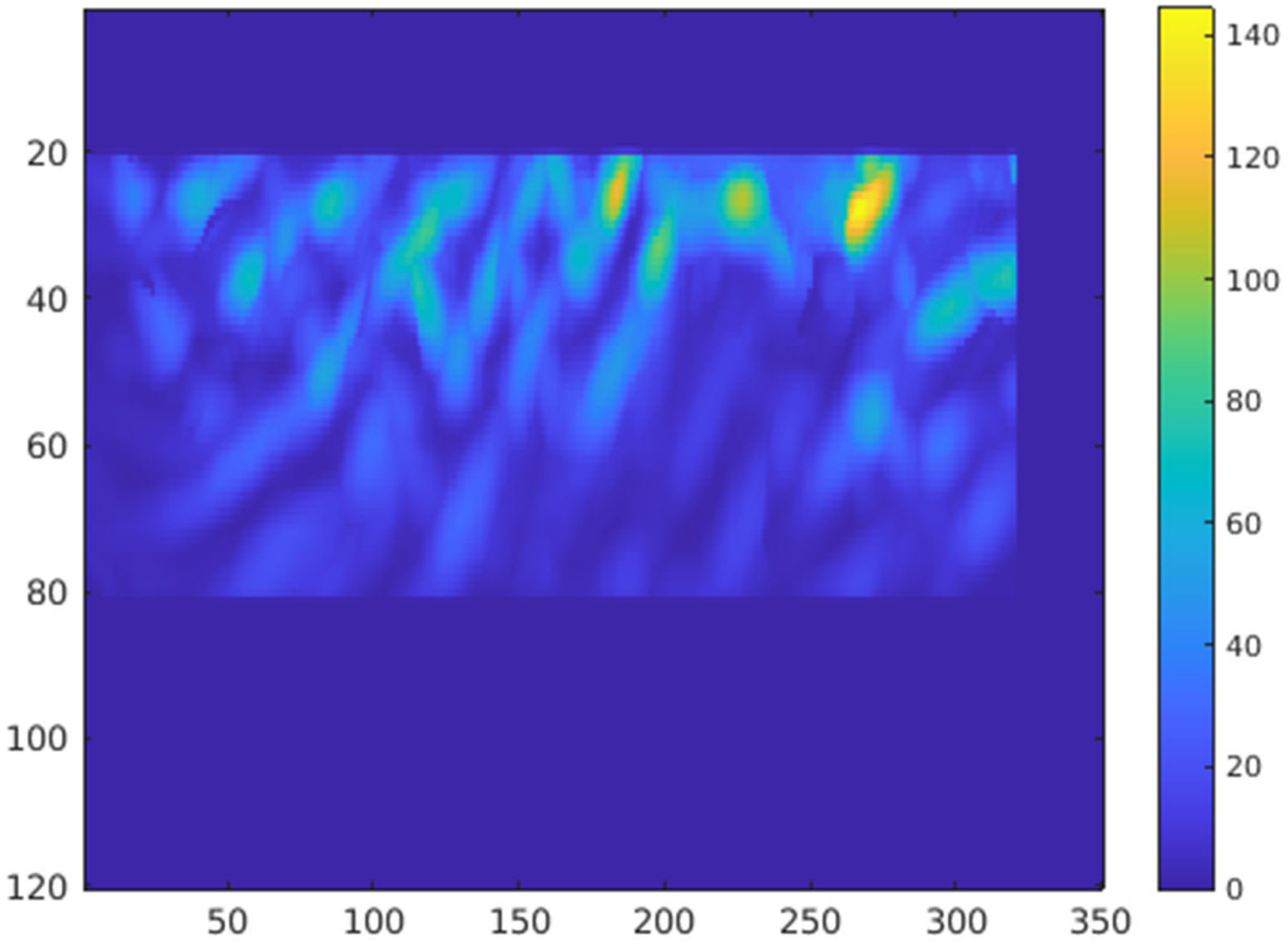
Angle domain FWI

Updated vel model, 60-90 degree, iter 1



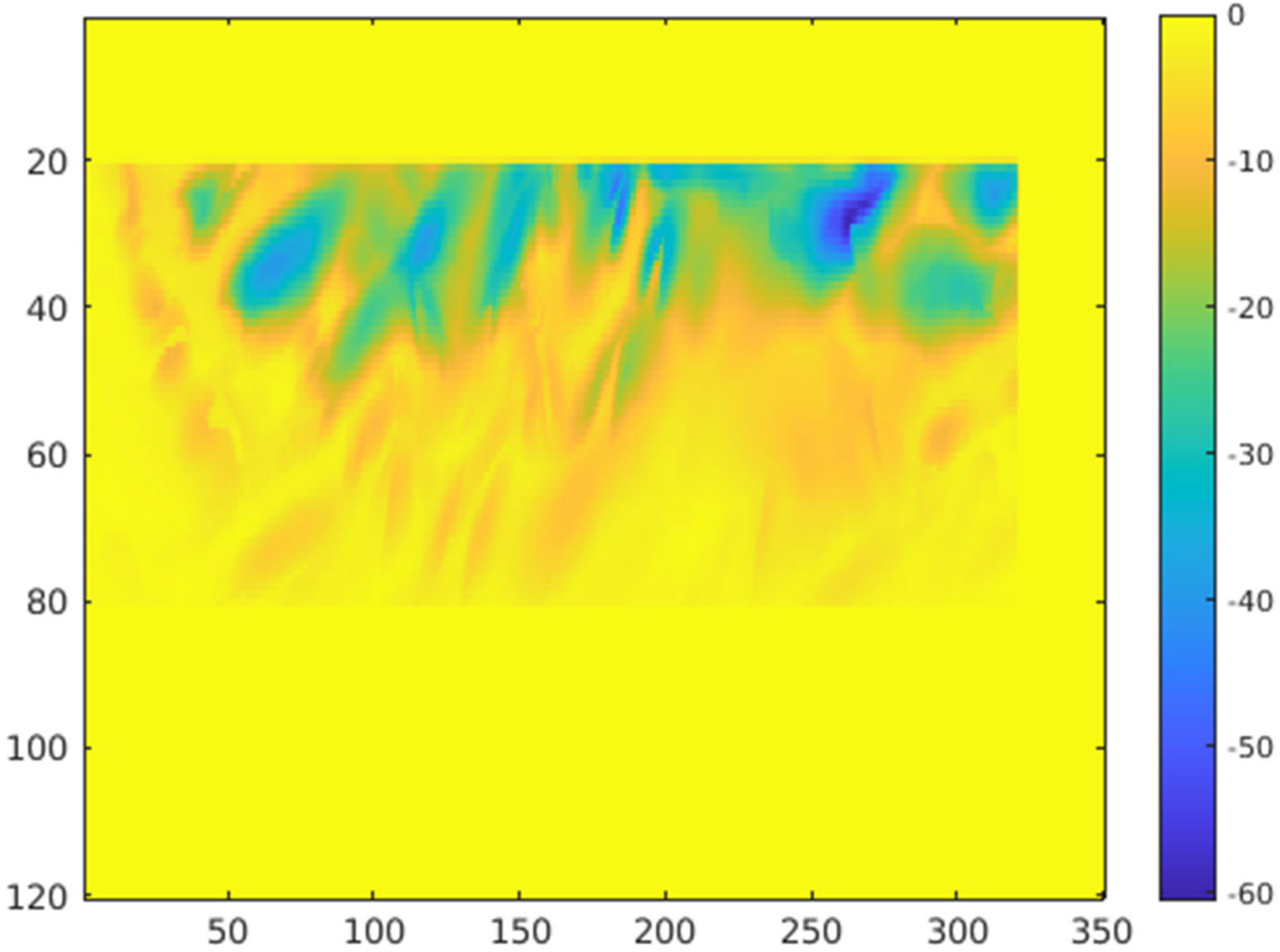
Angle domain FWI

Gradient 0-30 degree, iter 1



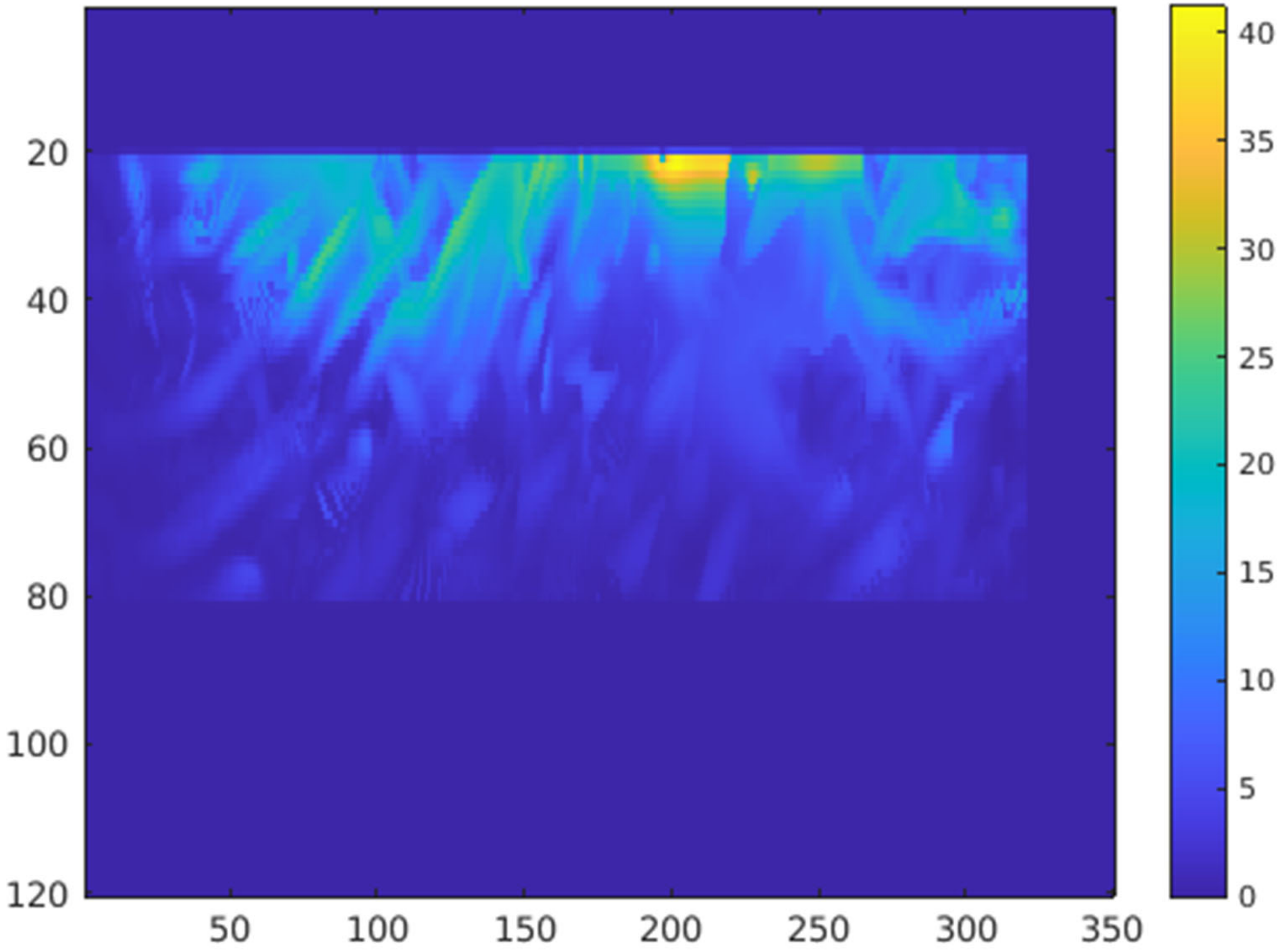
Angle domain FWI

