Evaluating the accuracy of wavelet estimates from Gabor deconvolution by VSP downgoing waves
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Summary
Accurate wavelet estimates in deconvolution can truly improve the resolution of a seismic section. Intuitively, nonstationary deconvolution will be more accurate than stationary, since a real wavelet is nonstationary. Gabor deconvolution (Margrave and Lamoureux, 2002) includes energy dissipation and velocity dispersion in its time-frequency operator to precisely estimate nonstationary wavelets. To verify the accuracy of wavelets estimated from surface data by Gabor deconvolution, we compare them to Q-filtered VSP wavelets from the same source. Synthetic propagating wavelets show that a Q-filtered VSP wavelet can perfectly match surface-estimated wavelets in both time and frequency domains. The results of a comparison based on real data are consistent with the hypothesis that Gabor deconvolution can accurately estimate the nonstationary wavelets embedded in seismic records. Our results also suggest that Gabor deconvolution is superior to both multi-window Wiener and frequency-domain spiking deconvolution.

Introduction
Wavelet estimates from Gabor deconvolution are affected by many factors, such as time-frequency smoothing methods, window length, stabilization factor, and operator length. Optimization of the effects of a smoothing approach, or other parameters, on the wavelet estimate is directly related to the accuracy of Gabor deconvolution. After removing the effects of geometric spreading, reflection and transmission, we can compare the propagating wavelet measured from VSP downgoing waves to the wavelet estimated from surface data, since the resulting wavelets are affected by the source signature and earth attenuation only. Here, we use wavelets obtained from VSP data to evaluate the accuracy of surface-estimated wavelets from the same source.

Wavelets from synthetic and real surface and VSP data are compared in this study. Synthetic VSP and surface seismograms with source signature and constant-Q attenuation effects only, were created by a complex ray approach. A simultaneous VSP and 2D surface seismic survey was provided by EnCana (formerly PanCanadian) for this research. Wavelet comparisons are carried out in both time and frequency domains.

Comparison of wavelets from synthetic VSP and surface seismograms
The wavelet comparison is based on the propagating wavelet model proposed by Margrave and Lamoureux (2002). Supposing the effects of geometric spreading and reflection/transmission on the wavelets has been removed, the propagating wavelet can now be modeled as an attenuated source signature which can be expressed as
\[ \omega_p(\tau, f) = \omega(f)\alpha_0(\tau, f), \]
where \( \omega(f) \) denotes the Fourier spectrum of the propagating wavelet at traveltime \( \tau \), \( \omega(f) \) is the spectrum of the stationary source signature, and \( \alpha_0(\tau, f) \) describes the time-frequency effects of attenuation. Assuming a constant-Q model, the attenuation term \( \alpha_Q(\tau, f) \) can be written as
\[ \alpha_Q(\tau, f) = e^{-\frac{\nu f}{Q(U)}}, \]
where \( H \) denotes the Hilbert transform and \( Q \) is the non-frequency dependent quality factor. Equation 2 shows the attenuation surface is a minimum-phase, time-variant low-pass filter.

Equation 2 assumes the direction of maximum attenuation is in the direction of increasing phase delay (Aki and Richards, 1980). For plane waves in inhomogeneous media, these two directions are different. To simulate a propagating wavelet passing through a sequence of anelastic horizontal layers, complex rays are used (Hearn and Krebes, 1990). The vertical component of displacement of a propagating wavelet at a receiver can be expressed by
\[ s_z(t, t-\tau) = \int_{-\infty}^{\infty} \omega(f) \exp[i2\pi f(\tau_c - t)]h_z df, \]
where traveltime \( \tau_c \) is a complex, frequency-dependent function, and \( h_z \) is the complex vertical component. The real part of \( \tau_c (\tau_c) \) is the actual traveltime of the ray, and the imaginary part of \( \tau_c \) is associated with absorption. If the direction of attenuation is the direction of wave
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propagation, $S_z$ is equivalent to the inverse Fourier transform of $\omega_p$ in equation (1).

