Introduction

A practitioner of AVO analysis might plausibly take an interest in anelastic reflection coefficients, which are the subject of this paper, for one of two reasons. One might wish to protect an elastic AVO procedure from errors due to unaccounted for anelastic influences. Or, one might seek to use the variations in such coefficients to infer additional properties of the target. The following discussion is largely insensitive to which of these two attitudes is taken: direct anelastic inverse theory (e.g., Innanen, 2011) leads both to formulas for estimation of fully anelastic parameters (e.g., $Q_P$ and $Q_S$), and, simultaneously, to formulas for estimation of more standard elastic parameters, and the latter are “protected” from that particular source of error. Its focus will be on anelastic AVO/AVF in a theoretical and experimental context, ongoing implementation efforts, and open issues whose resolution will be necessary for anelastic AVF/AVO/AVA inverse theory to be considered in any sense complete.

Anelastic reflections — theory and experiment

Theory. The fact that a contrast in lossy medium properties leads to a characteristic reflection signature was probably first established in optics, wherein a phase shift has long been known to accompany a reflection of light from a conducting target (Born and Wolf, 1999). The same idea has appeared sporadically in seismology since the 1960s (an early example being the discussion of White, 1965), sometimes as tangential discussions (e.g., Ch. 4, Kjartansson, 1979), or as a consequence of a viscoelastic treatment of the seismic wave problem, in the latter case both as applied in exploration (e.g., Samec and Blangy, 1992; Carcione, 2001), and more generally (Borcherdt, 2009). There seems to have been a marked increase in the number of discussions on this subject in recent years, both in geophysical contexts (Chapman et al., 2006; Lines et al., 2008; Quintal et al., 2009; Ren et al., 2009) as well as acoustic contexts (Lam et al., 2004; de Hoop et al., 2005). It may be that, ultimately, anelastic reflections will be most usefully understood in terms of poroelastic theory, which, in addition to providing a likely contributing mechanism (e.g., Müller and Gurevich, 2005), has been used to predict specific characteristics of reflection coefficients (e.g., Gurevich et al., 2004). With these developments, as well as an increase in the sophistication with which anelastic wave propagation is modelled numerically (e.g., Carcione, 2010), opportunities for quantitative anelastic AVO/AVF analysis may grow significantly in the near future.

Experimental and field evidence. Discussion of the characteristic reflectivity of anelastic targets and its influence on AVO is not merely hypothetical. The requirement of a suitable anelastic reflection coefficient to reproduce the angle dependence of the ultrasonic response of a stainless steel target is noted by Borcherdt (2009). Closer to the monitoring and E & P problem, the capability of fluid-bearing reservoir targets to cause measurable anelastic reflection responses has been discussed by Chapman (2008), and, critically, such “dispersion anomalies” have been positively identified in reservoir data (Odebeau et al., 2006).

Inverse & estimation methods. Estimation methods for seismic $Q$ primarily make use of the influence of lossy propagation on the waveform. These include all standard $Q$ estimation schemes, and tomographic/full-waveform inversion schemes that incorporate $Q$ (a full reckoning of which is beyond the scope of this paper). Inverse scattering, an inversion method closely connected to AVO and seismic amplitudes, will be focused on here. Several approaches to the attenuative inverse scattering problem have been taken, some involving an optimization-type inversion (e.g., Ribodetti and Virieux, 1998; Mulder and Hak, 2009), and others involving direct inversion, either assuming prior knowledge of the P-wave velocity structure of the Earth (Carrion and VerWest, 1987), or not (Innanen, 2003; Innanen and Weglein, 2004, 2007). The latter analysis has clarified that (1) anelastic reflectivity is the particular data variability constraining $Q$ in direct methods, and (2) target dispersion is a critical aspect of multiparameter inversion. The nonlinear form of the problem, as well as the use of anelastic reflectivity to form $Q$ compensation operators was discussed by Innanen and Lira (2010), and an alternative form of nonlinear
absorptive inversion, through iteration of the linear problem has been considered by Hak and Mulder (2010b). The same problem, studied for a fixed horizontal interface at known depth, transforms into an anelastic inversion scheme directly analogous to elastic AVO/AVA inversion (Innanen, 2011), which is our current interest.

**Practical issues: extraction of anelastic reflectivity information**

Several anelastic parameters undergoing simultaneous contrast at a target may be separately estimated, if (1) the target is correctly treated as dispersive, and (2) the frequency dependence of the dispersion is not arbitrary but constrained by the choice of $Q$ model (Innanen and Weglein, 2007; Innanen, 2011). Neglecting these issues can lead to ill-posed algorithms in larger scale inverse scattering problems (Mulder and Hak, 2009; Hak and Mulder, 2010a,b), as well as AVO/AVF amplitude analysis. For our current purposes, points (1)-(2) underscore the fact that the information content of an anelastic reflection coefficient is primarily spectral, which complicates the practical matter of analyzing reflection data to estimate the reflection coefficient input. How that issue is managed is discussed in this section.

**Wave methods.** Extraction of local reflectivity spectra is not a necessary step if the full scattered wave field is treated in direct anelastic or anacoustic inversion, which is how in general inverse scattering methods work. In that case anelastic reflectivity is an intrinsic part of the larger anelastic wave picture. Trivially then this practical issue is avoided if the full wave inverse problem is addressed.