Gradient 30-60 degree, iter 1



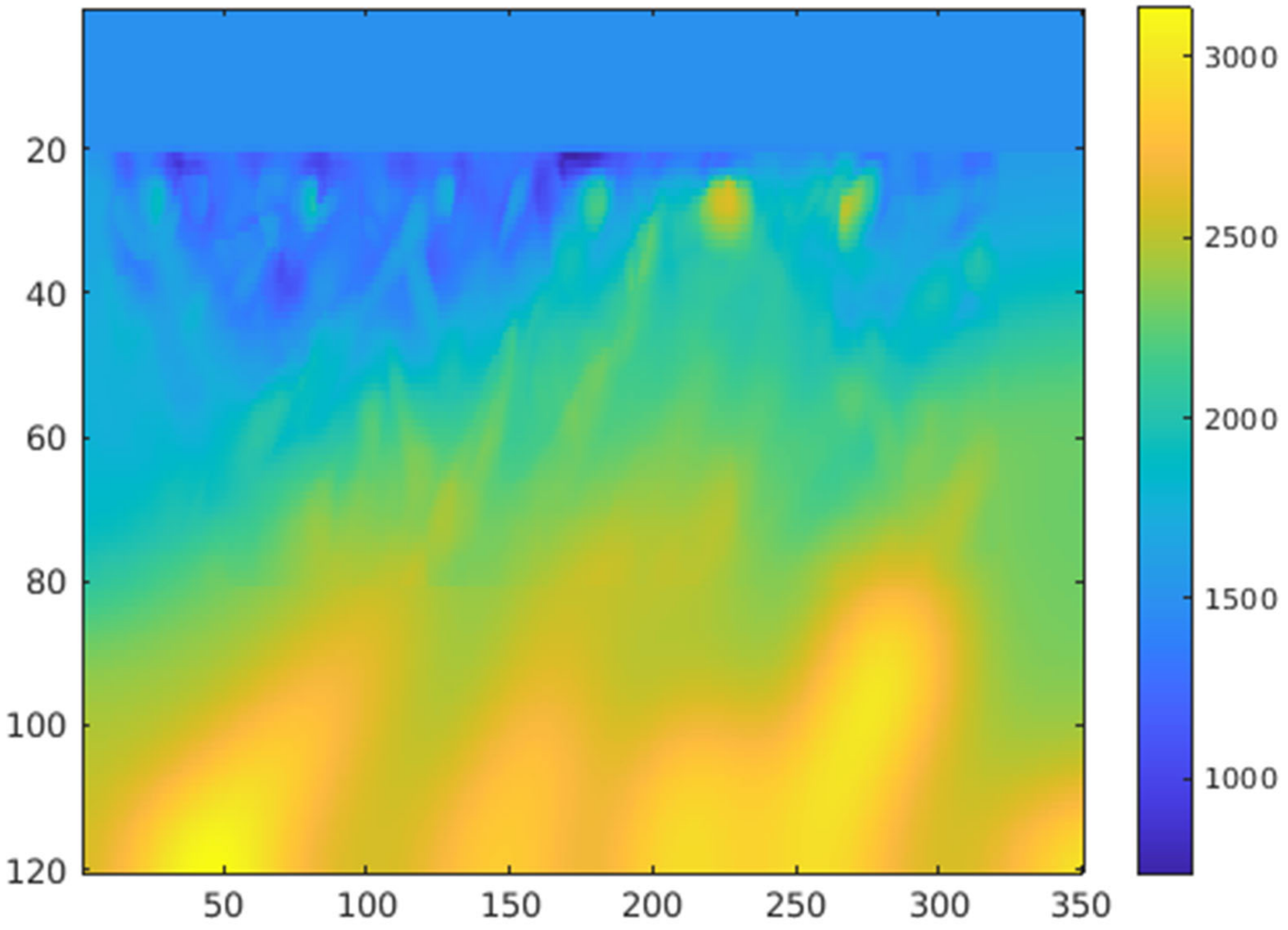
Angle domain FWI

Gradient 60-90 degree, iter 1



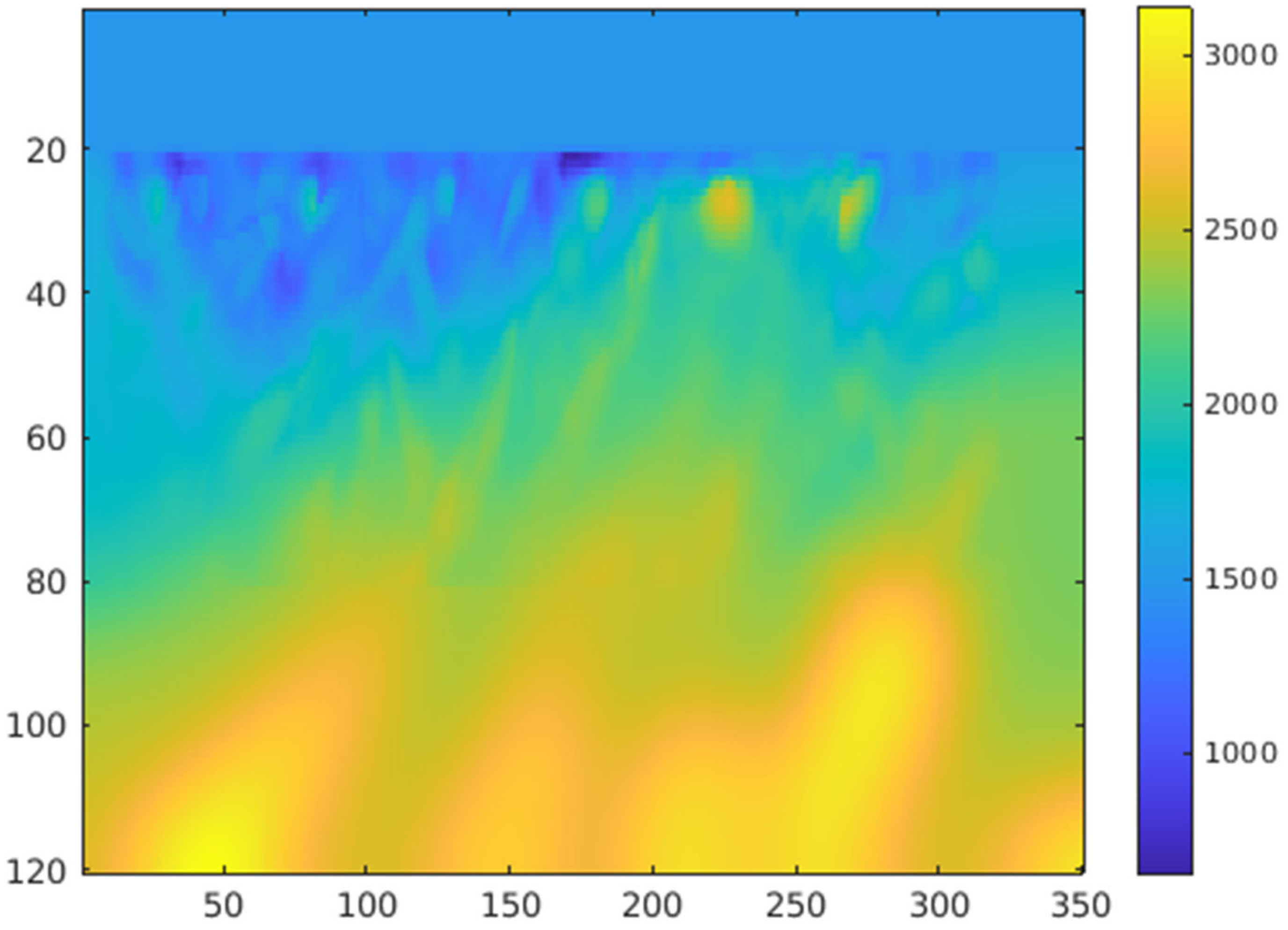
Angle domain FWI

Updated vel model, 0-30 degree, iter 9

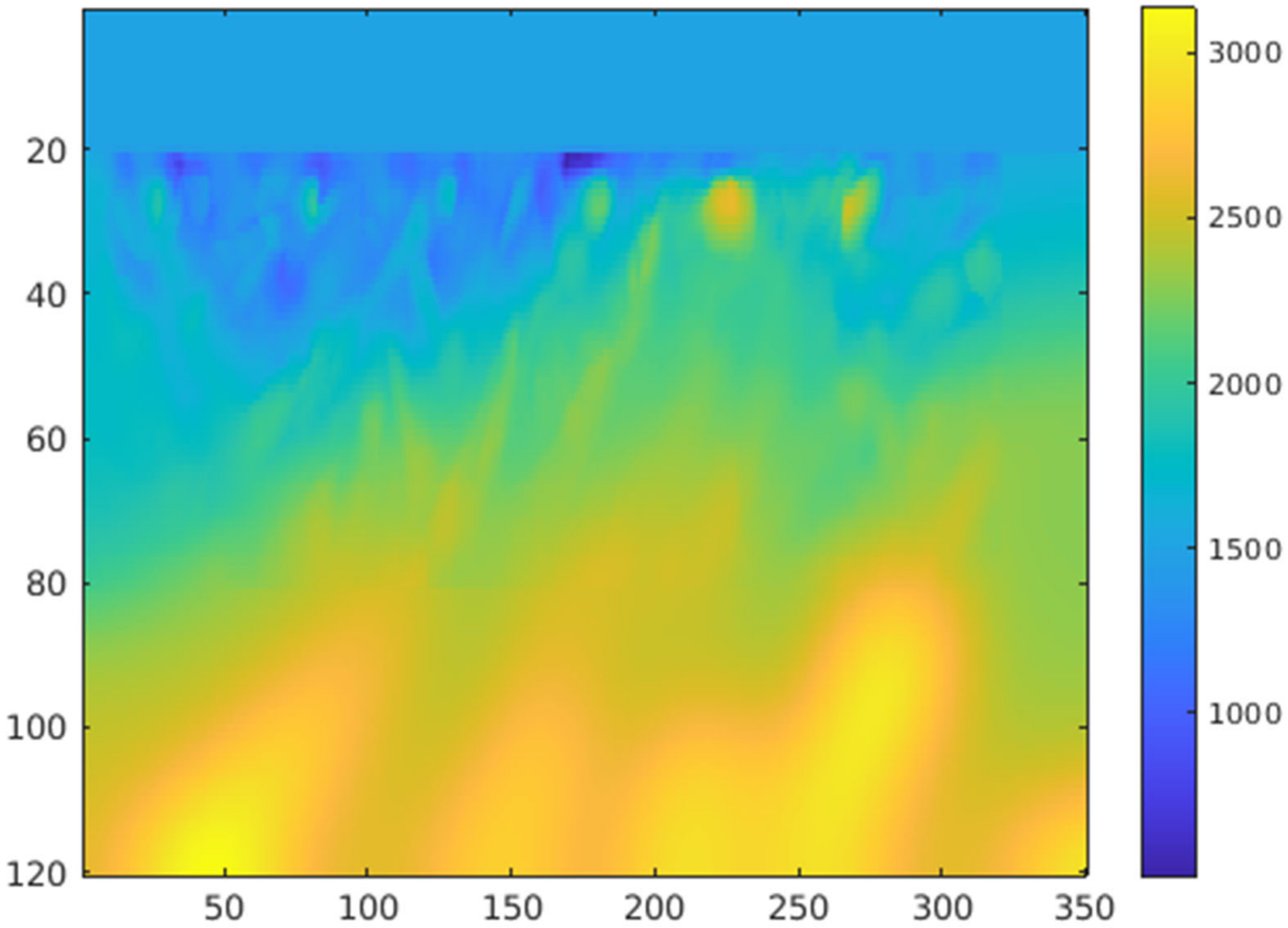


