Converted wave processing in the EOM domain

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Summary

A new approach for converted wave prestack migration and velocity analysis has been developed. That is based on the consideration of prestack migration by equivalent offset and common scatter points (CSP). During the process, a converted wave velocity \( V_c \) was estimated from the hyperbolic moveout on CSP gathers. Moveout (MO) correction and stacking complete the prestack migration. The intent of this paper is to see what additional uses can be made with \( V_c \).

Converted wave CSP are formed by summing all input traces at the equivalent offset migration. \( V_c \) can be quickly and accuracy estimated by gotten a limited converted wave CSP (LCCSP) gather. This \( V_c \) is valid only for zero offset data, however we can extend its application when there is an acceptable small error in the estimated travelt ime.

For a given trace with an acceptable time error, there may be still considerable trace energy to form a reasonable LCCSP.

Introduction

Reflection seismic exploration has been concerned predominately with P-wave energy for many reasons that include: compressional waves arrive first, usually they have high signal-to-noise ratios, particle motion that is usually close to rectilinear, are easily generated by a variety of sources, and propagate in fluid. Because many basins are or will be soon in mature stage, we need the use of new technologies, and converted shear wave seismic exploration is one of them. Converted wave exploration refers to a downward-propagating P-wave, converting on reflection at its deepest point of penetration to an upward-propagating S-wave.

Kirchhoff Prestack Migration concepts

Kirchhoff prestack migration is based on a model of the subsurface as an organized set of scattered points. The model assumes that energy may come from a source located anywhere on the surface to all receivers. The location of energy on a recorded trace is the total travelt ime along the ray path from the source down to the scatterpoint and back up to the receiver. Kirchhoff prestack migration assumes an output location, and then sums the appropriate energy from all available input traces.
The surface projection of a vertical array of scatterpoints is referred to as the common scatter point (CSP) location. From the raypaths showed in Figure 1, the traveltime $t$ is estimated by adding the time from the source to the scatterpoint $t_s$, and time from the scatterpoint to the receiver $t_r$, or

$$ t = t_s + t_r. \tag{1} $$

From the geometry, the total or two-way traveltime can be computed from:

$$ t = \left(\frac{t_0}{2}\right)^2 + \frac{(x+h)^2}{V_{mig}^2} \right)^{1/2} + \left(\frac{t_0}{2}\right)^2 + \frac{(x-h)^2}{V_{mig}^2} \right)^{1/2} \tag{2} $$

where $x$ is the location of the source-receiver midpoint (MP) relative to the scatterpoint (SP) located at $x=0$, and $V_{mig}$ is the RMS migration velocity evaluated at $t_0$. The time $t_0=t$ is the two-way zero-offset time and $t_o$ is defined from the data. The equation (2) is known as the double square root (DSR) equation and defines the traveltime surface over which the Kirchhoff summation or integration takes places.

**Equivalent Offset Migration**

The equivalent offset is defined by converting the DSR equation (2) into an equivalent single square root or hyperbolic form (Bancroft et al., 1998). This can be reformulated by defining a new source and receiver collocated at the equivalent offset position $E$ as illustrated figure 1. For convenient, the CSP gather is located at $x=0$. The equivalent offset $h_e$ is chosen to maintain the same traveltime from equation (14):

$$ t = 2t_e = t_s + t_r. \tag{3} $$

This traveltime can be written as:

$$ 2\left(\frac{t_0}{2}\right)^2 + \frac{h_e^2}{V_{mig}^2}\right)^{1/2} = \left(\frac{t_0}{2}\right)^2 + \frac{(x+h)^2}{V_{mig}^2} \right)^{1/2} + \left(\frac{t_0}{2}\right)^2 + \frac{(x-h)^2}{V_{mig}^2} \right)^{1/2}. \tag{4} $$

This equation may be solved for the equivalent offset $h_e$ to get:

$$ h_e^2 = x^2 + h^2 - \left(\frac{2xh}{V_{mig}}\right)^2. \tag{5} $$

The equivalent offset is a quadratic sum of the distance $x$ between the CSP and the CMP, and $h$, the source-receiver half offset.
**Converted wave migration using a single velocity**

Converted wave processing assumes the downward propagating energy is a P wave and reflection energy a shear wave. The processing methods start with the DSR equation (1) or (3), with the appropriate P and S velocities for each leg of the ray path, as illustrate in Figure 1.

