

Estimation of rock physics properties via FWI of VSP data recorded by accelerometer and fiberoptic sensors

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**NSERC
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FACULTY OF SCIENCE

Department of Earth, Energy, and Environment



**Carbon
Management
Canada**



- **Background**
- Elastic FWI
- Rock physics inversion



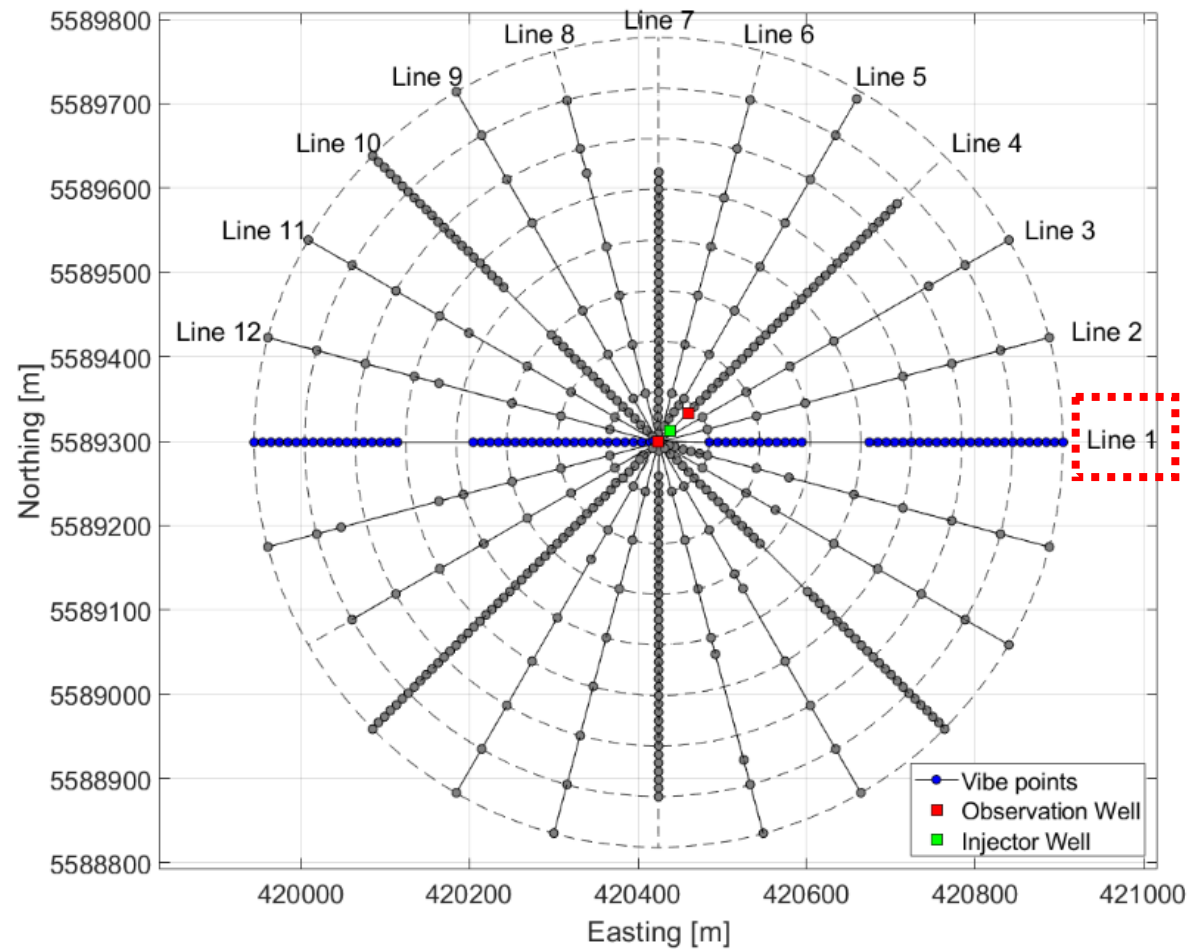
Carbon Management Canada's (CMC) Newell County Facility

A unique Field Research Station for in-field testing to accelerate the development and deployment of technologies to measure, monitor, and verify secure underground storage of CO₂.

