Vp/Vs from multicomponent seismic data and automatic PS to PP time mapping
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Summary

In this paper, we discuss a method of determining a depth-varying, zero-offset velocity ratio ($\gamma_0$) via velocity analysis, using Thomsen’s (1999) non-hyperbolic traveltime equation. We develop a velocity-analysis procedure to compute semblance as a function of three variables namely, the converted-wave (PS) stacking velocity, the zero-offset time, and ($v_p/v_s$). This process is used to scan for a depth-varying ratio. Use of this method on synthetic shot gathers from a four-layer geologic model provides the P-wave velocities and PS velocities used to stack the data sets. The PS stacked data is transformed to P-wave times using the derived velocity ratio values.

We find that post-critical angle reflected events cause errors and should not be used in the velocity analysis. The accuracy of the technique increases by decreasing the sampling interval in each variable used in the computation of semblance. This however, linearly increases the computational cost. By converting the $\gamma_0$ and the PS zero-offset times to the corresponding P-wave times, the correlation of P-wave and PS stacked sections can be implemented automatically. Correlation of the transformed synthetic data (PS to PP time using the estimated $\gamma_0$) is found to be good (the difference being less than 5%).

Introduction

A primary goal of exploration geophysics is to be able to translate acquired seismic data and other related information into drilling sites and hydrocarbon volumes. One of the critical values/parameters that govern the translating process is the stacking velocity. It constitutes a vital characteristic in exploration seismology; and its determination forms a key step in seismic data processing. The task is more challenging in PS propagation because the raypath is not symmetric about the conversion point. Tessmer and Behle (1988) derived expressions for the PS traveltime and the conversion point. Thomsen (1999) extended their work and derived PS traveltime equation for multi-layer, anisotropic, and inhomogeneous media. In these expressions, the conversion point (offset of the imaged point) depends on a $\gamma_0$ value (defined as the ratio of P-wave velocity to S-wave velocity measured at zero-offset time). Converted-wave traveltimes also depend on this parameter. Therefore, to obtain accurate PS stacking velocities via velocity analysis on gathers, it is important to accurately determine $\gamma_0$ values. In this paper, we demonstrate how to analyze for the velocity ratio ($\gamma_0$) using Thomsen’s (1999) non-hyperbolic traveltime equation by computing semblance in 3-dimensions. To perform velocity analysis with this equation, we make some simplifying assumptions to reduce the number of variables to three. In the non-hyperbolic NMO equation, $v_p \gamma_0$ is substituted for $v_p$ (Tessmer and Behle, 1988; Thomsen, 1999), where $v_p$ is the PS short-spread moveout velocity, and $v_p$ is the P-wave moveout velocity. Thus, instead of computing semblance as SC ($\gamma_0, v_p, v_p, t_{out}$), it will be computed as SC ($\gamma_0, v_p, v_p, t_{out}$).

Method

A four-layer geologic model, with the associated physical attributes as shown in Figure 1 was constructed using the GX2 raytracing modelling package. The dimensions of the model are 4000 m long by 4000 m deep. Simulated field acquisition parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Geophone spread length</th>
<th>4000 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geophone spacing</td>
<td>20 m</td>
</tr>
<tr>
<td>Shot interval</td>
<td>100 m</td>
</tr>
<tr>
<td>CDP interval</td>
<td>10 m</td>
</tr>
<tr>
<td>Total number of shots</td>
<td>41</td>
</tr>
<tr>
<td>Total CDP locations</td>
<td>400</td>
</tr>
<tr>
<td>Source wavelet</td>
<td>30 Hz Ricker</td>
</tr>
</tbody>
</table>

Table 1. Acquisition parameters

The resultant shot gathers (e.g. Figure 2) were exported to the Promax environment for processing. To process the PS data, asymptotic common conversion-point binning was used.
Vp/Vs scanning and PS-PP time mapping

