PP, PS Reflection and Transmission Coefficients for a Non-welded Interface Contact with Anisotropic Media
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Summary
Nowadays, most people have recognized that multi-component exploration is worthwhile; therefore, the integration of PP and PS seismic data becomes more valuable and popular for seismic inversion. In general, the effects on seismic wave propagation can be related to a particle motion in the boundary conditions. In an imperfect non-welded contact, the stresses are continuous across the interface, but the displacements are not. The discontinuous displacements are a linear function of stresses by the medium compliances. The compliances are a function of the rock physics properties (Lame’s constants) and can be characterized from the surface local seismic velocity. We use the extracted compliances to govern PP, PS reflection and transmission coefficients for the imperfect non-welded contact (e.g. fractured) embedding in VTI, TTI and HTI mediums. The imperfect non-welded interface contact of the two half sides can be either two different media or a single medium. We discuss the results of the methods and show not only the variation of the reflection and transmission coefficients, but also the phase changes with the incident angles. The discussions of the applications include two cases: 1) An imperfect non-welded interface contact embedded in the VTI media 2) An imperfect non-welded interface contact encountered in the TTI and HTI media.

Introduction
The reflections and transmissions of a seismic wave travelling through a welded contact interface of the two half spaces have been used extensively in the industry with different approximations. Aki and Richards (1980) simplified the Zoeppritz equations to describe the seismic reflected and transmitted amplitudes changing as a function of the source to receiver offset or the angle of incidence (AVO). Rügers (2002) updated AVO equations to describe how the amplitude of the reflected P wave varies at different incident angles and azimuths of the ray path in an anisotropic medium (AVAZ). However, coming from the observations of the borehole images, cores and outcrops, there are not perfect welded contact interfaces in the crust of the earth (due to faults, joints, and fractures). In general, the effects on seismic wave propagation can be related to particle motion at the boundaries. For a perfect welded contact interface, the seismic stresses and displacements can be modeled as being continuous across an interface of the two half spaces where they might be isotropic or anisotropic. For an imperfect non-welded contact interface, however, only the stresses are continuous across the interface, but the displacements are not. The discontinuous displacements are a linear function of the stresses multiplied by compliances. Schoenberg (1980) has given the elastic wave theory for the imperfect non-welded interface contact boundary conditions. Chaisri and Krebes (2000) presented the exact and approximate algebraic solutions for the elastic wave and for the reflection and transmission coefficients for an imperfect non-welded contact interface with isotropic media.

If one were sure that the LS (linear slip) assumption was valid, the parallel and perpendicular four velocities would have to be measured to fully characterize the model and sufficiently describe the medium (Hsu and Schoenberg, 1993). In this paper, we show that the medium compliances are a function of the elastic moduli and can be extracted from the surface seismic velocities that express the isotropic or anisotropic media properties. The non-welded contact interface can exist in either the two identical or two different media, it means that the upper and lower medium of the non-welded interface contact can have single rock properties or different properties with the impedance contrasts. If there is a welded interface contact in the single medium, there would be no reflected wave and there would only be a transmitted wave because there are no impedance contrasts. However, it occurs that both the reflected and transmitted waves propagate at the non-welded contact interface (Chaisri and Krebes 2000). We have updated the mathematical expressions from Chaisri and Krebes (2000) and we compare the application of the methods. These examples include welded and non-welded interface contacts embedded in either the two half spaces with the impedance contrasts or contacts in a single medium.

Theory
When a P plane wave is an incident wave, both reflected and transmitted waves propagate from the non-welded interface contact of the two half spaces in the VTI medium. We will set up a rectangular coordinate system in which the non-welded contact interface at z=0, and the interface parallel to the xy plane (x axis) and normal to the xz plane (y axis), then the boundary conditions at the non-welded interface contact for discontinuity displacements are

\[ u_{x1} - u_{x2} = S_{x VTI} v_{z1} \]

\[ u_{z1} - u_{z2} = S_{z VTI} v_{z1} \]

The continuity of stresses requires that

\[ S_{x z1} = S_{x z2} \]

\[ S_{z z1} = S_{z z2} \]

Where
PP, PS Reflection and Transmission Coefficients for a Non-welded Interface Contact with Anisotropic Media

Subscripts 1 and 2 represent the upper and lower medium, respectively; \( u_{1x}, u_{2x} \) and \( \sigma_{1x}, \sigma_{2x} \) denote the displacements and stresses parallel and normal to the non-welded interface contact. \( \lambda \) and \( \mu \) are the rock properties Lame’s constant. \( S_{x}^{VTI}, S_{z}^{VTI} \) are the horizontal and vertical compliances. We extract the compliance from the local velocity that refers to the VTI medium stiffness given by Sheriff (1991)

\[
\lambda_{+} + 2\mu_{e} = \lambda_{e} \quad \lambda_{-} + 2\mu_{o} = \lambda_{o} \\
\lambda_{o} - \lambda_{e} = 2\mu_{e} = \lambda_{o} - 2\mu_{o} = \lambda_{e} \\
0 \quad 0 \quad 0 \quad \mu_{o} \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad \mu_{e} \\
0 \quad 0 \quad 0 \quad 0 \quad 0
\]

