Wavefield extrapolation by nonstationary phase shift
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Summary
Wavefield extrapolation by phase shift is presented as an extension of nonstationary filter theory. We first derive phase shift plus interpolation (pspi) in the limit of continuous lateral velocity variation, and recognize it as equivalent to a linear nonstationary filter which is not the desired superposition of impulse responses (Huygens’ principle). To achieve a linear superposition of impulse responses we derive nonstationary phase shift (nspsi) as a better extrapolation. Both methods are presented in a mixed domain (space-wave number) and in the Fourier domain. We also demonstrate the effect of limiting dip and aperture for nspsi. The inclusion of the aperture limit is a further nonstationary filter and results in perfectly absorbing boundaries.

We then present a simple test scenario which extrapolates a set of impulses through a velocity field with an abrupt lateral velocity variation. This test clearly demonstrates the more desirable performance of nspsi compared to psgi.

Introduction
As stated by Berkhout (1981) nonstationary methods have direct application to wavefield extrapolation. Black et al. (1984) described an f-k wavefield extrapolation technique based on the equivalence of diffraction of optical plane-waves and depth migration. This approach is termed ‘standard’ by Grimbergen et al. (1995). We derive this extrapolation scheme as an extension of nonstationary filter theory as given by Margrave (1997).

Theory
The essence of the psgi method (Gazdag and Sguazzero, 1984) is to build an approximate extrapolation through v(x), say \( \psi(x,z_0+\Delta z,\omega) \), from a set of constant velocity extrapolations, say \( \psi(x,z_0+\Delta z,\omega) \), using a suitable set of reference velocities, \( \{v_j\} \). Each constant velocity extrapolation is done with the phase shift operator:

\[
\alpha(k_x,\omega) = \left\{ \begin{array}{ll}
\exp\left[i\Delta k_{x,j}\right] & \left|k_x\right| \leq \frac{\omega}{v_j} \\
\exp\left[-i\Delta k_{x,j}\right] & \left|k_x\right| > \frac{\omega}{v_j}
\end{array} \right.
\]

where, \( \omega \) is temporal frequency, \( k_x \) is wave number, and \( v_j \) is the \( j \)th reference velocity. The phase shift extrapolation of the Fourier transform \( \Phi(k_x,z_0) \) of the wavefield \( \psi(x,z_0,\omega) \) through \( \Delta z \) is:

\[
\psi(x,z_0+\Delta z,\omega) = \int_{-\infty}^{\infty} \Phi(k_x,z_0,\omega) \alpha(k_x,\omega) \exp[ik_x x] \, dk_x.
\]

(2)

where \( \psi(x,z_0+\Delta z,\omega) \) is the \( j \)th extrapolated wavefield. Linear (in velocity) interpolation (LI) is then used to approximate \( \psi(x,z_0+\Delta z,\omega) \):

\[
\text{LI}\left[ \psi(x,z_0+\Delta z,\omega), \psi_{j+1}(x,z_0+\Delta z,\omega) \right], v_j < v(x) < v_{j+1}
\]

(3)

As the number of reference velocities approaches the number of surface locations equation (3) generalizes to:

\[
\psi(x,z_0+\Delta z,\omega) = \int_{-\infty}^{\infty} \Phi(k_x,z_0,\omega) \alpha(k_x,\omega) \exp[ik_x x] \, dk_x
\]

(4)

where,

\[
\alpha(k_x,\omega) = \left\{ \begin{array}{lr}
\exp\left[i\Delta k_x\right] & \left|k_x\right| \leq \frac{\omega}{v(x)} \\
\exp\left[-i\Delta k_x\right] & \left|k_x\right| > \frac{\omega}{v(x)}
\end{array} \right.
\]

(5)

The definition of \( \alpha(k_x,\omega) \) (equation (5)) allows for infinite integration over \( k_x \) in equation (4), ensuring, evanescent energy will exponentially decay.

We recognize equation (4) as a type of nonstationary filter operation being applied in the mixed (kx,x) domain. Margrave (1997) calls this type of filter a nonstationary combination and shows it to be a linear filtering process which does not form the superposition of impulse responses of the nonstationary filter \( \alpha(k_x,\omega) \).

This mixed domain form, and therefore psgi, is not a linear superposition of impulse responses where in fact such a superposition is desirable (Huygens’ principle). The recognition that psgi becomes a nonstationary combination filter in the continuous limit allows us to deduce the mixed domain form of nonstationary convolution. Margrave (1997) shows that the latter does form the scaled superposition of impulse responses. Space variant convolution, as described by Margrave (1997) in mixed domain form, can be written:

\[
\Phi(k_x,x+\Delta z,\omega) = \int_{-\infty}^{\infty} \alpha(k_x,\omega) \psi(x,z,\omega) \exp[-ik_x x] \, dx.
\]

(6)

We propose equation (6) as the basis for a one way wavefield extrapolation which is superior to the best possible case for psgi. Comparing equations (4) and (6)
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sided and contains only those wavenumbers which cause wave fronting into the zero pad. Moving into the data interior, the operator rapidly changes to nearly symmetric. Since the aperture compensation has completely eliminated wavefronts from the data edge which go back into the data interior, it has the desirable effect of creating a perfectly absorbing boundary. Figure 6b is the corresponding spectral operator.

Figures 7a and 7b are respectively the extrapolated wavefield and \( A(k_x, k_y - k_x', \omega) \) for \( \text{nsps} \) and includes the absorbing boundaries and a 45 degree dip limit.

Conclusions

- In the limit of continuous lateral velocity variation, the \( \text{pspi} \) process can be formulated as a nonstationary combination filter. It has the non-wavelike behavior of preserving discontinuities in \( v(x) \) in the extrapolated solution.
- A superior alternative to \( \text{pspi} \), called \( \text{nsps} \), is easily deduced from nonstationary filtering theory. It is a nonstationary convolution and proceeds through the direct superposition of the impulse responses of the nonstationary wave propagator.
- Both of these processes can be formulated in the mixed \( (k_x, x) \) domain, the full Fourier domain \( (k_x, k_x') \) or the space domain \( (x, x') \) (though we did not present the latter).
- In the Fourier domain \( \text{nsps} \) is computed as a matrix multiplication for each frequency. The spectral propagation matrix is built from the Fourier transform of the phase shift model and maps input wavenumbers to output wavenumbers. In the stationary limit the spectral propagation matrix is diagonal with simple phase shift operators on the diagonal.
- At the expense of a slight increase in nonstationarity, the \( \text{nsps} \) propagator can be aperture compensated which gives it perfectly absorbing boundaries.

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References

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Figure 4a. Wavefield extrapolation by pspi.

Figure 4b. Nonstationary phase shift operator for pspi.

Figure 5a. Wavefield extrapolation by nsps.

Figure 5b. Nonstationary phase shift operator for nsps.

Figure 6a. Nsps with absorbing boundaries.

Figure 6b. Nsps operator with absorbing boundaries.

Figure 7a. Nsps with absorbing boundaries and 45 degree dip limit.

Figure 7b. Nsps operator with absorbing boundaries and 45 degree dip limit.