Angle domain FWI

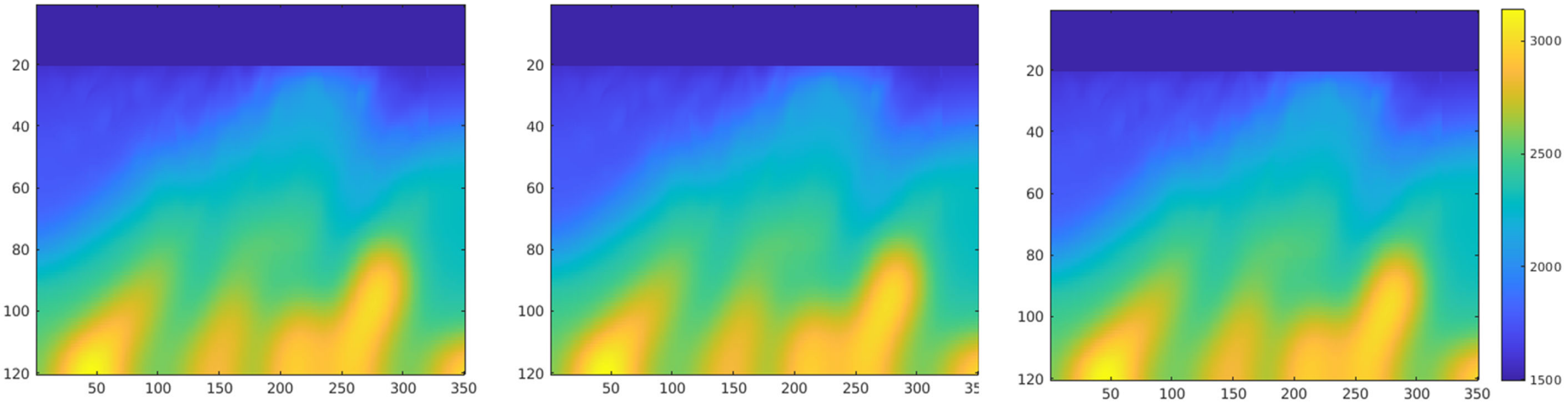
Updated vel model, 30-60 degree, iter 9



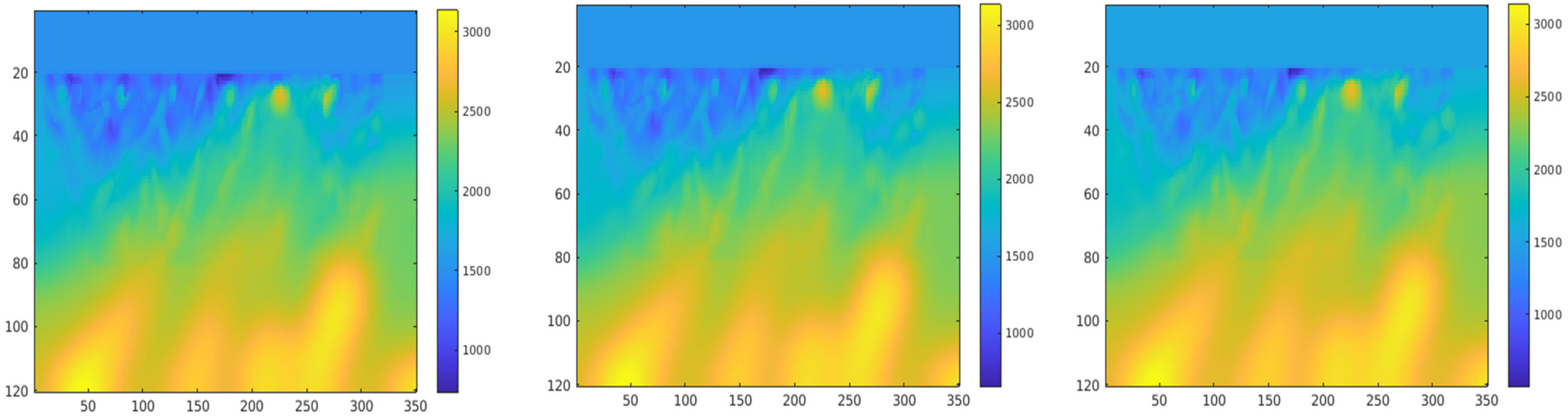
Updated vel model, 60-90 degree, iter 9



Updated vel model in 0-30, 30-60, 60-90 degree, iter 1