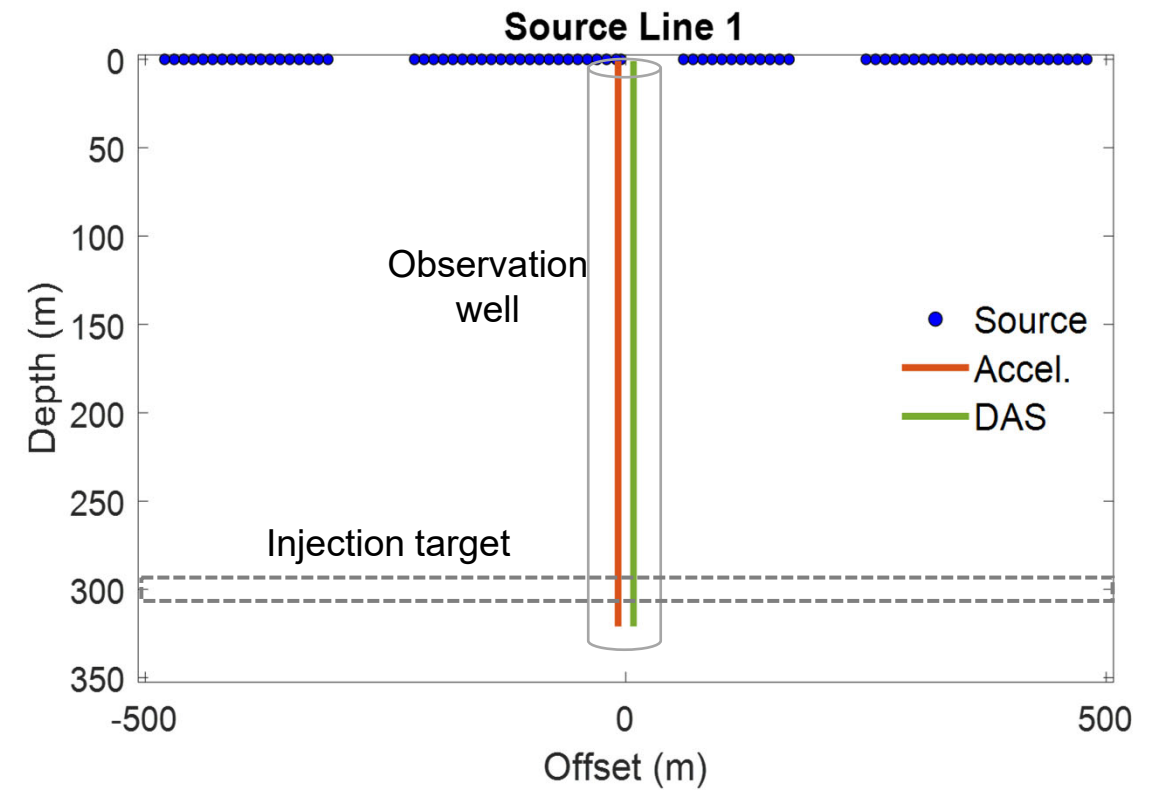




Shot geometry

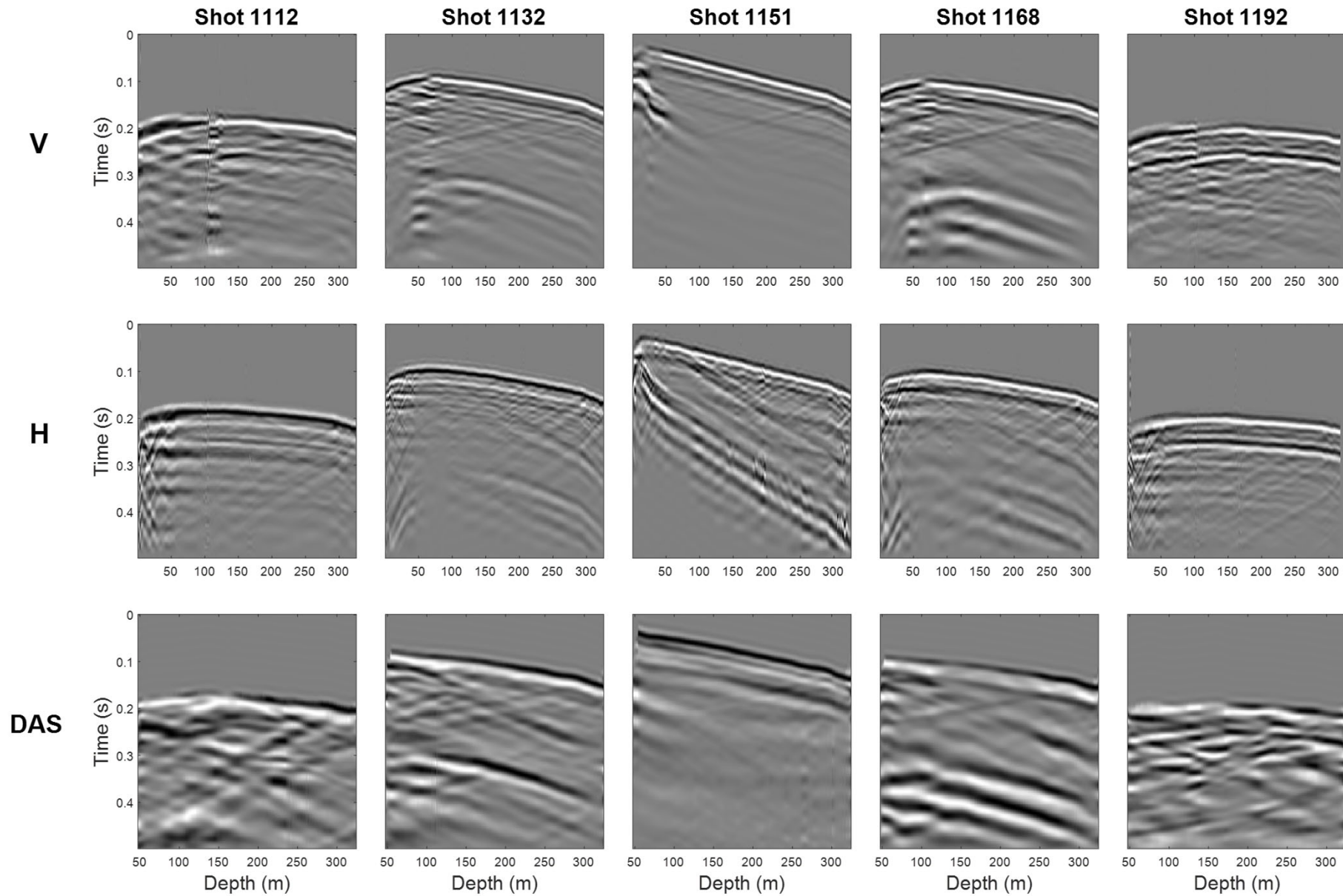


Section view

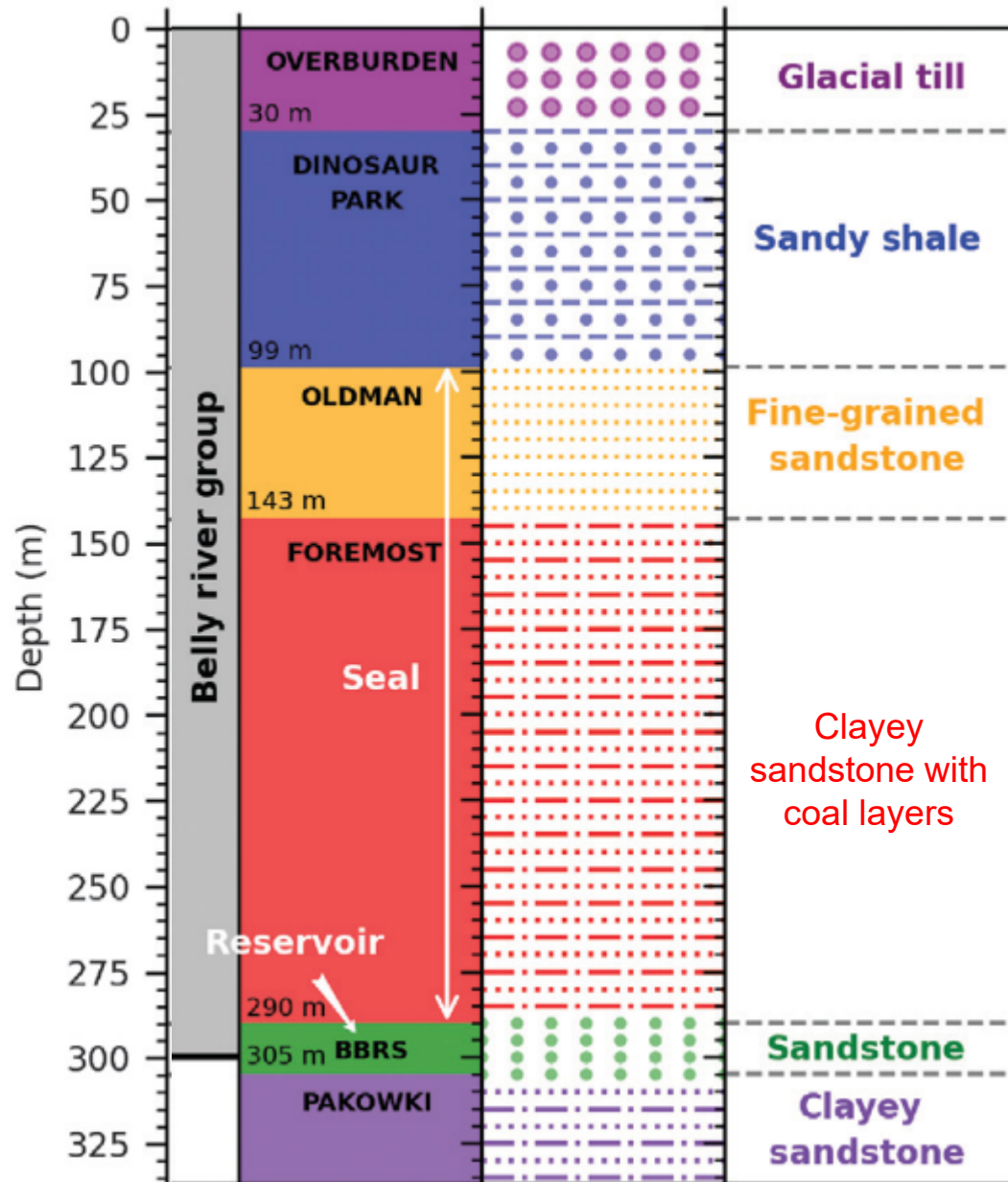




Processed seismic data



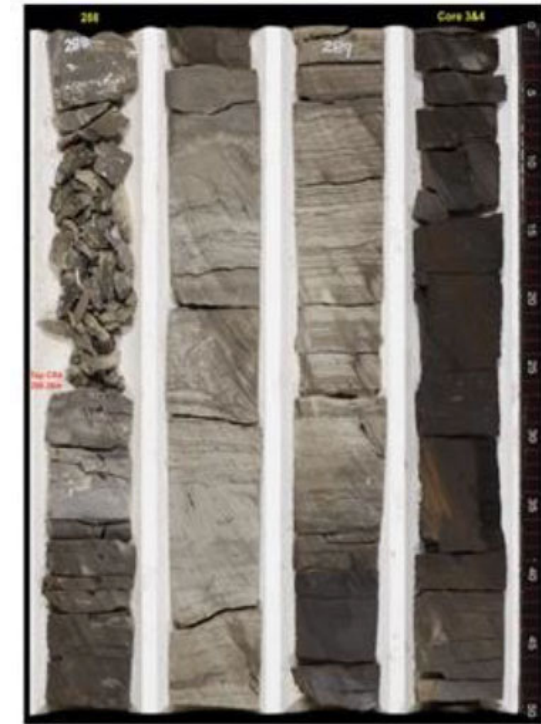
Eaid (2021)



BBRS core (296-298 m)



Caprock (288-290 m)

