Estimation of anisotropy parameters in VTI media
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Summary
Anisotropy parameters in a VTI medium can be obtained by anisotropy velocity analysis performed on short-spread or long-spread reflection-seismic data, in combination with check-shot or well-log data. Analysis of three traveltime approximations to the actual reflection traveltime in weak anisotropy media shows that each traveltime approximation has its own requirements for spread length and subsurface anisotropic parameters. The accuracy of the estimated Thomsen’s anisotropic parameter δ (or ε) depends not only on the accuracy of the picked NMO velocity (or horizontal velocity) but also on the absolute value of (ε - δ). The smaller the absolute value of (ε - δ), the higher the accuracy of estimated anisotropy parameter δ or ε. The results of the three traveltime inversions by semblance analysis for synthetic seismic examples demonstrate that nonhyperbolic estimation is better than the modified three-term Taylor series method, and the modified three-term Taylor series method better than hyperbolic estimation. None of these three approaches is suitable for estimating anisotropy parameters when the absolute value of (ε - δ) is large (i.e. |ε - δ| > 0.2 in this case).

Introduction
There are various reflection-traveltime inversion approaches for estimating anisotropy parameters but each has its own assumptions and limitations. It is necessary to understand these assumptions and limitations in order to guide their application. Thomsen (1986) has derived relations between normal-moveout (NMO) velocities and anisotropy parameters in a homogeneous anisotropic layer. We can use these approximations to obtain anisotropy parameters in VTI media by NMO-velocity analysis performed on short-spread or long-spread seismic data in combination with check-shot or well-log data. Besides hyperbolic NMO-velocity analysis, a popular approach for estimating anisotropy is a modified three-term Taylor series approximation to the reflection moveout curve. For convenience, we refer to this method as nonhyperbolic reflection-traveltime inversion.

In this paper, we compare the traveltime approximations of three reflection-traveltime inversion methods (hyperbolic traveltime inversion, modified three-term Taylor-series inversion and nonhyperbolic inversion) with the exact traveltimes in VTI media. We then carry out these three inversions on synthetic seismic data examples and compare the estimated anisotropy parameters with the true anisotropy parameters. Finally, we formulate some conclusions for guiding the application of these approximations.

Reflection traveltime approximations
Using an approximation of the exact eikonal equation in the quasi-compressional case for so-called weak anisotropy (Daley, 2001) and relations between phase velocity and group velocity, phase angle and group angle (Thomsen, 1986), we develop a multilayer ray-tracing code for modeling real traveltime-offset curves (solid line shown in Figure 1).

The traveltime approximations of three reflection-traveltime inversion methods are given by:
1) The hyperbolic reflection-traveltime approximation:
\[ t^2(x) = t_0^2 + \frac{x^2}{V_{NMO}^2} \] (1)
where \( V_{NMO}^2(P) = \alpha_0^2(1 + 2\delta) \) (2)
\( \alpha_0 \) is vertical velocity for P waves; \( \delta \) is Thomsen’s anisotropy parameter. \( V_{NMO} \) is NMO velocity for P-waves; and \( t_0, t \) are the two-way traveltimes for zero-offset and offset \( x \), respectively.

2) The modified three-term Taylor-series approximations for reflection traveltimes of the P- and SV-waves (Tsvankin and Thomsen, 1994) is:
\[ t^2(x) = t_0^2 + A_2x^2 + \frac{A_4x^4}{1 + A_3x^2} \] (3)
where \( A_2 = \frac{1}{V_{NMO}^2} = \frac{1}{\alpha_0^2(1 + 2\delta)} \)
and \( A = \frac{1}{(V_{NMO}t_0)^2} \).
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For P wave in the limit of weak anisotropy,

\[ A_4(P) = -\frac{2(\varepsilon - \delta)}{t_0 a_0^4} \]  

(4)

where \( v_0 \) is vertical velocity for P- or SV-waves; and the parameters \( A_2 \) and \( A_4 \) are Taylor-series coefficients.