From equation (4) and using the concepts of prestack time migration and RMS velocities for both, the P-wave and S-wave energy, the traveltime is defined by:

\[ t = \frac{1}{V_p} \sqrt{z_0^2 + h_s^2} + \frac{1}{V_s} \sqrt{z_0^2 + h_r^2} = \left( \frac{1}{V_p} + \frac{1}{V_s} \right) \sqrt{z_0^2 + h_e^2} = \frac{2}{V_c} \sqrt{z_0^2 + h_e^2}, \tag{6} \]

where \( V_c \) is defined as

\[ V_c = \frac{2V_pV_s}{V_p + V_s} = \frac{2V_p}{1 + \gamma}. \tag{7} \]

Note that \( V_c \) will be a solution of \( z_0 \) (or \( t_0 \)).

The equivalent offset \( h_e \) can also be redefined as:

\[ h_e^2 = \frac{V_c^2}{4} \left( \frac{1}{V_p} \sqrt{z_0^2 + h_s^2} + \frac{1}{V_s} \sqrt{z_0^2 + h_r^2} \right)^2 - z_0^2. \tag{8} \]

From the equation (8), \( h_e \) varies with the trace geometry \( h_s \), and \( h_r \), but also varies with depth \( z_0^2 \) and the velocity \( V_c(z_0) \), as \( V_p \) can also be a function of depth \( z_0^2 \).

An equivalent offset method of migrating converted wave data has been developed. During this process we estimate a converted wave velocity \( V_c \) from the hyperbolic moveout on a CSP gather. Moveout (MO) correction and stacking completes the prestack migration. We obtain \( V_c \) from equating the zero offset traveltimes with the original offset traveltimes.

Is it possible to ignore \( V_p \) and \( V_s \) and simply use \( V_c \) as a velocity for the entire input data.

\[ t_{p-s} = \frac{1}{V_p} \sqrt{z_0^2 + h_s^2} + \frac{1}{V_s} \sqrt{z_0^2 + h_r^2}, \tag{9} \]

and

\[ t_{Vc} = \frac{1}{V_c} \sqrt{z_0^2 + h_s^2} + \frac{1}{V_c} \sqrt{z_0^2 + h_r^2}. \tag{10} \]

Given \( h_s = x + h \) and \( h_r = x - h \), and if we assume either \( x = 0 \) or \( h = 0 \), then \( t_{h = 0 = x} = t_{Vc} \). For all other conditions, \( t_{h = 0} \neq t_{Vc} \).

The figure (2) represents the location of energy on a CSP gather and we can see where the limited offset data will lie. These figures (2) show the travelttime errors on CSP gathers views of various offsets of \( h = 50, 200, \) and 500.

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Figure 2: Traveltime difference on a CSP gather for various half offsets \( h \) equal to: a) 50 m, b) 200 m and c) 500 m. Note that the values on the time scale vary for each figure.

The previous figures assume the source is on the left and the receiver is on the right. The converted wave ray-paths are asymmetrical and produces and image that is asymmetrical about \( x = 0 \). Swapping the source-receiver locations will reverse the equivalent offsets. A center spread acquisition system will produce LCCSP gathers with opposite polarities of the traveltime difference that will tend to sum to zero and remove any bias in the gather.

**Conclusions**

Converted wave prestack migration by equivalent offset is based on the principles of Kirchhoff migration and uses equivalent offset to form limited converted CSP (LCCSP) gathers.

The DSR equation for prestack migration can be reformulated with an appropriate P and S velocities for each leg of the ray path. Using relation between these two velocities, a converted wave velocity can be estimated from the hyperbolic moveout on the CSP gathers.

An acceptable time error may be defined to form a LCCSP gather by assuming a constant converted wave velocity. The intended application is to rapidly form a LCCSP gather to provide an initial velocity model for converted wave prestack migration using the equivalent offset method.

The range of acceptable data is dependent on the P-wave velocity and an assumed S-wave velocity. However, the gather formed is independent of those velocities, and a more accurate S-wave velocity (or \( \gamma \)) is estimated.

**Acknowledgements**

We thank to the sponsors of the CREWES Project for their support.

**References**


