On the calibration of a fast S-transform with application to AVF inversion of anelastic reflectivity

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SUMMARY

The S-transform is a time-frequency decomposition technique with a wide range of applications in seismic signal analysis. However, it has been under-used due to the high computational cost needed to employ it. In this paper, we present a fast, non-redundant S-transform (FST). The ability of the FST to provide high-fidelity estimates of reflection seismic amplitudes is tested in this paper. Where the FST cannot provide accurate estimates of amplitudes, we calibrate it by normalizing the unit impulse response of the algorithm.

Strongly dispersive reflection coefficients associated with highly absorptive, hydrocarbon charged targets, have been observed in seismic data. A frequency by frequency method (AVF) for determining Q of a highly absorptive target from measurements of the dispersive reflection coefficient has been developed to invert data variations of this kind. In order to implement the AVF technique to invert for Q, it is necessary that we have a method of estimating the local spectrum of the reflection coefficient. We develop a method of implementing AVF inversion by using the calibrated FST to estimate the local spectrum of dispersive reflection coefficients. We test the effectiveness of the FST for estimating the spectrum of reflection coefficients by comparing with the analytic reflection coefficient as calculated by a fast Fourier transform. Using forward modeling to generate synthetic traces containing absorptive reflection coefficients, we observe accurate results of the AVF inversion for a range of Q values.

INTRODUCTION

Time-frequency decomposition methods provide a means to estimate the local spectrum of recorded seismic events. There are a range of time-frequency decomposition methods with applicability to seismic signal analysis (Margrave, 1997; Margrave et al., 2003). The S-transform is one such method which utilizes a Gaussian window and provides progressive resolution (Stockwell et al., 1996). The fast general Fourier family transform (FGFT) proposed by Brown et al. (2010) and applied to data reconstruction problems by Naghizadeh and Innanen (2010) provides a fast, non-redundant method of calculating the S-transform, henceforth referred to as the fast S-transform (FST). One of our immediate problems is the estimation of target absorption from anelastic reflectivity. In order for us to estimate target absorption, we must be able to obtain high-fidelity estimates of local reflection amplitudes as a function of frequency. It was our goal then, to test the ability of the FST to provide high-fidelity estimates of the local spectra of seismic reflections. Where it failed to do so, we calibrated the amplitudes by normalizing the unit impulse response of the FST algorithm.

The quality factor, Q, is a measure of how absorptive an Earth material is. A low Q value corresponds to a highly absorptive material. The ability to determine Q presents an important problem in exploration seismology because absorption is closely related to rock fluid properties such as viscosity, porosity and fluid saturation (Quan and Harris, 1997; Vasheghani and Lines, 2009). Absorption is not easy to measure in the field as it is difficult to isolate the effect of absorption on the seismic pulse from other attenuation mechanisms (Sheriff and Geldart, 1995) such as geometrical spreading and scattering. However, methods of determining Q exist such as the centroid frequency method which uses the shift in average frequency between two points to determine Q (Quan and Harris, 1997).

It has been observed that strong absorptive reflection coefficients, caused by reservoirs with very low Q, cause frequency dependent seismic anomalies (Odebeutu et al., 2006). In this paper a frequency-by-frequency (AVF) method of inverting for Q from absorptive reflections (Innanen, 2011) is reviewed and a method of implementing this technique using the calibrated FST is proposed. The method is tested using synthetic traces modeled with a single absorptive reflection. The ability of the FST to estimate the spectrum of the absorptive reflection coefficients is compared with the analytic result as calculated by a fast Fourier transform.

