Multigrid Deconvolution of seismic data
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Summary

Multigrid methods are an efficient and robust group of algorithms, used for the solution of large linear and non-linear systems of equations. The purpose of our study was to apply a multigrid style solver to the deconvolution problem, and study the effects of interpolation and downsampling on seismic data.

Included is the outline for 2 methods for seismic deconvolution. The spectral whitening results of these are tightly linked to the quality of wavelet estimation and the bandwidth of the data. Therefore, as we are most interested in the rate of convergence, a series of tests using known wavelets with frequencies up to Nyquist were used. As well, attempts were made to increase temporal resolution using interpolation above the recorded sample rate.

Introduction

Multigrid operates on the concept of decomposition of scale. It works by solving the system at a reduced resolution, then interpolating this coarse solution to the required accuracy and sample rate. The interpolated result is corrected using one of several possible iterative corrections.

One of the advantages of multigrid methods is that they are suited to a large class of problems. The main restriction is the ability to find an iterative method that has the ability to reduce the high frequency component of the error. As well, it is important that each point in the solution is dependent on other local points, as is the case in diagonally dominant matrix inversions.

Previous applications for this group of algorithms include finite-element and finite-difference approximations to differential systems. It is a common tool for numerical simulations in the field of fluid dynamics (Wesseling, 1992), but has not been studied extensively as applied to the seismic problem.

Iterative Methods

An iterative method is a solution scheme where an initial estimate of the solution vector is refined a specific number of times, or until some criterion of convergence is met. The key property of an iterative method for use in multigrid is that the high frequency component of the error is damped (Press et al., 1992). For this reason, Gauss-Seidel relaxation is a good choice. Conjugate Gradient, and other methods may have faster convergence rates when used alone, but they do not focus on the high frequency (Shewchuk, 2002), and so are less effective in multigrid applications. To demonstrate the Gauss-Seidel method, we use Laplace’s equation,

\[ \nabla^2 u = 0. \]  

(1)

In 2 dimensions, the second order finite difference approximation can be expressed as

\[ \frac{u(x+\Delta x, y) - 2u(x, y) + u(x-\Delta x, y)}{\Delta x^2} + \frac{u(x, y+\Delta y) - 2u(x, y) + u(x, y-\Delta y)}{\Delta y^2} = 0 \]  

(2)

To derive the Gauss-Seidel relaxation method, we solve equation (2) for \( u(x, y) \). For simplicity we set \( \Delta x = \Delta y \), yielding

\[ u(x, y) = \frac{u(x+\Delta x, y) + u(x-\Delta x, y) + u(x, y+\Delta y) + u(x, y-\Delta y)}{4} \]  

(3)

To apply the correction, each point in the solution is successively updated using equation (3). The process is repeated until the trial solution converges. Assigning homogeneous boundary conditions to equation (1), the solution becomes \( u = 0 \) across the whole domain, making it easy to track the amount of error in the solution after each iteration. Figure 1(a) shows the error of an initial estimate to the solution of equation (1). Figures 1(b), 1(c) and 1(d) show the error after 3, 10 and 30 iterations respectively. The high frequency component of the error is reduced very quickly, however the low wavelength error remains after many iterations of the correction. Figure
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Fig. 2: Spectral performance of the Gauss-Seidel correction, the error is attenuated by this factor with each iteration.

Fig. 3: Diagram showing the relationship between interpolation and restriction.

2 shows the spectral performance of the Gauss-Seidel operator for this problem. This figure was generated by dividing the radially averaged spectrum of the error after the correction by the spectrum before the correction, for each of the first five iterations. Errors components at or near Nyquist frequencies are attenuated by a factor of $\approx 0.2$ with each pass.

Multigrid

One strategy for effectively correcting the long wavelength component of the solution is to increase the grid size before applying the correction. We perform a restriction (opposite of interpolation, see Figure 3) of the data, including an anti-aliasing filter. This brings the long wavelength components of the error closer to the Nyquist frequency of the grid, and into the range where the Gauss-Seidel correction is more effective. The coarse grid correction process is demonstrated in Figure 4. The initial estimate (displayed again in 4(a)) to our sample problem was restricted to a sampling interval 8 times larger than the original. The restricted solution is shown in 4(b). From there, only 3 iterations of the Gauss-Seidel relaxation were performed, yielding the result in 4(c), then interpolated to the original sampling interval in 4(d). The number of computer operations is approximately equal to just one iteration of the Gauss-Seidel correction at full resolution. Were the restriction to continue to a coarser grid, it would take even less effort to reduce the error to zero.

Application to deconvolution

One of the possible applications of multigrid methods to seismic data is in deconvolution. We express the convolutional model as a system of equations, represented by the matrix multiplication

$$W r = s,$$  \hfill (4)

where $r$ is an unknown reflectivity series, $W$ is a wavelet matrix, which when multiplied with the reflectivity is the equivalent of convolution, yielding the seismic trace $s$.