\( S_{x}^{VTI} \) and \( S_{z}^{VTI} \) are elements of the compliance \( S^{VII} \). We used Thomsen’s (1986) phase velocities \( \alpha(\theta), \beta(\theta) \) to define the VTI medium compliance.

\[
\alpha(\theta) = \alpha_{e}(1 + \delta \sin^{2} \theta \cos^{2} \theta + \sin^{2} \theta) \\
\beta(\theta) = \beta_{e}(1 + \delta \sin^{2} \theta \cos^{2} \theta)
\]

\[
\varepsilon = \frac{C_{11s} - C_{33s}}{C_{11s}} \quad \quad \gamma = \frac{\varepsilon + 2\delta}{\varepsilon - 2\delta} \\
\delta = \frac{(C_{13s} + C_{44s}) - (C_{11s} + C_{33s})}{2(C_{11s} - C_{33s})}
\]

where \( \gamma, \delta, \varepsilon \) are Thomsen’s (1986) anisotropy parameters. \( \alpha_{e}, \beta_{e} \) are the vertical and horizontal velocities of the P and shear waves respectively. We see, if \( \varepsilon = 0, \delta = 0, \alpha_{e}, \beta_{e} \) are the velocities of the P and shear wave velocities in an isotropic medium. We choose \( \varepsilon = 0.2, \delta = 0.11 \).

We use the harmonic wave to represent a plane wave \( u = A \exp[i(\omega t - k \cdot \mathbf{x})] \cdot \mathbf{d} \) where \( A \) is the amplitude, \( s \mathbf{x} = s_{x} \mathbf{x} + s_{y} \mathbf{y} + s_{z} \mathbf{z} \) describes a harmonic plane wave in the direction of travel, and where \( \mathbf{d} \) is the polarization vector. Applying the above boundary conditions (Equations 1, 2) result in an incident and scattered wave relationship when a P plane wave incident from the medium 1 at the non-welded interface contact;

\[
M^{VII} \begin{bmatrix} u_{1} \\ S_{1} \\ P_{1} \\ S_{2} \end{bmatrix} = N \begin{bmatrix} P_{2} \end{bmatrix}
\]

The upper and down primers denote the up going and down going wave respectively.

\[
M_{VII}^{VTI} = \begin{bmatrix} -\alpha_{p} & -\cos \theta_{1} & S_{1u} & S_{1d} \\ \cos \theta_{1} & -\beta_{p} & S_{2u} & S_{2d} \\ 2\alpha_{e} \beta_{e} \cos \theta_{1} & \beta_{p}(1 - 2\beta_{e}^{2} p^{2}) & 2\alpha_{p} \beta_{e} \cos \theta_{1} & \beta_{p}(1 - 2\beta_{e}^{2} p^{2}) \\ -\alpha_{e} \beta_{e}(1 - 2\beta_{e}^{2} p^{2}) & 2\alpha_{e} \beta_{e} \cos \theta_{1} & -\beta_{p}(1 - 2\beta_{e}^{2} p^{2}) & 2\alpha_{p} \beta_{e} \cos \theta_{1} \end{bmatrix}
\]

where \( \alpha_{e}, \beta_{e}, \alpha_{p}, \beta_{p} \) approximate the medium velocities; \( \theta_{1}, \theta_{2} \) are the P wave incident and reflected angles. \( \varphi_{1}, \varphi_{2} \) are the shear wave reflected and transmitt ed angles. \( \rho_{1}, \rho_{2} \) are the upper and lower medium densities. If \( S_{x}^{VTI} \) and \( S_{z}^{VTI} \) are zero, then the reflected \( PP = 0 \) and \( PS = 0 \), the transmission \( PP = 1 \). This case would be equivalent to the case of the welded contact that we had expected (Chaisri and Krebes 2000). We change the subscripts 1 and 2 to one number for \( M^{VTI} \) if the non-welded interface contact is embedded in a single medium.

Consider another case of the non-welded interface, in which the contact is embedded in a horizontal symmetry transverse isotropic medium (HTI). The medium anisotropy for the symmetry axis directions are not always the most convenient form for all of the applications. Usually, the coordinate system used to analyze TTI or HTI anisotropy is rotated from the VTI. We modeled TTI or HTI compliance by rotation using the VTI compliance \( S^{VII} \) to obtain an arbitrary angle \( \varphi \) in respect to the vertical axis (Winterstein, 1990).

\[
S^{HTI} = M^{VII} M^{\varphi}
\]

Using \( M \) as the Bond transformation matrix, the elements of \( M = [a_{ij}] \), are given by the rotation coordinates axis. i, j=1, 2, 3.

\[
\begin{align*}
S_{11}^{HTI} & = \alpha_{e} \cos \theta_{1} \\
S_{12}^{HTI} & = \alpha_{e} \sin \theta_{1} \\
S_{21}^{HTI} & = \beta_{e} \cos \theta_{1} \\
S_{22}^{HTI} & = \beta_{e} \sin \theta_{1}
\end{align*}
\]
PP, PS Reflection and Transmission Coefficients for a Non-welded Interface Contact with Anisotropic Media

When $\theta = 90^\circ$, the symmetry axis parallel to the x axis, then.