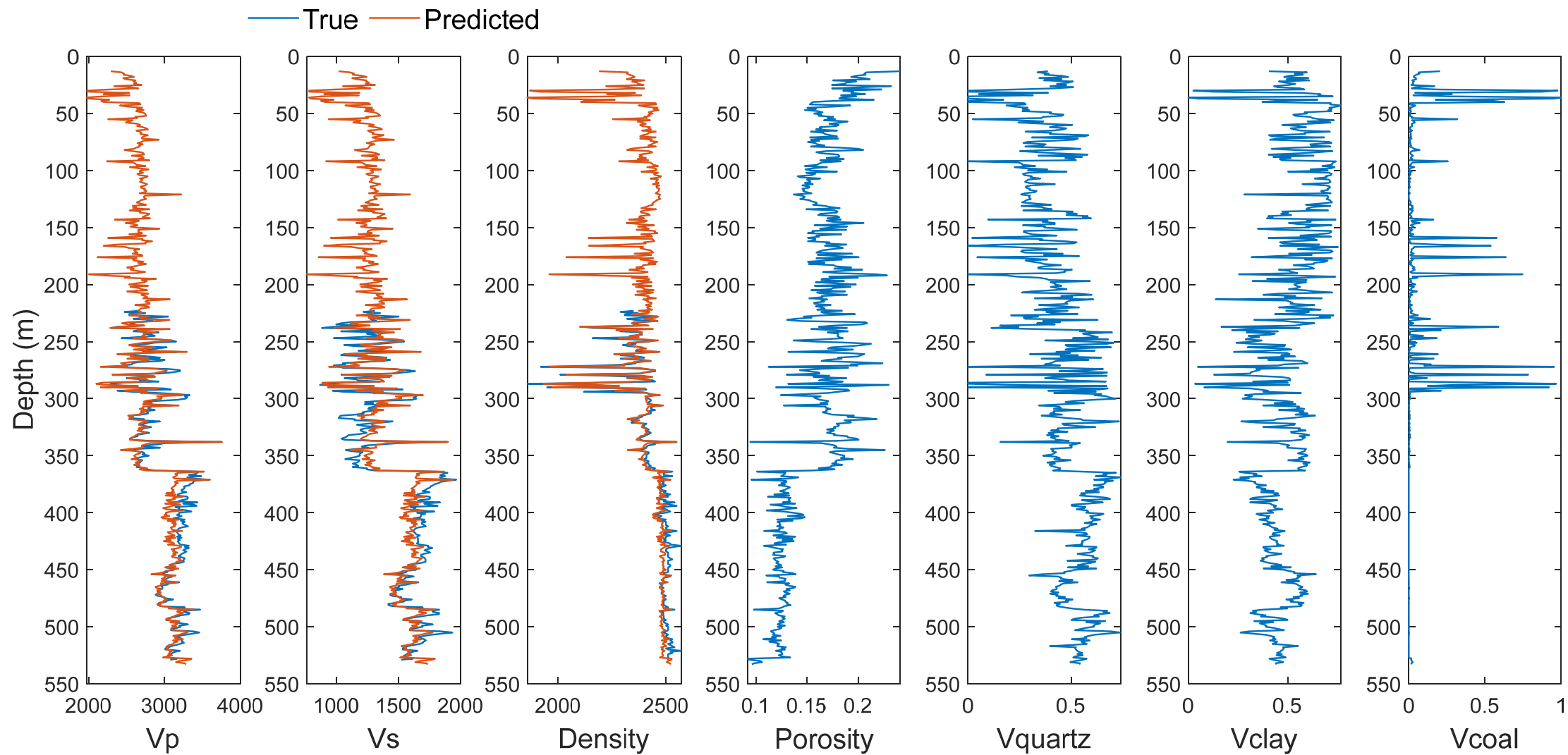


Macquet et al. (2022)

Pan et al (2023)



Well-log data



$$(V_P, V_S, \rho) = g(\phi, V_{qu}, V_{cl})$$

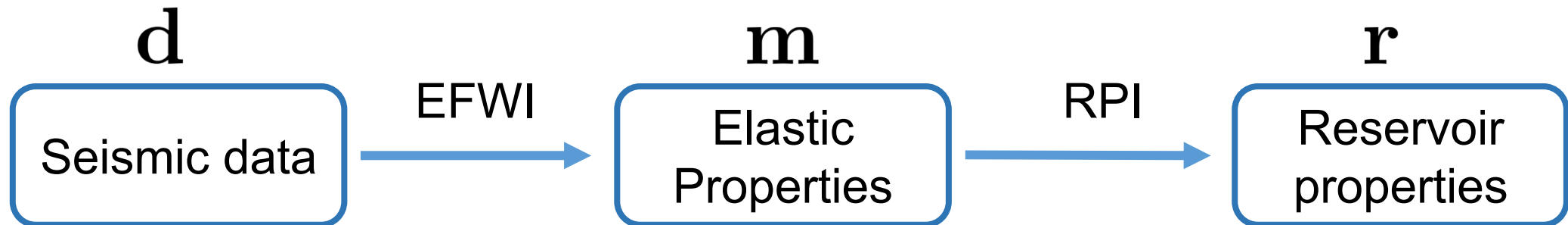


- **General problem**

f : Wave propagation model

g : Rock-physics relation

$$\mathbf{d} = \underset{\substack{\uparrow \\ \text{Elastic properties}}}{f(\mathbf{m})} + \mathbf{n} = \underset{\substack{\uparrow \\ \text{Reservoir properties}}}{f(g(\mathbf{r}))} + \mathbf{n}$$





- Background
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- Constrained optimization problem**

$$\min_{\mathbf{m}} E = \frac{1}{2} \|\mathbf{R}\mathbf{u} - \mathbf{d}\|_2^2 \quad \text{subject to} \quad \mathbf{A}(\mathbf{m})\mathbf{u} = \mathbf{f}(\omega),$$

Receiver matrix Impedance matrix Displacement wavefield

Data Source term

- Search direction**

$$\delta\mathbf{m} = -\mathbf{H}^{-1} \nabla_{\mathbf{m}} E,$$

Hessian Gradient

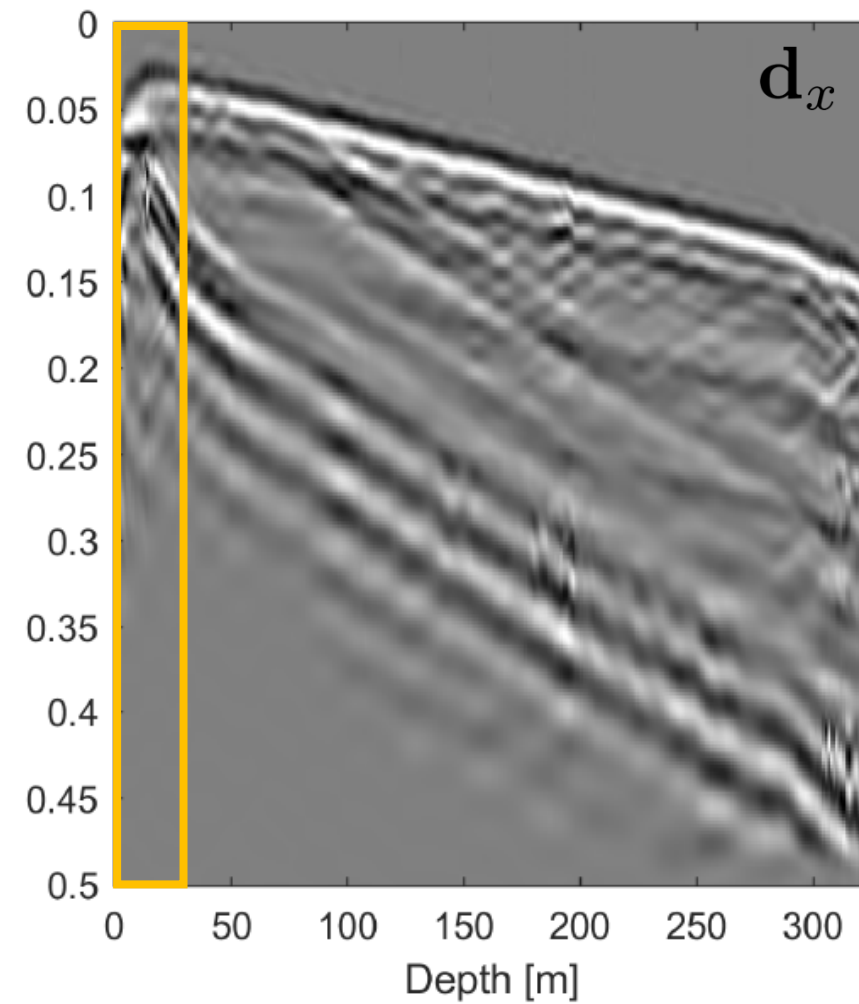
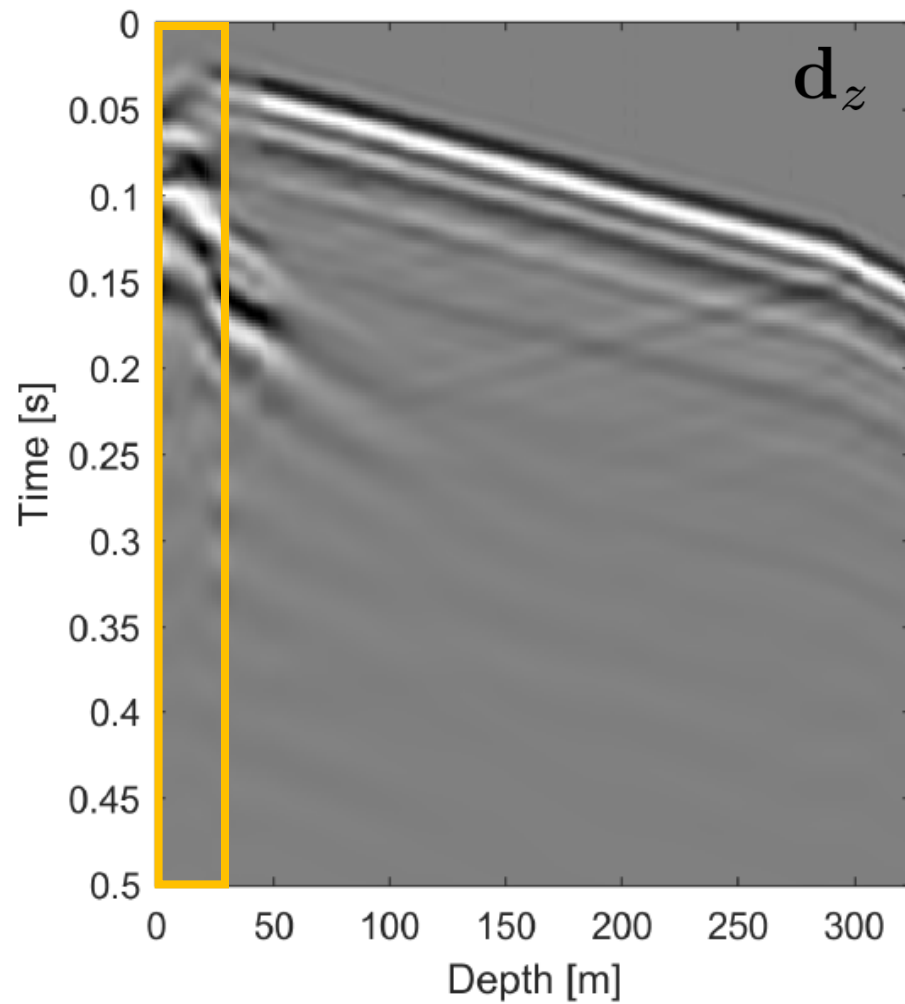
where

$$\nabla_{m_i} E = \Re \left\langle \frac{\partial \mathbf{A}}{\partial m_i} \mathbf{u}, \lambda \right\rangle.$$

