Modelling near-field effects in VSP-based Q-estimation

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Summary

The complete spherical wavefield emanating from a P-wave point source surrounded by a homogeneous isotropic medium is computed with the aid of Sommerfeld/Weyl integrals. In a resulting synthetic VSP, we observe a near-field, a far-field and a 90° phase rotation between the two. Depth dependence of magnitude spectra in these two depth regions is distinctly different. Log magnitude spectra show a linear dependence on frequency in the far-field but not in depth regions where the near-field becomes significant. Near-field effects are one possible explanation for large positive and even negative Q-factors in the shallow section that may be estimated from VSP data when applying the spectral ratio method.
Introduction

In the introduction to “Special section – seismic quality factor”, Best (2007) highlights the increased interest in recent years in seismic wave attenuation phenomena and seismic absorption (intrinsic attenuation). He writes, “it could be argued that understanding the physical processes giving rise to absorption is the last barrier to the full realization of the potential of seismic methods”. He also mentions the practical difficulties in attenuation (Q) estimation due to restricted bandwidth and poorly constrained multiple scattering as well as geometric spreading losses. The above very nicely sums up our motivation for undertaking the study presented here.

It has been observed that VSP-based Q-estimates can give negative values for the near surface. For example Raikes and White (1984) noted negative Q and attributed their result to a change in geophone coupling. This possibility is included in our own list (Haase and Stewart, 2006a) which comprises 1) acquisition issues, 2) analysis problems and 3) lithology model inadequacies. In pursuing the topic of stratigraphic attenuation, we recently computed synthetic VSP data with the aid of Weyl/Sommerfeld integrals (Haase and Stewart, 2007) and noted some peculiar behavior of magnitude spectra as a function of depth in near-surface situations. The cause for this strange departure from far-field expectations turns out to be, not surprisingly, a near-field contribution to the total wave field of a P-wave point source. The aim of this study is to show that near-field behavior is another possible explanation for negative Q-estimates and frequency-dependent Q in the near-surface.

Theory

From a wide variety of Q-estimation techniques developed over the years, the spectral ratio method (SRM) appears to be the most commonly used approach. The derivation of Q-estimation methods is usually based on the definition of the quality factor Q. For the spectral ratio method Tonn (1991) gives

$$\ln \left[ \frac{A_t(\omega)}{A_0(\omega)} \right] = -\frac{\omega t}{2Q}, \quad (1)$$

where the log-spreading ratio is ignored and Q >> 1 is assumed.

In case of a homogeneous medium, Q-factors can be recovered exactly (within numerical accuracy) under far-field conditions when the source-receiver separation is large. What are these far-field conditions and when do they break down? Aki and Richards (1980) use the phrase “many wavelengths” when describing the far-field distance from a point source. They give an equation for the P-wave potential caused by a point source (Weyl/Sommerfeld integral approach) in a 3D homogeneous medium. P-wave displacements can be obtained from such potentials by computing gradients, which for the direct wave gives

$$u_p(\omega) = i\omega e^{-i\omega t} \int_0^\infty \left[ \frac{\rho^2}{\xi} J_1(\omega pr) \sin(\theta) - ip J_0(\omega pr) \cos(\theta) \right] e^{i\omega \xi z} dp, \quad (2)$$

where $u_p(\omega)$ is the P-wave displacement along the ray at the current receiver and $\omega$,

- $\omega$ is the frequency in radians,
- $t$ is the time,
- $\rho$ is the horizontal slowness,
- $\xi$ is the vertical slowness,
- $J_0$ and $J_1$ are zero-order and first-order Bessel functions of the first kind,
- $r$ is the range (horizontal offset between source and receivers),
- $z$ is the source-to-receiver depth interval (to the current receiver) and
- $\theta$ is the incidence angle from the vertical (at the current receiver).
For zero offset (zo) VSP, the range \( r \) in both Bessel function arguments is zero. With \( J_0(0) = 1 \), \( J_1(0) = 0 \) and introducing \( \left[ \frac{p}{d p} = \frac{-\zeta}{d\zeta} \right] \) (obtained from \( \zeta^2 = \frac{1}{\alpha^2} - p^2 \) and \( d\zeta/dp = -p/\zeta \)), we find

\[
u_{p-zo}(\omega) = -\omega e^{-i\omega t} \int_{1/\alpha}^{\infty} \zeta e^{i\omega \zeta} d\zeta,
\]

which can be integrated analytically giving

\[
u\bigg|_{r=0} = \frac{i\omega}{\alpha} e^{-i\omega t} \left[ \frac{1}{z} + \frac{i\alpha}{\omega z^2} \right] e^{i\omega z/\alpha} = \frac{i\omega}{\alpha z} e^{-i\omega t} \left[ 1 + \frac{i\alpha}{\omega z} \right] e^{i\omega z/\alpha}.
\]

The first term in the brackets of Equation (4) is the familiar far-field term. It decays with \( 1/z \). The second term in brackets decays with \( 1/z^2 \). Because of this more rapid decay, it is noticeable only in the near-field where it can dominate the far-field term at low frequencies thereby mimic a frequency dependent \( Q \). There is a 90° phase difference between these two terms which should allow separation at least in this homogeneous modeling situation.