BACKGROUND

The Fourier transform (FT) does not enable us to examine the time-varying spectral content of a signal. This may be problematic because a seismic signal is often non-stationary, meaning it’s frequency content changes with time. The S-transform is a time-frequency decomposition method which utilizes a Gaussian window and provides progressive resolution (Brown et al., 2010). The S-transform of a time signal g(t) is defined by Stockwell et al. (1996) and presented by Brown et al. (2010) as

$$S(\tau, \omega) = \int_{-\infty}^{\infty} g(t) \frac{\omega}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2}} e^{-j2\pi\omega t} dt,$$  \hspace{1cm} (1)

where \(\tau\) and \(\omega\) are the time coordinate and frequency coordinate of the S-domain respectively and \(\frac{\omega}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2}}\) is the Gaussian window. Equation (1) demonstrates how the S-transform is calculated directly from the time domain.

As presented in Brown et al. (2010) and implemented by Naghizadeh and Innanen (2010) it is possible to calculate the ST using a fast and non-redundant method (for details see Brown et al., 2010; Bird et al., 2010; Naghizadeh and Innanen, 2010). Figure 1 shows a unit impulse (Figure 1a) and its corresponding S-domain generated by the FST (Figure 1b). Figure 1c shows a profile through the S-domain at the time sample of the unit impulse, hence Figure 1c shows the unit impulse response (UIR) of the FST. The Fourier spectrum of a unit impulse is well known to be a constant value at unity for all frequencies. By looking at Figure 1c it can be seen that the spectrum of the unit impulse is not a constant value but rather increases with a
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stair-case trend. The goal of this paper is to calibrate the amplitude spectrum of the FST by removing the fast transform algorithm footprint, as observed in Figure 1, by correcting its unit impulse response to be a constant value at unity.

![Figure 1: UIR of FST before calibration. (a) is a unit impulse input into the FST, (b) is its S-domain and (c) is a frequency profile through the S-domain at the time sample of the unit impulse.](image1)

**AMPLITUDE CALIBRATION OF THE FST ALGORITHM**

In order to calibrate the FST a code was developed which calculates and implements the corrections needed to make the spectrum of a unit impulse, at any location in time, a constant at unity. The corrections are stored in a matrix and then the matrix is multiplied with the S-domain. This matrix will hereafter be referred to as the FST spectrum fidelity correction matrix (SFCM). For every N-point signal there is a corresponding NxN point S-domain and SFCM. For a given input signal, the SFCM is found by first determining the UIR at every location in time. Then for each of these UIR’s a set of N scaling coefficients is produced by determining the multipliers needed to scale the UIR to a constant value at unity. This is carried out by dividing 1 by the UIR at all frequencies. Hence, the jth row of the SFCM contains the scaling coefficients that need to be multiplied with the S-domain spectrum at the jth time sample. The SFCM corrects the dependence of the UIR on position in time and it also takes care of all other residual algorithm footprint. With this correction matrix applied to the S-domain, a unit impulse input into the FST algorithm at any location will yield the desired constant value. Figure 2 Shows the S-domain for a unit impulse with the SFCM applied and it’s profile in the S-domain. As can be seen, the UIR is as desired.

![Figure 2: UIR of FST after calibration. In (a) we have a unit impulse in time. In (b) we have the calibrated S-transform of the unit impulse and in (c) we see the frequency profile of the unit impulse through the S-domain](image2)

**MOTIVATION**

If a plane wave is incident upon a planar boundary, oriented perpendicularly to z, and separating two media then the reflection coefficient will be given by

$$R = \frac{k_z - k_{z+1}}{k_z + k_{z+1}}, \quad (2)$$

where $R$ is the reflection coefficient, $k_z$ is the vertical wavenumber for the layer above the interface and $k_{z+1}$ is the vertical wavenumber for the layer below the interface. In order to model absorptive reflection coefficients, we use an expression for wavenumber which includes a model for nearly constant Q described by Aki and Richards (2002) as

$$k = \frac{\omega}{c} \left( 1 + \frac{i}{2Q} \frac{\log(\omega/\omega_r)}{\pi Q} \right). \quad (3)$$