$M_{\text{HTI}}(\phi) = \begin{bmatrix}
S_{11}^\text{HTI} & S_{12}^\text{HTI} & S_{13}^\text{HTI} & S_{14}^\text{HTI} \\
S_{21}^\text{HTI} & S_{22}^\text{HTI} & S_{23}^\text{HTI} & S_{24}^\text{HTI} \\
S_{31}^\text{HTI} & S_{32}^\text{HTI} & S_{33}^\text{HTI} & S_{34}^\text{HTI} \\
S_{41}^\text{HTI} & S_{42}^\text{HTI} & S_{43}^\text{HTI} & S_{44}^\text{HTI}
\end{bmatrix}$

Therefore, the reflected coefficients of the non-welded contact interface in an anisotropic HTI medium are given by

$M_{\text{HTI}}[R_1, S_1, R_2, S_2] = N[P_1]$ \hspace{1cm} (4)

The compliance elements in $M_{\text{HTI}}$ are $S^\text{HTI}$ instead of $S^\text{VTI}$.

For a single medium, the procedure is the same as the above VTI case.

Discussion of Applications

It is helpful to compare the results of the methods’ applications. We have sorted the solutions into VTI and HTI (TTI) in which they contain PP, PS reflected waves and PP, PS transmitted waves. Three color lines with marks dominate the figures below. The black solid line is for the perfect welded interface contact in the two different media with impedance contrasts. The blue dashed line is representing the non-welded interface contact in the two different media with impedance contrasts and the red dotted line denotes a non-welded interface contact in the single medium without impedance contrast. Each line combines circles representing the TTI medium ($\phi = 60^\circ$). Figure 1a and 1b contain PP reflection coefficients for the VTI and HTI (top part) and we see that the reflection coefficients of the welded (black solid line) and the non-welded interface contact (blue dashed line and red dotted line) have been separated sufficiently as well as the phase of the PP reflected wave in the VTI medium (Figure 1a lower part). Comparison with the HTI (TTI) media (Figure 1b) shows that the case of the reflection coefficients of the non-welded interface contact in a single medium have an opposite ratio relation to the cases of the non-welded and welded interface contacts with two different media with the large incident angles. This would help us to reduce the ambiguity of detection of the fracture features in the single medium because most of the fractures in the natural world are near to the vertical direction (HTI). From the PP reflection phase in the HTI medium, (Figure 1b lower part), we see that the phase trends appear to have clearly shown a difference for each case. In Figure 2a, 2b, they describe the PS reflection coefficients properties. We are sure that there is no converted wave (PS) in the VTI and HTI media when the P wave is normally incident. By increasing the incident angle, the curves of the PS reflection coefficients of the welded interface contact (black solid line) are far away from the anisotropic medium (blue dashed and red dotted lines). Meanwhile, the PS reflection coefficients in the VTI (Figure 2a) and HTI (Figure 2b) appear to have a different trend at the usual incident angles in practice. In the PS reflection phase (Figure 2a, 2b lower part), the different case phase trends appear to have been separated, and they also appear to have 180 degree difference with the PP reflection phase for each case. Figure 3 shows the PP transmission wave characterizations in the VTI and HTI media and it shows that the PP transmission coefficients of any interface contact (Figure 3a, 3b higher part) in the single medium are close to 1 because there are no impedance contrasts of the two half spaces on either side of the interface contact. Whereas when the interface contact exists between two different media, the transmission coefficients are not close to 1. This indicates that some of the features (e.g. fractures, faults) might be caused by the natural stress or unconformity. In the Figures 3a, and 3b (lower part), show the PP transmissions of the welded and non-welded interface contacts have a different phase angle. In Figures 4a, 4b the transmission coefficients are the properties of the PS transmissions in the VTI and HTI media. The figures show the type of coefficient of the interface contacts separating into two parts as well as into two phases.
Conclusions

These methods of extracting anisotropic compliances from the seismic data have been presented. Those variable compliances have been applied to resolve each of the reflection and transmission coefficients at the imperfect non-welded interface contact in the anisotropic mediums. We find that the PP, PS reflection and transmission coefficients have very different results in which the welded and non-welded interface contact is embedded in the anisotropic media (VTI, TTI, and HTI). The PP, PS reflection coefficients can help us to understand a type of the interface contact. The PP PS transmission confidents can imply a kind of embedding in a medium for a non-welded interface contact. Hence, this new method of the non-welded interface contact which is embedded in the anisotropic medium can accurately indicate some of the geological features such as fractures, faults.

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