Adjoint wavefield



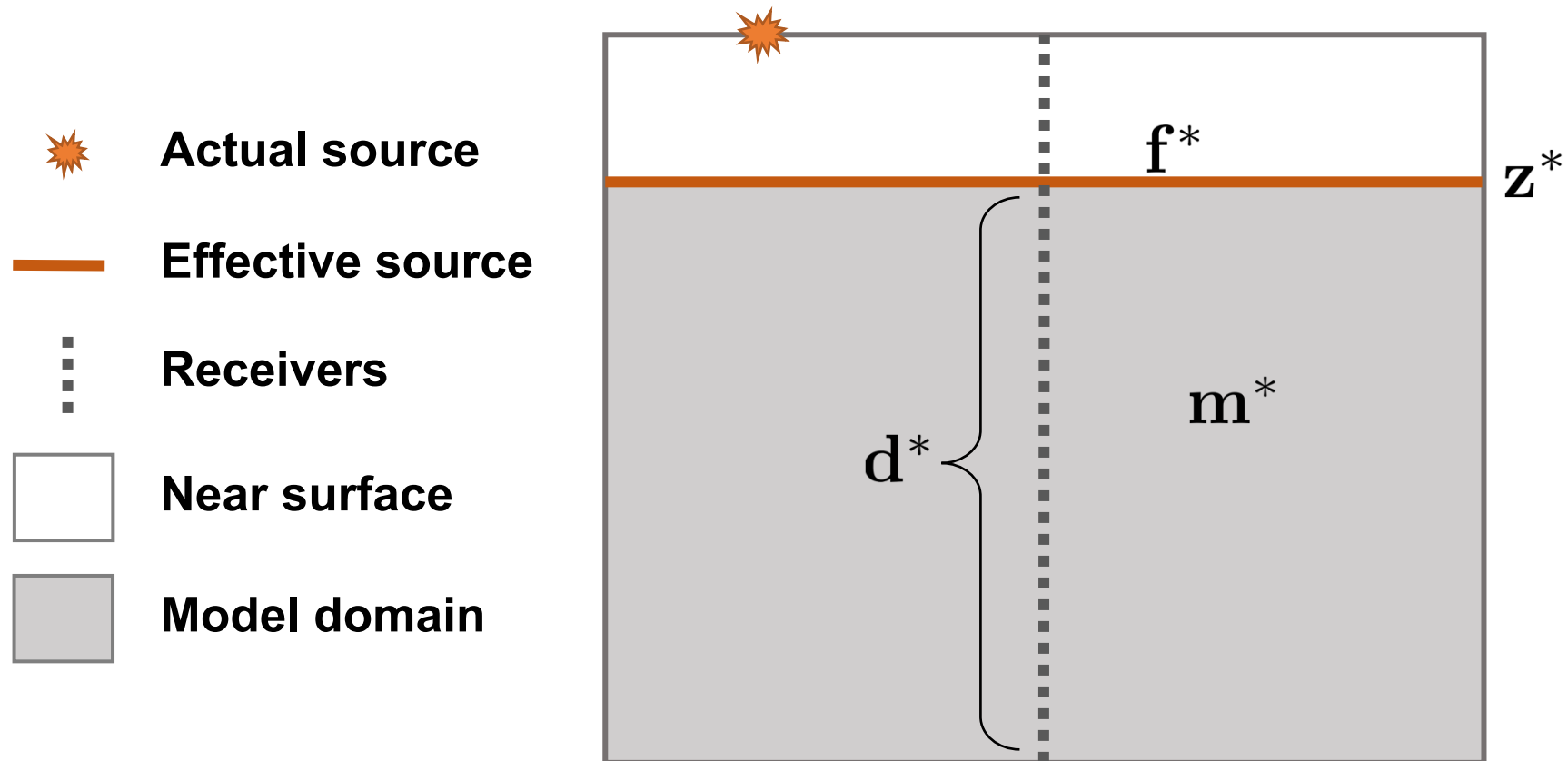
- Near-surface complications





- Effective source approach (*Keating, 2021*)

$$\min_{\mathbf{m}^*, \mathbf{f}^*} E = \frac{1}{2} \|\mathbf{R}^* \mathbf{u}^* - \mathbf{d}^*\|_2^2 \quad \text{subject to} \quad \mathbf{A}^*(\mathbf{m}^*) \mathbf{u}^* = \mathbf{f}^*,$$





- Incomplete nature of the data

Desirable data qualities for FWI	3C Geophone	DAS
Multicomponent	✓	✗
High S/N	✓	✗
Strong low frequency content	✗	✓
Cost effective dense sampling	✗	✓



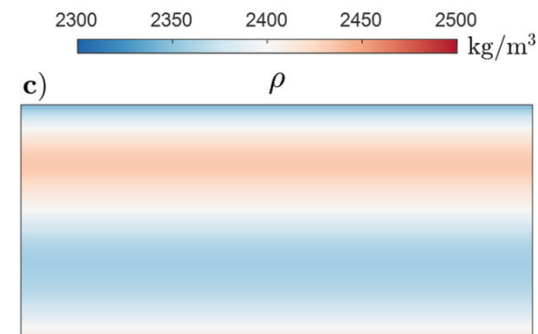
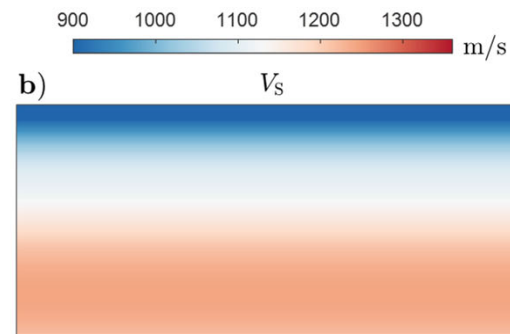
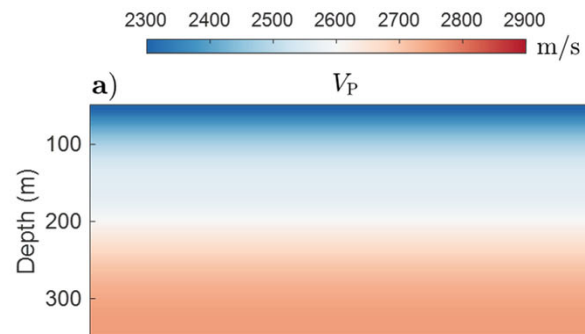
- Modeling strategy to include DAS data in FWI (*Eaid, 2020*)

$$\mathbf{d} = \mathbf{R}\mathbf{u}$$
$$\begin{array}{c} \mathbf{d}_x \\ \mathbf{d}_z \\ \mathbf{e}_t \end{array} \begin{bmatrix} d_x^1 \\ \vdots \\ d_x^{N_g} \\ \hline d_z^1 \\ \vdots \\ d_z^{N_g} \\ \hline e_t^1 \\ \vdots \\ e_t^{N_d} \end{bmatrix} = \begin{bmatrix} 1 & & & & & \\ \vdots & & & & & \\ & & & & 1 & \\ \hline & & 1 & & & \\ \vdots & & & & & \\ & & & & & 1 \\ \hline & w_1 & w_2 & \dots & w_3 & w_4 \\ \vdots & & & & & \\ & w_1 & w_2 & \dots & w_3 & w_4 \end{bmatrix} \begin{bmatrix} u_x^1 \\ u_z^1 \\ u_x^2 \\ u_z^2 \\ \vdots \\ u_x^N \\ u_z^N \end{bmatrix}$$

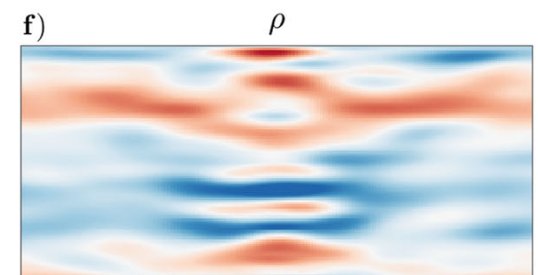
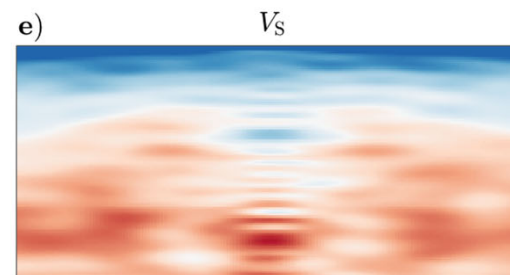
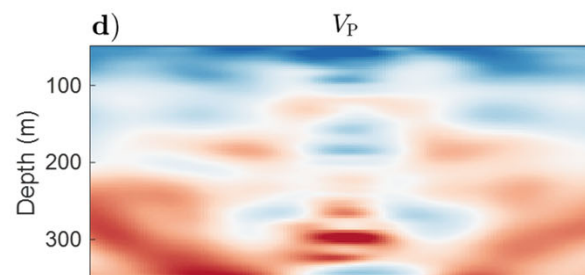