Here $c$ is the seismic velocity, $Q$ is quality factor, $\omega$ is the frequency component, and $\omega_r$ is a reference frequency. For waves at normal incidence the expression for wavenumber ($k$) given in (3) can be used as the vertical wavenumber ($k_z$) and implemented in equation (2). Now consider an elastic overburden overlying a highly attenuative target(with velocity $c_2$ and quality factor $Q$) then we may write the expression for anelastic reflection coefficients as

$$R(\omega) = \frac{1 - \Omega(\omega)}{1 + \Omega(\omega)}, \quad (4)$$

where

$$\Omega(\omega) = \frac{c_1}{c_2} \left[ 1 + \frac{i}{2Q} \frac{\log(\omega/\omega_r)}{\pi Q} \right] \quad (5)$$

now if we make the following substitutions (from Innanen, 2011) $F(\omega) = \frac{1}{2} - \frac{1}{2} \log(\omega/\omega_r)$, $a_C = 1 - \frac{c_1^2}{c_2^2}$ and $a_Q = \frac{1}{Q^2}$.

We can expand equation (4), and linearize (assuming small $a_Q$ an $a_C$) to obtain the following expression
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\[ R(\omega) \cong \frac{1}{2} a_Q F(\omega) + \frac{1}{4} a_C, \]  

(6)

where \( a_Q \) and \( a_C \) are the perturbation parameters in \( Q \) and acoustic seismic velocity respectively. It is also worth noting that \( F(\omega) \) is a known function. Equation (6) defines the forward problem of calculating \( R(\omega) \) given \( a_C \) and \( a_Q \). The inverse AVF problem is to determine \( a_Q \) or \( a_C \) from measurements of \( R(\omega) \). In this paper, we wish to obtain reliable measurements of \( R(\omega) \) in order to determine \( a_Q \). Equation (6) contains two unknowns, \( a_C \) and \( a_Q \). If we can determine \( R(\omega) \) for two different frequencies, \( \omega_1 \) and \( \omega_2 \), then we can take the difference between \( R(\omega_1) \) and \( R(\omega_2) \) and obtain an expression for \( a_Q \) given by

\[ a_Q = -2 \left( \frac{R(\omega_1) - R(\omega_2)}{F(\omega_1) - F(\omega_2)} \right). \]  

(7)

From \( a_Q \) we may calculate the target \( Q \) value.

**PROBLEM**

Equation (7) provides an expression for inverting for \( a_Q \) provided we have an estimate of \( R(\omega) \) for at least two different frequencies. The problem is how to determine \( R(\omega) \) from recorded seismic data. It is necessary that we be able to estimate the local spectra of seismic events. We employ the calibrated FST as discussed above to estimate the local spectra of absorptive reflections.

The FST generates blocky averages of the local spectra of a signal. We will write \( \tilde{R} \) to denote the average of the spectrum of a dispersive reflection \( R(\omega) \) over a given frequency band \( x \). We use forward modeling codes to generate synthetic traces with a single absorptive reflection and then implement the FST to estimate the local spectrum of the absorptive reflection. Finally, we invert for \( a_Q \) using a modified version of equation (7) to take input from the FST. For synthetic traces with a single reflection, the true spectrum of the reflection coefficient, \( R(\omega) \) may be obtained by using a fast Fourier transform (FFT). For our synthetic traces with a single absorptive reflection, we compare the spectrum of the reflection coefficient as estimated by the FST with the true spectrum as calculated by an FFT.

**METHOD**

We start by generating a synthetic trace with a single absorptive reflection coefficient (see Figure 3). Figure 3(a) is a plot of the synthetic trace, modeled using a plane wave normally incident upon a planar boundary separating an acoustic medium overlaying a medium with very low Q. Figure 3(b) is a plot of the synthetic trace in the S-domain. The black line in Figure 3(c) is the local spectrum of the reflection \( \tilde{R}(\omega) \) which is obtained by extracting the profile of the reflection through the S-domain. The blue line in Figure 3(c) is a plot of the true spectrum, \( R(\omega) \), calculated directly from equation (5). Note that \( \tilde{R} \) in Figure 3(c) is not a continuous spectrum; it has a blocky appearance due to the tiling associated with the FST algorithm (see Bird et al., 2010). Notice that the spectrum estimated by the FST is a very good approximation to the average of the true spectrum across the frequency band of each tile.