Figure 1. Geologic model with associated physical attributes

\[
\begin{align*}
V_p &= 3000\text{ m/s}, V_p' = 1395\text{ m/s}, V_p'/V_p &= 2.15, \rho = 2.5\text{ gm/cc, thickness} = 1000\text{ m} \\
V_p &= 3500\text{ m/s}, V_p' = 1635\text{ m/s}, V_p'/V_p &= 2.14, \rho = 2.52\text{ gm/cc, thickness} = 900\text{ m} \\
V_p &= 4000\text{ m/s}, V_p' = 1878\text{ m/s}, V_p'/V_p &= 2.13, \rho = 2.54\text{ gm/cc, thickness} = 1700\text{ m} \\
V_p &= 4500\text{ m/s}, V_p' = 2195\text{ m/s}, V_p'/V_p &= 2.05, \rho = 2.55\text{ gm/cc, thickness} = 400\text{ m}
\end{align*}
\]

Figure 2. Converted-wave shot record. (note the post-critical events)

The velocity ratio \((\gamma)\) used was 2.12, the arithmetic average of the four input velocity ratios. Post-critical events seen in the PS shot records were muted prior to velocity analysis. Subsequent CCP and CDP gathers were NMO-corrected and stacked after routine velocity analysis. The P-wave and PS stacked sections are shown in Figures 3 and 4. (note the differing time scales). Having obtained the stacked sections, the next step is to transform the PS section to P-wave times for the purposes of correlation and interpretation. This is a crucial stage for subsequent PS and P-wave interpretation. Transformation of PS data to P-wave times entails using the appropriate velocity ratios for the various geologic formations. This implies using depth-varying \(\gamma\) values. To derive this function, we turn to the non-hyperbolic equation (Thomsen, 1999).

Thomsen’s (1999) traveltime equation
Thomsen’s non-hyperbolic traveltime equation is given below as:

\[
t_p^2 = t_{\rho,0}^2 + \frac{X^2}{V_p^2} \left( \frac{(\gamma - 1)\left(V_p' - V_p\right)}{4(\gamma - 1)\rho V_p' + \rho V_p'} \right) X^2
\]

(1)

where \(V_p\) is the P-S shot-spread moveout velocity, defined by Tessmer and Behle (1988) and Thomsen (1999) as:

\[
V_p = \frac{V_p'}{\gamma}
\]

(2)

\(t_{\rho,0}\) is the PS zero-offset time, and \(X\) is the offset distance. \(V_p\) is the P-wave velocity and \(\gamma\) is the velocity ratio (values at zero-offsets). Substituting equation (2) in (1), we obtain:

\[
t_p^2 = t_{\rho,0}^2 + \frac{X^2}{V_p^2} \left( \frac{(\gamma - 1)}{4(\gamma - 1)\rho V_p' + \rho V_p'} \right) X^2
\]

(3)

this paper, equation (3) was used to compute semblance for various velocity ratios.

Figure 3. P-wave stacked section from model in Figure 1.

Figure 4. PS stacked section from model in Figure 1.
Vp/Vs scanning and PS-PP time mapping

Semblance computation and γ determination

The semblance coefficient is a statistical measure introduced into velocity analysis by Tanner and Koehler (1969). Simply stated, it is defined as the normalized output/input energy ratio, where the output trace is a simple compositing or sum of the input traces (Neidell and Taner, 1971). Mathematically, the semblance coefficient SC can be stated as:

$$SC = \frac{\sum_{j=k-(N/2)}^{k+(N/2)} \left( \sum_{i=1}^{M} f_{i,j(i)} \right)^2}{\sum_{j=k-(N/2)}^{k+(N/2)} \sum_{i=1}^{M} f_{i,j(i)}^2},$$

where, \( k \) is the time of the event calculated using the traveltime equation [in this case, equation (1)], \( N \) is the window length within which semblance is calculated, \( M \) is the number of traces, \( i \) is the channel (in this case the offset), and \( j \) is the time sample, and \( f_{i,j} \) is the seismic amplitude at offset \( i \), and at time sample, \( j \). Using equations (3) and (4), a Matlab code was developed to compute semblance as a function of PS velocity, velocity ratio \( \gamma \), and zero-offset, two-way time; i.e. semblance = \( SC(V_p/V_s,\gamma, t_{0s}) \). To execute the code, a range of values of PS velocities, velocity ratios, and zero-offset traveltimes are scanned. In a manner akin to routine velocity analysis, values corresponding to maximum semblance are extracted.