Elastic FWI results

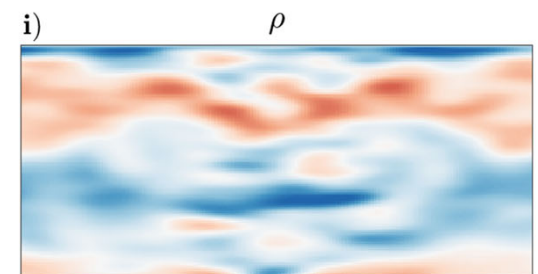
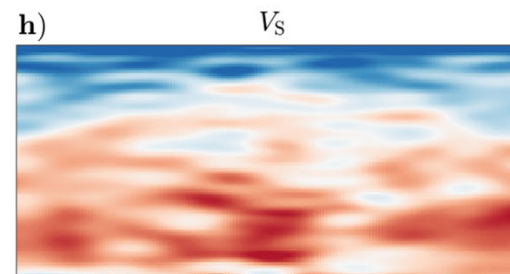
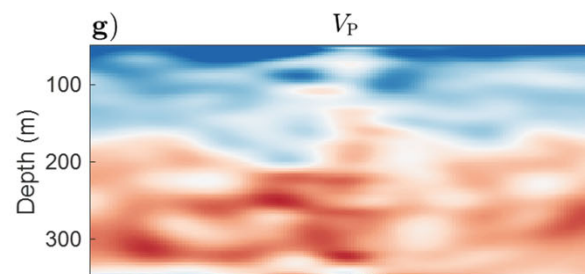
Initial



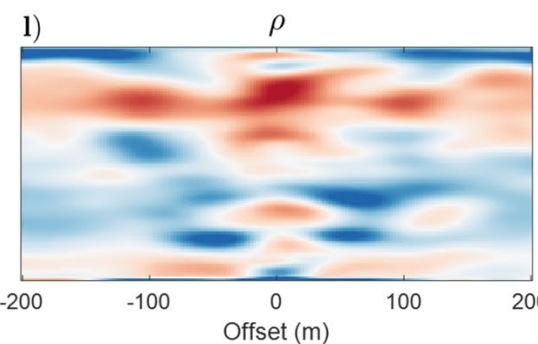
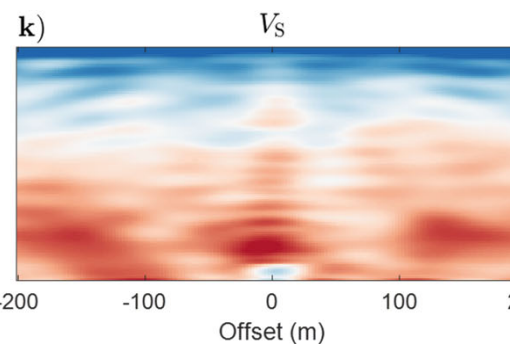
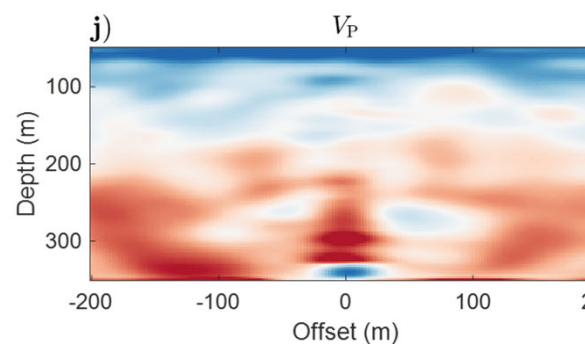
Inverted (Accel.)



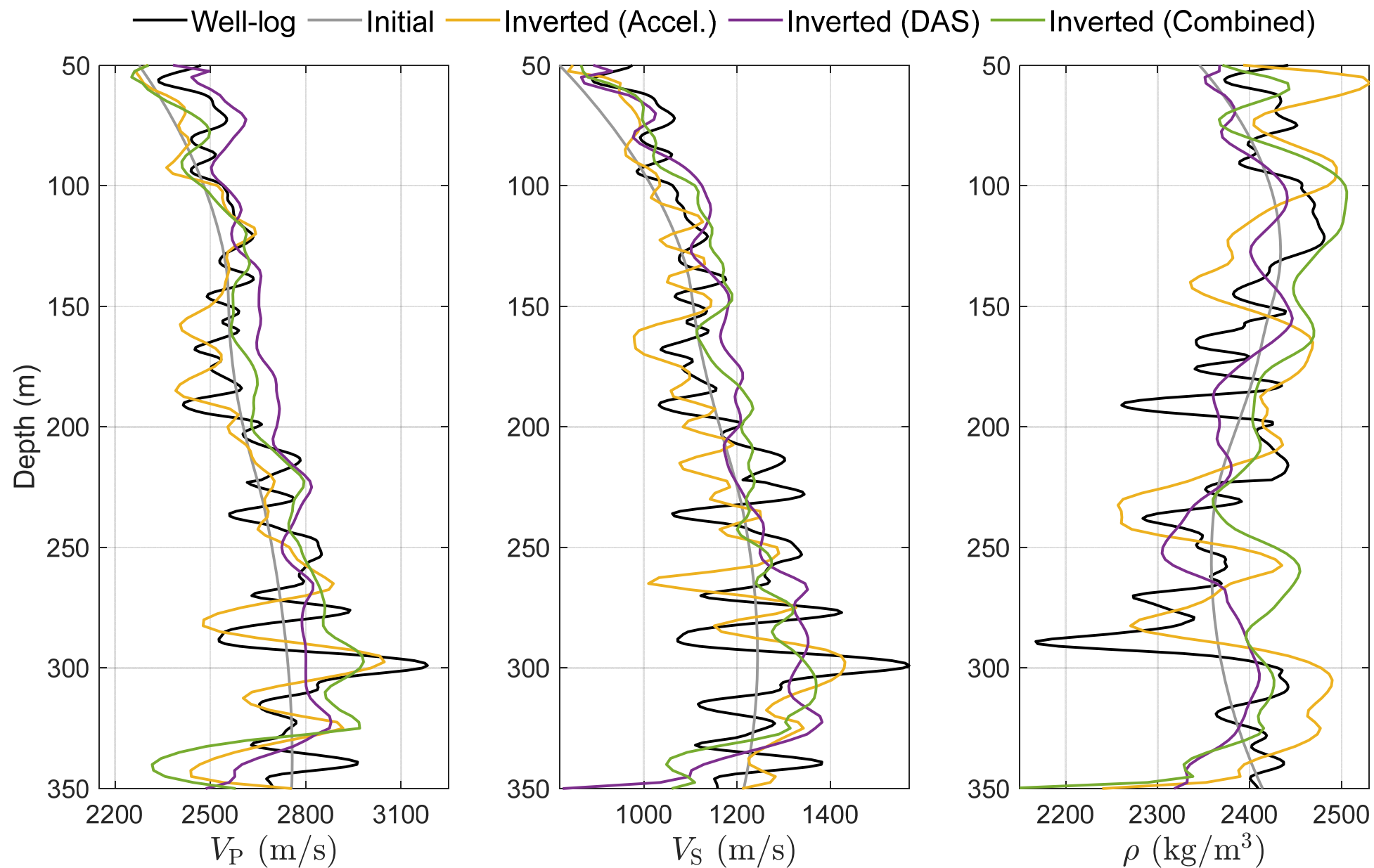
Inverted (DAS)



