Depth migration by nonstationary phase shift
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Summary
Depth migration by nonstationary phase shift is presented as an extension of conventional phase shift to handle lateral velocity variations. Using nonstationary filter theory, a Fourier domain wavefield extrapolator is presented which handles arbitrary lateral velocity variations. Applied recursively, vertical velocity variations are also addressed and a depth migration algorithm results. An example using data from the physical modeling facility at the University of Calgary shows that the method performs as expected.

Method
We present an extension of phase shift depth migration (Gazdag 1978) to handle media with arbitrary vertical or lateral velocity variations. Called nonstationary phase shift (NSPS), our method is a recursive wavefield extrapolation, done entirely in the Fourier domain, but adapted to handle lateral velocity variations (Margrave and Ferguson, 1997). Over each Δz step, we allow an arbitrary v(x) (lateral velocity variation) but assume no vertical gradient. Vertical velocity variations are addressed in the customary fashion by using a different v(x) in each Δz step.

Our method is related to the phase shift plus interpolation (PSPI) technique of Gazdag and Squazero (1984) but uses a different mathematical form for the wavefield extrapolator (Margrave and Ferguson, 1997). In essence, we formulated PSPI in the context of nonstationary filter theory (Margrave, 1998) and found it to be a nonstationary combination filter. Such filters are linear and highly nonstationary but do not form a simple scaled superposition of nonstationary impulse responses. We then modified the PSPI formula to be a nonstationary convolution and thereby developed a wavefield extrapolator which does form the scaled superposition of laterally variable impulse responses. Though nonstationary filter theory provides prescriptions to apply the extrapolation filter in the space, Fourier, or mixed domains, we currently use the Fourier domain formulation.

The NSPS extrapolation in the Fourier domain is done by

\[
q(k_x, z + \Delta z, \omega) = \int_{-\infty}^{\infty} q(k_x, \Delta z, \omega) \Lambda(k_x, k_x - k_x', \omega) \, dk_x
\]

(1)

where \(q(\omega, z, k_x)\) is the \((\omega, k_x)\) transform of post-stack seismic data at depth \(z\) and

\[
\Lambda(k_x, k_x - k_x', \omega) = \int_{-\infty}^{\infty} \alpha(k_x, x, \omega) \exp(-ix(k_x - k_x')) \, dx
\]

(2)

where

\[
\alpha(k_x, x, \omega) = \begin{cases} 
\exp(i\Delta z \sqrt{\frac{\omega^2}{v(x)^2} - k_x^2}), & \frac{\omega^2}{v(x)^2} > k_x^2 \\
0, & \text{otherwise}
\end{cases}
\]

(3)

The frequency domain extrapolator, \(\Lambda\), as given by equation (2) is an ordinary Fourier transform over the space axis of the nonstationary phase shift operator, \(\alpha\). When velocity is constant, it results that this series of equations produces exactly the familiar constant velocity phase shift extrapolator:

\[
q(k_x, z + \Delta z, \omega) = \begin{cases} 
q(k_x, \Delta z, \omega) \exp(i\Delta z \sqrt{\frac{\omega^2}{v^2} - k_x^2}), & \frac{\omega^2}{v^2} > k_x^2 \\
0, & \text{otherwise}
\end{cases}
\]

(4)

We note that this method is similar to one presented by Black et al. (1984) and Grimbergen et al. (1995).

Example
As a test of NSPS, we used a simple thrust fault model from the physical modeling facility at the University of Calgary. This consisted of a dipping slab of velocity 3060 m/s embedded in an approximately constant velocity background medium of 2300 m/s as shown in Figure a). An approximately zero-offset section is shown in Figure b). The reflection from the dipping slab is clearly visible as are a basal reflection, several reflections to the left of the
slab and some point scatterers are also visible. The intermediate reflections are caused by cement bonds and do not represent velocity contrasts while the point scatterers are imperfections in the model media. All reflectors, except the dipping slab and the small bump (reef, at x=3500 m) below the slab, are horizontal.

Figure c) shows the result from a constant velocity phase shift migration (using 2300 m/s). While the top of the slab is well imaged, the strong pull-ups beneath the slab are residual effects. The result from NSPS is shown in Figure d). The overall image quality is similar to that from phase shift but the pull-ups beneath the slab have now been approximately flattened. The shape of the basal reflector from c) has been overlaid for comparison. The additional wavefronting beneath the slab on the NSPS result is undesirable and is currently being investigated.

Conclusions
Depth migration by nonstationary phase shift can account for strong lateral velocity variations with an extrapolation technique done entirely in the Fourier domain.

Figure: Comparison of NSPS and ordinary phase shift. a) Velocity model. Gray is a velocity of 2300 m/s and black is 3060 m/s. b) Zero offset physical model data form the U of Calgary modeling facility c) phase shift migration of b) using 2300 m/s. d) NSPS migration of b) using the velocity model in a).

References
Margrave, G. F., 1998, Theory of nonstationary linear filtering in the Fourier domain with application to time variant filtering: Geophysics, 63, 244-259.

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