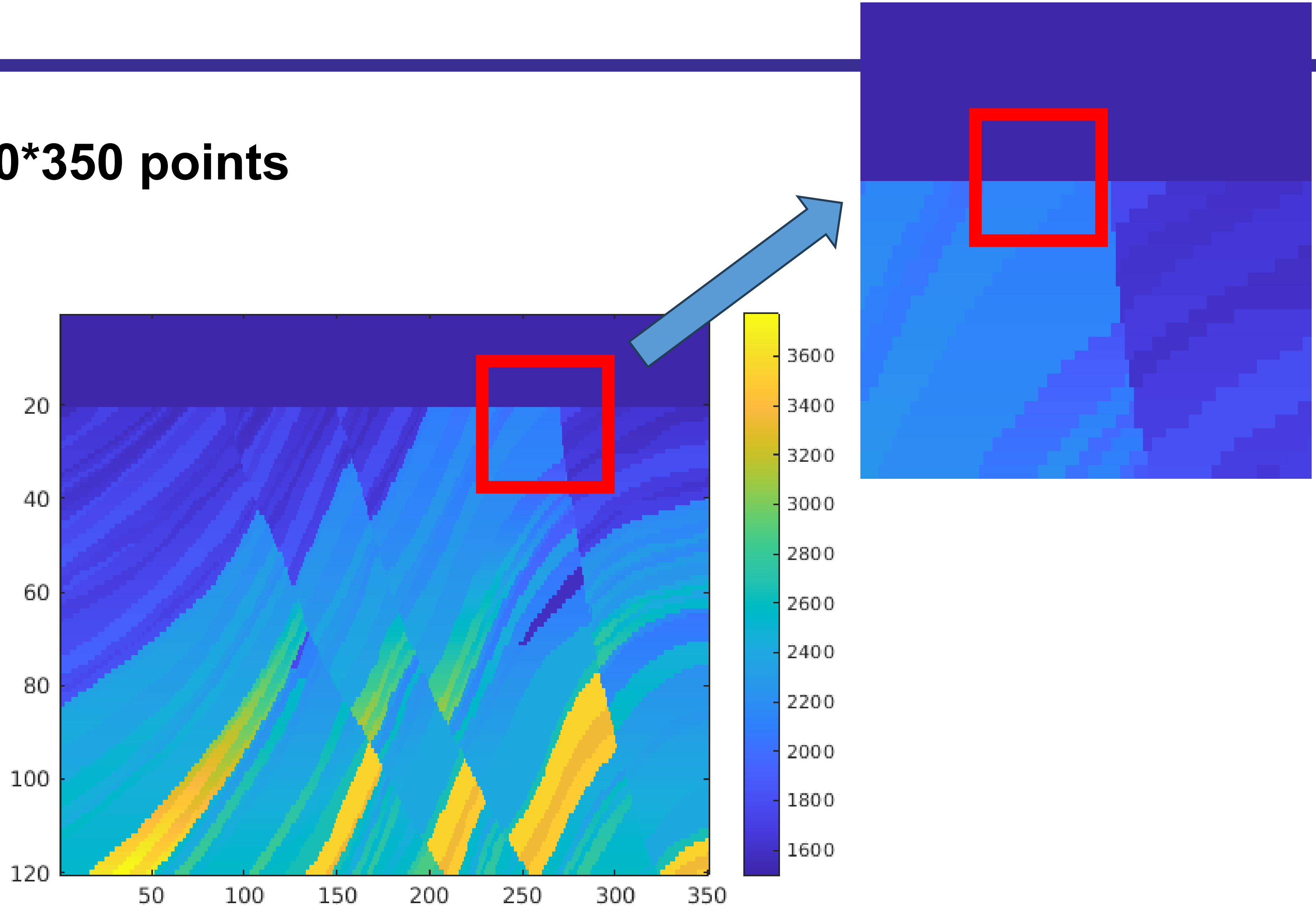


Updated vel model in 0-30, 30-60, 60-90 degree, iter 9

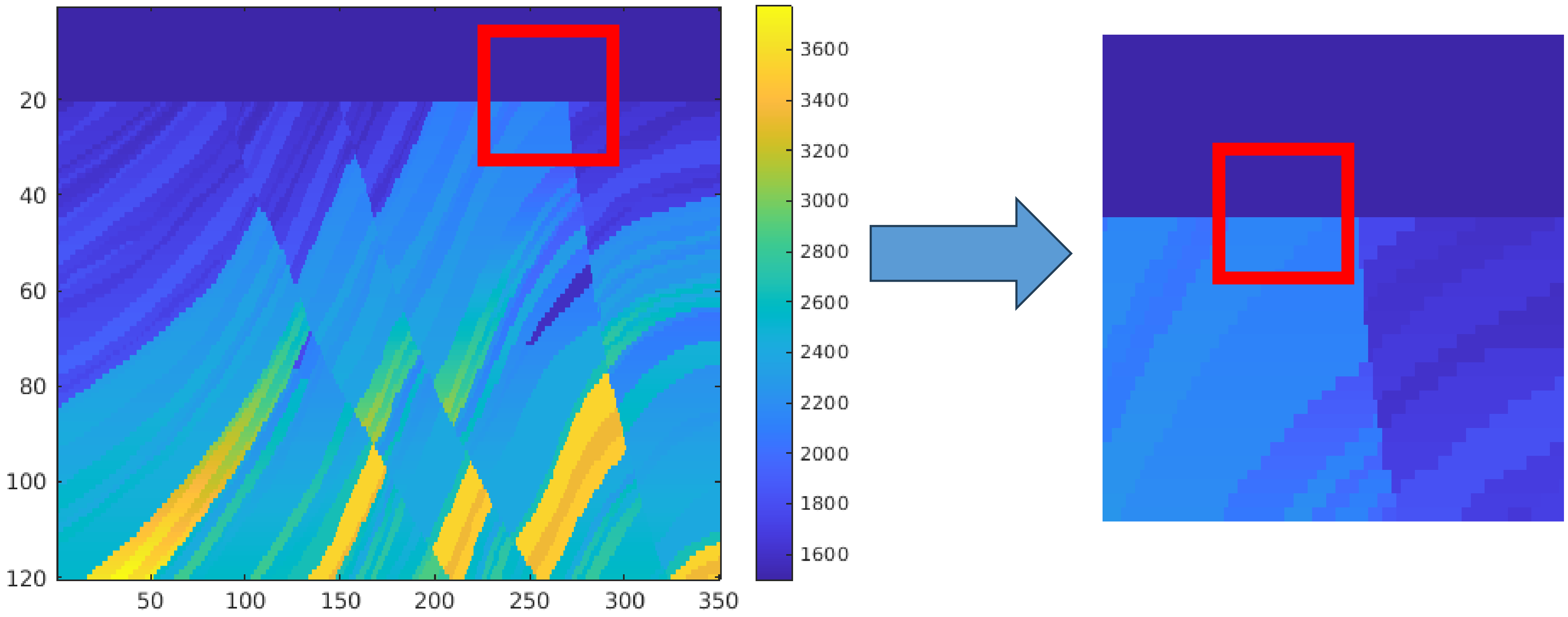


True velocity model, 130*350 points

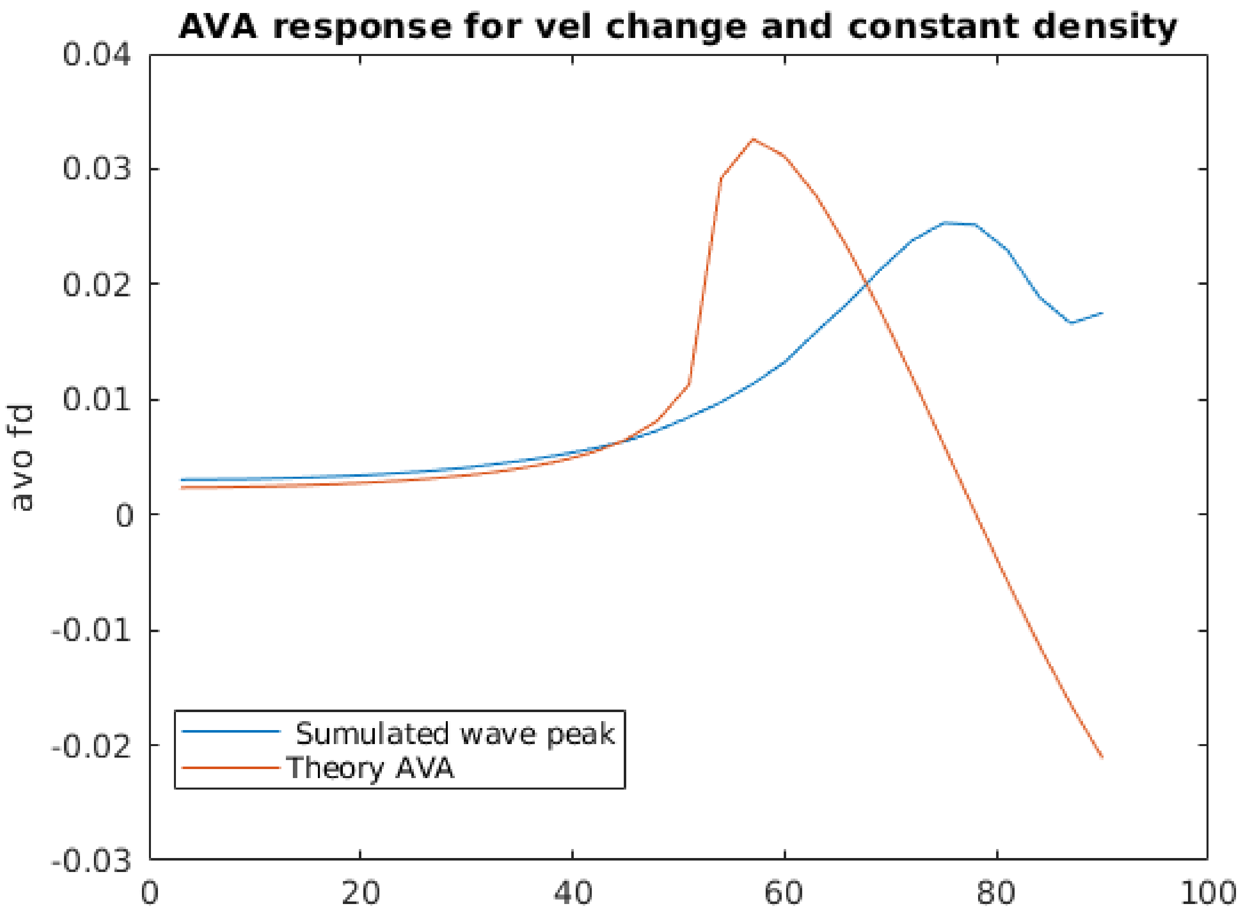
Vel1=1500; vel2=2000;