Results and discussions

The results from semblance analysis are shown in Figures 5 to 6. In these Figures, time is increasing upward. Figure 5a shows the 3D (volume) display of semblance at the three horizons. Notice that there are still some remnants of post-critical events at the shallow level due to incomplete muting. Figure 5b shows the time slice at horizon 3. From this Figure, the location of maximum semblance can easily be seen. Values corresponding to zero-offset time \( t_{0s} \), velocity ratio \( \gamma \), and PS velocity \( V_p \) can be extracted by taking vertical slices parallel to the time and velocity axes (see Figures 6a and 6b). The parameters for the other two horizons were determined in a similar manner. Results obtained from these displays are tabulated in Table 2. Runtime on a dedicated Sun Ultra 10 machine is about 10-15 minutes. Computed P-wave times for the PS events, match the actual P traveltimes reasonably well. Accuracy can be increased, by using a finer grid interval; but this would increase the computational cost. Test results from real data look promising.

Transformation of PS stacked data to P-wave times

Having stacked the data and computed the velocity ratios, we need to be able to correlate the PS stack section to the P-wave stacked section. To do this, we need an expression,

$$t_{ps} = \frac{1}{2}(t + \gamma_t) t_{ps},$$

which ties the sections at their zero-offset time. Tessmer and Behle (1988) derived this expression as:

where, \( t_{ps} \) and \( t_{ps} \) are respectively the P-wave PS zero-offset traveltime and \( \gamma_t = V_p/V_s \). To transform the PS stack data to P-wave times, three steps are involved:

1. Interpolate the \( \gamma_t \) values obtained from semblance analysis, to generate a \( \gamma_t \) function. I.E. \( \gamma_t(t_{ps}) \).
2. Next, using this function in equation (5), we compute P-wave times \( t_{ps} \). Results are shown in Table 2.
3. Finally, the PS amplitude at PS times are mapped to their
corresponding PP times which yields a transformed PS stacked section. Figure 7 shows the comparison of the transformed PS and the P-wave stacked sections.

![Figure 7](image)

Figure 7. The comparison of the transformed PS (7a) and P-wave (7b) stacked sections.

**Error analysis**

The differences between the computed P-wave times and the actual are shown in Table 2. These differences vary from 3.2% to 4.9%. This shows that the computed results very closely agree with the actual values.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Estimated $\gamma_s$</th>
<th>Input $\gamma_s$</th>
<th>Difference</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.147</td>
<td>2.15</td>
<td>-0.003</td>
<td>-0.14</td>
</tr>
<tr>
<td>2</td>
<td>2.057</td>
<td>2.14</td>
<td>-0.083</td>
<td>-3.9</td>
</tr>
<tr>
<td>3</td>
<td>2.12</td>
<td>2.13</td>
<td>-0.01</td>
<td>-0.47</td>
</tr>
</tbody>
</table>

Table 2. Shown here, is the % difference between actual and computed P-wave times.

From Table 3, the differences between the estimated and input $\gamma_s$ values vary from -3.9 to -0.14%. However, by using the derived $\gamma_s$ function to re-bin, and performing semblance analysis again, these errors could be made smaller.

**Conclusions**

It is possible to determine $\gamma_s$ and $\nu_p$ values kinematically via velocity analysis, using a non-hyperbolic traveltime equation from PS seismic data. Post-critical events cause errors and shouldn't be used in the velocity analysis. Accuracy increases by decreasing the sampling interval in each variable used in the computation of semblance. That is, increasing the number of samples in each variable. This however, linearly increases the computational cost. By converting the $\gamma_s$ and the PS zero-offset times to the corresponding P-wave times, the correlation of P-wave and PS stacked sections can be implemented automatically. Correlation of the transformed PS data and the P-wave stacked data sets is found to be good (the difference being less than 5%).

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**References**


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Thomsen, L., 1999, Converted-wave reflection seismology over inhomogeneous, anisotropic media: Geophysics, 64, 3, 678-690.