Figure 3: How to extract a frequency-dependent reflection coefficient from a seismic trace. In (a) the single primary reflection generated by a contrast from acoustic to highly attenuative media is plotted in the time domain; in (b) the calibrated fast S-transform is carried out on the trace in (a), identifying the location of the event and estimating its spectrum; in (c) the amplitudes picked from the S-transform (in black) is compared with the analytic reflection coefficient (in blue).

We likewise let \( \tilde{R}M_1 \) to be the average of \( R(\omega) \) from frequency \( M_1e \) to \( M_1s \) and \( F_{M1} \) be the sum of the known \( F(\omega) \) function over the same frequency band

\[ \tilde{R}_{M1} = \sum_{i=M1s}^{M1e} R(\omega_i), \]  

(8)

where \( \tilde{R}_{M1} \) is the average of \( R(\omega) \) from frequency \( M_1s \) to \( M_1e \). We now define \( F_{M1} \) as the sum of the known \( F(\omega) \) function over the same frequency band

\[ F_{M1} = \sum_{i=M1s}^{M1e} F(\omega_i), \]  

(9)

We likewise let \( \tilde{R}M_2 \) to be the average of \( R(\omega) \) from frequency \( M_2s \) to \( M_2e \) and \( F_{M2} \) be the sum of the known \( F(\omega) \) function over the same frequency samples as \( \tilde{R}_{M2} \). Now, let \( M_1 = (M_1e - M_1s) \) and \( M_2 = (M_2e - M_2s) \) and define \( Z \) such that \( Z \cdot M_2 = M_1 \). Then we can rewrite equation (7) in terms of \( \tilde{R}_{M1}, \tilde{R}_{M2}, F_{M1}, \) and \( F_{M2} \)

\[ a_Q = -2 \left( \frac{M_1 \tilde{R}_{M1} - Z M_2 \tilde{R}_{M2}}{F_{M1} - Z F_{M2}} \right). \]  

(10)

With equation (10), we may invert for \( a_Q \) after using the FST to estimate the local spectrum of absorptive reflection coefficients.
RESULTS

We implement equation (10) to invert for $Q$ by generating synthetic seismic traces with a single absorptive reflection, and using the FST to estimate the spectrum of the reflection. The traces are generated for a two-layer, single interface model in which $Q$ is very high above the interface and very low below the interface. We use the FST to estimate the spectrum of the reflection and then implement equation (10) to invert for $Q$. A number of traces were modeled in which $Q$ below the interface ranged from 120 to 1, the inversion was performed and compared with the actual value for accuracy. This is shown in Figure 4 and Figure 5, where the red line is the inverted $Q$ value and the black line is the actual $Q$ value used to model the reflection. From these figures, it appears that the inversion works best for $Q$ values greater than 15 but works satisfactorily until $Q$ drops below 8.

As can be seen from Figure 4 and Figure 5 the inversion works satisfactorily until $Q$ drops below 8 and then the difference between the inverted $Q$ value and the actual value becomes quite large. The failure of the inversion at very low $Q$ is likely due to linearization error and also to our choice of a high reference frequency in our chosen $Q$ model.

CONCLUSIONS

The FST provides a fast, non-redundant method of time-frequency decomposition with a wide range of applicability to seismic signal analysis. The amplitude spectrum of the FST algorithm was calibrated by normalizing the unit impulse response. The authors conclude that the calibrated FST, provides a useful seismic signal analysis tool.

We conclude that the calibrated FST is a promising time-frequency decomposition tool which may be used to extract AVF information. The FST was shown to produce a very good approx-
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REFERENCES


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