Table 1 shows the simple subsurface model used to generate the synthetic data. Figure 1 shows the VSP downgoing wave recorded between 200 and 1500 m depth at 100 m receiver spacing, for a source located 30 m from the borehole with a 10-90 Hz bandpass Klauder wavelet source signature. Figure 2 shows the equivalent surface data for receivers 60 to 900 m from the well at a 60 m interval.

Table 1. The subsurface model with velocity values for the 30 Hz reference frequency.

<table>
<thead>
<tr>
<th>Layer</th>
<th>h(m)</th>
<th>v(m/s)</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>Q</th>
</tr>
</thead>
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<tr>
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<td>1800</td>
<td>2.0</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>2300</td>
<td>2.1</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>3000</td>
<td>2.2</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 1. The subsurface model with velocity values for the 30 Hz reference frequency.

![FIG. 1. Synthetic wavelets received in the borehole.](image1)

![FIG. 2. Synthetic wavelets received at the surface. The energy dissipation and velocity dispersion are obvious.](image2)

Suppose wavelets from VSP and surface data share the same source and travel through the same anelastic medium, any differences between the wavelets are solely due to the distance the wave travels through the media if Q is independent of direction of rays. From equation 2 the spectral ratio of the VSP and surface wavelets can be written as

$$
\frac{S_{svp}(t_1, f)}{S_{svp}(t_1, f)} = e^{-\pi f (t_2 - t_1) / (v_1 v_2)} e^{i \phi (t_1, f)}
$$

(4)

where $S_{svp}(t_1, f)$ is the spectrum of the wavelet from VSP data and $S_{svr}(t_2, f)$ is the spectrum of the wavelet from surface data. Traveltime $t_1$ is the one-way time to a receiver in the borehole for an offset VSP, and $t_2$ is the two-way traveltime of a reflection recorded at the surface for a reflector at the same depth. The relationship between $t_1$ and $t_2$ can be expressed as

$$
t_2 = \sqrt{\frac{x^2}{v^2} + \frac{4}{v_{stk}^2}} + \frac{t_1^2 v_2^2 - c^2}{v_{stk}^2}
$$

(5)

where $v_2$ is the average velocity at time $t_1$, $v_{stk}$ is the stacking velocity estimated from the surface seismic, $c$ is the horizontal distance between the shot location and the borehole, and $x$ is the source-receiver offset on the surface.

Equation (4) can be used as a Q filter to match the VSP wavelets to surface wavelets. In the case of synthetic data, $t_1$ and $t_2$ are directly measured from synthetic seismograms and effective Q can be calculated from given interval Q. Figure 3 shows a qualitative comparison of Q-filtered VSP wavelets and wavelets received on the surface. The Q-filtered VSP wavelets are consistent with the surface wavelets, although there are small differences with increasing horizontal offsets.

![FIG. 3. Comparison of Q-filtered synthetic VSP wavelets to synthetic surface wavelets. Top six wavelets are three Q-filtered VSP wavelets received at 500 m depth and the other three wavelets reflected from the first reflector and received on the surface. Bottom six wavelets consist of three Q-filtered VSP wavelets received at 1500 m depth and three wavelets reflected from the second reflector. Similarity of waveforms decreases slightly with increasing offset from the borehole.](image3)
Evaluating the accuracy of wavelet estimates

Figure 4 shows the amplitude spectra of Q-filtered VSP wavelets have similar db down to these of surface-simulated wavelets at the same frequency.

Comparison of wavelets from real VSP and surface data

Receivers were positioned between 322 and 1820 m depth at a receiver interval of 20 m for a total of 75 receiver locations within the borehole. An additional 78 geophones were placed between 30 and 2310 m from the borehole at a 30 m interval. Five source points were used for this survey, located 27, 430, 960, 1350 and 1700 m from the borehole. A 12 s, 10-96 Hz non-linear sweep was used to record 16 second uncorrelated shot records at a 2 ms sample rate. Figure 5 shows a zero-offset VSP section and the corresponding shot gather. The synthetic results show that near-offset wavelets from the surface match the Q-filtered VSP wavelets slightly better than those recorded at far offsets. However, near-offset traces often contain more source noise, as shown in Figure 5 (bottom). As a result, we chose the less noisy traces 20 to 30 for wavelet estimation.

