Resolution of microseismic moment tensors: A synthetic modeling study

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Summary

Seismic moment tensors provide a general mathematical representation of point sources that can be used to distinguish between various microseismic source types. Here, I use synthetic tests with borehole receiver arrays to determine the geometrical conditions necessary to estimate reliably the six independent components of a full moment tensor. Complete determination requires 3-component receivers, analysis of both P- and S-waves, and an array geometry that subtends a nonzero solid angle ($\Omega$) viewed from the source. Moreover, the condition number of the generalized inverse matrix is linearly related to $\Omega^1$, indicating that moment-tensor inversion is more stable as the solid angle increases. This can be achieved using deviated boreholes and by placing receivers close to the source location.

Introduction

Microseismic methods have emerged as an important tool for monitoring fluid processes at the reservoir scale. Microseismic activity in a subsurface reservoir may result from brittle deformation of reservoir rocks due to fluid injection. Applications of microseismic monitoring have included mapping microseismic activity induced during a program of cyclic steam stimulation (McGillivray, 2005) or CO$_2$ sequestration (White et al., 2004), and monitoring and characterization of hydraulic fracturing (Nolen-Hoeksema and Ruff, 2001; Sasaki and Kaieda, 2002; Rutledge and Phillips, 2003).

As illustrated in Figure 1, microseismic source mechanisms may include slip on a pre-existing fracture surface (no net change in porosity) or opening of a new crack under tension, which increases porosity. The ability to distinguish reliably between these types of source mechanisms using far-field seismic observations would be highly beneficial for many applications, particularly for monitoring of hydraulic fracture treatments.

Seismic moment tensors provide a general mathematical description of a point source in terms of force couples. They are routinely estimated for earthquakes, but are rarely considered in microseismic applications, although well constrained moment-tensor solutions can be used to infer microseismic source type (Nolen-Hoeksema and Ruff, 2001). The purpose of this study is to provide a better understanding of the geometrical conditions required to obtain robust moment-tensor solutions for microseismic events. Only borehole receiver arrays are considered here, as the borehole environment typically provides the highest signal-to-noise ratio for recording microseismic activity.

Theory

Seismic moment tensors can be represented as a 3x3 array, normalized to unit amplitude (e.g., Lay and Wallace, 1995),

$$
M = M_0 \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix},
$$

where $M_0$ is the seismic moment and each element $M_{ij}$ represents a force couple composed of opposing unit forces pointing in the $i$-direction, separated by an infinitesimal distance in the $j$-direction (Figure 2). Conservation of angular momentum imposes the condition that $M$ is symmetric, reducing the number of independent moment-tensor elements to 6 in any co-ordinate system.

A particularly simple moment tensor is the so-called double couple, which describes the radiation pattern for slip on a fracture. A double couple can be represented using two equal, nonzero off-diagonal elements. Although most earthquakes can be well represented by double-couple mechanisms, other mechanisms are sometimes used. For example, the identity matrix corresponds to an isotropic volumetric expansion (explosion) and the compensated linear vector dipole (CLVD) has a double-strength force couple in the direction of one eigenvector and unit-strength force couples in the directions of the other two (Lay and Wallace, 1995). The latter has been used to represent tensile events that occur in hydraulic fracturing due to

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Figure 1: Examples of microseismic focal mechanisms. a. Slip on a fracture surface, which may be represented by a double couple. b. Crack opening under tension, a non double-couple source.
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nucleation and propagation of the fracture into a competent rockmass (Nolen-Hoeksema and Ruff, 2001).

For a homogeneous region around a source located at the origin, the $i$th component of particle motion arising from the radiated $P$ wave can be written as (Lay and Wallace, 1995)

$$u_i^P(x,t) = \left(\frac{\rho_0 a^3}{\gamma i}\right) \frac{1}{\gamma i} \delta_{jk} \left(\frac{r}{\rho} \right) , \quad (2)$$

where $\rho$ and $\alpha$ are density and $P$-wave velocity, $\gamma$, $i = 1,2,3$ are direction cosines of the receiver position vector, $r$ is the source-receiver offset, the dot notion denotes time derivative and the summation convention for repeated indices is used. Similarly, the radiated $S$-wave may be written as

$$u_i^S(x,t) = \left(\frac{\rho_0 a^3}{\gamma ij} \beta^{-1}\right) \delta_{ij} \left(\frac{r}{\rho} \right) , \quad (3)$$

where $\beta$ is shear-wave velocity and $\delta_{ij}$ is the Kronecker delta.

The relationship between the data and model components can be written in matrix form,

$$d = Am \quad , \quad (4)$$

where $d = (a_1^P, a_2^P, a_3^P, a_1^S, a_2^S, a_3^S)^T$ defines the observed amplitudes of the $P$- and $S$-wave direct arrivals and $m = (M_{11}, M_{22}, M_{33}, M_{12}, M_{13}, M_{23})^T$ defines the independent components of the seismic moment tensor. For a single receiver, the 6x6 matrix $A$ can be easily surmised from equations (2) and (3). For $n$ receivers, the $6n \times 6$ system can be formed by appending additional rows, as required, onto $d$ and $A$. A least-squares solution to this over-determined system can be obtained using the generalized inverse,

$$m = A^{-g} d = (A^T A)^{-1} A^T d \quad . \quad (5)$$

In most of the examples below, the generalized inverse is ill-conditioned. In such cases, the number of resolvable components of $m$ is assumed to be equal to the number ($N$) of significant eigenvalues of $A^T A$. Significant eigenvalues are defined here as those with absolute value greater than one ten thousandth of the absolute value of the largest eigenvalue. The best resolved component of $m$ is taken as the largest component of the eigenvector that corresponds to the largest eigenvalue.