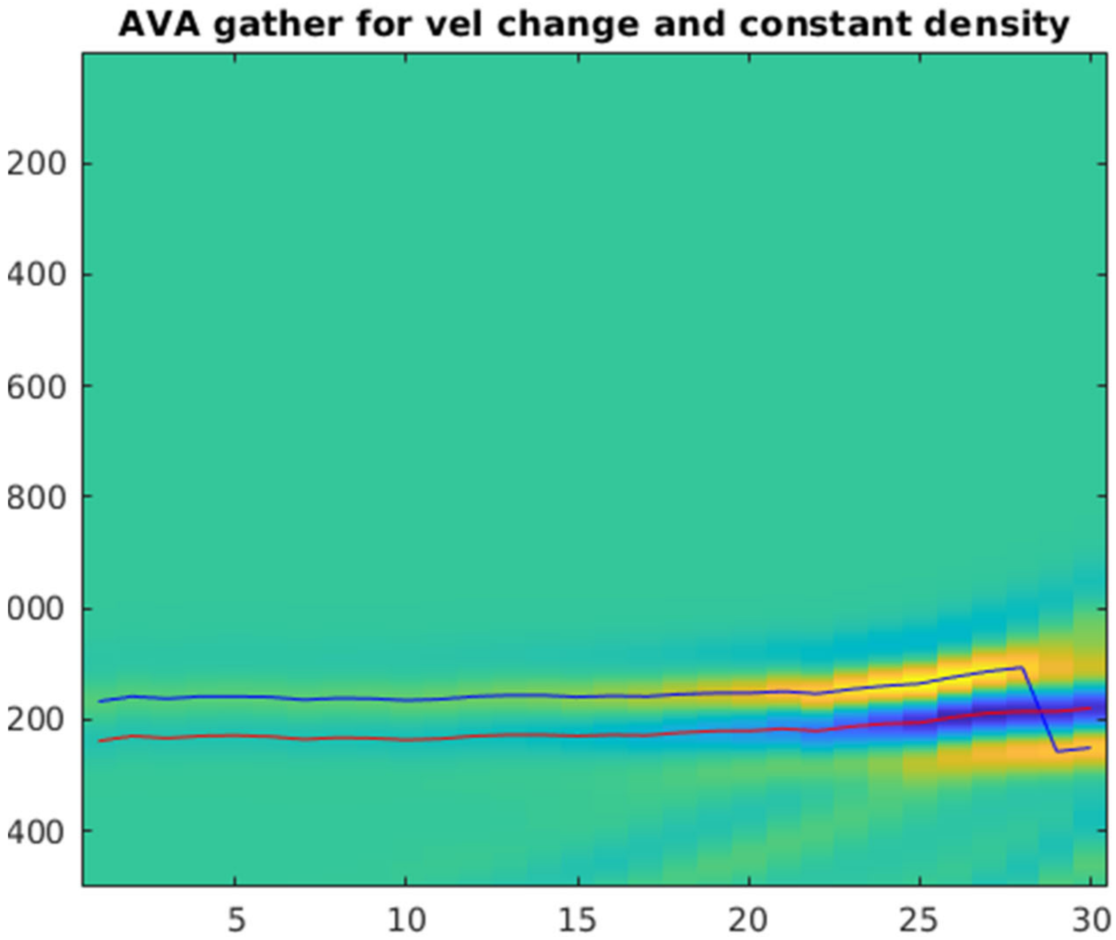


Angle domain FWI

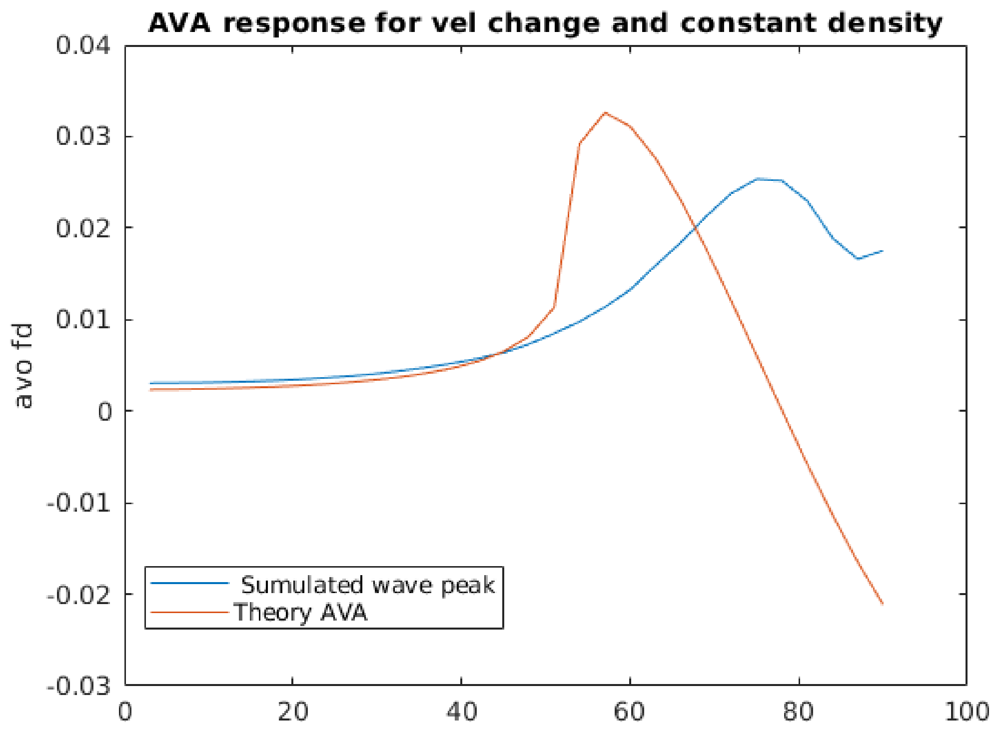


Angle domain FWI

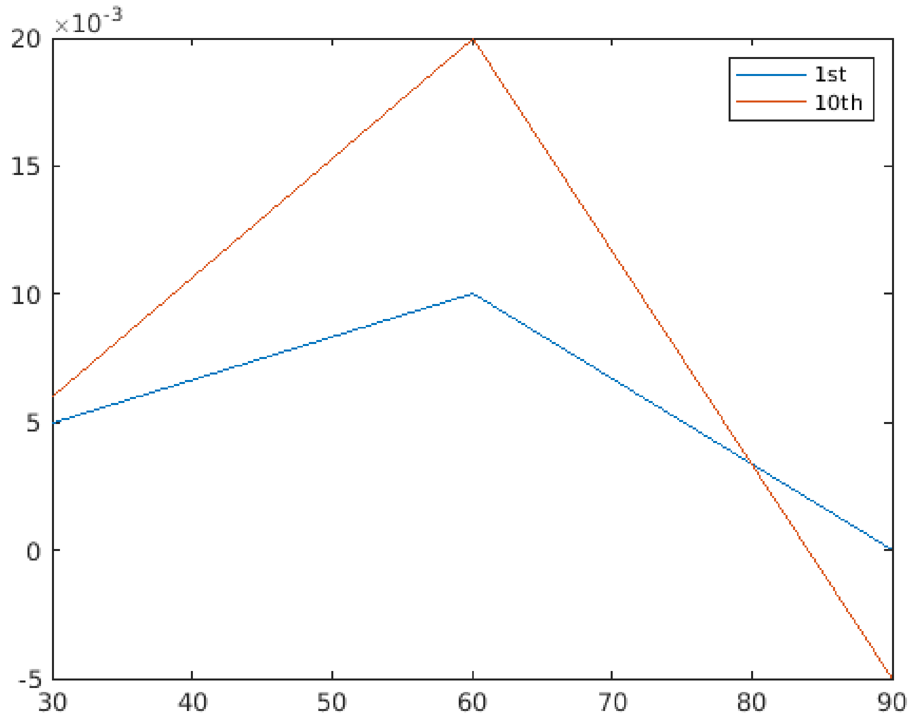


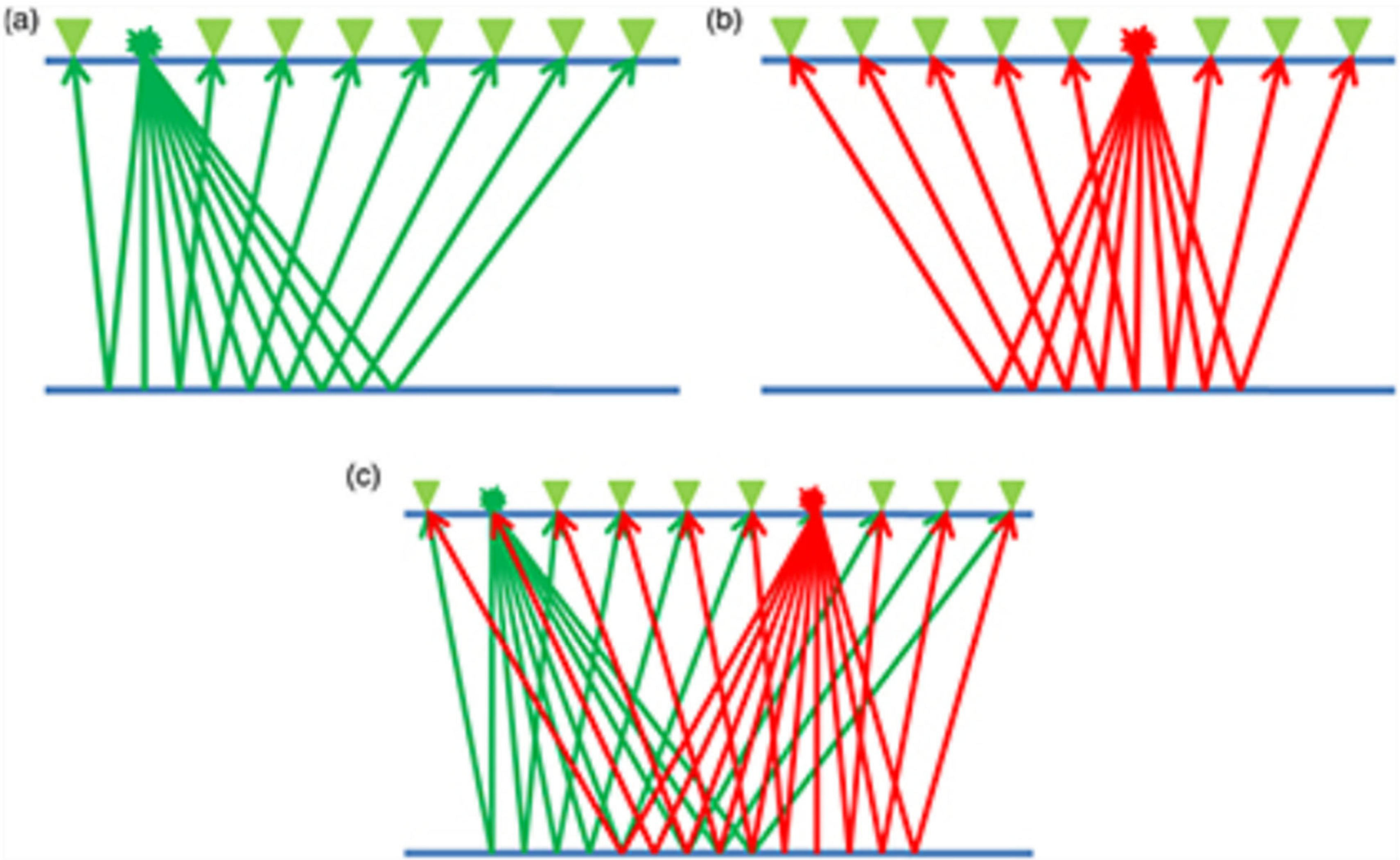


Angle domain FWI



AVA response from FWI in 0-30, 30-60, and 60-90 degree

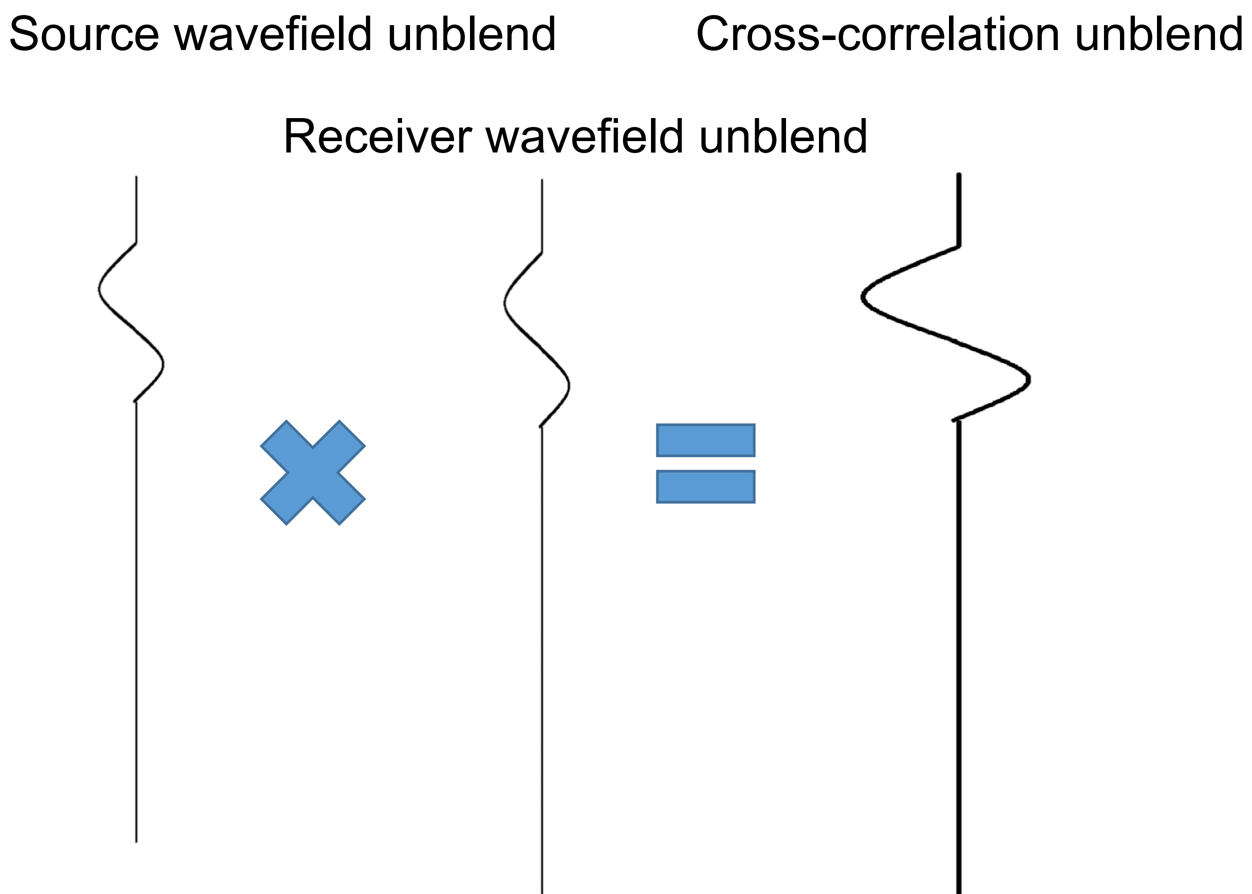



(a)
(b)
(c)

Blended RTM & ADCIGs

How to solve RTM?

The image condition/IC in conventional RTM is the cross-correlation in the image point containing all reflection angles for unblended acquisition.



$$\text{image}(x,z) = \sum_{\text{time}} S(x,z,t)R(x,z,t),$$

Claerbout, 1971; Kaelin and Guitton, 2006

Blended RTM & ADCIGs