For the Gauss-Seidel algorithm to work, it requires a diagonally dominant system (Trottenberg et al., 2001). In order to ensure that this is the case, we use instead

$$W^T W r = W^T s,$$  \hfill (5)

where the superscripted $T$ denotes the transpose.

To derive the Gauss-Seidel correction for this system of $N$ equations, we inspect the $i^{th}$ equation,

$$\sum_j [W^T W](i, j)r(j) = [W^T s](i), \quad j = 1, N.$$  \hfill (6)

Solving (6) for $r(i)$,

$$\sum_j [W^T W](i, j)r(j) - [W^T s](i) \quad \frac{[W^T W](i, i)}{[W^T W](i, i)} = r(i), \quad j = 1, N, \quad j \neq i.$$  \hfill (7)

To apply this correction, we cycle through the entire trace, updating each sample in the trace with equation 7. Results are far superior if the direction of the cycle changes...
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Fig. 5: Results of Gauss-Seidel deconvolution. Leftmost trace is reflectivity, followed by seismic, then progressive iterations of the correction to the right, with the number of iterations at the top of the trace.

with each iteration, i.e. the first iteration starts at time zero and moves down the trace, the second iteration starts at maximum time and moves up the trace.

The results of the Gauss-Seidel deconvolution are satisfactory. In the case of an exact known wavelet, with frequency right up to the Nyquist frequency, the deconvolution recovers the reflectivity series exactly. This can be seen in Figure 5. As is predicted, the high frequency component of the reflectivity is resolved quite quickly, while the lower frequency errors tend to be more difficult to reduce, taking many iterations for an improvement in the solution.

For the multigrid deconvolution, a coarse grid correction approach was used. The seismic data and the wavelet were anti-alias filtered and reduced to a sample rate 8 times that of the original. This restricted wavelet was used to perform a Gauss-Seidel correction on the trace. The solution was interpolated, and corrected again. Alternating interpolations and corrections were made until the trace was at the original sample rate. Several iterations of the Gauss-Seidel correction were made at the finest grid level. The results are shown in Figure 6, showing how the solution is refined after each interpolation.

Trace 1 is the seismic data reduced to 8 times the sample rate. Traces 2-4 show the trace after each interpolation and correction pair. The remaining traces show the result after progressive iterations at full resolution. As predicted, at the coarser grid spacing the long wavelength errors are removed very quickly. The last seven traces are successive passes of the correction at the fine grid spacing, however very little improvement is made after the first pass.

Resolution

Tests were performed to determine if it was possible to improve temporal resolution by interpolating the recorded seismic trace past the recorded sample rate. This was done by calculating a synthetic seismic trace at 0.1ms intervals with a sparse spike reflectivity, then restricting the data without any anti-aliasing filtering to 1.6ms data. This 1.6ms data was then re-interpolated to 0.1ms data, and was deconvolved using a wavelet that was estimated from the 1.6ms data.

The results are displayed in Figures 7, where a reflectivity spike sits directly in between 2 samples at the lower sample rate, and Figure 8 where the reflectivity spike is not centered in between the samples.

Both the multigrid and the Gauss-Seidel deconvolution were unable to add any content to the amplitude spectrum. The multigrid deconvolution was able to correct the phase of the wavelet, to place the apex of the deconvolved trace much closer to the actual location of the reflectivity spike.

Conclusions

Results show that multigrid methods are an effective way to deconvolve seismic data. The quality of the deconvolution rests on both the ability to estimate the wavelet, and the frequency content. The usefulness of the method will rely heavily on the ability to quickly estimate the embedded wavelet. Tests comparing the relative speed of multigrid to other deconvolution algorithms have not been conducted.

One strength of the method that has not been explored in this paper is the ease with which a non-stationary wavelet could be used. The modifications to the code is minor, and the run time will not be affected greatly.

The multigrid deconvolution algorithm also provides a method to resolve a reflectivity in between samples. This may be of use in determining to a greater accuracy the thickness and depth of thin beds.
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Fig. 7: Deconvolution of seismic data, interpolated past the original sampling interval. a) Seismic at $\Delta t = 0.1\text{ms}$ b) Reflectivity c) Seismic down-sampled to $\Delta t = 1.6\text{ms}$ d) Deconvolution of 0.1ms data e) Gauss-Seidel deconvolution of interpolated data f) Multigrid Deconvolution of interpolated data

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References


Fig. 8: Deconvolution of seismic data, interpolated past the original sampling interval. a) Seismic at $\Delta t = 0.1\text{ms}$ b) Reflectivity c) Seismic down-sampled to $\Delta t = 1.6\text{ms}$ d) Deconvolution of 0.1ms data e) Gauss-Seidel deconvolution of interpolated data f) Multigrid Deconvolution of interpolated data