Results

As illustrated in Figure 3, several scenarios are considered here to investigate the resolution of $m$ using realistic borehole receiver arrays. The source is located at $x = y = 0$ and a depth of 1000 m. Array 1 represents a vertical set of receivers between 200-700 m depth, located 100 m north of the source. Array 2 represents a set located between 500-700 m depth and includes a 100 m borehole deviation to the east. Receivers are shown every 20 m in Figure 3, but numerical tests show that 2-3 receivers at the corner points give equivalent results (i.e., sparse observations are sufficient to invert for $m$).
Different seismic moment tensors predict different relative amplitudes and polarities of P- and S-wave arrivals. This concept is illustrated in Figure 4, which shows synthetic seismograms for two different moment tensors, a double couple and a CLVD source. It is these differences in amplitude and polarity that render it possible to distinguish different seismic sources. The insets in each part of Figure 4 represent so-called beach-ball diagrams, which are standard in earthquake seismology. They indicate the P-wave polarity (black = out, white = in) on the lower unit hemisphere, centered on the source.

Table 1 summarizes a series of tests conducted using both arrays in conjunction with either vertical and three-component (3-C) receivers. In addition, analyses of P-wave only, or both P- and S-waves are considered. The results of these tests indicate that the full moment tensor (6 independent components) can only be obtained using 3-C receivers using array 2, coupled with the analysis of both P-waves and S-waves. In general, numerical tests indicate that a necessary condition for the moment tensor to be fully resolved is that the receiver array subtends a solid angle $\Omega > 0$, viewed from the source. The solid angle is given by the surface integral

$$\Omega = \int_{S} \frac{\hat{r} \cdot \hat{n}}{r^3} dS.$$  

As shown in Figure 5, for cases where $\mathbf{m}$ can be fully resolved (i.e., 3-C receivers, both P and S analyzed, $\Omega > 0$), synthetic tests show that the condition number of $\mathbf{A}^T\mathbf{A}$ is linearly related to $\Omega^{-1}$. Thus, the geometry of the receiver array should be designed to maximize the field of view, as seen from the source. Vertical arrays of geophones do not accomplish this, since the solid angle subtended by the array is zero. In general, $\Omega$ can be increased by using a deviated borehole and/or multiple boreholes, as well as by placing the receivers as close as possible to the source.

Although other scenarios are ill-conditioned and so do not allow the full model vector to be determined, the structure of the generalized inverse reveals a number of interesting features. For the receiver geometry considered here, where the array is situated above the source, the best-resolved moment-tensor component with P-waves only is $M_{33}$. This is not surprising, since this component corresponds with a vertically oriented force couple. When S waves are also considered, the best resolved component is always a shear couple. For vertical-component receivers, the best resolved component represents shear forces that produce SV waves, whereas the use of 3-C receivers makes the best resolved component that which produces SH waves.

<table>
<thead>
<tr>
<th>Receiver Type</th>
<th>Wave Type</th>
<th>Rec. Array</th>
<th>$N^1$</th>
<th>Best resolved component of $\mathbf{M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-C</td>
<td>P</td>
<td>1</td>
<td>2</td>
<td>33</td>
</tr>
<tr>
<td>3-C</td>
<td>P+S</td>
<td>1</td>
<td>3</td>
<td>23</td>
</tr>
<tr>
<td>3-C</td>
<td>P</td>
<td>2</td>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>3-C</td>
<td>P+S</td>
<td>2</td>
<td>5</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 1. Summary of test results.

$^1$ Number of significant eigenvalues; see text.
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Conclusions

Obtaining full moment-tensor solutions from borehole observations of microseismic activity is potentially useful for discriminating between deformation that yields no net change in porosity (e.g., slip on fracture surfaces) and other types of deformation (e.g., opening of cracks). Numerical tests considered here indicate that there are several conditions that must be satisfied in order for the inversion to be well-conditioned. These are: 1) the receiver array must subtend a nonzero solid angle, viewed from the source; 2) 3-C observations are required; 3) both $P$- and $S$-wave amplitudes are required. The first condition is never satisfied by a linear array of receivers.

Further work is needed to assess these results under realistic conditions. For example, noise, incomplete knowledge of the source waveform, amplitude and timing uncertainties are not considered. These factors are currently being investigated.

Acknowledgments

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Figure 5: Condition number of the matrix $A^TA$ used in the generalized inverse, versus solid angle $\Omega$ subtended by the receiver array, in steradians (sr). The condition number is the ratio of the largest to smallest eigenvalue, and is a measure of ill-conditioning of the inverse. The slope of the graph is -1, indicating that the condition number is proportional to $\Omega^{-1}$. 