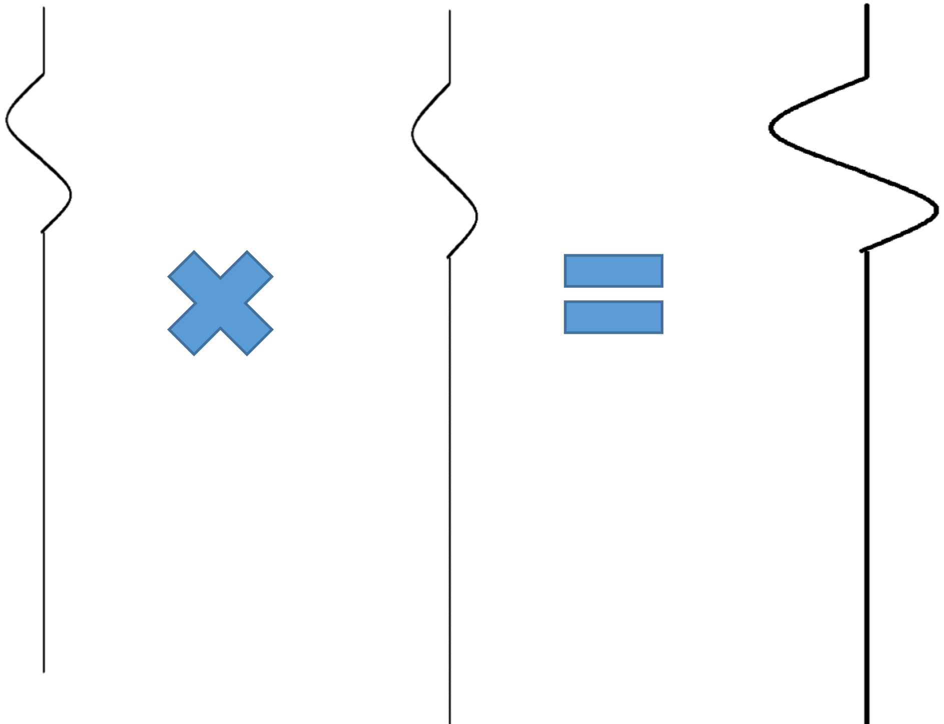
The deconvolution IC can be obtained by normalizing the square of the source illumination strength.

It contains all reflection angles for unblended acquisition.

Source wavefield unblend

Cross-correlation unblend

Receiver wavefield unblend



$$R(\vec{x}) = \int p_B(\vec{x};t)p_F^{-1}(\vec{x};t)dt$$

$$\text{image}(x,z) = \frac{\sum_{\text{time}} S(x,z,t)\, R(x,z,t)}{\sum_{\text{time}} S^2(x,z,t)},$$

Claerbout, 1971; Kaelin and Guitton, 2006

Blended RTM & ADCIGs

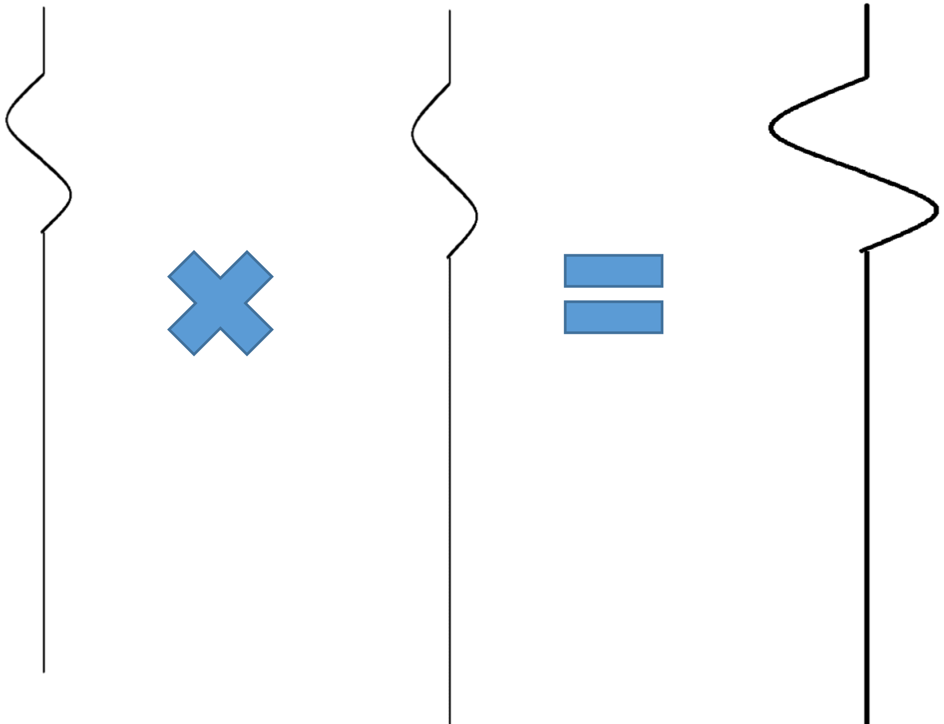
The ADCIG IC adds the angle dimension to the output

It contains the given reflection angles for unblended acquisition.