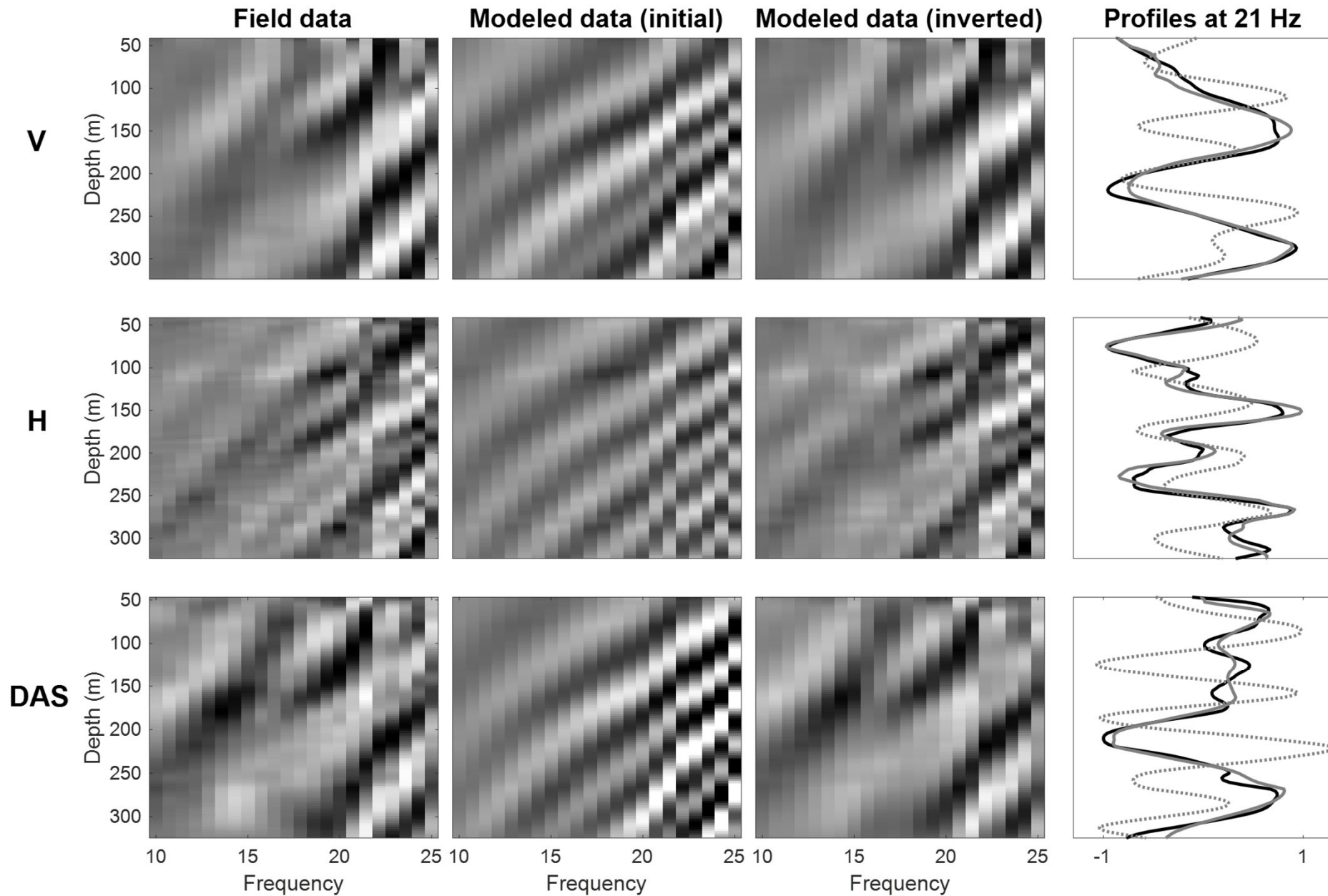
Inverted
(Accel.+DAS)



Model profiles at the well location

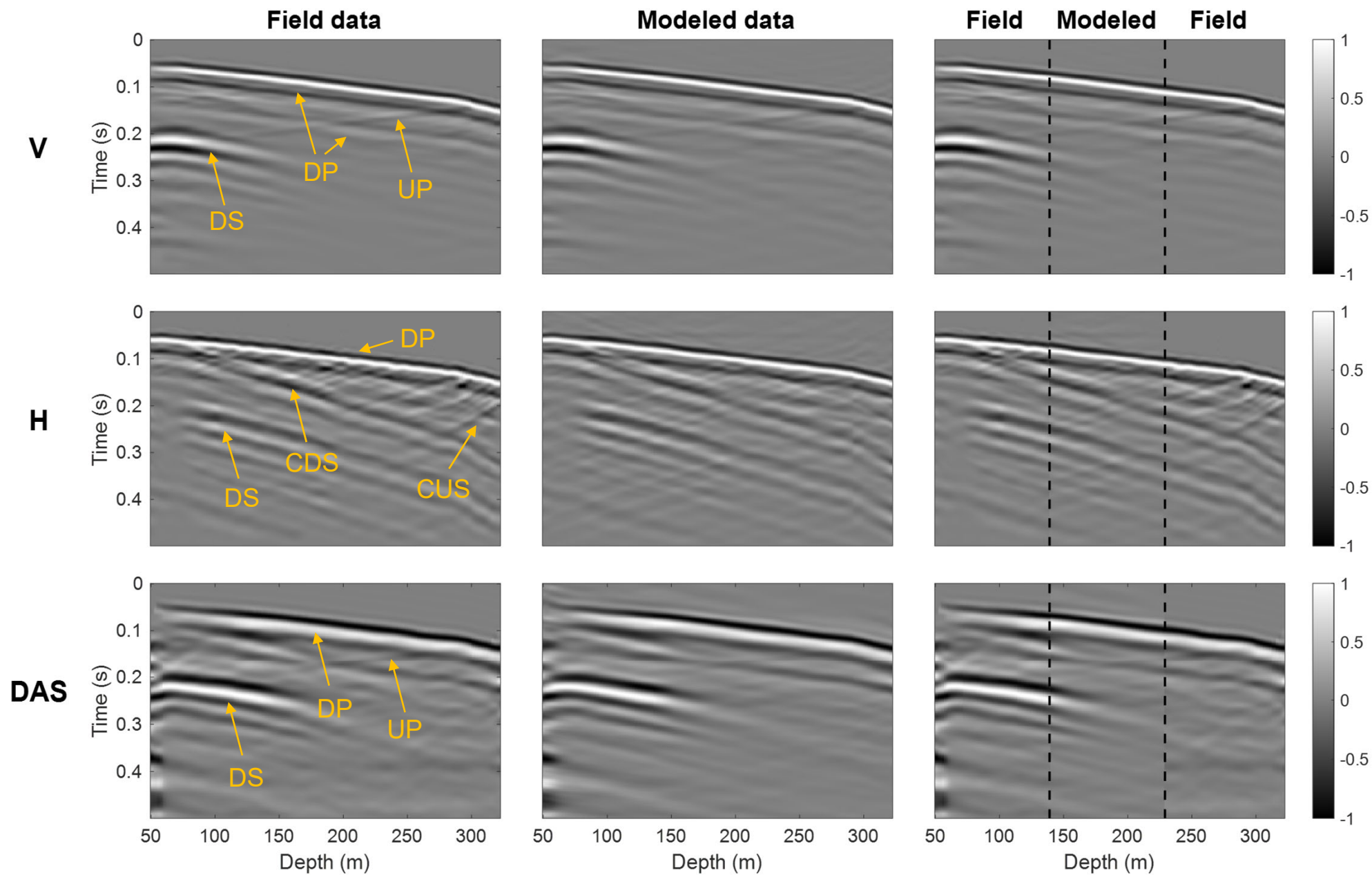


Data fitting (frequency domain)





Data fitting (time domain)





