Rayleigh wave modelling by finite difference
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Summary

Finite difference modelling of elastic wave fields is a practical method for elucidating features of records obtained for exploration seismic purposes, including surface waves. This has become possible because of the major increase in computing power available at reasonable cost. To take advantage of this power a 2D finite difference modelling program for elastic displacements has been written in Matlab. Two techniques for representing realistic boundaries are presented in some detail. Examples are provided of the program’s use to propagate waves at the surface of the earth through shallow lateral and vertical velocity changes. It can be seen that surface wave reflections and transmissions have similarities to body wave reflections and transmissions.

Introduction

Modelling by finite difference methods is now a very practical endeavor because of the wide availability of computer power and user friendly languages. Modelling of surface waves is one possibility for finite difference methods, especially since ray-tracing methods can’t be used. To do this, physical boundaries must be realistically represented.

Surface waves are an inevitable feature on seismic records and have usually been considered noise. However recent studies have found the shallow penetration of surface waves to be ideal for near surface investigations, especially for environmental purposes (Park et al, 1996). Seismic modelling in general has proved very useful for showing how changes in the earth affect body waves, and it is reasonable to expect that modelling of surface waves can also be useful by relating changes in their character to specific changes in surface conditions.

An understanding and correct interpretation of surface wave anomalies will likely lead to improved statics for shear wave processing. The reason is that the large static shifts found on shear wave traces are usually caused by the shallow conditions within surface wave range, and shear wave velocities are closely related to surface wave velocities.

Matlab finite difference code

Finite difference code was written in Matlab using staggered grids for the x and z components, similar to the Virieux (1986) method except that only displacements are kept in memory between time steps. The program is second order in space and time, satisfying the two dimensional elastic wave equation

\[ (\lambda + 2\mu) \frac{\partial^2 u_x}{\partial t^2} + (\lambda + \mu) \frac{\partial^2 u_x}{\partial x \partial t} + \mu \frac{\partial^2 u_x}{\partial z^2} = \rho \frac{\partial^2 u_x}{\partial t^2} \]  

with a similar equation for z acceleration. Figure 1a shows the relative positions of the density and elastic parameters and all the displacements which, at the next time step, will change the x displacement at the centre of the diagram. The boxes in Figure 1a show where diagrams 1b and 1c apply.

It is important to treat the second term of the equation (which couples the x and z displacements) carefully because the \( \lambda \) and \( \mu \) terms are applied at different points in the staggered grid method. Figures 1b and 1c show how the \( \partial^2 U_z/\partial x \partial z \) term is most properly represented by two sets of finite difference terms centered on the shear \( \mu \) parameter position, and the compressional \( \lambda \) (and \( \lambda + 2\mu \)) parameter positions. Figure 1b shows how a net compression in the z direction causes a force in the x direction. Figure 1c shows how a net torque caused by a change of \( z \) displacements in the x direction causes a force in the x direction. These conditions are handled more simply with the Virieux velocity-stress method.

Figure 2 shows how the free surface boundary and the top of the model are represented. Two ‘pseudo’ rows of displacements are assumed to be above the top of the grid, with elastic parameters identical to those within the grid. The material within these pseudo rows is then assumed to have no shear or compressional traction across horizontal planes in accordance with the free surface boundary condition. The lower pseudo row of \( x \) displacements is determined by the requirement that \( \partial U_x/\partial z = -\partial U_z/\partial x \) which ensures that the horizontal shear force is zero. The upper pseudo row of \( z \) displacements is determined by the requirement that \( \lambda \partial U_x/\partial x = (\lambda + 2\mu) \partial U_z/\partial z \), or that the vertical compressional force be zero. The pseudo displacements can then be used in the same formulae as used within the grid to calculate the real near surface effects.

Surface wave source wavelet

A specialized source wavelet was designed for study of surface waves, and a snapshot of it is shown in the vector plot of figure 3. The reasons for a special wavelet are: the limited depth penetration of a surface wave allows a relatively thin model and so lowers run times, the low level of body wave energy ensures that the effects seen are mainly from surface waves, and the short surface wave
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The surface wavelet was constructed with the following set

\[ u_x = \left[ e^{\frac{-t}{\xi}} - \left( \frac{1}{1 - \frac{c}{2\beta^2}} \right)^{\frac{t}{\xi}} \right] f(t) \]  \hspace{1cm} (2)

\[ u_z = \sqrt{1 - \frac{c^2}{\alpha^2}} \left[ e^{\frac{-z}{\xi}} - \left( \frac{1 - \frac{c^2}{2\beta^2}}{1 - \frac{c^2}{\alpha^2}} \right)^{\frac{z}{\xi}} \right] g(t) \]  \hspace{1cm} (3)

Where \( \xi = \frac{\omega}{c} \sqrt{1 - \frac{c^2}{\alpha^2}} \) and \( \xi = \frac{\omega}{c} \sqrt{1 - \frac{c^2}{\beta^2}} \)

and \( f(t) \) is a source wavelet with \( g(t) \) as its derivative.

The Rayleigh wave velocity \( c \) was calculated from the formula by Achenbach (1973) p192. The wavelet constructed with these equations is only an approximate surface wave but works well in practice if the frequency \( \omega \) is assumed to be the peak frequency of the input wavelet \( f(t) \). The time wavelet can be translated into a pair of spatial wavelets at time 0 and \( \Delta t \) to define the wave form and initiate propagation. It propagates almost unchanged and does not create body waves with significant amplitude. The wavelet shown in figure 3 was based on a zero phase 30 Hz Ricker wavelet and was designed to move toward the right (it has retrograde particle motion at the surface if moving right). Figure 4 shows the wavelet after 40 ms and it can be seen to have largely preserved its shape.

Surface wave modelling examples

Figures 5 to 8 all show seismograms of vertical displacements at the free surface.

Figure 5 shows the wavelet designed above propagating on a uniform half space with compressional velocity of 2867 m/sec and shear velocity of 1000 m/sec. This is the base case and can be seen to have almost no change in wavelet character with time. The velocity of propagation is slightly less than the shear wave velocity - consistent with theory.

In figure 6 the propagation is into a material with lower shear velocity and great thickness. A line marks the boundary. A small reflected surface wave propagates left and the transmitted surface wave propagates more slowly to the right undispersed.

The dispersive effects of layering on surface waves is shown in figure 7 where the wavelet propagates into a low velocity layer 10 metres thick (the position marked by a box). The reflected wave and first part of the transmitted wave are very similar to the great thickness case (figure 6) but then the low-speed portion of the wavelet receives interference from the high velocity half space below.

Conclusions

Careful design of free surface boundary conditions can provide useful models of surface waves. A surface wave ‘wavelet’ is an efficient way to provide useful insights into some aspects of surface wave behavior. In a qualitative sense, surface wave behavior at velocity boundaries has parallels to that of body waves.

Acknowledgements

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References

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Figure 1: Relative positions of the parameters in the finite difference scheme
   a - all displacements which affect the evaluation point
   b – z displacement influence through compression
   c – z displacement influence through shear

Figure 2: Free surface pseudo displacements derived from model displacements. d derived from a, b, c. f derived from c, d, e.
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Figure 3: wavelet as designed.

Figure 4: wavelet after propagation.

Figure 5: base case.

Figure 6: propagation into low velocity.

Figure 7: propagation into 10 metre layer

Figure 8: propagation into 5 metre layer.