3) For the P-waves, the fourth-order Taylor series with coefficients valid for arbitrary transverse isotropy (Tsvankin and Thomsen, 1994) is:

\[ A_4(P) = -\frac{2(\varepsilon - \delta)}{t_0 a_0^4} \left( \frac{1 + \frac{2\delta}{1 - \beta_0^2 / a_0^2}}{(1 + 2\delta)^2} \right) \]  

(5)

\[ A(P) = \frac{A_4}{V_h^{-2} - A_2} \]  

(6)

\[ V_h = \alpha_0 \bar{a} (1 + 2\varepsilon) \]  

(7)

where \( \beta_0 \) is vertical velocity for SV-waves; \( \varepsilon \) is Thomsen’s anisotropy parameter and \( V_h \) is horizontal velocity for P-waves. If one ignores the contribution of the vertical shear-wave velocity, which is negligible (Tsvankin and Thomsen, 1994; Alkhalifah and Lerner, 1994; Tsvankin, 1995), we have reflection traveltine approximation for P-waves in VTI media (here so-called nonhyperbolic approximation) by substituting equation (5), (6) and (7) into (3):

\[ t^2(x) = t_0^2 + \frac{x^2}{V_{NMO}^2} \left( \frac{[V_h^2 - V_{NMO}^2]x^4}{V_{NMO}^4[t_0 V_{NMO}^2 + V_h^2 x^2]} \right) \]  

(8)

Figure 1 shows the hyperbolic traveltine approximation (dotted line), the modified three-term Taylor series approximation (dashdot line) and nonhyperbolic approximation (dashed line) to the exact traveltine (solid line). It can be seen from Figure 1 that i) when \( \varepsilon - \delta = 0 \) (elliptically anisotropy), the three approximations are the same as the exact traveltine (see Figure 1(a)); ii) When \( 0 < \varepsilon - \delta \leq 0.2 \), these three closely approximate the exact traveltine well for the short spread (left side of figure 1(b)), but deviation of hyperbolic and the modified three-term Taylor series traveltine from the actual traveltine increases with spread length (right side of Figure 1(b)); iii) When \( \varepsilon - \delta > 0.2 \), nonhyperbolic traveltine approximates the actual reflection traveltine while the traveltimes of hyperbolic and modified three term Taylor series approximations largely deviate from the actual reflection traveltine, even for a short spread (see Figure 1(c)). This demonstrates that none of these three traveltine approximations is suitable for estimating anisotropic parameters when \( \varepsilon - \delta > 0.2 \).
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Estimation of Thomsen’s anisotropy parameters

Using equations (1), (3) and (8), we can pick up $A_2$, $A_4$, $V_{NMO}$ and $V_h$ from semblance plots (see Figure 2), and then obtain anisotropic parameters $\varepsilon$ and $\delta$ by using equations (2), (4) and (7).

The input CMP gather for anisotropy-parameter estimation contains a single reflection from a flat interface. The depth of this interface is 500 m. Vertical P- and S-wave velocities above the reflector are 3000 m/s and 1500 m/s, respectively.

Semblance scanning is employed to generate semblance plots. The scanning increments are: for normal moveout velocity, $V_{NMO}$, 5 m/s; for zero-offset two-way traveltime, $t_0$, 0.001 s, and for horizontal velocity, $V_h$, 5 m/s.

Figure 2 shows the semblance plots using nonhyperbolic traveltime approximation for anisotropic parameters: (a) $\varepsilon = 0.2$, $\delta = -0.2$; (b) $\varepsilon = 0.2$, $\delta = 0.1$; (c) $\varepsilon = 0.2$, $\delta = 0.2$. The symbol o represents the position of true values and + represents the position of picked values. It can be seen that the picked values approximate the true values when $|\varepsilon - \delta| < 0.2$ (Figure 2a, 2b), and the picked values largely deviate from the true values when $|\varepsilon - \delta| = 0.4 > 0.2$ (Figure 2c).