- Background
- Elastic FWI
- **Rock physics inversion**



- General problem

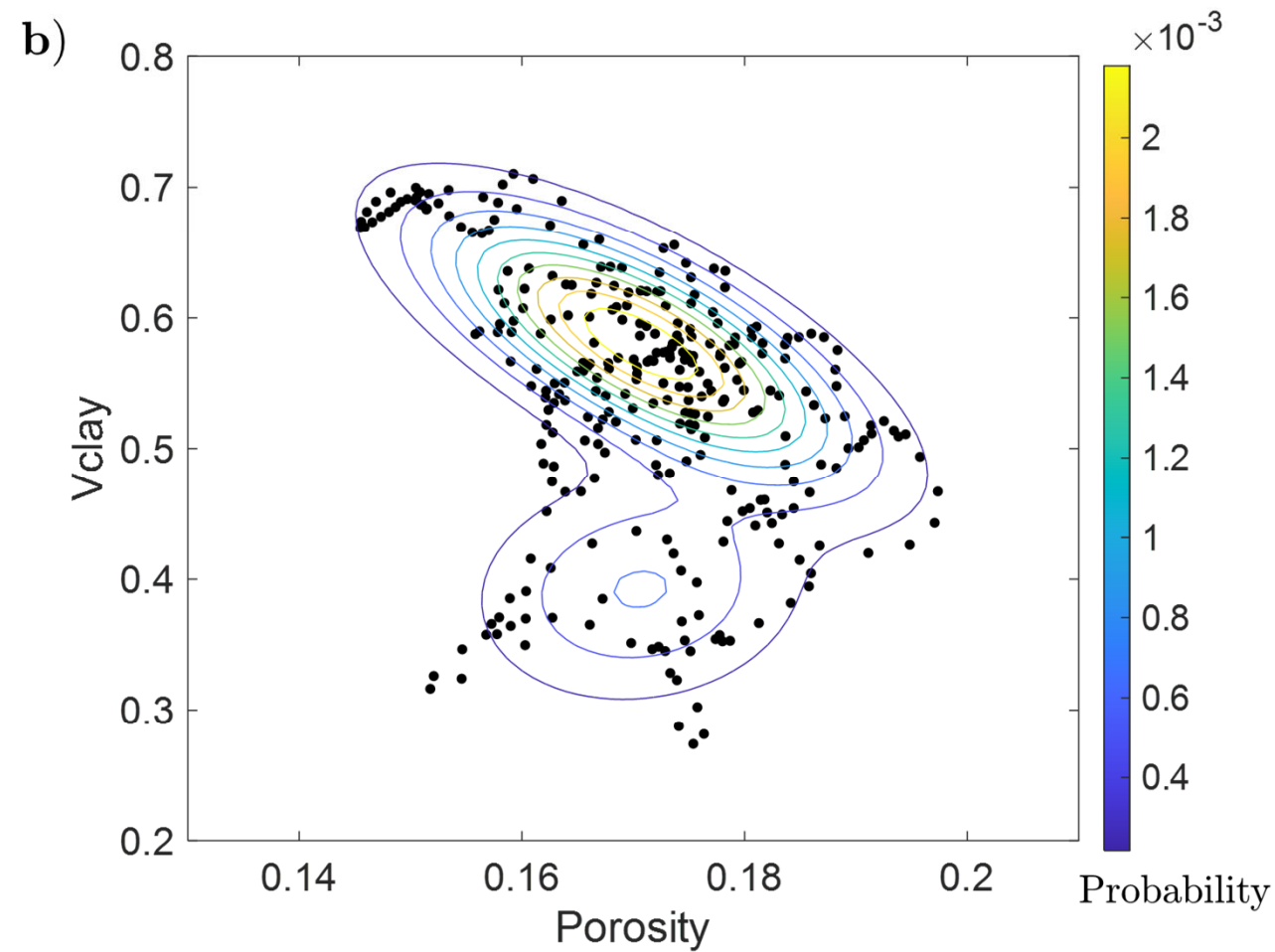
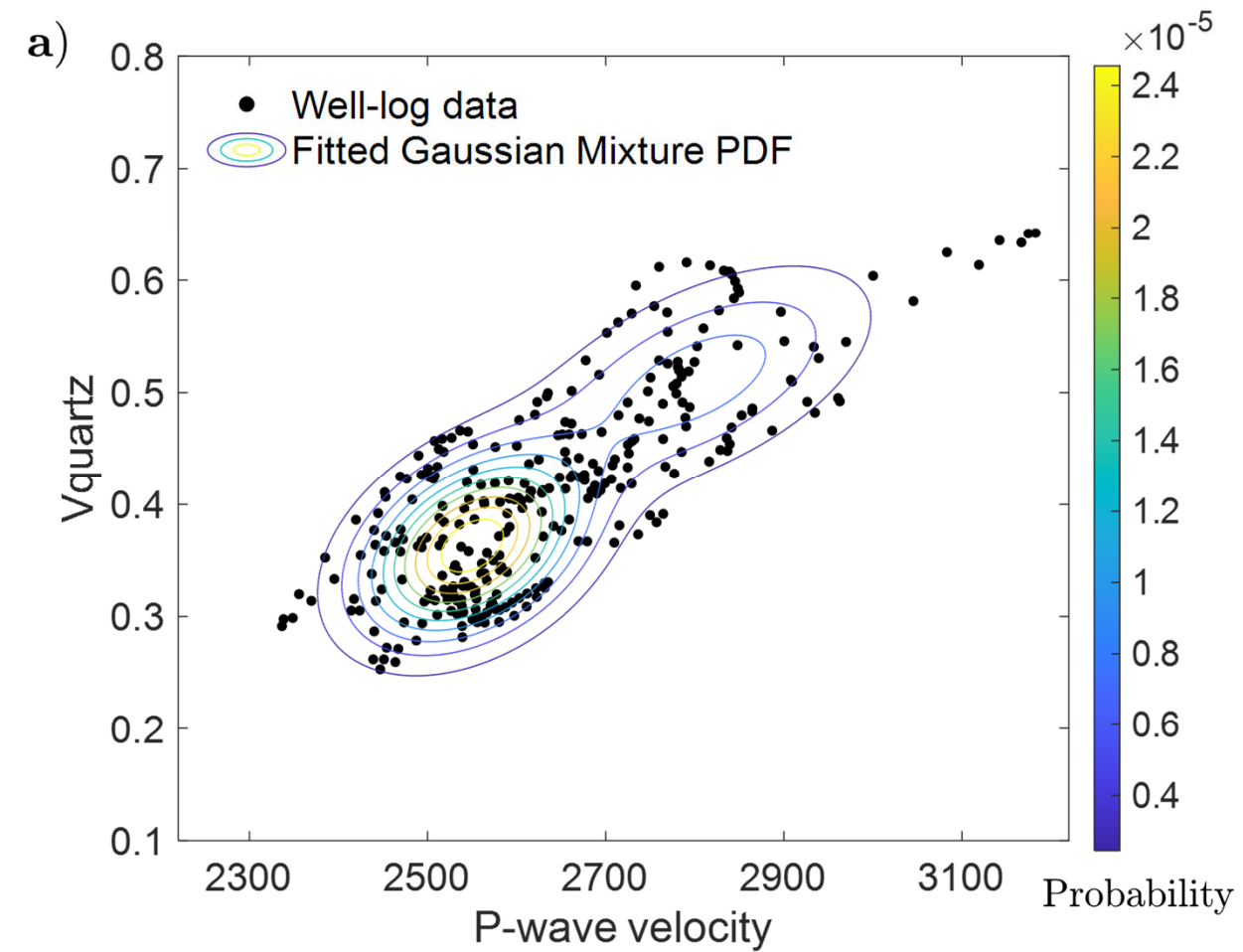
$$\mathbf{m} = g(\mathbf{r}) + \mathbf{e},$$

- Bayesian setting

$$P(\mathbf{r}|\mathbf{m}) = \frac{P(\mathbf{r}, \mathbf{m})}{P(\mathbf{m})} = \frac{P(\mathbf{m}|\mathbf{r})P(\mathbf{r})}{P(\mathbf{m})},$$



Rock physics inversion





- Gaussian mixture approach *(Grana and Rossa, 2010)*

Prior :
$$P(\mathbf{r}) = \sum_{k=1}^{N_f} \lambda_k \mathcal{N}(\mathbf{r}; \boldsymbol{\mu}_r^k, \boldsymbol{\Sigma}_r^k),$$

Joint :
$$P(\mathbf{r}, \mathbf{m}) = \sum_{k=1}^{N_f} \lambda_k \mathcal{N}(\mathbf{y}; \boldsymbol{\mu}_y^k, \boldsymbol{\Sigma}_y^k), \quad \text{where } \mathbf{y} = (\mathbf{r}, \mathbf{m}),$$

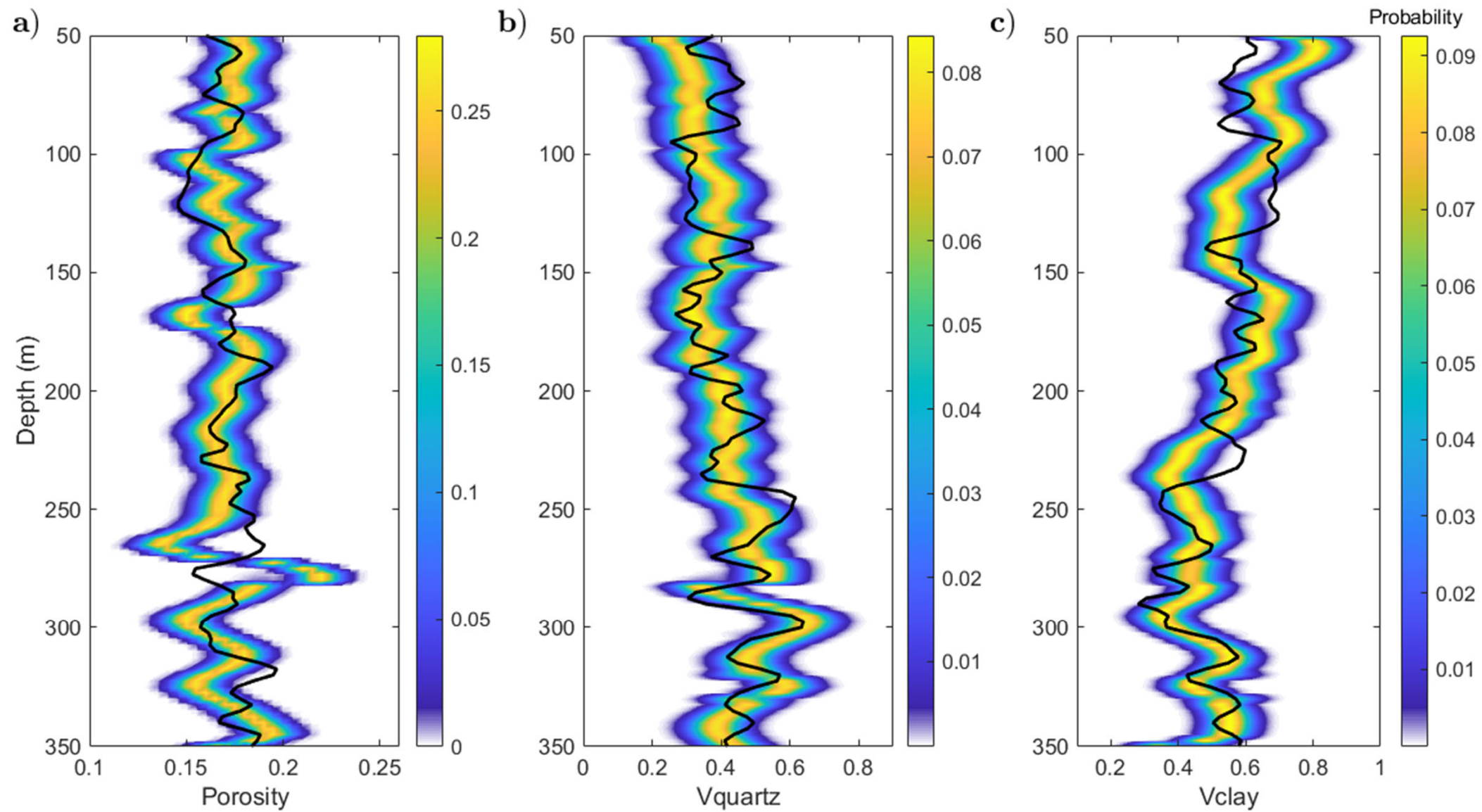
Posterior :
$$P(\mathbf{r}|\mathbf{m}) = \sum_{k=1}^{N_f} \lambda_k' \mathcal{N}(\mathbf{r}; \boldsymbol{\mu}_{r|m}^k, \boldsymbol{\Sigma}_{r|m}^k),$$