Source wavefield unblend

Cross-correlation unblend

Receiver wavefield unblend



$$R(\overrightarrow{x}) = \int p_B(\overrightarrow{x};t)p_F^{-1}(\overrightarrow{x};t)dt$$

$$R(\overrightarrow{x},\theta) = \int p_B(\overrightarrow{x},\theta;t)p_F^{-1}(\overrightarrow{x},\theta;t)dt$$

Claerbout, 1971; Kaelin and Guitton, 2006

Blended acquisition in ADCIG

For a given reflection angle at an image point, reflectivity coefficient refl(θ) is constant, reflection amplitude R is proportional to the incident amplitude S.

R= refl(θ) $\times S$

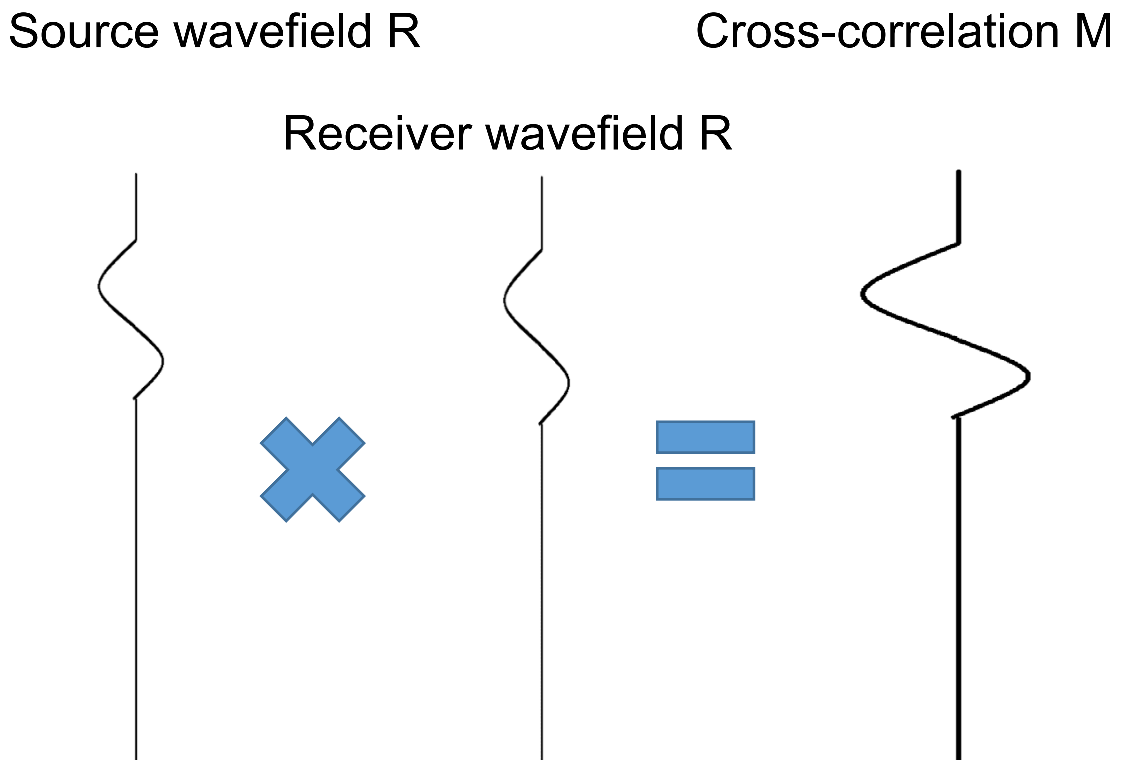
For a given reflection angle at an image point, that reflection may happens more than once, especially in blended data.

S1, S2, S3 … Si

R1, R2, R3 … Rj

Blended acquisition in ADCIG

For a given reflection angle at an image point, reflection amplitude is proportional to the incident amplitude.
For a given reflection angle at an image point, that reflection may happens more than once, especially in blended data.



Conventional cross-correlation IC is the sum of all IC values. Its output amplitude will be affected by subsurface fold/illumination.

Blended acquisition in ADCIG

For a given reflection angle at an image point, reflectivity coefficient refl(θ) is constant, reflection amplitude R is proportional to the incident amplitude S.

$$R = refl(\theta) \times S$$

For a given reflection angle at an image point, that reflection may happens more than once, especially in blended data.

S1, S2, S3 … Si

R1, R2, R3 … Rj

The largest amplitude in R correspond to largest amplitude in S

Blended acquisition in ADCIG

The largest amplitude in R correspond to largest amplitude in S
Cross-correlation of unmatched source and receiver amplitude will cause crosstalk.

$M = S_i \times R_j$ $\quad$ $i \neq j$ crosstalk

$\quad\quad\quad\quad\quad\;$ $i = j$ target

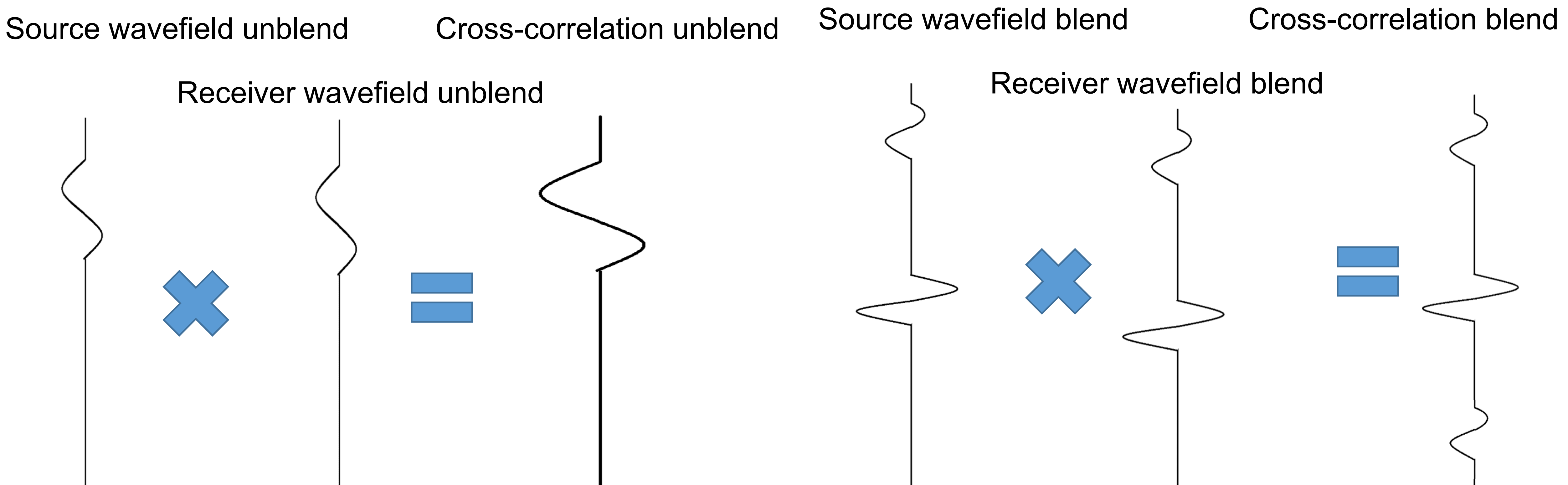
$M \leq Smax \times Rmax$

$Mmax = Smax \times Rmax$

Blended RTM & ADCIGs

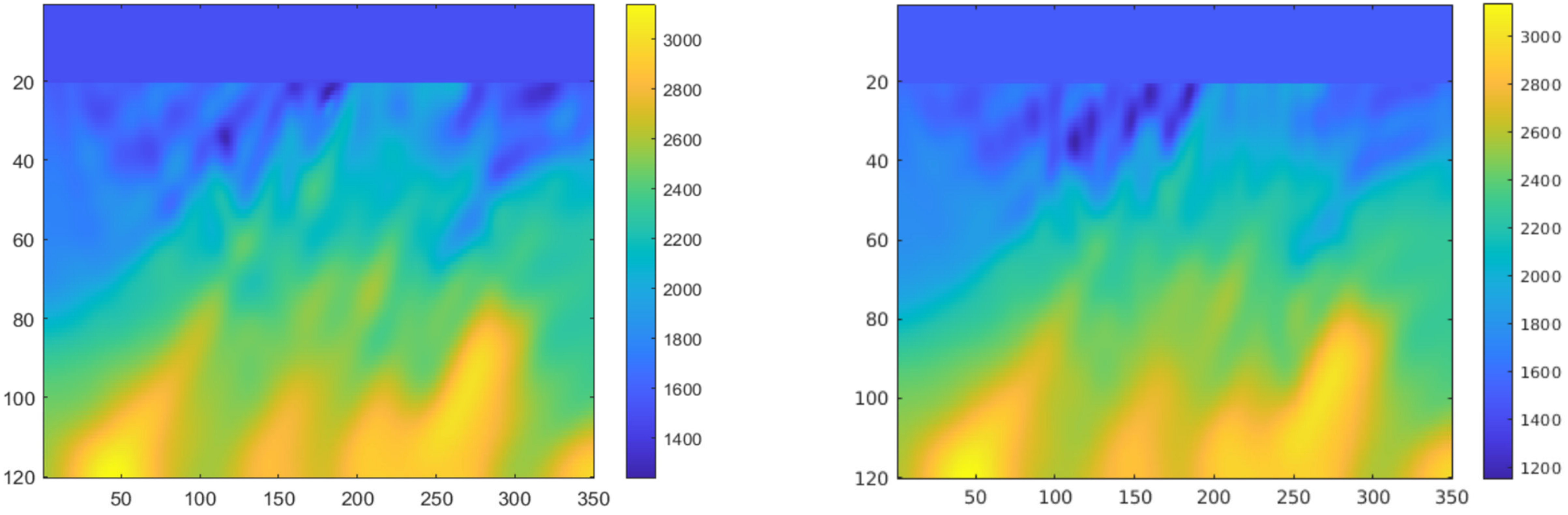
Blended acquisition in ADCIG

For a given reflection angle at an image point, reflection amplitude is proportional to the incident amplitude.
Wrong source-receiver pairs cause crosstalk, <u>yet the maximum value in the cross-correlation always indicates the right source and receiver signal pair.</u>