Figure 3 and Figure 4 show the crossplots of estimated $\delta$ versus true $\delta$ and estimated $\varepsilon$ versus true $\delta$ when the anisotropy parameter $\varepsilon$ is fixed at 0.2 (Figure 3a, 4a), 0.1 (Figure 3b, 4b) and 0.0 (Figure 3c, 4c), respectively, and the values of the anisotropy parameter $\delta$ range from −0.2 to 0.2 at an increment of 0.02.

Mathematically, the error of the estimated anisotropic parameter $\delta$ is given by:

$$\Delta\delta = \frac{\sqrt{1 + 2\delta}}{a_0} \Delta V_{NMO}$$  \hspace{1cm} (9)

It is shown that the error in the estimate of the anisotropy parameter $\delta$ mainly depends on the accuracy of the picked NMO velocity. It can be seen from Figure 3 that the errors of the estimated anisotropic parameters $\delta$ depend not only on the accuracy of the picked NMO velocity but also on the absolute value of $(\varepsilon - \delta)$. The smaller the absolute value of $(\varepsilon - \delta)$, the higher the accuracy of estimated anisotropy parameter $\delta$.

The error in the estimate of the anisotropy parameter $\varepsilon$ for the modified three-term Taylor-series inversion is given by:
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\[
\Delta \varepsilon = \left( \frac{1 + 4 \varepsilon - 6 \delta}{\sqrt{1 + 2 \delta}} \right) \frac{1}{a_0} \Delta V_{\text{NMO}} - \frac{1}{2(1 + 2 \delta)^2} \Delta t
\]

(10)

where \( A = -2(\varepsilon - \delta)(1 + 2 \delta)^3 \). The dash-dot lines in Figure 4 show that the error in the estimate of the anisotropic parameter \( \varepsilon \) is more complicated than for the dashed line, and mainly depends on the combined accuracy of picked \( V_{\text{NMO}} \) and \( A \).

The error in the estimate of anisotropy parameter \( \varepsilon \) for the nonhyperbolic inversion is given by:

\[
\Delta \varepsilon = \frac{\sqrt{1 + 2 \varepsilon}}{a_0} \Delta V_h
\]

(11)

The error in the estimated anisotropic parameter \( \varepsilon \) (dashed line in Figure 4) depends not only on the accuracy of the picked horizontal velocity but also on the absolute value of \( (\varepsilon - \delta) \). The smaller the absolute value of \( (\varepsilon - \delta) \), the higher the accuracy of estimated anisotropy parameter \( \varepsilon \).

Conclusions

The accuracy of the estimated anisotropic parameter \( \delta \) (or \( \varepsilon \)) depends not only on the accuracy of the picked NMO velocity (or horizontal velocity) but also on the absolute value of \( (\varepsilon - \delta) \). The smaller the absolute value of \( (\varepsilon - \delta) \), the higher the accuracy of estimated anisotropy parameter \( \delta \) or \( \varepsilon \). The results of the three traveltime inversions by semblance analysis for the seismic examples demonstrate that nonhyperbolic estimation is better than the modified three-term Taylor series method, which in turn is better than hyperbolic estimation. None of these three approaches is suitable for estimating anisotropy parameters when the absolute value of \( (\varepsilon - \delta) \) is large (i.e. \( |\varepsilon - \delta| > 0.2 \) in this case).

![Figure 3](image1.png)

Figure 3 The cross plots of estimated \( \delta \) vs. true \( \delta \). Solid line: the exact anisotropy parameters; dotted line: hyperbolic travelt ime inversion; dash-dot line: the modified three-term Taylor-series inversion; dashed line: nonhyperbolic inversion.

![Figure 4](image2.png)

Figure 4 The cross plots of estimated \( \varepsilon \) vs. true \( \delta \). Solid line: the exact travelt ime. Dash-dot line: the modified three-term Taylor-series inversion; dashed line: nonhyperbolic inversion.

References


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