with

$$\begin{aligned} \boldsymbol{\mu}_{r|m}^k &= \boldsymbol{\mu}_r^k + \boldsymbol{\Sigma}_{r,m}^k (\boldsymbol{\Sigma}_{m,m}^k)^{-1} (\mathbf{m} - \boldsymbol{\mu}_m^k) \\ \boldsymbol{\Sigma}_{r|m}^k &= \boldsymbol{\Sigma}_{r,r}^k - \boldsymbol{\Sigma}_{r,m}^k (\boldsymbol{\Sigma}_{m,m}^k)^{-1} \boldsymbol{\Sigma}_{m,r}^k. \end{aligned}$$

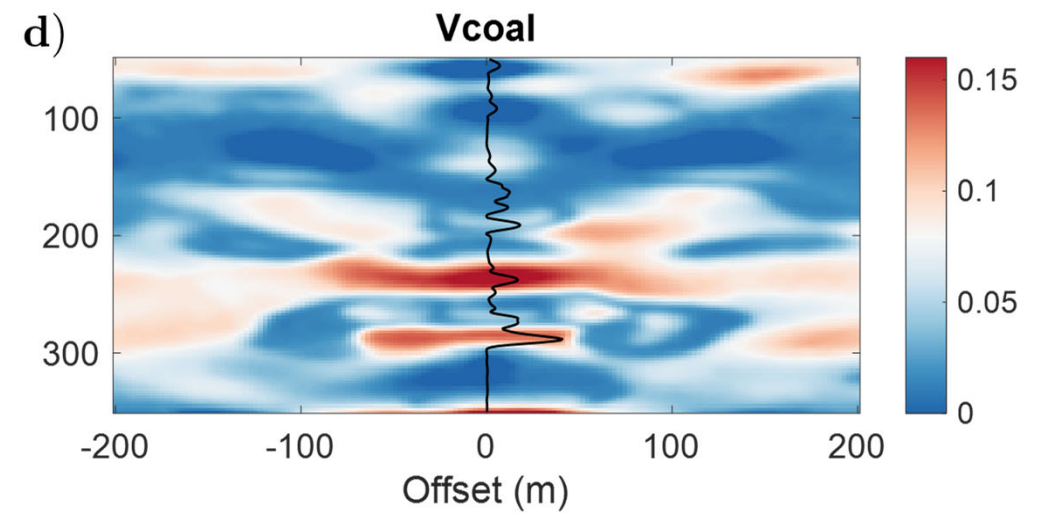
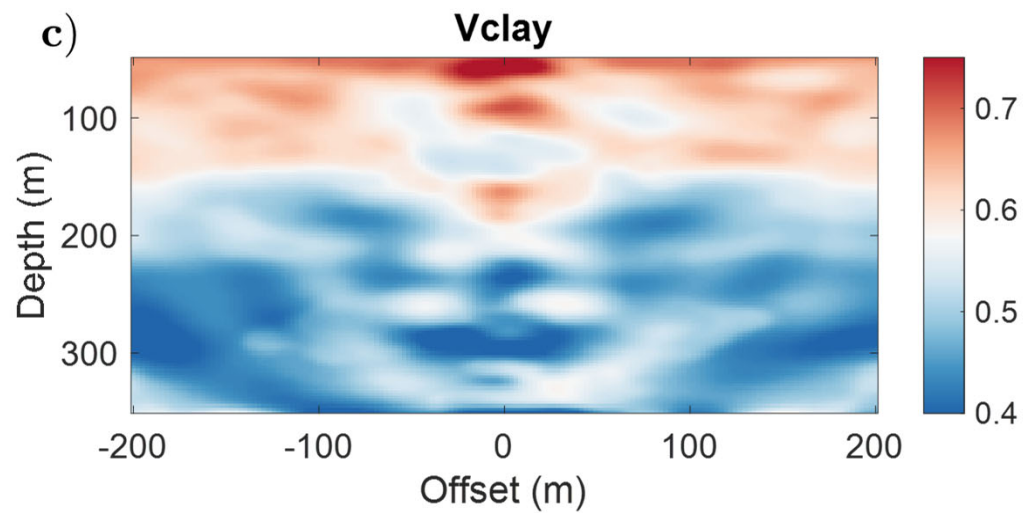
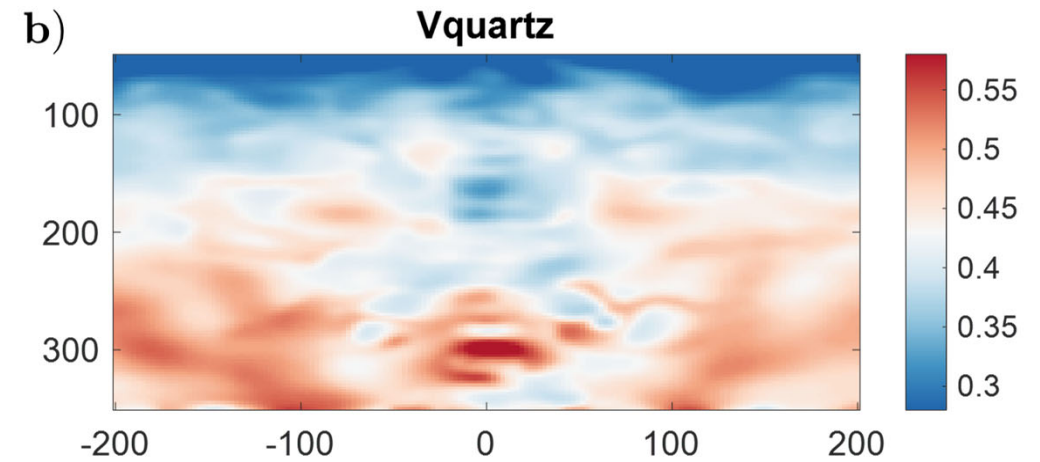
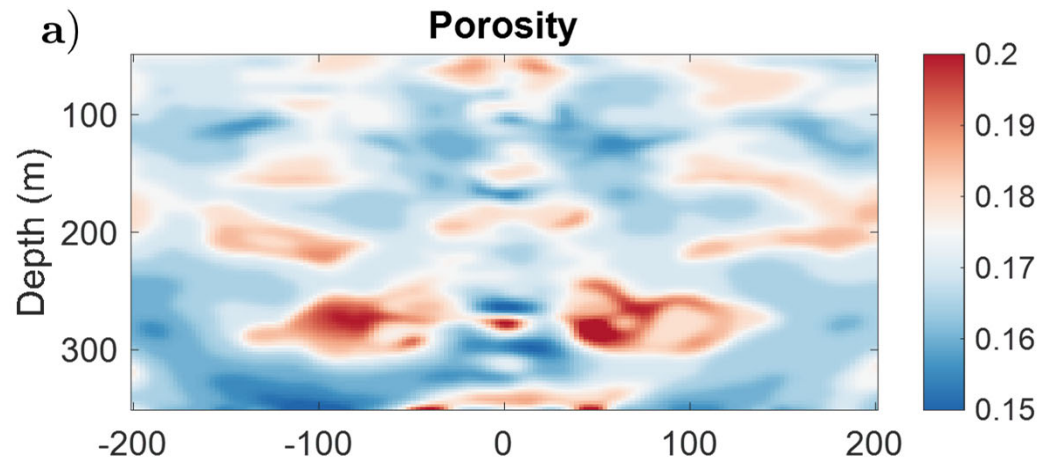


Inversion results at the well location





MAP solution (2D)





- We applied a sequential inversion scheme combining elastic FWI and rock physics to a VSP dataset including accelerometer and DAS measurements.
- Our key strategies include an effective source approach to cope with near-surface complications, a modeling strategy to simulate DAS data, and a Bayesian Gaussian mixture approach to predict rock properties.
- The proposed methodology can be extended to time-lapse analysis, to predict dynamic reservoir properties such as CO₂ saturation and pore pressure.



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