**Signal processing methods.** Absorptive reflection coefficients may be analysed in terms of their amplitude variation with frequency (AVF) or angle (AVA), but, regardless of which is considered, for local inversion to proceed the spectral characteristics of specific events must be exposed through some form of time-frequency decomposition (e.g., Wu et al., 2010). Recently, a fast S-transform algorithm has been used to produce practical input to an anelastic AVO/AVF algorithm (Bird et al., 2011). The fast S-transform generates blocky averages of local spectra, which, when the algorithm has been properly calibrated, faithfully track the raw spectra of the reflection coefficients (Figure 1). Thus, if the event in question bears an anelastic AVF imprint, the imprint emerges as a local trend in the amplitudes of the S-transform. The inversion procedure, adjusted to accept this local reflectivity data information, averaged over the S-transform bands, then has its required input.

**Open issues**

**Theoretical issues.** Even excluding issues of rock-physics mechanism (such as poroelasticity), a fully-realized wave theory for anelastic seismology is not yet available, and this may represent the largest single obstacle to practical anelastic AVO. For example, coupling anelasticity with anisotropy opens questions about appropriate definitions of $Q$ (Červený and Pšenčík, 2005, 2008); for another example, there currently exist unresolved discrepancies in theoretical forms for anelastic reflection coefficients, wherein no one definition or sign choice for vertical slowness leads to consistent post-critical behaviour (Krebes and Daley, 2007; Morozov, 2011). Our scattering representation and inversion research thus far has avoided regimes where such theoretical uncertainty exists, but a meaningful anelastic AVO/AVF methodology will, ultimately, rely on the presence of a solid theoretical foundation.

**Q model and parametrization.** In the $(k_z, \omega)$ domain, an anacoustic example of the type of reflectivity considered in AVF inversion is $R = (1 - \Omega)/(1 + \Omega)$, where $\Omega = k_{z1}/k_{z0}$ is the ratio of depth wavenumbers $k_{z1} = (K_1^2 - k_x^2)^{1/2}$ and $k_{z0} = (K_0^2 - k_x^2)^{1/2}$. If the incidence medium is acoustic we have $K_0 = \omega/c_0$; and, following a common choice of $Q$ model, the target propagation constant $K_1$ is

$$K_1 = \frac{\omega}{c_1} \left[ 1 + \frac{i}{2Q_1} - \frac{1}{\pi Q_1} \log \left( \frac{\omega}{\omega_0} \right) \right]. \quad (1)$$

In this equation it is possible to discern behaviour important to a well-posed inversion. First, the dispersive behaviour of the target medium is specified by the model type, through the logarithm. In inverting
Figure 1  Three examples of fast S-transform decomposition of events bearing the spectral imprint of anelastic (i.e., dispersive) reflection. L-R columns: reflections from a target with a fixed P-wave velocity contrast and a contrast from $Q = \infty$ to $Q = 20$, $Q = 14$, and $Q = 8$. In the bottom panels spectral profiles are extracted and compared to the (in practice unknown) exact reflection coefficients $R(\omega)$. The blocky approximated spectra track the average of $R(\omega)$ and thus form input for practical AVF inversion.

we distinguish between knowing that $K_1$ follows this model and knowing the value of $Q_1$. Knowing or assuming the former, we may form a well-posed inverse AVF problem to determine $Q_1$ and $c_1$ (in this case). However, if we were to collapse the two rightmost terms in equation (1) into an arbitrary, frequency dependent $Q_1$ such as $K_1' = \omega c_1^{-1} \{ 1 + i[2Q_1(\omega)]^{-1} \}$, separate inversion for $c_1$ and $Q_1(\omega)$ becomes ill-posed absent prior information. The open question, then, is how to know in advance what $Q_1$ model type to choose, granting that we will not know the value of the parameters in the model until after the inversion is complete? Also, should auxiliary parameters such as $\omega_0$ be estimated/fixed prior to inversion, or included as additional unknowns?

Sources of ambiguity.  Finally, thus far we have also made the assumption that a complex, frequency-dependent reflectivity must be due to an anelastic contrast and cannot have some other origin. It is well-known that a sequence of thin layers can mimic the influence of attenuation on wave propagation, through scattering. It is also true that small phase changes occurring in a wave as it reflects from a sequence of thin layers can present as a single bulk reflection with a complex reflection coefficient (e.g., Brekhovskikh, 1960), and thus mimic attenuative reflection. It is not clear that thin layer reflectivity vs. anelastic reflectivity can be distinguished without some level of geological prior knowledge.

Conclusions

A pursuit of well-posed equations for anelastic AVO/AVF analysis and inversion is warranted by theoretical predictions, coupled with experimental and field evidence that such characteristic reflectivity appears at measurable levels in hydrocarbon relevant data. Direct inverse theory appropriately deployed generates methods to map from reflection coefficient data to Earth property contrasts at the point of reflection, and time-frequency decomposition methods provide estimates of those data from reflection seismic traces & gathers. Several open issues remain, ranging from foundational/theoretical to practical and geological, nevertheless the potential for increasing our ability to infer subsurface properties in
reservoir settings appears to be high.

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References