There is another less obvious approximation hidden in the derivation of Equation (1). From the definition of \( Q \) (Aki and Richards, 1980), we have

\[
A(t) = A_0 \left( 1 - \frac{\pi}{Q} \right)^n
\]

which Aki and Richards (1980, page 168) replace by

\[
A(t) = A_0 e^{-\pi t/Q}
\]

for large \( n \) (i.e., for large times because \( t = 2\pi\omega/\alpha \)). However, the resulting \( Q \)-estimation error is independent of \( n \) and increases with decreasing \( Q \)-factors. At \( Q = 100 \) the departure between actual and estimated \( Q \) is only a fraction of a percent which increases to approximately 6% at \( Q = 10 \).

**Modelling Results**

Zero-offset synthetic VSP traces computed with Equation (2) for a homogeneous situation are displayed in Figure 1. The parameters for this earth model are \( \alpha = 2000 \text{m/s} \), \( \beta = 879.88 \text{m/s} \), \( \rho = 2400 \text{kg/m}^3 \) and \( Q_p = 100 \). Amplitude decay with increasing depth is clearly visible. Also note the phase rotation between shallower and deeper traces where deep traces are roughly zero-phase but shallow traces are \( \sim 90° \) phase rotated. As the near-field decays with \( 1/z^2 \) (in contrast to \( 1/z \) for the far-field) its influence quickly disappears with increasing depth. At shallow depths, where the near-field predominates, we can expect time-domain \( Q \)-estimation methods to underestimate \( Q \) (overestimate attenuation) because of the \( 1/z^2 \) amplitude decay. Figure 2 shows far-field log magnitude spectra as generated with Equation (2) for depths from 61m to 1113m. The zero-phase (non-causal) Ormsby wavelet employed for these computations has the parameters 5/15-80/100 Hz. \( Q \)-factor recovery in this band is expected to be almost perfect because spectral ratio methods are derived for these circumstances with linear log ratio slopes. The deeper the receiver, the steeper the log magnitude curve. Note that all spectra in Figures 2 and 3 are normalized to the same zero dB maximum but the plotting scales differ between the Figures. The near-field equivalent to Figure 2, now for depths from 15m to 61m, is shown in Figure 3. For the near-field the slopes show an opposite trend: the shallower the receiver, the steeper the log magnitude curve. The only exception can be seen for the deepest receivers (at 53.3m and 60.9m depths) above 40Hz or 50Hz. \( Q \)-estimation leads to negative \( Q \)-factors near the source and large positive \( Q \) some distance away in the
near-field where the slope trend is reversed but the log magnitude curves are still almost parallel. As far-field conditions are approached the estimated Q converges with the model Q of 100 as displayed in Figure 4. It is interesting to note that this kind of “near-field behavior” has also been observed when estimating Q from actual VSP data. A real data example (Haase and Stewart, 2006b) is shown in Figure 5. Where do the near-field stop and the far-field begin? Aki and Richards (1980) use the term “many wavelengths” when describing the distance from source points to where far-field approximations start to be accurate. Common practice is to assume that “several wavelengths” are sufficient for far-field approximation accuracy (E. Krebes, 2007, personal communication). From Equation (4), it is clear that near-fields diminish with increasing depth z and increasing frequency ω relative to the far-field. The term α/(oz) in Equation (4) is proportional to λ/z where λ is the wave length. In Figure 6 λ/z is plotted as a function of z and ω. The maximum value of 1.0 represents the case of λ=z and the minimum of 0.1 stands for z=10λ. At 10Hz, the 10% threshold is as deep as z=2000m; in the other extreme at z=200m the frequency must be increased to 100Hz in order to reach the same 10% threshold. If we take the “many wavelengths” to mean 10λ then this prescribes a minimum evaluation frequency of 40Hz for a depth of 500m. “Near-field” and “near-surface” may not be strictly interchangeable. In reality, a 10λ cut-off is probably too restrictive because of the phase difference between near-field and far-field.

Conclusions
The near-field of a P-wave point source decays faster with distance from the source than the far-field (1/z^2 versus 1/z) and also decays with frequency. The combined near-field factor is α/(oz)=λ/(2πz) and for α/(oz)<1 far-field conditions apply. Because of the more rapid decay of low frequencies close to the source there is a “relative” increase in strength of high frequencies in this region leading to negative Q-estimates when the spectral ratio method is applied here. At some distance from the source, where log magnitude curves are parallel, the estimated Q would be infinite. Just beyond that (further away from the source point) Q-estimates show large positive values which will decrease towards the model-Q as the source-to-receiver distance is further increased and far-field conditions are approached. There is also a bias error in SRM Q-estimates that depends on 1/Q only and not on source-to-receiver separation. For Q=100 this error is only a fraction of a percent; it increases to approximately 6% for Q=10.

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References
FIG. 1. Zero-offset synthetic VSP computed with Equation (2) for a zero-phase (non-causal) 5/15-80/100Hz Ormsby wavelet.

FIG. 2. Far-field magnitude spectrum for depth range from 61m to 1113m (note the zero dB scaling for Figures 2. and 3.).

FIG. 3. Near-field magnitude spectrum for depth range from 15m to 61m. The down going wave appears to gain high frequency strength (0dB scaling).

FIG. 4. Spectral ratio method Q-estimate computed from the synthetic VSP traces shown in FIG. 1 compared to the original model-Q of 100.

FIG. 5. Q versus depth for zero offset VSP data acquired at Ross Lake, Saskatchewan.

FIG. 6. Relative near-field strength ($\lambda/z$) plotted as function of depth $z$ and frequency $\omega/(2\pi)$. Note the 10% cutoff.