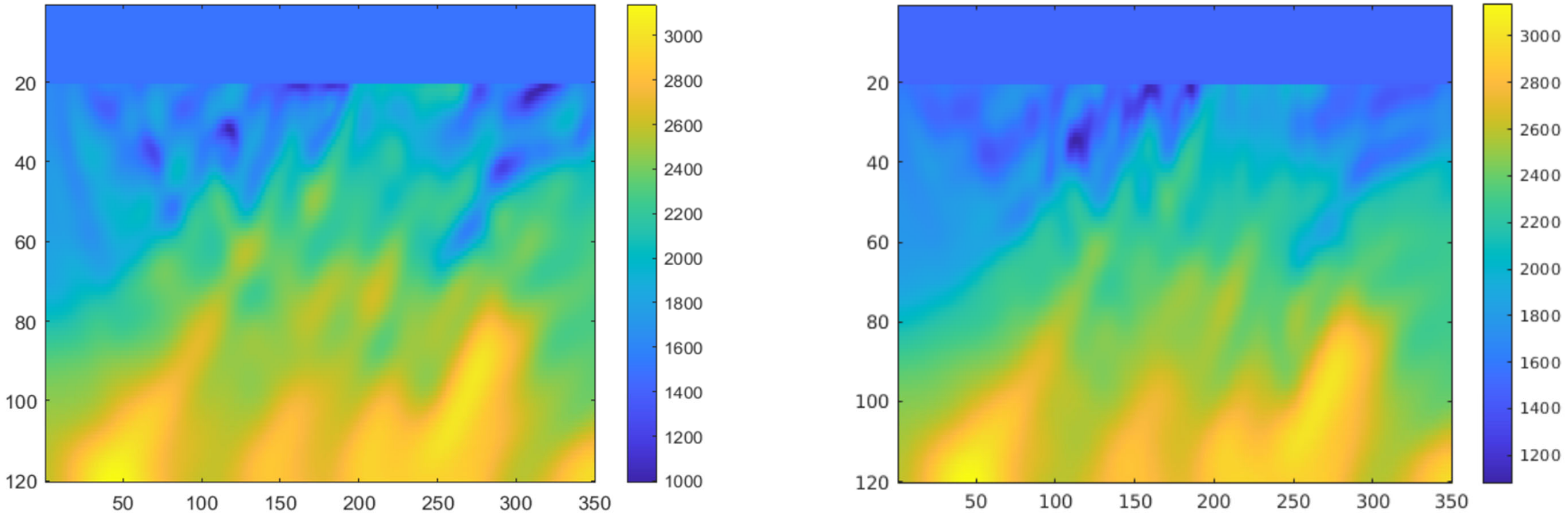
FWI from blended and unblended data

Updated velocity model, iteration 10



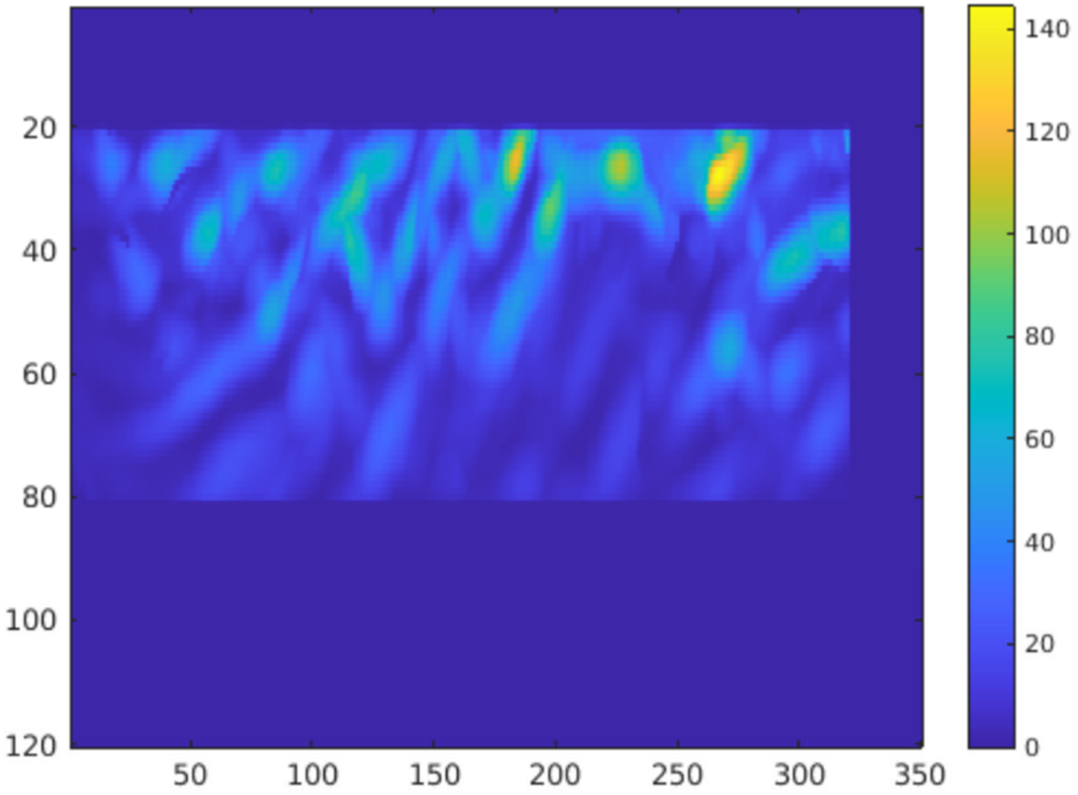
FWI from blended and unblended data

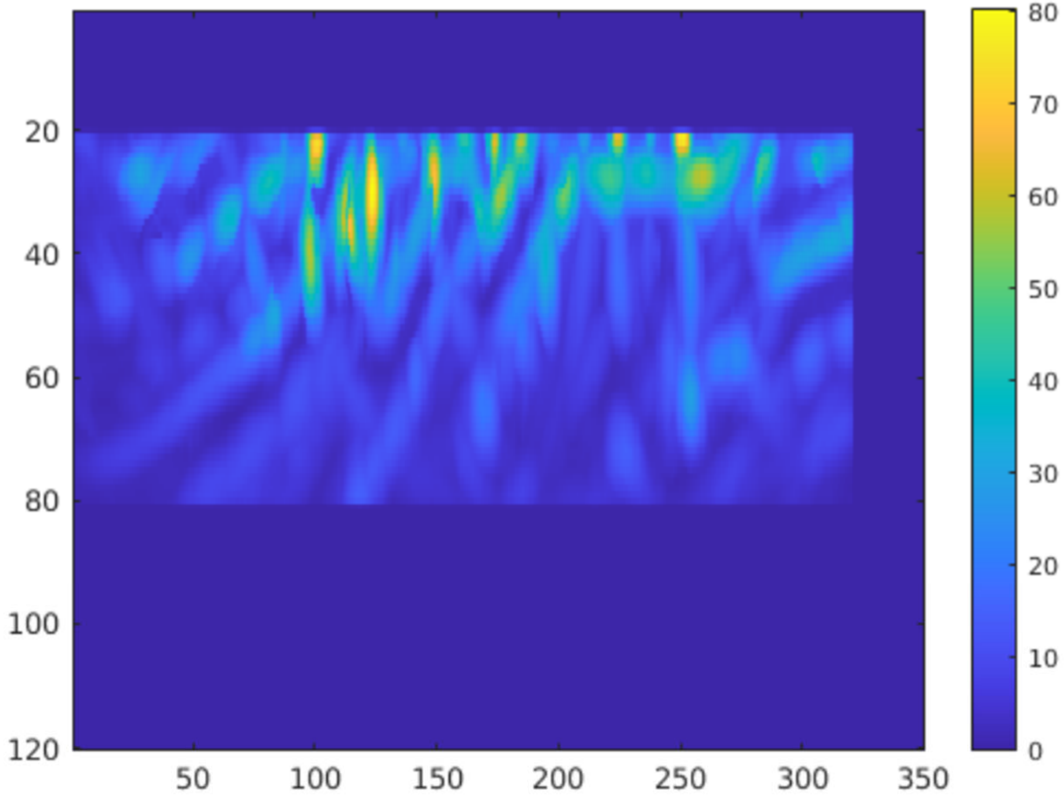
Updated velocity model, iteration 20



FWI from blended and unblended data

Gradient 0-30 degree, iter 1





Conclusions

Extract amplitude variation with angle /AVA information from ADCIGs

RTM / two-way wave-equation migration used for ADCIGs extraction.

Wave-equation migration with velocity and reflectivity updated simultaneously in acoustic media with density

Simultaneous prediction of velocity and angle-dependent reflectivity in time domain FWI for blended data

Acknowledgement

Thanks for the help with He Liu, Tianze Zhang, Qi Hu.

We thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 543578-19.

Future work

3D for AVAz

Extend to elastic media.