Major steps in VSP wavelet estimation are: 1) vertical sum, 2) geometric spreading correction, 3) downgoing wave flattening and Fk filter, and 4) windowing downgoing waves to extract wavelets. The geometric spreading correction removes nonstationarity relating to wavefront divergence, which is independent of frequency. The velocity function used in the geometric spreading correction should be the same as used for surface seismic processing. After flattening downgoing waves, a f-k filter is applied to separate downgoing waves from upcoming waves.

Three methods were applied to estimate the propagating wavelet from the surface data. To estimate wavelets with Gabor deconvolution, we apply the Gabor transform using Gaussian windows with an 80 percent window overlap, and then smooth the Gabor magnitude spectrum along hyperbolae \((t \cdot f = \text{constant})\) to estimate the magnitude of the attenuation function. The source signature is then estimated by dividing the Gabor magnitude spectrum by the attenuation, and averaging over time. A Hilbert transform over frequency at constant time, applied to the logarithm of the product of the attenuation surface and the source signature, provides the associated minimum phase estimate. Wavelet estimation from Wiener and frequency domain spiking deconvolution is well known. Parameters such as window length, stabilization factor, operator length, and frequency smoothing length were the same for all three methods.

Figure 6 shows wavelet and spectrum estimates from VSP data and their counterparts at traveltime \(t_2\) from surface data by Gabor deconvolution. \(t_2\) is calculated by equation 5 from \(t_1\), the first break time of VSP downgoing waves.
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Inspection shows that the propagating wavelets experience attenuation of high frequencies and dispersion with increasing time. These effects are more pronounced on the surface data.

In Q filtering, Q estimated from VSP data is usually more reliable than Q estimated from surface data (White 1992). Q-values are estimated from VSP wavelets by spectral ratios. Figure 7 shows the wavelets estimated from surface data compared to Q filtered wavelets from the VSP data. For a wavelet estimated in a window starting at 0.486 s two-way time on the surface data, the equivalent Q filtered VSP wavelet contains higher frequencies (Figure 7).

FIG. 7. Comparison of Q filtered VSP wavelets to wavelets estimated from surface data. Two-way travel times for wavelets from surface data are shown just below the curves.

For a wavelet estimated in a window starting at 0.486 s two-way time on the surface data, the equivalent Q filtered VSP wavelet contains higher frequencies (Figure 7).

With increasing time, high frequency components are attenuated and the Q filtered wavelets gradually conform to the wavelets estimated from surface data. Figure 8 shows the amplitude spectra corresponding to the wavelets in Figure 7. The wavelet from Gabor deconvolution is the closest match to the Q filtered wavelet from the VSP, especially at greater travel times. Figure 9 shows cross-correlations of wavelets from the three deconvolution methods with the Q filtered VSP wavelet. Wavelets estimated by Gabor deconvolution are most similar to the VSP wavelets.

FIG. 8. Comparison of amplitude spectra for wavelets shown in Figure 7. Since Gabor deconvolution uses a Gaussian window and smoothes the time-frequency spectrum, the amplitude spectra derived from Gabor deconvolution are smoother than spectra from Wiener and frequency domain spiking deconvolution.

FIG. 9. Peak curves of cross-correlation between Q filtered VSP wavelets and wavelets estimated from surface data by three different deconvolution methods.

Conclusions

The result of wavelet comparison on synthetic seismograms shows that the wavelets from VSP and surface data are comparable and the filter used in wavelet comparison is accurate. Our results are consistent with the hypothesis that Gabor deconvolution can accurately estimate the nonstationary wavelets embedded in real seismic records. These results also suggest that Gabor deconvolution is superior to both multi-window Wiener and frequency domain spiking approaches.

References


White, R. E., 1992, The accuracy of estimating Q from seismic data: Geophysics, 57, 1508-1511.

Acknowledgments

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